

Comment on “Exact three-dimensional wave function and the on-shell t matrix for the sharply cut-off Coulomb potential: Failure of the standard renormalization factor”

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The results by W. Glöckle *et al.* [Phys. Rev. C **79**, 044003 (2009)] are shown to be consistent with the screening and renormalization theory, and they do not invalidate previous numerical calculations of physical observables.

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In a recent work [1], W. Glöckle *et al.* derived a relation between the full and sharply cut-off Coulomb scattering amplitudes and wave functions. We do not discuss here the mathematical rigorousness of the results of Ref. [1] (see the accompanying comment by K. A. Kouzakov *et al.*). However, the comparison with previous works on the screening and renormalization theory is not fully correct and has already led to serious misinterpretations by other authors [2]. Therefore we feel the need for an additional discussion.

The scattering amplitudes for the full and sharply cut-off Coulomb, $A_C(\theta)$ and $\tilde{f}_R(\theta)$, respectively, are related in Eq. (55) of Ref. [1]; in a more compact notation, convenient for our considerations, that relation looks like

$$\tilde{f}_R(\theta) = \left[e^{-2i\Phi_R} - \frac{1}{2} e^{2ipR \sin\theta + i\chi_+(\theta)} - \frac{1}{2} e^{-2ipR \sin\theta + i\chi_-(\theta)} \right] A_C(\theta). \quad (1)$$

Due to the presence of the second and third terms on the right-hand side, that result seems to be in contradiction with the screening and renormalization theory as developed in previous works, e.g., Ref. [3]. However, Ref. [3] proves for a general screening that in the limit of infinite screening radius R , the screened Coulomb scattering amplitude $\tilde{f}_R(\theta)$, renormalized by the infinitely oscillating phase factor $e^{2i\Phi_R}$, approaches the full Coulomb amplitude $A_C(\theta)$ in general as a distribution, i.e.,

$$\lim_{R \rightarrow \infty} e^{2i\Phi_R} \int_0^\pi \tilde{f}_R(\theta) \varphi(\theta) \sin\theta d\theta = \int_0^\pi A_C(\theta) \varphi(\theta) \sin\theta d\theta \quad (2)$$

for any test function $\varphi(\theta)$ with the properties given in Ref. [3]; in particular, $\varphi(0) = 0$. Reference [3] uses partial-wave expansion, which for the full Coulomb amplitude itself converges only as a distribution, and it therefore is unable to make any conclusions on the possible pointwise convergence, i.e.,

$$\lim_{R \rightarrow \infty} e^{2i\Phi_R} \tilde{f}_R(\theta) = A_C(\theta). \quad (3)$$

Although Eq. (3) is obviously not fulfilled for the sharply cut-off Coulomb, as the authors of Ref. [1] point out, for nonzero on-shell momentum $p \neq 0$, Eq. (1) is consistent with Eq. (2) because

$$\lim_{R \rightarrow \infty} \int_0^\pi e^{\pm 2ipR \sin\theta + i\chi_\pm(\theta)} A_C(\theta) \varphi(\theta) \sin\theta d\theta = 0, \quad (4)$$

owing to the infinitely rapid oscillations of the integrand with θ caused by $e^{\pm 2ipR \sin\theta}$; the logarithmic singularities in $\chi_\pm(\theta)$ at $\theta = 0$ or π do not change that result. Thus, Ref. [1] does not contradict the theory of screening and renormalization at all, but for one particular choice of screening, the sharp cut-off, it demonstrates the absence of the pointwise convergence (3). Furthermore, the remark given in Ref. [1] that the terms of the partial-wave series for $\tilde{f}_R(\theta)$ with orbital angular momenta $l \approx pR$ are out of control is correct when the series is considered as a function but not when considered as a distribution as done in Ref. [3]; the partial-wave series converges as a distribution sufficiently rapidly; i.e., for R large enough, only partial waves with $l \ll pR$ provide nonvanishing contributions.

The absence of the pointwise convergence (3) is not unexpected in the case of the sharply cut-off Coulomb potential

$$\langle \mathbf{p}' | w_R | \mathbf{p} \rangle = \frac{\alpha_e [1 - \cos(|\mathbf{p}' - \mathbf{p}|R)]}{2\pi^2 (\mathbf{p}' - \mathbf{p})^2}, \quad (5)$$

which itself in the $R \rightarrow \infty$ limit converges to the full Coulomb potential

$$\langle \mathbf{p}' | w_C | \mathbf{p} \rangle = \frac{\alpha_e}{2\pi^2 (\mathbf{p}' - \mathbf{p})^2} \quad (6)$$

as a distribution only. In contrast, the screened Coulomb potential with Yukawa screening [4] or with the more rapid but still smooth screening of Ref. [5] in the $R \rightarrow \infty$ limit converges pointwise to the full potential. Of course that is no proof that the corresponding scattering amplitude does so also. Unfortunately, an analytic study of the screening limit, as given so nicely for sharp cut-off in Ref. [1], is missing for the screening forms employed in practice [4,5]. For sharp cut-off, the absence of pointwise convergence (3) clearly shows up numerically in the scattering amplitude [1], whereas for the smoother screenings there is no numerical evidence for it [5,6].

Finally, as argued already in Ref. [3], we emphasize that the convergence of the renormalized screened Coulomb scattering amplitude to the full Coulomb amplitude in the sense of distributions is sufficient for the description of physical observables: In the step from a theoretical scattering amplitude to an experimental cross section, one has to go through the conceptual exercise of averaging the scattering amplitude over the initial-state physical wave packet, being normalized and quite sharply centered around the experimental beam momentum. For a fixed final-state observation direction

θ_f , that averaging implies an integration of the scattering amplitude $\tilde{f}_R(\theta)$ with $\varphi_i(\theta, \theta_f)$ over the scattering angle θ , the $\varphi_i(\theta, \theta_f)$ being determined by the angular spread of the initial wave packet and being peaked around θ_f . Thus, on the right-hand side of Eq. (2), $A_C(\theta_f)$ itself is picked out in that average. In addition, the outgoing wave packet is never observed in the forward direction $\theta_f = 0$; the necessary property $\varphi_i(0, \theta_f) = 0$ can therefore always be fulfilled by the sharpness of the initial wave packet. Note that Refs. [4,5] carry

out the above averaging implicitly, replacing the renormalized screened Coulomb amplitude in the $R \rightarrow \infty$ limit by the full one; furthermore, none of the Refs. [4,5] used the sharply cut-off Coulomb potential.

In conclusion, the relation between the full and sharply cut-off Coulomb scattering amplitudes derived by W. Glöckle *et al.* in Ref. [1] is consistent with the screening and renormalization theory [3] and has no implications for previous numerical calculations [4,5].

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