# Calculation of Proton-Deuteron Breakup Reactions including the Coulomb Interaction between the Two Protons

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The Coulomb interaction between the two protons is fully included in the calculation of proton-deuteron breakup with realistic interactions for the first time. The hadron dynamics is based on the purely nucleonic charge-dependent (CD) Bonn potential and its realistic extension CD Bonn $+\Delta$ to a coupled-channel two-baryon potential, allowing for single virtual $\Delta$-isobar excitation. Calculations are done using integral equations in momentum space. The screening and renormalization approach is employed for including the Coulomb interaction. The Coulomb effect on breakup observables is seen at all energies in particular kinematic regimes.

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Experimentally, hadronic three-nucleon scattering is predominantly studied in proton-deuteron ($pd$) reactions, i.e., in $pd$ elastic scattering and breakup: Proton and deuteron beams and targets are available, with and without polarization. In contrast, the Coulomb interaction between the two protons is a nightmare for the theoretical description of three-nucleon reactions. The Coulomb interaction is well known, unlike the strong two-nucleon and three-nucleon potentials mainly studied in three-nucleon scattering. However, because of its $1/r$ behavior, the Coulomb interaction does not satisfy the mathematical properties required for the formulation of standard scattering theory. Therefore the inclusion of the Coulomb interaction in the description of the three-nucleon continuum is one of the most challenging tasks in theoretical nuclear physics [1]. Whereas it has already been solved for elastic $pd$ scattering with realistic hadronic interactions using various procedures [1–4], there are only a very few attempts [5–7] to calculate $pd$ breakup, and none of them use realistic potentials allowing for a meaningful comparison with the experimental data. The work presented here is a major breakthrough in the description of $pd$ breakup and therefore may be helpful for the understanding of few-nucleon systems microscopically and of the underlying forces that constrain their dynamics.

Recently in Ref. [4] we included the Coulomb interaction between the protons in the description of three-nucleon reactions with two-body initial and final states. The description is based on the Alt-Grassberger-Sandhas (AGS) equation [8] in momentum space. The Coulomb potential is screened around the separation $r = R$ between the two charged baryons, and the resulting scattering amplitudes are corrected by the renormalization technique of Refs. [9,10] to recover the unscreened limit. The treatment is applicable to any two-nucleon potential without separable expansion. Reference [4] and this Letter use the purely nucleonic charge-dependent (CD) Bonn potential [11] and its coupled-channel extension CD Bonn $+\Delta$ [12], allowing for a single virtual $\Delta$-isobar excitation and fitted to the experimental data with the same degree of accuracy as CD Bonn itself. In the three-nucleon system the $\Delta$ isobar mediates an effective three-nucleon force and effective two- and three-nucleon currents, both consistent with the underlying two-nucleon force. The treatment of Ref. [4] is technically highly successful, but still limited to the description of $pd$ elastic scattering and of electromagnetic (EM) reactions involving $^3$He with $pd$ initial or final states only. This Letter extends the treatment of Coulomb to breakup in $pd$ scattering. In that extension we follow the ideas of Refs. [5,9,10], but avoid approximations on the hadronic potential and in the treatment of screened Coulomb. Thus, our three-particle equations, including the screened Coulomb interaction, are completely different from the quasiparticle equations solved in Ref. [5] where the screened Coulomb transition matrix is approximated by the screened Coulomb potential.

Relative to Ref. [5] there are two important differences that are paramount to the fast convergence of the calculations in terms of the screening radius $R$ and the effective use of realistic interactions: (a) We work with a new type of screened Coulomb potential,

$$w_R(r) = w(r) e^{-(r/R)^n}, \tag{1}$$

where $w(r) = \alpha_e/r$ is the true Coulomb potential, $\alpha_e$ being the fine structure constant and $n$ controlling the smoothness of the screening. We prefer to work with a sharper screening than the Yukawa screening ($n = 1$) of Refs. [1,5]. We want to ensure that the screened Coulomb potential $w_R$ approximates well the true Coulomb one $w$ for distances $r < R$ and simultaneously vanishes rapidly for $r > R$, providing a comparatively fast convergence of the partial-wave expansion. As in Ref. [4], $n = 4$ is our choice for the results of this Letter. (b) Although the choice of the screened potential improves the partial-wave convergence, the practical implementation of the solution of the AGS equation still places a technical difficulty; i.e., the calculation of the AGS operators for nuclear plus screened Coulomb potentials requires two-nucleon partial waves with pair orbital angular momentum considerably higher than required for the hadronic potential alone. In this context the perturbation theory for high two-nucleon partial waves developed in Ref. [13] is a very efficient and reliable technical tool for treating the screened Coulomb interaction in high partial waves.

As a result of these two technical implementations, the method [14] that was developed before for solving three-particle AGS equations without Coulomb could be successfully used in the presence of screened Coulomb. As in Ref. [4] we choose an isospin description for the three baryons in which the nucleons are considered identical. The full breakup transition matrix $U_0^{(R)}(Z)$ follows by quadrature

$$U_0^{(R)}(Z) = (1 + P)G_0^{-1}(Z) + (1 + P)T_\alpha^{(R)}(Z)G_0(Z)U^{(R)}(Z) \tag{2a}$$

from the multichannel transition matrix $U^{(R)}(Z)$ of elastic $pd$ scattering, satisfying the standard symmetrized form of the AGS integral equation

$$U^{(R)}(Z) = PG_0^{-1}(Z) + PT_\alpha^{(R)}(Z)G_0(Z)U^{(R)}(Z). \tag{2b}$$

The superscript $(R)$ denotes the dependence on the screening radius $R$ of the Coulomb potential, $G_0(Z) = (Z - H_0)^{-1}$ is the free resolvent, and $P = P_{231} + P_{312}$ is the sum of two cyclic permutations of three baryons. The two-baryon transition matrix $T_\alpha^{(R)}$ is derived from the full channel interaction $v_\alpha + w_{\alpha R}$

$$T_\alpha^{(R)}(Z) = (v_\alpha + w_{\alpha R}) + (v_\alpha + w_{\alpha R})G_0(Z)T_\alpha^{(R)}(Z), \tag{3}$$

where $v_\alpha$ is the nuclear interaction between baryons and $w_{\alpha R}$ the screened Coulomb potential between charged baryons ($w_{\alpha R} = 0$ otherwise). In the isospin description chosen by us the two-baryon transition matrix $T_\alpha^{(R)}(Z)$ becomes an operator coupling total isospin $\mathcal{T} = \frac{1}{2}$ and $\frac{3}{2}$ states as described in detail in Ref. [4].

Matrix elements of $U_0^{(R)}(Z)$ are taken between the initial $pd$ channel state $|\phi_\alpha(\mathbf{q}_i)\nu_{\alpha_i}\rangle$ of relative $pd$ momentum $\mathbf{q}_i$, energy $E_\alpha(q_i)$, and additional discrete quantum numbers $\nu_{\alpha_i}$ and the final breakup channel states $|\phi_0(\mathbf{p}_f\mathbf{q}_f)\nu_{0_f}\rangle$, $\mathbf{p}_f$ and $\mathbf{q}_f$ being three-nucleon Jacobi momenta, $E_0(p_f q_f)$ its energy, and $\nu_{0_f}$ additional discrete quantum numbers. Equations (2) and (3) are solved for given values of the screening radius $R$ using standard numerical techniques [4] much like what is commonly done for neutron-deuteron ($nd$) breakup.

As explained in Refs. [5,10,15], $U_0^{(R)}(Z)$ has to get renormalized, much like the corresponding amplitude for $pd$ elastic scattering [4,10], in order to obtain the results appropriate for the unscreened Coulomb limit. According to Refs. [5,10,15], the full breakup transition amplitude for initial and final states $|\phi_\alpha(\mathbf{q}_i)\nu_{\alpha_i}\rangle$ and $|\phi_0(\mathbf{p}_f\mathbf{q}_f)\nu_{0_f}\rangle$, $E_\alpha(q_i) = E_0(p_f q_f)$, referring to the strong potential $v_\alpha$ and the unscreened Coulomb potential $w_\alpha$, is obtained via the renormalization of the on-shell breakup transition matrix $U_0^{(R)}(E_\alpha(q_i) + i0)$ in the infinite $R$ limit

$$\begin{aligned}\langle\phi_0(\mathbf{p}_f\mathbf{q}_f)\nu_{0_f}|U_0|\phi_\alpha(\mathbf{q}_i)\nu_{\alpha_i}\rangle = \lim_{R\to\infty}\{&z_R^{-1/2}(p_f)\\ &\times\langle\phi_0(\mathbf{p}_f\mathbf{q}_f)\nu_{0_f}\\ &\times|U_0^{(R)}(E_\alpha(q_i)\\ &+ i0)|\phi_\alpha(\mathbf{q}_i)\nu_{\alpha_i}\rangle\\ &\times \mathcal{Z}_R^{-1/2}(q_i)\}.\end{aligned} \tag{4}$$

The renormalization factors $\mathcal{Z}_R(q_i)$ and $z_R(p_f)$ in the initial and final channels are diverging phase factors

$$\mathcal{Z}_R(q_i) = e^{-2i\kappa(q_i)[\ln(2q_iR) - C/n]}, \tag{5a}$$

$$z_R(p_f) = e^{-2i\kappa(p_f)[\ln(2p_fR) - C/n]}, \tag{5b}$$

$\kappa(q_i) = \alpha_e M/q_i$ and $\kappa(p_f) = \alpha_e\mu/p_f$ being the $pd$ and $pp$ Coulomb parameters, $M$ and $\mu$ the reduced $pd$ and $pp$ masses, $C \approx 0.577\,215\,664\,9$ the Euler number, and $n$ the exponent in Eq. (1). In $pd$ elastic scattering [4], the renormalization factors were used in a partial-wave dependent form, which yielded a slight advantage on convergence with $R$ compared to the partial-wave independent form (5). In breakup, the operator $T_\alpha^{(R)}(Z)G_0(Z)U^{(R)}(Z)$ is calculated in a partial-wave basis, but the on-shell elements of the full breakup operator $U_0^{(R)}(Z)$ are calculated in a plane wave basis. Therefore the renormalization is applicable only in the partial-wave independent form of Eq. (5).

The limit in Eq. (4) has to be performed numerically, but, due to the finite-range nature of the breakup operator discussed in Refs. [10,15], the infinite $R$ limit is reached with sufficient accuracy at rather modest screening radii $R$. Convergence with screening radius $R$ is the internal criterion for the reliability of our Coulomb treatment. Configuration-space approaches [6,7] may provide a viable alternative to our momentum-space calculation, but they still involve approximations in the treatment of Coulomb and the employed hadronic dynamics is not realistic. Thus a benchmark comparison between our breakup results and corresponding configuration-space results is, in contrast to $pd$ elastic scattering [16], not possible yet. Therefore at this time we rely solely on our internal criterion for the convergence of breakup observables with the screening radius $R$, which proved highly reliable for $pd$ elastic scattering and related EM reactions [4].

The practical implementation of the outlined calculational scheme faces the technical difficulty of slow partial-wave convergence due to the screened Coulomb potential. In the employed perturbation theory for high two-baryon

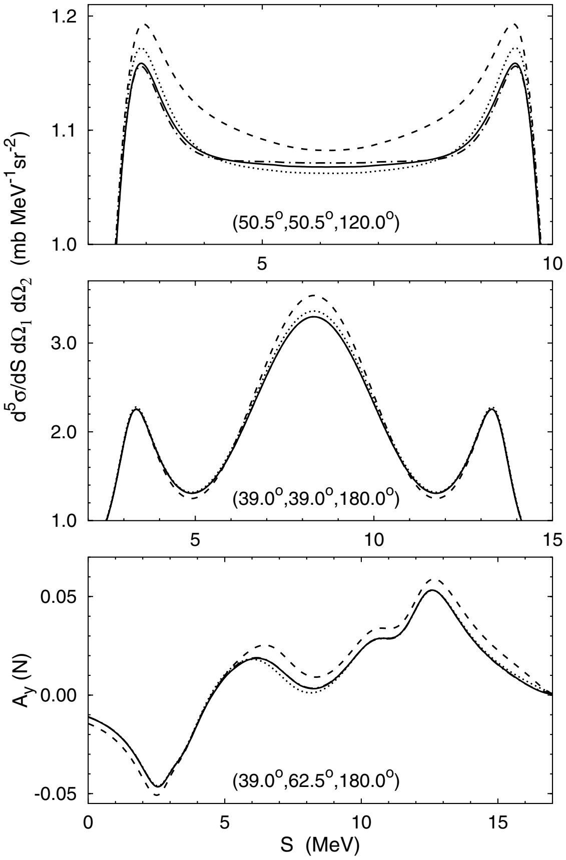


FIG. 1. Convergence of the *pd* breakup observables with screening radius $R$. The differential cross section and the proton analyzing power $A_y(N)$ for *pd* breakup at 13 MeV proton lab energy are shown as functions of the arclength $S$ along the kinematical curve. Results for CD Bonn potential obtained with screening radius $R = 10$ fm (dotted curves), 20 fm (dash-dotted curves), and 30 fm (solid curves) are compared. Results without Coulomb (dashed curves) are given as reference for the size of the Coulomb effect.

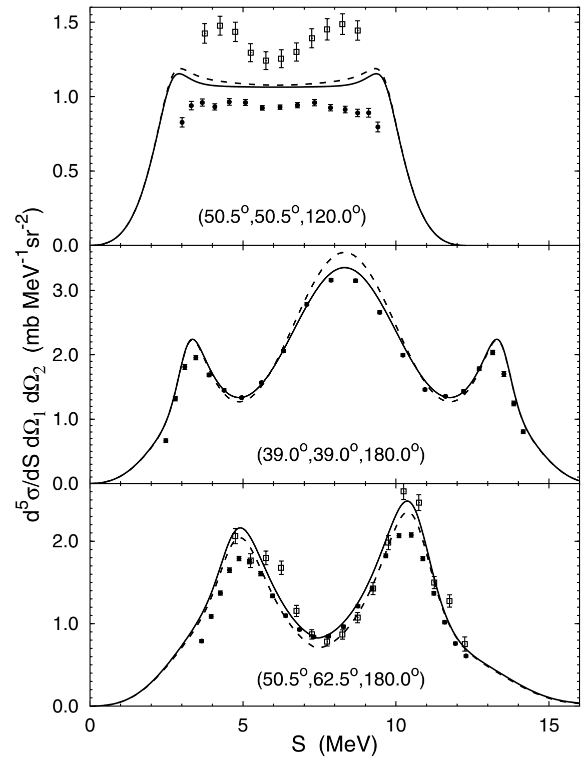


FIG. 2. Differential cross section for *pd* breakup at 13 MeV proton lab energy for space star, quasi-free scattering, and collinear configurations (from top to bottom). Results for CD Bonn $+\Delta$ potential including the Coulomb interaction (solid curves) are compared to results without Coulomb (dashed curves). The experimental *pd* data (circles) are from Ref. [18] and *nd* data (squares) are from Ref. [19].

partial waves [13], we vary the dividing line between partial waves included exactly and perturbatively in order to test the convergence and thereby establish the validity of the procedure.

With respect to the partial-wave expansion in the actual calculations of this Letter, we obtain fully converged results by taking into account the screened Coulomb interaction in two-baryon partial waves with pair orbital angular momentum $L < 15$; orbital angular momenta $L \geq 9$ can safely be treated perturbatively. The above values refer to the screening radius $R = 30$ fm; for smaller screening radii the convergence in orbital angular momentum is faster. The hadronic interaction is taken into account in two-baryon partial waves with total angular momentum $I \leq 5$. Both three-baryon total isospin $\mathcal{T} = \frac{1}{2}$ and $\frac{3}{2}$ states are included. The maximal three-baryon total angular momentum $\mathcal{J}$ considered is $\frac{61}{2}$.

The results of our calculations are presented in Figs. 1–4. The kinematical final-state configurations are characterized in a standard way by the polar angles of the two protons and by the azimuthal angle between them, $(\theta_1, \theta_2, \varphi_2 - \varphi_1)$. In Fig. 1 we show the convergence of calculated *pd* breakup observables at 13 MeV as the Coulomb screening radius increases from $R = 10$ to 30 fm; dashed lines correspond to results without Coulomb. In most configurations we study, results with $R = 20$ fm are already indistinguishable from $R = 30$ fm. A characteristic Coulomb effect on *pd* breakup at low energies is shown and compared with data in Fig. 2 for the differential cross section in space star, quasi-free scattering, and collinear configurations; the proton lab energy is 13 MeV. The inclusion of Coulomb appears to be unable to resolve the so-called *space star anomaly*, but improves the description of the data in the vicinity of the quasi-free scattering and collinear points. The disagreement around the peaks in the collinear configuration is probably due to the finite geometry, not taken into account in our calculations owing to the lack of information on experimental details, but may also be due to the underlying hadronic interaction. At higher energies, in most configurations for

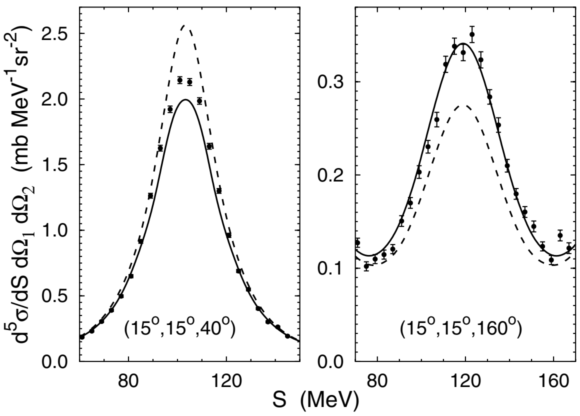


FIG. 3. Differential cross section for *pd* breakup at 130 MeV deuteron lab energy. Curves as in Fig. 2. The experimental data are from Ref. [17].

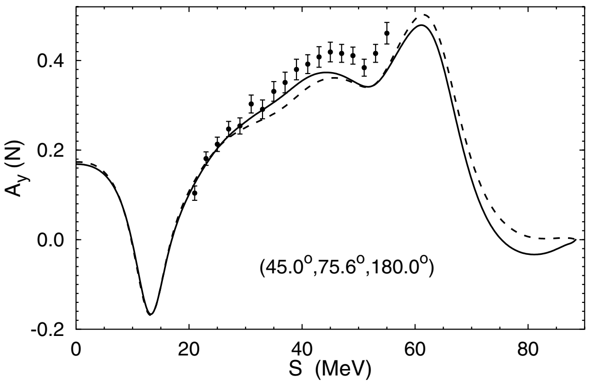


FIG. 4. Proton analyzing power for *pd* breakup at 65 MeV proton lab energy in the collinear configuration. Curves as in Fig. 2. The experimental data are from Ref. [20].

which there are available data, we find the effect of Coulomb to be rather small except in recently measured configurations of *pd* breakup at 130 MeV deuteron lab energy [17] shown in Fig. 3 where, depending on the azimuthal angle, Coulomb either increases or decreases the differential cross section. By and large, the agreement between theoretical predictions and experimental data is improved. Other configurations where the Coulomb effect is very important correspond to the proton-proton final-state interaction (*pp*-FSI) regime with very low relative *pp* energy (not shown here) for which there are no available data. In there *pp* repulsion is responsible for decreasing the cross section, converting a *pp*-FSI peak obtained in the absence of Coulomb into a minima [5,15]. This effect is partially responsible for lowering the peak in the configuration (15°, 15°, 40°) in Fig. 3, where the relative *pp* energy is rather low at the peak. The relative *pp* energy gets considerably increased as one changes the azimuthal angle to 160°; adding Coulomb there increases the cross section. Finally, Fig. 4 shows the Coulomb effect on the nucleon analyzing power in the collinear configuration at 65 MeV nucleon lab energy.

In conclusion, we have been able for the first time to calculate *pd* breakup observables using realistic hadronic potentials and the full Coulomb interaction between charged baryons. Coulomb effect may be important even at higher energies, depending on the kinematical configuration, but it is unable to resolve the space star anomaly at low energies.

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