FOCUSED WAVE GENERATION IN LABORATORY FLUMES OVER UNEVEN BOTTOM

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A proper design of offshore and coastal structures requires further knowledge about extreme wave events. Such waves are highly nonlinear and may occur unexpectedly due to diverse reasons. One of these reasons is wave-wave interaction and the wave focusing technique represents one option to generate extreme wave events in the laboratory. The underlying mechanism is the superimposition and phasing of wave components at a predefined location. To date, most of the existing methods to propagate target wave profile backwards to the position of the wave generator apply linear wave theory. The problem is that the generated waves with different frequencies generate new components which do not satisfy the linear dispersion relation. As a result, small changes in the wave board control signal generally induce large and random shifts in the resulting focused wave. This means that iterations are necessary to get the required wave profile at the correct position in the flume. In this study, a Self Correcting Method (SCM) is applied to optimize the control signal of the wave maker in a Numerical Wave Tank (NWT). The nonlinearities are included in the control signal and accurate wave focusing is obtained irrespective of the prevailing seabed topography (horizontal or sloping) and type of structure (reflective or absorbing). The performance of the proposed SCM is numerically investigated for a wide variety of scenarios and validated by scale model tests in the Large Wave Flume (Großer Wellen Kanal, GWK), Hannover, Germany. The strengths and limitations of the proposed SCM are discussed, including the potential for further developments.

Keywords: Extreme waves; Wave focusing; Nonlinear waves; Self correcting method; Numerical wave tank; Tsunami generation

INTRODUCTION

In harsh weather, extreme events will lead to damages to ships, offshore and coastal structures. The design of reliable and economic coastal and offshore structures requires further knowledge of such episodic waves. These are unpredictable waves that are out of the statistical logic Osborne et al., (2000). Extreme waves are considered like a strange phenomenon because they are highly asymmetric and nonlinear waves (high ratio between wave crest and wave trough) that can occur in a relative calm sea state. This is not only occurring in deep water, but more recently in shallow water depth, for instance the extreme wave recorded near the Cape Olga, Kamchatka (Russia), Fig. 1. Extreme wave events may occur due to four main processes: wave-current interactions, wave-bottom interactions, wave-wave interaction and wind-wave interactions. The generation of extreme wave events in laboratory flumes by means of nonlinear superposition and phasing of wave components (known as wave focusing) is particularly considered in this work. Several different approaches to generate such episodic waves were suggested in the past, but they are based on linear theory to transform the target wave from the focal point backwards to the position of the wave maker (Rapp and Melville, (1990),Clauss, (2002),Clauss and Klein, (2011),Hofland et al., (2010)). A methodology to create 3D wave packets in an irregular wave field was elaborated by Ducrozet et al., (2012), but linear dispersion relation is used to calculate the wave component celerity. Shemer et al., (2007) developed a computational model based on the unidirectional spatial Zakharov equation to describe the evolution of steep wave groups over constant water depth and they validated the theoretical model at the GWK, Hannover, Germany. The NewWave theory was used by Westphalen et al., (2012),Ma et al., (2010) to produce extreme wave events from a measured or theoretical spectrum. Also Borthwick et al., (2006) generated normal and oblique focused wave groups at the toe of a plane beach by using the New Wave Theory. Liu et al., (2011) employed a method based on Longuet-Higgins theory to simulate extreme waves at a certain location preserving the statistical properties of a realistic sea state. On the other hand, Funke and Mansard, (1988) developed a method of reversible dispersive technique to generate extreme waves (or episodic), that can also deal with variable water depth. However, they used linear theory during the process of wave assemblage. Thus, one needs to conduct trial and error to obtain the profile at the required position in the flume. Baldock and Swan, (1996) presented a series of experimental tests in which they generated focused wave groups over shallow and intermediate water depths. To avoid the shifts at the focal point, \( x_p \), they empirically determined the value of \( x_p \) in order to generate the focused wave at the desired position.
The problem is that when a wave packet containing different wave frequencies is generated, the waves interact and new components that are not satisfying the linear dispersion relation are created (due to wave-wave interaction). This means that small changes in the associated wave maker control signal can lead to large and unpredictable changes in the generated focused wave, resulting in premature wave focusing. Additionally, as linear dispersion relation is used to back scatter the target wave elevation to the position of the wave maker, when a wave packet is generated, the wave is travelling faster as compared to the celerity calculated by means of linear wave theory, therefore focusing occurs after the theoretical focal point. Other reason for premature wave focusing is due to the presence of spurious components generated by the wave paddle Sriram et al., (2013). Furthermore, most of these methodologies cannot deal with varying topography and cannot account for wave reflection (when testing a fully reflective wall for instance).

As most of the extreme wave tests with offshore or coastal structures are conducted with an uneven bottom and with reflective structures like vertical breakwaters or seawalls, the nonlinear effects due to the presence of an uneven bottom or wave reflection increase. This means that larger shifts at the focal point occur. Hence, the effects of the complex bathymetry and wave reflection have to be included in the wave board control signal in order to ensure the generation of an accurate focused wave at the desired location in the flume.

In this work, a Self Correcting Method (SCM) is proposed. The pioneering work on the SCM was performed in the '80s at the University of Hannover, for the generation of irregular Daemrich et al., (1980) and higher order regular waves Daemrich and Gotschenberg, (1988) by establishing amplitude and phase transfer functions. It was found out that this method may not always work due to the generated spurious, free sub or super harmonics components when using linear paddle displacements (Prof. Daemrich personal communication). For this reason automation of the method was not implemented. Later, Chaplin, (1996) used a similar approach for focused wave generation employing phase corrections alone and reported good results. An iterative approach was also used by Do et al., (2004) for Gaussian wave packet simulation. Schmittner et al., (2009) developed both amplitude correction and phase correction, and showed excellent results for improving deterministic wave trains. Recently, this method was also employed for tsunami wave simulation in a small wave flume Buldakov, (2013). However, the potential and disadvantages of this method were not reported previously. In our previous investigations Fernandez et al., (2013) the performance of the SCM was demonstrated to be dependent on the correction scheme (phase only or phase and amplitude correction steps) and the type of wave profile used for the development of the correction steps (first or second order). In this paper, the SCM is further tested in a Numerical Wave Tank (NWT), the objective is its implementation at the GWK for the generation of accurate focused waves at the predefined location irrespective of the bathymetry and the kind of structure.

The goals of this paper are: (i) to analyse the performance of the SCM by means of using different correction schemes, (ii) to demonstrate the capability of the SCM to reproduce accurate wave focusing in presence of wave reflection and sloping bottom, and (iii) to verify the methodology for generating non breaking and breaking focused waves.

The paper is arranged as follows. First, an overview of the SCM and the NWT is provided. At the second part the results of the numerical simulations and laboratory tests are discussed. Finally the concluding remarks and outlook are addressed.

SELF CORRECTING METHOD

The SCM is a proper way to improve the quality of the control signal by means of taking into account the inherent nonlinear interactions in the resulting wave train. The efficiency of the wave focusing is iteratively improved after some correction steps. If one knows the experimental set up (focal point, bathymetry, etc.) to be employed at the physical tests, the SCM can be applied by following these steps:

1. The target wave profile to be generated at the predefined location is back transformed to the position of the wave maker by means of linear theory without considering any nonlinear interactions.
2. Using (1) as a initial control signal the simulation is being carried out. Later, the control signal is corrected in frequency domain (in terms of wave phases and amplitudes) by means of the differences between the target and the recorded wave profile.
3. After applying 2 or 3 correction steps, the control signal is improved by the SCM and an required target wave is generated at the predefined location irrespective the water depth present in between the wave maker and the focal point. Thus, the missing nonlinear interaction that was not taken into considerations will be taken care automatically.

The physical testing is expensive and time consuming, so the above steps can be carried out in a Numerical Wave Tank (NWT) and the final control signal will be used in the experimental wave paddle. Further details about the SCM and the NWT can be found at our last publication, Fernández et al., (2014).

RESULTS AND DISCUSSION

The validation of the numerical model with the experimental measurements, particularly from GWK data is reported in our previous publications Fernandez et al., (2013),Sriram et al., (2006),Hildebrant et al., (2013). Hence, this section presents the validity of the SCM in order to generate predefined time series at the desired point for different cases including: (i) constant water depth, (ii) a fully reflective wall, (iii) wave reflection and uneven bottom, (iv) a submerged bar; and (v) experimental testing of wave focusing.

Focused wave in constant water depth

The aim of this section is to evaluate the performance of the SCM. Linear displacements were used for the motion of the wave maker. The SCM was developed with a linear and a second order target wave signal. The second order wave profile was computed by means of the frequency difference and sum terms from the linear components in the frequency domain Schäffer H.A., (1996). The performance of the methodology is evaluated with different correction approaches. Amplitude and phase corrections were applied simultaneously as well as independently to check the sensitivity of the SCM. The parameters defining the first control signal for starting the process are: \( d = 4 \) m water depth, the focal point is located at \( x_f = 75.43 \) m from the wave paddle, the focusing time is \( t_f = 86 \) s, the frequency bandwidth is \( \Delta f = 0.33 \) Hz, the central frequency \( f_c = 0.45 \) Hz and the gain factor \( G = 0.002 \pi \). The wave packet is built with a constant steepness spectrum, this avoids premature wave breaking as all the components within the wave packet maintain the same scale respect to wave steepness, Roux de Reilhac et al., (2011). The experimental layout used for the tests is shown in Fig. 3, a total of six wave probes were employed to track the motion of the free surface, whereas \( Wg4 \) was used to measure the free surface elevation at the focal point. The dimension of the tank is selected in accordance with the GWK in Hannover.

![Figure 1. Numerical layout during the tests conducted by means of flat horizontal bottom (All dimensions in meter).](image-url)

The performance of the method using a second order target wave profile is presented in Fig. 2. For the first measurement at the focal point, Fig. 2a, a shift between the second order target signal and the recorded wave profile is noticed. This is due to the nonlinear interactions within the wave packet are not taken into account in the control signal and the main wave is travelling faster than the target wave.
profile predicted by second order theory, this phenomenon was also reported in Stansberg, (2002). In order to quantify the agreement between the wave profiles, a correlation coefficient is used.

![Figure 2](image1.png)

**Figure 2.** Phase and amplitude correction steps. The solid blue line represents the second order target wave profile whereas the red solid line corresponds to the recorded wave elevation at the focal point (75.43 m from the wave maker).

The comparison between the correction schemes by means of the correlation coefficient is reported in Fig. 3, this represents the correlation coefficient versus the number of correction step, where the step 0 is the first measurement at the focal point. When a linear wave profile is used as target wave elevation, it is evident that performing amplitude corrections alone does not provide a substantial improvement. This is due to the fact that in the wave focusing process, the phases play a more important role than the amplitudes. This can be demonstrated by performing a phase correction scheme alone: after two correction steps a higher agreement between profiles is reached. However, phase and amplitude correction scheme gives worse performance than conducting phase corrections alone. This might be the reason, why a phase correction scheme was used in previous studies to optimize wave focusing Chaplin, (1996).

![Figure 3](image2.png)

**Figure 3.** Correlation coefficient described by equation 11 versus the number of correction steps for the different cases.

The evolution of the amplitude spectrum with the correction steps is reported in Fig. 4. Phase and amplitude wave components were corrected and a second order wave profile was employed as target.
The cross marker represents the amplitude spectra for the first measurement at the focal point, although the major part of the energy is concentrated within the fundamental frequencies, it can be found that a certain amount of energy at low ($f< f_{\text{min}}$) and at high frequencies ($f>f_{\text{max}}$) are present, this is due to the fact that when a wave packet containing different frequencies travels towards the focal point, the waves interact and new wave components that do not satisfy linear dispersion relation are created. As reported in Baldock and Swan, (1996), the additional high frequencies harmonics appear due to “local” non-linearities that sharpen the wave crest and flatten the troughs, whereas the low frequency components can be considered as “global” non-linearities causing the set-down beneath the largest waves within the group. The rest of series (triangle marker, point marker and black solid line) show the amplitude spectra for the correction steps, a higher amount of energy at high and low frequencies can be observed with respect to the target wave profile, this is due to the higher order nonlinear interactions Baldock et al., (1996), Ma et al., (2010). Furthermore, it can be observed that although convergence is achieved in time domain (Fig. 4) some deviations between the target (circle marker) and the corrected wave profile (black line) are reported in frequency domain, Fig. 4. This is due to the wave-wave interactions given in the wave focusing process are higher than the second order and one can not force the fully nonlinear model to be second order theory, as wave-wave interactions were classified in the past as 3rd or 4th order Baldock et al., (1996).

**Focused wave in presence of wave reflection and uneven seabed**

Fig. 5 shows a typical case where the combination of an uneven bottom and wave reflection is given. It is important to mention that none of the existing methods for generating focused waves can deal with such scenarios. The same set up with respect to the fully reflective wall case (Fig. 6) is used, but a berm is set in the domain between $x = 60$ m and $x = 76$ m. The first control signal is the same as in the test presented in the previous section.

- **Target**
- **Recorded**
- **Corrected 1**
- **Corrected 2**
- **Corrected 3**

**Figure 4.** Amplitude spectrum of the wave elevation at the focal point, evolution with phase and amplitude correction steps and a second order target wave profile.

**Figure 5.** Numerical layout for the fully reflective wall and a variable water depth configuration (All the dimensions in meter).
Fig. 6 shows the development of the correction steps. Fig. 6a presents the first wave measurement at the focal point (red line) and the second order theoretical wave elevation (blue). A difference between the signals can be noticed, due to the effect of the bottom topography and wave reflection. The energy is spread around the focal point and is not concentrated at the predefined location. When applying the correction steps, the control signal is iteratively improved by considering the additional nonlinearities resulting from the uneven bottom and wave reflection. With one correction step, Fig. 6b, improvement is yielded but there is still a little deviation between the target and the measured wave profile. The best result is reached after three phase and amplitude correction steps, with a correlation coefficient, $R = 0.9678$, after applying a further iteration the agreement is worse, with $R = 0.9671$.

Figure 6. Correction steps of the SCM (phase and amplitude correction) in presence of wave reflection and variable water depth. In the first subplot the solid red line represents the recorded surface elevation whereas the solid blue line is the second order target wave profile at the focal point. The underlined correlation coefficient represents the maximum achieved $R$.

Verification of the SCM in the GWK

The test cases presented above demonstrate the ability of the SCM to improve wave focusing in the presence of uneven bottom, wave reflection or the combination of both. In this section the validation of the presented methodology is carried out in the GWK of the Coastal Research Centre (Forschungszentrum Küste), Germany, whose main dimensions are: 330 m length, 7 m height and 5 m width. The correction steps are performed in the NWT until convergence is reached. The resulting stroke time series is transferred to the wave maker in GWK. These verification tests were conducted within the WAVESLAM project whose scope was to investigate slamming forces on a truss structure. The physical model is deployed in 2 m water depth at $x = 198.4$ m from the wave maker, Fig. 7. Then the water depth linearly increases with a 1:10 slope from $d = 2.0$ m to $d = 4.3$ m where it remains constant until the position of the wave maker.
The parameters for the non breaking focused wave case are: $x_f = 198.40$ m, $t_f = 67.40$ s, $\Delta f = 0.2$ Hz, $f_c = 0.204$ Hz, $G = 0.001*\pi$, $N = 32$. The control signal for the non breaking focused wave is improved in the NWT by means of the SCM developing phase and amplitude correction steps with a second order wave signal as target, Fig. 8, the maximum correlation coefficient between wave profiles is given after two correction steps, $R = 0.9749$, later the correlation decreases with $R = 0.9744$. Afterwards, the improved paddle stroke is generated at the flume. The free surface elevation recorded at the physical experiments is shown in Fig. 9; the wave elevation measured in the physical experiments is represented by the red solid line while the blue line is the wave profile recorded in the NWT (both measurements were taken at the focal point). Very good agreement between the experimental and the numerical data is reported, the slight deviation after the focusing is due to the wave reflection from the truss structure is not considered in the NWT.

For the purpose of completeness in the validation procedure, a non breaking focused wave over constant water depth ($x < 175$ m) was also generated with the model setup presented in Fig. 7, but the water depth was lowered until $d = 3.5$ m. The parameters defining the wave focusing were: $d = 3.5$ m, $x_f = 140$ m, $t_f = 85$ s, $\Delta f = 0.31$ Hz, $f_c = 0.31$ Hz, $G = 0.003*\pi$, $N = 32$. The SCM was performed in the NWT until convergence was achieved and then the optimized control signal was generated in the flume. The comparison between the experimental and the numerical results are reported in Fig. 10, looking at the experimental measurement one can notice a steep focused wave with a crest height of $A = 0.83$ m, forming a total wave height $H = 1.31$ m. It is important to mention that in general both (numerical and experimental) signals present a relative good agreement for the main wave crest with slight deviations at the wave troughs. This validation test confirms that the proposed methodology is capable to produce accurate steep non breaking focused waves also over constant water depth.
Figure 9. Time series recorded at the focal point \((x_f = 198.40\, m)\) from the wave maker, the thicker line corresponds to the experimental free surface elevation whereas the finer line is the data recorded in the numerical model.

Figure 10. Time history at the focal point \((x_f = 140\, m)\) from the wave generation, the numerical result is represented by the blue line while the experimental one is the red line.

The second validation test corresponds to a breaking focused wave case, the procedure is repeated and the control signal is improved by means of the SCM within the NWT. Following the guidelines given at Fernández et al., (2014), the correction steps are developed in a position upstream respect to the focal point, the ratio between the position upstream and the considered wavelength of the focused wave was set, \(x_1/L = 0.5\) (\(x_1\) is the distance between the focal point and the position upstream and \(L\) is the wavelength of the focused wave). Using the model set up presented in Fig. 7, the wave parameters defining the test case are: \(x_f = 190\, m\), \(t_f = 64\, s\), \(A_f = 0.2\, Hz\), \(f_c = 0.204\, Hz\), \(G = 0.0025\pi\), \(N = 32\) and \(x_f = 14\, m\), ensuring \(x_1/L = 0.5\). The focal point is located over the slope, where the water depth is \(d = 2.81\, m\) and the water depth at the position upstream, \(d = 4.21\, m\), Fig. 7.
Figure 11. a) Free surface elevation recorded at x = 55 m from the wave maker, the red line represents the physical data while the blue one corresponds to the numerical result. b) Free surface elevation recorded at the focal point, xf = 190 m, for the breaking focused wave, the blue line is the second order theoretical wave profile whereas the red line is the data recorded during the physical experiments.

The comparison between the experiments and numerical simulation at 55m is shown in Fig. 11a. At this point, x = 55 m, a wave height of \( H = 1.66 \) m with a crest of 1.16 m is reported. It is important to mention that here the agreement between the physical and the numerical data is quite satisfying nevertheless the numerical model slightly underestimates the extreme values, like the main wave crest located at \( t = 35 \) s and the next wave trough, \( t = 37 \) s. The comparison between the experimental and the theoretical wave elevation at the focal point is given (measurements in the NWT were not feasible due to wave breaking at this location) in Fig. 11b, here one can see a wave crest height of 1.16 m with a total wave height of \( H = 1.84 \) m (red line), that will break at around x = 197 m, immediately before the structure. Although the agreement between profiles is lower with respect to the previous cases (the experimental result is steeper, higher and asymmetric respect to the theoretical one), the energy is concentrated at the predefined location. The reported differences are due to the target wave profile located upstream is back propagated by \( x_1 \) meters from the focusing location by means of linear theory; this means that the nonlinearities along this distance are not included in the control signal, leading to the reported deviations. Furthermore, as this case is a breaking wave test, the target wave profile to be
employed should be steeper and more nonlinear, otherwise the process is forced to be second order and the wave breaking will be altered, this is also one of the reasons for differences between wave signals (Fig. 11b).

CONCLUSIONS

The design of safe and economic offshore and coastal structures requires knowledge of extreme wave events. Thus, it is necessary to develop accurate laboratory techniques for generating this kind of wave sequences. Among the different methodologies, wave focusing by means of wave-wave interaction is one option to generate extreme waves in laboratory flumes. However, most of the existing techniques use linear wave theory to conduct the transformation from the target location back to the position of the wave maker. The nonlinear effects due to wave-wave interaction are not included in the wave maker driving signal, leading this to shifts at the focusing location. In order to take into account these nonlinear effects and to avoid differences at the focal point, a Self Correcting Method (SCM) is developed to obtain more accurate extreme wave events at a predefined location. The methodology is tested in a Numerical Wave Tank (NWT) and verified in the GWK. It is important to mention that by the assumption of linear backwards transformation from the target location to the wave maker, there will be always a shift at the focal point (between the theoretical and the recorded focused wave), irrespective of the theory employed for the wave paddle displacements (linear or second order wave board motions). The proposed SCM allows us to avoid these shifts by means of correction steps. The required control signal to generate the focused wave at the desired point is corrected stepwise by means of the differences (in terms of spectral wave phases and amplitudes) between the target and the recorded wave signal at the focal point. The method also removes the spurious free sub- and super harmonics components generated by the assumption of linear paddle displacements. Moreover, the method does not modify the wave focusing process and the high order nonlinearity of the wave profile is not altered, as shown by different test cases as summarised below.

First, a sensitivity analysis of the method was performed, i.e. its performance in terms of the correction scheme and the target wave profile was investigated. The results of this analysis are summarised as follows. If one uses a linear wave profile as a target, a phase correction scheme gives better results than combined phase and amplitude corrections; whereas when a second order target wave elevation is used, the differences between phase and phase and amplitude correction scheme can be neglected, i.e., the same correlation coefficient is reached after applying the same number of correction steps. This means that for an optimum performance of the methodology, the target wave profile to be employed should be realistic.

The SCM was proven to hold a good performance for fully reflective structures and the combination of both (i.e. focused wave tests with a composite breakwater). The capability of the SCM for generating breaking and non breaking waves (both for constant as well as variable water depth) was successfully validated in the laboratory (GWK). The performance of the SCM coupled with a NWT has been proven for a wide variety of scenarios. Hence, it will be implemented in the wave generation routines of the GWK in order to generate focused waves.

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