Essays on Nonlinear and Explosive Time Series – With Applications to Financial Markets

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Kurzfassung

In den letzten Jahrzehnten wurde eine Vielzahl von Ansätzen entwickelt, um Nichtlinearitäten in Zeitreihenmodelle zu integrieren. Motiviert werden diese Ansätze durch empirische Beobachtungen wie Blasen, Rezessionen oder Politikwechsel. Weiterhin implizieren viele ökonomische Theorien nichtlineare Zusammenhänge. Diese Arbeit enthält Beiträge zur Spezifikation dieser Zeitreihenmodelle. Dabei werden zwei eng verzahnte Literaturstränge betrachtet: nichtlineare Regimewechselmodelle und die Schätzung zeitabhängiger Parameter.

In Kapitel 2 wird eine bootstrap-basierte Version des Spezifikationstests von Cox vorgestellt, um eine Entscheidung zwischen dem exponentiellen Smooth Transition Autoregressive Modell (STAR) und dem Markov Switching Modell treffen zu können. Beide Modelle werden häufig genutzt, um reale Wechselkurse zu modellieren. Wir zeigen, dass der Test gute Eigenschaften in endlichen Stichproben aufweist. Weiterhin wird der Test auf 24 reale Wechselkurse angewendet, um eine Modellempfehlung aussprechen und die dominierenden Einflüsse bewerten zu können. In Kapitel 3 wird ein einfaches Prozedere vorgestellt, um zwischen verschiedenen Übergangsfunktionen in nichtlinearen autoregressiven Modellen zu unterscheiden. Der Ansatz basiert komplett auf Hilfsregressionen von Einheitswurzeltests und nutzt Informationskriterien zur Modellselektion. Monte-Carlo-Simulationen zeigen, dass der Ansatz in realistischen Szenarien gut funktioniert. Zwei Anwendungen (S&P500 Preis-Gewinn-Verhältnis und US-Zinsspanne) verdeutlichen die empirische Relevanz. Kapitel 4 betrachtet Linearitätstests gegen STAR Modelle unter Berücksichtigung eines potentiellen deterministischen Trends in den Daten. Linearitätstests sind ein elementarer Schritt zur Modellbildung, besonders bei der Frage, ob ein komplexes nichtlineares Modell angemessen ist. Im Gegensatz zu den Ergebnissen in Zhang (2012) zeigen wir, dass Linearitätstests in diesem Modellrahmen zu nützlichen Resultaten führen.

Kapitel 5 steuert einen umfassenden Monte-Carlo-Vergleich zwischen verschiedenen Verfahren bei, welche den Schätzbias in autoregressiven Modellen korrigieren. Wir betrachten stationäre, nicht-stationäre und mild explosive Szenarien. Unsere Ergebnisse zeigen, dass ein Schätzer beruhend auf indirekter Inferenz die ausgewogensten Eigenschaften aufweist. Eine empirische Anwendung auf die US-Verschuldungsquote unterstreicht die Ergebnisse. Kapitel 6 liefert empirische Evidenz für zeitvariierende Persistenz im S&P500 Preis-Dividenden-Verhältnis. Die Persistenz verhält sich prozyklisch und ist abhängig von volkswirtschaftlichen Fundamentalwerten. Neben erwarteter Inflation sind der Zustand des Bankensektors sowie die Verbraucherstimmung wichtige Indikatoren. In Übereinstimmung mit dem Fed-Modell finden wir einen negativen Zusammenhang zu Anleiherenditen. Außerdem sind die Resultate in Einklang mit einem heterogenen Agentenmodell zur Bewertung von Wertpapieren.

Schlagwörter: Bias Korrektur, explosives Verhalten, Nichtlinearität, Modellselektion, Persistenz, Spezifikationstests

Short summary

In the last decades many approaches have been proposed to include nonlinearities in time series models. The approaches are motivated by the empirical observation of structural instabilities caused by bubbles, recessions or policy changes. Moreover, economic theory often implies a nonlinear relationship of variables. This thesis contributes to the specification of these time series models in a univariate framework. Two closely related strands of literature are considered: nonlinear regime switching models and time-varying parameter estimation.

Chapter 2 offers a bootstrap-based version of the Cox specification test for non-nested hypothesis to discriminate between exponential smooth transition autoregressive (STAR) and Markov switching models. Both models are commonly used for modeling real exchange rate dynamics. We show that the proposed test has good size and power properties in finite samples. In an application, we analyze 24 real exchange rates to shed light on the question which model is more appropriate. This allows us to draw conclusions about the driving forces of real exchange rates. In Chapter 3 a simple specification procedure for the switching mechanism in nonlinear autoregressive models is provided. The approach entirely relies on OLS estimation and is based on auxiliary regressions of unit root tests. We use information criteria for the selection of the unknown transition function. Monte Carlo simulations reveal that the approach works well in practice. The procedure is applied to the S&P500 price-earnings ratio and an US interest rate spread. Chapter 4 considers linearity testing against STAR models when there is the potential that deterministic trends are present in the data. Testing for linearity is an elementary step in the modeling cycle and of great importance when it comes to nonlinear model building. In contrast to results recently reported in Zhang (2012), our findings show that linearity tests against STAR models lead to useful results in this framework.

Chapter 5 provides a comprehensive Monte Carlo comparison of different finite-sample bias-correction methods for autoregressive processes. We consider situations where the process is stationary, exhibits a unit root or is mildly explosive. Our findings suggest that the indirect inference approach has the most balanced properties in terms of bias and root mean squared error. An empirical application of the US Debt/GDP series underlines its usefulness. Chapter 6 provides empirical evidence for time-variation in the persistence of the S&P500 price-dividend ratio and shows that the persistence is pro-cyclical and related to macroeconomic fundamentals. Besides expected inflation, the main drivers are the condition of the banking sector and consumer sentiment. Consistent with the Fed model, we find persistence being negatively related to bond yields via expected inflation. In addition to that, the results are in line with a heterogenous asset pricing model.

Keywords: Bias correction, explosive behavior, non-linearity, model selection, persistence, specification testing

Contents

Chapter 1

Introduction

Introduction

Until the end of the 1980s, linear time series models dominated in applied econometrics. One essential characteristic of these models is that the impact of previous observations is constant over time. Although this way of modeling has the advantage of simplicity, limits are easily reached. Examples for such limits are structural instabilities caused by bubbles, recessions or around policy changes – time periods where economic or financial time series behave differently. Moreover, economic theory often implies non-linear relationship of variables. For instance, market imperfections such as transportation costs or taxes lead to a non-linear behavior of real exchange rates and interest rate spreads. In the last two decades many approaches have been proposed to include these events and theoretical considerations into modeling economic time series. This thesis contributes to the specification of these time series models in a univariate framework. Two closely related strands of literature are considered (see Granger, 2008): nonlinear regime switching models and time-varying parameter estimation.

Nonlinear regime switching models have become increasingly popular in applied econometrics since the works of Hamilton (1989), Tong (1990) and Teräsvirta (1994). The basic idea of these models is the connection of two or more linear models with a transition function. This transition function can be the indicator function, so that the process shifts abruptly from one regime to another if a pre-specified threshold is exceeded. It can be a continuous function in the unit interval, which allows for an infinite amount of combinations of the regimes. Whereas the latter case results in a so called smooth transition autoregressive or STAR model, an abrupt change characterizes the class of threshold autoregressive or TAR models. The state might also depend on the outcome of an unobservable Markov chain (the Markov switching autoregressive or MSAR model). These models are not only able to connect different linear processes and thereby change the behavior (and persistence) over time; they also allow for local non-stationarity while maintaining global stationarity. Although the behavior seems relatively similar in this regard, the detailed dynamics driving these models differ: endogenous versus exogenous regime switching and visible versus latent transition variables lead to completely different economic interpretations. A recent reference for nonlinear modeling is Teräsvirta et al. (2010). The first three chapters of this work focus on the specification of the aforementioned nonlinear models.

Chapter 2 offers a bootstrap-based version of the Cox specification test for non-nested hypothesis to discriminate between exponential STAR and MSAR models. Both models are commonly used for modeling real exchange rate dynamics. We show that the proposed test has good size and power properties in finite samples. In an application, we analyze several major real exchange rates to shed light on the question of which model describes the data best. This allows us to draw conclusions about the driving forces of real exchange rates.

In Chapter 3 a simple specification procedure for the switching mechanism in nonlinear autoregressive models is provided. The approach entirely relies on OLS estimation and is based on auxiliary regressions of unit root tests. Therefore, it is applicable to a variety of transition functions. We use standard information criteria for the selection of the unknown transition function. Monte Carlo simulations reveal that the approach works well in practice. Empirical applications to the S&P500 price-earnings ratio and an US interest rate spread highlight the limits and merits of the suggested technique. In contrast to other procedures, complicated and computer-intensive estimation of the candidate models is not necessary.

Chapter 4 considers linearity testing against STAR models when deterministic trends are potentially present in the data. Testing for linearity is an elementary step in the modeling cycle and of great importance when it comes to nonlinear model building. In contrast to results recently reported in Zhang (2012), our findings show that linearity tests against STAR models lead to useful results in this framework. Additionally, the power of the specification test is analyzed in empirical settings.

The second strand of literature to which this thesis contributes is the analysis of time-varying persistence in a rolling window scheme. The work by Stock and Watson (1996) tests against the instability of parameters using a comprehensive data set. These authors find that the persistence of most economic time series is instable over time. Rolling window estimation is a simple setup to address this stylized fact. A specific model is estimated several times using only some portion, i.e. a window of the observations. Going from the very beginning to the end of the sample with this window leads to an estimation of the persistence of the series over time. Due to estimation uncertainty a perfectly stable persistence cannot be expected. However, a linear model is clearly misspecified if the persistence follows a dynamic path. An interesting issue is the change between stationarity, unit roots and explosiveness over time. In particular, mild explosiveness in time series has received some attention in the last two decades since the work of Phillips (1987). A recent and important contribution using rolling window estimation is the detection of bubbles, see Phillips et al. (2011). This improves the classic linear analysis which is not able to model events like the dot-com bubble or the recent financial crisis appropriately, where explosiveness instead of unit root behavior can be observed. Other popular examples are commodity and food prices. But the investigation of economic time series in a rolling window scheme is less straightforward as it seems in the first place. The high persistence of economic time series in general and the small number of observations per window lead to a serious estimation bias that needs to be addressed in applications. Another recent research question is the relation of time-varying persistence and the business cycle.

Chapter 5 provides a comprehensive Monte Carlo comparison of different finite-sample bias-correction methods for autoregressive processes. We consider classic situations where the process is either stationary or exhibits a unit root. Importantly, the case of mildly explosive behavior is studied as well. We compare the empirical performance of an indirect inference estimator (Phillips et al., 2011), a jackknife approach (Chambers, 2013), the approximately median-unbiased estimator by Roy and Fuller (2001) and the bootstrap-aided estimator by Kim (2003). Our findings suggest that the indirect inference approach offers a valuable alternative to other existing techniques. Its performance (measured by its bias and root mean squared error) is balanced and highly competitive across many different settings. A clear advantage is its applicability to mildly explosive processes. In an empirical application to a long annual US Debt/GDP series we consider rolling window estimation of autoregressive models. We find substantial evidence for time-varying persistence and periods of explosiveness during the Civil War and World War II. Further applications to commodity and interest rate series are considered as well.

Chapter 6 provides empirical evidence for pronounced time-variation in the persistence of the S&P500 price-dividend ratio. It addresses the question whether these movements can be directly related to cyclical macroeconomic activity. A flexible econometric framework is applied to study the role of 138 variables including survey data. We handle the high dimensional data set by model averaging techniques. The persistence is found to be pro-cyclical and related to macroeconomic fundamentals. Besides expected inflation, the main drivers are the condition of the banking sector and consumer sentiment. In general, favorable economic conditions are tied to high levels of persistence and vice versa. Consistent with the Fed model, we find persistence being negatively related to bond yields via expected inflation. Moreover, the results are consistent with a heterogenous asset pricing model, where a positive economic outlook leads to an increased fraction of chartists through lowered risk premia.

Chapter 2

The dynamics of real exchange rates – A reconsideration

The dynamics of real exchange rates – A reconsideration

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Chapter 3

A simple specification procedure for the transition function in persistent nonlinear time series models

A simple specification procedure for the transition function in persistent nonlinear time series models

Co-authored with Robinson Kruse and Philipp Sibbertsen.

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Chapter 4

On tests for linearity against STAR models with deterministic trends

On tests for linearity against STAR models with deterministic trends

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Chapter 5

Bias-corrected estimation in potentially mildly explosive autoregressive models

Bias-corrected estimation in potentially mildly explosive autoregressive models

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5.1 Introduction

Measuring and estimating the persistence of time series is a long standing issue in econometrics. The most common framework for assessing the persistence is the autoregressive model. But, a major practical problem is the inherent bias of the conventional OLS estimator. Its bias increases amongst two dimensions: a small sample size and a true autoregressive parameter in the vicinity of unity are disadvantageous. Given a relatively small sample size, it is a complicated task to estimate the persistence if the process is (i) either stationary, but highly persistent, (ii) exhibits a unit root or (iii) is mildly explosive. As we argue, these situations are likely to occur in practice.

In economics, it is a well established fact that most time series are characterized by high persistence or even stochastic trends, see e.g. Nelson and Plosser (1982) and Schotman and van Dijk (1991). Another important empirical issue is the instability of parameters, which is often observed and documented (see e.g. Stock and Watson, 1996). During the past decade, a literature on structural changes in persistence emerged, see e.g. Chong (2001), Kim (2000), Leybourne et al. (2007) and Harvey et al. (2006) amongst many others. In order to cope with potential time-variation in the parameters, users often apply the popular rolling window technique. Under these empirically relevant circumstances, the issue of unbiased and efficient estimation of persistence becomes particularly important: Typically, a relatively small window size is chosen. If a bubble or a crisis occurs in this particular window, some economic time series are likely to exhibit explosive behavior. Leading examples for time series with at least local explosive roots are stock prices (as caused by bubbles, see Diba and Grossman, 1988), price-dividend and price-earnings ratios (as caused by a dominant regime of chartist traders, see Lof, 2012), house and oil prices (due to speculation, see Homm and Breitung, 2012, Clark and Coggin, 2011 and Shi and Arora, 2012), hyperinflation (due to a collapse of a country's monetary system, see Casella, 1989), exchange rates (due to speculation, see van Norden, 1996 and Pavlidis et al., 2012) and the US Debt/GDP ratio (due to unsustainable fiscal policies, see Yoon, 2011) amongst others.

The complicated estimation of autoregressive processes in finite-samples sparked a fruitful area of research. Kendall (1954), Shaman and Stine (1988), Tjøstheim and Paulsen (1983), Tanaka (1984) and Abadir (1993) provide analytic derivations of asymptotic expansions which can be used for bias-correction. Approximately median-unbiased estimation is proposed in e.g. Andrews

(1993), Andrews and Chen (1994) and Roy and Fuller (2001). Restricted maximum likelihood estimation is considered in Cheang and Reinsel (2000). Bootstrap-based bias-correction procedures have been suggested by e.g. Hansen (1999) and Kim (2003). Recently, Engsted and Pedersen (2011) compare analytical bias formulas and bootstrapping for stationary VAR models. Indirect inference has been put forward in MacKinnon and Smith (1998) and Gouriéroux et al. (2000). Jackknifing based on Efron (1979) is recently studied in Chambers (2013). Importantly, we note that the main body of the literature focusses on stationary autoregressive models and on the unit root case while the case of (mildly) explosive behavior has received less attention.

This work compares the analytic median-bias-correction by Roy and Fuller (2001), the bootstrap technique by Kim (2003) and the Jacknife approach by Chambers (2013) to the indirect inference approach by Phillips et al. (2011), who propose a technique for autoregressive processes, based on the work of MacKinnon and Smith (1998) and Gouriéroux et al. (2000). Indirect inference estimators to correct the small sample bias have a long tradition, e.g. see Gouriéroux et al. (1993) and Smith (1993). In a recent contribution, Gouriéroux et al. (2010) extend this principle to dynamic panel data models. The indirect inference estimator allows for explosiveness in addition to highly persistent and unit root behavior, see also Phillips (2012) for a recent contribution on its limit theory. Most competing methods rule out explosive behavior by construction (i.e. Roy and Fuller, 2001 and Kim, 2003). This feature renders the indirect inference estimation approach to autoregressive models particularly attractive. However, the finite-sample properties of the indirect inference estimator are not fully explored and a comprehensive comparison to other popular and successful bias-correction techniques has not been conducted yet.

In our Monte Carlo study, we consider various sample sizes, normal and fat-tailed innovations, ARCH disturbances and misspecification of the autoregressive lag structure. Furthermore, we also study the case where a linear deterministic trend is included in the autoregressive model. We evaluate the performance of the estimators by means of bias and root mean squared errors (RMSE). Our results suggest that all procedures lead to a substantial bias-reduction in most non-explosive cases. The best procedure in terms of bias-reduction is the jackknife, but comes with the costs of an increase in the variance. The indirect inference estimator provides almost the same level of bias-reduction with a remarkably low variance.

We provide a detailed empirical application to a long annual US Debt/GDP ratio from 1791-2011, where we use rolling window estimation to investigate potential instabilities. Our results suggest that persistence is characterized by strong time-variation. Episodes of stationarity, unit root and explosive behavior are observed. These episodes are related to major wars, peace movements during the Sixties and Seventies, and recent activities in the aftermath of 9/11. Moreover, we consider three further applications to Oil prices, Gold prices and the spread between long-term interest rates in Germany and Greece. All applications stress the importance of bias-correction. In addition, accounting for locally explosive behavior is relevant in all cases.

The paper is organized as follows. Section 5.2 briefly describes the different estimation techniques. Our simulation results are presented in Section 5.3. The empirical applications are located in Section 5.4 while conclusions are drawn in Section 5.5. The Appendix contains further simulation results.

5.2 Bias-correction procedures

Point of departure is the inherent bias of the OLS estimator. In order to illustrate the problem, we simulate the empirical performance of the OLS estimator. Therefore, we focus on finite samples and the possibility of mild explosiveness in a simple autoregressive framework:

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t \, . \tag{5.1}$$

We consider the cases of stationarity and unit roots, i.e. $|\rho| < 1$ and $\rho = 1$, and the case where ρ satisfies $\rho = 1 + c/k_T$, with $c > 0$ and k_T being a sequence tending to infinity such that $k_T = o(T)$ as $T \to \infty$. In the latter case, the autoregressive parameter is local-to-unity in the sense that $\rho \to 1$ as $T \to \infty$. For finite T (as considered in this work), ρ deviates moderately from unity. Asymptotic theory for this case is developed in Phillips and Magdalinos (2007).

The left panel of Figure 5.1 shows the AR(1) case as in equation (5.1) for four different sample sizes, i.e. $T = \{30, 60, 120, 240\}$. The true autoregressive parameter ρ (on the x-axis) ranges from 0.6 to 1.2 which measures the persistence of the process. The bias is given on the y-axis. The results confirm the theoretical finding that the bias depends on the true value of the autoregressive parameter. The smaller the sample size, the more severe is the bias. The vicinity of unity is the region where the bias is strongest. Furthermore, it can be seen that the bias reduces for explosive processes and approaches zero at some point, but that the estimation of mildly explosive processes is still heavily biased.

As expected, the bias problem persists if we consider the AR(2) process, i.e.

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \, . \tag{5.2}$$

Since our primary interest is the persistance of the time series, we work with an alternative representation which gathers the persistence in the parameter ρ:

$$y_t = \mu + \rho y_{t-1} + \beta \Delta y_{t-1} + \varepsilon_t \, , \tag{5.3}$$

where $\rho = \phi_1 + \phi_2$ and $\beta = -\phi_2$. The usefulness of this approach stems from the fact that a direct relationship to the cumulative impulse response $(1/(1-\rho))$ exists (for stationary autoregressive processes). Moreover, it is also directly connected to the spectrum at frequency zero which measures the low-frequency autocovariance. It is given by $var(\varepsilon_t)/(1-\rho)^2$.[1] The right panel

[1] Alternative measures of persistence are the largest autoregressive root, see Stock (1991) for its median-unbiased estimation, and the half life of a unit shock, see Rossi (2005).

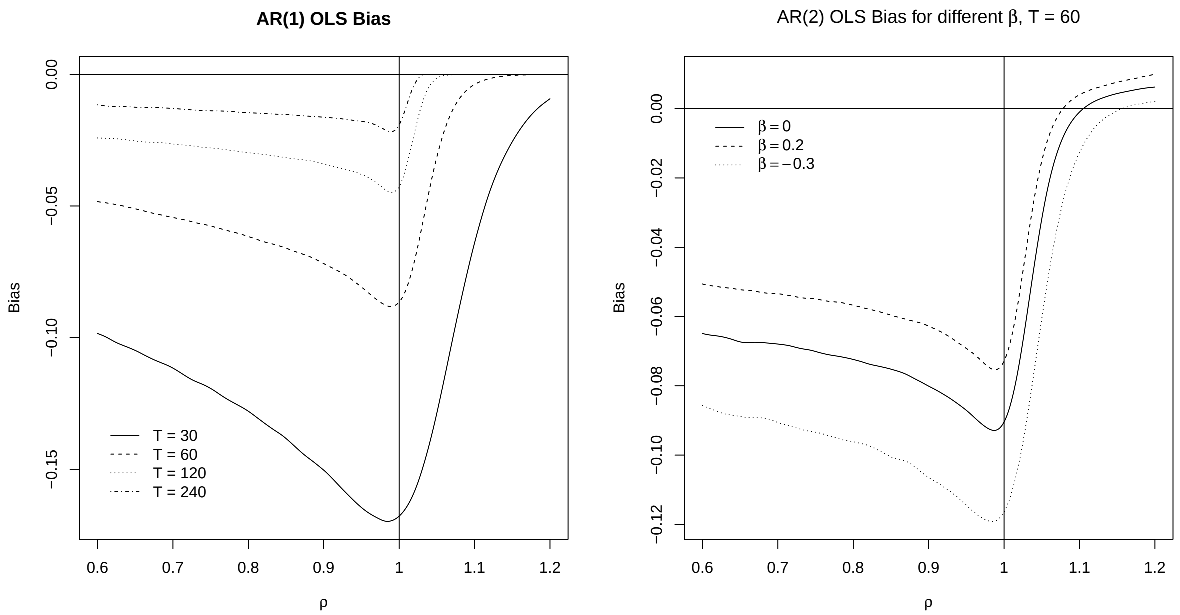

Figure 5.1: OLS Bias for different values of ρ, β and sample sizes for AR(1) and AR(2) processes (constant included).

of Figure 5.1 shows the bias for ρ for three different values of β: -0.2, 0 and 0.3. The bias depends substantially on the value of β. Positive values decrease the bias and vice versa. A comparison between the AR(1) case for $T = 60$ and the AR(2) case for $T = 60$ and $\beta = 0$ shows that the estimation of an additional, but unnecessary, parameter increases the bias slightly. These results motivate the development of bias-correction techniques. Four different methods are briefly discussed in the following.

5.2.1 Roy-Fuller median-unbiased estimator

The first bias-correction method we consider is the approximately median-unbiased[2] Roy-Fuller estimator which has been proven to be of empirical usefulness (see Kim, 2003). The Roy and Fuller (2001) estimator provides an analytic modification of the OLS estimator for the persistence parameter ρ. Let $\widehat{\rho}$ denote the OLS estimator for ρ in $\bar{y}_t = \rho\bar{y}_{t-1} + \beta\Delta\bar{y}_{t-1} + \varepsilon_t$, where $\bar{y}_t$ is the previously de-meaned time series y_t, i.e., $\bar{y}_t \equiv y_t - (1/T)\sum_{t=1}^{T} y_t$. Furthermore, $\widehat{\sigma}$ denotes the standard error of $\widehat{\rho}$ and $\widehat{\lambda} = (\widehat{\rho} - 1)/\widehat{\sigma}$ is the usual Dickey and Fuller (1979) unit root test statistic. The Roy-Fuller estimator[3] $\widehat{\rho}^{RF}$ is now given by $\widehat{\rho}^{RF} = \min(\widetilde{\rho}, 1)$, where

$$\widetilde{\rho} = \widehat{\rho} + C(\widehat{\lambda})\widehat{\sigma}\,.$$

[2] An estimator $\widetilde{\rho}$ for ρ is said to be median-unbiased if $P(\widetilde{\rho} \geq \rho) \geq 1/2$ and $P(\widetilde{\rho} \leq \rho) \geq 1/2$.

[3] The original Roy-Fuller estimator corrects positive and negative autocorrelation bias in AR(p) processes. In this work only substantial positive autocorrelations of AR(1) and AR(2) processes are considered. The given formulas are simplified for this case.

Related to the asymptotic bias of the OLS estimator, the function $C(\widehat{\lambda})$ is constructed to make $\widetilde{\rho}$ approximately median-unbiased at $\rho = 1$. The function is given by

$$C(\widehat{\lambda}) = \begin{cases} 0, & \text{if } \widehat{\lambda} \leq -\sqrt{2T} \\ T^{-1}\widehat{\lambda} - 2\widehat{\lambda}^{-1}, & \text{if } -\sqrt{2T} < \widehat{\lambda} \leq -K \\ T^{-1}\widehat{\lambda} - 2\left[\widehat{\lambda} + k(\widehat{\lambda} + K)\right]^{-1}, & \text{if } -K < \widehat{\lambda} \leq \lambda_{0.5} \\ -\lambda_{0.5} + d_n(\widehat{\lambda} - \lambda_{0.5}), & \text{if } \widehat{\lambda} > \lambda_{0.5}, \end{cases}$$

where $\lambda_{0.5} = -1.57$ denotes the median of the limiting distribution of $\widehat{\lambda}$ if $\rho = 1$ and data is demeaned prior to testing, K is some fixed number (set to 5), d_n is a slope parameter (set to 0.1111) and $k = \left(2 - T^{-1}\lambda_{0.5}^2\right)\left[(1 + T^{-1})\lambda_{0.5}(\lambda_{0.5} - K)\right]^{-1}$. The function $C(\widehat{\lambda})$ accounts for different asymptotics and convergence rates for different persistence levels of ρ. Further details can be found in Roy and Fuller (2001). After the bias-corrected estimation of ρ the other parameters of the process, μ in the AR(1) case given in equation (5.1) and μ, ϕ_1 and ϕ_2 in the AR(2) case given in equation (5.2), can be estimated subject to the restriction $\rho = \widehat{\rho}^{RF}$.

5.2.2 Bootstrap bias-corrected estimator

The second competitor is the bootstrap-based procedure by Kim (2003). This method involves the generation of a large number of pseudo-data sets using the estimated coefficients and re-sampled residuals. Pseudo-data sets shall resemble the dependence structure that is present in the original data set. The bias of the OLS estimator can be estimated as follows: Estimate the model via OLS and obtain the estimates $\widehat{\theta} = \left(\widehat{\mu}, \widehat{\rho}, \widehat{\beta}\right)'$. Generate a pseudo-data set $\{y_t^b\}_{t=1}^T$ based on these estimates according to

$$y_t^b = \widehat{\mu} + \widehat{\rho} y_{t-1}^b + \widehat{\beta} \Delta y_{t-1}^b + u_t^b,$$

where u_t^b is a random draw with replacement from the OLS residuals $\{\widehat{u}_t\}_{t=1}^T$. B sets of pseudo-data are generated. Each pseudo-data set gives a bootstrap parameter estimate $\widehat{\theta}^b = (\widehat{\mu}^b, \widehat{\rho}^b, \widehat{\beta}^b)'$ by estimating the model $y_t^b = \mu + \rho y_{t-1}^b + \beta \Delta y_{t-1}^b + v_t$, $b = 1, \ldots, B$. We obtain the sequence $\{\widehat{\theta}^b\}_{b=1}^B$ and the average bias of $\widehat{\theta}^b$ is estimated as $\widetilde{\theta} - \widehat{\theta}$, where $\widetilde{\theta}$ is the sample average of $\{\widehat{\theta}^b\}_{i=1}^B$, i.e.

$$\widetilde{\theta} \equiv \frac{1}{B} \sum_{b=1}^{B} \widehat{\theta}^b.$$

Using this bootstrap-based estimator for the bias, a bias-correction for $\widehat{\theta}$ can be directly obtained via

$$\widehat{\theta}^{KIM} = \widehat{\theta} - \left(\widetilde{\theta} - \widehat{\theta}\right) = 2\widehat{\theta} - \widetilde{\theta}.$$

If $\widehat{\theta}^{KIM}$ does not fulfill the stationarity condition $\widehat{\rho} < 1$, the iterative filter

$$\widehat{\theta}_i^{KIM} = \widehat{\theta} - \prod_{j=1}^{i} (1 - 0.01i)\left(\widetilde{\theta} - \widehat{\theta}\right), \; i = 1, 2, 3, \ldots$$

is applied until $\widehat{\rho} < 1$ is ensured. Denote by $\bar{i}$ the index where the iteration stops. Thus, $\widehat{\theta}^{KIM} = \widehat{\theta}_{\bar{i}}^{KIM}$. For further details regarding this estimator, the interested reader is referred to Kim (2003). This estimator computes the OLS estimation bias for a process with parameter values $\widehat{\theta}$ and uses this bias as approximation for the true bias of $\widehat{\theta}$. In contrast to the former procedure all parameters of the model are estimated simultaneously.

5.2.3 Indirect inference estimator

We now turn to a simulation-based estimator relying on the concept of indirect inference. The following exposition draws heavily from Phillips et al. (2011). The basic idea of this simulation-based estimator is to consider initially the OLS estimator labeled as $\widehat{\rho}$. Consider a set of simulated series with AR(1) coefficient equal to some ρ, i.e. $\{y_t^h(\rho)\}_{h=1}^H$, $h = 1, 2, \ldots, H$. H denotes the total number of available simulation paths.[4] For each single $h \in 1, 2, ..., H$, we obtain an OLS estimate denoted as $\widehat{\rho}^h(\rho)$. The indirect inference estimator (which belongs to the class of extremum estimators) is given by

$$\widehat{\rho}_H^{II} = \arg\min_{\rho\in\Theta} \left\| \widehat{\rho} - \frac{1}{H}\sum_{h=1}^{H} \widehat{\rho}^h(\rho) \right\| ,$$

where Θ is a compact parameter space and $\|\cdot\|$ is a distance metric. For $H \to \infty$ one obtains

$$\widehat{\rho}^{II} = \arg\min_{\rho\in\Theta} \left\| \widehat{\rho} - q(\rho) \right\| ,$$

where $q(\rho) = E\left(\widehat{\rho}^h(\rho)\right)$ is the so-called binding function. Given invertibility of q, the indirect inference estimator results as

$$\widehat{\rho}^{II} = q^{-1}(\widehat{\rho}) .$$

So the idea of this estimator is to have a grid of possible true values for ρ and the corresponding average OLS estimates $(1/H)\sum_{h=1}^{H} \widehat{\rho}^h(\rho)$. The estimate $\widehat{\rho}$ is compared to the average OLS estimates. $\widehat{\rho}^{II}$ is now the value which leads to the average OLS estimate with the minimal distance to $\widehat{\rho}$. The finite-sample bias-correction stems from the simulation of $q(\rho)$. Precision is naturally expected to be increased with rising H, although it can be computationally costly. Nonetheless, the binding function has to be simulated only once and can thus be applied afterwards without any further simulation or re-sampling. This is a fundamental difference to the bootstrap approach. Furthermore, the indirect inference estimator is applicable even for mildly explosive processes. This is not the case for the Roy-Fuller and the bootstrap-based estimator by Kim (2003). Estimation of all other parameters of the process can be done analogously to the Roy-Fuller estimator.

[4] In order to generate $\{y_t^h(\rho)\}_{h=1}^H$, we assume normal errors in the following. The importance of this assumption is investigated later in Section 5.3.2.

5.2.4 Jackknife estimators

In general, Bao and Ullah (2007) show that the expected value of the OLS estimator $\widehat{\theta} = \left(\widehat{\mu}, \widehat{\rho}, \widehat{\beta}\right)$ has the form

$$E\left(\widehat{\theta}\right) = \theta + \frac{a}{T} + O\left(T^{-2}\right).$$

Shaman and Stine (1988) show that the vector $a = -(1+3\rho)$ for $\widehat{\rho}$ in the AR(1) process and $a = -(1+\rho, 2-4\beta)'$ for $\left(\widehat{\rho}, \widehat{\beta}\right)'$ in the AR(2) process. If the full sample y is divided into m sub-samples Y_j of same length l, $j = 1, \ldots, m$, and $\widehat{\theta}^j$ is the OLS estimate for θ in sub-sample Y_j then the jackknife statistic

$$\widehat{\theta}^J = \left(\frac{T}{T-l}\right)\widehat{\theta} - \left(\frac{l}{T-l}\right)\widetilde{\theta}$$

with $\widetilde{\theta} = \frac{1}{m}\sum_{j=1}^{m}\widehat{\theta}^j$ satisfies $E(\widehat{\theta}^J) = \theta + O(T^{-2})$ and is thus able to reduce the bias. Chambers (2013) proposes and compares various jackknife techniques to reduce the small sample bias. In this work we focus on one of the methods in the comparison of Chambers (2013): the non-overlapping sub-samples jackknife. This estimator has good bias-correction properties without the considerable increase of the RMSE of higher order jackknife estimators. Here the time series is splitted in m non-overlapping sub-samples,

$$Y_j = (y_{[(j-1)T/m+1]}, \ldots, y_{[jT/m]})', \qquad j = 1, \ldots, m.$$

In the following we work with $m = 2$ sub-samples, because the procedure with this particular choice of m has the best bias-correction properties according to Chambers (2013), see his Table 1. This simplifies the jackknife statistic to

$$\widehat{\theta}^J = 2\widehat{\theta} - \widetilde{\theta}.$$

The intuition behind this approach is almost the same as in the bootstrap approach of Kim (2003). The average bias in the sub-samples is higher because of the smaller sample size and therefore a bias-reduction is induced. The difference to the bootstrap procedure is that the average bias is calculated on sub-samples of the true process and not on pseudo-data. In the following we abbreviate this procedure as J(2). It should be noted that the introduced jackknife procedure is only valid as long as the process is stationary, see Chambers (2013). The unit root case is tackled in Chambers and Kyriacou (2012). To our best knowledge, the (mildly) explosive case has not been under consideration so far.

5.3 Finite-sample properties

In this section we investigate the properties of various bias-correction methods via Monte Carlo simulation. The foci of this analysis are the bias-reduction and the RMSE of these estimators for AR(1) and AR(2) models in various settings. The simulation setup is as follows: We consider autoregressive models of the structure

$$y_t = \mu + \rho y_{t-1} + \beta \Delta y_{t-1} + \varepsilon_t,$$

with $\varepsilon_t \sim N(0,1)$. Non-normal and heteroscedastic errors are studied in Section 5.3.2. The case of a linear deterministic trend in addition to the intercept μ is located in Section 5.3.3. The autoregressive parameter ρ measures the persistence of y_t and takes values $\rho = \{0.85, 0.9, 0.95, 0.99, 1, 1.01, 1.02\}$. The considered samples sizes are $T = \{30, 60, 120, 240\}$. The mildly autoregressive process is characterized by $\rho = 1 + \frac{c}{T^\gamma}$ with $0 < \gamma < 1$ and $c > 0$. Following Breitung and Kruse (2013)[5], $\gamma = 0.75$ corresponds to $c = \{0.13, 0.22, 0.36, 0.61\}$ and $c = \{0.26, 0.43, 0.73, 1.22\}$ for $\rho = 1.01$ and for $\rho = 1.02$, respectively. Thus, the degree of explosiveness is in fact very mild in our setup. The intercept μ is set equal to zero without loss of generality. If the data is generated by an AR(2) process, β is set to $\beta = \{-0.2, 0.3\}$. The number of Monte Carlo repetitions is set to 10,000 for each single experiment. The number of bootstrap repetitions for the procedure of Kim (2003) is set to 499. The binding function for the indirect inference estimator was simulated with $\rho = \{0.60, 0.61, \dots, 1.20\}$ and $\beta = \{-0.90, -0.89, \dots, 0.90\}$. The number of simulation paths H equals 10,000 in the AR(1) case and $H = 100$ for AR(2) models. In an unreported comparison between different values for H, we find that there are only marginal changes in the results as long as $H \geq 100$. That means that the indirect inference procedure can be applied at low computational costs with negligible loss of precision.

Summary results are reported in Section 5.3.5. Detailed results are reported in Tables 5.1–5.7. Table 5.1 shows the results for the case where the estimated model coincides with the true DGP which is an AR(1). The next subsection discusses the performance for GARCH and heavy-tailed innovations (see Tables 5.2 and 5.6). Results for processes with deterministic trends are given in Table 5.3. Finally Tables 5.4, 5.5 and 5.7 contain results for correctly specified AR(2) models, under-fitted AR(2) models and over-fitted AR(1) models.

5.3.1 First-order autoregressive model with i.i.d. Normal innovations

Our benchmark case is the AR(1) process with a constant as in equation (5.1). The left-hand side of Table 5.1 provides the average bias of the OLS estimator and all discussed bias-correction procedures. Every procedure leads to a substantial bias-reduction compared to the OLS estimator. For $T = 60$, the jackknife estimator J(2) has the best bias-correction capabilities in nearly all cases. The indirect inference estimator is second-best followed by the approximately median-unbiased Roy-Fuller estimator and the bootstrap-based approach (Kim). In smaller samples ($T = 30$), the jackknife is still the best procedure for unit root and explosive cases, but the results for stationary autoregressive models are mixed. In larger samples ($T = 120$), the indirect inference estimator is the best method for stationary processes whereas the jackknife wins for $\rho = 1$ and $\rho = 1.01$. Interestingly, for $\rho = 1.02$ the bias of the J(2) approach changes its sign and

[5] Breitung and Kruse (2013) consider values for c in the range of one half to five when simulating the empirical performance of Chow-type tests for bursting bubbles.

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.135	0.012	**0.000**	-0.028	-0.002	0.201	**0.145**	0.165	0.162	0.218
	0.90	-0.148	**-0.002**	-0.018	-0.044	-0.010	0.206	**0.141**	0.153	0.155	0.217
	0.95	-0.162	**-0.019**	-0.043	-0.067	-0.020	0.213	**0.135**	0.143	0.153	0.217
	0.99	-0.168	-0.029	-0.065	-0.089	**-0.021**	0.214	**0.128**	0.138	0.155	0.214
	1.00	-0.166	-0.030	-0.069	-0.094	**-0.018**	0.212	**0.125**	0.136	0.156	0.211
	1.01	-0.163	-0.030	-	-	**-0.014**	0.209	**0.122**	-	-	0.207
	1.02	-0.157	-0.028	-	-	**-0.009**	0.204	**0.117**	-	-	0.203
60	0.85	-0.066	0.004	0.007	-0.006	**0.003**	0.113	**0.097**	0.104	0.098	0.123
	0.90	-0.072	**0.001**	0.004	-0.011	0.002	0.113	0.093	0.095	**0.091**	0.121
	0.95	-0.081	-0.006	-0.010	-0.023	**-0.003**	0.114	0.084	**0.081**	0.083	0.119
	0.99	-0.088	-0.016	-0.029	-0.041	**-0.007**	0.114	0.072	**0.070**	0.078	0.113
	1.00	-0.086	-0.016	-0.033	-0.046	**-0.005**	0.111	0.068	**0.068**	0.078	0.111
	1.01	-0.081	-0.015	-	-	**0.000**	0.106	**0.063**	-	-	0.107
	1.02	-0.071	-0.012	-	-	**0.004**	0.099	**0.058**	-	-	0.101
120	0.85	-0.032	**0.000**	0.001	-0.002	0.002	0.065	0.059	0.061	**0.059**	0.069
	0.90	-0.034	**0.001**	0.003	-0.002	0.002	0.062	0.055	0.057	**0.054**	0.066
	0.95	-0.038	**0.000**	0.003	-0.005	0.002	0.059	0.048	0.048	**0.046**	0.063
	0.99	-0.045	-0.007	-0.012	-0.018	**-0.003**	0.059	0.038	**0.037**	0.040	0.059
	1.00	-0.044	-0.009	-0.017	-0.023	**-0.002**	0.058	0.036	**0.036**	0.040	0.057
	1.01	-0.036	-0.006	-	-	**0.003**	0.051	**0.031**	-	-	0.052
	1.02	-0.021	**-0.002**	-	-	0.009	0.037	**0.023**	-	-	0.043
240	0.85	-0.015	0.000	0.000	**0.000**	0.001	0.040	0.038	0.038	**0.038**	0.042
	0.90	-0.016	0.000	0.001	**0.000**	0.001	0.036	0.033	0.034	**0.033**	0.038
	0.95	-0.018	0.001	0.002	**0.000**	0.001	0.032	0.028	0.029	**0.027**	0.034
	0.99	-0.022	-0.002	-0.003	-0.006	**-0.001**	0.030	0.020	**0.019**	0.020	0.030
	1.00	-0.022	-0.004	-0.008	-0.011	**-0.001**	0.029	0.018	**0.017**	0.020	0.029
	1.01	-0.011	**-0.001**	-	-	0.005	0.019	**0.011**	-	-	0.022
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.017

Table 5.1: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant included).

yields a very small, but positive bias. While this behavior may not seem striking at first sight, it becomes more important when $\rho > 1.02$ (not reported). The higher ρ, the more obvious is the overcorrection even in small samples. For $T = 240$, the OLS bias is quite small and the need for bias-correction procedures becomes less important. Nevertheless, a reduction of the bias to levels very close to zero is possible with any method.

The second important statistic we investigate is the RMSE. It is reported at the right-hand side of Table 5.1. For $T = 60$, the bootstrap procedure has the highest RMSE reduction for stationary cases, the Roy-Fuller method for processes close to and at the unit root and the indirect inference estimator for explosive cases. All three techniques are highly competitive in terms of variance reduction whereas the J(2) causes an increase in the variance compared to the OLS estimator. This pattern remains the same for larger samples. For $T = 30$, the indirect inference estimator is always the best procedure in terms of the RMSE. This shows that the jackknife estimator provides the best bias-correction on average, but comes along with a fairly large variance. This result is in line with Chambers (2013) where only stationary autoregressive

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.141	0.007	-0.007	-0.035	**-0.007**	0.208	**0.147**	0.170	0.168	0.227
	0.90	-0.153	**-0.007**	-0.024	-0.051	-0.014	0.213	**0.144**	0.158	0.162	0.226
	0.95	-0.167	-0.024	-0.049	-0.074	**-0.024**	0.220	**0.139**	0.149	0.160	0.225
	0.99	-0.173	-0.034	-0.070	-0.095	**-0.026**	0.221	**0.133**	0.144	0.163	0.220
	1.00	-0.171	-0.035	-0.074	-0.100	**-0.023**	0.220	**0.130**	0.143	0.165	0.218
	1.01	-0.168	-0.034	-	-	**-0.019**	0.216	**0.127**	-	-	0.215
	1.02	-0.162	-0.032	-	-	**-0.015**	0.212	**0.123**	-	-	0.210
60	0.85	-0.068	0.002	0.005	-0.008	**-0.001**	0.116	**0.099**	0.107	0.101	0.126
	0.90	-0.074	**-0.001**	0.002	-0.013	-0.002	0.115	0.095	0.097	**0.094**	0.123
	0.95	-0.083	-0.008	-0.012	-0.025	**-0.007**	0.117	0.086	**0.084**	0.086	0.121
	0.99	-0.091	-0.018	-0.032	-0.044	**-0.011**	0.117	0.076	**0.075**	0.083	0.117
	1.00	-0.089	-0.019	-0.036	-0.049	**-0.007**	0.115	**0.072**	0.072	0.082	0.116
	1.01	-0.083	-0.017	-	-	**-0.003**	0.110	**0.068**	-	-	0.111
	1.02	-0.073	-0.013	-	-	**0.002**	0.101	**0.061**	-	-	0.103
120	0.85	-0.033	-0.001	**0.000**	-0.003	0.000	0.069	0.063	0.064	**0.062**	0.074
	0.90	-0.036	-0.001	0.002	-0.004	**0.000**	0.064	0.057	0.059	**0.056**	0.069
	0.95	-0.040	-0.001	0.001	-0.006	**0.000**	0.061	0.050	0.050	**0.048**	0.065
	0.99	-0.046	-0.008	-0.013	-0.019	**-0.004**	0.061	0.040	**0.039**	0.042	0.062
	1.00	-0.045	-0.009	-0.018	-0.024	**-0.001**	0.059	0.037	**0.036**	0.042	0.060
	1.01	-0.037	-0.007	-	-	**0.003**	0.051	**0.031**	-	-	0.053
	1.02	-0.022	**-0.002**	-	-	0.009	0.038	**0.023**	-	-	0.045
240	0.85	-0.017	-0.001	-0.002	-0.002	**-0.001**	0.044	0.042	0.042	**0.042**	0.047
	0.90	-0.018	-0.001	-0.001	-0.002	**0.000**	0.040	0.036	0.037	**0.036**	0.042
	0.95	-0.019	-0.001	0.001	-0.002	**0.000**	0.034	0.030	0.031	**0.029**	0.036
	0.99	-0.022	-0.003	-0.004	-0.007	**-0.001**	0.031	0.022	**0.021**	0.022	0.032
	1.00	-0.023	-0.005	-0.009	-0.012	**0.000**	0.030	0.019	**0.018**	0.021	0.031
	1.01	-0.011	**-0.001**	-	-	0.005	0.019	**0.012**	-	-	0.023
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.018

Table 5.2: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant included) with GARCH(1,1) errors.

models are considered. Our results indicate that the general conclusion remains to hold for unit root and mildly explosive autoregressive models as well. On the contrary, the indirect inference estimator offers a similar performance in terms of bias-reduction (even though somewhat less effective) and does not suffer from an increased variance.

5.3.2 Heteroscedastic and heavy-tailed innovations

So far all results are based on $\varepsilon_t \sim N(0,1)$ innovations. As a robustness check on the normality assumption we also investigate the performance of the bias-reduction methods under heteroscedasticity and heavy-tailed error distributions. In order to investigate the influence of heteroscedasticity we generate highly persistent GARCH disturbances as follows:

$$\begin{aligned}\varepsilon_t &= \sigma_t z_t \\ \sigma_t &= a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2,\end{aligned}$$

where $z_t \sim N(0,1)$ and the parameters are set equal to $a_0 = 0.05$, $a_1 = 0.1$ and $b_1 = 0.85$. The simulation results for those DGPs are given in Table 5.2. For $T = 60$, the OLS bias is slightly higher (in absolute value) than in the standard *iid* case. All procedures still offer a substantial bias-reduction, and the remaining bias is usually smaller than in the benchmark case. This means that the GARCH disturbances affect all estimators in a similar way. The general ranking of the bias-correction methods stays the same as in the benchmark case. For all other sample sizes in this setup, the jackknife estimator has the best bias-correction abilities. The RMSE is on average slightly higher than in the benchmark case, but the pattern remains exactly the same.

In order to investigate whether heavy-tailed innovations may lead to problems, we use stable distributed errors which are generated as $\varepsilon_t \sim S(\alpha = 1.85, \beta = 0, \gamma = 1, \delta = 0)$. This distribution exhibits much fatter tails than the standard Normal distribution: $P(|\varepsilon_t| > 2.5758) = 8.6\%$ instead of 1% as for the $N(0,1)$ distribution. Remarkably, the change in the error distribution has hardly any impact on the bias and RMSE results compared to the benchmark case. Therefore, the corresponding Table 5.6 is located in the Appendix.

5.3.3 Inclusion of a linear deterministic trend and misspecified AR(1)

In this subsection we study autoregressive models with an additional linear trend term of the form

$$y_t = \mu + \delta t + \rho y_{t-1} + \varepsilon_t \; .$$

In all simulations we set $\delta = 0$ (in addition to $\mu = 0$) without loss of generality. Table 5.3 shows that the additional uncertainty about the trend parameter causes a rise of the OLS bias. As expected, all procedures perform worse than in the benchmark case (see Table 5.1). Further deviations from the benchmark case are the better overall performance of the Roy-Fuller estimator in stationary setups and the superior performance of the J(2) procedure in small samples ($T = 30$). An interesting development is the reduction of the variance of the indirect inference, Roy-Fuller and Kim's bootstrap estimator. The performance in terms of RMSE is not as convincingly good as in the benchmark case, but the average raise of the RMSE for the OLS estimator is higher than for the bias-correction procedures. Even the J(2) estimator is now able to a lower RMSE than the OLS estimator in most cases, although not in a competitive way.

Almost the same pattern is visible if the AR(1) process is misspecified as an AR(2) process. This means that the data is generated as in the benchmark case, but an AR(2) model with the additional parameter β is estimated. Instead of the trend parameter δ an additional autoregressive parameter adds uncertainty to the estimation. All the effects caused by the inclusion of a linear trend are also visible in the misspecified case, but in a much milder form. The detailed results are gathered in Table 5.7 in the Appendix.

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.227	0.027	-0.033	-0.074	**0.002**	0.282	**0.183**	0.200	0.200	0.278
	0.90	-0.247	0.008	-0.056	-0.096	**-0.005**	0.297	**0.178**	0.195	0.203	0.283
	0.95	-0.271	-0.018	-0.086	-0.124	**-0.018**	0.317	**0.175**	0.197	0.212	0.291
	0.99	-0.300	-0.048	-0.117	-0.156	**-0.040**	0.342	**0.178**	0.208	0.229	0.297
	1.00	-0.309	-0.057	-0.127	-0.165	**-0.049**	0.350	**0.180**	0.214	0.236	0.299
	1.01	-0.320	-0.068	-	-	**-0.059**	0.359	**0.184**	-	-	0.302
	1.02	-0.331	-0.079	-	-	**-0.072**	0.369	**0.188**	-	-	0.305
60	0.85	-0.109	0.010	**0.001**	-0.018	0.009	0.148	0.119	0.119	**0.111**	0.150
	0.90	-0.119	**0.007**	-0.008	-0.028	0.008	0.153	0.116	0.109	**0.107**	0.152
	0.95	-0.134	-0.005	-0.027	-0.046	**0.004**	0.163	0.107	**0.101**	0.105	0.155
	0.99	-0.155	-0.026	-0.053	-0.072	**-0.008**	0.179	**0.104**	0.104	0.114	0.160
	1.00	-0.163	-0.034	-0.062	-0.081	**-0.016**	0.187	**0.106**	0.108	0.119	0.161
	1.01	-0.174	-0.045	-	-	**-0.027**	0.196	**0.109**	-	-	0.162
	1.02	-0.183	-0.053	-	-	**-0.034**	0.204	**0.113**	-	-	0.164
120	0.85	-0.049	0.002	**0.002**	-0.002	0.007	0.078	0.066	0.067	**0.064**	0.080
	0.90	-0.054	0.004	**0.004**	-0.005	0.008	0.078	0.065	0.064	**0.060**	0.080
	0.95	-0.062	0.004	**-0.002**	-0.012	0.008	0.080	0.060	0.056	**0.054**	0.081
	0.99	-0.075	-0.009	-0.021	-0.030	**0.002**	0.089	0.053	**0.050**	0.055	0.083
	1.00	-0.083	-0.017	-0.030	-0.039	**-0.005**	0.096	**0.054**	0.054	0.059	0.084
	1.01	-0.093	-0.027	-	-	**-0.014**	0.105	**0.057**	-	-	0.085
	1.02	-0.063	**-0.013**	-	-	0.037	0.081	**0.043**	-	-	0.099
240	0.85	-0.025	**0.000**	-0.002	-0.001	0.002	0.046	0.040	0.040	**0.040**	0.046
	0.90	-0.026	**0.000**	-0.001	-0.001	0.003	0.043	0.036	0.037	**0.036**	0.044
	0.95	-0.029	0.001	**0.001**	-0.003	0.005	0.041	0.033	0.033	**0.031**	0.042
	0.99	-0.036	**-0.002**	-0.008	-0.012	0.004	0.044	0.027	**0.026**	0.027	0.043
	1.00	-0.042	-0.009	-0.015	-0.020	**-0.001**	0.049	**0.027**	0.028	0.030	0.044
	1.01	-0.032	**-0.007**	-	-	0.020	0.041	**0.022**	-	-	0.052
	1.02	-0.004	**0.001**	-	-	0.028	0.014	**0.008**	-	-	0.040

Table 5.3: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant and trend included).

5.3.4 Higher-order and misspecified autoregressive models

Finally, we extend our analysis to the AR(2) model as in equation (5.3). As visible in Figure 5.1, the OLS bias depends on the value of β. We work with $\beta = \{0.2, -0.3\}$, typical values in macroeconomic time series. In order to save space only results for $T = 60$ are reported in Table 5.4, results for all other sample sizes can be found in Table 5.8 in the Appendix. All procedures are able to reduce the OLS bias for higher order models and the order in terms of bias-correction does not deviate from the AR(1) case. The jackknife is the best method, in particular for the unit root and stationary near unit root setups. The procedure is also the only one which does not depend on β. All other methods gain strictly better results for $\beta = 0.2$. The same pattern appears if $T = 30$. For larger sample sizes the results are more mixed in favor of the indirect inference estimator.

The RMSE results show that the indirect inference estimator has the highest RMSE reduction for most cases. In comparison to the benchmark case, the typical pattern appears only for samples sizes of $T = 120$ or larger, in smaller samples the indirect inference estimator is the best

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
60	0.2	0.85	-0.059	**0.001**	0.004	-0.004	0.010	0.102	**0.089**	0.095	0.089	0.117
		0.90	-0.063	-0.001	0.002	-0.008	**0.009**	0.099	**0.083**	0.086	0.081	0.112
		0.95	-0.069	-0.006	-0.008	-0.017	**0.005**	0.099	**0.074**	0.073	0.072	0.109
		0.99	-0.075	-0.014	-0.025	-0.035	**-0.001**	0.098	**0.063**	0.062	0.068	0.104
		1.00	-0.073	-0.014	-0.029	-0.039	**0.004**	0.095	**0.059**	0.059	0.067	0.102
		1.01	-0.067	-0.012	-	-	**0.006**	0.090	**0.055**	-	-	0.097
		1.02	-0.054	**-0.007**	-	-	0.010	0.079	**0.047**	-	-	0.089
	-0.3	0.85	-0.099	**0.002**	0.005	-0.012	0.010	0.155	**0.119**	0.133	0.126	0.172
		0.90	-0.106	**-0.006**	-0.005	-0.022	0.007	0.154	**0.115**	0.119	0.116	0.168
		0.95	-0.113	-0.015	-0.021	-0.037	**0.002**	0.153	0.105	**0.103**	0.106	0.163
		0.99	-0.117	-0.024	-0.041	-0.057	**-0.002**	0.152	**0.095**	0.095	0.104	0.158
		1.00	-0.114	-0.023	-0.044	-0.060	**0.002**	0.147	**0.090**	0.091	0.103	0.154
		1.01	-0.111	-0.024	-	-	**0.004**	0.146	**0.089**	-	-	0.151
		1.02	-0.102	**-0.020**	-	-	0.009	0.139	**0.083**	-	-	0.144

Table 5.4: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes (constant included).

procedure in terms of RMSE reduction. It is also notable that the J(2) estimator leads to a significant raise in the variance compared to the OLS estimator in all setups.

This result changes if the order of the model is underestimated. The results for a simulated AR(2) process but an estimated AR(1) model are given in Table 5.5 and for other sample sizes in Table 5.9 in the Appendix. For $T = 60$, the jackknife is the best bias-correction method. In particular, if $\beta = -0.3$ it is significantly better than its competitors. Although the ranking of the other bias-correction procedures remains the same, it is not as obvious as before. All methods perform worse than in the correctly specified model. In one setup, $\beta = 0.2$ and $\rho = 0.85$, all bias-corrected estimators have a higher bias than the OLS estimator. For larger samples the results depend on the value of β. If $\beta = 0.2$, no best procedure can be identified but in more and more setups bias-correction is not successful at all. If $\beta = -0.3$, the J(2) estimator offers the highest bias-reduction.

In terms of RMSE the standard pattern from the AR(1) case is visible for $\beta = 0.2$, whereas for $\beta = -0.3$ the indirect inference estimator is the best procedure. But, no procedure is able to offer a constant reduction of the RMSE and if this reduction is much less than in the correctly specified model. These results lead to the recommendation to choose the model order with a parameter friendly information criterion like the AIC when bias-correction should be applied.

5.3.5 Summary of simulation results

Our main results are as follows: (i) bias-correction plays an important role for all considered levels of persistence (i.e. stationarity, unit roots and explosive behavior), in particular for samples sizes up to $T = 120$, (ii) the most effective bias-correction is obtained when applying the jackknife estimator for small and moderate sample sizes; in terms of RMSE, the indirect inference approach is generally recommendable. It performs particulary well for small sample sizes

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
60	0.2	0.85	**-0.023**	0.051	0.056	0.040	0.030	**0.073**	0.093	0.097	0.085	0.097
		0.90	-0.034	0.043	0.044	0.029	**0.023**	0.073	0.082	0.081	**0.073**	0.095
		0.95	-0.048	0.027	0.019	**0.007**	0.013	0.078	0.066	0.058	**0.057**	0.095
		0.99	-0.062	0.008	-0.013	-0.024	**-0.002**	0.084	0.051	**0.047**	0.056	0.094
		1.00	-0.062	0.005	-0.020	-0.033	**-0.002**	0.084	0.047	**0.046**	0.061	0.092
		1.01	-0.058	0.004	-	-	**0.002**	0.079	**0.043**	-	-	0.087
		1.02	-0.047	**0.005**	-	-	0.006	0.071	**0.039**	-	-	0.082
	-0.3	0.85	-0.189	-0.108	-0.129	-0.137	**-0.086**	0.236	**0.156**	0.202	0.203	0.206
		0.90	-0.179	-0.105	-0.113	-0.124	**-0.069**	0.224	**0.161**	0.188	0.189	0.197
		0.95	-0.172	-0.099	-0.103	-0.115	**-0.055**	0.212	**0.157**	0.171	0.175	0.186
		0.99	-0.161	-0.087	-0.095	-0.108	**-0.036**	0.197	**0.144**	0.153	0.158	0.174
		1.00	-0.154	-0.081	-0.091	-0.104	**-0.027**	0.191	**0.139**	0.147	0.153	0.169
		1.01	-0.145	-0.073	-	-	**-0.016**	0.182	**0.131**	-	-	0.163
		1.02	-0.132	-0.064	-	-	**-0.007**	0.171	**0.122**	-	-	0.155

Table 5.5: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes when the model is misspecified as AR(1).

and explosive processes. (iii) Under the presence of a unit root, the Roy-Fuller and the indirect inference estimator perform best in terms of RMSE, while the bootstrap-based estimator by Kim (2003) performs well for stationary models. (iv) Heteroscedastic and heavy-tailed errors hardly affect the former conclusions. (v) In case of correct specification, the exact order of the autoregressive model does not alter our main findings. Overfitting of the autoregressive model is not harmful, while underfitting turns out to be an important issue. Therefore, the lag length shall be carried out on the basis of liberal selection procedures like the AIC. (vi) The performance of all estimators weakens when a deterministic trend in addition to an intercept is included. But, the ranking of estimators remains unaffected.

5.4 Empirical applications

We apply the different bias-correction methods to four economic time series using the popular rolling window technique. In Section 5.4.1 we analyze a long annual ratio of the US Debt/GDP series in detail. Recently, there has been an extensive discussion on lifting the US government debt ceiling. The sustainability of US fiscal policy hinges on the persistence properties of the US Debt/GDP series: only when the series exhibits stationarity, fiscal policies are sustainable.

Further empirical applications are considered in Section 5.4.2 where the following three series are studied: (1) log Oil price, (2) log Gold price and (3) spread between long-term interest rates in Germany and Greece. Figure 5.2 contains time series plots of all four variables. All series are strongly autocorrelated. The first three series are even likely to exhibit locally explosive behavior due to expansions during war times (US Debt) and speculation (Oil and Gold). The situation is different for the interest rate series whose persistence properties have not been studied extensively yet. Data for the debt series is available at http://www.econ.ucsb.edu/~bohn/data.html while the remaining data has been obtained from the FRED and the ECB database. Bias-

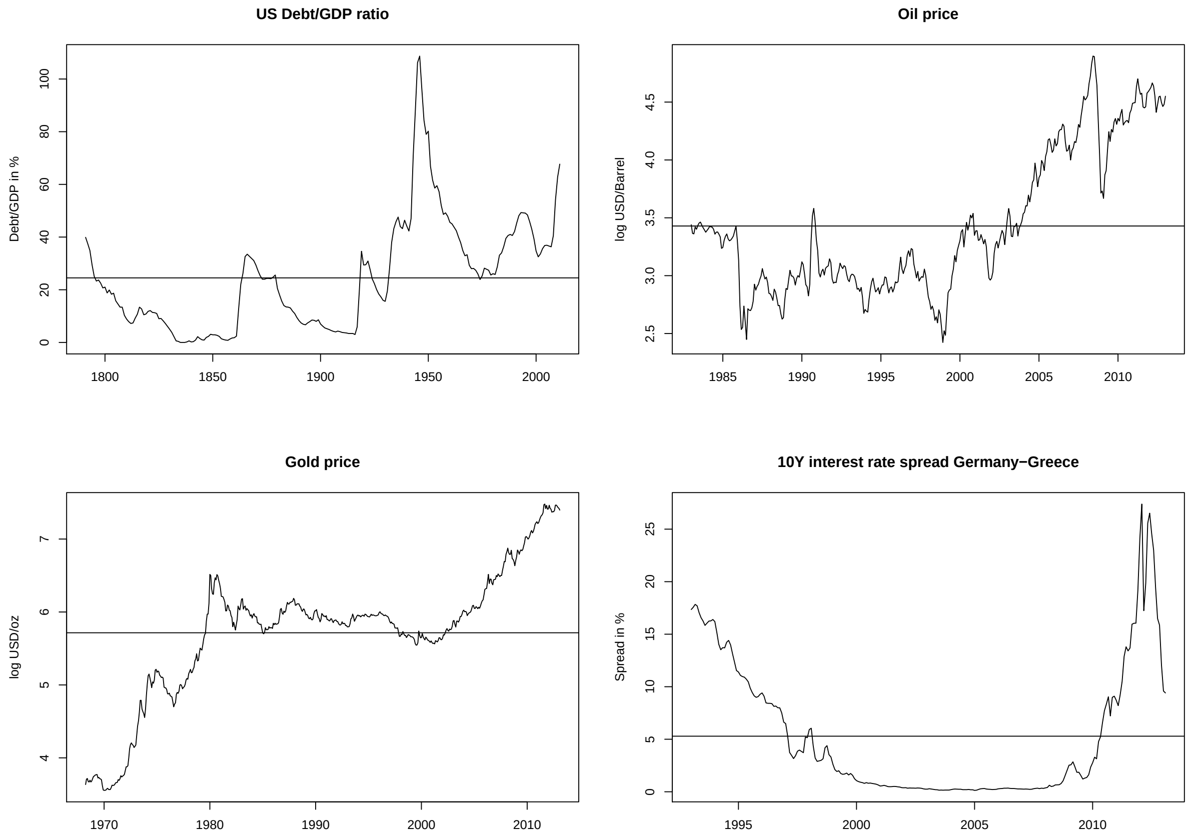

Figure 5.2: Time series under consideration.

corrected rolling window estimation (with 60 observations per window) is compared to classic OLS estimation. The lag length is chosen via the Akaike information criterion as underfitting is a problematic issue. For each series, an intercept is included in the autoregressive model due to a non-zero mean.

5.4.1 US Debt/GDP ratio

The US Debt/GDP ratio series is measured in percent. The sample ranges from 1791-2011, yielding 221 annual observations. Given a window size of 60, we obtain the first estimates for the period from 1791 to 1850.[6] The second estimates are based on the sample ranging from 1792-1851 and so on. The last estimates use the sample from 1952 to 2011. According to the AIC, an AR(2) model is fitted to the data.

The estimated values of ρ for the different bias-correction techniques are given in Figure 5.3, each in comparison to the OLS estimator. First, bias-correction obviously plays an important role in this application as differences between OLS and bias-corrected estimates are clearly visible. Second, the Roy-Fuller, bootstrap and indirect inference estimator agree on the general

[6] The choice of 60 observations has been made in accordance to the simulations in the previous section. However, our calculations for 50 observations (half a century of data per window) lead to very similar conclusions.

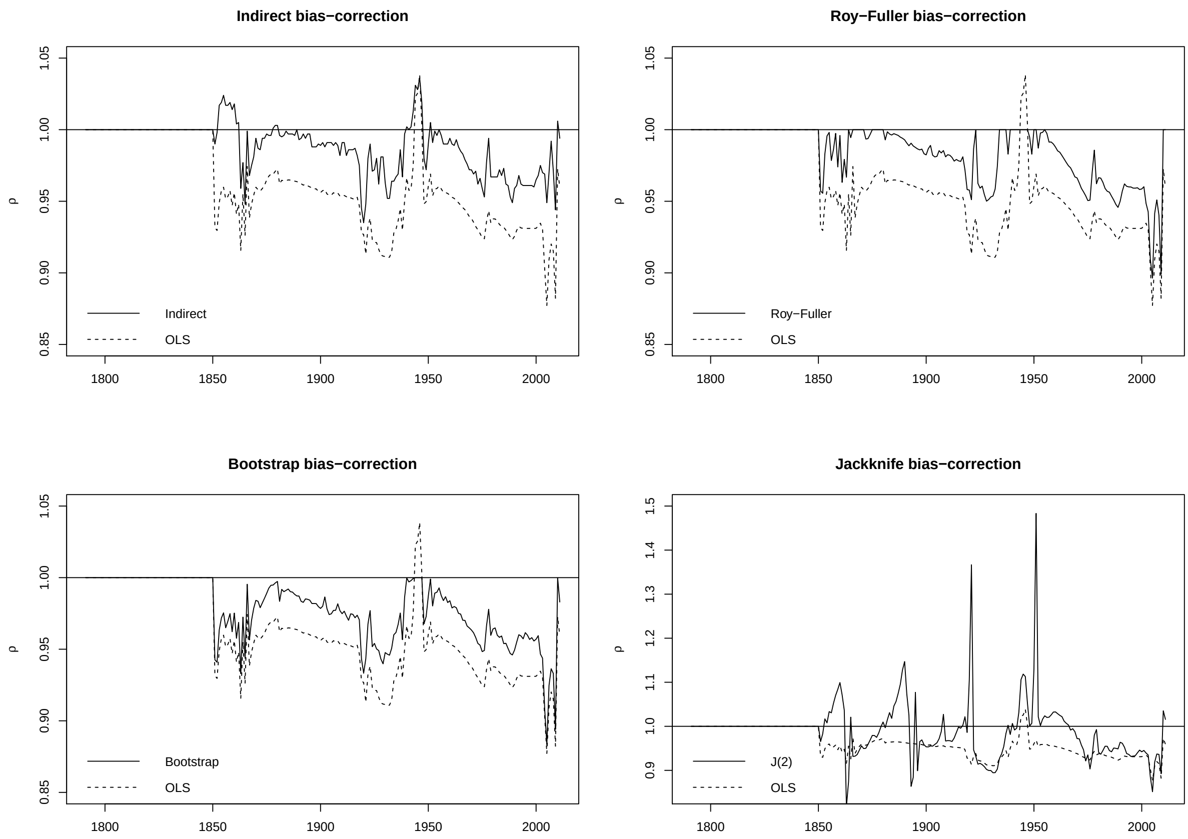

Figure 5.3: Rolling window AR(2) estimation for the US Debt/GDP series with different bias-correction methods.

evolution of the persistence over time, whereas the jackknife estimator shows a more volatile behavior. An obvious shortcoming of the Roy-Fuller bias-correction and the bootstrap technique by Kim (2003) is their limitation to the parameter space $\widehat{\rho} \leq 1$. The results for the OLS, indirect inference and jackknife estimator clearly suggest the need of relaxing this restriction for obtaining meaningful estimates of the persistence. Therefore, we focus on the indirect inference estimator in comparison to the jackknife estimator.

The indirect inference estimator displays explosiveness during major wars (Civil War and World War II), where the autoregressive parameter estimates reach a maximum of $\widehat{\rho}^{II} = 1.036$. After 1950, persistence dropped remarkably, but recovered during the recent years since 2001 possibly in response to the patriot act and related policies after 9/11. The very last point estimates indicate a high persistence and a possible unit root. Parameter estimates for the J(2) bias-correction method show explosive behavior during the Civil War, in the late 18th century and both World Wars. Estimated persistence is relatively close around the unit root with an interval from $\widehat{\rho}_{J,2} = [0.818, 1.483]$.[7] These results support the Monte Carlo analysis, where the jackknife estimators show a very good bias-reduction but at the costs of high standard errors. The OLS

[7] The higher-order J(2,3) bias-correction (not reported to conserve some space) yields very volatile results with many highly explosive phases but also some major drops down to $\widehat{\rho}_{J(2,3)} = 0.558$.

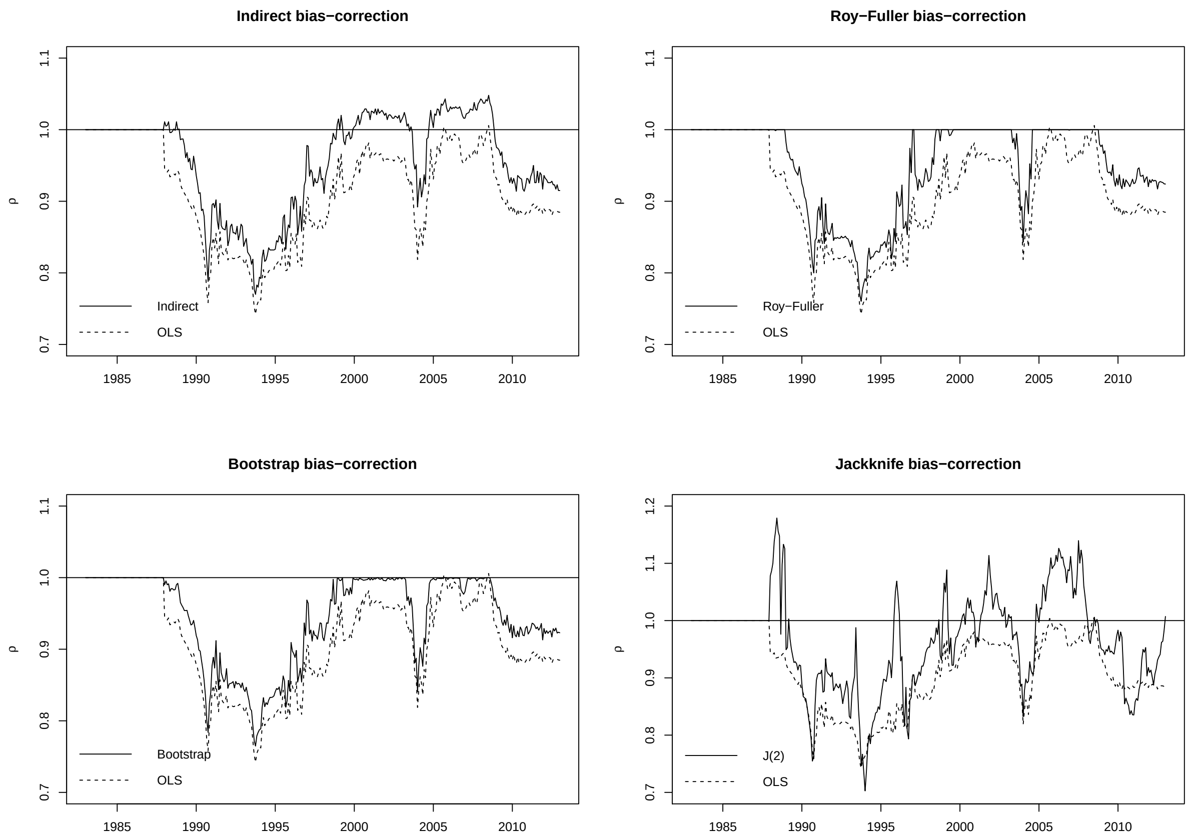

Figure 5.4: Rolling window AR(2) estimation for the log Oil price series with different bias-correction methods.

estimation results only indicate a single period of explosive behavior, i.e. the second World War. Moreover, the OLS results for the Civil War period are in clear discrepancy to the ones obtained by bias-corrected estimators.

Our results suggest that a lifting of the US government debt ceiling may easily end up in unsustainable fiscal policies as the persistence of the series is non-stationary and nearly explosive during the most recent years. In general, our findings are in line with Yoon (2011) who applies the recursive right-tailed unit root test of Phillips et al. (2011) to test the hypothesis of a unit root against explosive behavior. His main result is that the US Debt/GDP ratio is explosive and that the explosiveness is linked to the high increase in the ratio during and after the World War II. Our study complements Yoon (2011) as the author did not consider bias-corrected estimation for the series.

5.4.2 Further applications: Oil, Gold and European interest rates

In this subsection we analyze some further time series which potentially exhibit phases of explosiveness due to pronounced growth rates. We start with the spot oil price series (West Texas Intermediate), which is measured in US Dollars per barrel. Episodes of explosive behavior hint

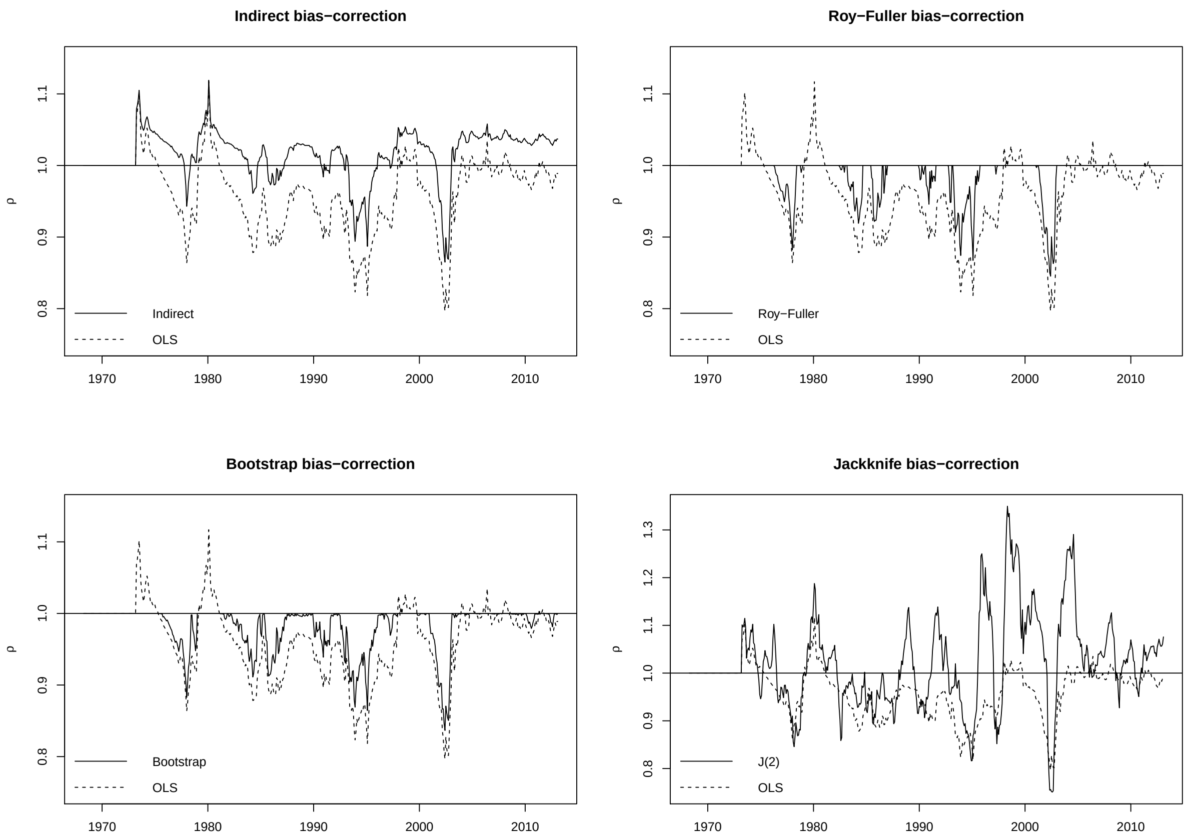

Figure 5.5: Rolling window AR(1) estimation for the log Gold price series with different bias-correction methods.

at strong speculation activities in the market. The sample ranges from 1983:01 to 2013:01 ($T = 361$). An AR(2) model is fitted to the data. The window size equals 60 months (5 years).

The estimated values of ρ for the different bias-correction techniques are given in Figure 5.4, each in comparison to the OLS estimator. The general evolution of all estimators suggests that persistence has undergone remarkable changes. Bias-correction is of importance in this application, too. The OLS estimates do not indicate explosive behavior (and thus phases of pronounced speculation) at all. When looking at the results for the indirect inference estimator, one observes that oil prices have been much less persistent (and presumably stationary) during the Nineties. Persistence increased towards the year 2000 and stayed above, but close to, unity. Around 2004, persistence dropped again whilst recovering quickly to high levels indicating mild explosiveness. Interestingly, there has been another drop to values around 0.9 in the recent years. The rolling window estimation results reflect the movements in the series, see Figure 5.2 (upper right panel). The Roy-Fuller and the bootstrap bias-correction techniques suggest similar findings expect of the important periods of explosiveness. The jackknife estimator provides results which are in general accordance to the ones for the indirect inference estimator. However, estimated persistence is much higher in explosive phases and the persistence path is more volatile. This behavior is confirmed by our simulation results which show that the jackknife estimator has a fairly large

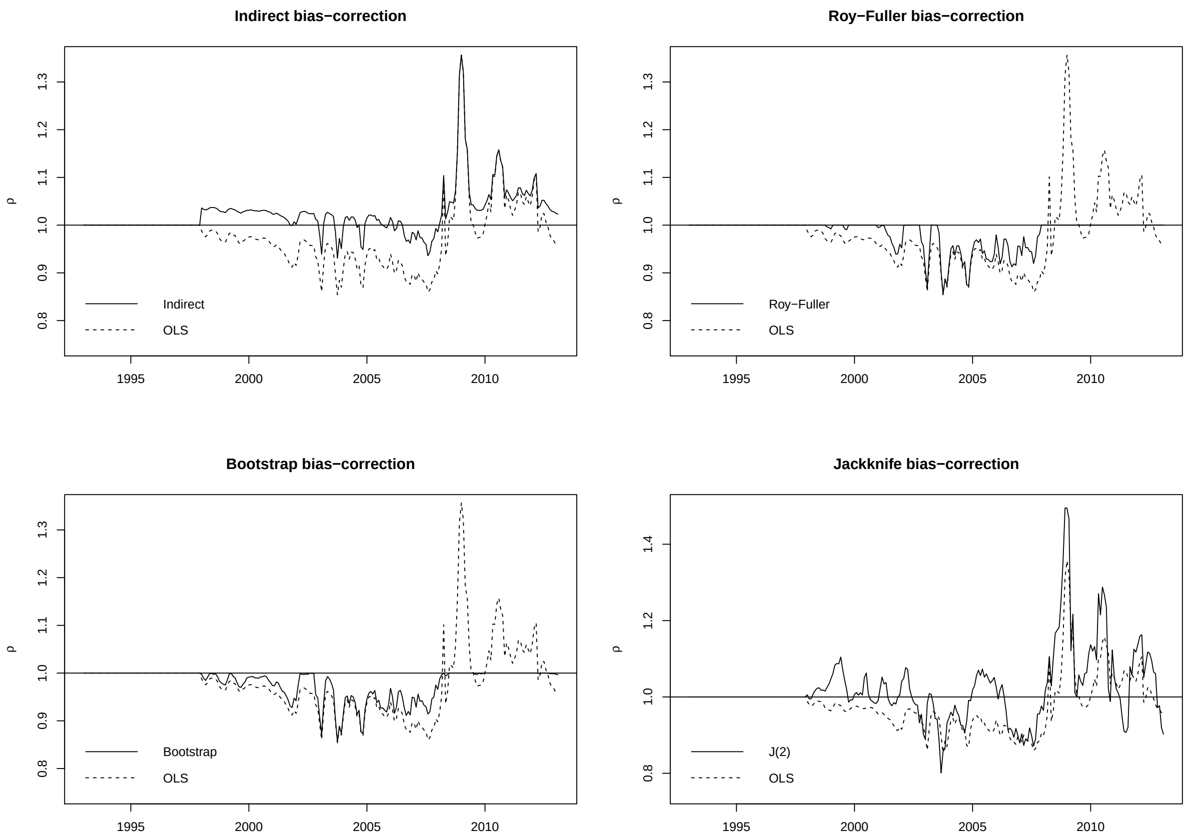

Figure 5.6: Rolling window AR(1) estimation for the interest rate spread series with different bias-correction methods.

variance.

Next, we study another important commodity series. The presence of bubbles (characterized by explosive price paths) in gold prices (measured in US Dollars per ounce) has implications with respect to its safe haven property, see Baur et al. (2012) and Baur and McDermott (2010). During periods of explosive behavior, the stabilizing effect of Gold vanishes which may endanger the financial system to a certain extent. Monthly data is sampled from 1968:04 to 2013:01, yielding 539 observations. An AR(1) model is fitted to the data.

The results are reported in Figure 5.5. As a first clear result, the series is strongly persistent and exhibits many and long phases of mild explosiveness. Even the rolling window OLS estimates clearly indicate two such phases in the beginning of the Seventies and the Eighties, respectively. When comparing different bias-correction techniques, we find a similar picture as for the previous applications. The importance of bias-correction and the simultaneous allowance for explosive behavior is further underlined.

Finally, we consider the spread between long-term interest rates in Germany and Greece. The series spans 1993:01–2013:02, thereby giving a total number of 242 observations. The selected lag length equals one. The spread has remarkably declined during the European economic integration and reached levels near zero after the Euro introduction. During the following years (up to 2007), long-term interest rates remained nearly the same in Germany and Greece and only a minor risk premium for investing in Greece has been paid. After the beginning of the financial crisis, however, the spread reached historic values above 25% reflecting the increased default risk. Results for bias-corrected estimation of persistence in this series are reported in Figure 5.6. In the beginning of the sample, estimated persistence indicate a unit root followed by lower persistence caused by European monetary integration efforts. But, the results also show a dramatic increase in persistence at the beginning of the global financial crisis and even the OLS estimates take values above 1.3 which is remarkably high. Obviously, it is of major importance to allow for explosiveness in this application. Towards the end of the sample, persistence lowered considerably to values near unity indicating one of the outcomes of the European Stability Mechanism. The indirect inference estimator and the jackknife estimator yield similar results as they agree on the general evolution of persistence.

5.5 Conclusion

This paper compares four different bias-correction techniques for autoregressive processes. Among these are the approximately median-unbiased estimator by Roy and Fuller (2001), a bootstrap-based estimator by Kim (2003), an indirect inference estimator by Phillips et al. (2011) and a jackknife estimator suggested in Chambers (2013). We thus compare established techniques to newly proposed procedures in a comprehensive way. In particular, we focus on situations where the sample size is relatively small and data is highly persistent, exhibits a unit root or is even mildly explosive. When the popular rolling window framework is applied for assessing the possibly time-varying persistence of a time series, sample sizes are typically small. Moreover, it is reasonable to expect that time series undergo changes in persistence during different regimes and episodes. These changes can be either driven by episodes of speculation (leading to temporary bubbles) or policy induced (typically leading to a reduction in persistence). Therefore, we study an empirically relevant situation and provide practical recommendations for further applications.

A large-scale simulation study of bias and root mean squared errors of estimators reveals the following results: The substantial bias of the OLS estimator can be remarkably reduced across the whole range of considered autoregressive parameter values. The most promising approaches are the indirect inference estimator and the jackknife estimator. The indirect inference estimator provides excellent bias-correction in various settings (i.e. heavy-tailed errors, GARCH errors, linear trend and misspecified autoregression) together with a reasonably low variance, while the jackknife estimator performs often best in terms of bias-correction, but has a clearly larger variance rendering this estimator less recommendable in terms of RMSE.

As the main empirical application, we consider a long annual US Debt/GDP series in a rolling window estimation framework. Remarkable evidence for time-varying persistence and periods of explosiveness during the Civil War and World War II are documented. The results clearly suggest substantial differences for various estimation techniques and thus, different policy implications. Further empirical applications consider Oil prices, Gold prices and the spread between long-term interest rates in Germany and Greece. In all cases, the importance of bias-correction and the simultaneous allowance for locally explosive behavior is further stressed.

5.A Appendix

5.A.1 Stable errors

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.135	0.012	**0.000**	-0.030	-0.008	0.199	**0.144**	0.163	0.160	0.220
	0.90	-0.148	**-0.002**	-0.017	-0.046	-0.014	0.205	**0.140**	0.150	0.155	0.221
	0.95	-0.162	**-0.018**	-0.042	-0.069	-0.023	0.213	**0.134**	0.142	0.154	0.222
	0.99	-0.167	-0.028	-0.064	-0.090	**-0.022**	0.215	**0.127**	0.139	0.160	0.221
	1.00	-0.166	-0.029	-0.068	-0.095	**-0.019**	0.213	**0.125**	0.137	0.160	0.219
	1.01	-0.162	-0.029	-	-	**-0.015**	0.210	**0.123**	-	-	0.215
	1.02	-0.157	-0.028	-	-	**-0.012**	0.206	**0.120**	-	-	0.210
60	0.85	-0.064	0.006	0.010	-0.005	**0.002**	0.108	0.094	0.100	**0.093**	0.119
	0.90	-0.070	0.004	0.007	-0.009	**0.001**	0.108	0.089	0.091	**0.087**	0.118
	0.95	-0.078	**-0.003**	-0.006	-0.021	-0.003	0.109	0.080	**0.077**	0.079	0.116
	0.99	-0.086	-0.014	-0.028	-0.041	**-0.008**	0.112	0.071	**0.071**	0.079	0.116
	1.00	-0.085	-0.015	-0.033	-0.046	**-0.006**	0.111	**0.068**	0.070	0.081	0.115
	1.01	-0.079	-0.013	-	-	**-0.001**	0.104	**0.062**	-	-	0.107
	1.02	-0.069	-0.010	-	-	**0.003**	0.095	**0.056**	-	-	0.099
120	0.85	-0.030	0.002	0.003	**0.000**	0.002	0.062	0.057	0.059	**0.057**	0.068
	0.90	-0.033	0.002	0.005	**-0.001**	0.002	0.059	0.053	0.055	**0.052**	0.065
	0.95	-0.037	0.002	0.004	-0.004	**0.002**	0.057	0.047	0.047	**0.045**	0.062
	0.99	-0.044	-0.006	-0.011	-0.018	**-0.003**	0.059	0.038	**0.037**	0.040	0.061
	1.00	-0.044	-0.008	-0.016	-0.022	**-0.002**	0.057	**0.035**	0.035	0.040	0.059
	1.01	-0.035	-0.005	-	-	**0.003**	0.049	**0.028**	-	-	0.051
	1.02	-0.021	**-0.002**	-	-	0.009	0.037	**0.022**	-	-	0.044
240	0.85	-0.015	0.000	**0.000**	0.000	0.001	0.039	0.036	0.037	**0.036**	0.041
	0.90	-0.016	0.001	0.001	**0.000**	0.001	0.035	0.032	0.032	**0.032**	0.038
	0.95	-0.017	0.001	0.003	**0.000**	0.002	0.031	0.027	0.028	**0.026**	0.034
	0.99	-0.021	-0.001	-0.002	-0.006	**0.000**	0.029	0.020	**0.019**	0.020	0.031
	1.00	-0.022	-0.004	-0.008	-0.011	**0.000**	0.030	**0.019**	0.019	0.022	0.032
	1.01	-0.010	**-0.001**	-	-	0.005	0.018	**0.011**	-	-	0.022
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.017

Table 5.6: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant included) with stable error distribution.

5.A.2 Misspecified AR(1) process

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.162	**0.003**	-0.021	-0.044	0.024	0.237	**0.151**	0.190	0.186	0.280
	0.90	-0.173	-0.014	-0.038	-0.061	**0.013**	0.240	**0.149**	0.178	0.180	0.273
	0.95	-0.183	-0.030	-0.060	-0.083	**0.003**	0.242	**0.145**	0.167	0.176	0.269
	0.99	-0.186	-0.039	-0.078	-0.105	**0.002**	0.241	**0.140**	0.160	0.179	0.262
	1.00	-0.182	-0.037	-0.079	-0.106	**0.009**	0.236	**0.136**	0.155	0.177	0.259
	1.01	-0.178	-0.036	-	-	**0.009**	0.232	**0.134**	-	-	0.255
	1.02	-0.170	-0.033	-	-	**0.016**	0.226	**0.130**	-	-	0.254
60	0.85	-0.075	**0.002**	0.004	-0.007	0.008	0.125	**0.104**	0.113	0.106	0.141
	0.90	-0.080	-0.003	**-0.001**	-0.013	0.007	0.123	0.099	0.102	**0.098**	0.137
	0.95	-0.086	-0.010	-0.013	-0.025	**0.004**	0.121	0.087	**0.085**	0.086	0.130
	0.99	-0.094	-0.020	-0.034	-0.045	**-0.003**	0.122	0.079	**0.078**	0.085	0.127
	1.00	-0.090	-0.018	-0.036	-0.048	**0.003**	0.116	**0.072**	0.073	0.083	0.123
	1.01	-0.086	-0.018	-	-	**0.004**	0.114	**0.070**	-	-	0.118
	1.02	-0.074	-0.013	-	-	**0.009**	0.103	**0.062**	-	-	0.112
120	0.85	-0.035	**-0.001**	0.001	-0.002	0.003	0.069	0.063	0.064	**0.062**	0.075
	0.90	-0.037	**0.000**	0.002	-0.003	0.003	0.065	0.057	0.059	**0.056**	0.071
	0.95	-0.041	-0.001	**0.001**	-0.006	0.003	0.062	0.050	0.050	**0.048**	0.066
	0.99	-0.046	-0.008	-0.013	-0.019	**-0.001**	0.061	0.040	**0.038**	0.041	0.063
	1.00	-0.045	-0.009	-0.017	-0.023	**0.000**	0.059	0.036	**0.036**	0.040	0.060
	1.01	-0.037	-0.006	-	-	**0.005**	0.051	**0.031**	-	-	0.055
	1.02	-0.022	**-0.002**	-	-	0.010	0.039	**0.024**	-	-	0.046
240	0.85	-0.017	**0.000**	0.000	0.000	0.001	0.043	0.040	0.040	**0.040**	0.044
	0.90	-0.018	0.000	**0.000**	-0.001	0.001	0.038	0.034	0.035	**0.034**	0.040
	0.95	-0.019	**0.000**	0.002	-0.001	0.002	0.033	0.029	0.030	**0.028**	0.035
	0.99	-0.022	-0.003	-0.003	-0.007	**0.000**	0.031	0.021	**0.020**	0.021	0.031
	1.00	-0.022	-0.004	-0.008	-0.011	**0.000**	0.029	0.017	**0.017**	0.020	0.030
	1.01	-0.011	**-0.001**	-	-	0.005	0.019	**0.012**	-	-	0.023
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.018

Table 5.7: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes when the model is misspecified as AR(2) (constant included).

5.A.3 AR(2) process

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.2	0.85	-0.134	**-0.001**	-0.015	-0.032	0.023	0.199	**0.138**	0.165	0.160	0.239
		0.90	-0.142	**-0.012**	-0.028	-0.045	0.017	0.201	**0.136**	0.153	0.152	0.235
		0.95	-0.151	-0.025	-0.047	-0.065	**0.012**	0.201	**0.128**	0.140	0.146	0.232
		0.99	-0.153	-0.030	-0.063	-0.084	**0.008**	0.200	**0.120**	0.132	0.147	0.224
		1.00	-0.150	-0.030	-0.066	-0.089	**0.011**	0.196	**0.118**	0.129	0.150	0.222
		1.01	-0.144	-0.027	-	-	**0.018**	0.191	**0.113**	-	-	0.219
		1.02	-0.139	-0.026	-	-	**0.018**	0.188	**0.111**	-	-	0.214
	-0.3	0.85	-0.217	**-0.001**	-0.040	-0.072	0.008	0.304	**0.167**	0.232	0.232	0.335
		0.90	-0.229	-0.022	-0.063	-0.093	**-0.007**	0.309	**0.167**	0.222	0.229	0.329
		0.95	-0.240	-0.043	-0.087	-0.119	**-0.016**	0.311	**0.168**	0.213	0.228	0.328
		0.99	-0.237	-0.048	-0.101	-0.135	**-0.007**	0.303	**0.163**	0.202	0.226	0.320
		1.00	-0.231	-0.045	-0.101	-0.137	**0.003**	0.297	**0.158**	0.196	0.227	0.317
		1.01	-0.228	-0.046	-	-	**0.005**	0.295	**0.157**	-	-	0.313
		1.02	-0.224	-0.046	-	-	**0.007**	0.293	**0.157**	-	-	0.309
120	0.2	0.85	-0.029	-0.001	**-0.001**	-0.002	0.001	0.060	0.055	0.057	**0.055**	0.064
		0.90	-0.029	**0.000**	0.001	-0.002	0.002	0.055	0.049	0.051	**0.048**	0.059
		0.95	-0.032	**0.000**	0.002	-0.003	0.003	0.050	0.042	0.043	**0.041**	0.055
		0.99	-0.037	-0.006	-0.009	-0.014	**-0.001**	0.050	0.033	**0.032**	0.034	0.051
		1.00	-0.036	-0.007	-0.014	-0.019	**0.002**	0.047	0.029	**0.029**	0.033	0.050
		1.01	-0.027	-0.004	-	-	**0.004**	0.040	**0.024**	-	-	0.042
		1.02	-0.012	**0.000**	-	-	0.011	0.027	**0.017**	-	-	0.036
	-0.3	0.85	-0.048	**-0.001**	0.002	-0.004	0.003	0.087	0.076	0.080	**0.076**	0.092
		0.90	-0.050	**-0.001**	0.004	-0.005	0.004	0.082	0.070	0.073	**0.069**	0.088
		0.95	-0.054	-0.004	-0.002	-0.011	**0.002**	0.080	0.061	0.060	**0.059**	0.084
		0.99	-0.059	-0.011	-0.017	-0.025	**0.000**	0.078	0.051	**0.048**	0.053	0.081
		1.00	-0.059	-0.012	-0.022	-0.030	**-0.001**	0.076	0.047	**0.047**	0.053	0.078
		1.01	-0.050	**-0.009**	-	-	0.005	0.069	**0.042**	-	-	0.072
		1.02	-0.037	**-0.005**	-	-	0.009	0.058	**0.035**	-	-	0.064
240	0.2	0.85	-0.013	0.000	0.000	**0.000**	0.001	0.037	0.035	0.035	**0.035**	0.038
		0.90	-0.013	0.000	0.000	**0.000**	0.001	0.032	0.030	0.030	**0.030**	0.034
		0.95	-0.015	**0.000**	0.001	0.000	0.002	0.028	0.025	0.025	**0.024**	0.029
		0.99	-0.017	-0.001	-0.002	-0.004	**0.000**	0.024	0.017	**0.016**	0.017	0.025
		1.00	-0.018	-0.003	-0.007	-0.009	**0.000**	0.023	0.014	**0.014**	0.016	0.024
		1.01	-0.006	**0.000**	-	-	0.006	0.013	**0.008**	-	-	0.017
		1.02	0.000	**0.000**	-	-	0.006	0.005	**0.004**	-	-	0.013
	-0.3	0.85	-0.023	-0.001	-0.001	-0.002	**0.000**	0.051	0.047	0.047	**0.047**	0.053
		0.90	-0.024	-0.001	0.000	-0.001	**0.000**	0.046	0.041	0.042	**0.041**	0.048
		0.95	-0.024	**0.001**	0.003	-0.001	0.002	0.041	0.034	0.036	**0.033**	0.043
		0.99	-0.029	-0.004	-0.006	-0.010	**-0.001**	0.040	0.027	**0.025**	0.027	0.041
		1.00	-0.029	-0.005	-0.011	-0.015	**0.000**	0.037	0.023	**0.022**	0.025	0.038
		1.01	-0.019	**-0.003**	-	-	0.004	0.029	**0.017**	-	-	0.031
		1.02	-0.006	**0.000**	-	-	0.011	0.016	**0.010**	-	-	0.026

Table 5.8: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes (constant included).

5.A.4 Misspecified AR(2) process

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.2	0.85	-0.081	0.066	0.054	**0.027**	0.028	0.147	0.142	0.140	**0.131**	0.184
		0.90	-0.097	0.047	0.027	**0.002**	0.015	0.154	0.127	0.119	**0.119**	0.185
		0.95	-0.116	0.024	-0.009	-0.032	**-0.002**	0.164	0.112	**0.105**	0.116	0.187
		0.99	-0.126	**0.007**	-0.040	-0.065	-0.009	0.169	**0.100**	0.101	0.128	0.186
		1.00	-0.126	**0.005**	-0.046	-0.075	-0.008	0.168	**0.097**	0.101	0.136	0.185
		1.01	-0.123	**0.003**	-	-	-0.005	0.166	**0.094**	-	-	0.182
		1.02	-0.118	0.003	-	-	**-0.001**	0.162	**0.091**	-	-	0.177
	-0.3	0.85	-0.314	**-0.122**	-0.202	-0.223	-0.145	0.382	**0.187**	0.325	0.329	0.340
		0.90	-0.313	-0.136	-0.197	-0.219	**-0.134**	0.378	**0.205**	0.315	0.320	0.334
		0.95	-0.309	-0.143	-0.193	-0.215	**-0.120**	0.372	**0.217**	0.301	0.309	0.327
		0.99	-0.298	-0.141	-0.186	-0.209	**-0.098**	0.360	**0.218**	0.286	0.296	0.316
		1.00	-0.292	-0.137	-0.182	-0.205	**-0.090**	0.354	**0.216**	0.280	0.291	0.311
		1.01	-0.285	-0.133	-	-	**-0.080**	0.348	**0.213**	-	-	0.306
		1.02	-0.277	-0.128	-	-	**-0.070**	0.340	**0.209**	-	-	0.300
120	0.2	0.85	**0.002**	0.036	0.038	0.034	0.026	**0.043**	0.058	0.061	0.056	0.058
		0.90	**-0.007**	0.030	0.034	0.027	0.018	**0.039**	0.052	0.055	0.049	0.052
		0.95	-0.017	0.023	**0.024**	0.016	0.011	0.038	0.042	0.041	**0.037**	0.048
		0.99	-0.029	0.008	-0.001	-0.006	**0.001**	0.041	0.026	**0.022**	0.026	0.046
		1.00	-0.031	0.003	-0.009	-0.015	0.000	0.041	0.022	**0.021**	0.030	0.045
		1.01	-0.023	**0.003**	-	-	0.003	0.035	**0.019**	-	-	0.041
		1.02	-0.008	**0.006**	-	-	0.014	0.023	**0.015**	-	-	0.035
	-0.3	0.85	-0.124	-0.092	-0.095	-0.097	**-0.069**	0.153	**0.125**	0.134	0.134	0.130
		0.90	-0.108	-0.076	-0.076	-0.079	**-0.047**	0.136	**0.113**	0.116	0.116	0.116
		0.95	-0.095	-0.060	-0.058	-0.063	**-0.029**	0.120	**0.098**	0.099	0.099	0.105
		0.99	-0.086	-0.049	-0.049	-0.056	**-0.016**	0.107	0.083	**0.083**	0.085	0.096
		1.00	-0.080	-0.043	-0.046	-0.052	**-0.007**	0.100	**0.075**	0.076	0.079	0.091
		1.01	-0.068	-0.033	-	-	**0.002**	0.089	**0.066**	-	-	0.084
		1.02	-0.051	-0.024	-	-	**0.007**	0.075	**0.055**	-	-	0.072
240	0.2	0.85	**0.014**	0.030	0.030	0.030	0.025	**0.031**	0.041	0.042	0.041	0.040
		0.90	**0.006**	0.023	0.023	0.022	0.018	**0.025**	0.033	0.034	0.033	0.033
		0.95	**-0.003**	0.016	0.018	0.014	0.010	**0.020**	0.026	0.028	0.025	0.026
		0.99	-0.012	0.007	0.004	**0.001**	0.003	0.020	0.015	**0.013**	0.013	0.023
		1.00	-0.015	0.002	-0.004	-0.007	**0.000**	0.020	0.011	**0.010**	0.015	0.022
		1.01	-0.004	**0.003**	-	-	0.007	0.012	**0.007**	-	-	0.018
		1.02	**0.004**	0.005	-	-	0.010	0.006	**0.006**	-	-	0.015
	-0.3	0.85	-0.093	-0.079	-0.079	-0.079	**-0.066**	0.110	0.099	0.100	0.100	**0.094**
		0.90	-0.074	-0.059	-0.059	-0.059	**-0.044**	0.091	0.080	0.080	0.080	**0.076**
		0.95	-0.056	-0.039	-0.039	-0.040	**-0.022**	0.071	**0.060**	0.060	0.060	0.061
		0.99	-0.046	-0.026	-0.026	-0.029	**-0.008**	0.057	0.045	**0.045**	0.046	0.051
		1.00	-0.041	-0.022	-0.023	-0.026	**-0.002**	0.052	**0.039**	0.039	0.040	0.048
		1.01	-0.026	-0.012	-	-	**0.004**	0.038	**0.028**	-	-	0.038
		1.02	-0.012	**-0.007**	-	-	0.008	0.022	**0.016**	-	-	0.028

Table 5.9: Bias and RMSE for OLS. indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes when the model is misspecified as AR(1) (constant included).

Chapter 6

Macroeconomic determinants of time-varying persistence in the S&P500 price-dividend ratio

Macroeconomic determinants of time-varying persistence in the S&P500 price-dividend ratio

Co-authored with Robinson Kruse.

6.1 Introduction

Asset pricing models are mainly concerned with the connection between prices and dividends. According to standard models, asset prices are determined by discounted expected future dividends which is a measure of the fundamental value. A key variable receiving much attention in the academic and the financial world is the price-dividend ratio (PD ratio). Financial theory suggests under a set of standard assumptions and by imposing a no-bubble condition that the PD ratio is stationary as prices and dividends are cointegrated in the long-run. When stock prices escalate their fundamental value in a systematic (and possibly rational) way, the PD ratio is no longer stationary. This fact arises from stock prices becoming explosive and dividends still being difference-stationary. It is natural to assume that the persistence (and thereby the order of integration) of the PD ratio exhibits a dynamic structure. Time-varying persistence of the PD ratio is consistent with e.g. the existence of periodically collapsing bubbles, see Evans (1991). Popular tests for (rational) asset price bubbles are built directly on the persistence properties of the PD ratio, see e.g. Craine (1993).

Another field where the PD ratio and its time-varying persistence are of importance is the prediction of stock returns, see Campbell and Shiller (1988) and Fama and French (1988). The ability of the PD ratio to successfully predict stationary future stock returns is still controversial, see e.g. Spiegel (2008). However, recent studies investigating the link between stock returns and the dividend yield emphasize that the evidence for the predictive relationship is heavily dependent on the considered sample period. The general finding that the PD ratio has been an important predictor before the 1990s, but lost its predictive abilities afterwards is supported by a number studies in this field, see for instance Chen (2009) and Park (2010). Dangl and Halling (2012) find that time-variation in the coefficients of return prediction models is very important. Moreover, they argue that return predictability appears to be linked to the business cycle, whereas decreasing risk premia are associated with expansions and vice versa. During the 1990s, the relationship between the PD ratio and stock returns became fragile, presumably due to the emergence and burst of the dot-com bubble. A possible explanation is that risk premia are time-varying and may be related to business cycles, see Guidolin et al. (2013). Welch and Goyal (2008) find that increased persistence is related to declined predictive power. In a recent contribution, Kim and Park (2013) explain the highly persistent dynamics in the PD ratio by a time-varying long-run relationship between stocks and dividends. They argue that the change in persistence arises from the decreasing number of firms with a traditional dividend-payout policy.

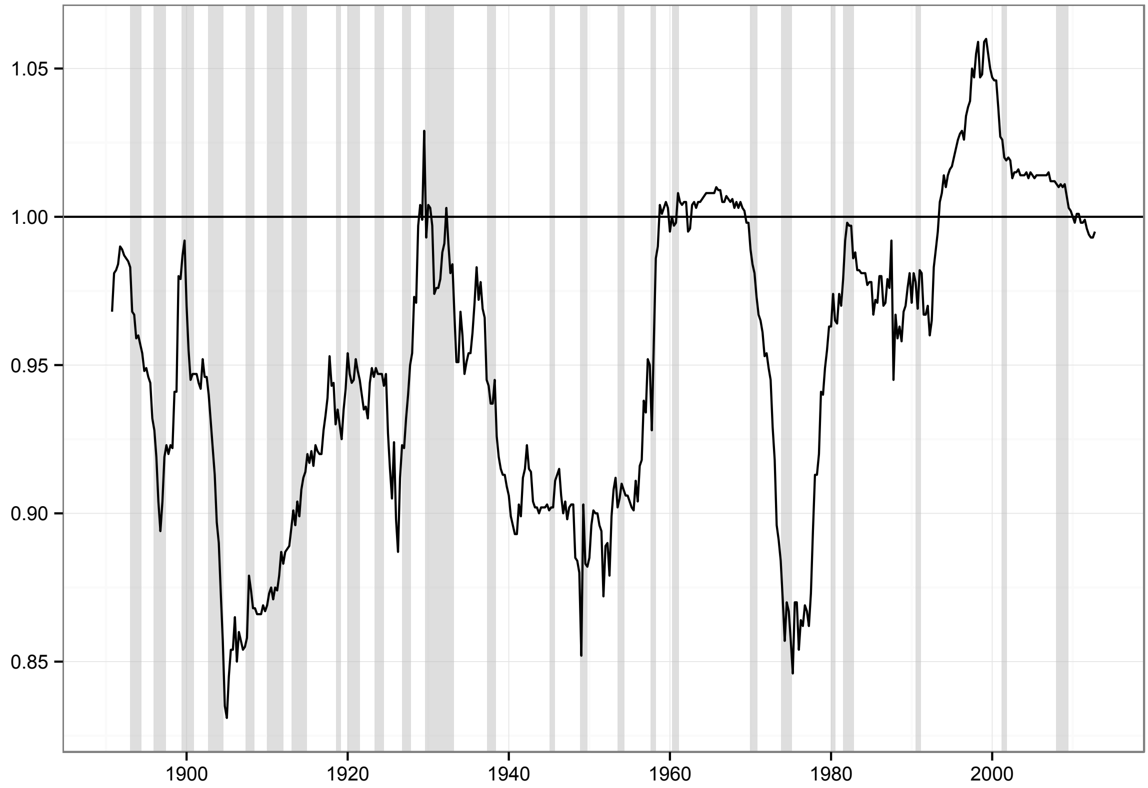

Figure 6.1: Dynamic persistence of the S&P500 PD ratio. Time spans from 1890:Q4 to 2012:Q4 ($T = 489$). Grey areas mark NBER recession periods.

For illustration of the time-varying nature of the PD ratio, we show its dynamic persistence in Figure 6.1. The estimates are based on a rolling window regression with a window size of 20 years corresponding to a long-run perspective. A detailed description on how the estimates are obtained is given in Section 6.3. Eyeballing the series suggests a clear pattern: From the beginning of the sample in the late 19th century until the beginning of the 1990s, the PD ratio appears to be strongly dependent, but stationary most of the time. There are only two deviations, a short peak in 1929 just before the Black Thursday and persistence around the unit root during the 1960s. Moreover, we observe that persistence declines during recession periods in almost all recessions since the 1930s. However, from the 1990s onwards, the PD ratio is clearly non-stationary. There is even some indication for the series to be mildly explosive. Therefore, this period can be seen as the most remarkable in terms of persistence. Recently, Phillips et al. (2013b) propose a testing procedure for multiple bubbles and provide evidence for a bubble emerging in November 1996 and lasting until May 2002. Further evidence is provided in Phillips et al. (2013a).

This work addresses the empirical question if movements in persistence of the PD ratio can be directly related to cyclical macroeconomic activity. A possible channel is the so-called Fed model which is based on the empirical regularity that US government bond yields are surprisingly highly correlated with the S&P500 earnings and dividend yields. This finding has been seen as incompatible with rational valuation of the stock market. In a recent contribution, Bekaert and Engstrom (2010) find that "[...] expected inflation is indeed the primary bond yield component responsible for the high stock–bond yield correlation." Besides, the authors provide an explanation (alternative to money illusion) that is based on their finding that high levels of expected inflation are connected with pronounced economic uncertainty. Recessions take a special role as stagflation periods are responsible for these high correlations. The Fed model immediately suggests a link between the PD ratio (which is highly correlated with its persistence) and factors driving long-term interest rates in the US. According to the Gordon model and to the results in Bekaert and Engstrom (2010), this would be expected inflation and inflation risk premia. In addition, based on Fama's (1981) proxy hypothesis, stock returns and inflation are negatively linked. In a related study Ludvigson and Ng (2009) find that the risk premium of bonds can be predicted by macroeconomic fundamentals. By taking all these results together, macroeconomic fundamentals have the potential to affect persistence in multiple ways.

We apply a flexible econometric framework to study the role of some of the most important US macroeconomic variables for movements in the persistence of the S&P500 PD ratio. In particular, we first estimate the time-varying persistence in a rolling window scheme. Estimation of persistence is a complicated task as a heavy bias is present when the sample size is small and the true and unknown persistence is simultaneously in the vicinity of the unit root. Both features are predominant in our analysis and we tackle the bias problem by applying a suitable indirect inference estimator, recently proposed by Phillips et al. (2011). As a second step, we relate the estimated persistence over time to a large number of potential macroeconomic determinants measuring the condition of the monetary system (by using e.g. inflation series and term spreads), condition of the banking sector (as measured by return on average equity and net interest margin for all US banks) and general business cycle variables (i.e. industrial production and consumption amongst others). Moreover, we exploit the Survey of Professional Forecasters (SPF) which serves as a rich data set on forecasts (one to four quarters ahead) for many macroeconomic and financial variables. Thus, in addition to the current state of the economy we also investigate the role of its expected future development which is probably even more important.

We deal with the high dimensional data set by model averaging techniques. The main advantage is that the final results are not driven by a single model which can be easily biased. Instead, the final outcome is a weighted average of all estimated models including all possible subset combinations of variables. By using this approach, we can handle a large set of economic variables and still retain the standard interpretation of estimated coefficients, which is an advantage in comparison to e.g. factor models. The fact that dynamic persistence is not observed, but a generated regressand is taken into account by a correction of standard errors and the coefficient of determination based on Dumont et al. (2005). In total, we have 138 variables at our disposal and we provide a comprehensive study of whether estimated persistence varies with macroeconomic fundamentals.

We find a pronounced pro-cyclical variation in the persistence. Moreover, the movements can be linked to US macroeconomic fundamentals. In particular, a low expected inflation, high asset returns of all banks, positive expected consumption growth and increasing consumer sentiment are related to high levels of persistence. For some of these variables, the effect changes during recession periods. Around 66% of the variation in persistence (estimated over rolling windows covering 20 years of data) can be explained. Our results are robust to several variations in the data set and to changes in specifications. The findings are new to the literature and are discussed in the light of the Fed model. We argue that our main finding, namely the positive link between expected inflation and the persistence of the PD ratio, is consistent with the Fed model. Most of our findings are also consistent with a heterogenous agent asset pricing model that features chartist and fundamentalist traders. In a paper related to ours, Lof (2012) finds strong evidence in favor of the hypothesis that financial agents base their expectation about future stock market outcomes on macroeconomic information. He studies a nonlinear dynamic time series model and finds that persistence increases during favorable economic conditions and vice versa.

The rest of this article is organized as follows: We review further related literature in Section 6.2. In Section 6.3 the econometric procedure is presented in detail. Section 6.4 describes the data set while the empirical results including discussion are presented in Section 6.5. Robustness checks are given in Section 6.6. Section 6.7 concludes.

6.2 Further related literature

Our paper is also related to another strand of literature featuring the macro-finance link. In particular, many of the related papers consider the link between *financial volatility* and economic variables. Paye (2012) and Christiansen et al. (2012) investigate the importance of economic variables for the prediction of realized financial volatility measures. Their main result is that evidence for the predictive power of macroeconomic determinants is given. We follow their approach to a certain extent and also consider a dimension reduction of the initial set of determinants. Secondly, we also make use of model averaging techniques in our study. Conrad and Loch (2012) use a GARCH-MIDAS model to investigate the relationship between the business cycle and stock market volatility. They find a strong counter-cyclical behavior using FRED data and SPF survey data.

Other articles are investigating the connection between (time-varying) persistence and economic variables: Imbs et al. (2003) follow up on Obstfeld and Taylor (1997) and analyze the impact of economic variables on estimated persistence of relative prices. Spierdijk et al. (2012) provide evidence for time-varying persistence in stock markets by using rolling window methods. Their results suggest that the speed at which stocks revert to their fundamental value is higher in periods of high economic uncertainty. Conrad and Eife (2012) consider rolling window estimation of persistence for inflation-gap series. They relate its time-varying persistence to estimated reaction coefficients on inflation and the output gap in the context of a forward-looking Taylor rule.

Stengos and Yazgan (2013) consider long memory models and find trade variables and sticky prices to be mainly responsible for the slow adjustment of real exchange rates to Purchasing Power Parity. Rengel et al. (2013) use a nonlinear state space model to allow for a time-varying steady state in the PD ratio. They conclude that the current state of the PD ratio can be linked to macroeconomic factors.

The OLS estimator is known to be heavily downward-biased for autoregressive processes in small samples. This is especially the case when using rolling window techniques. The bias increases when the roots of the process are close to unity. The problem persists for mildly explosive processes. We apply the indirect inference estimator as proposed by Phillips et al. (2011) to correct the bias. In a companion paper, Kaufmann and Kruse (2013) compare a variety of different approaches for bias-correction in a large-scale Monte Carlo study. They compare an analytic correction method (see Roy and Fuller, 2001), a bootstrap-based estimator (see Kim, 2003) and jackknifing (see Chambers, 2013) to the indirect inference estimator. Their results demonstrate the usefulness of the indirect inference estimator over the other approaches, in particular in empirical applications. The estimator is also robust against various kinds of misspecifications. Furthermore, it shows excellent performance in terms of mean squared error (MSE) for highly persistent and possibly mildly explosive processes.

6.3 Econometric approach

The central goal of this work is to select variables explaining the time variation in the persistence of the PD ratio. Because of the large number of possible potential determinants, we set up a three step procedure. The matrix of determinants is denoted by Z, which is a $T \times K$ matrix with possibly $K > T$, where K denotes the number of variables and T is the length of the series. Typically, T depends on the particular variable, i.e. T differs across variables leading to an unbalanced data set. With the PD ratio and this type of explanatory data the procedure can be summed up as follows:

Step 1: Estimate the dynamic persistence ρ_t of the PD ratio via indirect inference using a rolling window scheme.

Step 2: Run OLS on the regression model

$$\widehat{\rho}_t = \gamma^i Z_t^i + \epsilon_t^i$$

for $i = 1, 2, ..., K$ and select

$$\mathcal{Z} = \left\{ Z^i \middle| |t_{\gamma^i=0}| > cv_\alpha \right\}$$

with $k = \dim(\mathcal{Z}) \leq \dim(Z) = K$, $t_{\gamma^i=0}$ being the t-statistic of the hypothesis $H_0 : \gamma^i = 0$ and cv_α denoting the critical value for a chosen significance level α.

Step 3: Run $M = 2^k - 1$ OLS regressions of all possible combinations of subsets of $\mathcal{Z}$ on $\widehat{\rho}_t$. Then do model averaging across all estimated models.

In the following, we describe the steps of the procedure in more detail.

Step 1: Persistence estimation

For our rolling window estimation scheme, an autoregressive (AR) model of order p is specified:

$$y_t = \mu + \rho y_{t-1} + \sum_{i=1}^{p-1} \tau_i \Delta y_{t-i} + v_t, \tag{6.1}$$

where y_t denotes the price-dividend ratio and ρ equals the sum of autoregressive coefficients. Lag selection is done via AIC based on the full sample as suggested in Kaufmann and Kruse (2013). The window size w is set to 80 (corresponding to 20 years of quarterly recorded data) in the benchmark case.

The OLS estimation of ρ in the AR model (6.1) is heavily biased in small samples and if

$$\rho \in \left(1 - \frac{c}{T}, 1 + \frac{d}{T^\delta}\right),$$

with $c, d > 0, \delta \in (0,1)$ (i.e. in the vicinity of unity), see Phillips and Magdalinos (2007). In order to cope with the OLS bias, we apply the indirect inference estimator. In particular, the indirect inference estimator (see Phillips et al., 2011) is given by

$$\widehat{\rho}_H^{II} = \arg\min_{\rho \in \Theta} \left\| \widehat{\rho} - \frac{1}{H} \sum_{h=1}^{H} \widehat{\rho}^h(\rho) \right\|,$$

where Θ is a compact parameter space and $\|\cdot\|$ is a quadratic distance metric. The h-th simulated OLS estimate depending on the true and unknown parameter value ρ is denoted as $\widehat{\rho}^h(\rho)$. H denotes the total number of simulated paths and for $H \to \infty$ one obtains

$$\widehat{\rho}^{II} = \arg\min_{\rho \in \Theta} \left\| \widehat{\rho} - q(\rho) \right\|,$$

where $q(\rho) = E\left(\widehat{\rho}^h(\rho)\right)$ is the so-called binding function. In our empirical analysis, the number of simulated paths equals 1,000. Given invertibility of q, the indirect inference estimator results as

$$\widehat{\rho}^{II} = q^{-1}(\widehat{\rho}).$$

For convenience, we use the short-hand notation $\widehat{\rho}_t$ instead of $\widehat{\rho}_t^{II}$ in the following.

Step 2: Dimension reduction of Z

We regress every single element of Z_t on the dynamic persistence. The estimation of the regression model $\rho_t = \gamma^i Z_t^i + u_t^i$ is infeasible because ρ_t is unobserved. From step 1, we obtain $\widehat{\rho}_t = \rho_t + \varepsilon_t$, where ε_t is the estimation error. Let σ_ρ^2 denote its variance. Therefore, a feasible regression is

(see Dumont et al., 2005)

$$\widehat{\rho}_t = \gamma^i Z_t^i + (u_t^i + \varepsilon_t).$$

Neglecting the fact that the regressand is estimated leads to an upward-bias in absolute t-statistics and the R^2. Under the assumption that u_t^i and ε_t are independent of each other, a correction factor for the t-statistics and for the coefficient of determination R^2 can be constructed along the lines of Dumont et al. (2005):

$$\lambda^i = \frac{\sigma_\rho^2 + \sigma_{u^i}^2}{\sigma_{u^i}^2} \geq 1.$$

As ρ is estimated in a rolling window fashion, a sequence of estimated variances for $\widehat{\rho}$ is obtained. We use the median of the sequence to measure the overall estimation uncertainty. Another issue is the widely acknowledged problem of heteroscedasticity and autocorrelation in the residuals. We additionally employ HAC standard errors following the suggestions made in Andrews (1991). The intercept is omitted as data are standardized.

Dimension-reduction is achieved by considering the absolute value of the robust t-statistic for testing $H_0 : \gamma^i = 0$. We construct the reduced set of determinants as follows: $\mathcal{Z} = \{Z^i \big| |t_{\gamma^i=0}| > cv_\alpha\}$ meaning that only variables with a t-statistic being significant at the nominal $\alpha = 30\%$ level are further considered. Typically, we obtain $k = \dim(\mathcal{Z}) < \dim(Z) = K$ and $k < T$. As some variables in $\mathcal{Z}$ are likely to be highly correlated with each other, we also exclude further variables from $\mathcal{Z}$. A variable is dropped from the final set if:

1. the correlation between the variable and any variable with a larger absolute t-value exceeds 0.7,
2. if another forecast horizon from the same variable exhibits a larger absolute t-value,
3. it has less than 100 non-NA observations.

The first restriction deals with the potentially upcoming multi-collinearity problem in the subsequent multiple regression models. The second restriction ensures that we only consider the most important variable amongst different horizons for data, while the third requirement ensures a balanced sample in the end for ease of comparison.[1]

Step 3: Estimation of all possible models and model averaging

We achieve final results by model averaging. In contrast to model selection, where a single model is selected and interpreted, all models contribute to the final parameter estimates. For the construction of a model averaging estimator all possible models are estimated and a smoothed weight is assigned to each model. The weight depends on the relative performance of the model in terms of an information criterion. The appeal of this method stems from the fact that it

[1] As these values are carefully chosen but quite liberal, we experiment with more conservative settings as a robustness check. It has hardly any impact on the estimated coefficients and conclusions.

provides some kind of insurance against the selection of a poor model without losing the standard and straightforward interpretation of estimated regression coefficients. The smoothed AIC model averaging approach has been suggested by Buckland et al. (1997) and is further developed in contributions by Burnham and Anderson (2002) and Hjort and Claeskens (2003).

Following the model averaging approach, all possible sub-models of the reduced data set $\mathcal{Z}$ are estimated via OLS. This leads to $M = 2^{k'} - 1$ models, where k' denotes the number of variables in $\mathcal{Z}$. The model containing only an intercept is dropped due to standardization. For each estimated model, we compute information criteria (AIC and BIC) and the corresponding smoothed weight. The weight for a model $m \in \{1, 2, ..., M\}$ is given by, see e.g. Hansen (2007):

$$\omega^m = \frac{\exp(-\frac{1}{2} IC^m)}{\sum_{m=1}^{M} \exp(-\frac{1}{2} IC^m)} \in (0, 1),$$

where IC^m denotes the value of an information criterion for model m and $\sum_{m=1}^{M} \omega^m = 1$. The model averaging (MA) estimator for the k'-dimensional parameter vector is given by:

$$\widehat{\gamma}_{MA} = \sum_{m=1}^{M} \omega^m \widehat{\gamma}_0^m,$$

with $\widehat{\gamma}_0^m$ being $\widehat{\gamma}^m$ augmented with zeros in the case of $m < M$ (due to zero-restrictions on a number of coefficients). For comparison, we also consider the best performing models in terms of AIC and BIC.

6.4 Data

We obtain publicly available data from Robert Shiller's website at Yale University (iedata.xls, see http://www.econ.yale.edu/~shiller/data.htm), the Federal Reserve of St. Louis database (FRED, see http://research.stlouisfed.org/fred2/) and the Survey of Professional Forecasters (SPF, see http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/). Regarding the SPF data, we use the mean of all individual forecasts. The data spreads broadly over the following categories: stock market data, banking sector, monetary system, economic activity and sentiments. In total we have 138 explanatory variables at our disposal. We use quarterly data from 1984:Q1 to 2012:Q4 ($T = 116$ observations) for our explanatory variables (FRED and SPF) and 195 observations for the PD ratio to account for the rolling window estimation. In principle, we could have started our analysis from 1968:Q4, but some important variables are only available from 1984:Q1 onwards. Monthly data is aggregated to quarterly frequency by averaging. All series are differenced if needed to ensure stationarity. Besides, a recession dummy according to the NBER dating is included. A detailed description of all series is provided in the Appendix 6.A.1 to this paper.

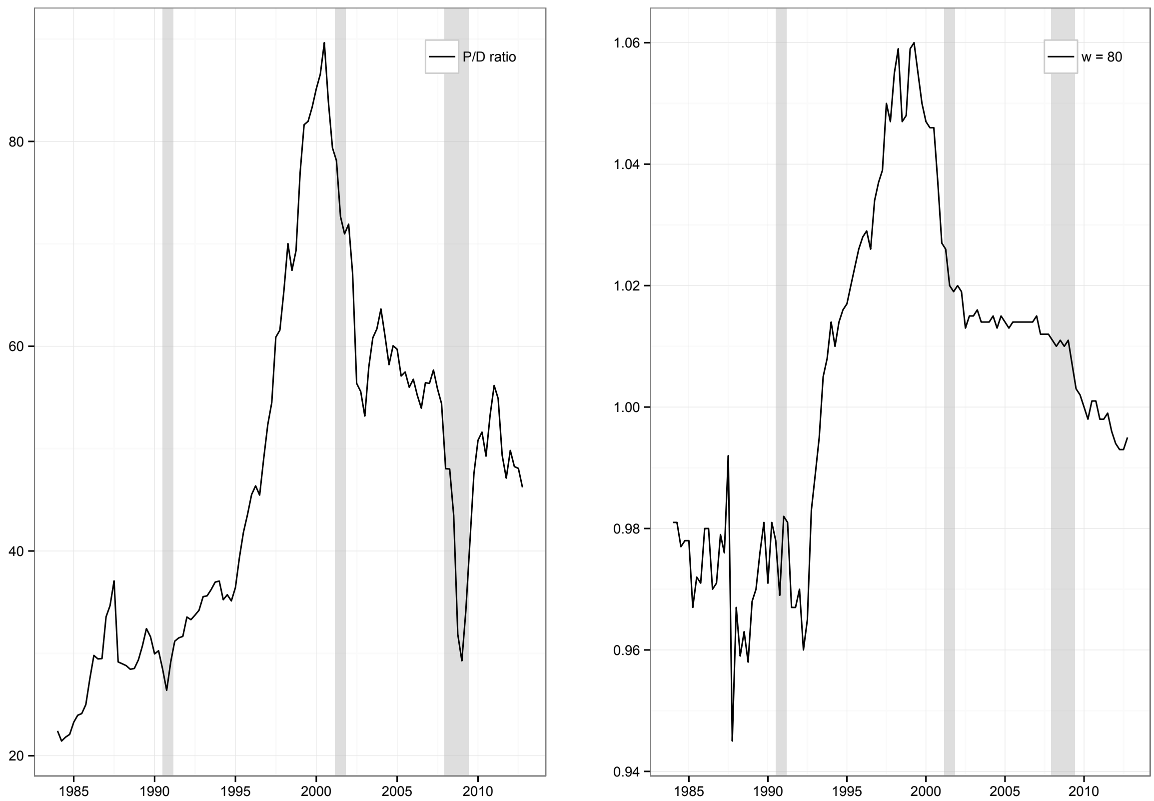

Figure 6.2: PD ratio (left) and estimated persistence (right). Time span from 1984:Q1 to 2012:Q4 ($T = 116$).

6.5 Empirical results

6.5.1 Dynamic persistence

We obtain the dynamic persistence of the PD ratio in a rolling window scheme with $w = 80$ observations. The autoregressive lag structure equals $\widehat{p} = 2$ according to AIC.[2] The PD ratio and the time series of estimated persistence are presented in Figure 6.2. In the very beginning of the sample relatively strong persistence is observed (around 0.97) with a considerable degree of fluctuations. During the mid to late Nineties, persistence increases remarkably. Towards the end of the 1990s, two spikes at 1.06 are suggesting mild explosiveness. Such behavior indicates that prices and dividends are presumably no longer cointegrated rendering the PD ratio non-stationary. The period of significant explosiveness (based on a 95% confidence interval, see Phillips et al. (2011) and Phillips (2012) for details on the exact construction) lasts 13 quarters and ranges from 1997:Q3 to 2000:Q3. During this period, point estimates of the indirect inference estimator are significantly different from unity. This finding is in line with previous results on the emergence and the burst of the dot-com bubble. Confirmative results are obtained by running the Phillips et al. (2011) test on the individual price and dividend series. The persistence of the PD ratio lowered considerably after the burst of the dot-com bubble in

[2]The maximal lag length is given by $p_{\max} = [12(T/100)^{0.25}] = 14$ with $T = 195$.

	Min	Median	Mean	Max	n
PD ratio	16.6492	33.2925	38.9521	89.6442	195
Estimated persistence	0.9450	1.0105	1.0045	1.0600	116
… during recessions	0.9690	1.0110	1.0069	1.0260	11
… during non-recessions	0.9450	1.0100	1.0042	1.0600	105

Table 6.1: Summary statistics for the PD ratio and estimated persistence.

2000. During the recent global financial crisis the persistence appears to be close to unity and the last three years (2010-2012) indicate that it tends against the level as in early Nineties again.

The summary statistics in Table 6.1 indicate high persistence on average. Moreover, when splitting the sample into recession and non-recession periods, it becomes clear that persistence is near unity during recessions. In contrast, explosive and stationary regimes are present during non-recession periods. Figure 6.2 suggests that both the PD ratio and its persistence drop during ongoing recessions as expected. There are three recessions (according to the NBER classification) in this sample and the total duration equals eleven quarters. We investigate the asymmetric behavior of persistence below in more detail. When estimating the persistence over the entire sample period, i.e. by using a single window with size of $T = 195$ observations, the resulting estimate is $\widehat{\rho}^{II} = 1.01$. This result indicates high persistence in general and is in line with the mean and median statistics for the window size of 80 observations.

6.5.2 Preliminary analysis

In a next step, we consider simple regression models, see Section 6.3 Step 2. In these regressions, only a single element of Z is analyzed. The regression model without recession dummy D_t is given by:

$$\widehat{\rho}_t = \gamma^i Z_t^i + \epsilon_t.$$

De-standardized OLS point estimates and corrected t-statistics of all variables which are significant at the nominal level of $\alpha = 0.3$ are given in Table 6.2. Further results are located in the Appendix 6.A.2. The reported coefficient estimates can be interpreted via the relationship $\widehat{\gamma}^i = \partial \widehat{\rho}_t / \partial Z_t^i$. All variables Z_t^i except of one-year ahead consumer sentiment (UMCSENT) are measured in percentages which eases interpretation. For example, a one-unit increase in one-quarter ahead expected inflation (e.g. dpgdp3) comes along with a decrease of persistence by 0.018. The corresponding t-value is -5.467 and the estimated correction factor equals 1.035 suggesting a highly significant variable and a mild increase in the variance due to first-stage estimation of persistence.

The variables are sorted by the absolute value of the t-statistic in descending order. Amongst the 51 significant variables are mostly series measuring (i) inflation (dpgdp2-6, CPI2-6, MICH,

Variable	γ^i	t-stat	λ^i	Variable	γ^i	t-stat	λ^i
dpgdp3	-0.018	-5.467	1.035	drresinv4	-0.002	-1.669	1.012
dpgdp6	-0.018	-5.358	1.029	SPR-Tbond-Tbill6	-0.013	-1.659	1.003
dpgdp5	-0.018	-5.287	1.030	BOND4	-0.006	-1.659	1.005
dpgdp4	-0.018	-5.217	1.030	drnresin5	0.003	1.647	1.011
CPI6	-0.018	-5.033	1.027	dhousing3	-0.001	-1.640	1.012
CPI5	-0.018	-4.712	1.026	SPR-Tbond-Tbill2	-0.010	-1.619	1.003
CPI4	-0.017	-4.589	1.026	drresinv5	-0.002	-1.617	1.014
dpgdp2	-0.017	-4.535	1.025	SPR-Tbond-Tbill5	-0.012	-1.593	1.003
CPI3	-0.016	-4.316	1.026	CUSR0000SA0L2	-0.006	-1.581	1.009
PSAVERT	-0.009	-3.774	1.016	SPR-Tbond-Tbill4	-0.011	-1.559	1.003
dhousing2	-0.001	-3.375	1.050	GS10	-0.005	-1.556	1.005
CPILFESL	-0.015	-2.865	1.009	SPR-Tbond-Tbill3	-0.011	-1.547	1.003
PCEPI	-0.015	-2.633	1.009	BOND3	-0.006	-1.534	1.004
USROA	0.041	2.561	1.008	STLFSI	-0.003	-1.473	1.039
MICH	-0.020	-2.368	1.016	dhousing4	-0.001	-1.445	1.011
drconsum6	0.018	2.351	1.028	RCBI6	0.001	1.439	1.005
CPI2	-0.008	-2.148	1.013	BOND2	-0.006	-1.394	1.004
drnresin6	0.004	2.106	1.022	drfedgov2	-0.001	-1.388	1.054
drconsum4	0.012	2.090	1.023	drconsum2	0.004	1.337	1.022
drresinv3	-0.001	-2.084	1.028	drconsum3	0.007	1.317	1.018
USROE	0.003	2.019	1.011	RCBI5	0.001	1.263	1.004
BAA	-0.007	-1.927	1.006	SP500-ABS-RET	0.001	1.232	1.013
BOND6	-0.007	-1.806	1.005	drnresin4	0.002	1.109	1.008
BOND5	-0.007	-1.781	1.005	drnresin3	0.002	1.099	1.009
USACPIALLQINMEI	-0.009	-1.761	1.008	UMCSENT	0.001	1.098	1.004
GDPDEF	-0.018	-1.721	1.004				

Table 6.2: Regression of dynamic persistence on a single variable.

USACPIALLQINMEI, CPILFESL, PCEP, CUSR0000SA0L2, GDPDEF), (ii) conditions of the banking sector (USROA, USROE, STLFSI, SP500-ABS-RET), (iii) interest rates (BAA, BOND2-6, SPR-Tbond-Tbill2-6, GS10), investments (drnresin6, drresinv3), (iv) consumer related variables (PSAVERT, drconsum6, dhousing3, UMCSENT) and (v) others (RCBI6, drfedgov2). Somewhat surprising, prominent variables like industrial production, real GDP growth, term spreads and credit risk variables are apparently not significant. However, related variables are contained in the list. For example, the correlation between industrial production and significant investment variables is up to 0.74 and real GDP growth has a correlation of over 0.70 with consumer sentiment, real consumption growth and investment.

The left panel of Table 6.3 reports all selected variables after imposing restrictions on correlation, balancedness and sample size as described in Section 6.3. The estimation uncertainty factor λ is moderate (i.e. less than 1.05) for most series. The dimension-reduced data set now covers eight variables. These are inflation expectations (one quarter ahead), housing starts, return on average assets for all US banks, expected real personal consumption expenditures (one year ahead), expected changes in private inventories (one year ahead), expected real federal government consumption and gross investment, absolute returns of the S&P500 index and University of Michigan: Consumer Sentiment (one year ahead). A visualization of these variables (and

	Rec. dummy *excluded*			Rec. dummy *included*							
Variable	γ^i	t-stat	λ	η^i	t-stat	γ^i	t-stat	δ^i	t-stat	$\gamma^i+\delta^i$	F-pval
dpgdp3	-0.018	-5.467	1.035	-0.015	-0.699	-0.019	-4.482	0.005	0.850	-0.013	0.000
dhousing2	-0.001	-3.375	1.050	0.001	0.040	-0.001	-3.077	0.001	1.266	0.000	0.044
USROA	0.041	2.561	1.008	0.043	2.284	0.050	3.368	-0.035	-2.136	0.014	0.003
drconsum6	0.018	2.351	1.028	0.008	0.212	0.018	2.170	-0.001	-0.093	0.017	0.329
RCBI6	0.001	1.439	1.005	0.033	1.665	0.001	1.923	-0.001	-0.944	0.001	0.036
drfedgov2	-0.001	-1.388	1.054	0.004	0.550	-0.001	-1.384	0.000	-0.239	-0.001	0.089
SP500-ABS-RET	0.001	1.232	1.013	0.005	0.276	0.001	1.076	0.000	-0.310	0.000	0.269
UMCSENT	0.001	1.098	1.004	0.070	1.244	0.001	1.924	-0.001	-0.952	0.001	0.986

Table 6.3: Selected variables from single regressions.

some others, appearing in robustness checks later on) is given in Figure 6.3.

We briefly describe the patterns: (i) There is some discrepancy amongst expected inflation measures over different horizons in the second half of the sample. Expected consumer price inflation is considerably higher than GDP deflator growth. (ii) Expected real growth of private business inventories shows a clear cyclical pattern with a dramatic drop to negative values in the last and most severe recession. (iii) There is only little difference between the actual and expected growth rate of housing starts, but during recessions, the expectations are too optimistic. The dynamic pattern of the series shows an important jump right after the end of the last recession. The growth rate switched its sign and rose by about eighty percentage points. (iv) Changes in federal government expenditures are volatile in the beginning, but much smoother after the beginning of the 1990s where the series fluctuates around zero percent. In response to the worsened economic situation in the US, the growth rates reached positive values and stayed above the zero line most of the time until 2010. (v) The average returns on assets and equities for all US banks show an interest-related pro-cyclical pattern. The first out of two distinct periods is the stock market crash of 1987, where the series took negative values. Returns rose smoothly during the 1990s and did not respond to recessions. During the recent global financial crisis, however, returns dropped in a dramatic way over several quarters reaching its lowest value during this period towards the end of the last recession. Afterwards, a quick recovery of the returns can be observed. (vi) Absolute returns of the S&P500 stock market index are quite volatile and reflect the stock market crash in the late Eighties, the emergence and burst of the dot-com bubble and the ups and downs of the stock prices during the global financial crisis. (vii) Real expected consumption growth and consumer sentiment share a related cyclical path in particular during the last ten years. Before 2003, the relationship between these two series is less pronounced.

The estimated coefficients reported in Table 6.3 hint to the following connection between persistence of the S&P500 PD ratio and the (expected) state of the US economy: Favorable economic conditions (i.e. low inflation, high returns for banks and large consumption growth) are associated with high persistence and vice versa. The positive signs of the coefficients for the expected changes in private business inventories and the University of Michigan Consumer Sentiment Index further support this notion. The positive coefficient estimate for absolute returns can be

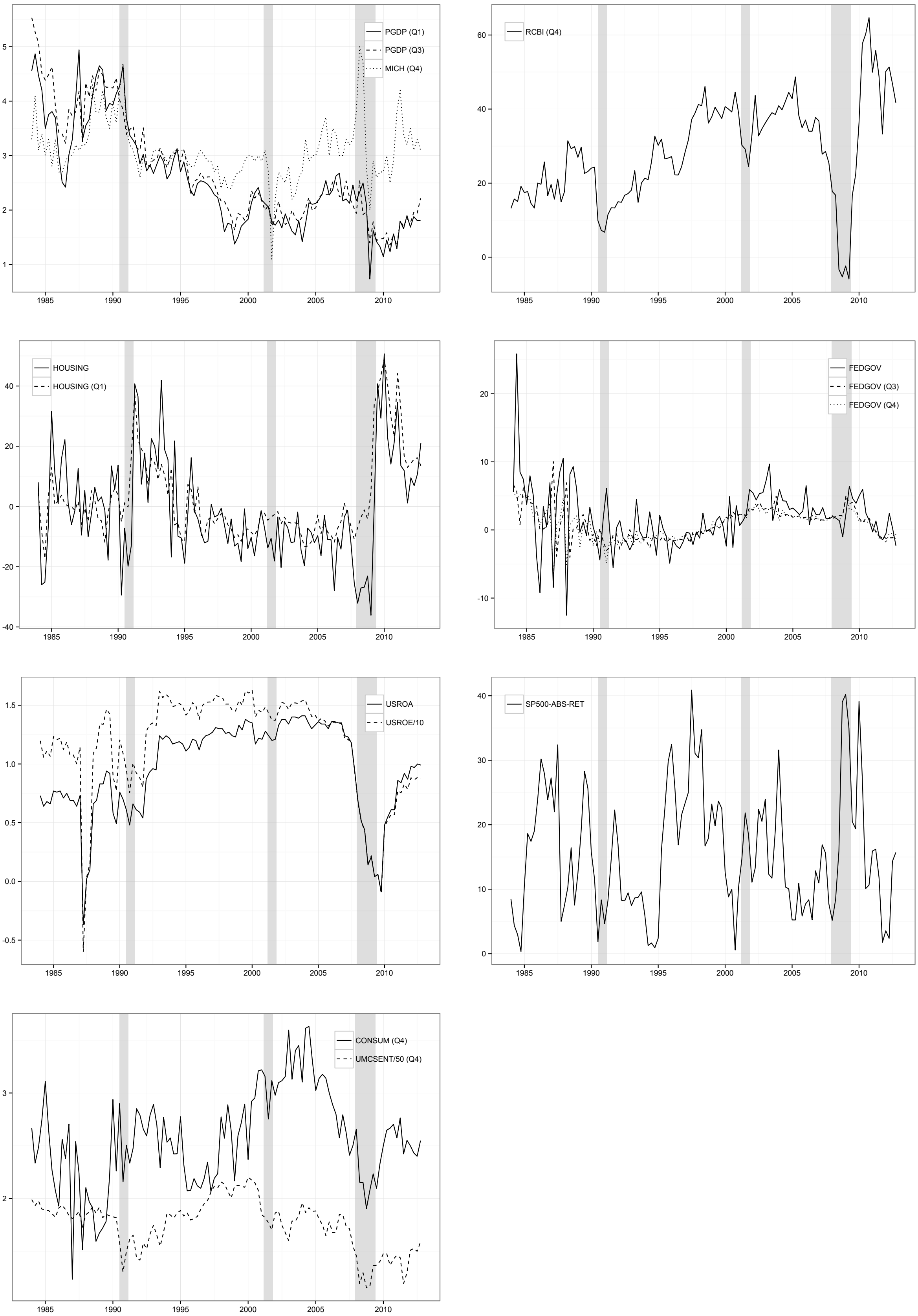

Figure 6.3: Selected explanatory variables.

interpreted as follows: As traders are not restricted to hold long positions in their portfolios only, the one-period absolute returns reflect the role of short-term investment on the S&P500 which positively impacts persistence. This result is in line with the return on average assets for all US banks, which captures different types of investments and horizons.

Expected increases in government expenditures appear to have a stabilizing effect on the persistence of the price-dividend ratio. A possible explanation is that expected increases in growth rates of government spending can be seen as a sign of upcoming unfavorable economic circumstances which requires governmental actions. In fact, when considering the path of governmental consumption, one observes that growth rates increase particularly during recessions and shortly afterwards. Moreover, a clear time trend in growth rates can be seen for the period after the burst of the dot-com bubble until 2003 when the economy was stabilized again. Housing starts played an important role in the recent financial crisis: according to a recent study by Phillips and Yu (2011), "A bubble emerged in the real estate market in February 2002. After the subprime crisis erupted in 2007, the phenomenon migrated selectively into the commodity market and the bond market, creating bubbles which subsequently burst at the end of 2008, just as the effects on the real economy and economic growth became manifest." During the housing bubble, housing starts and housing prices were rising by a large extent. This high degree of co-movement outlived the end of the recession, where both series were rapidly rising again after their dramatic fall in the previous quarters. Therefore, the negative sign of the estimated coefficient for the growth rate of housing starts is in line with the previous conclusions.

In order to investigate the influence of recessions, we also run augmented regressions including the recession dummy D_t and an interaction term, see also Guidolin et al. (2013) for a similar regression,

$$\widehat{\rho}_t = \eta^i D_t + \gamma^i Z_t^i + \delta^i D_t Z_t^i + \epsilon_t. \tag{6.2}$$

Results for these regressions are given in the right panel of Table 6.3. In addition to point estimates and t-values of the coefficients, we also report the sum $\gamma^i + \delta^i$ which measures the impact of Z_t^i on persistence in recession periods. Finally, we test the null hypothesis $H_0 : \gamma^i + \delta^i = 0$ by using a corrected F-statistic (p-values are given in the last column of Table 6.3). A non-rejection of H_0 would indicate that the variable Z_t^i has no impact on persistence during recession periods.

The signs of $\widehat{\gamma}^i$ do not change in comparison to the previous regression results. The intercept shift dummy variable is significant in three cases, for the return on average assets for all US banks, private inventories and consumer sentiment. The slope change is only important for housing starts and the banking variable. The most striking results are obtained for the average return of all US banks. In this case, all regressors are significantly impacting persistence. During recession periods, the impact of the return on average assets drops remarkably as the sum of $\widehat{\gamma}^i$ and $\widehat{\delta}^i$ indicates, but the p-value of the F-statistic suggest that the impact is non-zero even during recession periods. The F-statistic further indicates that most variables are important

	Recession dummy excluded				*Recession dummy included*			
	Model Averaging		Model selection		Model Averaging		Model selection	
Variable	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}
dpgdp3	-0.0097	-0.0097	-0.0190	-0.0200	-0.0092	-0.0091	-0.0169	-0.0169
dhousing2	-0.0002	-0.0002	-0.0004	-0.0004	-0.0002	-0.0002		
USROA	0.0144	0.0144	0.0122		0.0165	0.0164	0.0181	0.0181
drconsum6	0.0014	0.0014	-0.0060		0.0014	0.0014		
RCBI6	0.0001	0.0001			0.0002	0.0002		
drfedgov2	-0.0003	-0.0003			-0.0003	-0.0003		
SP500-ABS-RET	0.0003	0.0003			0.0002	0.0002		
UMCSENT	0.0004	0.0004	0.0008	0.0010	0.0005	0.0005	0.0012	0.0012
dhousing2-rec					0.0000	0.0000		
USROA-rec					-0.0159	-0.0157	-0.0312	-0.0312
dummy-rec					0.0283	0.0282	0.0458	0.0458
λ	1.0427	1.0426	1.0572	1.0891	1.0421	1.0420	1.0340	1.0340
R^2	0.6612	0.6600	0.7291	0.7116	0.7021	0.7009	0.7472	0.7472
AIC			-1.2286	-1.2005			-1.2978	-1.2978
BIC			-1.1099	-1.1293			-1.1791	-1.1791
$100 \cdot \omega_{AIC}$			0.5065	0.4994			0.5069	0.5069
$100 \cdot \omega_{BIC}$			0.5014	0.5063			0.5141	0.5141

Table 6.4: Model averaging and model selection results.

during both, recession and non-recession periods. We find for the majority of series that $\widehat{\gamma}^i$ and $\widehat{\delta}^i$ are of different sign and that the intercept shift dummy is positive which confirms our descriptive statistics in Table 6.1.

While most of the selected variables come along with an increase in persistence during favorable economic conditions and vice versa, some kind of asymmetric behavior is found when considering recession periods separately. The next step is to extend the regression setup and analyze all possible subsets of combinations of the variables collected in Table 6.3 via model averaging.

6.5.3 Main results

The model averaging approach is conducted in two versions: one is based on the smoothed AIC, while the other one uses the BIC. Moreover, we consider the two different cases where a recession dummy and its interaction terms are either excluded or included. Interaction terms are only included for those variables whose t-statistic for $H_0 : \delta^i = 0$ is significant, see Table 6.3. In addition to the model averaging results, we report the outcome for the best performing individual model in terms of AIC and BIC. This allows a direct comparison between the model averaging and the model selection approach. Results are reported in Table 6.4. In addition to the OLS point estimates γ, we report the average correction factor λ, the coefficient of determination R^2, the AIC and BIC themselves as well as scaled model weights ω^m. The left panel presents results where the recession dummy and its interactions with regressors are excluded, while the right panel contains extended regression results.

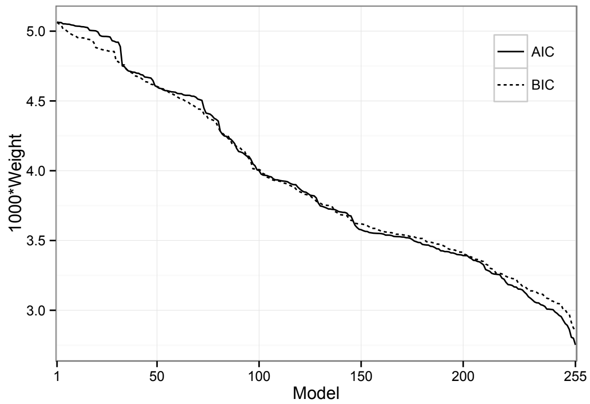

Figure 6.4: Model averaging weights.

The model averaging results are mainly in line with the previous analysis. We observe that the outcomes are robust in the sense that they do not depend on the particular information criterion in use. This finding continues to hold throughout the whole analysis. In order to judge the importance of different variables, we consider the standardized regression coefficients. They suggest the following ranking: expected inflation (-0.348), average returns over assets (0.212), consumer sentiment (0.174), housing starts (-0.142), absolute returns (0.097), real business inventories (0.062), federal expenditures (-0.046) and lastly, expected consumption (0.024). Thus, the three important determinants are expected inflation, average returns of all US banks and consumer sentiment. The first one has a negative impact on persistence, while the latter two have a positive one. The averaged point estimate for absolute returns is in line with the sign for the average returns for all US banks. A positive economic outlook with a horizon between a quarter and a year (e.g. consumption growth, consumer sentiment and private business inventories) has also a positive effect on the persistence properties of the S&P500 PD ratio. Governmental activity is found to have a down-calming effect on persistence and thereby supporting mean reversion in stock markets. Housing starts have a negative effect on persistence which is in line with our preliminary analysis.

The model weights ω^m are shown in Figure 6.4. As $k' = 8$, we estimate 255 models in total. The graph shows weights of the individual models, sorted from the best to the worst performing model. It is clearly seen that even the model weights for the best performing models are rather small which is due to the fact that we consider all possible subset combinations of the variables. Essentially, each variable enters the $\left(2^{k'} - 1\right)$ different models $2^{k'-1}$ times. Because the information criteria of all models are very close to each other, the model selection via AIC or BIC is a rather risky task. Therefore, model averaging is of great importance to achieve robust results.[3] When looking at the results for the individually top performing models in terms of AIC and

[3]One could expect larger differences between the model weights when considering the full data set. We have selected a number of eight promising variables in an automatic way before entering the step of model averaging and thus it is obvious that all variables bear some relation to the estimated persistence. It is therefore less surprising that the model weights are small and close to each other. This is supported by the R^2, which is greater than 0.5 for more than half of the models.

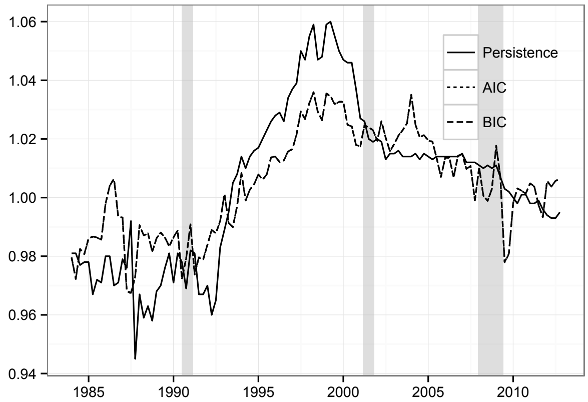

Figure 6.5: Model averaging fit (AIC and BIC, recession dummies included).

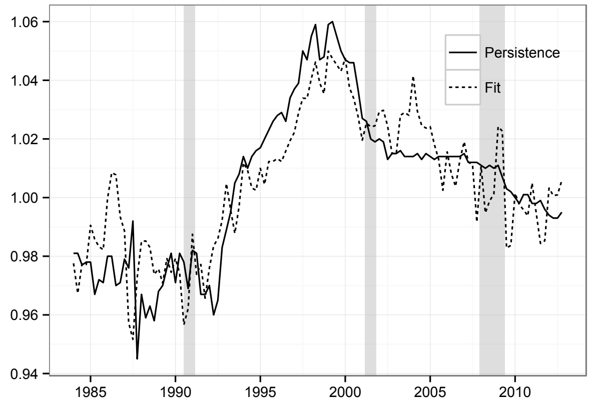

Figure 6.6: Fit of the best model chosen by AIC (recession dummies included).

BIC, we find that expected inflation, housing starts and consumer sentiment are included. The best AIC model also includes the average returns of all US banks and consumption growth. All signs of estimated coefficients are the same except of consumption in the best AIC model. The main conclusions remain unchanged when considering these models, but the model averaging approach provides more insights in the relation between persistence and economic variables.

We now turn to the right panel of Table 6.4 and consider the results when the regression models are augmented with a recession dummy and its interactions with some regressors. The results for the model averaging approach are fairly robust. Signs do not change and averaged point estimates remain nearly the same as before. Newly included are the recession dummy with an expected positive coefficient estimate and the interaction term of the average return of all US banks and housing starts. The effect of the banking variable on persistence during recessions is close to zero, but still positive. An opposite picture is drawn for housing starts: the effect of recessions is nearly invisible and the effect is still negative. The recession dummy and its interaction with the return on average assets of all US banks are also included in the top performing models according to AIC and BIC model selection. For these we find that the effect of the bank returns is negative during recessions, which is partly accommodated by a large point estimate for the recession dummy. Similar to the case without the recession dummy, we find inflation and consumer sentiment to be important in all models.

Figures 6.5 and 6.6 present the fit of the model averaging approach and for the best AIC model, respectively, with included recession dummies. Unsurprisingly, the fit of the best AIC model is better than the one of the model averaging. The corresponding coefficients of determination are 0.75 and 0.70, respectively. The general evolution is, however, quite similar. The best AIC model is driven by three variables: inflation, average returns of all US banks and consumer sentiment. While the impact of bank returns is almost the same as in the model averaging case, it is doubled for the other two. The dynamic persistence is well captured with some pronounced deviations in the end of the Eighties, around 2003 and after the recession in 2009. Both models underestimate the persistence during the explosive period, which might indicate that bubbles are not explainable by macroeconomic determinants. Another interesting point is the deviation in 1987. Both models expect a fall in persistence a quarter before the Black Monday, basically the macroeconomic indicators are one quarter ahead around this point in time. This is another example of the difficulty to model such an extreme event.

6.5.4 Discussion

We discuss our results in the light of the Fed model and an asset pricing model with heterogenous beliefs. The Fed model describes a relation between nominal bond yields and real equity yields. The observed strong correlation between the dividend or earnings yields and long-term government bond yields (as measured by bonds with a maturity of ten years) is reported and discussed in many studies, see e.g. Bekaert and Engstrom (2010). For our data set, we compute correlations between (i) the dividend yield (i.e. the inverse of the PD ratio) and bond yields which gives 0.754, (ii) the PD ratio and bond yields resulting in -0.564, (iii) the PD ratio and its dynamic persistence (0.824) and finally, (iv) the dynamic persistence and bond yields (-0.469). The results confirm previous findings on older data sets. The last correlation coefficient is negative and close to the one for case (ii), which is in line with our results reported in Table 6.2: All estimated coefficients for (expected) bond yield variables are negative. In addition, the estimated coefficients are pretty stable over the expectation horizon.

As suggested by the Gordon model, the components of the equity cash yield (EY) and the nominal bond yield (BY) are as follows:

$$EY = -EDIV + RRF + ERP$$

and

$$BY = EINF + RRF + IRP$$

respectively. $EDIV$ denotes expected growth of real equity dividends, RRF is the real risk free interest rate, ERP is the equity risk premium and $EINF$ stands for expected inflation and IRP is the corresponding inflation risk premium. According to this decomposition, expected inflation ($EINF$) should also have a negative impact on persistence as BY and $EINF$ are positively related. In fact, long-term government bond yields reflect long-term inflation expectations to a certain degree. Our findings are fully consistent with this interpretation.

In contrast to BY, EY is driven by real components instead of nominal ones.[4] The positive comovement between $EINF$ and ERP, as found by Bekaert and Engstrom (2010), links bond yields to equity yields. Furthermore, expected inflation correlates positively with risk aversion (based

[4] According to the financial literature it is not possible to rationally argue why expected inflation shall impact real components determining the equity yield. So far, the main explanations are money illusion and behavioral biases of investors, see Bekaert and Engstrom (2010).

on consumption) and real economic uncertainty (based on GDP forecast dispersion) which both can be seen as a rational time-varying risk premia. These two variables are key ingredients of sophisticated asset pricing models, see Bansal and Yaron (2004) and Campbell and Cochrane (1999), where risk premia rise when the economy is growing slowly or even contracting. It is reasonable to assume that consumption-based risk aversion is negatively related to consumer sentiment, see Cooper and Priestley (2009). Our results for the consumer confidence over the next year suggest a positive impact on persistence. Therefore, risk aversion is expected to negatively impact persistence, similar to expected inflation. Regarding real economic uncertainty, we use a cross-sectional forecast dispersion measure from the SPF data set.[5] The correlation between expected inflation and real economic uncertainty is found to be equal to 0.337, thereby confirming a mild positive relationship. Importantly, persistence and uncertainty are negatively correlated (-0.340). We conclude that our results are consistent with the Fed model and the predictions made by Bekaert and Engstrom (2010).

We provide an alternative explanation for our results which builds on a heterogenous agent model for financial markets, see Brock and Hommes (1997, 1998). The model features two distinct types of traders with bounded rationality. Chartists and fundamentalists react differently to observed mispricings in the market: while chartists believe in continued and even larger mispricings in the next period, fundamentalists expect a correction towards the fundamental value. Therefore, chartists would take a long position in an over-valued asset, whereas fundamentalists would take a short position instead. The model leads to a dynamic equation for the price-dividend ratio,

$$y_t = \alpha_C y_{t-1} \cdot G_t + \alpha_F y_{t-1} \cdot (1 - G_t) + u_t,$$

where $\alpha_C \geq 1$ and $\alpha_F < 1$ are autoregressive parameters depending on the demand function of chartists and fundamentalists, respectively. The time-varying fraction of traders is denoted by $G_t \in [0, 1]$. The previous equation can be slightly re-written as $y_t = \rho_t y_{t-1} + u_t$, where ρ_t denotes a time-varying autoregressive parameter in the model for the PD ratio. As long as $\rho_t \geq 1$ holds, chartists are dominating the market in period t. If $\rho_t < 1$, the market is dominated by the group of traders believing in pricing error correction.

A first and important insight form this model is that the persistence of the PD ratio can be time-varying due to a dynamic composition of traders in the market with heterogenous beliefs about future developments. Second, this model is also able to explain asset price bubbles which would be possible when chartists are dominant for a certain while and when their demand function parameters are large enough to generate further mispricings in the market. Empirical evidence for the model and its ability to explain the bubble-type characteristics during the 1990s is provided in Boswijk et al. (2007). While the main strand of the literature followed the idea that the fraction of traders G_t is entirely based on evolutionary dynamics (i.e. past realized profits of the trading strategies), Lof (2012) investigates the possibility that traders update

[5] It is defined as the difference between the 75th percentile and the 25th percentile of the individual projections for real GDP growth one year ahead, expressed in annualized percentage points.

their beliefs according to macroeconomic factors like e.g. inflation, real GDP growth and the term spread. His main findings are that the persistence of the PD ratio increases during times of positive economic conditions (as measured by industrial production growth) due to chartists dominance. On the contrary, persistence decreases in times of bad economic news and the degree of mean reversion is strengthened due to traders who believe in the importance of fundamentals. The results can be explained by traders being less risk averse during economic upswings and following chartism, see Cooper and Priestley (2009) and Chiarella et al. (2009). An increased speed of mean reversion during times of high uncertainty, possibly due to an enlarged fraction of fundamentalists, is found also by Spierdijk et al. (2012). Our results are consistent with these findings and conclusions.

6.6 Robustness checks

In order to investigate the robustness of the main results, we consider a variety of different settings. The first robustness check deals with the stability of estimated coefficients over time. To this end, a recursive analysis of the multiple regressions is undertaken. It can possibly happen that the relationship between expected inflation, average returns over assets for banks and the consumer sentiment is unstable. The second robustness check is concerned with the possibility that lagged macroeconomic fundamentals have predictive power. Therefore, one-quarter ahead predictive regressions are studied.

Due to specific settings in the data construction and the econometric modeling approach, some variations are considered. A key issue regarding the SPF data set is the aggregation of individual forecasts. In contrast to using the mean, we use median forecasts which are less influenced by a stronger disagreement among the professional forecasters. This might be especially important during recessions. Next, we tackle the measurement of recession periods by using recession probabilities instead. The merit of using these stems from the fact that they provide further information on the strength of recessions. Moreover, the rolling window size for estimating the persistence is varied.

The observation from all considered variations is that the results are robust and that main conclusions are not changing. The notion that increases in persistence over time can be linked to favorable economic expectations regarding the monetary sector, the banking industry and consumption is supported throughout the whole analysis. Moreover, the role of asymmetry during recession periods persists.

6.6.1 Recursive regression

We start by estimating the best AIC model including the recession dummy and its interaction with banking returns for the period 1984:Q1 to 1996:Q2 ($T = 50$). Next, we add the observations for the subsequent period to our data set and re-estimate the model. We proceed in this way until we obtain the full sample estimates reported already in Table 6.4. The evolution of estimated

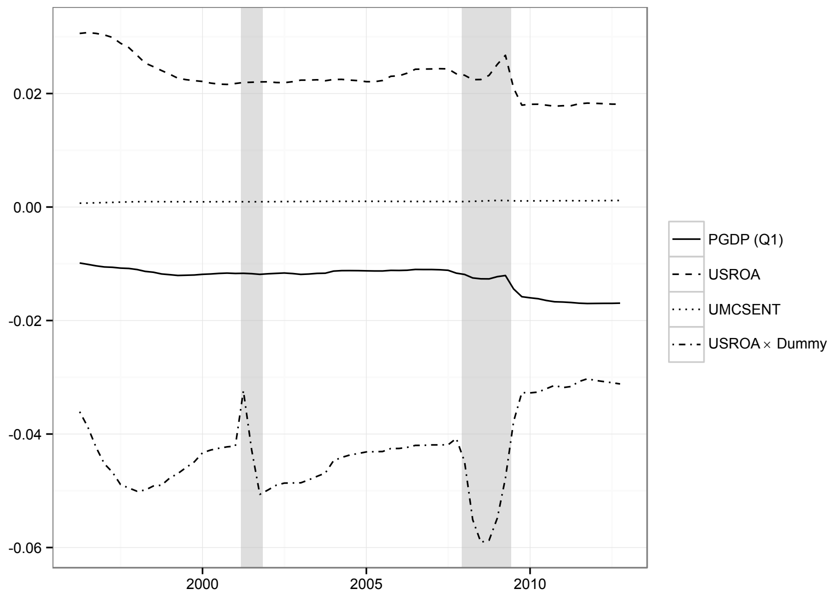

Figure 6.7: Recursive regression of the best AIC model.

coefficients over time is displayed in Figure 6.7. In general, the point estimates are stable. The recession periods, however, clearly influence the results as suggested by our previous findings. First, return on average assets has a strikingly distinct behavior when recession periods are included in the sample. Second, inflation remains nearly unaffected. Third, consumer sentiment has no different impact in recessions than in non-recession periods. These findings are consistent with our single regressions with recession dummys provided in Table 6.3.

6.6.2 Predictive regression

We expect a certain degree of predictability of persistence by lagged determinants for the following reasons: (i) the selected variables mainly stem from the SPF data set and reflect expectations which are important in asset pricing models and (ii) the series of dynamic persistence and regressors are autocorrelated. The results are reported in Table 6.5. We find only minor differences when comparing these results to the benchmark case. The composition of variables for the model averaging approach is almost the same. We comment on two minor differences: (i) for expected inflation a shorter horizon (one quarter ahead) becomes most relevant and (ii) when accounting for asymmetry, consumer sentiment is found to play an opposite role during recessions. The individual models selected by AIC and BIC are richer in terms of included variables. The model weights are also somewhat higher and the coefficient of determination suggests a good degree of predictability.

6.6.3 SPF data set

The results for median SPF forecasts are given in Table 6.6. In comparison to the mean forecasts, two new variables from the SPF data set enter the final selection, namely the change

	Recession dummy excluded				*Recession dummy included*			
	Model Averaging		Model selection		Model Averaging		Model selection	
Variable	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}
dpgdp5	-0.0111	-0.0110	-0.0223	-0.0223	-0.0108	-0.0108	-0.0207	-0.0191
USROA	0.0156	0.0156	0.0166	0.0166	0.0174	0.0173	0.0193	0.0188
dhousing2	-0.0002	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001		
drconsum6	-0.0006	-0.0006	-0.0087	-0.0087	-0.0002	-0.0002	-0.0067	-0.0070
RCBI6	0.0001	0.0001	-0.0004	-0.0004	0.0001	0.0001	-0.0002	
drfedgov2	-0.0003	-0.0003			-0.0003	-0.0003		
UMCSENT	0.0005	0.0005	0.0011	0.0011	0.0007	0.0006	0.0013	0.0013
SP500-ABS-RET	0.0002	0.0002			0.0002	0.0002		
USROA-rec					-0.0049	-0.0050	0.0195	0.0119
dhousing2-rec					0.0001	0.0001		
UMCSENT-rec					-0.0008	-0.0008	-0.0018	-0.0017
dummy-rec					0.0766	0.0758	0.1370	0.1384
λ	1.0526	1.0524	1.0854	1.0854	1.0442	1.0440	1.0378	1.0426
R^2	0.7105	0.7091	0.8088	0.8088	0.7492	0.7474	0.8257	0.8217
AIC			-1.5594	-1.5594			-1.6175	-1.6120
BIC			-1.4170	-1.4170			-1.4276	-1.4458
$100 \cdot \omega_{AIC}$			0.5680	0.5680			0.5637	0.5621
$100 \cdot \omega_{BIC}$			0.5560	0.5560			0.5556	0.5607

Table 6.5: Model averaging and model selection results, predictive regression.

	Recession dummy excluded				*Recession dummy included*			
	Model Averaging		Model selection		Model Averaging		Model selection	
Variable	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}
dpgdp5	-0.0102	-0.0102	-0.0174	-0.0195	-0.0097	-0.0097	-0.0181	-0.0176
dhousing2	-0.0002	-0.0002	-0.0003	-0.0003	-0.0001	-0.0001		
drnresin6	0.0001	0.0001			0.0002	0.0002		
USROA	0.0132	0.0132	0.0128		0.0143	0.0142	0.0121	0.0137
drresinv6	-0.0004	-0.0004			-0.0004	-0.0004		
drconsum4	-0.0005	-0.0005	-0.0046		0.0006	0.0006		
RCBI6	0.0002	0.0002			0.0003	0.0003		
drfedgov2	-0.0003	-0.0003			-0.0003	-0.0003	-0.0002	
SP500-ABS-RET	0.0003	0.0003	0.0002		0.0002	0.0002		
UMCSENT	0.0005	0.0005	0.0011	0.0012	0.0006	0.0006	0.0014	0.0013
dhousing2-rec					0.0000	0.0000		
drnresin6-rec					-0.0018	-0.0018		
USROA-rec					-0.0096	-0.0096	-0.0262	-0.0328
drfedgov2-rec					-0.0008	-0.0008	-0.0051	
dummy-rec					0.0320	0.0319	0.0516	0.0417
λ	1.0555	1.0553	1.0786	1.1158	1.0589	1.0586	1.1000	1.0538
R^2	0.7197	0.7185	0.7870	0.7669	0.7531	0.7520	0.8061	0.7928
AIC			-1.4516	-1.4133			-1.5285	-1.4964
BIC			-1.3092	-1.3421			-1.3624	-1.3778
$100 \cdot \omega_{AIC}$			0.1325	0.1300			0.1341	0.1320
$100 \cdot \omega_{BIC}$			0.1312	0.1334			0.1360	0.1370

Table 6.6: Model averaging and model selection results, SPF data (median).

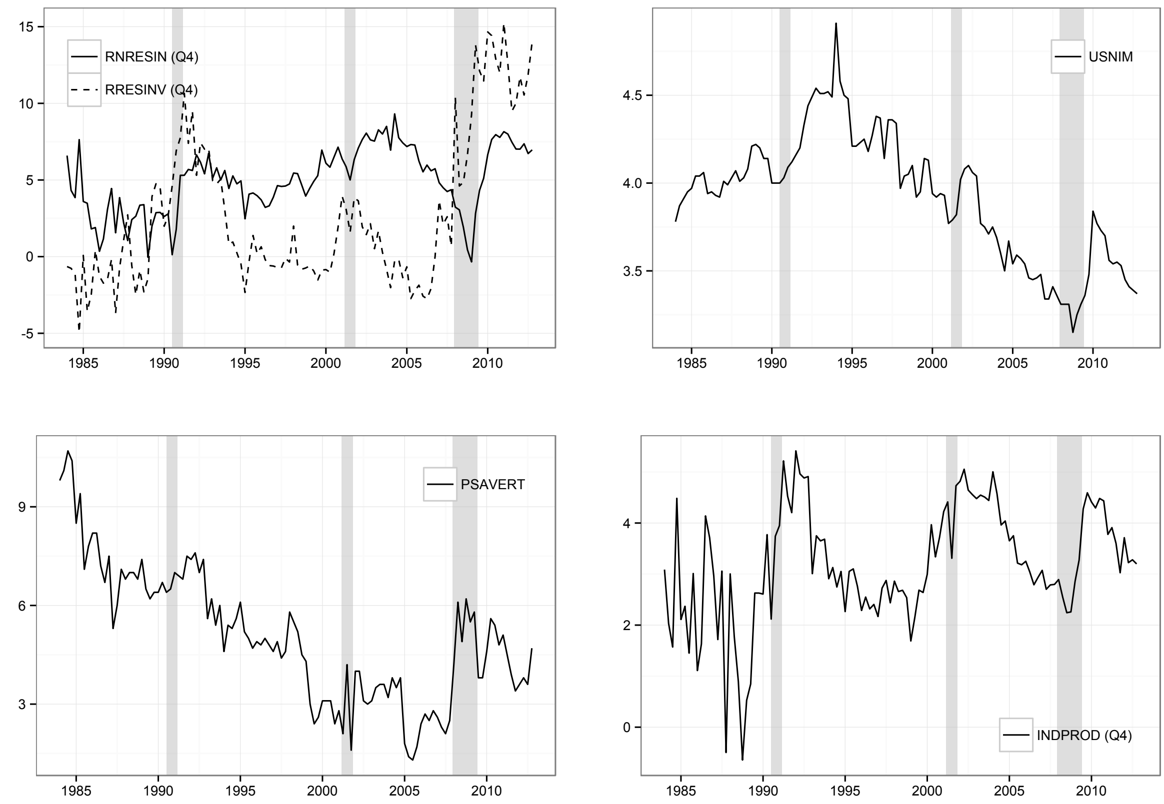

Figure 6.8: Additional explanatory variables from robustness regressions.

in real (non)-residential fixed investment (RNRESIN and RRESINV). Both are one-year-ahead forecasts and presented in Figure 6.8. Residential investment is much more volatile and has, in contrast to non-residential investments, no clear downward trend before and in the beginning of recessions. In addition to these variables, the recession interaction terms of nonresidential investment and government spending are selected. The estimated coefficients of the formerly selected variables are very close to the benchmark case, with consumption being the only exception. Non-residential investment growth impacts persistence positively during non-recession periods which is in line with our previous results. During recessions, however, the effect vanishes. Residential investments are positively correlated with housing starts and therefore, the negative sign is not surprising. Interestingly, the coefficient of determination is larger even though the estimation uncertainty arising from persistence estimation is larger as well. This observation can be explained from the fact that median forecasts are less volatile.

6.6.4 Measuring recessions

Our main results demonstrate the importance of recessions. Obviously, the results hinge on the classification of recession periods. To this end, we consider smoothed recession probabilities (FRED code RECPROUSM156N). The results are reported in Table 6.7. The selection of variables in comparison to the benchmark case is unchanged and their coefficients are nearly the same. Moreover, the overall effect of recessions is somewhat smaller. This can be explained

Variable	Model Averaging γ_{AIC}	γ_{BIC}	Model selection γ_{AIC}	γ_{BIC}
dpgdp3	-0.0097	-0.0096	-0.0192	-0.0182
dhousing2	-0.0001	-0.0001	-0.0001	
USROA	0.0163	0.0162	0.0158	0.0162
drconsum6	0.0012	0.0012	-0.0050	
RCBI6	0.0002	0.0002		
drfedgov2	-0.0003	-0.0003		
SP500-ABS-RET	0.0002	0.0002		
UMCSENT	0.0006	0.0006	0.0011	0.0012
dpgdp3-rec	0.0041	0.0041	0.0093	0.0091
USROA-rec	-0.0050	-0.0050	-0.0061	-0.0103
dummy-rec	0.0151	0.0151	0.0056	0.0149
λ	1.0403	1.0402	1.0643	1.0508
R^2	0.7002	0.6988	0.7708	0.7613
AIC			-1.3440	-1.3377
BIC			-1.1541	-1.1953
$100 \cdot \omega_{AIC}$			0.5096	0.5080
$100 \cdot \omega_{BIC}$			0.4991	0.5095

Table 6.7: Model averaging and model selection results, recession probabilities.

by the relatively low recession probabilities in the first two recessions in the sample (around 70% and 50%) in comparison to the last one (around 100%). This exercise also uncovers an asymmetric effect of expected inflation during recessions: we find a considerable decline of the effect when recession probabilities increase.

Due to a different weighting of a small fraction of data points, the AIC for instance selects a model with three more variables than before (housing, real consumption growth and the recession interaction term with inflation). This is a clear indication that model selection is less robust than model averaging.

6.6.5 Rolling window sizes

In this robustness check the window size w is lowered to 15 years of data and increased to 25 years, corresponding to 60 and 100 observations. These choices are typical for many related applications. To ensure a fair comparison, we use $T = 175$ and $T = 205$ observations of the PD ratio to compute 116 persistence estimates matching with our explanatory data. The results are presented in Tables 6.8 and 6.9. The estimated persistence is shown in Figure 6.9. As expected, the smaller the window size the more volatile the persistence estimation. In particular, the Black Monday in 1987:Q4 is clearly visible if $w = 60$. Remarkable differences can be observed after this major event: only for $w = 60$ a mild degree of explosiveness is suggested. From the year of 2000 onwards, the persistence paths are strikingly similar. This is reflected by the estimation results where many similarities are visible. New selected variables are (see Figure 6.8): (i) the personal savings rate (PSAVERT), which is strongly co-moving with expected inflation, (ii) industrial production growth (dindprod6) which shows a clear cyclical behavior and (iii) the net interest margin (USNIM), which peaked in the Nineties and followed inflation on a linear downward

	Recession dummy excluded				*Recession dummy included*			
	Model Averaging		Model selection		Model Averaging		Model selection	
Variable	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}
USROE	0.0016	0.0015	0.0029	0.0031	0.0020	0.0020	0.0028	0.0034
drfedgov2	-0.0007	-0.0007	-0.0011	-0.0011	-0.0007	-0.0007	-0.0009	
PSAVERT	-0.0032	-0.0032	-0.0048	-0.0052	-0.0033	-0.0033	-0.0065	-0.0078
MICH	-0.0044	-0.0044	-0.0081		-0.0084	-0.0084	-0.0090	
dhousing3	-0.0001	-0.0001			0.0000	0.0000	0.0006	0.0007
drconsum6	0.0034	0.0034			0.0044	0.0044	0.0070	
USNIM	0.0053	0.0053			0.0077	0.0077	0.0162	0.0189
dindprod6	0.0018	0.0018			0.0017	0.0017		
UMCSENT	0.0002	0.0002			0.0005	0.0005	0.0009	0.0009
USROE-rec					-0.0021	-0.0021	-0.0052	-0.0059
MICH-rec					0.0088	0.0088	0.0128	
dummy-rec					0.0123	0.0123	0.0381	0.0846
λ	1.0691	1.0691	1.0897	1.0935	1.0748	1.0751	1.0730	1.0690
R^2	0.4595	0.4589	0.4822	0.4619	0.5702	0.5688	0.6473	0.6085
AIC			-0.5980	-0.5767			-0.8612	-0.8259
BIC			-0.5030	-0.5055			-0.6001	-0.6597
$100 \cdot \omega_{AIC}$			0.2161	0.2138			0.2335	0.2294
$100 \cdot \omega_{BIC}$			0.2176	0.2178			0.2217	0.2284

Table 6.8: Model averaging and model selection results, $w = 60$.

	Recession dummy excluded				*Recession dummy included*			
	Model Averaging		Model selection		Model Averaging		Model selection	
Variable	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}	γ_{AIC}	γ_{BIC}
dpgdp5	-0.0096	-0.0096	-0.0147	-0.0140	-0.0094	-0.0094	-0.0142	-0.0166
drconsum6	0.0008	0.0008	-0.0055		0.0007	0.0008		
dhousing2	-0.0002	-0.0002	-0.0003	-0.0004	-0.0002	-0.0002	-0.0004	-0.0004
USNIM	-0.0041	-0.0041			-0.0027	-0.0027		
USROA	0.0135	0.0134	0.0187	0.0150	0.0144	0.0143	0.0152	0.0158
RCBI6	0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0003	
SP500-ABS-RET	0.0003	0.0003	0.0004	0.0005	0.0003	0.0003	0.0005	0.0005
drfedgov6	0.0005	0.0005	0.0012		0.0004	0.0004		
USROA-rec					-0.0071	-0.0071	-0.0197	0.0065
RCBI6-rec					-0.0001	-0.0001	0.0006	
drfedgov6-rec					0.0010	0.0010		
dummy-rec					0.0105	0.0104	0.0050	-0.0124
λ	1.0318	1.0317	1.0457	1.0464	1.0327	1.0326	1.0389	1.0537
R^2	0.7159	0.7149	0.7779	0.7678	0.7276	0.7265	0.7691	0.7573
AIC			-1.3924	-1.3827			-1.3364	-1.3211
BIC			-1.2262	-1.2641			-1.1465	-1.1786
$100 \cdot \omega_{AIC}$			0.5181	0.5157			0.4987	0.4949
$100 \cdot \omega_{BIC}$			0.5010	0.5106			0.4910	0.4990

Table 6.9: Model averaging and model selection results, $w = 100$.

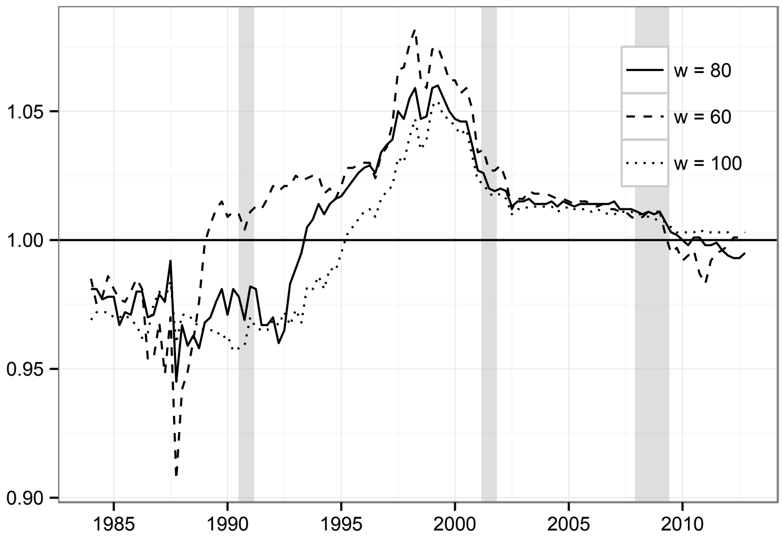

Figure 6.9: Estimated persistence for different window sizes.

trend afterwards. Main differences arise from the distinct behavior of persistence during the late Eighties and mid-Nineties: The rapid rise in persistence can be related to industrial production growth and interest rate margins for banks which both affect persistence positively.

6.7 Conclusion

This work investigates the connection between the persistence of the price-dividend ratio and macroeconomic determinants. We capture the time-varying persistence with a rolling window approach using the indirect inference estimator. This bias-corrected estimator is suitable for stationary, unit root and even mildly explosive time series. The results show that the persistence is in the local-to-unity region most of the time, but displays explosive behavior during the late 1990s. We study the role of more than a hundred macroeconomic and financial variables for the dynamics of persistence. Importantly, our data set covers expectations from SPF survey data which turn out to be influential. We deal with the comprehensive data set by model averaging techniques. The persistence is pro-cyclical and connected to macroeconomic fundamentals. The main drivers are expected inflation, the average returns of banks and consumer sentiment. Our results are discussed in the light of the Fed model which originally relates the dividend-price ratio to bond yields. Additionally, we provide an alternative explanation by linking our result to an asset pricing model with heterogeneous agents. Both ways show that the determinants explaining the persistence of the price-dividend ratio are in line with economic theory. Various robustness checks underline these results.

6.A Appendix

6.A.1 Explanatory variables

No	Mnemonic	Transition	Database	Start	End	Obs.	Description
1	UNEMP2-6	lv	SPF	1968-4	2013-1	178	Forecasts for the quarterly average unemployment rate
6	HOUSING2-6	lv	SPF	1968-4	2013-1	178	Forecasts for the quarterly average level of housing starts
11	TBILL2-6	lv	SPF	1981-3	2013-1	127	Forecasts for the quarterly average three-month Treasury bill rate.
16	BOND2-6	lv	SPF	1981-3	2013-1	127	Forecasts for the quarterly average level of Moody's AAA corporate bond yield
21	RCBI2-6	lv	SPF	1981-3	2013-1	127	Forecasts for the quarterly level of real change in private inventories
26	CPI2-6	lv	SPF	1981-3	2013-1	127	Forecasts for the CPI inflation rate
31	SPR_Tbond_Tbill2-6	lv	SPF	1992-1	2013-1	85	(Implied) Forecasts for the spread between the nominal rate on 10-year Treasury bonds and the nominal rate on three-month Treasury bills
36	SPR_AAA_TBOND2-6	lv	SPF	1992-1	2013-1	85	(Implied) Forecasts for the spread between the nominal rate on Moody's AAA bonds and the nominal rate on 10-year Treasury bonds
41	RR1_TBILL_PGDP_2-6	lv	SPF	1981-3	2013-1	127	(Implied) Forecasts for the real rate on the 3-Month Treasury Bill Minus By Same-Quarter GDP Price Inflation
46	RR2_TBILL_PGDP_2-6	lv	SPF	1981-3	2013-1	127	(Implied) Forecasts for the real rate on the 3-Month Treasury Bill Minus Next-Quarter GDP Price Inflation
50	RR3_TBILL_PGDP_2-6	lv	SPF	1981-3	2013-1	127	(Implied) Forecasts for the real rate on the 3-Month Treasury Bill Minus Average of Same-Quarter GDP Price Inflation and Next-Quarter GDP Price Inflation
54	RR1_TBILL_CPI_2-6	lv	SPF	1981-3	2013-1	127	(Implied) Forecasts for the real rate on the 3-Month Treasury Bill Minus Same-Quarter CPI Inflation
59	RR2_TBILL_CPI_2-5	lv	SPF	1981-3	2013-1	127	(Implied) Forecasts for the real rate on the 3-Month Treasury Bill Minus Next-Quarter CPI Inflation
63	RR3_TBILL_CPI_2-5	lv	SPF	1981-3	2013-1	127	(Implied) Forecasts for the real rate on the 3-Month Treasury Bill Minus Average of Same-Quarter CPI Inflation and Next-Quarter CPI Inflation Next-Quarter CPI Inflation
67	dpgdp2-6	Δlv	SPF	1968-4	2013-1	178	Differenced Forecasts for the quarterly and annual level of the GDP price index
72	dindprod2-6	Δlv	SPF	1968-4	2013-1	178	Differenced Forecasts for the quarterly average and annual average level of the index of industrial production
77	dhousing2-6	Δlv	SPF	1968-4	2013-1	178	Differenced Forecasts for the quarterly average and annual average level of housing starts
82	drgdp2-6	Δlv	SPF	1968-4	2013-1	178	Differenced Forecasts for the quarterly level of real GDP
87	drconsum2-6	Δlv	SPF	1981-3	2013-1	127	Differenced Forecasts for the quarterly level of real personal consumption expenditures
92	drnresin2-6	Δlv	SPF	1981-3	2013-1	127	Differenced Forecasts for the quarterly level of real non-residential fixed investment
97	drresinv2-6	Δlv	SPF	1981-3	2013-1	127	Differenced Forecasts for the quarterly level of real residential fixed investment.
102	drfedgov2-6	Δlv	SPF	1981-3	2013-1	127	Differenced Forecasts for the quarterly level of real federal government consumption and gross investment

No	Mnemonic	Transition	Database	Start	End	Obs.	Description
107	TB3MS	lv	FRED	1968-4	2013-1	178	3-Month Treasury Bill: Secondary Market Rate
108	GS10	lv	FRED	1968-4	2013-1	178	10-Year Treasury Constant Maturity Rate
109	STLFSI	lv	FRED	1994-1	2013-1	77	St. Louis Fed Financial Stress Index
110	ANFCI	lv	FRED	1973-1	2013-1	161	Chicago Fed Adjusted National Financial Conditions Index
111	BAA	lv	FRED	1968-4	2013-1	178	Moody's Seasoned Baa Corporate Bond Yield
112	UNRATE	lv	FRED	1968-4	2013-1	178	Civilian Unemployment Rate
113	MICH	lv	FRED	1978-1	2013-1	141	University of Michigan Inflation Expectation
114	UMCSENT	lv	FRED	1978-1	2012-3	139	University of Michigan: Consumer Sentiment
115	USROA	lv	FRED	1984-1	2012-4	116	Return on Average Assets for all U.S. Banks
116	USROE	lv	FRED	1984-1	2012-4	116	Return on Average Equity for all U.S. Banks
117	USNIM	lv	FRED	1984-1	2012-4	116	Net Interest Margin for all U.S. Banks
118	EQTA	lv	FRED	1988-1	2012-4	100	Total Equity / Total Assets
119	PSAVERT	lv	FRED	1968-4	2013-1	178	Personal Saving Rate
120	RECPROUSM156N	lv	FRED	1968-4	2012-4	177	Smoothed U.S. Recession Probabilities
121	USRECQ	lv	FRED	1968-4	2013-1	178	U.S. Recession Dummy
122	SP500	Δlv	FRED	1968-4	2013-1	178	S&P 500 Stock Price Index
123	INDPRO	Δlv	FRED	1968-4	2013-1	178	Industrial Production Index
124	GDPC1	Δlv	FRED	1968-4	2013-1	178	Real Gross Domestic Product, 1 Decimal
125	GDPDEF	Δlv	FRED	1968-4	2013-1	178	Gross Domestic Product: Implicit Price Deflator
126	USACPIALLQINMEI	Δlv	FRED	1968-4	2012-4	177	Consumer Price Index of All Items in United States
127	PCECC96	Δlv	FRED	1968-4	2013-1	178	Real Personal Consumption Expenditures
128	PCEPI	Δlv	FRED	1968-4	2013-1	178	Personal Consumption Expenditures: Chain-type Price Index
129	PPIACO	Δlv	FRED	1968-4	2013-1	178	Producer Price Index: All Commodities
130	PPIFGS	Δlv	FRED	1968-4	2013-1	178	Producer Price Index: Finished Goods
131	UMCSENT	Δlv	FRED	1979-1	2012-3	135	University of Michigan: Consumer Sentiment
132	CPILFESL	Δlv	FRED	1968-4	2013-1	178	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
133	CUSR0000SA0L2	Δlv	FRED	1968-4	2013-1	178	Consumer Price Index for All Urban Consumers: All items less shelter
134	USSTHPI	Δlv	FRED	1976-1	2012-4	148	All-Transactions House Price Index for the United States
135	USNUM	Δlv	FRED	1985-1	2012-4	112	Number of Commercial Banks in the U.S.
136	M2	Δlv	FRED	1982-1	2013-1	125	M2 Money Stock
137	CMDEBT	Δlv	FRED	1968-4	2012-4	177	Households and Nonprofit Organizations; Credit Market Instruments; Liability
138	SP500_ABS_RET	Δlv	-	1968-4	2013-1	178	Absolute Returns of the S&P 500

6.A.2 Insignificant variables from single regressions

Variable	γ^i	t-stat	λ^i	Variable	γ^i	t-stat	λ^i
drresinv6	-0.001	-1.019	1.011	USNIM	-0.005	-0.271	1.008
drnresin2	0.001	1.008	1.008	HOUSING4	0.005	0.269	1.007
RCBI4	0.001	1.004	1.003	RR1_TBILL_CPI_2	-0.001	-0.266	1.008
drconsum5	0.006	0.928	1.038	CMDEBT	-0.001	-0.255	1.004
dhousing6	0.000	-0.889	1.015	USRECQ	0.003	0.249	1.032
SPR_AAA_TBOND4	-0.011	-0.875	1.010	drfedgov5	0.000	-0.226	1.031
SPR_AAA_TBOND3	-0.010	-0.873	1.010	GDPC1	0.001	0.225	1.007
drgdp6	0.007	0.866	1.016	HOUSING5	0.004	0.220	1.008
SPR_AAA_TBOND5	-0.011	-0.836	1.009	RR3_TBILL_CPI_2	-0.001	-0.181	1.004
dhousing5	0.000	-0.831	1.009	M2_pc1	0.001	0.171	1.004
UNEMP6	-0.010	-0.822	1.001	drfedgov3	0.000	0.169	1.023
SPR_AAA_TBOND2	-0.008	-0.814	1.010	HOUSING6	0.004	0.167	1.008
RCBI3	0.000	0.808	1.003	dindprod5	-0.001	-0.144	1.005
RCBI2	0.000	0.790	1.004	RR2_TBILL_CPI_2	-0.001	-0.124	1.002
UNEMP5	-0.009	-0.769	1.001	dindprod3	0.000	-0.118	1.026
drgdp2	0.002	0.734	1.030	RR1_TBILL_CPI_3	-0.001	-0.093	1.002
SPR_AAA_TBOND6	-0.011	-0.725	1.008	RR1_TBILL_PGDP_2	-0.001	-0.088	1.003
UNEMP4	-0.009	-0.702	1.001	drfedgov4	0.000	-0.088	1.020
TB3MS	-0.003	-0.693	1.003	RR3_TBILL_PGDP_2	-0.001	-0.080	1.002
TBILL2	-0.003	-0.686	1.003	RR2_TBILL_PGDP_2	-0.001	-0.080	1.003
TBILL6	-0.003	-0.676	1.003	RR3_TBILL_CPI_3	-0.001	-0.065	1.002
TBILL5	-0.003	-0.668	1.003	RR1_TBILL_CPI_6	0.000	-0.065	1.003
EQTA	0.005	0.660	1.002	RR1_TBILL_CPI_4	0.000	-0.063	1.002
UNEMP3	-0.009	-0.657	1.001	RR1_TBILL_CPI_5	0.000	-0.052	1.002
UNRATE	-0.009	-0.647	1.001	RR2_TBILL_CPI_3	0.000	-0.052	1.002
TBILL4	-0.003	-0.642	1.003	RR3_TBILL_CPI_4	0.000	-0.044	1.002
TBILL3	-0.003	-0.639	1.003	drgdp5	0.000	0.043	1.014
UNEMP2	-0.009	-0.617	1.001	RR2_TBILL_PGDP_5	0.000	0.043	1.003
PCECC96	0.003	0.581	1.005	RR1_TBILL_PGDP_3	0.000	-0.042	1.003
ANFCI	0.004	0.581	1.014	dindprod2	0.000	-0.042	1.041
dindprod6	0.002	0.552	1.013	RR3_TBILL_CPI_5	0.000	-0.040	1.002
drresinv2	0.000	-0.546	1.030	dindprod4	0.000	-0.036	1.015
HOUSING2	0.009	0.534	1.008	RR2_TBILL_CPI_4	0.000	-0.029	1.002
drgdp3	0.002	0.486	1.028	PPIACO_pc1	0.000	-0.024	1.004
drgdp4	0.002	0.453	1.027	RR2_TBILL_CPI_5	0.000	-0.023	1.002
USNUM	-0.003	-0.446	1.006	RR3_TBILL_PGDP_3	0.000	-0.018	1.002
USSTHPI	0.001	0.418	1.009	RR1_TBILL_PGDP_5	0.000	-0.017	1.004
HOUSING3	0.007	0.385	1.007	RR2_TBILL_PGDP_4	0.000	0.015	1.004
UMCSENT	0.000	0.330	1.029	RR3_TBILL_PGDP_5	0.000	0.014	1.003
RECPROUSM156N	0.000	0.301	1.032	RR1_TBILL_PGDP_6	0.000	0.007	1.003
INDPRO	0.001	0.300	1.005	RR1_TBILL_PGDP_4	0.000	-0.003	1.002
drfedgov6	0.001	0.299	1.028	RR2_TBILL_PGDP_3	0.000	0.003	1.002
SP500	0.000	0.295	1.004	RR3_TBILL_PGDP_4	0.000	0.003	1.002
PPIFGS	-0.001	-0.285	1.005				

Table 6.10: Regression of dynamic persistence on a single variable.

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