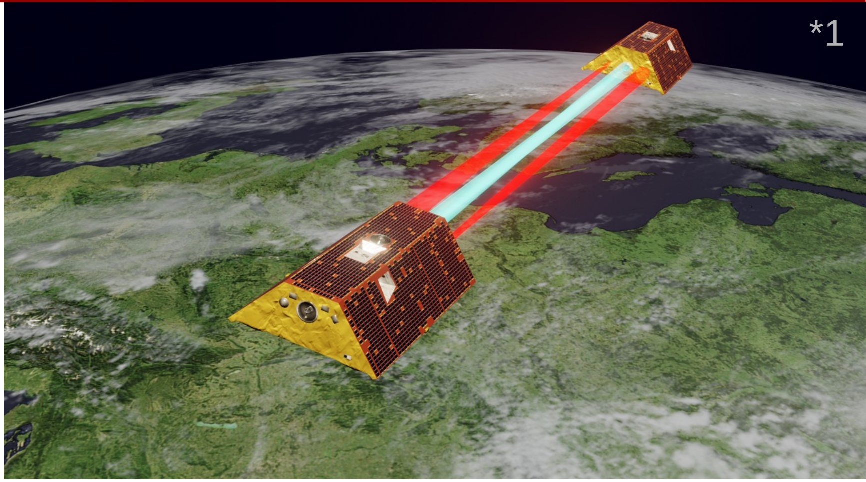
Comparing GRACE Follow-On Inter-Satellite Pointing Angles from Star Camera and LRI Fast Steering Mirror

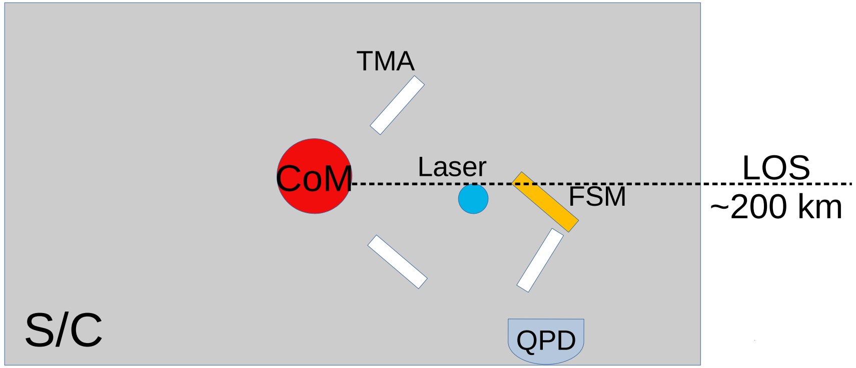
by **Laura Müller**, Vitali Müller, Malte Misfeldt, and Gerhard Heinzel

Abstract: IUGG23-3347

LRI Setup

GRACE-FO hosts a Laser Ranging Interferometer (LRI) which measures the distance changes between the spacecraft (S/C).



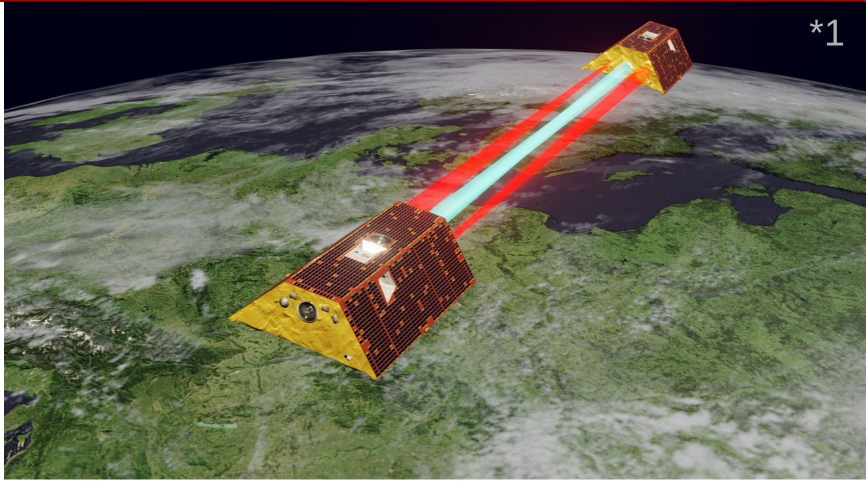


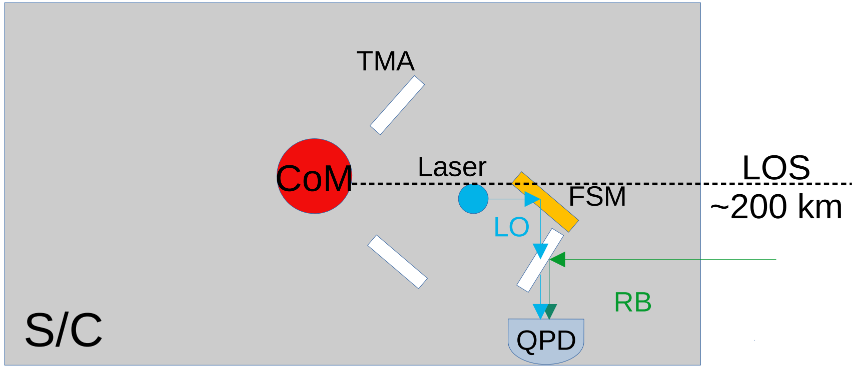
*1) Image: Malte Misfeldt. Textures: NASA, the blue marble

LRI Setup

GRACE-FO hosts a Laser Ranging Interferometer (LRI) which measures the distance changes between the spacecraft (S/C).

- Received beam (RX) from distant S/C interferes with Local Oscillator (LO) on Quadrant Photodiode (QPD).



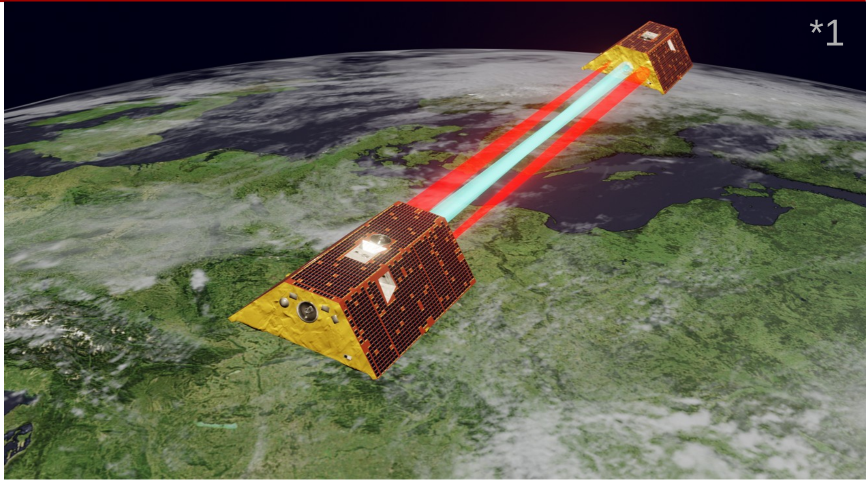


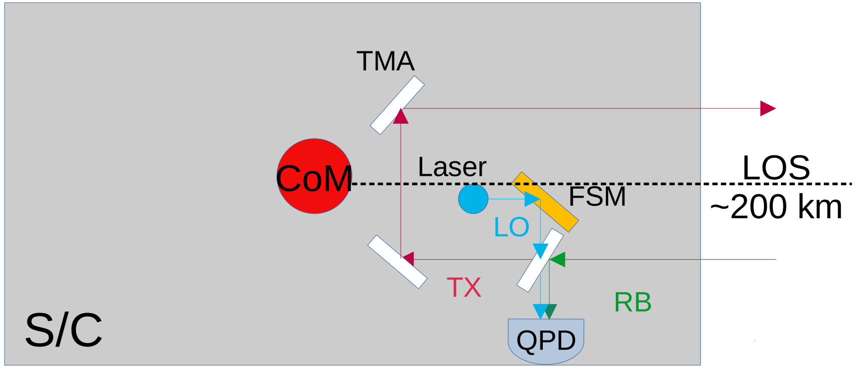
*1) Image: Malte Misfeldt. Textures: NASA, the blue marble

LRI Setup

GRACE-FO hosts a Laser Ranging Interferometer (LRI) which measures the distance changes between the spacecraft (S/C).

- Received beam (RX) from distant S/C interferes with Local Oscillator (LO) on Quadrant Photodiode (QPD).
- Transmitted beam (TX) is routed by Triple Mirror Assembly (TMA) and sent back to distant S/C.
- TMA‘s virtual intersection point is co-located with Center of Mass (CoM).
- The LRI measures changes along the Line of Sight (LOS) – connecting line between both CoMs.





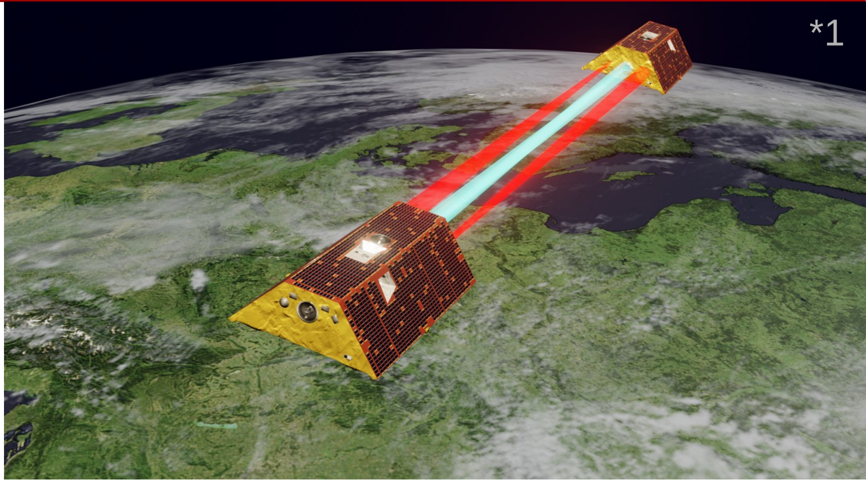
*1) Image: Malte Misfeldt. Textures: NASA, the blue marble

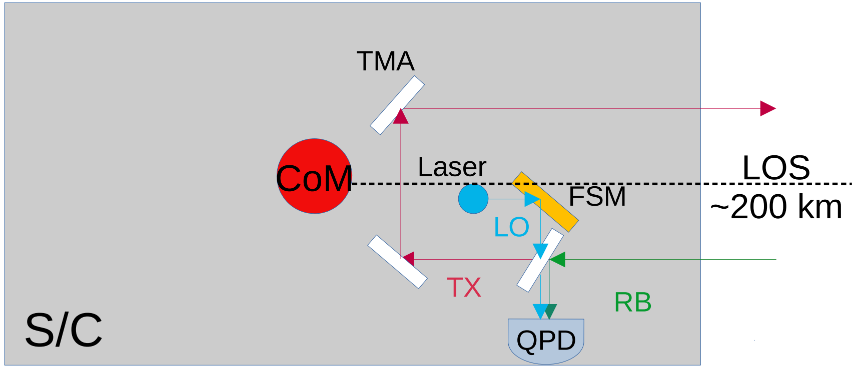
LRI Setup

GRACE-FO hosts a Laser Ranging Interferometer (LRI) which measures the distance changes between the spacecraft (S/C).

- Received beam (RX) from distant S/C interferes with Local Oscillator (LO) on Quadrant Photodiode (QPD).
- Transmitted beam (TX) is routed by Triple Mirror Assembly (TMA) and sent back to distant S/C.
- TMA‘s virtual intersection point is co-located with Center of Mass (CoM).
- The LRI measures changes along the Line of Sight (LOS) – connecting line between both CoMs.

This ranging signal is provided in LRI1B products and is used for gravity field recovery in Level2+3. Albert Einstein Institute derives alternative LRI1B sets, which are publicly available at https://www.aei.mpg.de/grace-fo-ranging-datasets





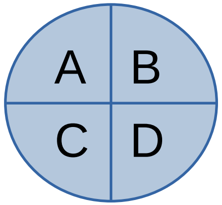
*1) Image: Malte Misfeldt. Textures: NASA, the blue marble

LRI Setup / FSM Working Principle

Attitude changes:

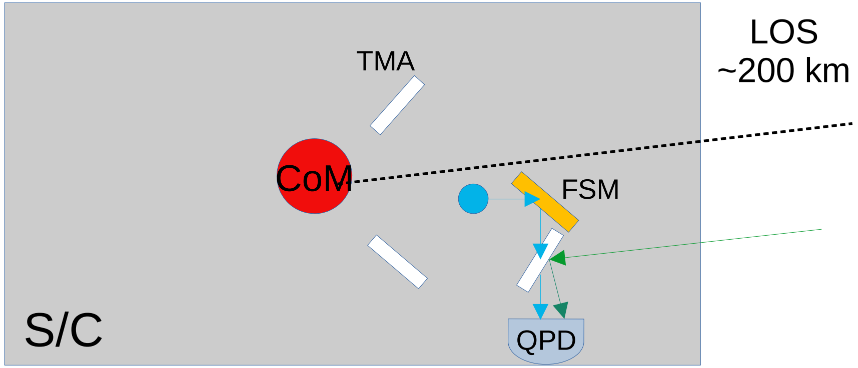
- **RX** beam enters S/C under certain angle
- Wavefront of **RX** and **LO** beam are slightly tilted on QPD → interferometric contrast not optimal
- QPD has four quadrants, which signals are used for Differential Wavefront Sensing (DWS).

QPD channels



$$DWS_v = \frac{\phi_A + \phi_B - \phi_C - \phi_D}{2} \rightarrow 0$$

$$DWS_h = \frac{\phi_A - \phi_B + \phi_C - \phi_D}{2} \rightarrow 0$$

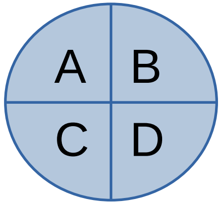


LRI Setup / FSM Working Principle

Attitude changes:

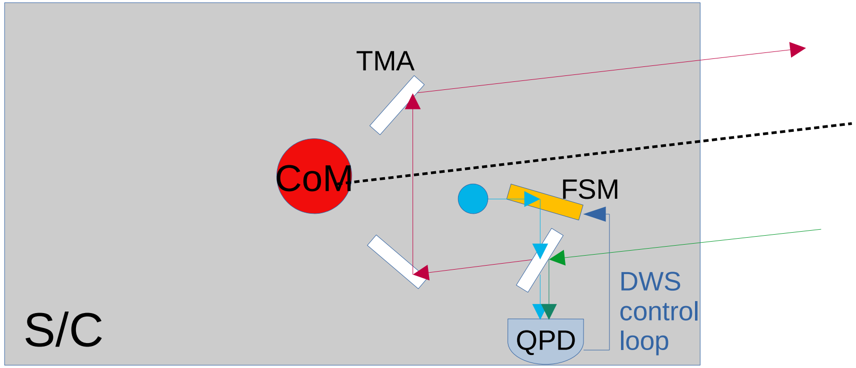
- **RX** beam enters S/C under certain angle
- Wavefront of **RX** and **LO** beam are slightly tilted on QPD
  → interferometric contrast not optimal
- QPD has four quadrants, which signals are used for Differential Wavefront Sensing (DWS).
- DWS control loop give commands to Fast Steering Mirror (FSM), to compensate for the beam misalignment.
  → zeros the DWS signals
- FSM is steerable around two axis.
- Commanded (COM) values from DWS loop are provided in units of counts – convertible to S/C pointing angles
- Position Sensing System (PPS) provides FSM position in units of counts – convertible to S/C pointing angles
- FSM can only provide information about yaw and pitch rotation

QPD channels



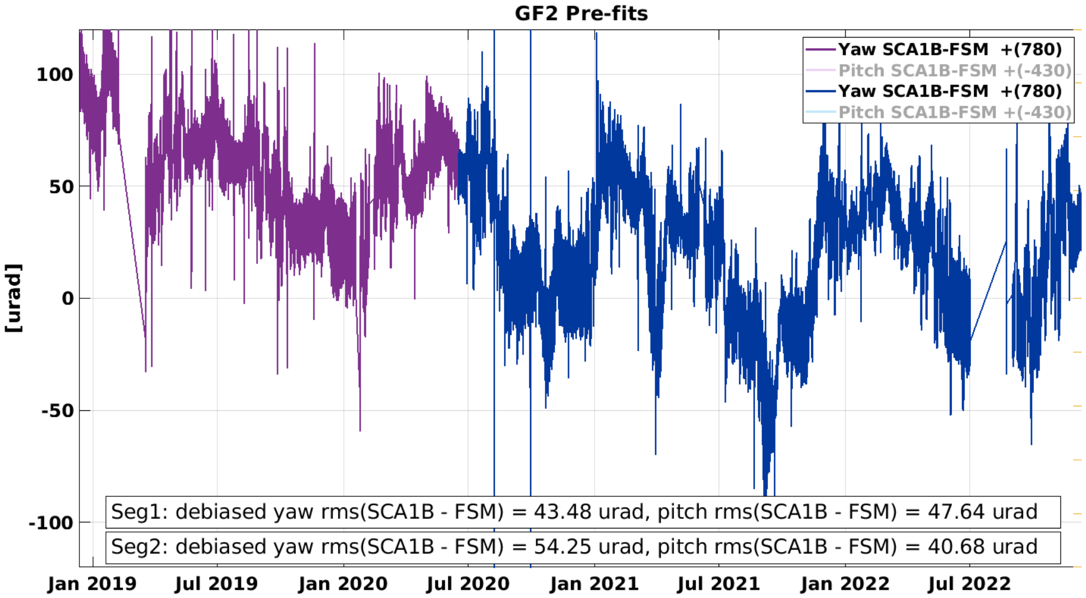
$$DWS_v = \frac{\phi_A + \phi_B - \phi_C - \phi_D}{2} \rightarrow 0$$

$$DWS_h = \frac{\phi_A - \phi_B + \phi_C - \phi_D}{2} \rightarrow 0$$



SCA1B-FSM Prefit Residuals

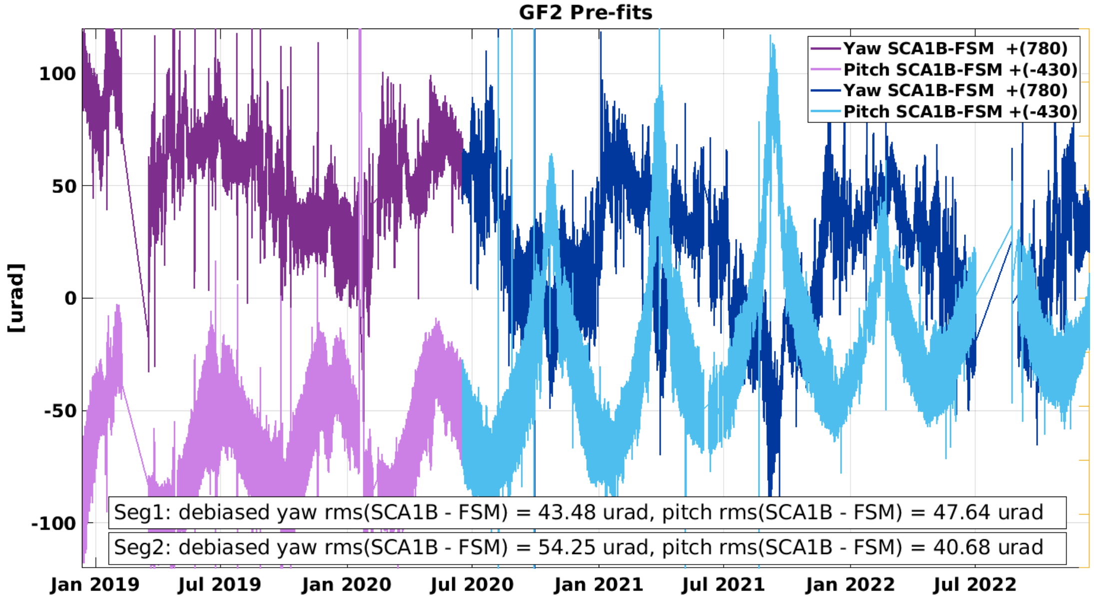
- Pointing Angles between Satellite Reference Frame (SRF) and Line of Sight Frame (LOSF) can be computed from LRI Fast Steering Mirror (FSM) orientation or from Star Camera quaternions (SCA1B, attitude product) & satellite positions (GNI1B).



- SCA1B-FSM residuals are splitted in two segments.
- **Before** and **after** QSA1B update.
- Shown angles are debiased.

SCA1B-FSM Prefit Residuals

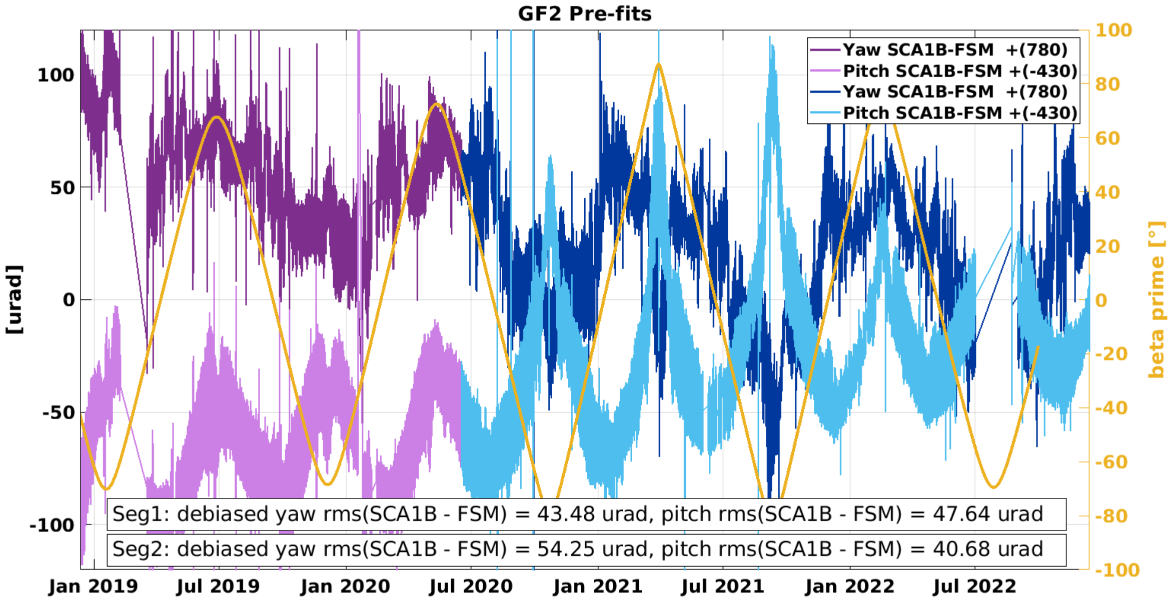
- Pointing Angles between Satellite Reference Frame (SRF) and Line of Sight Frame (LOSF) can be computed from LRI Fast Steering Mirror (FSM) orientation or from Star Camera quaternions (SCA1B, attitude product) & satellite positions (GNI1B).

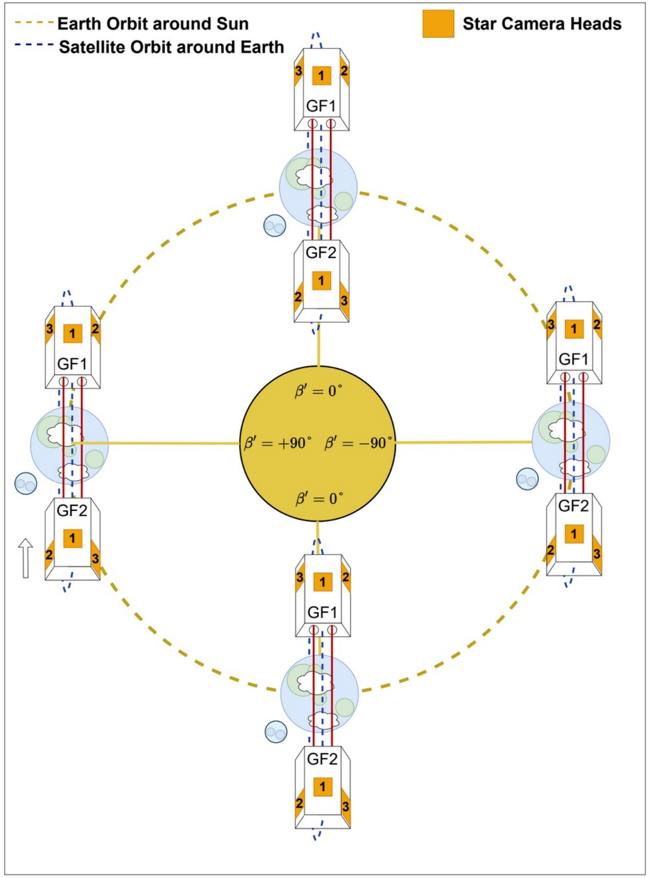


- SCA1B-FSM residuals are splitted in two segments.
- **Before** and **after** QSA1B update.
- Shown angles are debiased.

SCA1B-FSM Prefit Residuals

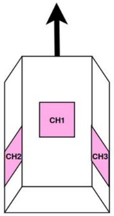
- β‘ is the angle between the orbital plane and the sun vector.
- Earth-satellite system stays the same when orbiting around the sun.
- β‘ =+/- 90°: sun-blindings in SCAs on side panels
- β‘ = 0°: sun-blindings in SCA1 and also in the LRI for a few seconds per orbit.

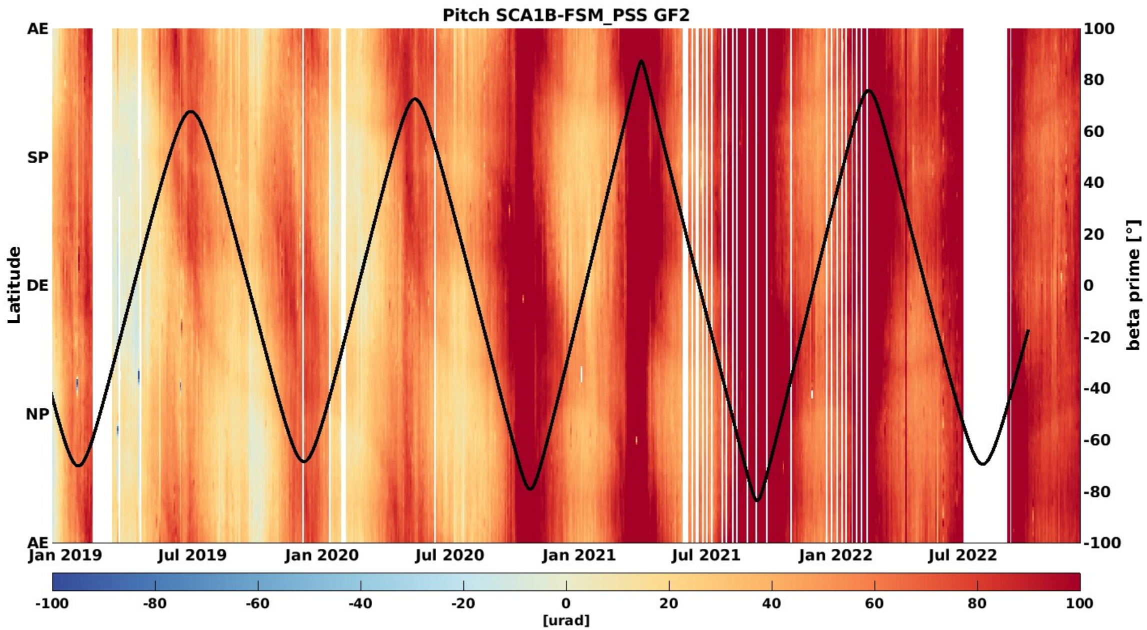




Comparison at low-frequencies

- Argument of Latitude Plot: each vertical line = one orbit
- Difference in pitch angle between FSM and SCA1B dataset show correlation to:
  - changing orientation of orbital plane to sun: β‘ angle
  - temperatures aboard the spacecraft (not shown in this talk)

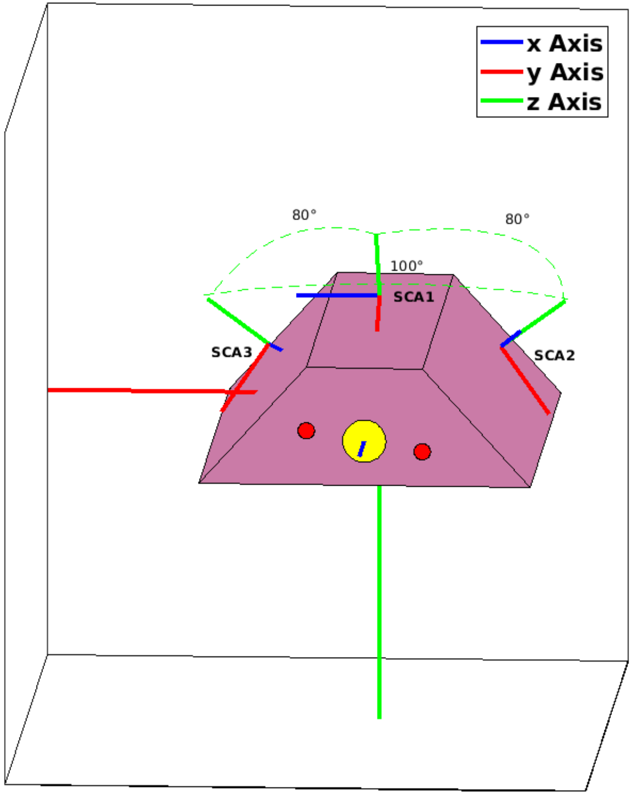




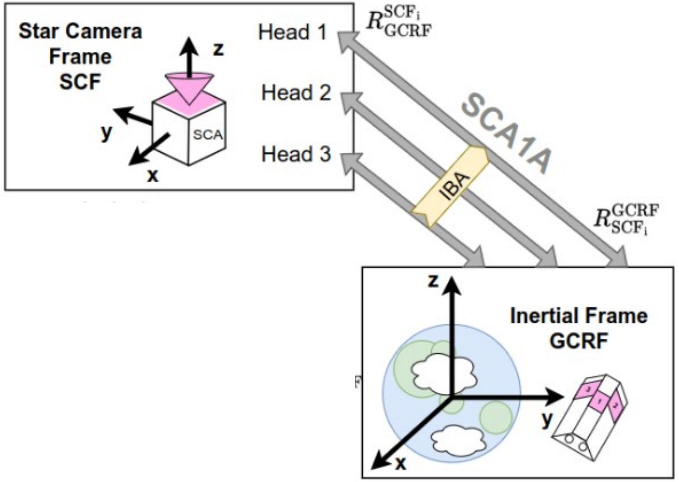
- The attribution of FSM-SCA1B residuals to one instrument is difficult as β‘ angle and temperatures affect the FSM and the star camera heads.
- Can we find some data streams that might allow attribution of residuals?

Detailed comparison for high-frequencies shown by S. Goswami 2021, see *2.

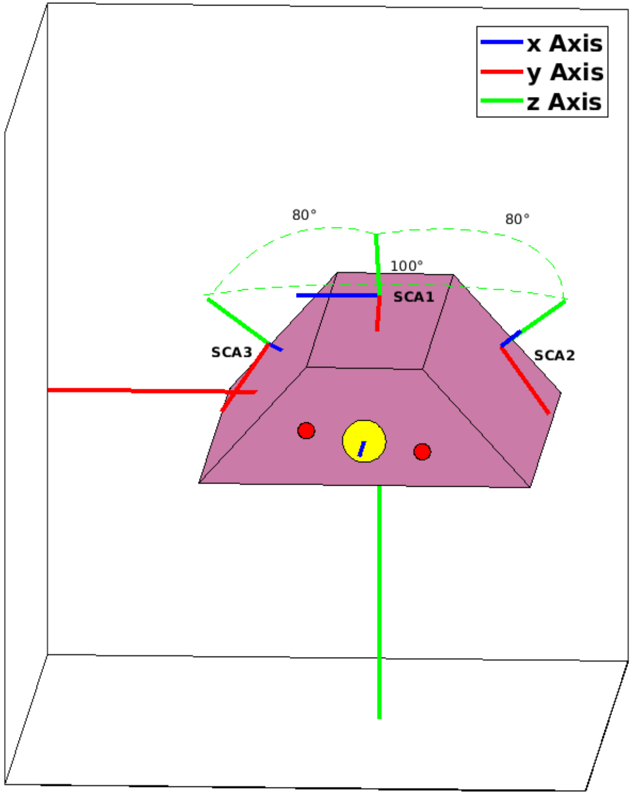
Inter-Boresight Angle (IBA)



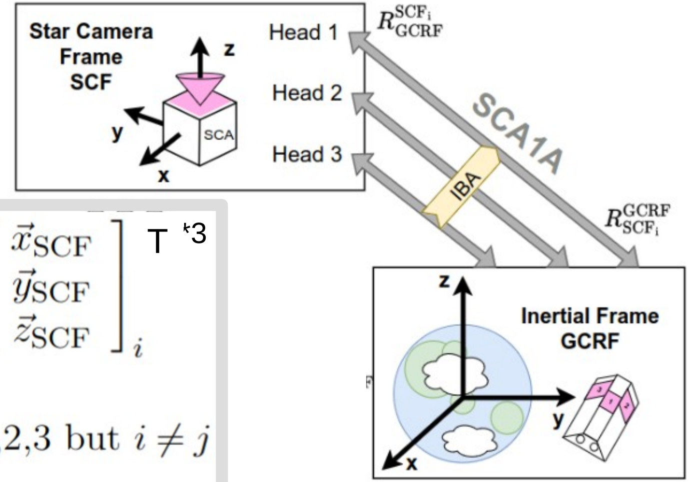
- The Inter Boresight Angle (IBA) can be computed between the SCA z-axes.
- Quaternions of single SCA heads describe rotation from inertial to corresponding SCA frame.



Inter-Boresight Angle (IBA)



- The Inter Boresight Angle (IBA) can be computed between the SCA z-axes.
- Quaternions of single SCA heads describe rotation from inertial to corresponding SCA frame.
- They are used to compute cosine-direction matrix $R_{\mathrm{GCRF}}^{\mathrm{SCF}_i}$.
- Its z-components describe the optical axis in coordinates of the inertial frame.



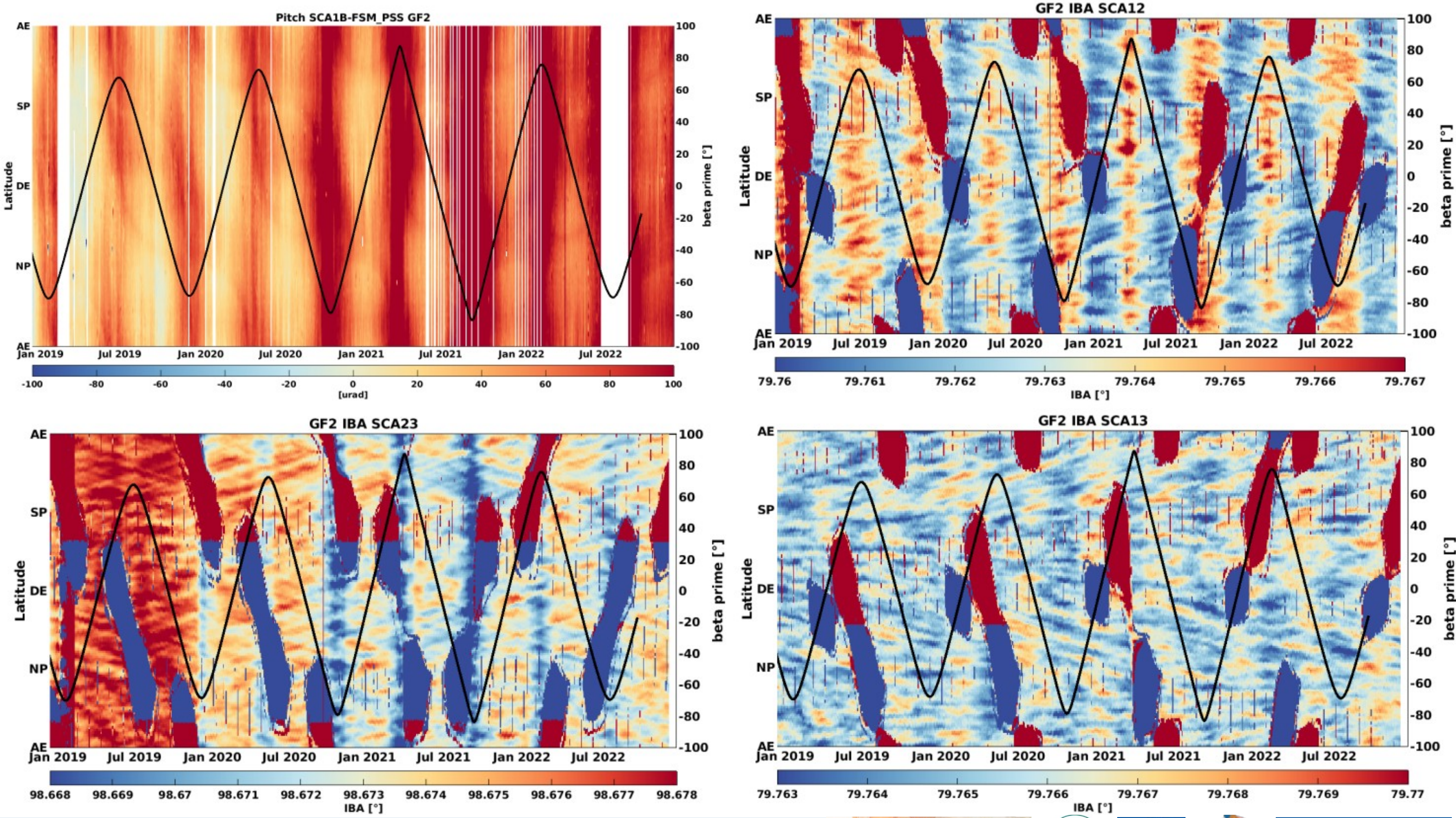
$$\vec{Q}_{\mathrm{SCA1A}} \rightarrow R_{\mathrm{GCRF}}^{\mathrm{SCF}_i} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}_i = \begin{bmatrix} \vec{x}_{\mathrm{SCF}} \\ \vec{y}_{\mathrm{SCF}} \\ \vec{z}_{\mathrm{SCF}} \end{bmatrix}_i^{\mathrm{T}} \quad [3]$$

$$\mathrm{IBA}_{i,j} = arccos\left(\frac{\vec{z}_{\mathrm{SCF},i} \cdot \vec{z}_{\mathrm{SCF},j}}{|\vec{z}_{\mathrm{SCF},i}| \cdot |\vec{z}_{\mathrm{SCF},j}|}\right) \quad i,j = 1{,}2{,}3 \text{ but } i \neq j$$

3) Tamara Bandikova, 2015, PhD Thesis: The role of attitude determination for inter-satellite ranging

Inter-Boresight Angle (IBA)

- IBAs seem correlated with SCA1B-FSM residuals.
- Similar patterns for GF2 in IBA between SCA 1+2 & 2+3.
- Huge red and blue spots indicate SCA head Sun and Moon blindings.
- Extract one mean value per orbit (one vertical line), but neglect sun- & moon-blindings.



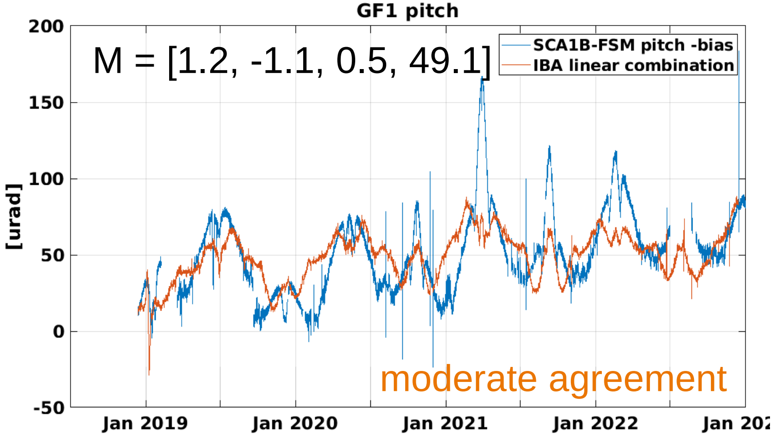
Inter-Boresight Angle (IBA)

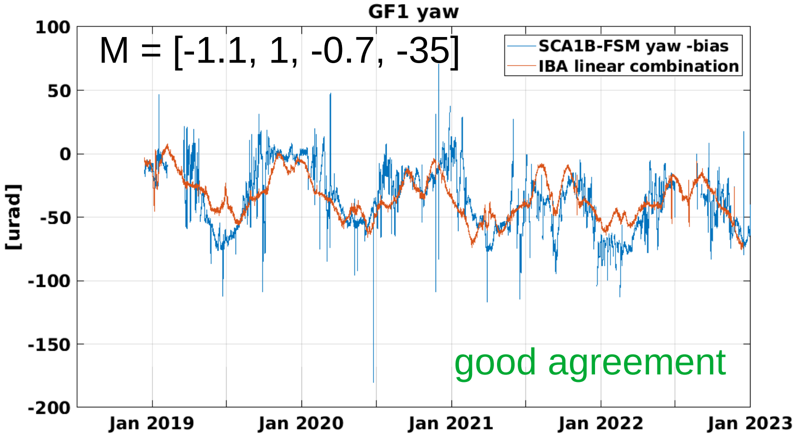
- **Blue curves:** SCA1B-FSM residuals (one value / orbit).
- **Red curves:** fitted linear combination of IBAs to SCA-FSM residuals.

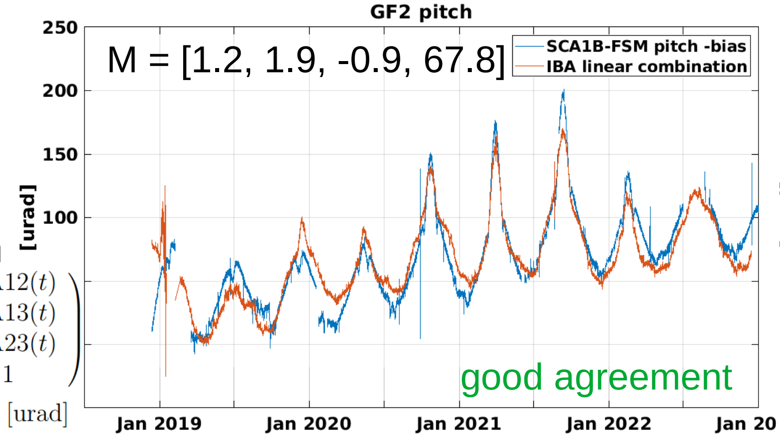
- GF2 and also GF1 yaw residuals can be reconstructed with a combination of IBA variations.
  → might be related to thermal distortions of star camera mounting platform

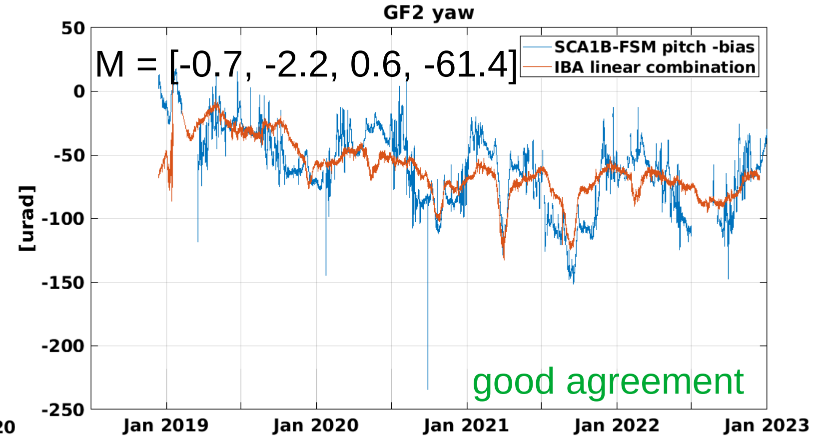
$$\begin{pmatrix} \mathrm{SCA(t)} - \mathrm{FSM(t)} \end{pmatrix} = \underbrace{\begin{pmatrix} c_1 & c_2 & c_3 & a \end{pmatrix}}_{:=\mathrm{M}} \cdot \begin{pmatrix} \mathrm{IBA12}(t) \\ \mathrm{IBA13}(t) \\ \mathrm{IBA23}(t) \\ 1 \end{pmatrix}$$

with c_i : [urad/urad] and a : [urad]



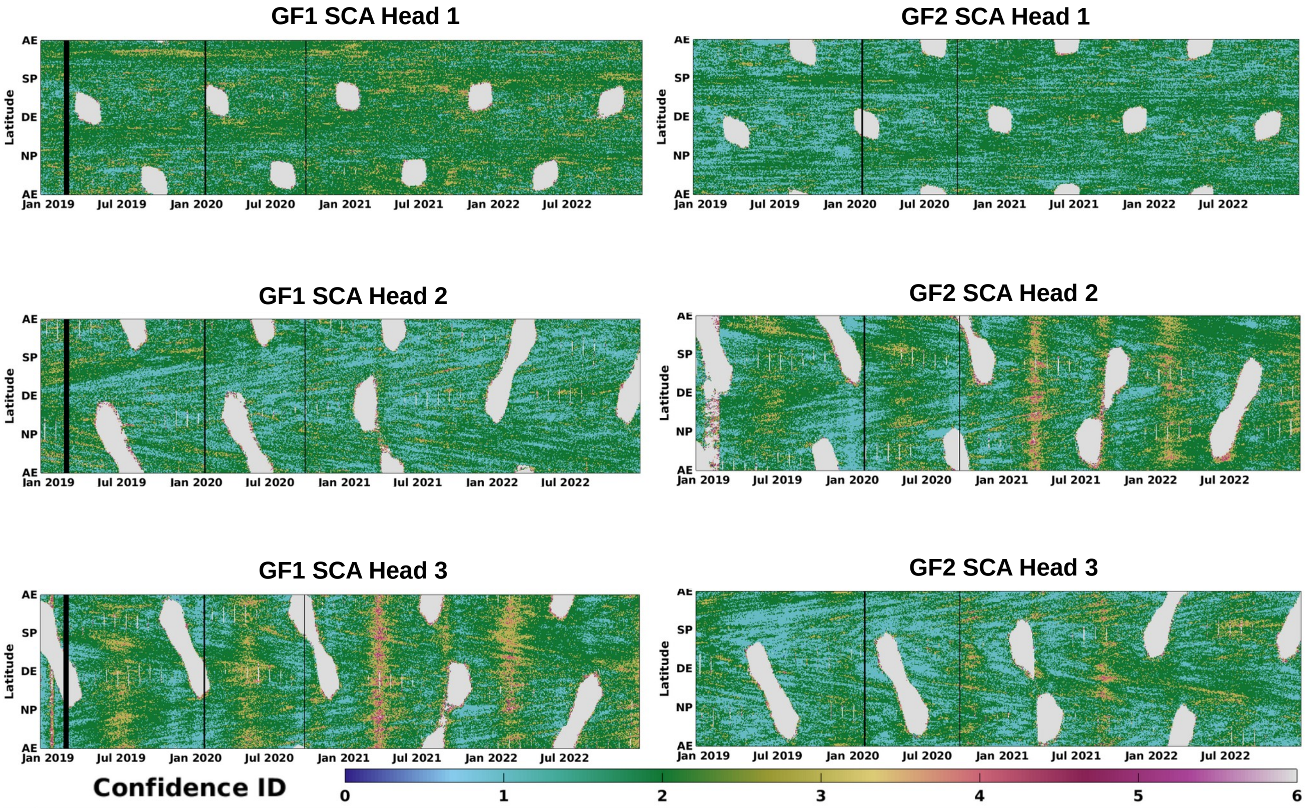






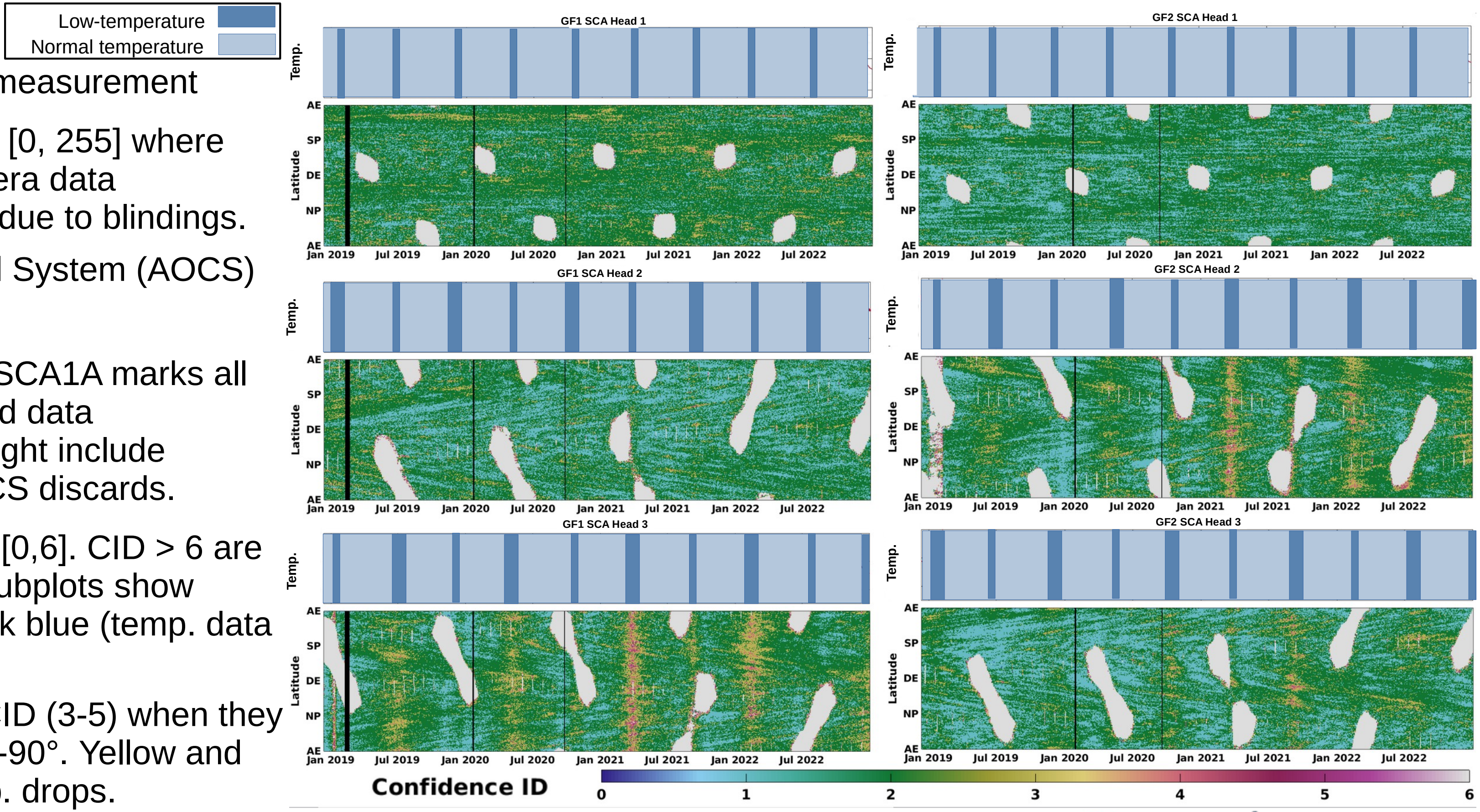
Star Camera Confidence ID (CID) from SCA1A

- CID for each SCA head measurement
- Values are in the interval [0, 255] where
  0: reliable star camera data
  255: invalid data, e.g. due to blindings.
- Attitude & Orbital Control System (AOCS) reject CID > 6.
- additional quality flag of SCA1A marks all values (CID < 60) as valid data
  → SCA1B processing might include inaccurate data that AOCS discards.
- Plot range is adjusted to [0,6]. CID > 6 are shown in gray.



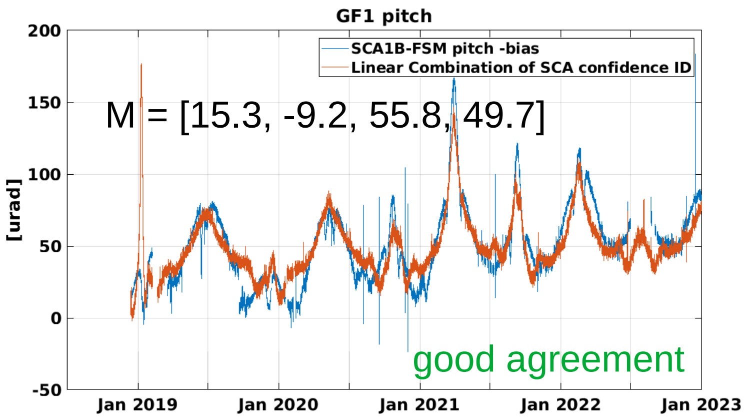
Star Camera Confidence ID (CID) from SCA1A

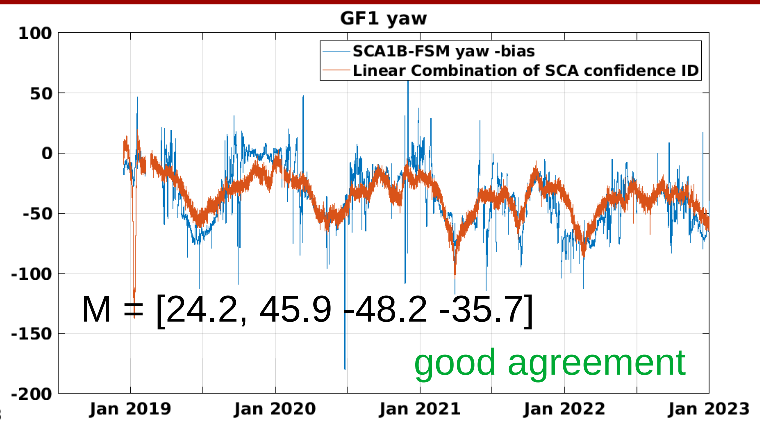
- CID for each SCA head measurement
- Values are in the interval [0, 255] where
  0: reliable star camera data
  255: invalid data, e.g. due to blindings.
- Attitude & Orbital Control System (AOCS) reject CID > 6.
- additional quality flag of SCA1A marks all values (CID < 60) as valid data
  → SCA1B processing might include inaccurate data that AOCS discards.
- Plot range is adjusted to [0,6]. CID > 6 are shown in gray. Smaller subplots show temperature drops in dark blue (temp. data not publicly available).
- SCA2 & 3 show higher CID (3-5) when they are on shadow side β' +/-90°. Yellow and pinkish areas when temp. drops.



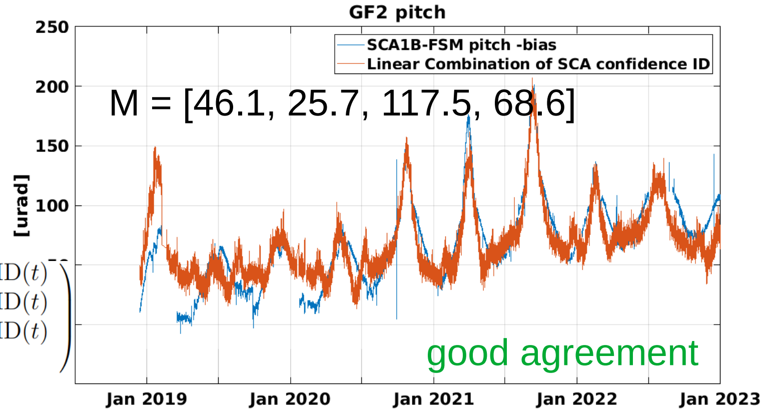
Star Camera Confidence ID (CID)

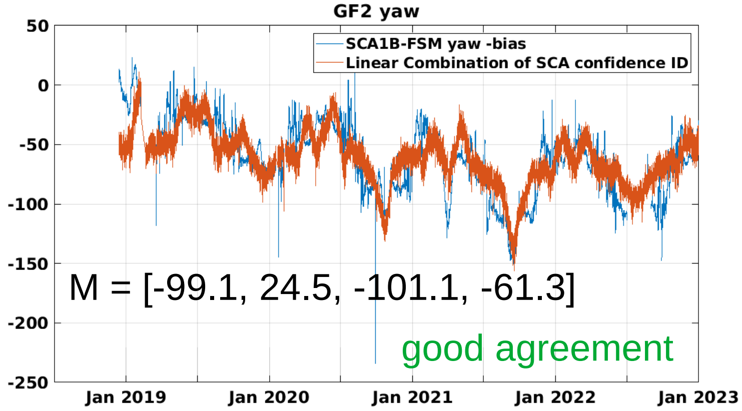
- **Blue curves:** SCA1B-FSM residuals (1 value / orbit)
- **Red curves:** fitted linear combination of CIDs to SCA-FSM residuals.





- GF1 & GF2 residuals are highly correlated with Star Camera Confidence IDs.





$$\left(\mathrm{SCA(t)} - \mathrm{FSM(t)} \right) = \underbrace{\left(c_1 \quad c_2 \quad c_3 \quad a \right)}_{:=\mathrm{M}} \cdot \begin{pmatrix} \mathrm{SCA1\_ConfiID}(t) \\ \mathrm{SCA2\_ConfiID}(t) \\ \mathrm{SCA3\_ConfiID}(t) \\ 1 \end{pmatrix}$$

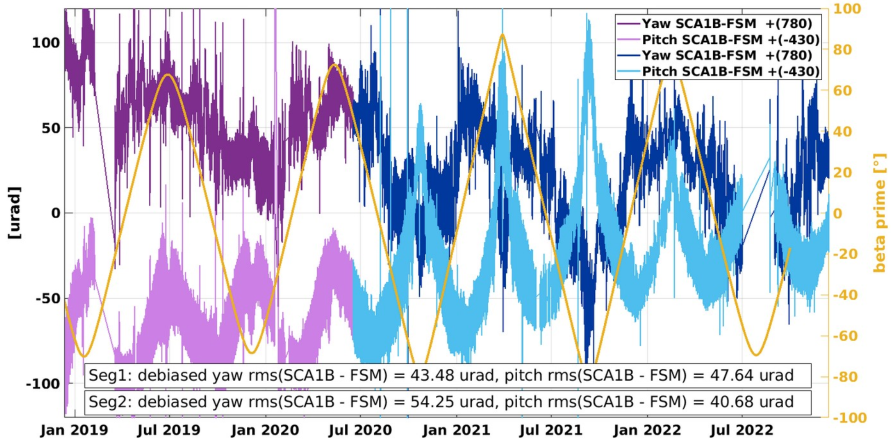
with c_i : [CID/urad] and a : [urad]

Modelling Residuals with IBA and CID

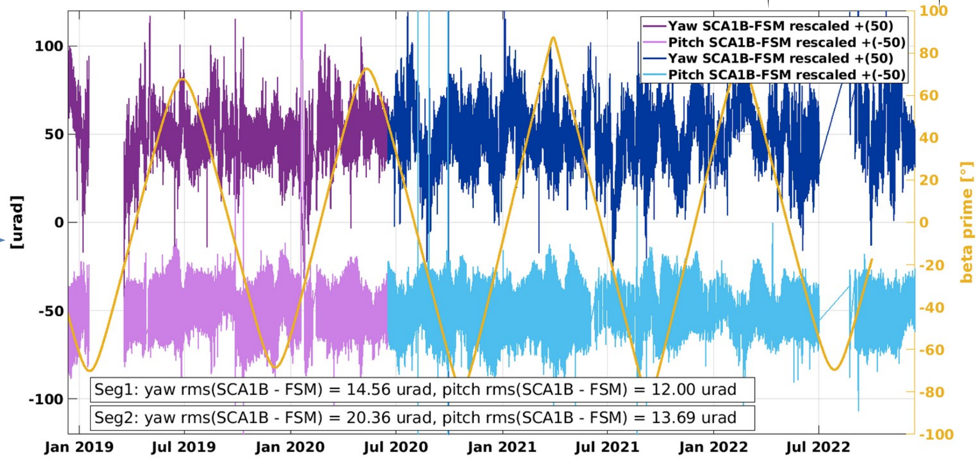
Least Squares Fit considers Coupling Factors for
- scale, cross-coupling between yaw and pitch
- Offset between SCA1B and FSM
- Drift
- Variations in inter boresight angles IBA12 & IBA13 & IBA23
- Confidence IDs of three star camera heads

$$\begin{pmatrix} Yaw'_{AEI}(t) \\ Pitch'_{AEI}(t) \end{pmatrix} = \begin{pmatrix} c\,c_{yy} & c\,c_{py}\,a_y & d_y\,c_{y,IBA12} & \dots c_{y,CID3} \\ c\,c_{yp} & c\,c_{pp}\,a_p & d_p\,c_{p,IBA12} & \dots c_{p,CID3} \end{pmatrix} \begin{pmatrix} Yaw_{AEI}(t) \\ Pitch_{AEI}(t) \\ 1 \\ t \\ \delta IBA12(t) \\ \delta IBA13(t) \\ \delta IBA23(t) \\ STR1_{ConfID}(t) \\ STR2_{ConfID}(t) \\ STR3_{ConfID}(t) \end{pmatrix}$$

GF2 Prefits



GF2 Postfits



Conclusion / Outlook

Conclusion:

1) We analyzed yaw and pitch pointing angle differences between the LRI fast steering mirror (FSM) and the star camera.

2) The residuals show patterns with approx. 2/year frequency and reaching an amplitude up to 100-200 urad.

3) Analysis shows that FSM-SCA residuals have high correlation to star camera quantities, such as inter-boresight angles and confidence IDs.

4) We derived a model based on IBAs and ConfIDs which can reduce the residual variations (amplitudes reduced to 50 urad, 2/year pattern vanished).

Outlook:

AEI wants to produce an alternative attitude dataset, which considers FSM data and new insights about star camera IBAs and ConfIDs.

Thank you for your attention!

References

[2] Sujata Goswami et al, 2021, Analysis of GRACE Follow-On Laser Ranging Interferometer derived inter-satellite pointing angles, https://doi.org/10.48550/arXiv.2107.02260

[3] Tamara Bandikova, 2015, PhD Thesis: The role of attitude determination for inter-satellite ranging, https://dgk.badw.de/fileadmin/user_upload/Files/DGK/docs/c-758.pdf

Acknowledgements

This work has been supported by:

- the **European Space Agency**’s third-party mission support for GRACE-FO and in the framework of Next Generation Geodesy Mission development;
- The **Deutsche Forschungsgemeinschaft** (DFG, German Research Foundation, Project-ID 434617780, SFB 1464 TerraQ); Clusters of Excellence “QuantumFrontiers: Light and Matter at the Quantum Frontier: Foundations and Applications in Metrology” (EXC-2123, project number: 390837967);

Appendix

How to compute inter-satellite pointing angles w.r.t. the Line of Sight from FSM PSS counts and Star Camera Quaternions and orbit positions?

Satellite Pointing Angles w.r.t. LOS

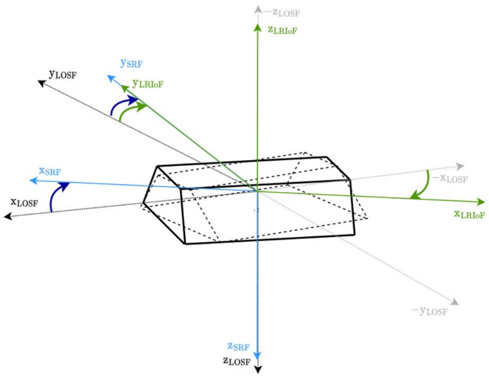
SC	s_x [urad/cts]	s_y [urad/cts]	p_0 [cts]	p_1 [cts]	c_0 [cts]	c_1 [cts]	γ $[\frac{\mu\text{rad}}{\text{cts}} \cdot \frac{\text{cts}}{\mu\text{rad}}]$
GF1	4.023	4.065	1829.2	1832.2	1.62	-1.23	$\frac{1}{0.3157952} \cdot \frac{1}{2.7999}$
GF2	3.951	4.017	1829	1830.4	1.4	-3.25	conversion factors

$$\begin{pmatrix} \text{Yaw\_AEI} \\ \text{Pitch\_AEI} \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \text{PSS}_0 - p_0 \\ \text{PSS}_1 - p_1 \end{pmatrix} \cdot \gamma$$

Rotation around z-axis: Yaw

Rotation around $z_{\text{SRF}} > 0 \Rightarrow \text{yaw}_{\text{SRF}} = \theta_z$

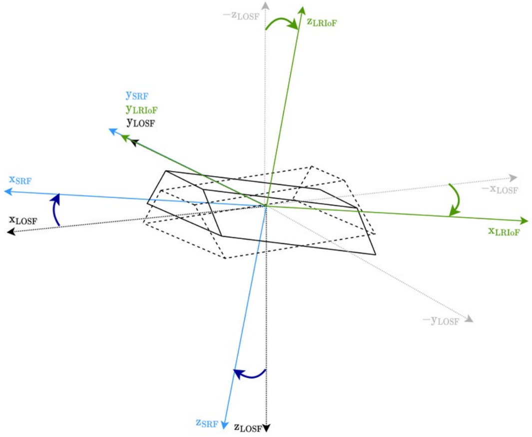
Rotation around $z_{\text{LRIoF}} < 0 \Rightarrow \text{yaw}_{\text{LRIoF}} = -\theta_z$



Rotation around y-axis: Pitch

Rotation around $y_{\text{SRF}} > 0 \Rightarrow \text{pitch}_{\text{SRF}} = \theta_y$

Rotation around $y_{\text{SRF}} > 0 \Rightarrow \text{pitch}_{\text{LRIoF}} = \theta_y$



- PSS counts are converted to yaw and pitch angles with calibration factors from ground-calibrations.
- The derived yaw and pitch angles describe the rotation between the LRI optical Frame (LRIoF) and the Line of Sight Frame (LOSF).
- This values are then converted to the Satellite Reference Frame (SRF) by using the relation

  yaw_SRF = – yaw_LRIoF,
  pitch_SRF = pitch_LRIoF

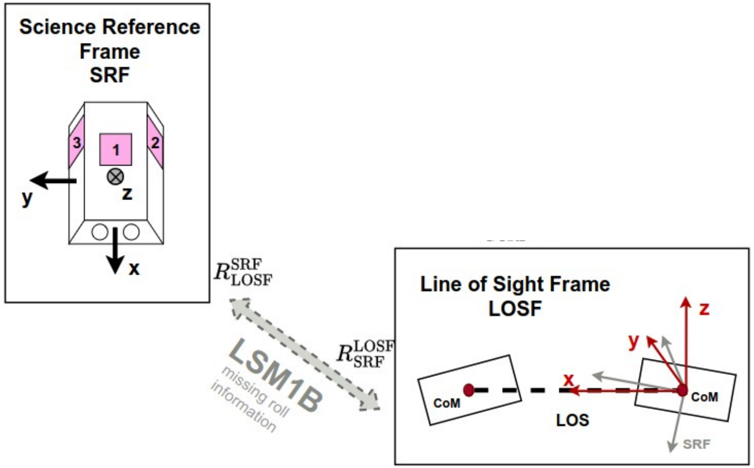
  which provides the rotation around z- and y-axis respectively.
- Roll: rotation around x-axis cannot be measured by FSM

→ FSM yaw_SRF and pitch_SRF angles are stored in LSM1B.

Other conversion shown by S. Goswami 2021, see *2.

Pointing Angles from FSM

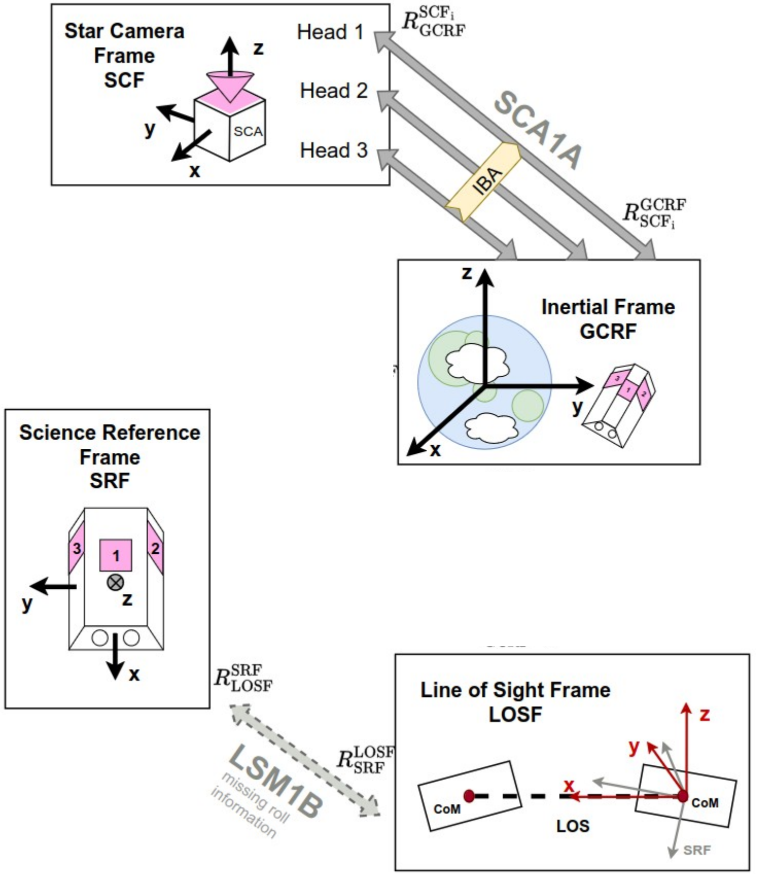
- LSM1B describes the rotation from the Line of Sight Frame (LOSF) to the Science Reference Frame (SRF). However, due to the missing roll information the rotation around the x-axis can not be considered correctly.
- To compare yaw and pitch angles from Star Camera Quaternions and FSM, one can use the following relations shown by the diagram.



Pointing Angles from Star Cameras

Star Cameras:

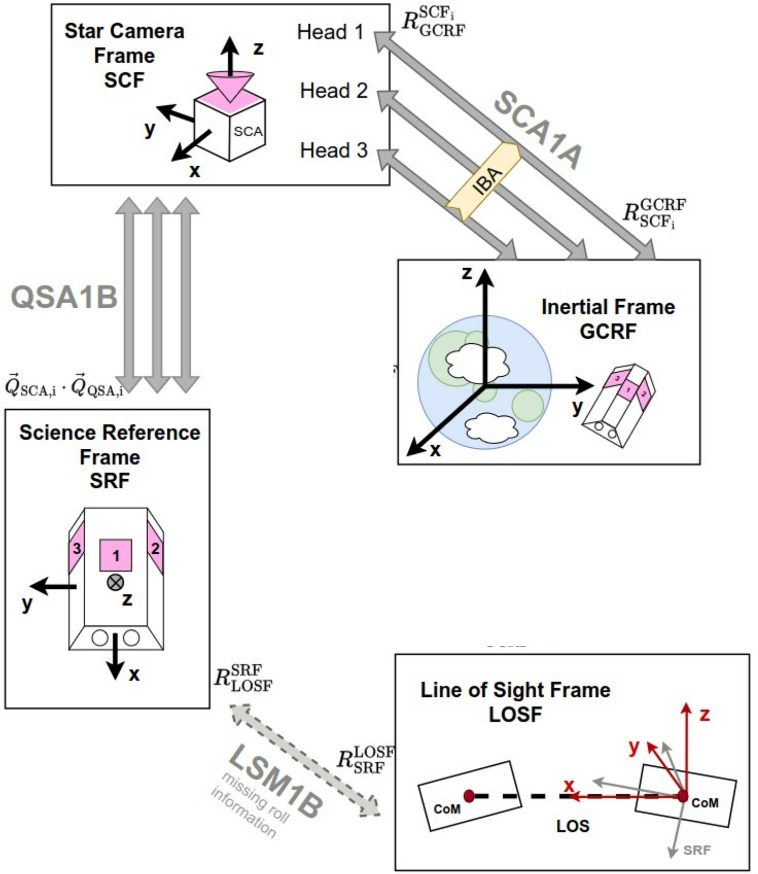
- SCA1: top panel, SCA2: left side panel, SCA3: right side panel
- take pictures of the dark sky and compare the star patterns with internal star catalogs to determine their own position in inertial frame.
- Data is stored as a set of Quaternions.



Pointing Angles from Star Cameras

Star Cameras:

- SCA1: top panel, SCA2: left side panel, SCA3: right side panel
- take pictures of the dark sky and compare the star patterns with internal star catalogs to determine their own position in inertial frame.
- Data is stored as a set of Quaternions.
- Calibration values (QSA1B) are used, such that 3 heads describe the same orientation of the S/C.



Pointing Angles from Star Cameras

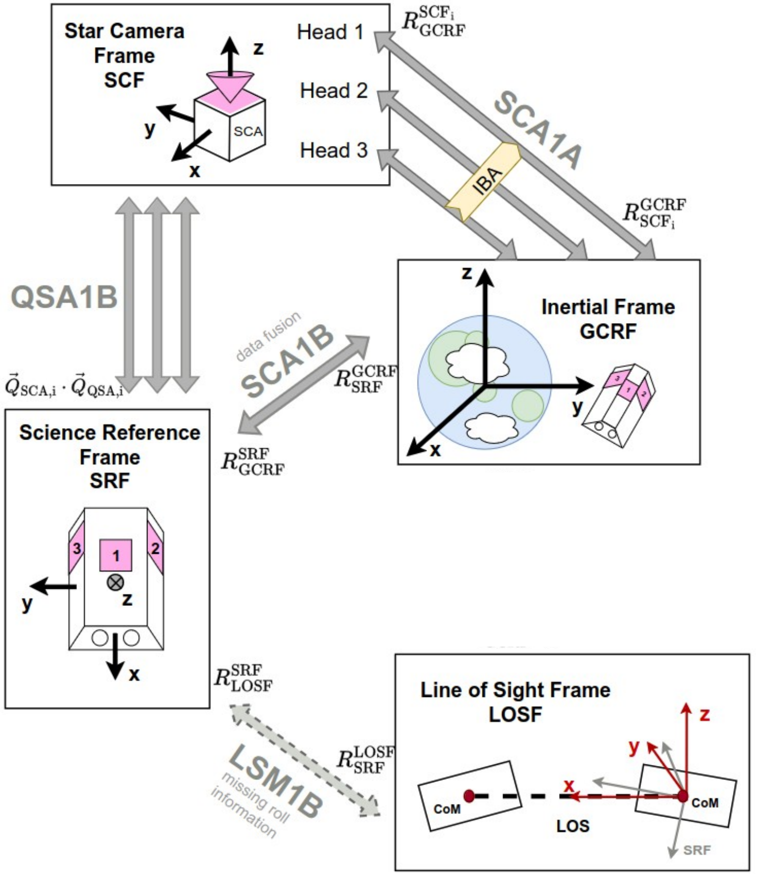
Star Cameras:

- SCA1: top panel, SCA2: left side panel, SCA3: right side panel
- take pictures of the dark sky and compare the star patterns with internal star catalogs to determine their own position in inertial frame.
- Data is stored as a set of Quaternions.
- Calibration values (QSA1B) are used, such that 3 heads describe the same orientation of the S/C.

Attitude Data Product SCA1B:

- Data fusion of Internal Measurement Unit (IMU, fiber optic gyroscopes) and SCAs.
- Provided as a set of Quaternions [q0 q1 q2 q3].

$$R_{\mathrm{GCRF}}^{\mathrm{SRF}} = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2\cdot(q_1\cdot q_2+q_0\cdot q_3) & 2\cdot(q_1\cdot q_3-q_0\cdot q_2) \\ 2\cdot(q_1\cdot q_2-q_0\cdot q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2\cdot(q_2\cdot q_3+q_0\cdot q_1) \\ 2\cdot(q_1\cdot q_3+q_0\cdot q_2) & 2\cdot(q_2\cdot q_3-q_0\cdot q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}$$



Pointing Angles from Star Cameras

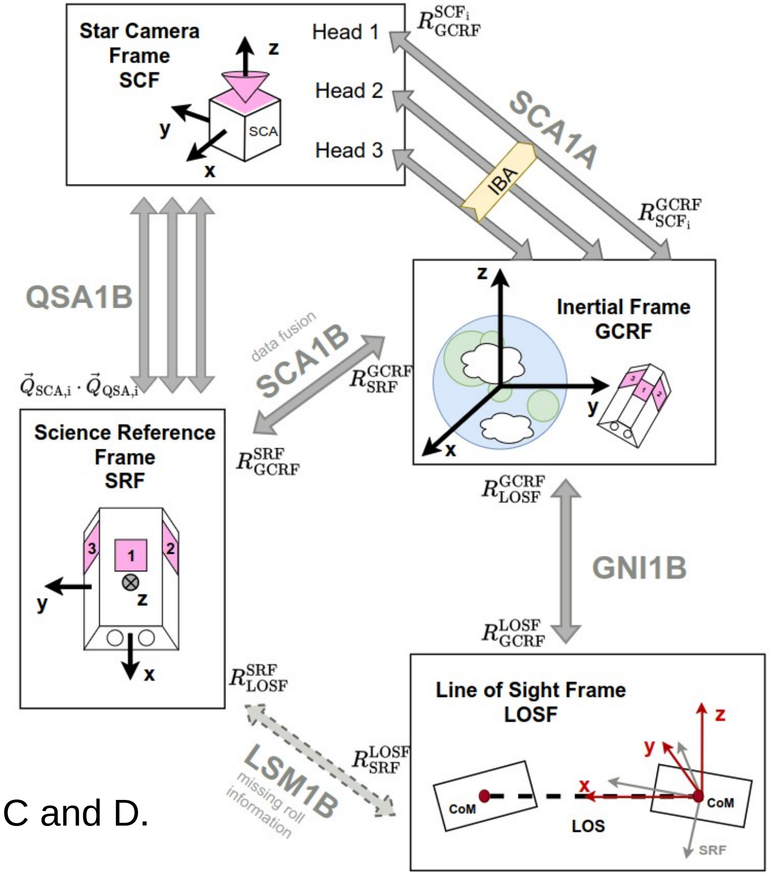
SCA1B or single SCA data can be used to compute inter-satellite pointing angles w.r.t. the Line of Sight, by considering inertial positions of the spacecraft.

$$R_{\mathrm{GCRF}}^{\mathrm{LOSF}} = \begin{bmatrix} x_{\mathrm{LOSF}} & y_{\mathrm{LOSF}} & z_{\mathrm{LOSF}} \end{bmatrix}$$

$$x_{\mathrm{LOSF_C}} = \frac{r_\mathrm{D} - r_\mathrm{C}}{|r_\mathrm{D} - r_\mathrm{C}|}$$

$$y_{\mathrm{LOSF_C}} = \frac{x_{\mathrm{LOSF_C}} \times r_\mathrm{C}}{|x_{\mathrm{LOSF_C}} \times r_\mathrm{C}|}$$

$$z_{\mathrm{LOSF_C}} = x_{\mathrm{LOSF_C}} \times y_{\mathrm{LOSF_C}}$$



The rotation matrix for satellite D can be computed by exchanging the indices C and D.

Pointing Angles from Star Cameras

SCA1B or single SCA data can be used to compute inter-satellite pointing angles w.r.t. the Line of Sight, by considering inertial positions of the spacecraft.

$$R^{\mathrm{LOSF}}_{\mathrm{SRF}} = R^{\mathrm{LOSF}}_{\mathrm{GCRF}} \cdot R^{\mathrm{GCRF}}_{\mathrm{SRF}} = R^{\mathrm{LOSF}}_{\mathrm{GCRF}} \cdot {R^{\mathrm{SRF}}_{\mathrm{GCRF}}}^{T}$$

$$\psi = atan2\left(\frac{R^{\mathrm{LOSF}}_{\mathrm{SRF},23}}{R^{\mathrm{LOSF}}_{\mathrm{SRF},33}}\right)$$

ψ : roll

θ : pitch

ϕ : yaw

$$\theta = -asin\left(R^{\mathrm{LOSF}}_{\mathrm{SRF},13}\right)$$

$$\phi = atan2\left(\frac{R^{\mathrm{LOSF}}_{\mathrm{SRF},12}}{R^{\mathrm{LOSF}}_{\mathrm{SRF},11}}\right)$$