The cosmological constant, branes and non-geometry

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Abstract

In this thesis we derive an equation for the classical cosmological constant in general string compactifications by employing scaling symmetries present in string theory. We find that in heterotic string theory, a perturbatively small, but non-vanishing, cosmological constant is impossible unless non-perturbative and/or string loop corrections are taken into account. In type II string theory we show that the classical cosmological constant is given by a sum of two terms, the source actions evaluated on-shell, and a certain combination of non-vanishing fluxes integrated over spacetime. In many cases we can express the classical cosmological constant in terms of only the source contributions by exploiting two scaling symmetries. This result can be used in two ways. First one can simply predict the classical cosmological constant in a given setup without solving all equations of motion. A second application is to give constraints on the near brane behavior of supergravity fields when the cosmological constant is known. In particular we motivate that energy densities of some fields diverge in the well-known KKLT scenario for de Sitter solutions in type IIB string theory. More precisely, we show, using our results and minimal assumptions, that energy densities of the three-form fluxes diverge in the near-source region of internal space. This divergence is unusual, since these fields do not directly couple to the source, and has been interpreted as a hint of instability of the solution. In the last chapter of the thesis we discuss the worldvolume actions of exotic five-branes. Using a specific chain of T- and S-dualities in a spacetime with two circular isometries, we derive the DBI and WZ actions of the so-called 5^2_2- and 5^2_3-brane. These actions describe the dynamics of the branes as well as their couplings to the ten-dimensional gauge potentials. We propose a modified Bianchi identity for the non-geometric Q-flux due to one of the branes. Q-flux often appears when geometric backgrounds with non-trivial NSNS flux are subject to a chain of T-dualities. Finally we argue that using S-duality also leads to exotic branes and modified Bianchi identities for associated non-geometric RR flux. Some non-geometric flux compactifications have been shown to give rise to a positive cosmological constant from the dimensionally reduced point of view. The study of exotic branes is a step towards complete understanding of non-geometric fluxes from a ten-dimensional point of view.

Keywords String theory, Compactification, Cosmological constant, D-Branes, String dualities

Zusammenfassung

In dieser Arbeit leiten wir aus Skalensymmetrien der Stringtheorie eine allgemeine Gleichung für die klassische kosmologische Konstante einer Stringkompaktifizierung her. Dabei stellt sich heraus, dass eine kleine, nicht-verschwindende kosmologische Konstante in der heterotische Stringtheorie unmöglich ist, solang nicht-perturbative und/oder String-Schleifenkorrekturen unberücksichtigt bleiben. Für die Typ II Stringtheorie zeigen wir, dass die klassische kosmologische Konstante durch zwei Terme gegeben ist; der eine ist die on-shell ausgewertete Quellenwirkung und der andere ist eine Kombination von nicht-verschwindenen Flusstermen integriert über die Raumzeit. Durch Benutzung von zweier Skalensymmetrien kann die kosmologische Konstante in vielen Fällen nur durch Quellenbeiträge ausgedrückt werden. Das Ergebnis kann auf zwei Weisen benutzt werden. Zum einen kann die kosmologische Konstante für einen gegebenen Fall bestimmt werden, ohne die Bewegungsgleichungen zu lösen. Zum anderen können Einschränkungen an das Verhalten von Supergravitationsfeldern in der Nähe von Branen angegeben werden, wenn die kosmologische Konstant bekannt ist. Insbesondere finden wir, dass die Energiedichten mancher Feldern in dem bekannten KKLT-Szenario für de Sitter-Vakua in der Typ IIB Stringtheorie divergieren. Mit minimalen Annahmen sind wir daher in der Lage auf die Divergenz der Energiedichten des Drei-Form-Flusses nahe der Quelle zu schließen. Dieses divergente Verhalten ist ungewöhnlich, da diese Felder nicht direkt an die Quelle koppeln und wurde als Instabilität interpretiert. Im letzten Kapitel dieser Arbeit diskutieren wir die Weltvolumen-Wirkung von exotischen 5-Branen. Mit Hilfe einer spezifischen Abfolge von T- und S-Dualitäten in einer Raumzeit mit zwei Isometrien leiten wir die DBI- und WZ-Wirkung dieser 5-Branen. Diese Wirkungen beschreiben die Dynamik der Branen und deren Kopplung an die zehn-dimensionalen Eichpotentiale. Wir sind daher in der Lage, eine modifizierte Bianchi-Identität für den nichtgeometrischen Q-Fluss herzuleiten. Ein solcher Q-Fluss tritt oft in Erscheinung, wenn ein geometrischer Hintergrund mit nichttrivialem NSNS-Fluss unter einer Reihe von T-Dualitäten abgebildet wird. In analoger Weise können wir schließen, dass S-Dualität zu exotischen Branen und modifizierten Binchi-Identitäten für den zugehörigen nichtgeometrischen RR-Fluss führt. In vier Dimensionen wurde bereits gezeigt, dass manche nichtgeometrische Fluss-Kompaktifizierungen zu einer positiven kosmologischen Konstanten führen. Die Untersuchung von exotischen Branen ist ein Schritt in Richtung eines vollständigen Verständnisses von nichtgeometrischen Flüssen in zehn Dimensionen.

Schlüsselwörter Stringtheorie, Kompaktifizierung, Kosmologische Konstante, D-Branen, Stringdualitäten

Contents

Introduction

One of the most interesting developments in recent experimental physics is the discovery of the accelerated expansion of the universe [1–6]. Stars and galaxies are pulled together via the gravitational attraction which counteracts the expansion of the universe. This led most twentieth century physicists to the conviction that the expansion was slowing down. Distant galaxies are, however, observed to be receding from us at increasing rate. This accelerated expansion can be modelled by adding an extra parameter to the equations of general relativity. The latter is a successful theory of gravity presented by Einstein in 1915 [7]. Einstein actually included the cosmological constant in his equations in 1917 to solve a problem he faced when studying cosmological solutions [8]. The cosmological constant provides non-zero vacuum energy density which can counteract or reinforce the overall gravitational pull due to matter in the universe. Like many physicists at the time, Einstein believed that the universe was completely static. General relativity is not able to predict such a universe without a non-vanishing cosmological constant. Even including the effect of a non-zero cosmological constant, Friedmann showed in 1922 that static solutions are not stable and small perturbations always lead to expansion or contraction [9]. It is then clear that general relativity cannot describe a stable, static universe. This problem was solved in the late 1920's when Edwin Hubble determined a law that now bears his name. Hubble was able to relate the distance to galaxies with the observed redshift [10]. Hubble's law indicates that the universe is expanding, a fact that obviated the need for a cosmological constant at that time. It was not until 1998 when the acceleration of the expansion was measured, that the inclusion of the cosmological constant in Einstein's equations became necessary once again. The discovery of the non-zero vacuum energy came as a big surprise to many physicists, especially since the naive estimate of the vacuum energy density,

$$\frac{c^7}{G^2\hbar} \sim 10^{118}\ \mathrm{GeV/cm^3},$$

where c is the speed of light in vacuum, G is the gravitational constant and $\hbar$ is the Planck's constant, is roughly 122 orders of magnitudes larger than the observed value. This was also a mystery when the vacuum energy was believed to vanish, but one could argue that some unknown symmetry forced the cosmological constant to be zero. Now, however, we have a much more difficult question on our hands. Namely, why is the cosmological constant not exactly zero?

General relativity is extremely successful in describing all cosmological, galactic and solar system dynamics. The ΛCDM model describes the present day cosmology in the framework of general relativity. To agree with experiments, the ΛCDM includes huge amount of dark matter. Dark matter is some matter that does not interact with light and therefore cannot be observed directly. Together with the cosmological constant Λ, these two phenomena constitute major theoretical challenges for complete understanding of late time cosmology. General relativity, despite its success, cannot be the final answer, since it is incompatible with quantum mechanics. Moreover it cannot be made into a renormalizable quantum theory [11]. This means that if general relativity is quantized in the same way that other field theories are, an infinite set of counterterms are required and the theory loses its predictive power. The standard model of particle physics, on the other hand, is a renormalizable theory and therefore needs only a finite set of input parameters to be able to predict results from experiments [12, 13]. It has proven extremely successful ever since it was finalized in the 1970's. It predicted the discovery of the W and Z bosons at CERN in 1983 and later the discovery of the t quark at Fermilab in 1995 (see [14] for an overview). Finally in 2012, the first scalar particle ever discovered was found at the Large Hadron Collider (LHC) in CERN [15, 16]. The particle is widely believed to be the standard model Higgs boson, the last missing piece in the standard model construction. The standard model is not without its own problems. Neutrino masses have to be incorporated to agree with experiments which have found neutrino oscillations [17, 18]. The standard model does not include candidates to explain the dark matter predicted by the ΛCDM model. it also suffers from a hierarchy problem. The mass of scalar particles such as the Higgs boson receive huge quantum loop corrections. These quantum corrections can be cancelled by fine-tuning of the bare mass or by some kind of cancellation mechanism provided by physics beyond the standard model (see for example [19] for a review). The main problem of the standard model is however the fact that gravity is in no way taken into account. This does not lead to any significant problems for predicting outcomes of experiments made in particle accelerators such as the LHC in CERN. But in more extreme situations, such as close to black hole singularities, the standard model is not valid. A new unifying theory, a UV completion of the standard model, is needed.

String theory is a mathematically consistent candidate theory of quantum gravity [20–22]. We can view string theory as a quantum field theory in two dimensions with non-linear interactions. Mathematical consistency imposes strong constraints on the number of matter fields and the form of their interactions. The scalar fields present in the theory can then be interpreted as coordinates of a target space which must be ten dimensional and moreover the effective theory in ten dimensions is a complicated extension of general relativity. In the expansion of string length, the leading order terms in the ten dimensional effective action is that of a ten dimensional supergravity. Higher order terms give stringy corrections to the leading order supergravity action. In order to make contact with four dimensional physics the ten dimensional spacetime is taken to be a spacetime with six compact and small spatial directions and four macroscopic spacetime directions. This enables us to make an expansion in the inverse volume of the six dimensional space and write down an effective theory in the four dimensional spacetime. This procedure is called compactification, and is just a more complicated version of Kaluza-Klein theory studied in the 1920's [23, 24]. With this construction we obtain not only four dimensional general relativity but a host of matter fields and interactions. The matter content depends on how the compactification is performed but most of the fields have very large masses, related to the inverse volume of internal space, and therefore play no role in low-energy physics (note that low-energy in this context contains the energy scale of particle accelerators.) A successful compactification of string theory should describe the standard model, and necessary additions to explain dark matter and non-zero neutrino masses but no visible exotic matter. Furthermore it should have positive cosmological constant and most importantly be stable.

In the 1980's most of the effort was concentrated on compactifying the heterotic string with $E_8 \times E_8$ or $SO(32)$ gauge group on a Calabi-Yau manifold which results in $\mathcal{N} = 1$ supergravity in four dimensions [25]. Recent constructions of heterotic compactification on Calabi-Yau manifolds were able to produce interesting supersymmetric extensions of the standard model[1] [31]. These models have the severe drawback that Calabi-Yau spaces have a large number of moduli, scalar fields that are associated to the shape and the size of the internal space. These moduli

[1] Similar progress has also been made in compactifying the heterotic string on orbifolds [26–30].

are a priori massless, and can lead to many problems such as long-range forces which could be observed in experiments. A safe solution to these problems is to give large masses to the moduli fields in a consistent manner. One way to progress is to allow for a vacuum expectation value of the three-form field H, one is then forced to break supersymmetry at high scale (see however [32]) or abandon Calabi-Yau internal spaces and consider non-Kähler spaces which are not as well studied as the Calabi-Yau manifolds [33]. After Polchinski's discovery of D-branes in 1995 [34], the focus of the community shifted from heterotic to type II string theory for model building and phenomenology. The standard model can be constructed on the worldvolume of intersecting D-branes (for reviews see [35–37]). Furthermore, D-branes act as sources for form fields and give rise to non-trivial vacuum expectation values for them. These fields, called fluxes, can partially break supersymmetry and provide a mechanism to give masses to many moduli [38–40].

The models discussed so far have all had either zero or negative cosmological constant and can therefore not be considered realistic. However, in 2003 Kachru, Kallosh, Linde and Trivedi (KKLT) proposed a method to obtain meta-stable de Sitter vacuum in string theory [41]. Their construction includes fluxes together with non-perturbative effects to stabilize the moduli of a Calabi-Yau orientifold in a supersymmetric anti de Sitter vacuum. Then a small number of anti D3-branes are included that break supersymmetry and lift the vacuum energy to a positive value which can be tuned to the small observed value by a choice of parameters. An improved setup is provided by the large volume scenario (LVS) that includes quantum corrections to stabilize the internal volume at exponentially large values [42]. A positive cosmological constant is achieved essentially by the same procedure as in the KKLT scenario, that is by inclusion of anti D3-branes or by the inclusion of dilaton dependent non-perturbative effects [43]. Both the KKLT and LVS constructions have the advantage of being meta-stable, meaning that the lifetime of the state is more than the age of the observable universe [41], but the drawback that they rely heavily on non-perturbative effects which are usually not treated from a string theory perspective, but rather in the four dimensional field theory limit [41, 44–46]. Some progress has been made in discussing non-perturbative effects in ten dimensions but a complete understanding is still missing [47–51]. Another drawback is that in [41] the anti D3-branes are treated only in the probe brane approximation, which means that implicitly some simplifications are made. A more careful analysis of the equations of motion indicates an unusual singularity of the full solution. There still is a debate on whether the KKLT scenario is singular or not and what the interpretation of the singularity is [52–76]. In chapter 4 we will discuss this problem in some detail and provide evidence that the full solution does indeed exhibit a singularity in the energy density of some fields [62]. Other methods exist to obtain positive cosmological constant, for example the so-called Kähler uplifting scenario trades off non-perturbative effects and perturbative corrections to engineer a small window in parameter space where the vacuum energy is positive [77–79]. This method again relies heavily on non-perturbative effects but moreover the examples found so far seem to be on the margin of validity of the supergravity limit of string theory. Finally, some attempts have been made to construct classical de Sitter solutions that do not rely on perturbative or non-perturbative quantum effects [80–93]. These solutions have their own problems, some do not solve the complete set of ten dimensional equations of motion and others have been shown to be perturbatively unstable (see [94] for a discussion on tachyons in classical de Sitter constructions.)

Although flux compactification has had great success over the past years it is safe to say that all currently known de Sitter constructions in string theory are not without problems or open questions. One approach we have not discussed so far is to generalize flux compactifications to include so-called non-geometric flux [87, 95–104]. The effect of these fluxes in the reduced four dimensional theories can be inferred from duality arguments, but there has been some difficulty in establishing their precise role in the ten dimensional theory. Much progress has however been made, both in the context of double field theory [105,106] (see also [107–109] for recent reviews) and generalized geometry [110,111]. The non-geometric fluxes arise for example from T-duality, a symmetry of string theory which we will now discuss. Because of the extended nature of the string it can wind compact dimensions in a non-trivial way. Let for example the ten dimensional spacetime be a direct product of nine flat spacetime dimensions and one circular direction, then states where the string winds the circular direction W times are said to have winding number W. States with different winding numbers are all different, that is the energy of the state depends on the winding number. A careful analysis of the mode expansion and using the fact that the momentum of the string in the circular direction is quantized leads to the conclusion that a state with certain winding number W and momentum K, in the circular direction is equivalent to a state with winding number K and momentum W if the radius of the circular direction is also inverted. This procedure is called T-duality and is a symmetry of string theory. We should mention that T-duality maps between the ten dimensional string theories, for example type IIA compactified on a circle of radius R is T-dual to type IIB compactified on a circle of radius l_s^2/R where l_s is the string length. In 1987 Buscher derived a set of transformation rules for the target space fields which implements T-duality at the level of the effective theory [112]. However, the low energy effective theory is not T-duality invariant. Therefore when the rules of [112] are applied to solutions one can end up with field configurations that do not solve the low energy effective equations of motion globally. A common example to demonstrate this behaviour is to compactify type IIA string theory on a three-torus and turn on H-flux in the compact space. Now we have three circle isometries to perform T-dualities, which leads to the T-duality chain

$$H_{123} \xmapsto{T_1} f^1{}_{23} \xmapsto{T_2} Q^{12}{}_3 \xmapsto{T_3} R^{123}.$$

The second entry in this chain indicates a torus with geometric torsion or a twisted torus, then we have an entry with Q-flux. This is a configuration on the torus where the metric and the Kalb-Ramond two-form B are globally ill-defined, a jump in the fields can be observed when going around the third direction of the torus, and this jump cannot be undone by gauge or coordinate transformations. However by a proper change of variables inspired by generalized geometry this setup can be made geometric, that is, globally well defined. The new transformed variables are called again the metric (sometimes written with a tilde to distinguish from the globally ill-defined metric) and the two-vector β. The field strength associated with β is called Q which explains the notation above. Finally if T-duality is performed in the third direction at least formally[2], a so called R-flux is obtained which can also be written in terms of β [113–115]. One reason to study non-geometric fluxes is to understand whether compactifications of string theory are allowed with fluxes for which no T-duality chain can lead us to the standard fluxes which can be described in supergravity. Some evidence suggests that this may be the case [116–118] and if so then this calls for a reformulation of low energy string theory that can deal with such setups like generlized geometry [110,111,113–115] or double field theory [105,106].

Another interesting consequence of string dualities is the result when dualities are applied to branes in the theory. D-branes are known to exist in the full theory as endpoints of open strings. Their existence was supported by T-duality which maps the spacetime filling D9-brane to all other D-branes in the theory [119]. The fundamental string is of course well known to source the Kalb-Ramond two-form B. Its magnetic cousin, the NS5 brane, can be constructed as the S-dual of a D5 brane (see chapter 5). The Kaluza-Klein monopole is also a well established brane in string theory, and is a magnetic source for Kaluza-Klein gauge fields which appears upon

[2]The third direction of the torus loses its isometry after two T-dualities and the third T-duality is not well justified, however we will not discuss R-flux extensively in this thesis so this does not concern us here.

compactification. The Kaluza-Klein monopole is the T-dual of a NS5 brane in nine dimensions and is the first so-called exotic brane we encounter. Exotic branes are branes whose existance can only be inferred from T-duality in the dimensionally reduced theory and not in the full ten dimensional theory. As one can imagine, a huge ammount of exotic branes exist in various spacetime dimensions. These branes have been classified [120–122] and some of these branes can be showed to be sources for the non-geometric fluxes discussed above [123, 124]. This is particularly interesting since we know from the above discussion on flux compactifications that D-branes play a central role in building phenomenologically interesting models. The same might then be possible using non-geometric fluxes and exotic branes as ingredients. As mentioned above, non-geometric fluxes are known to stabilize some moduli and give positive contribution to the vacuum energy which are two disirable effects.

This thesis is organized as follows, in chapter 1, we discuss string theory in the low energy effective limit and give a short introduction to ten dimensional supergravity needed for the rest of the thesis. In chapter 2 which is based on [125], we show that α' correction to the low energy effective action of heterotic string theory cannot induce perturbatively small cosmological constant. In chapters 3-4 which are based on [62], we introduce scaling symmetries as tools to generate on-shell constraints on observables and apply these constraints to the type II cosmological constant. We also discuss how the backreaction of brane sources can lead to singular energy densities for fields not directly coupled to the sources. This is discussed in particular in the context of the KKLT scenario, where we motivate singularities in the energy density of some fields. Finally in chapter 5 which is based on [124], we derive the world volume actions of exotic fivebranes present in type IIB string theory. We find to which non-geometric fields these branes couple to and write down modified Bianchi identities for the fields due to the brane sources.

Chapter 1

Supergravity

In this thesis we are concerned mostly with the low energy effective theory for the massless degrees of freedom of string theory. This is a ten dimensional supergravity together with brane sources. For completeness we will give a short description of how supergravity arises in the low energy limit from the two dimensional worldsheet perspective. We will then list the supergravity theories used in this thesis. This chapter then also serves to fix the notation for the rest of the thesis.

1.1 Low energy effective theory

Perturbative string theory [20–22] is a two dimensional non-linear sigma model with D bosonic target space coordinates and in the case of type I or type II superstring in the RNS formulation D fermionic superpartners of the coordinates. The main feature of the theory is the fact that it is invariant under (super)conformal transformations which makes the quantization of the theory consistent. Vanishing of the trace anomaly gives a condition on the number of target space dimensions, $D = 26$ in purely bosonic theory and $D = 10$ for the superstring. Further demanding that the conformal invariance be a quantum symmetry of the theory implies that the β functions vanish, which can be interpreted as target space equations of motion for the massless modes of the string. In the case of the superstring, these massless modes can be grouped together into supersymmetry multiplets of the four 10D supergravity theories (which theory we obtain in 10D depends on the details of the worldsheet theory and how the so-called GSO projection, which is demanded by modular invariance, is performed on the string spectrum.) In fact, vanishing of the β functions reproduces exactly the correct equations of motions for these multiplets. The construction of the heterotic string is essentially a combination of the bosonic and RNS superstring where the left moving degrees of freedom are purely bosonic but the right moving degrees of freedom are of the ten dimensional RNS superstring. The result is an $N = 1$ superstring theory in ten dimensions which has either $SO(32)$ or $E_8 \times E_8$ gauge invariance. Vanishing of the β functions again gives non-trivial equations of motion for the massless fields which are identical to those of a 10D supergravity theory.

The solutions of the low energy effective theories in 10D therefore determine consistent vacuum states of string theory, essentially the stages on which the full quantum theory can take place. The vacuum state is of primary importance for applications of string theory to late time cosmology and phenomenology in general. It determines for example the amount of supersymmetry and the gauge group, both of which are inherited from the full 10D theory but can be spontaneously broken at low energies, and also the cosmological constant. The above described procedure is seemingly dependent on the background around which the expansion is made. No consistent background independent formulation is known for string theory which is sometimes considered one of the weak points of the theory. In this thesis we will work only with the bosonic degrees of freedom. Below we only give the bosonic part of the supergravity actions from which the equations of motions for the bosonic degrees of freedom can be derived. Effectively the vacuum state of the fermions is taken to be trivial.

Finally we should mention that there are many aspects of string theory that are not captured by supergravity that we use in later parts of this thesis, some of which are not discussed in this chapter. A primary example are string dualities which we will discuss in chapter 5.

1.2 Heterotic supergravity

The low energy effective action of the heterotic string is given by

$$S = \int \mathrm{e}^{-2\phi} \star_{10} \left\{ R + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\mathrm{tr}|F|^2 \right\}, \tag{1.1}$$

where ϕ is the dilaton, H is the heterotic 3-form and $F = \mathrm{d}A + A \wedge A$ is the Yang-Mills field strength corresponding to the $E_8 \times E_8$ or $SO(32)$ gauge field A. Furthermore, R is the curvature scalar associated with the 10D metric G (with mostly plus signature), and the gauge theory trace appearing in (1.1) is evaluated using the fundamental representation of the gauge group. Notice that in the action (1.1) and below in the type II action (1.3) we absorb the prefactor $2\kappa_{10}^2$ into the definition of the metric.

In this thesis we make extensive use of form notation, and we will summarize our conventions here for convenience. A general p-form ω_p is decomposed in the basis forms $\mathrm{d}x^M$ as

$$\omega_p = \frac{1}{p!}\omega_{M_1\cdots M_p}\mathrm{d}x^{M_1} \wedge \cdots \wedge \mathrm{d}x^{M_p}$$

and our 10D Hodge star is defined such that

$$\omega_p \wedge \star_{10}\omega_p = \star_{10}|\omega_p|^2 = \frac{1}{p!}\omega_{M_1\cdots M_p}\omega_{M_1'\cdots M_p'}G^{M_1M_1'}\cdots G^{M_pM_p'}\sqrt{-\det G}\ \mathrm{d}x^0 \wedge \cdots \wedge \mathrm{d}x^9.$$

This means in particular that

$$\star_{10}\mathrm{d}x^0 \wedge \cdots \wedge \mathrm{d}x^p = \sqrt{-G}\ \mathrm{d}x^{p+1} \wedge \cdots \wedge \mathrm{d}x^9.$$

We often consider warped product spaces $\mathcal{M}^{(10)} = \mathcal{M}^{(d)} \times_w \mathcal{M}^{(10-d)}$, where $\star_d$ and $\star_{10-d}$ then denote the Hodge operators of the corresponding warped metric factors. For factorizing forms $\omega_p \wedge \psi_q$, where ω_p is a p-form on $\mathcal{M}^{(d)}$ and ψ_q a q-form on $\mathcal{M}^{(10-d)}$, these Hodge operators satisfy the useful identity

$$\star_{10}(\omega_p \wedge \psi_q) = (-1)^{p(10-d-q)}(\star_d\omega_p) \wedge (\star_{10-d}\psi_q).$$

In general, we have $(\star_D)^2\omega_p = (-1)^{p(D-p)+t}\omega_p$ for any p-form on a D-dimensional manifold with t timelike directions.

The heterotic 3-form H satisfies the Bianchi identity

$$\mathrm{d}H = \frac{\alpha'}{4}\left(\mathrm{tr}R_2 \wedge R_2 - \mathrm{tr}F \wedge F\right),$$

where R_2 is the curvature 2-form. This implies that locally we can express

$$H = \mathrm{d}B + \frac{\alpha'}{4}\left(\omega_{\mathrm{L}} - \omega_{\mathrm{YM}}\right) \tag{1.2}$$

where the Chern-Simons terms are given by

$$\omega_{\rm L} = \mathrm{tr}\left(\omega \wedge \mathrm{d}\omega + \frac{2}{3}\omega \wedge \omega \wedge \omega\right), \qquad \omega_{\rm YM} = \mathrm{tr}\left(A \wedge \mathrm{d}A + \frac{2}{3}A \wedge A \wedge A\right),$$

and ω is the spin connection. The right hand side of the Bianchi identity (1.2) is required for anomaly cancellation, but the Lorentz Chern-Simons term is higher order in derivatives and therefore does not appear in the two-derivative action (1.1). The Yang-Mills Chern-Simons term $\omega_{\rm YM}$, on the other hand, is of leading order [22].

1.3 Type II supergravity

Using the democratic formulation [126] we can write both the type IIA and type IIB action at the same time

$$S = \int \mathrm{e}^{-2\phi} \star_{10} \left\{ R + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 - \frac{1}{4}\mathrm{e}^{2\phi}|\mathbf{F}|^2 \right\}, \tag{1.3}$$

where the first three terms are familiar from the heterotic action (1.1) but the last term contains the kinetic terms for all RR fields. We use the so-called polyform notation

$$\mathbf{F} = \sum_n F_n \qquad \text{and} \qquad |\mathbf{F}|^2 = \sum_n |F_n|^2$$

where F_n is an n-form field strength. The field $\mathbf{F}$ satisfies the Bianchi identity, given in terms of the twisted derivative,

$$\mathrm{d}_{-H}\mathbf{F} = \mathrm{d}\mathbf{F} - H \wedge \mathbf{F} = 0. \tag{1.4}$$

Note that the derivative and product of a form with a polyform is a linear extension of the action on forms. The twisted derivative satisfies the convenient property

$$\mathrm{d}^2_{-H} = 0,$$

provided that the Bianchi identity for H is satisfied

$$\mathrm{d}H = 0.$$

This means that locally we can express the field strength in terms of the RR potential

$$\mathbf{F} = \mathrm{d}_{-H}\mathbf{C}.$$

The action (1.3) includes not only the physical degrees of freedom F_n with $n \leq 5$ but also the dual fields with $n > 5$. This means that on-shell one has to relate the fields to each other with [127]

$$\star_{10}\, \sigma(\mathbf{F}) = \mathbf{F}, \tag{1.5}$$

where σ is the reversal operator given by the action on forms

$$\sigma(\omega \wedge \psi) = \sigma(\psi) \wedge \sigma(\omega), \qquad \sigma(A) = A,$$

where ω and ψ are any forms but A is a 1-form. Clearly σ then only serves to give a sign depending on the form degree it acts on. The action of σ extends linearly on polyforms. The duality rule (1.5) in particular implies

$$\langle \mathbf{G} \wedge \star_{10}\mathbf{F} - \sigma(\mathbf{G}) \wedge \mathbf{F} \rangle_{10} = 0,$$

for any polyform $\mathbf{G}$. Here we have introduced the $\langle \mathbf{A} \rangle_p$ operator that projects out the p-form A_p from the general polyform $\mathbf{A}$.

1.4 D-branes

Finally we mention the worldvolume actions of D-branes which play an important role in flux compactifications of type II string theory. From a world sheet point of view the D-branes are the endpoints of open strings and the DBI action can in fact be obtained from a β function calculation just like the 10D actions [128, 129]. The DBI action of a Dp-brane takes the form

$$S^{(p)}_{\text{DBI}} = -T_p \int \mathrm{d}^{p+1}\sigma \ \mathrm{e}^{-\phi}\sqrt{-\det\left(G_{ij} + B_{ij} + \mathrm{d}\check{\mathcal{A}}_{ij}\right)}, \tag{1.6}$$

where G and B with lower case Latin indices $i, j, k, l, \ldots$ indicate the pullbacks of the target space fields

$$G_{ij} = \partial_i X^M \partial_j X^N G_{MN} \quad \text{and} \quad B_{ij} = \partial_i X^M \partial_j X^N B_{MN},$$

and X^M are the embedding coordinates of the brane. Finally, $\check{\mathcal{A}}$ is the worldvolume abelian gauge field living on the brane. The Dp-brane is charged under the RR fields, it couples electrically to the RR gauge potential C_{p+1} or equivalently, by the duality relation (1.5), magnetically to C_{7-p}. The coupling is determined by the Wess-Zumino action

$$S^{(p)}_{\text{WZ}} = \mu_p \int \left\langle \mathrm{e}^{-\check{\mathcal{F}}} \wedge \mathbf{C} \right\rangle_{p+1}, \tag{1.7}$$

where

$$\check{\mathcal{F}} = B + \mathrm{d}\check{\mathcal{A}},$$

and the polyform $\mathrm{e}^{-\check{\mathcal{F}}}$ is defined by its power series. Notice that in the expressions above, both $\mathbf{C}$ and B should be understood as the pullbacks of the relevant fields. Although we use the same symbols as the target space fields this should not cause any confusion and should be understood from the context. We can introduce a δ form that enables us to integrate the localized actions (1.6) and (1.7) over full 10D space which are otherwise only integrated over the worldvolume of the brane in question. Let Σ be the worldvolume of a Dp-brane, then $\delta_{9-p} = \sigma(\star_{9-p}1)\delta(\Sigma)$ and $\delta(\Sigma)$ is the normalized δ distribution with support on the worldvolume. We use $\star_{9-p}1$ to denote the volume form transverse to the brane such that $\star_{10}1 = \star_{9-p}1 \wedge \star_{p+1}1$. For these definitions we can rewrite the WZ action

$$S_{\text{WZ}} = \sum_p \mu_p \int_\Sigma \left\langle \mathrm{e}^{-\check{\mathcal{F}}} \wedge \mathbf{C} \right\rangle_{p+1} = \sum_p \mu_p \int \sigma(\delta_{9-p}) \wedge \left\langle \mathrm{e}^{-\check{\mathcal{F}}} \wedge \mathbf{C} \right\rangle_{p+1}.$$

The D-brane charge μ_p can be related to its tension via $\mu_p = T_p$ for positively charged branes. In fact we will always take μ_p to be positive but put in the minus sign by hand when dealing with anti D-branes ($\bar{\text{D}}$-brane). The world volume action of orientifold planes or O-planes can be obtained from the DBI and WZ actions above by setting B and $\mathrm{d}\check{\mathcal{A}}$ to zero on the brane. For O-planes we still have the identification $\mu_p = T_p$ but O-planes have negative tension, so T_p is negative. O-planes are not dynamical objects but they are charged under the target space fields as described by the DBI and WZ actions.

The Bianchi identities for the RR fields in the presence of sources are

$$\mathrm{d}_{-H}\mathbf{F} + \mathbf{j} = 0, \tag{1.8}$$

where $\mathbf{j}$ is the polyform containing the sum over all source contributions of the different Bianchi identities, where $\mathbf{j} = \sum_p \mu_p \langle \delta \wedge \mathrm{e}^B \rangle_{9-p}$ for D-branes and $\mathbf{j} = -\sum_p \mu_p \delta_{9-p}$ for O-planes. The polyform $\mathbf{j}$ has the convenient property that

$$\int \sigma(\mathbf{j}) \wedge \mathbf{C} = S_{\text{WZ}} = \sum_p S^{(p)}_{\text{WZ}}. \tag{1.9}$$

Chapter 2

The heterotic cosmological constant

At sufficiently low energies and for small string coupling, perturbative string theory is well approximated by an effective two-derivative supergravity Lagrangian supplemented by small corrections coming from a double expansion in the slope parameter α' and the string coupling g_s. The terms of the α' expansion are higher derivative corrections to the supergravity action that account for the extended nature of the strings. They are negligible if the curvature of the background manifold and derivatives of the fields are small in units of α'. The terms coming from the g_s expansion are loop corrections due to nontrivial topologies of the string world sheet, which are negligible in the semi-classical regime when the string coupling is small.

From a phenomenological point of view, such sub-leading corrections to the leading supergravity action can have important consequences, as they may allow for solutions with properties that are forbidden at the two-derivative supergravity level. A well-known example in type IIB string theory is the AdS_4 solutions at large internal volume [42], where α' corrections [130] break the no-scale structure of the leading order Minkowski solutions found in [131] (and also [132–135]) and contribute to a nonzero cosmological constant. In this example, however, the α' corrections alone are not sufficient, and also non-perturbative quantum corrections from localized sources are needed in order to generate the AdS vacuum.

For the heterotic string, an analogous scenario was investigated in [136], where the authors found that an interplay of the lowest order α' correction [137] and non-perturbative effects could give rise to a similar large volume AdS vacuum in 4D, while the classical two-derivative supergravity action only admits Minkowski ground states.

In view of these constructions, one might wonder whether there could also be situations where the perturbative α' corrections alone already suffice to generate a small non-vanishing cosmological constant in a controlled compactification scheme. This question should be easiest to study for the heterotic string, where D-branes and orientifold planes are absent, and the leading α'-corrections are completely known and already appear at order $\mathcal{O}(\alpha')$. Looking at the heterotic effective action at string tree level, however, one might quickly conclude that α' corrections alone can never suffice to generate vacua other than Minkowski space. All terms in the action come from world sheets with the same topology such that this action scales uniformly with the dilaton ϕ:

$$S = \int \mathrm{d}^{10}x\sqrt{-g}\,\mathrm{e}^{-2\phi}\{\dots\} \tag{2.1}$$

(cf. (2.4)). As a consequence, the four dimensional effective scalar potential likewise scales uniformly with the dilaton zero mode, and one would expect the 4D dilaton equation to be solved either if the potential vanishes on the solution or if there is a runaway to a free vacuum [138,139]. It therefore seems obvious that heterotic string theory at string tree level can only lead to

Minkowski solutions, and that a non-vanishing cosmological constant also requires string loop or non-perturbative quantum corrections.

In this chapter which is based on [125] we re-address this question and in particular the seemingly trivial counterargument against non-Minkowski vacua sketched in the previous paragraph. The reason is that the higher curvature terms among the α' corrections (e.g. the $\alpha' \mathrm{tr}|R^+|^2$ terms in the heterotic string) also lead to contributions to the four dimensional Einstein equation and the equations of motion for the moduli that involve higher powers of external Riemann tensors and hence can *not* be interpreted as a part of the effective scalar potential. It is therefore a priori not clear whether the scaling argument sketched above is still valid or whether nontrivial effects might emerge from such higher order terms.

That these effects exist follows from explicitly known AdS_4 compactifications of the heterotic string when the effective action is truncated after the lowest order α' corrections (see e.g. [47, 140, 141]). In these solutions, the 4D cosmological constant turns out large, $\Lambda \sim \frac{1}{\alpha'}$, so that the effects of even higher α' corrections are difficult to estimate offhand and would require more explicit calculations [140].

In this chapter we investigate to what extent the usual scaling analysis of the 4D effective potential is invalidated by higher curvature terms in the α' expansion and check whether this expansion can yield perturbatively small cosmological constants of order $\mathcal{O}(\alpha')$ or higher. The main result of our analysis is that this is in general not possible at string tree level. This follows from the four dimensional Einstein equation and the dilaton equation, which can be combined to yield a constraint of the form

$$\Lambda = \sum_{m,n} c_{mn} \alpha'^m \Lambda^n, \qquad m, n > 0, \tag{2.2}$$

where c_{mn} are numerical coefficients containing integrals over internal fields and their derivatives. Assuming a perturbative α' expansion for Λ, one then obtains $\Lambda = 0$ as the only solution to all orders in α', as we will explain in more detail below.

2.1 A "no-go theorem"

In this section, we discuss a simple argument showing that tree level heterotic string theory with its first order α' corrections does not have 4D de Sitter or anti-de Sitter vacua with a perturbatively small cosmological constant at this order [142]. We then show that the argument can in fact be extended to all orders in the α' expansion. Our assumptions throughout the chapter are as follows:

- We consider compactifications to four dimensions that respect maximal four dimensional spacetime symmetry, i.e.:
  - The 10D metric is a warped product of a maximally symmetric 4D spacetime (parameterized by coordinates x^μ; $\mu, \nu, \ldots = 0, \ldots, 3$) and a 6D compact manifold (parameterized by y^m; $m, n, \ldots = 4, \ldots, 9$),

    $$\mathrm{d}s^2 = \mathrm{e}^{2A} \mathrm{d}s_4^2 + \mathrm{d}s_6^2, \tag{2.3}$$

    where the warp factor, e^{2A}, depends on the 6D coordinates only, and $\mathrm{d}s_4^2$ describes an unwarped 4D Minkowski, de Sitter or anti-de Sitter spacetime.
  - All 4D parts of tensor and spinor fields vanish (up to gauge choices) except for combinations that can be built from the 4D (unwarped) metric, its Riemann tensor or

its volume form. This means, in particular, that there are no spacetime filling fluxes[1] and that all 4D covariant derivatives of all tensor fields, including the dilaton and the Riemann tensor, can be set to zero on the solution.[2] Furthermore, the Lorentz-Chern-Simons 3-form does not contribute to the equations of motion in maximally symmetric backgrounds [143].

- String loop and/or non-perturbative corrections to the action are disregarded.
- α' is a meaningful expansion parameter in the sense that all field variations are small over a string length and the α' corrections can be organized in a perturbative expansion about the zero-slope limit.[3]

2.1.1 Heterotic supergravity with leading α' corrections

In string frame, the heterotic supergravity action with leading α' corrections reads (cf. (1.1))

$$S = \int \mathrm{e}^{-2\phi} \star_{10} \left\{ R + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left[\mathrm{tr}|F|^2 - \mathrm{tr}|R^+|^2\right] + \mathcal{O}(\alpha'^2) \right\} \tag{2.4}$$

with $\mathrm{tr}|R^+|^2 = \frac{1}{2} R^+_{MNPQ} R^{+MNPQ}$ and R^+_{MNRS} is the Riemann tensor constructed from the torsionful connection $\Gamma^{+M}_{NR} = \Gamma^M_{NR} - \frac{1}{2} H^M{}_{NR}$. For our argument, it is sufficient to look at the field equations of the dilaton and the external metric.

In the effective 4D theory, we can restrict our attention to the zero mode, τ, of the dilaton, which we define by separating off the higher Kaluza-Klein modes,

$$\mathrm{e}^{-\phi} = \tau \mathrm{e}^{-\phi_{\mathrm{KK}}}. \tag{2.5}$$

Here ϕ_{KK} denotes the sum of all remaining KK modes, which we integrate out by simply setting them equal to their on-shell values. It does not matter for our argument whether τ or one of the KK modes has the lowest mass (or whether they even combine with other degrees of freedom in the low energy EFT as suggested in [144].) This can be seen directly from the equivalent ten dimensional analysis which we will come to in Sec. 3.3.

On-shell, all fields in 4D must be covariantly constant by maximal symmetry, so we can henceforth ignore any x^μ-dependence of τ and only need to keep track of τ itself in the action, but not of its derivatives.

The only other field whose dynamics we need to consider is the external metric $g_{\mu\nu}$. Switching to four dimensional Einstein frame, we define a new 4D metric $\tilde{g}_{\mu\nu}$ by

$$\tilde{g}_{\mu\nu} \equiv \mathcal{V}\tau^2 \mathrm{e}^{-2A} g_{\mu\nu}. \tag{2.6}$$

Here we have defined the volume modulus

$$\mathcal{V} \equiv \int \mathrm{d}^6 y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}+2A}, \tag{2.7}$$

which can again be treated as constant in 4D by maximal symmetry.

[1] We express everything in terms of the Yang-Mills field strength F and the NS 3-form H, which have a too small rank to be spacetime filling in 4D. The Hodge duals of purely 6D fluxes of these fields would of course generically have spacetime filling components, but they do not appear explicitly in our formalism.

[2] Note that for maximally symmetric spaces, the Riemann tensor is covariantly constant.

[3] The α' expansion differs from the derivative expansion in that some terms appear at higher orders than suggested by the number of their derivatives. An example is the term $\mathrm{tr}|F|^2$ which, although a two derivative term, appears at $\wr\alpha'$. It should be noted though that our analysis does not depend on which of the two expansion schemes is used.

Performing this rescaling, we then obtain an effective 4D action for $\tilde{g}_{\mu\nu}$ and τ of the form

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}_4}\left\{\tilde{R}_4 - V + W\right\}, \tag{2.8}$$

where we have split the action into the Einstein-Hilbert term and two extra contributions. V contains all terms that are constructed from fields without external indices, whereas W contains all terms that include fields with 4D spacetime indices. In the absence of W, V is just the usual effective potential.

Using (2.4), these two terms are given by

$$\begin{aligned} V = &-\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{-2\phi_{\mathrm{KK}}+4A}\frac{1}{\tau^2\mathcal{V}^2} \\ &\times\left[R_6 - 20|\mathrm{d}A|^2 - 8\nabla^2 A + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left(\mathrm{tr}|F|^2 - |R_6^+|^2\right)\right] + \mathcal{O}(\alpha'^2) \end{aligned} \tag{2.9}$$

and

$$W = \int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{-2\phi_{\mathrm{KK}}}\left[\frac{\alpha'\tau^2}{4}\mathrm{tr}|\tilde{R}|_4^2 - \frac{\alpha'}{2\mathcal{V}}\mathrm{e}^{2A}\tilde{R}_4|\mathrm{d}A|^2\right] + \mathcal{O}(\alpha'^2), \tag{2.10}$$

where we have evaluated the curvature terms R and $\mathrm{tr}|R^+|^2$ for the tilded metric (2.6) and expressed them in terms of $\tilde{R}_4$ and $\mathrm{tr}|\tilde{R}|_4^2 = \frac{1}{2}\tilde{R}_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$ as well as a term $|R_6^+|^2$ containing various internal fields. Further details and the definition of $|R_6^+|^2$ can be found in App. A.

Using the scaling $V \sim \tau^{-2}$, one finds the 4D dilaton equation,

$$2V + \frac{\alpha'\tau^2}{2}\mathrm{tr}|\tilde{R}|_4^2\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{-2\phi_{\mathrm{KK}}} = 0, \tag{2.11}$$

and the trace of the four dimensional Einstein equation,

$$\tilde{R}_4 - 2V - \frac{\alpha'}{2\mathcal{V}}\tilde{R}_4\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{-2\phi_{\mathrm{KK}}+2A}\,|\mathrm{d}A|^2 = 0, \tag{2.12}$$

where we have neglected the variation with respect to the connection as it would give rise to covariant derivatives upon partial integration, which vanish due to maximal symmetry. Combining the two equations such that V cancels out and substituting $\tilde{R}_{\mu\nu\lambda\rho} = \frac{2}{3}\Lambda\tilde{g}_{\lambda[\mu}\tilde{g}_{\nu]\rho}$ then yields an equation of the form

$$\Lambda = \alpha'\left(c_{11}\Lambda + c_{12}\Lambda^2\right) + \mathcal{O}(\alpha'^2), \tag{2.13}$$

where c_{11} and c_{12} are given by

$$c_{11} = \frac{1}{2\mathcal{V}}\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{-2\phi_{\mathrm{KK}}+2A}\,|\mathrm{d}A|^2, \qquad c_{12} = -\frac{\tau^2}{3}\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{-2\phi_{\mathrm{KK}}}. \tag{2.14}$$

Given our assumption that we are in the regime of validity of the perturbative α' expansion, (2.13) must be solved order by order with an ansatz of the form

$$\Lambda = \Lambda_0 + \alpha'\Lambda_1 + \mathcal{O}(\alpha'^2) \tag{2.15}$$

for the cosmological constant, where Λ_0 denotes the solution of the leading order supergravity equations without α' corrections, $\alpha'\Lambda_1$ is a correction due to next-to-leading order terms in the α' expansion, and so on. It is straightforward to see that plugging this ansatz into (2.13) yields

$$\Lambda = \mathcal{O}(\alpha'^2) \tag{2.16}$$

as the only solution. Thus, perturbative heterotic string theory does not yield solutions with a nonzero cosmological constant up to corrections of order $\mathcal{O}(\alpha'^2)$.

2.1.2 General argument

Let us now generalize the above argument to the heterotic string with α' corrections of arbitrarily high order. The effective action for the massless fields then reads

$$S = \int \mathrm{e}^{-2\phi} \star_{10} \left\{ R + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 + \alpha'\text{-corrections} \right\}, \tag{2.17}$$

where all terms scale identically with respect to the dilaton if we neglect string loop or non-perturbative corrections as initially stated.

Rescaling the metric as in (2.6), we obtain the action in four dimensional Einstein frame

$$S = \int \mathrm{d}^4 x \sqrt{-\tilde{g}_4} \left\{ \tilde{R}_4 - V + W \right\}. \tag{2.18}$$

As in the previous section, we have split the action into an Einstein-Hilbert term $\tilde{R}_4$, a term V containing all terms that are constructed from fields without external spacetime indices, and a term W containing everything else.

In the absence of string loop or non-perturbative corrections, all terms in V scale again as $V \sim \tau^{-2}$ such that the dilaton equation yields

$$2V + \tau \partial_\tau W = 0. \tag{2.19}$$

Taking the trace of the four dimensional Einstein equation, we furthermore find

$$\tilde{R}_4 - 2V - W' = 0, \qquad W' \equiv \frac{\tilde{g}^{\mu\nu}}{\sqrt{-\tilde{g}_4}} \frac{\delta}{\delta \tilde{g}^{\mu\nu}} \left(\int \mathrm{d}^4 x \sqrt{-\tilde{g}_4}\, W \right), \tag{2.20}$$

where, as indicated, W' denotes all terms that are due to the variation of W with respect to the external metric.

Combining the two Eqs. (2.19) and (2.20), we then find

$$\tilde{R}_4 = -\tau \partial_\tau W + W'. \tag{2.21}$$

Although an explicit expression for the right hand side of this equation is only known for the first few orders in the α' expansion, the general structure is rather simple: it is a sum of positive powers of the cosmological constant with coefficients built from integrals over internal fields and their derivatives.

To see this, recall that our assumption of maximal 4D spacetime symmetry implies that only the metric, the epsilon tensor and the Riemann tensor are nontrivial, all with vanishing covariant derivative. Considering first the metric variations of W that come from variations of connections (either within covariant derivatives or curvature tensors or Lorentz-Chern-Simons forms), one sees that these variations do not contribute to the right hand side of (2.21), as they would lead to terms with a total 4D covariant derivative, which vanish by assumption. The only contributions to W' are therefore from variations of metric tensors that appear algebraically in W or in the metric determinant. As there are no nontrivial contractions of just the epsilon tensor and/or the metric, all these terms must contain at least one Riemann tensor.[4] Similar remarks also apply to the dilaton variation of W, so that the right hand side of (2.21) is a sum of terms that each involves at least one Riemann tensor. Because of $\tilde{R}_{\mu\nu\lambda\rho} = \frac{2}{3}\Lambda \tilde{g}_{\lambda[\mu}\tilde{g}_{\nu]\rho}$, these then translate into positive powers of the cosmological constant, as claimed.

[4] Note that there is no constant term in W: a constant has no external spacetime indices and hence would be part of V, which however cancels out in (2.21).

Since at leading order the supergravity action does not contain any terms that depend on the Riemann tensor except for the Einstein-Hilbert term, the terms in W and W' are of order $\mathcal{O}(\alpha')$ or higher. We can therefore schematically rewrite Eq. (2.21) as

$$\Lambda = \sum_{m,n} c_{mn} \alpha'^m \Lambda^n, \qquad m, n > 0, \tag{2.22}$$

with some numerical coefficients c_{mn} that in general contain integrals over contractions of warp factor terms, internal field strengths and curvatures, and so on.

Assuming again the validity of a perturbative α' expansion, we need to solve (2.22) order by order with an ansatz of the form

$$\Lambda = \Lambda_0 + \alpha' \Lambda_1 + \alpha'^2 \Lambda_2 + ... \tag{2.23}$$

as in Sec. 2.1.1. This yields

$$\Lambda = 0 \tag{2.24}$$

as the only solution to all orders in the perturbative α' expansion.[5] Hence, heterotic string theory yields Minkowski spacetime as the only maximally symmetric solution to all orders in the perturbative α' expansion, unless one introduces loop and/or non-perturbative corrections. In particular, we don't find α' generated AdS_4 vacua with perturbatively small curvatures to be possible.

2.2 Discussion

Let us now discuss several implications of our findings. In particular, we will discuss possibilities to evade our above no-go argument, its relation to the Dine-Seiberg problem and the violation of the effective potential description due to higher order corrections to the supergravity action.

2.2.1 Evading the no-go theorem

In Sec. 2.1.2, we have shown that heterotic string compactifications at string tree level yield 4D Minkowski spacetime as the only maximally symmetric solution to all orders in a perturbative α' expansion, unless one violates one of our initial assumptions. Let us now discuss these possible violations and how they evade our argument.

Loop and non-perturbative corrections/extended sources

An obvious possibility to circumvent the argument of Sec. 2.1.2 is the inclusion of terms that scale differently with respect to the dilaton than the tree level terms considered here. Natural candidates are string loop or non-perturbative corrections e.g. from gaugino condensation [44, 45]. With such terms turned on, the dilaton and Einstein equations read

$$-\tau \partial_\tau V + \tau \partial_\tau W = 0, \qquad \tilde{R}_4 - 2V - W' = 0 \tag{2.25}$$

and can in general not be combined such that V cancels out. The right hand side of Eq. (2.22) may then contain terms which are independent of Λ, making solutions other than $\Lambda = 0$ possible.

[5] We might also try to solve Eq. (2.22) without expanding Λ as in (2.23). Assuming that $\Lambda \neq 0$, we can then divide by Λ to get $1 \leq \sum |c_{mn} \alpha'^m \Lambda^{n-1}|$. But this is again a contradiction to the assumption made in the beginning of Sec. 2.1.

It would be interesting to see whether including the first loop correction at order $\mathcal{O}(\alpha'^3 g_s)$ could allow for purely perturbative solutions with a non-zero cosmological constant for the heterotic string.

A different dilaton scaling may also be introduced if one includes extended sources such as the various types of D-branes and orientifold planes in type II string theory. Being an open string tree level action, the DBI action scales only with $e^{-\phi}$ and so would in general also invalidate our argument. In fact, in type II string theory, a large number of compactification scenarios with a nonzero cosmological constant have been proposed using D-branes and orientifolds as well as non-perturbative quantum corrections starting with [41] (see also chapter 4). Heterotic string theory, on the other hand, is much more limited in this respect, as it does not contain D-branes and O-planes. We would require dealing with NS5-branes or Kaluza-Klein monopoles or non-standard branes some of which we discuss in chapter 5 albeit in the context of type II string theory.

Spacetime filling fluxes

Since spacetime filling fluxes are in general not forbidden by maximal symmetry, they can be used to invalidate our argument around Eq. (2.21), where we explained that all terms in W are contractions of Riemann tensors and must therefore contain factors of the cosmological constant. In heterotic string theory, there are no spacetime filling fluxes if spacetime is assumed to be four dimensional. Compactifying to three dimensions, however, allows for solutions with a nonzero cosmological constant, if spacetime components of H are turned on (see e.g. [145]). In type II string theory, spacetime filling RR fluxes may also be present in compactifications to four dimensions and may lead to solutions with a nonzero cosmological constant, we will explicitly see this in chapter 4.

Large higher derivative terms

Another way to circumvent our no-go theorem is to leave the perturbative regime of the α' expansion and consider solutions for which higher derivative terms are not small in units of α'. A truncation of the action at a finite order is then in general not guaranteed to be a good approximation to the full theory, because higher order terms are not automatically suppressed.[6] This problem does of course not apply when supergravity is studied in its own right instead of being considered the low energy effective field theory of string theory. In any case, allowing curvature and derivatives of the fields to be large in units of α', it is indeed possible to construct solutions with a nonzero cosmological constant that is large in units of α'. A good example are the heterotic AdS compactifications studied in [140, 141], which are solutions to the heterotic supergravity action with linear α' corrections that feature a curvature of order $\mathcal{O}(\frac{1}{\alpha'})$. By construction, our argument does not make statements in this regime.

Breaking maximal symmetry

Requiring spacetime to be maximally symmetric implies a very limited field content such that, in the absence of spacetime filling fluxes, all terms showing up on the right hand side of Eq. (2.21) contain contractions of spacetime components of the Riemann tensor. All of these terms can then be rewritten as a power of Λ times some numerical factor, regardless of how the Riemann tensors are contracted. As explained earlier, this property ensures that the higher derivative curvature terms on the right hand side of Eq. (2.21) are much smaller than the Ricci scalar on

[6] This does of course not rule out that the truncated action could still capture the essential features of a solution or that the higher order terms happen to be small or even vanish in certain cases.

the left hand side, leading to constraint (2.22) and the conclusion that only Minkowski solutions are possible.

For spacetimes without maximal symmetry, however, this need not be the case. The presence of various (spacetime) tensor fields then leads to new terms in Eq. (2.21) which can be of the same order as the 4D Ricci scalar and thus generate a nonzero cosmological constant. Furthermore, it is not guaranteed anymore that higher derivative curvature terms in (2.21) are negligible, since whether they are much smaller than the 4D Ricci scalar or can compete with it depends on how they are contracted. This is due to the well-known fact that for general spaces the magnitude of individual components of the Riemann tensor and the Ricci scalar need not be the same, so that the Riemann tensor can have large components even when the Ricci scalar is very small. In heterotic string theory, the Ricci scalar may then compete, for example, with the $\alpha'|\tilde{R}_{\mu\nu\lambda}{}^{\rho}|^2$ term and thus become nonzero.

This is also the reason why it is not possible to extend our analysis to make a statement about the curvature of the internal space. An exception are compactifications on maximally symmetric spaces such as the six-sphere, which can be ruled out using an argument along the lines of Sec. 2.1.2, unless there exist six-form fluxes filling internal space. Since this only concerns a very restricted class of compactifications, our discussion does unfortunately not add much to the discussion of [146], where it is suggested that higher derivative corrections (or strong warping, see also [147]) could in principle support an everywhere negative internal Ricci scalar, which is difficult to realize otherwise.

2.2.2 The Dine-Seiberg problem

In [138, 139], Dine and Seiberg used the dilaton behavior of the effective 4D scalar potential in the weak coupling limit to argue that, unless the effective potential is identically zero, there must in general either be a runaway to the free vacuum or a minimum at strong coupling. Using an analogous scaling analysis for the universal volume modulus, one may argue for similar difficulties regarding compactifications at large volume (cf. e.g. [148] for a recent discussion). Progress in moduli stabilization techniques have since then led to many interesting scenarios where an interplay of various scalar potential contributions suggest the existence of weakly coupled minima at controllably large volumes. Still many of the difficulties and complexities one encounters in these attempts can be traced back to the issues pointed out in [138, 139].

The argument given in the present chapter, although somewhat similar in its consequences, differs from the argument of [138, 139] in several ways. First of all we do not really use or discuss moduli stabilization. Nor do we trace the dependence of the scalar potential on the volume modulus. In fact, the detailed form of the scalar potential and its moduli dependence play no role for our argument (except that we exploit the overall dilaton scaling to eliminate the scalar potential completely from the equation of interest (Eq. (2.21))). Instead, the only terms that matter for our argument are higher order products of 4D Riemann tensors, which did not play a role for the arguments in [138, 139].

Moreover, it could have been the case that terms that appear to be of lower order in the α' expansion compete with terms that are explicitly of higher order in α' without that the perturbative α' expansion breaks down. An example for this are the $|H|^2$ and $|F|^2$ terms appearing in the heterotic supergravity action or gradient terms of the warp factor or the dilaton. As reviewed in App. B, they are forced to be zero by the leading order equations of motion, if our initially stated assumptions hold. Including α' corrections to the action, however, the equations of motion are modified such that the above terms can become nonzero and thus compete with higher order terms in the α' expansion. This could have postponed the emergence of a nontrivial cosmological constant to a higher order than suggested by [149]. Our argument from Sec. 2.1.2, however, shows that this can not happen at any order in α', regardless of the scalar potential.

2.2.3 Violation of effective scalar potential description

The effective scalar potential description is a standard tool in effective field theory which is widely used in the moduli stabilization literature. For solutions yielding a maximally symmetric spacetime, the effective potential is usually expected to fulfill two assumptions:

- The equations of motion are satisfied at a point in moduli space which is an extremum of V.
- The value of V at this point is proportional to the cosmological constant.

These assumptions are true if the effective action can be written in the form

$$S = \int \mathrm{d}^4 x \sqrt{-\tilde{g}_4} \left\{ \tilde{R}_4 - V \right\}, \tag{2.26}$$

where $\tilde{R}_4$ is the only term in the Lagrangian that depends on the external metric, and V is the only term that depends on the moduli.

It is interesting to note that both assumptions are generically violated by higher order effects in the α' expansion, unless the cosmological constant is zero. This follows from Eq. (2.25) which on-shell yields

$$\partial_\tau V \neq 0, \qquad V \not\propto \Lambda. \tag{2.27}$$

Hence, the equations of motion are in general satisfied at a point in moduli space which is not an extremum of V. Moreover, V is not proportional to the cosmological constant anymore. This effect is usually completely negligible when the cosmological constant is small. For inflation scenarios with a very high energy scale, these corrections might be more sizeable, but when they are, the validity of the perturbative α' expansion would also be less obvious.

Chapter 3

Scaling symmetries

In the previous chapter we saw how the scaling of the 4D equations of motion with respect to the dilaton led to a strong restriction on the cosmological constant for the heterotic string at tree level. We will now analyse how this result arises from the 10D theory and generalize the method used. A systematic treatment of the 10D scaling symmetries leads to integral constraints on the on-shell observables which can be useful when combined with a subset of the equations of motion. In this spirit we rederive the result from last chapter and in next chapter we will apply this method to the 10D effective theory of type II string theory and derive a simple formula for the cosmological constant.

In type II string theory it is particularly interesting to be able to calculate observables such as the cosmological constant for setups where the full solutions to the differential equations are unknown. The difficulty in solving the equations is due mostly to the fact that localized sources such as D-branes and orientifold planes complicate the equations significantly. There exists a well known procedure to simplify the task of finding solutions called smearing. The sources encountered in many type II setups are spacetime filling but localized in the internal space. Smearing is the procedure of replacing the delta function encountered in the brane actions (1.6,1.7) by a smooth function in internal space that integrates to the same value as the localized source. This procedure may introduce its own problems and need not capture essential features of the true localized solution as we will discuss in the next chapter (see e.g. [64,69,146,147,150]).

In this chapter, which is based on [125] and [62], we will focus on the general argument for using scaling symmetries to derive relation for the on-shell fields and discuss some of the subtleties one encounters in more complicated setups.

3.1 Two scaling symmetries

It has been known since the 1980s [151] that the terms in the low energy effective action of string theory must satisfy simple scaling properties when the dilaton or equivalently the string coupling constant is scaled. This property is inherited from the simple coupling of the dilaton to the world sheet curvature in string perturbation theory and is manifest in the string frame of the 10D effective action. In this chapter we work mainly with the 10D effective actions in Einstein frame, where the scaling does not only affect the dilaton ϕ, but also the metric G_{MN}. Remember the mapping from string frame to Einstein frame is accomplished by the conformal transformation of the metric

$$G_{\text{String}} = \mathrm{e}^{\phi/2} G_{\text{Einst}},$$

which leads to the following relation between the curvature scalars in the two frames (see appendix A)

$$\star_{10}\mathrm{e}^{-2\phi}R_{\text{String}} = \tilde{\star}_{10}\left\{R_{\text{Einst}} - \frac{9}{2}\tilde{\nabla}^2\phi - \frac{9}{2}|\mathrm{d}\phi|^2\right\},$$

in this expression we have used tilde for Einstein frame quantities but in the following we will omit writing the tilde, as we will mostly work in Einstein frame in this and next chapter. In type II string theory we also encounter RR $(n-1)$-forms C_{n-1} that have been rescaled with the dilaton so that the 10D action takes the form (1.3). This is a standard form of the action although all terms in (1.3) have been obtained from a spherical worldsheet [22]. In these variables the so-called dilaton scaling can be written as:

$$\mathrm{e}^{-\phi} \mapsto s\mathrm{e}^{-\phi}, \quad G_{MN} \mapsto \sqrt{s}G_{MN}, \quad C_{n-1} \mapsto sC_{n-1}, \tag{3.1}$$

where s is a scaling parameter. This then leads to

$$S^{(\chi)} \mapsto s^{\chi}S^{(\chi)}, \tag{3.2}$$

where χ is the Euler characteristic of the worldsheet from which the contribution, $S^{(\chi)}$, to the effective action was derived [22]. This can be observed directly from the two dimensional world sheet theory. For simplicity we consider the world sheet action for the bosonic string, the coupling of the string world sheet to the dilaton is accomplished by adding a term of the form

$$-\int \star_2 R_2\phi$$

where R_2 and $\star_2$ are the two dimensional curvature scalar and Hodge operator respectively. Shifting the dilaton by a constant scalar

$$\phi \mapsto \phi - \frac{1}{2}\log s$$

generates a topological term in the two dimensional path integral

$$-\int \star_2 R_2\phi + \frac{1}{2}\log s\int \star_2 R_2 = -\int \star_2 R_2\phi + \log s^{\chi}$$

where we have used the well-known fact that the integral of the curvature scalar over any Riemann surface is proportional to the Euler characteristic of the surface. The string path integral is then scaled by the same factor s^{χ}. The low energy effective action is directly related to the world sheet path integral [152], or indirectly via the vanishing of the beta functions [153]. This then shows that the low energy effective action satisfies the claimed scaling law.

For a standard low energy effective action consisting of the classical two-derivative action for the bulk supergravity fields, S_{bulk}, and the lowest order action due to the presence of localized sources, S_{loc}, we then get

$$S = S_{\text{bulk}} + S_{\text{loc}} \mapsto s^2 S_{\text{bulk}} + sS_{\text{loc}}. \tag{3.3}$$

Thus, in absence of localized sources, the effect of (3.1) is to rescale the tree level supergravity action by an overall factor s^2. The transformations (3.1) are then a symmetry of the theory, since they leave the equations of motion invariant.

A second scaling symmetry [154] can be obtained from the mass dimension of the fields, which can be determined from the fact that the effective action is a derivative expansion and has mass dimension zero. Let us scale the coordinates in the action by a fixed parameter

$$x^M \mapsto t^{-1}x^M,$$

This has the effect that p-form fields scale as t^{-p} and the metric, being a 2-tensor scales as $G \mapsto t^{-2}G$. The action on the other hand is not affected by general coordinate invariance. If we now scale only the fields according to this rule, but leave the coordinates unchanged, we obtain a non-trivial scaling of the terms in the action. The corresponding scaling of the bosonic fields in type II string theory is[1]

$$G_{MN} \mapsto t^{-2}G_{MN}, \quad C_{n-1} \mapsto t^{-(n-1)}C_{n-1}, \quad B \mapsto t^{-2}B, \tag{3.4}$$

where we reuse t as the associated scaling parameter. This yields the following scaling of the terms in the low energy action

$$S_i^D \mapsto t^{i-D}S_i^D, \tag{3.5}$$

where D denotes the number of dimensions that are integrated over (usually $D = 10$, but D is less than ten for source terms) and i denotes the number of derivatives of the terms involved. For a two-derivative bulk action and zero-derivative source terms with $(p+1)$-dimensional world volume, we thus get

$$S = S_{\text{bulk}} + S_{\text{loc}} \mapsto t^{-8}S_{\text{bulk}} + \sum_p t^{-p-1}S_{\text{loc}}^{(p)}. \tag{3.6}$$

In absence of localized sources, the transformations (3.4) are a symmetry, since they rescale the bulk action by an overall factor t^{-8} and thus leave the equations of motion invariant. Together with Eq. (3.3), this implies that the type II supergravity action at tree level has two global scaling symmetries, which are explicitly broken by terms that are due to the presence of localized sources.

3.2 Scaling rules and constraints

We will now sketch the method for deriving a generalized integral constraint on the on-shell observables. This on-shell expression can then be substituted into the integrated Einstein equation to give an equation for the cosmological constant Λ. This is essentially the same method as used in last chapter for the heterotic string but in a generalized framework and directly in the 10D effective theory. Before we discuss how to derive the on-shell constraint in the general case, let us at first review the basic principle [156] using a simple example. Consider an action $S[\psi_i]$ that depends on a number of form fields ψ_i and that satisfies a scaling symmetry,

$$S[\tau^{k_i}\psi_i] = \tau^K S[\psi_i], \tag{3.7}$$

where the scaling parameter τ is a real number, and k_i and K are assumed to be non-vanishing. We can then take the τ derivative of (3.7) and let $\tau \to 1$ to obtain

$$\int \sum_i k_i \frac{\delta S[\psi_i]}{\delta \psi_i} \wedge \psi_i = K S[\psi_i], \tag{3.8}$$

where we have written the result in terms of the usual functional derivative. Note that in the above expression we have assumed that the fields in the action are all forms. This in itself is consistent since the supergravity actions can be written in the tedrad formalism where the metric is replaced by frame fields which are 1-forms. Furthermore the dilaton is a scalar and therefore a 0-form, of course we can also just use the metric formalism of gravity but then the

[1]This symmetry is sometimes called *Trombone* symmetry in the context of supergravity, see for example [155]. Note that in our conventions the exponent of t in (3.4) actually corresponds to the length (i.e. the inverse mass) dimension of the field.

wedge product above should be replaced by contraction of indices in the obvious way. Using the fact that the fields satisfy the equations of motion $\delta S[\psi_i]/\delta\psi_i = 0$, we then find that the left hand side of Eq. (3.8) vanishes and

$$S[\psi_i] = 0 \tag{3.9}$$

on-shell.

In deriving (3.9) we made two simplifications that do in general not hold in the context of string compactifications. The right hand side of the equation is therefore often more complicated than in this simple example. First, we assumed that all terms in the action $S[\psi_i]$ scale uniformly with τ. When we identify τ with the scaling parameters s or t of the previous subsection, this is not true when localized sources or higher order corrections are taken into account. Second, when we evaluated $\delta S[\tau^{k_i}\psi_i]/\delta\tau$ to arrive at (3.8), we had to integrate by parts all those terms in $S[\tau^{k_i}\psi_i]$ that involve derivatives of ψ_i. In general the effective action is not written in terms of the potentials ψ_i, but rather in terms of the field strengths ω_i. In string theory, these field strengths are accompanied by Bianchi identities which, when solved, give local expressions of ω_i in terms of ψ_i. However, many compactifications involve the presence of non-trivial background fluxes. The corresponding field strengths ω_i then have a non-exact part such that, globally, they cannot be written in terms of the gauge potential ψ_i. This is not really a problem though since globally we can write

$$\omega_i = \mathrm{d}\psi_i + \omega_i^b,$$

where ω_i^b denotes the non-trivial flux part of ω_i. When we repeat the above calculation for a general action $S = \sum_a S_a$, where each term S_a scales uniformly, we get

$$S[\tau^{k_i}\psi_i, \tau^{k_i}\omega_i^b] = \sum_a \tau^{K_a} S_a[\psi_i, \omega_i^b].$$

Upon differentiation with respect to τ for $\tau = 1$, we find

$$\int \sum_i k_i \frac{\delta S}{\delta\omega_i} \wedge \omega_i^b = \sum_j K_a S_a, \tag{3.10}$$

on-shell. We therefore see that the right hand side of Eq. (3.9) receives two contributions: one contribution due to the presence of localized sources or higher order correction terms which behave differently with respect to the scaling rules and another due to non-trivial background fluxes.

3.3 Heterotic string revisited

Returning again to the tree level heterotic string we see that the two issues mentioned above do not concern us. This is because even though we can have potential 3-form flux H, it does not transform under the dilaton scaling (3.3) and we get the result

$$S = 0, \tag{3.11}$$

where the action S is the full tree level action (2.17). The result, (3.11), might not seem particulary surprising at first glance, and is actually well known for the leading order terms in the tree level action [153]. However, the new result here is the fact that this extends to all orders in the α' expansion. When higher loop corrections are included the result, (3.11), is modified, but as can be noted from the discussion above, the structure of g_s corrections to Eq. (3.11) are easily inferred from the above procedure.

Schematically we can write the full perturbative tree level action as

$$S = \int \mathrm{d}^{10}x \, \sqrt{-G} \sum_{n=0} \alpha'^n L_n. \tag{3.12}$$

The Lagrangians L_n depend on the same fields as the leading order expression appearing in Eq. (1.1) albeit with higher order derivatives present. From the action (3.12) we can compute the 10 dimensional Einstein equation, it has the following structure

$$E_{MN} - \frac{1}{2} G_{MN} \sum_{n=0} \alpha'^n L_n = 0, \tag{3.13}$$

the second factor comes from varying only the determinant of the metric while the first term appears from varying the Lagrangian. The tensor E_{MN} is a short hand expression for a general two-tensor which structure is only known to leading orders. We can write

$$E_{MN} = \sum_{n=0} \alpha'^n E^{(n)}_{MN},$$

but then $E^{(0)}_{MN}$ is easily calculable from the leading order action, (1.1). We can now use the Einstein equation together with the result $S = 0$ to compute the leading order contribution to the four dimensional cosmological constant. To this end we make the same assumptions as in last chapter, that the four dimensional space has maximal symmetry, i.e.:

- The 10D spacetime is a warped product space of a maximally symmetric 4D spacetime and a compact 6D space. The details of the internal manifold are not important for our discussion, and the results are completely independent of them. The metric describing the 10D space can be written as

$$\mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}s_6^2, \tag{3.14}$$

  where the external metric $g_{\mu\nu}$ only depends on the external coordinates and the internal metric only depends on the internal coordinates. Being maximally symmetric the metric of the 4D space $g_{\mu\nu}$ must be either Minkowski, de Sitter or anti de Sitter spacetime.

- All external (i.e. 4D) components of tensors can be built from the 4D metric, the Riemann tensor and the 4D volume form. This means that any tensor fields (of positive tensor degree) describing 4D matter such as the Yang-Mills gauge field A or the three-form H must have vanishing external components. This also implies that the dilaton must be constant in space time. This is not really a additional requirement to the one above since any matter field with preferred direction would generate a preferred direction in the 4D energy momentum tensor which would break the symmetry of 4D spacetime.

Using the external metric $g_{\mu\nu}$ we trace the Einstein equation (3.13) to get

$$\sum_{n=0} \alpha'^n \left[\mathrm{e}^{-2A} g^{\mu\nu} E^{(n)}_{\mu\nu} - 2L_n \right] = 0.$$

If we now integrate this over 10 dimensional spacetime and use the equation (3.11) to simplify we get

$$\sum_{n=0} \alpha'^n \int \mathrm{d}^{10}x \, \sqrt{-G} \mathrm{e}^{-2A} g^{\mu\nu} E^{(n)}_{\mu\nu} - 2S = \sum_{n=0} \alpha'^n \int \mathrm{d}^{10}x \, \sqrt{-G} \mathrm{e}^{-2A} g^{\mu\nu} E^{(n)}_{\mu\nu} = 0. \tag{3.15}$$

We have now reduced the Einstein equation to an equation that puts strong constraint on the external components of the tensor $E_{\mu\nu}$. One can easily confirm that the leading order contribution to $E_{\mu\nu}$, in the α' expansion, is the Ricci tensor

$$E^{(0)}_{\mu\nu} = R_{\mu\nu}$$

where we used the fact that H and $\mathrm{d}\phi$ have no non-vanishing external components. We will now argue that the higher order contribution to $E_{\mu\nu}$ all contain positive power of the Riemann tensor, or is a total derivative. The argument goes as follows: Assume that $E_{\mu\nu}$ can be written as a sum of two terms,

$$E_{\mu\nu} = E_{1\mu\nu} + E_{2\mu\nu}$$

where $E_{2\mu\nu}$ is independent of the Riemann tensor, we will show that $E_{2\mu\nu}$ must be a total derivative. Given our second assumption above we must be able to express $E_{2\mu\nu}$ in terms of the 4D metric $g_{\mu\nu}$ and the anti-symmetric tensor $\epsilon_{\mu\nu\rho\sigma}$. We can immediately eliminate the ϵ tensor since it has more than two indices which must contract with something in order to give non-vanishing contribution, the only anti-symmetric tensor available besides $\epsilon_{\mu\nu\rho\sigma}$ is the Riemann tensor, but this is forbidden in $E_{2\mu\nu}$ by construction. The only possibility therefore is that

$$E_{2\mu\nu} = E_2 g_{\mu\nu},$$

where E_2 is some scalar. A term of this kind could be a result of two different terms in the 10D action. First possibility is the metric determinant, however the tensor E_{MN} was derived by varying the Lagrangian

$$L = \sum_n \alpha'^n L_n,$$

which is a tensor density of weight 0. Since any additional powers of the metric determinant would disturb the tensor density weight of that term the term $E_2 g_{\mu\nu}$ can not be a result of variation of the metric determinant. The only possibility we are left with is when $E_2 g_{\mu\nu}$ is derived from a term in the action involving some derivatives of the metric. This however will always lead to a total derivative in the equations of motion, for example, the variation of the Christoffel symbols is

$$\delta\Gamma^R_{MN} = \frac{1}{2}G^{RS}\left(\nabla_M \delta G_{NS} + \nabla_N \delta G_{MS} - \nabla_S \delta G_{MN}\right).$$

It is now clear that the tensor $E_{\mu\nu}$ can be written as a sum of two terms, one that contains positive powers of the Riemann tensor, and a second that is a total derivative. The Eq. (3.15) can now be further simplified, using the above argument, to give

$$\int \sum_{n=0} \mathrm{e}^{-2\phi} \alpha'^n g^{\mu\nu} E^{(n)}_{1\mu\nu} \mathrm{d}^{10}x = 0. \tag{3.16}$$

We can now use the first assumption above to relate the 4D Riemann tensor to the cosmological constant. For a maximally symmetric spacetime the Riemann tensor takes the form

$$R_{\mu\nu\rho\sigma} = g_{\mu[\sigma} g_{\rho]\nu} \frac{2}{3}\Lambda.$$

Inserting this into Eq. (3.16) we get the final result

$$\sum_{n,m} \alpha'^n c_{nm} \Lambda^m = 0, \tag{3.17}$$

where the coefficients c_{nm} can be related to integrals of internal fields over the internal space, the first coefficent c_{01} for example is related to the Einstein-Hilbert term of the action (1.1) in Einstein frame.

Before continuing we note that the above analysis was done assuming a trivial spacetime factorization

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}s_6^2,$$

where $g_{\mu\nu}$ is independent of the internal coordinates y. We can redo the analysis assuming warped product space

$$\mathrm{d}s^2 = \mathrm{e}^{2A(y)}\tilde{G}_{MN}\mathrm{d}X^M \mathrm{d}X^N = \mathrm{e}^{2A(y)}\left(\tilde{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}\tilde{s}_6^2\right), \tag{3.18}$$

where now the *unwarped* metric $\tilde{g}_{\mu\nu}$ is independent of y and is assumed to be maximally symmetric. The structure of the argument is the same as before, we rewrite the action (3.12) in terms of $\tilde{G}_{MN}$, the leading order terms take the form (cf. (1.1))

$$S = \int \mathrm{e}^{8A} \star_{10} \left\{ \tilde{R} - 72|\mathrm{d}A|^2 - \frac{1}{2}|\mathrm{d}\phi|^2 - \frac{1}{2}\mathrm{e}^{-\phi}|H|^2 - \frac{\alpha'}{4}\mathrm{e}^{-\phi/2}\mathrm{tr}|F|^2 + \mathcal{O}(\alpha') \right\}. \tag{3.19}$$

The scaling symmetry is unaffected, and we still get the result $S = 0$, next the Einstein equation is derived, but it takes the same form as before (3.15) after having used that $S = 0$. Following the same logic as before and using that the unwarped metric is maximally symmetric, we get the result (3.17) also for the warped product space.

The rest of the argument to show that $\Lambda = 0$ is the same as in Sec. 2.1.2 and we will not repeat it here. Using the dilaton scaling (3.3) alone we have been able to rederive the result from last chapter which serves as a warm up exercise for our next task: to apply this framework to the considerably more complicated type II theory. There both non-trivial fluxes[2] and scaling laws that do not leave the action invariant play a role, but we also make use of the two scaling rules (3.3) and (3.6) to perform essentially similar analysis as here.

[2] The H-flux was not a problem in this section since it does not scale with the dilaton, however the RR fluxes in type II do enter the dilaton scaling (3.1).

Chapter 4

Type II cosmological constant and brane singularities

In the first part of this chapter, we will derive, in the context of type II supergravity coupled to D-branes and O-planes, an exact formula for the cosmological constant. Using results from the previous chapter, we will argue that the cosmological constant can often be expressed as a sum of terms that are due to the action of localized sources,

$$\Lambda \propto \sum_p c_p \left(S_{\rm DBI}^{(p)} + S_{\rm WZ}^{(p)} \right) , \tag{4.1}$$

where $S_{\rm DBI}^{(p)}$ and $S_{\rm WZ}^{(p)}$ are the on-shell evaluated DBI and Wess-Zumino actions of the Dp-branes and/or Op-planes present in the corresponding supergravity solution, and c_p are p-dependent constants. Thus, in compactification scenarios where our reasoning holds, Λ is entirely specified by the classical boundary conditions of some of the bulk fields at the positions of the sources and independent of the details of the ten-dimensional bulk dynamics. This result is an extension of previous results from the literature [125,156,157]. Using a single scaling symmetry of the action the authors of [156] were able to relate Λ to boundary terms involving the supergravity fields that have to be evaluated in the near source region in setups without background fluxes. They also pointed out that topologically nontrivial background fluxes can give contributions that arise from the patching of gauge charts. In principle one might be able to use the formulas derived in [156] for flux compactification on a case by case basis encoding the fluxes in Dirac strings. In this chapter, however, we derive an expression that explicitly spells out both the source and the background flux contributions. Furthermore we also use two scaling symmetries in stead of just one in [156] which often allows us to choose parameters such that the cosmological constant is only given by source contributions.

The results of this chapter which is based on [62] is also obviously a generalization of our result from chapter 2 where we found that the cosmological constant in solutions of perturbative heterotic string theory is zero to all orders in the α' expansion [125]. We discussed at the end of chapter 2 that spacetime filling fluxes, sources or non-perturbative effects will alter the conclusion and essentially that is what we will observe in this chapter. It turns out, however, that the intuitive scaling argument of chapter 2 is complicated in the type II string by a subtlety related to the RR fields and localized sources. As noted in chapter 3 the fluxes lead to form fields non-trivial in cohomology. It is then easy to see that these form fields lead to non-vanishing contribution to Λ.

It remains true in many cases that Λ is completely determined by a sum of source terms. The reason is that classical type II (and also heterotic) supergravity exhibits a two-parameter scaling symmetry, related to the dilaton scaling and the mass scaling of the classical action (see chapter 3). Both the scaling symmetry exploited in [156] and the one implicitly used in chapter 2 [125] are special cases of this more general symmetry.

In the second part of this chapter, we discuss an application of our result to the idea of placing $\overline{\text{D3}}$-branes at the bottom of the Klebanov-Strassler solution [131, 158, 159], a setup that has been suggested for the construction of meta stable de Sitter vacua in string theory starting with KKLT [41]. As briefly discussed in the introduction, the KKLT scenario is constructed by compactifying the type IIB string on a warped Calabi-Yau orientifold [131] or its F-theory generalizations [160]. The Calabi-Yau orientifold, on which the theory is compactified, is such that it has a so-called warped throat region where the topology is approximately a cone over $S^3 \times S^2$, and the apex of the cone is deformed in to a finite S^3 by placing F_3 flux on the so-called A cycle of the throat (see Fig. 4.1) [161, 162]. In addition to H flux, non-perturbative

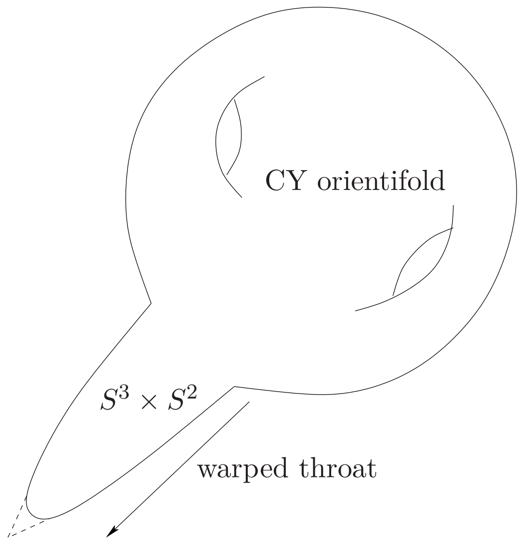


Figure 4.1: A sketch of a Klebanov-Strassler deformed conifold glued to compact Calabi-Yau orientifold.

effects are included to stabilize all moduli at supersymmetric AdS. Finally a small number of anti D3-branes ($\overline{\text{D3}}$) are added to break supersymmetry and lift the vacuum energy, leading to a meta-stable de Sitter solution (see figure 4.2 for a sketch of the effective potential before and after adding the branes).

The backreaction of $\overline{\text{D3}}$-branes on the Klebanov-Strassler (KS) geometry has recently been subject of intense discussions [52–63] (see also related discussion for $\overline{\text{D6}}$-branes [64–69], and $\overline{\text{M2}}$-branes [70–76]). Part of this debate concerns the computational evidence for a singularity in fields that do not directly couple to the anti-branes as it emerged in several approaches.

More precisely, the presence of this singularity has so far been demonstrated in simplified setups that use certain approximations. In earlier works on the subject, this involved a partial smearing of the branes and a perturbative treatment of the equations of motion around the KS background [53, 54, 56, 57]. [55] therefore also discusses the possibility that the singularity might just be an artifact of perturbation theory and disappear in the full setup (see however [58]). Although it could recently be shown in [59] that also the non-linear equations of motion necessarily lead to a singular solution, the analysis still required partially smeared branes. An analysis of the fully localized case could only be carried out for a simplified toy model with $\overline{\text{D6}}$-branes [64, 65, 67, 68]. In this simplified setup, it was shown that fully localized branes in a non-BPS flux background lead to a singularity in the energy density of the H flux, which is not

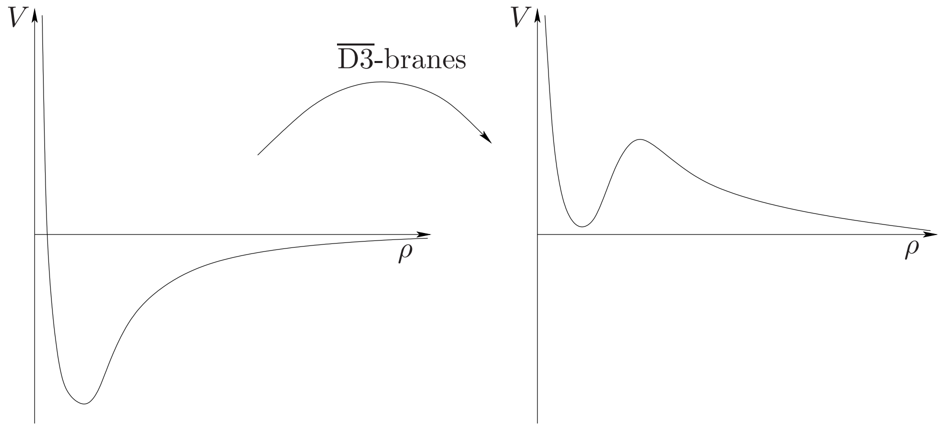


Figure 4.2: A sketch of the effective potential as a function of the volume modulus for the KKLT scenario before and after adding the $\overline{\text{D3}}$-branes. The small vacuum energy of the meta-stable point on the right hand graph gives rise to an effective positive cosmological constant [41].

directly sourced by the $\overline{\text{D6}}$-branes. Recently a class of new AdS_7 solutions have been found in type IIA with D6- or $\overline{\text{D6}}$-branes [68] which includes the solution discussed in [64, 65, 67]. The solutions of [68] were constructed to be supersymmetric, but still were found to have diverging energy density at the brane positions. This is a rather interesting result because it has been speculated in the past that the divergence in the energy density of H might be a signature of instability of the solution and might perhaps lead to a decay into other stable solutions with different branes (brane polarization) [60, 67, 159]. Earlier work, using the non-compact approximation, found this not to be the case [67]. However, in [68] compact solutions with $\overline{\text{D8}}$-branes replaced by $\overline{\text{D6}}$-branes were found which did not posess the diveregent energy density of the $\overline{\text{D6}}$-brane solutions but did carry the necessary $\overline{\text{D6}}$-brane charge to allow for polarization. Finally in [150] it was showed that brane polarization does indeed occur. One may wonder if similar process is possible for the KKLT scenario where the $\overline{\text{D3}}$-branes polarizes to a nonsingular NS5-branes solution as the probe approximation suggests [159]. Recent work strongly hints that this is not the case [60].

As our result from the first part of the chapter relates the near brane behavior of the supergravity fields to the effective cosmological constant, it is natural to try to apply this to $\overline{\text{D3}}$-branes in the KS background. We show that under a few assumptions this would indeed be possible and confirm the presence of a non-standard singularity at the $\overline{\text{D3}}$-brane similar to the one discussed before, but now without the approximation of any smearing and by using the full non-linear supergravity equations.

4.1 Type II supergravity in Einstein frame

The low energy effective action of type II string theory presented in Sec. 1.3 and 1.4 can be written in Einstein frame as

$$S = S_{\text{bulk}} + S_{\text{loc}} \tag{4.2}$$

with

$$S_{\text{bulk}} = S_{\text{NSNS}} + S_{\text{RR}} = \int \star_{10} \left\{ R - \frac{1}{2}|\mathrm{d}\phi|^2 - \frac{1}{2}\mathrm{e}^{-\phi}|H|^2 - \frac{1}{4}\sum_n \mathrm{e}^{\frac{5-n}{2}\phi}|F_n|^2 \right\}. \tag{4.3}$$

Here, R is the curvature scalar of the Einstein frame metric G,[1] $\star_{10}$ denotes the ten-dimensional Hodge operator also associated with G. Remember the relation of the string frame metric to the Einstein frame metric

$$G_{\text{String}} = \mathrm{e}^{\phi/2} G_{\text{Einst}}.$$

In Einstein frame the duality relation of the RR fields (1.5) takes a slightly different form:

$$\mathrm{e}^{\frac{5-n}{2}\phi} F_n = \star_{10}\, \sigma(F_{10-n}). \tag{4.4}$$

Again, this relation has to be imposed by hand on-shell.

The term S_{loc} denotes the action of localized sources corresponding to either Dp-branes or Op-planes and reads[2]

$$S_{\text{loc}} = \sum_p S_{\text{loc}}^{(p)} = \sum_p \left(S_{\text{DBI}}^{(p)} + S_{\text{WZ}}^{(p)} \right) \tag{4.5}$$

with

$$S_{\text{DBI}}^{(p)} = \mp \mu_p \int \star_{10} \mathrm{e}^{\frac{p-3}{4}\phi} \delta(\Sigma_{p+1}), \qquad S_{\text{WZ}}^{(p)} = \begin{cases} +\,\mu_p \int \sigma(\delta_{9-p}) \wedge \langle \mathbf{C} \wedge \mathrm{e}^{-B} \rangle_{p+1} \\ -\,\mu_p \int \sigma(\delta_{9-p}) \wedge C_{p+1} \end{cases}, \tag{4.6}$$

where the upper line is for Dp-branes and the lower line for Op-planes, and $\mu_p > 0$ is the absolute value of the Dp-brane/Op-plane charge. For $\overline{\text{D}p}$-branes and $\overline{\text{O}p}$-planes, the Wess-Zumino terms would have the opposite sign.

4.1.1 Compactification and equations of motion

Throughout this chapter, we restrict ourselves to warped compactifications to $d \geq 4$ dimensions that preserve maximal symmetry in the non-compact d-dimensional spacetime. Accordingly, we only consider spacetime filling sources extending in $p+1 \geq d$ dimensions. Furthermore, all fields are assumed to depend only on the internal coordinates x^m. The form fields are allowed to have legs in external directions only if they are spacetime filling, in other words they have to be of rank d or higher. All other form fields are purely internal. We assume a warped metric of the form

$$\mathrm{d}s_{10}^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn} \mathrm{d}x^m \mathrm{d}x^n, \qquad g_{\mu\nu} = \mathrm{e}^{2A} \tilde{g}_{\mu\nu}, \tag{4.7}$$

where A is the warp factor and $\tilde{g}_{\mu\nu}$ is the unwarped d-dimensional metric corresponding to a Minkowski or (A)dS spacetime. We will also put a tilde on quantities such as Hodge operators, covariant derivatives or contractions of tensors if they are constructed using the unwarped metric instead of the warped one.

We now list the relevant equations of motion needed for this chapter. The trace of the external Einstein equation reads

$$R_d = \frac{d}{2}\Big(\mathcal{L} - \sum_p \mathcal{L}_{\text{WZ}}^{(p)} \Big) + \frac{d}{4} \sum_n \mathrm{e}^{\frac{5-n}{2}\phi} |F_n^{\text{ext}}|^2, \tag{4.8}$$

[1] In this chapter we only use the Einstein frame, and therefore use the same symbols to denote Einstein frame quantities without confusion.

[2] In this chapter we do not include the NSNS 2-form in the DBI action, because in all the examples we discuss in detail the sources are either point like in the internal space or they are wrapped O-planes, so that a B-field along the world volume cannot occur. Likewise we do not consider D-branes with world volume fluxes in our examples and hence also omit them in the DBI action. It is easy to check that omitting the NSNS 2-form in the DBI action does not lead to a missing term in the H-equation of motion, because $\delta S_{DBI}/\delta B_{\mu\nu}$ also vanishes if B and F are set to zero after the field equations are derived (cf. also the explicit expressions in [128]).

where $R_d = R_{\mu\nu}g^{\mu\nu}$ is the d-dimensional curvature scalar and we denote the spacetime filling RR field strengths by F_n^{ext}. $\mathcal{L}$ is the Lagrangian including all bulk terms and the DBI and WZ terms due to the localized sources, and $\mathcal{L}_{\text{WZ}}^{(p)}$ are the WZ parts of the source Lagrangian. For the warped metric (4.7), one finds

$$R_d = \frac{2d}{d-2}\mathrm{e}^{-2A}\Lambda - \mathrm{e}^{-dA}\tilde{\nabla}^2\mathrm{e}^{dA}, \tag{4.9}$$

where Λ is the d-dimensional cosmological constant. Substituting this into (4.8) and integrating over ten-dimensional spacetime then yields

$$\frac{8v\mathcal{V}}{d-2}\Lambda = 2\Big(S - \sum_p S_{\text{WZ}}^{(p)}\Big) + \sum_n \int \star_{10}\, \mathrm{e}^{\frac{5-n}{2}\phi}|F_n^{\text{ext}}|^2, \tag{4.10}$$

where we have introduced the volume factors

$$v = \int \tilde{\star}_d 1, \qquad \mathcal{V} = \int \star_{10-d}\, \mathrm{e}^{(d-2)A}. \tag{4.11}$$

We will also need the H equation of motion

$$\mathrm{d}\left(\mathrm{e}^{-\phi} \star_{10} H\right) + \frac{1}{2}\left\langle \sigma(\mathbf{F}) \wedge \mathbf{F}\right\rangle_8 = 0. \tag{4.12}$$

4.1.2 Cosmological constant in the absence of sources

Let us now start by using the scaling symmetries and on-shell constraints discussed in last chapter to calculate the on-shell d-dimensional cosmological constant.

In order to account for the possibility of flux, we explicitly divide the NSNS and RR field strengths into a flux part, which is closed but not exact, and a fluctuation, which is exact and given in terms of a globally defined gauge potential. For H, we thus write

$$H = \mathrm{d}B + H^b, \tag{4.13}$$

where H^b denotes the background flux and B is the fluctuating globally defined NSNS potential. Since H^b is closed, the Bianchi identity $\mathrm{d}H = 0$ is satisfied such that our definition is consistent.[3] The RR Bianchi identity when no sources are present reduces to (1.4), which implies that we can then globally write

$$\mathbf{F} = \mathrm{d}_{-H}\mathbf{C} + \mathrm{e}^B \wedge \mathbf{F}^b, \tag{4.14}$$

where $\mathbf{F}^b$ is d_{-H^b}-closed, but non-trivial in cohomology. We can now use the general constraint derived in last chapter (3.10) applied to our present setup. For the dilaton scaling (3.1) we get

$$\begin{aligned} 2S &= \int \left\langle \frac{\delta S}{\delta \mathbf{F}} \wedge \mathrm{e}^B \wedge \mathbf{F}^b \right\rangle_{10} \\ &= -\frac{1}{2}\int \left\langle \sigma(\mathrm{e}^B \wedge \mathbf{F}^b) \wedge \mathbf{F} \right\rangle_{10} \\ &= -\frac{1}{2}\int \left\langle \sigma(\mathbf{F} - \mathrm{d}_{-H}\mathbf{C}) \wedge \mathbf{F}\right\rangle_{10} \\ &= -\frac{1}{2}\int \left\langle \sigma(\mathbf{C}) \wedge \mathrm{d}_{-H}\mathbf{F}\right\rangle_{10} \\ &= 0, \end{aligned}$$

[3] We do not consider compactifications involving NS5-branes in this chapter, i.e. the Bianchi identity for H does not contain a source term.

where we used the Bianchi idendity and that $\langle\sigma(\mathbf{A})\wedge\mathbf{A}\rangle_{10}=0$ for any polyform $\mathbf{A}$. Inserting this result into the integrated Einstein equation (4.10) we see that the cosmological constant is given solely by the spacetime filling flux present in the setup. Using the mass scaling (3.4) we would be able to derive a relation for the spacetime filling fluxes, but we will not do this here since this is only a special case of the more general derivation carried out in next section.

4.2 The cosmological constant as a sum of source terms

In this section, we will use the two independent scaling symmetries introduced in chapter 3, which are satisfied by the action (4.2), to derive an expression for the cosmological constant Λ in terms of the (on-shell evaluated) action of localized sources. This analysis is in many ways similar to what we did in Sec. 3.3 but because of specifics to the present setup we will repeat all the steps here.

4.2.1 Type II flux

For the RR field strengths, separating off the non-exact part is subtle in the presence of sources. This is related to the fact that their Bianchi identities are more complicated and, in particular, that some of them receive contributions from localized sources. Since we only consider spacetime filling sources in this chapter, they enter the Bianchi identities as delta forms whose legs are always in some of the internal directions. Thus, a source term can only show up in the Bianchi identity for the purely internal part of the corresponding RR field strength. It is therefore convenient to split the polyform $\mathbf{F}$ into a part $\mathbf{F}^{\text{int}}$, which contains all RR field strengths that are purely internal and may have a source term in their Bianchi identity, and a part $\mathbf{F}^{\text{ext}}$, which contains all RR field strengths that are spacetime filling (and possibly also have legs in the internal part) and, accordingly, do not have a source term in their Bianchi identity,

$$\mathbf{F}=\mathbf{F}^{\text{int}}+\mathbf{F}^{\text{ext}}. \tag{4.15}$$

For $\mathbf{F}^{\text{ext}}$, the Bianchi identity (1.8) then simplifies to

$$\mathrm{d}_{-H}\mathbf{F}^{\text{ext}}=0. \tag{4.16}$$

This allows us to write

$$\mathbf{F}^{\text{ext}}=\mathrm{d}_{-H}\mathbf{C}^{\text{ext}}+\mathrm{e}^{B}\wedge\mathbf{F}^{b}, \tag{4.17}$$

where $\mathbf{F}^{b}$ is again a d_{-H^b}-closed but non-exact polyform containing the sum over the spacetime filling background fluxes only and $\mathbf{C}^{\text{ext}}$ is a polyform containing the sum over the spacetime filling RR potentials. In a (maximally symmetric) type IIB compactification to 4 dimensions, for example, we would have $\mathbf{F}^{b}=F_5^b+F_7^b+F_9^b$ and $\mathbf{C}^{\text{ext}}=C_4^{\text{ext}}+C_6^{\text{ext}}+C_8^{\text{ext}}$, since only forms of rank 4 or higher would be allowed to be spacetime filling.

The Bianchi identities of the internal field strengths, $\mathbf{F}^{\text{int}}$, however, contain source terms

$$\mathrm{d}_{-H}\mathbf{F}^{\text{int}}+\mathbf{j}=0, \tag{4.18}$$

where as before $\mathbf{j}=\sum\mu_p\left\langle\mathrm{e}^{B}\wedge\delta\right\rangle_{9-p}$ for D-branes and $\mathbf{j}=-\sum\mu_p\delta_{9-p}$ for O-planes. This means that these field strengths can in general not be written in a way similar to (4.17) everywhere on the compact space. We will circumvent this problem in this section by simply expressing, at the level of the equations of motion, $\mathbf{F}^{\text{int}}$ in terms of their dual field strengths

$\mathbf{F}^{\text{ext}}$, which then in turn can be expressed in terms of (4.17). With the present definitions the duality relation (4.4) now reduces to

$$\mathrm{e}^{\frac{5-n}{2}\phi} F_n^{\text{int}} = \star_{10}\sigma\left(F_{10-n}^{\text{ext}}\right).$$

If, for example, $F_3 = F_3^{\text{int}}$ is internal, we can express it in terms of the spacetime filling $F_7 = F_7^{\text{ext}}$ via the duality relation $F_3^{\text{int}} = -\mathrm{e}^{-\phi} \star_{10} F_7^{\text{ext}}$ and then use (4.17) to split F_7^{ext} into an exact and a non-exact part.[4]

Using the internal Bianchi identity (4.18) together with (1.9) and the duality relation just described, we immediately find the relation

$$S_{\text{WZ}} - \sum_n \int \star_{10}\mathrm{e}^{\frac{5-n}{2}\phi}|F_n^{\text{ext}}|^2 = -\int \left\langle \sigma(\mathbf{F}^b) \wedge \mathrm{e}^{-B} \wedge \mathbf{F}^{\text{int}} \right\rangle_{10}, \tag{4.19}$$

which will become useful later.

Finally, let us note that, since we put the non-exact parts of the NSNS and RR field strengths into H^b and $\mathbf{F}^b$, we can assume that the gauge potentials B and $\mathbf{C}^{\text{ext}}$ are globally defined. This implies that total derivatives involving B and $\mathbf{C}^{\text{ext}}$ integrate to zero on a compact space, which will be used below. It should also be mentioned that, under the scalings (3.3) and (3.6), the flux terms H^b and $\mathbf{F}^b$ behave in the same way as the corresponding gauge potentials do. This follows from the fact that the mass dimension and the coupling to the dilaton is the same for the exact and the non-exact parts of the NSNS and RR field strengths.

4.2.2 On-shell action and cosmological constant

Let us now discuss how to derive the on-shell expression for the action (4.2) that will later be used in the integrated Einstein equation (4.10) to obtain our result for Λ. Contrary to the simple example sketched in the previous section, the calculation is rather involved if one considers the general case including sources and fluxes. Let us therefore note that there is an alternative way to obtain our result, which only uses the equations of motion instead of exploiting the scaling symmetries. This second derivation may serve as a double check of our results and is detailed in App. C. In the following, we will continue to discuss the first method, using the scaling symmetries.

Let τ denote the scaling parameter, where τ equals s if we consider the dilaton scaling (3.1) and t in case of the mass scaling (3.4). Moreover, we will use primes to denote the τ transformed fields and the corresponding τ transformed action. Thus, if $\tau = s$, we have, for example, $G'_{MN} = \sqrt{s}G_{MN}$, and if $\tau = t$, we have $G'_{MN} = t^{-2}G_{MN}$. According to (3.3) and (3.6), the action (4.2) then scales as

$$S' = S'_{\text{bulk}} + S'_{\text{loc}} = \tau^k S_{\text{bulk}} + \sum_p \tau^{l_p} S_{\text{loc}}^{(p)}, \tag{4.20}$$

where $k = 2$, $l_p = 1$ for $\tau = s$ and $k = -8$, $l_p = -p-1$ for $\tau = t$. Taking the τ derivative and evaluating the equation at $\tau = 1$, we find

$$\left.\frac{\mathrm{d}S'_{\text{bulk}}}{\mathrm{d}\tau}\right|_{\tau=1} + \left.\frac{\mathrm{d}S'_{\text{loc}}}{\mathrm{d}\tau}\right|_{\tau=1} = kS_{\text{bulk}} + \sum_p l_p S_{\text{loc}}^{(p)}. \tag{4.21}$$

[4] A subtlety occurs for F_5, which is self dual, and F_4, which can have both internal and spacetime filling components in compactifications to 4 dimensions. In these cases, only the internal components F_4^{int}, F_5^{int} can have a source term in the Bianchi identity. We therefore express those in terms of their duals F_6^{ext}, F_5^{ext}, which can in turn be written in terms of (4.17).

We now proceed as in the simple example discussed in Sec. 3.2: we first evaluate the terms on the left hand side of the equation and integrate by parts to express them in terms of a functional derivative of the action with respect to the fields. We then substitute the equations of motion to simplify the expressions.

The first term on the left hand side of (4.21) yields

$$\frac{\mathrm{d}S'_{\mathrm{bulk}}}{\mathrm{d}\tau}\bigg|_{\tau=1} = \int \left[\frac{\delta S_{\mathrm{bulk}}}{\delta G_{MN}}\frac{\mathrm{d}G'_{MN}}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{bulk}}}{\delta \phi}\frac{\mathrm{d}\phi'}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{NSNS}}}{\delta H}\wedge\frac{\mathrm{d}H'}{\mathrm{d}\tau} + \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}}\wedge\frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}\tau}\right\rangle_{10}\right]\bigg|_{\tau=1}, \tag{4.22}$$

where we have implicitly used partial integration to write the first two terms in the integrand as functional derivatives of S_{bulk} with respect to the metric and the dilaton. These functional derivatives are equivalent to the variation of the bulk action, which will later allow us to use the equations of motion to simplify the expression. Similarly, we should also rewrite the remaining two terms in above equation as variations with respect to the NSNS and RR potentials. This is more involved since H and $\mathbf{F}$ may contain flux (cf. (4.13) and (4.17)), and so we will consider these terms separately later. Let us at first evaluate the $\mathrm{d}S'_{\mathrm{loc}}/\mathrm{d}\tau$ term in (4.21),

$$\begin{aligned}\frac{\mathrm{d}S'_{\mathrm{loc}}}{\mathrm{d}\tau}\bigg|_{\tau=1} &= \int \left[\frac{\delta S_{\mathrm{loc}}}{\delta G_{MN}}\frac{\mathrm{d}G'_{MN}}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{loc}}}{\delta \phi}\frac{\mathrm{d}\phi'}{\mathrm{d}\tau} + \left\langle \frac{\delta S_{\mathrm{loc}}}{\delta \mathbf{C}}\wedge\frac{\mathrm{d}\mathbf{C}'}{\mathrm{d}\tau}\right\rangle_{10} + \frac{\delta S_{\mathrm{loc}}}{\delta B}\wedge\frac{\mathrm{d}B'}{\mathrm{d}\tau}\right]\bigg|_{\tau=1} \\ &= \int \left[\frac{\delta S_{\mathrm{loc}}}{\delta G_{MN}}\frac{\mathrm{d}G'_{MN}}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{loc}}}{\delta \phi}\frac{\mathrm{d}\phi'}{\mathrm{d}\tau}\right]\bigg|_{\tau=1} + \sum_p \frac{\mathrm{d}S'^{(p)}_{\mathrm{CS}}}{\mathrm{d}\tau}\bigg|_{\tau=1}.\end{aligned} \tag{4.23}$$

Since S_{loc} does not depend on any field derivatives but only on the fields themselves, we did not have to integrate by parts here. We can now combine (4.22) and (4.23) and use the equations of motion $\delta S/\delta G_{MN} = \delta S/\delta\phi = 0$ to obtain

$$\frac{\mathrm{d}S'_{\mathrm{bulk}}}{\mathrm{d}\tau}\bigg|_{\tau=1} + \frac{\mathrm{d}S'_{\mathrm{loc}}}{\mathrm{d}\tau}\bigg|_{\tau=1} = \int \left[\frac{\delta S_{\mathrm{NSNS}}}{\delta H}\wedge\frac{\mathrm{d}H'}{\mathrm{d}\tau} + \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}}\wedge\frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}\tau}\right\rangle_{10}\right]\bigg|_{\tau=1} + \sum_p \frac{\mathrm{d}S'^{(p)}_{\mathrm{CS}}}{\mathrm{d}\tau}\bigg|_{\tau=1}. \tag{4.24}$$

The two terms involving δH and $\delta\mathbf{F}$ are evaluated as follows. Substituting (4.13) into the $\delta S_{\mathrm{NSNS}}/\delta H$ term in (4.24), we can integrate by parts to obtain

$$\begin{aligned}\int \frac{\delta S_{\mathrm{NSNS}}}{\delta H}\wedge\frac{\mathrm{d}H'}{\mathrm{d}\tau}\bigg|_{\tau=1} &= \int \left[\mathrm{d}\frac{\delta S_{\mathrm{NSNS}}}{\delta H}\wedge\frac{\mathrm{d}B'}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{NSNS}}}{\delta H}\wedge\frac{\mathrm{d}H'^b}{\mathrm{d}\tau}\right]\bigg|_{\tau=1} \\ &= \int \left[\frac{\delta S_{\mathrm{NSNS}}}{\delta B}\wedge\frac{\mathrm{d}B'}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{NSNS}}}{\delta H}\wedge\frac{\mathrm{d}H'^b}{\mathrm{d}\tau}\right]\bigg|_{\tau=1}.\end{aligned} \tag{4.25}$$

The $\delta S_{\mathrm{RR}}/\delta\mathbf{F}$ term in (4.24) can be computed in a similar fashion but is more complicated due to the subtleties explained in Sec. 3.2. We first use (4.15) and write

$$\int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}}\wedge\frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}\tau}\right\rangle_{10}\bigg|_{\tau=1} = \int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}^{\mathrm{ext}}}\wedge\frac{\mathrm{d}\mathbf{F}'^{\mathrm{ext}}}{\mathrm{d}\tau} + \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}^{\mathrm{int}}}\wedge\frac{\mathrm{d}\mathbf{F}'^{\mathrm{int}}}{\mathrm{d}\tau}\right\rangle_{10}\bigg|_{\tau=1}. \tag{4.26}$$

We now have to replace all RR field strengths F_n^{int} by their dual field strengths F_{10-n}^{ext} in order to be able to write them in terms of the globally defined gauge potentials $\mathbf{C}^{\mathrm{ext}}$ using (4.17), which in turn will allow us to integrate by parts in (4.26). Using the duality relations (4.4) as well as the scalings (3.1) and (3.4), we find for the two cases $\tau = s$ and $\tau = t$:

$$\begin{aligned}
\int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}} \wedge \frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}s} \right\rangle_{10}\Bigg|_{s=1} &= \sum_n \int \left(\frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{ext}}} \wedge F_n^{\mathrm{ext}} + \frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{int}}} \wedge F_n^{\mathrm{int}} \right) \\
&= \sum_n \int \left(\frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{ext}}} \wedge F_n^{\mathrm{ext}} - \frac{\delta S_{\mathrm{RR}}}{\delta F_{10-n}^{\mathrm{ext}}} \wedge F_{10-n}^{\mathrm{ext}} \right) \\
&= 0, \qquad (4.27)\\
\int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}} \wedge \frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}t} \right\rangle_{10}\Bigg|_{t=1} &= \sum_n (1-n) \int \left(\frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{ext}}} \wedge F_n^{\mathrm{ext}} + \frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{int}}} \wedge F_n^{\mathrm{int}} \right) \\
&= \sum_n (1-n) \int \left(\frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{ext}}} \wedge F_n^{\mathrm{ext}} - \frac{\delta S_{\mathrm{RR}}}{\delta F_{10-n}^{\mathrm{ext}}} \wedge F_{10-n}^{\mathrm{ext}} \right) \\
&= \sum_n (10-2n) \int \frac{\delta S_{\mathrm{RR}}}{\delta F_n^{\mathrm{ext}}} \wedge F_n^{\mathrm{ext}}. \qquad (4.28)
\end{aligned}$$

These two expressions can now be rewritten in a way that will become convenient further below. In order to do so, we again exploit the scalings (3.1) and (3.4) and make use of the identity $\delta S_{\mathrm{RR}}/\delta F_n^{\mathrm{ext}} \wedge F_n^{\mathrm{ext}} = -\frac{1}{2} \star_{10} \mathrm{e}^{(5-n)\phi/2} |F_n^{\mathrm{ext}}|^2$, which can be derived from (4.3). We thus find

$$\begin{aligned}
\int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}} \wedge \frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}\tau} \right\rangle_{10}\Bigg|_{\tau=1} &= 2 \int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}^{\mathrm{ext}}} \wedge \frac{\mathrm{d}\mathbf{F}'^{\mathrm{ext}}}{\mathrm{d}\tau} \right\rangle_{10}\Bigg|_{\tau=1} - 2k \int \left\langle \frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}^{\mathrm{ext}}} \wedge \mathbf{F}^{\mathrm{ext}} \right\rangle_{10} \\
&\quad - \frac{k}{2} \sum_n \int \star_{10} \mathrm{e}^{\frac{5-n}{2}\phi} |F_n^{\mathrm{ext}}|^2, \qquad (4.29)
\end{aligned}$$

where $k = 2$ for $\tau = s$ and $k = -8$ for $\tau = t$ as in (4.20).

We now integrate by parts on the right hand side of Eq. (4.29). Taking into account (4.13) and (4.17), this yields[5]

$$
\begin{aligned}
&\int \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \frac{\mathrm{d}\mathbf{F}'^{\rm ext}}{\mathrm{d}\tau} \right\rangle_{10}\Big|_{\tau=1} \\
&= \int \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \left(\mathrm{d}_{-H} \frac{\mathrm{d}\mathbf{C}'^{\rm ext}}{\mathrm{d}\tau} + \mathrm{e}^B \wedge \frac{\mathrm{d}\mathbf{F}'^b}{\mathrm{d}\tau} - \frac{\mathrm{d}(\mathrm{d}B' + H'^b)}{\mathrm{d}\tau} \wedge \mathbf{C}^{\rm ext} + \frac{\mathrm{d}B'}{\mathrm{d}\tau} \wedge \mathrm{e}^B \wedge \mathbf{F}^b \right) \right\rangle_{10}\Big|_{\tau=1} \\
&= \int \left\langle \frac{\delta S_{\rm RR}}{\delta C^{\rm ext}} \wedge \frac{\mathrm{d}C'^{\rm ext}}{\mathrm{d}\tau} + \frac{\delta S_{\rm RR}}{\delta F^{\rm ext}} \wedge \left(\mathrm{e}^B \wedge \frac{\mathrm{d}F'^b}{\mathrm{d}\tau} + \frac{\delta F^{\rm ext}}{\delta B} \wedge \frac{\mathrm{d}B'}{\mathrm{d}\tau} + \frac{\delta F^{\rm ext}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau} \right) \right\rangle_{10}\Big|_{\tau=1} \\
&= \int \Bigg\langle \left(\frac{\delta S}{\delta \mathbf{C}^{\rm ext}} - \frac{1}{2} \frac{\delta S_{\rm loc}}{\delta \mathbf{C}^{\rm ext}} \right) \wedge \frac{\mathrm{d}\mathbf{C}'^{\rm ext}}{\mathrm{d}\tau} + \frac{1}{2} \frac{\delta S_{\rm RR}}{\delta B} \wedge \frac{\mathrm{d}B'}{\mathrm{d}\tau} \\
&\qquad + \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \left(\mathrm{e}^B \wedge \frac{\mathrm{d}\mathbf{F}'^b}{\mathrm{d}\tau} + \frac{\delta \mathbf{F}^{\rm ext}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau} \right) \Bigg\rangle_{10}\Bigg|_{\tau=1} \\
&= \int \Bigg\langle \left(\frac{\delta S}{\delta \mathbf{C}^{\rm ext}} - \frac{1}{2} \frac{\delta S_{\rm loc}}{\delta \mathbf{C}^{\rm ext}} \right) \wedge \frac{\mathrm{d}\mathbf{C}'^{\rm ext}}{\mathrm{d}\tau} + \frac{1}{2} \left(\frac{\delta S}{\delta B} - \frac{\delta S_{\rm NSNS}}{\delta B} - \frac{\delta S_{\rm loc}}{\delta B} \right) \wedge \frac{\mathrm{d}B'}{\mathrm{d}\tau} \\
&\qquad + \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \left(\mathrm{e}^B \wedge \frac{\mathrm{d}\mathbf{F}'^b}{\mathrm{d}\tau} + \frac{\delta \mathbf{F}^{\rm ext}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau} \right) \Bigg\rangle_{10}\Bigg|_{\tau=1} ,
\end{aligned}
\tag{4.30}
$$

where we also used

$$
2 \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \frac{\delta \mathbf{F}^{\rm ext}}{\delta B} \right\rangle_8 = \left\langle \mathbf{F}^{\rm ext} \wedge \sigma(\mathbf{F}^{\rm int}) \right\rangle_8 - \frac{\delta S_{\rm loc}}{\delta B} = \frac{\delta S}{\delta B} - \frac{\delta S_{\rm NSNS}}{\delta B} - \frac{\delta S_{\rm loc}}{\delta B} = \frac{\delta S_{\rm RR}}{\delta B}, \tag{4.31}
$$

which can be derived using (4.3), (4.12), (4.13) and (4.17). With the equations of motion, $\delta S/\delta \mathbf{C}^{\rm ext} = \delta S/\delta B = 0$, one finally obtains

$$
\begin{aligned}
\int \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \frac{\mathrm{d}\mathbf{F}'^{\rm ext}}{\mathrm{d}\tau} \right\rangle_{10}\Big|_{\tau=1} &= -\frac{1}{2} \sum_p \frac{\mathrm{d}S'^{(p)}_{\rm WZ}}{\mathrm{d}\tau}\Big|_{\tau=1} - \frac{1}{2} \int \frac{\delta S_{\rm NSNS}}{\delta B} \wedge \frac{\mathrm{d}B'}{\mathrm{d}\tau}\Big|_{\tau=1} \\
&\quad + \int \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \left(\mathrm{e}^B \wedge \frac{\mathrm{d}\mathbf{F}'^b}{\mathrm{d}\tau} + \frac{\delta \mathbf{F}^{\rm ext}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau} \right) \right\rangle_{10}\Big|_{\tau=1} .
\end{aligned}
\tag{4.32}
$$

Evaluating this for $\tau = s$ using (3.1) then also implies

$$
\int \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \mathbf{F}^{\rm ext} \right\rangle_{10} = -\frac{1}{2} \sum_p S^{(p)}_{\rm WZ} + \int \left\langle \frac{\delta S_{\rm RR}}{\delta \mathbf{F}^{\rm ext}} \wedge \mathrm{e}^B \wedge \mathbf{F}^b \right\rangle_{10} . \tag{4.33}
$$

[5]The factor $\frac{1}{2}$ that appears when rewriting $\delta S_{\rm RR}/\delta \mathbf{C}^{\rm ext}$ in terms of $\delta S/\delta \mathbf{C}^{\rm ext}$ and $\delta S_{\rm loc}/\delta \mathbf{C}^{\rm ext}$ is related to a subtlety regarding the variation of the CS action of the RR fields. One only obtains the correct equations of motion if one takes the coupling of the RR fields to the sources as being half the coupling that one would get from the "naive" variation of the action. One can think of this as being due to the fact that one half of $\sum_p S^{(p)}_{\rm WZ}$ represents an electric coupling of the RR fields to the sources, whereas the other half is due to a magnetic coupling of the dual RR fields to the sources. This subtlety is known in the literature and has, for example, been discussed in footnote 6 of [131] but also in [126, 163].

Substituting (4.32) and (4.33) into (4.29) then leads to

$$\begin{aligned}\int \left\langle \frac{\delta S_{\text{RR}}}{\delta \mathbf{F}} \wedge \frac{\mathrm{d}\mathbf{F}'}{\mathrm{d}\tau} \right\rangle_{10}\bigg|_{\tau=1} = &-\sum_p \frac{\mathrm{d}S'^{(p)}_{\text{WZ}}}{\mathrm{d}\tau}\bigg|_{\tau=1} + k\sum_p S^{(p)}_{\text{WZ}} - \int \frac{\delta S_{\text{NSNS}}}{\delta B} \wedge \frac{\mathrm{d}B'}{\mathrm{d}\tau}\bigg|_{\tau=1} \\ &+ 2\int \left\langle \frac{\delta S_{\text{RR}}}{\delta \mathbf{F}^{\text{ext}}} \wedge \left(\mathrm{e}^B \wedge \frac{\mathrm{d}\mathbf{F}'^b}{\mathrm{d}\tau} - k\, \mathrm{e}^B \wedge \mathbf{F}^b + \frac{\delta \mathbf{F}^{\text{ext}}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau} \right) \right\rangle_{10}\bigg|_{\tau=1} \\ &- \frac{k}{2}\sum_n \int \star_{10}\, \mathrm{e}^{\frac{5-n}{2}\phi} |F^{\text{ext}}_n|^2. \end{aligned} \tag{4.34}$$

Putting everything together, we now use (4.34) together with (4.25) in (4.24) to arrive at

$$\begin{aligned}\left[\frac{\delta S'_{\text{bulk}}}{\delta\tau} + \frac{\delta S'_{\text{loc}}}{\delta\tau}\right]_{\tau=1} =& k\sum_p S^{(p)}_{\text{WZ}} - \frac{k}{2}\sum_n \int \star_{10}\, \mathrm{e}^{\frac{5-n}{2}\phi}|F^{\text{ext}}_n|^2 + \int \frac{\delta S_{\text{NSNS}}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau}\bigg|_{\tau=1} \\ &+ 2\int \left\langle \frac{\delta S_{\text{RR}}}{\delta \mathbf{F}^{\text{ext}}} \wedge \left(\mathrm{e}^B \wedge \frac{\mathrm{d}\mathbf{F}'^b}{\mathrm{d}\tau} - k\, \mathrm{e}^B \wedge \mathbf{F}^b + \frac{\delta \mathbf{F}^{\text{ext}}}{\delta H} \wedge \frac{\mathrm{d}H'^b}{\mathrm{d}\tau} \right) \right\rangle_{10}\bigg|_{\tau=1}. \end{aligned} \tag{4.35}$$

Using (4.21) and the two scaling symmetries (3.1) and (3.4) and evaluating the functional derivatives then leads to the two equations

$$\begin{aligned} 2S_{\text{bulk}} + S_{\text{loc}} =& 2S_{\text{WZ}} - \sum_n \int \star_{10}\, \mathrm{e}^{\frac{5-n}{2}\phi}|F^{\text{ext}}_n|^2 \\ &- \int \left\langle \sigma(\mathbf{F}^{\text{int}}) \wedge \mathrm{e}^B \wedge \mathbf{F}^b \right\rangle_{10}, \end{aligned} \tag{4.36}$$

$$\begin{aligned} -8S_{\text{bulk}} - \sum_p (p+1) S^{(p)}_{\text{loc}} =& -8\sum_p S^{(p)}_{\text{WZ}} + 4\sum_n \int \star_{10}\, \mathrm{e}^{\frac{5-n}{2}\phi}|F^{\text{ext}}_n|^2 \\ &+ \sum_n (9-n) \int \left\langle \sigma(\mathbf{F}^{\text{int}}) \wedge \mathrm{e}^B \right\rangle_{10-n} \wedge F^b_n \\ &- 2\int H^b \wedge \left(\mathrm{e}^{-\phi} \star_{10} H - \kappa \left\langle \sigma(\mathbf{F}^{\text{int}}) \wedge \mathbf{C}^{\text{ext}} \right\rangle_7 \right), \end{aligned} \tag{4.37}$$

where κ equals +1 for type IIA and -1 for type IIB. We can now linearly combine (4.36) and (4.37) introducing a free parameter c and rearrange the source terms using $S = S_{\text{bulk}} + S_{\text{loc}}$ and $S^{(p)}_{\text{loc}} = S^{(p)}_{\text{DBI}} + S^{(p)}_{\text{WZ}}$, which yields

$$\begin{aligned} 2S - 2\sum_p S^{(p)}_{\text{WZ}} + \sum_n \int \star_{10}\, \mathrm{e}^{\frac{5-n}{2}\phi}|F^{\text{ext}}_n|^2 =& \sum_p \left(1 + \frac{p-3}{2}c\right) \left[S^{(p)}_{\text{DBI}} + S^{(p)}_{\text{WZ}}\right] \\ &- \sum_n \left(1 + \frac{n-5}{2}c\right) \int \left\langle \sigma(\mathbf{F}^{\text{int}}) \wedge \mathrm{e}^B \right\rangle_{10-n} \wedge F^b_n \\ &- c\int H^b \wedge \left(\mathrm{e}^{-\phi} \star_{10} H - \kappa \left\langle \sigma(\mathbf{F}^{\text{int}}) \wedge \mathbf{C}^{\text{ext}} \right\rangle_7 \right). \end{aligned} \tag{4.38}$$

Substituting this into the integrated Einstein equation (4.10) and collecting all contributions from background fluxes into a single term $\mathcal{F}(c)$, we find the result

$$\frac{8v\mathcal{V}}{d-2}\Lambda = \sum_p \left(1 + \frac{p-3}{2}c\right) \left[S^{(p)}_{\text{DBI}} + S^{(p)}_{\text{WZ}}\right] + \int \mathcal{F}(c) \tag{4.39}$$

with the volume factors v and $\mathcal{V}$ defined as in (4.11). Note that all terms on the right hand side of (4.39) contain an implicit factor of the external "volume" v such that it cancels out in the equation, and Λ does not depend on it. The flux term $\mathcal{F}(c)$ takes the form

$$\begin{aligned}\mathcal{F}(c) = &-\sum_{n\geq d}\left(1+\frac{n-5}{2}c\right)\left\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge \mathrm{e}^{B}\right\rangle_{10-n}\wedge F_n^b \\ &- cH^b\wedge\left(\mathrm{e}^{-\phi}\star_{10}H-\kappa\left\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\right\rangle_7\right),\end{aligned} \tag{4.40}$$

where the summation range is determined by the fact that the background fluxes F_n^b are space-time filling by definition and must therefore be of rank d or higher (cf. the discussion in Sec. 3.2). Note that the form of (4.40) is slightly different to the corresponding equation in [62], this is merely because we use a different conventions for the hodge operator compared to [62].

As stated earlier, the contribution of the flux term $\mathcal{F}(c)$ can often be gauged away in (4.39) by choosing an appropriate numerical value for the free parameter c. Up to an overall volume factor $\mathcal{V}$ (whose sign is known to be positive), Λ is then completely determined by the on-shell actions of the localized sources that appear in the corresponding solution. If only one of the fluxes in (4.40) is non-zero, it is straightforward to see that $\mathcal{F}(c)$ can be set to zero, since then one can simply choose c such that the c-dependent prefactor of the corresponding term vanishes in (4.40).[6] For a compactification with non-zero H^b, for example, one would choose $c=0$, and, for a compactification with non-zero F_7^b, one would choose $c=-1$.

Even if the NSNS flux H^b and one of the RR fluxes (other than F_5^b) are both non-zero, it is still often possible to find a c such that $\mathcal{F}(c)$ vanishes. The reason is that the term multiplying H^b in (4.40) is proportional to

$$\frac{\delta S_{\mathrm{NSNS}}}{\delta H}+2\left\langle\frac{\delta S_{\mathrm{RR}}}{\delta \mathbf{F}^{\mathrm{ext}}}\wedge\frac{\delta\mathbf{F}^{\mathrm{ext}}}{\delta H}\right\rangle_7 = -\mathrm{e}^{-\phi}\star_{10}H-\kappa\left\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\right\rangle_7. \tag{4.41}$$

If the H equation of motion implies that $\mathrm{d}\left[\mathrm{e}^{-\phi}\star_{10}H-\kappa\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\rangle_7\right]=0$, which is the case in many interesting examples, then we can write

$$-\mathrm{e}^{-\phi}\star_{10}H-\kappa\left\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\right\rangle_7=\omega_7, \tag{4.42}$$

where ω_7 is a closed but not necessarily exact 7-form. Note that only a possible non-exact part of ω_7 can contribute to (4.39) since any exact part of ω_7 would reduce to zero when inserted into (4.40) and integrated over. If a gauge transformation of the RR potentials can be employed to cancel ω_7 in (4.42), the term multiplying H^b in (4.40) vanishes for any c, and we can choose the value for c such that also the RR flux term in (4.40) vanishes. Consider, for example, a compactification of type IIA supergravity with non-zero H^b and F_0. The non-trivial background fluxes appearing in (4.40) are then H^b and F_{10}^b,

$$\mathcal{F}(c)=-\left(1+\frac{5}{2}c\right)F_{10}^b\wedge F_0-cH^b\wedge\left(\mathrm{e}^{-\phi}\star_{10}H-\kappa\left\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\right\rangle_7\right). \tag{4.43}$$

Assuming that $\mathrm{d}\left[\mathrm{e}^{-\phi}\star_{10}H-\kappa\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\rangle_7\right]=0$ by the H equation of motion, (4.41) and (4.42) now imply that the term multiplying H^b can be canceled by a gauge transformation $C_7\mapsto C_7-\omega_7/F_0$. This is a valid gauge transformation that leaves all RR field strengths

[6] F_5 flux is an exception, because it does not have a c-dependent prefactor in $\mathcal{F}(c)$ and can therefore not be gauged away in (4.39). This is the reason for the existence of the Freund-Rubin solutions of type IIB supergravity on $\mathrm{AdS}_5\times S^5$ [164].

unchanged. In the new gauge, we then have $\mathrm{e}^{-\phi}\star_{10} H - \kappa\,\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\rangle_7 = 0$ such that (4.43) reduces to $\mathcal{F}(c) = -(1+5c/2)F_{10}^b\wedge F_0$. We can therefore choose $c=-2/5$ so that $\mathcal{F}=0$.[7]

In presence of more than one type of RR flux, this reasoning does not work anymore, since it is then not possible to choose an appropriate c such that each term in $\mathcal{F}(c)$ is set to zero individually. We may still be able to find a $c=c_0$ that solves the equation $\int\mathcal{F}(c_0)=0$ such that $\int\mathcal{F}(c_0)$ vanishes as a whole, but the numerical value of c_0 then depends on the bulk fields that appear in (4.40). This will in general not be useful, since it just has the effect of trading the explicit dependence of Λ on the bulk dynamics for an implicit dependence hidden in the value of c_0. We will explain this in more detail in Sec. 4.3 where we discuss several examples for string compactifications in which $\mathcal{F}(c)$ can be set to zero and one counterexample in which it cannot be set to zero.

4.2.3 Validity of the supergravity approximation

Before we proceed with applying the above results to some explicit examples, a comment on their regime of validity is in order. In the vicinity of localized sources, field derivatives and the string coupling often blow up such that α' and loop corrections can become large, making the reliability of the supergravity approximation questionable. Given that the right hand side of (4.39) is evaluated directly at the positions of the sources, one might therefore wonder about the self consistency of our expression for Λ.

In order to clarify the meaning of our result, it is important to recall that (4.39) has been derived by using the two-derivative supergravity action (4.2), (4.3), (4.5), (4.6). Within this theory, (4.39) is an *exact* expression that can serve as well as any other method for calculating the cosmological constant in the supergravity approximation. The only question now is what happens to (4.39) if one takes into account the various types of stringy corrections, because these may significantly affect the strong field region at the sources.

The answer to this question depends on how (4.39) is used. If one reads it as an expression that calculates the cosmological constant in terms of the near source behavior, one has to use the near source behavior in the supergravity approximation and then gets the cosmological constant in the supergravity approximation. Let us, for simplicity, focus on the case with only one type of sources present in the compactification. We can then schematically write $\Lambda^{\mathrm{class}}=\lambda S_{\mathrm{loc}}^{\mathrm{class}}$, where the superscript $^{\mathrm{class}}$ denotes the values in the supergravity appoximation, and λ is some constant. If classical supergravity provides a good approximation for the lower dimensional effective theory, e.g. in the usual regime of large volume and small string coupling, the full cosmological constant, Λ^{full}, is well approximated by the lowest order expression, $\Lambda^{\mathrm{full}}\approx\Lambda^{\mathrm{class}}$, and one therefore also has $\Lambda^{\mathrm{full}}\approx\lambda S_{\mathrm{loc}}^{\mathrm{class}}$. Note that this is true even when $S_{\mathrm{loc}}^{\mathrm{class}}$ is not a good approximation to $S_{\mathrm{loc}}^{\mathrm{full}}$. This is the way we will use (4.39) in Sec. 4.3.

In Sec. 4.4, on the other hand, we also use (4.39) backwards, i.e. we extract information on the near brane behavior in a setup where Λ is known. Here it is important to stress that this will only give us information on $S_{\mathrm{loc}}^{\mathrm{class}}$, i.e. on the near brane behavior in the supergravity approximation. In particular, the singularity in the H and F_3 energy density we find is a priori only a feature of the supergravity approximation, and our result just confirms the singularity exactly like other people have seen the singularity in the supergravity approximation [53, 54, 56, 57, 59]. Whether the singularity gets resolved by stringy effects can not be inferred from our argument and is beyond the scope of our work. The useful advantage of our method is that it shows that this singularity survives the full supergravity analysis and is not an artifact of the partial smearing or a linearization around the BPS background.

[7] Note that, even though $\mathcal{F}(c)$ is not gauge invariant, one can convince oneself that the full expression for Λ in (4.39) is gauge invariant.

4.3 Examples

In this section, we discuss different solutions of type IIA and IIB supergravity that have appeared in the literature and show how (4.39) can be evaluated in our framework to obtain an explicit expression for the cosmological constant.

4.3.1 The GKP Solutions

Here we consider warped compactifications of type IIB supergravity to 4-dimensional Minkowski space with H flux and F_3 flux and the necessary sources for tadpole cancelation along the lines of [131] (GKP) and related work [132–135]. For simplicity, we specialize to models involving only O3-planes as sources. In [131], the authors also discussed models with D7-branes and O7-planes along with their F-theory description. The discussion of models with 7-branes in our framework is analogous albeit more lengthy.

Following [131], we find that the non-vanishing fields must satisfy

$$F_3 = -\mathrm{e}^{-\phi} \star_6 H, \qquad F_5 = -(1+\star_{10})\mathrm{e}^{-4A} \star_6 \mathrm{d}\alpha, \qquad C_4^{\text{ext}} = \tilde{\star}_4(\alpha + a), \qquad \alpha = \mathrm{e}^{4A}, \tag{4.44}$$

where the warp factor A and the dilaton ϕ are functions on the compact space, and a is an integration constant corresponding to a gauge transformation. Also note that $F_5 = \star_{10} F_5 = F_5^{\text{int}} + F_5^{\text{ext}}$ with $F_5^{\text{ext}} = \mathrm{d}C_4^{\text{ext}}$. The topologically non-trivial fluxes canceling the O3-tadpoles are F_3 flux and H flux, so that the relevant fluxes appearing in the definition of $\mathcal{F}(c)$, given by (4.40), are

$$H^b \quad \text{and} \quad F_7^b, \tag{4.45}$$

whereas all other terms in (4.40) vanish. Thus (4.40) reduces to

$$\mathcal{F}(c) = -c\, H^b \wedge \left[\mathrm{e}^{-\phi} \star_{10} H - F_3 \wedge C_4^{\text{ext}}\right] + (1+c)\, F_7^b \wedge F_3. \tag{4.46}$$

Using (4.44), we find that the first term can be put to zero by gauge fixing $a = 0$.[8] Furthermore, F_3 and H are related by a special condition which is given in (4.44). This condition can be shown to saturate a BPS-like bound and is equivalent to the ISD condition of the complex three-form field strength in the notation of [131]. It follows from this condition that also the second term in (4.46) is zero, as can be checked:

$$\begin{aligned} \int F_7^b \wedge F_3 &= \int \left(F_7 - \mathrm{d}C_6^{\text{ext}} + H \wedge C_4^{\text{ext}}\right) \wedge F_3 \\ &= \int \left(F_7 \wedge F_3 + \mathrm{e}^{\phi} \star_6 F_3 \wedge (\tilde{\star}_4 \mathrm{e}^{4A}) \wedge F_3\right) = 0, \end{aligned} \tag{4.47}$$

where in the last step we used that $F_7 = -\mathrm{e}^{\phi} \star_{10} F_3 = -\mathrm{e}^{\phi} \star_6 F_3 \wedge \tilde{\star}_4 \mathrm{e}^{4A}$. Thus $\mathcal{F}(c)$ reduces to zero for any choice of c. This is expected in this model, since also the contribution of localized source terms to Λ is independent of c for sources with $p = 3$.

We therefore find that (4.39) yields

$$\Lambda = \frac{1}{4v\mathcal{V}} \left(S_{\text{DBI}}^{(3)} + S_{\text{CS}}^{(3)}\right). \tag{4.48}$$

[8] Note that, although $\mathcal{F}(c)$ is not gauge invariant, the full expression for the cosmological constant Λ is, since it contains a term $C_4 \wedge \mu_3 \delta_6$ which changes such that the total a-dependence of Λ cancels out as it should.

Spelling out the contributions from the O3-planes and using (4.44) in (4.6), we arrive at

$$\Lambda = \frac{1}{4v\mathcal{V}}\,\mu_3 \int \left(\tilde{\star}_4 \mathrm{e}^{4A} - C_4^{\mathrm{ext}}\right) \wedge \sigma(\delta_6) = \frac{1}{4\mathcal{V}}\,N_{\mathrm{O3}}\,\mu_3\Big(\mathrm{e}^{4A_0} - \alpha_0\Big)\,, \tag{4.49}$$

where A_0, α_0 denote the values of A, α at the position of the O3-plane(s) and $\mu_3 > 0$ is the absolute value of the O3 charge. Since $\alpha = \mathrm{e}^{4A}$, the DBI and Chern-Simons parts of the source action cancel out such that

$$\Lambda = 0 \tag{4.50}$$

as expected.

4.3.2 $\overline{\text{D6}}$-branes on $\text{AdS}_7 \times S^3$

Let us now consider type IIA supergravity with $\overline{\text{D6}}$-branes on $\text{AdS}_7 \times S^3$, i.e. the setup studied in [64, 65, 67, 68].[9] While a smeared solution can be constructed explicitly for this setup, it was argued in [64, 65] that in the supergravity approximation a solution with fully localized branes, if existent at all, necessarily yields a singularity in the energy density of the H flux at the location of the $\overline{\text{D6}}$-branes. As we will see below, it is rather straightforward to reproduce this result in our framework.

It was shown in [64] that the non-vanishing fields in this setup must satisfy the ansatz

$$F_0 = \text{const.}, \qquad H = \alpha F_0 \mathrm{e}^{\phi - 7A} \star_3 1, \qquad F_2 = \mathrm{e}^{-3/2\phi - 7A} \star_3 \mathrm{d}\alpha, \qquad C_7^{\mathrm{ext}} = \tilde{\star}_7(\alpha + a), \tag{4.51}$$

where the warp factor A, the dilaton ϕ and α are functions on the internal space, and a is an integration constant related to a gauge freedom. The tadpole for the $\overline{\text{D6}}$-branes is canceled by a non-zero H flux on the 3-sphere and a non-zero Romans mass, i.e. F_0 "flux". The relevant fluxes appearing in $\mathcal{F}(c)$ are therefore

$$H^b \quad \text{and} \quad F_{10}^b, \tag{4.52}$$

and (4.40) reduces to

$$\mathcal{F}(c) = -c\,H^b \wedge \left[\mathrm{e}^{-\phi} \star_{10} H - F_0 \wedge C_7^{\mathrm{ext}}\right] - \left(1 + \frac{5}{2}c\right) F_{10}^b \wedge F_0. \tag{4.53}$$

Using (4.51), one can see that the first term vanishes by a convenient gauge choice, $a = 0$. We are then left with the second term which can be set to zero choosing $c = -\frac{2}{5}$.

We can now substitute this into (4.39) to find

$$\Lambda = \frac{1}{4v\mathcal{V}}\left(S_{\mathrm{DBI}}^{(6)} + S_{\mathrm{CS}}^{(6)}\right). \tag{4.54}$$

Spelling out the contributions of the $\overline{\text{D6}}$-branes and using (4.51) then yields

$$\Lambda = \frac{1}{4v\mathcal{V}}\,\mu_6 \int \left(-\tilde{\star}_7 \mathrm{e}^{3/4\phi + 7A} - C_7^{\mathrm{ext}}\right) \wedge \sigma(\delta_3) = -\frac{1}{4\mathcal{V}}\,N_{\overline{\mathrm{D6}}}\,\mu_6\Big(\mathrm{e}^{3/4\phi_0 + 7A_0} + \alpha_0\Big)\,, \tag{4.55}$$

where A_0, α_0, ϕ_0 denote the values of A, α, ϕ at the brane position and $\mu_6 > 0$ is the absolute value of the $\overline{\text{D6}}$ charge. Assuming that at leading order in the distance r to the brane, the dilaton and the warp factor diverge as they would in flat space [165],

$$\mathrm{e}^{2A} \sim r^{1/8}, \qquad \mathrm{e}^{\phi} \sim r^{3/4}, \tag{4.56}$$

[9]Note that, unlike in the scenario considered in [41], the anti-branes are here not added to uplift an existing AdS solution to dS, but to cancel the tadpole and guarantee the existence of an AdS solution in the first place.

it is straightforward to show that the first term in Eq. (4.55) (which comes from the DBI part of the brane action) is actually zero. That this assumption is correct was explicitly proven in the analysis carried out in [65].

The cosmological constant is therefore exclusively determined by α_0:

$$\Lambda \sim -\mu_6 \alpha_0. \tag{4.57}$$

Since Λ is negative, it then follows that α has to be non-zero and positive at the source. Together with Eq. (4.56), this implies that near the source the energy density of the H flux diverges like the inverse of the warp factor,

$$\mathrm{e}^{-\phi}|H|^2 = \alpha^2 \mathrm{e}^{-14A} \mathrm{e}^{\phi} F_0^2 \sim \mathrm{e}^{-2A}. \tag{4.58}$$

This is consistent with the result found in [64, 65] by other methods, where it was also argued that finite α_0 implies a singular energy density of the H flux. As we will show in Sec. 4.4, a similar argument holds for meta-stable de Sitter vacua that are obtained by placing $\overline{\text{D3}}$-branes in the Klebanov-Strassler throat embedded into a compact space. Under few assumptions we will discuss in detail, one would find a singularity similar to the one observed in the $\overline{\text{D6}}$ model.

4.3.3 SU(3) structure Manifolds with O6-planes

Here we discuss a particular model of compactifications of type IIA supergravity on SU(3) structure manifolds that was studied in [84], namely O6-planes on $\mathrm{dS}_4 \times \mathrm{SU}(2) \times \mathrm{SU}(2)$ (see also [92] for more examples of this type). This setup allows (unstable) critical points with positive Λ.

According to [84], the form fields satisfy

$$F_0 = m, \qquad F_2 = m^i Y_i^{(2-)}, \qquad H = p\left(Y_1^{(3-)} + Y_2^{(3-)} - Y_3^{(3-)} + Y_4^{(3-)}\right), \tag{4.59}$$

where $Y_i^{(2-)}, Y_i^{(3-)}$ are certain 2-forms and 3-forms, respectively, and m, m^i, p are constant coefficients that are not relevant for the following discussion. The tadpole generated by the O6-planes is canceled by non-zero H and F_0 flux. However, while there is a non-trivial field strength F_2 (induced by the presence of the O6-planes), there is no topological F_2 flux, since it is not allowed by the cohomology of $\mathrm{SU}(2) \times \mathrm{SU}(2)$. For the same reason, $F_8^b = 0$, and the non-zero background fluxes appearing in $\mathcal{F}(c)$ are

$$H^b \quad \text{and} \quad F_{10}^b. \tag{4.60}$$

Considering (4.40) for this setup, we thus find

$$\mathcal{F}(c) = -c\, H^b \wedge \left[\mathrm{e}^{-\phi} \star_{10} H - F_0 \wedge C_7^{\text{ext}}\right] - \left(1 + \frac{5}{2}c\right) F_{10}^b \wedge F_0. \tag{4.61}$$

As discussed in Sec. 4.2.2, the H equation of motion

$$\mathrm{d}\left[\mathrm{e}^{-\phi} \star_{10} H - F_0 \wedge C_7^{\text{ext}}\right] = 0 \tag{4.62}$$

implies that we can choose a gauge for C_7^{ext} such that the first term on the right hand side of Eq. (4.61) vanishes. The second term can be set to zero by choosing $c = -\frac{2}{5}$.

Evaluating (4.39), we therefore find that the cosmological constant is given by

$$\Lambda = \frac{1}{10 v \mathcal{V}}\left(S_{\text{DBI}}^{(6)} + S_{\text{CS}}^{(6)}\right) = \frac{1}{10 v \mathcal{V}}\, \mu_6 \int \left(\mathrm{e}^{3/4\phi} \star_4 1 \wedge \star_3 1 - C_7^{\text{ext}}\right) \wedge \sigma(\delta_3), \tag{4.63}$$

where the right hand side should be understood as a sum over the various O6-plane terms, and $\mu_6 > 0$ is the absolute value of the O6 charge. In [84], the setup was considered in the smeared limit, where the delta forms δ_3 are replaced by volume forms of the space transverse to the corresponding sources. If a localized version of this solution exists, (4.63) would give a constraint on the possible field behavior at the O-planes.

4.3.4 The DGKT solutions

Finally, we look at type IIA supergravity compactified on $T^6/\mathbf{Z}_3^2$, which is an explicit example for the type IIA flux compactifications considered in [166, 167].[10] In order to stabilize the moduli, the model requires the presence of NSNS flux as well as several RR fluxes of different ranks. As discussed in Sec. 4.2.2, it is therefore a counterexample, where it is in general not possible to set the flux dependent terms in (4.39) to zero and write Λ as a sum of localized source terms only.

The NSNS and RR field strengths in this model are given by

$$H^b = -p\beta_0, \quad F_0 = m_0, \quad F_2 = 0, \quad F_4 = F_4^{\text{int}} + F_4^{\text{ext}} = e_i \tilde{\omega}^i + \star_4 e_0, \tag{4.64}$$

where p, m_0, e_0, e_i are numbers, β_0 is an odd 3-form and $\tilde{\omega}^i$ are even 4-forms under the orientifold involution.[11] The non-trivial fluxes appearing in Eq. (4.40) are thus

$$H^b, \quad F_{10}^b, \quad F_6^b \quad \text{and} \quad F_4^b \tag{4.65}$$

such that

$$\begin{aligned} \mathcal{F}(c) = & -c\, H^b \wedge \left[\mathrm{e}^{-\phi} \star_{10} H - F_0 \wedge C_7^{\text{ext}}\right] - \left(1 + \frac{5}{2}c\right) F_{10}^b \wedge F_0 \\ & - \left(1 + \frac{1}{2}c\right) F_6^b \wedge F_4^{\text{int}} + \left(1 - \frac{1}{2}c\right) F_4^b \wedge F_6^{\text{int}}, \end{aligned} \tag{4.66}$$

where we used that the fluctuation B is zero on-shell. The first term on the right hand side can be made to vanish by choosing a gauge for the C_7^{ext} field. Since the other terms do in general not vanish, however, we cannot choose c such that all of them are set to zero simultaneously.

As pointed out in Sec. 4.2.2, we can still solve the equation $\int \mathcal{F}(c) = 0$ for some $c = c_0$ (unless its c-dependence coincidentally cancels out on-shell) and use it in (4.39) to arrive at an expression for Λ which formally only depends on source terms,

$$\Lambda = \frac{2 + 3c_0}{8v\mathcal{V}} \left(S_{\text{DBI}}^{(6)} + S_{\text{CS}}^{(6)}\right). \tag{4.67}$$

However, the resulting numerical value for c_0 then implicitly depends on the bulk fields appearing in $\mathcal{F}(c)$. It is therefore hard to approximate its numerical value or even its sign in compactification scenarios with more than one type of RR flux, unless the full solution is already known (as in the present example). This is contrary to the previous examples, where c could be fixed to a known number such that, up to a volume factor, Λ was completely determined by the boundary conditions of the fields in the near source region.

[10] As discussed in [167], the sources are smeared in order to obtain a solution. The discussion whether a corresponding localized solution exists or how it differs from the smeared solution [64, 146, 147, 167–170] does not concern us here. We only consider this model to give an example of a solution where many fluxes are turned on.

[11] Note that the spacetime filling part of F_4, which is given by F_4^{ext}, is treated as internal F_6 in the conventions of [166].

4.4 Singular $\overline{\text{D3}}$-branes in the Klebanov-Strassler throat

In this section, we discuss to what extent our previous results can be applied to meta-stable de Sitter vacua in type IIB string theory obtained by placing $\overline{\text{D3}}$-branes at the tip of a warped throat geometry along the lines of [41]. We spell out and discuss the assumptions under which one can give a simple topological argument for a singularity in the energy density of H and F_3 due to the brane backreaction.

4.4.1 Ansatz

Following [41], we consider type IIB no-scale Minkowski solutions obtained by embedding the Klebanov-Strassler solution [158] into a compact setting [131]. In order to stabilize the geometric moduli, we also include non-perturbative effects which may come from Euclidean D3-brane instantons or gaugino condensation. The resulting supersymmetric AdS vacuum is then uplifted to a meta-stable de Sitter vacuum by putting a small number of $\overline{\text{D3}}$-branes at the tip of the Klebanov-Strassler throat [41, 159].

In order to apply the results of Sec. 4.2 to this scenario, we split the total cosmological constant into a part, Λ^{class}, which is due to the classical equations of motion and given by evaluating (4.39) at the solution, and the rest, Λ^{np}, which contains all corrections from non-perturbative effects that are not captured by the classical computation, i.e., we write

$$\Lambda = \Lambda^{\text{class}} + \Lambda^{\text{np}}. \tag{4.68}$$

Let us now discuss the explicit form of Λ^{class} in the present setup. For simplicity, we will restrict ourselves to the case, where the no-scale solutions of [131] are realized in a model with O3-planes, and the non-perturbative effects come from Euclidean D3-brane instantons. In [131], also orientifold limits of F-theory compactifications involving D7-branes and O7-planes are discussed. We checked that it is also possible to study such models in our framework, but the discussion becomes more involved, since the presence of these sources induces a non-trivial F_1 field strength.

Our ansatz for the different fields thus reads[12]

$$C_4^{\text{ext}} = \tilde{\star}_4(\alpha + a), \quad F_5 = -(1+\star_{10})\mathrm{e}^{-4A}\star_6 \mathrm{d}\alpha, \quad H = \mathrm{e}^{\phi-4A}\star_6(\alpha F_3 + X_3), \quad F_1 = 0, \tag{4.69}$$

where A, α, ϕ are functions on the internal space, a is an integration constant corresponding to a gauge freedom, and X_3 is an a priori unknown 3-form satisfying $\mathrm{d}X_3 = 0$. One can check that this ansatz follows from the form equations of motion and the requirement that the non-compact part of spacetime be maximally symmetric, if only sources with $p = 3$ are present.

As in the examples discussed in Sec. 4.3, the flux dependent terms $\mathcal{F}(c)$ in (4.39) can now be simplified by a convenient choice of the parameter c. To see this recall that the relevant fluxes in the present case are

$$H^b \quad \text{and} \quad F_7^b \tag{4.70}$$

and thus (4.40) reduces to

$$\mathcal{F}(c) = -\,c\,H^b \wedge \left[\mathrm{e}^{-\phi}\star_{10} H - F_3 \wedge C_4^{\text{ext}}\right] + (1+c)\,F_7^b \wedge F_3. \tag{4.71}$$

[12] If one no longer assumes the BPS condition of Sec. 4.3.1, the function α need not be related to the warp factor, and X_3 may be non-vanishing.

Using (4.69), we find that the first expression on the right hand side of (4.71) cancels out for $a = 0$ except for a term $\sim X_3$. The second term in (4.71) can be set to zero by the choice $c = -1$, yielding[13]

$$\mathcal{F}(-1) = -\tilde{\star}_4 1 \wedge H^b \wedge X_3. \tag{4.72}$$

We will argue below that, upon a certain choice for the UV boundary conditions of the three-form field strengths, the integral of (4.72) gives a contribution to the cosmological constant in (4.39) that is negligible compared to the contribution from the anti-D3-brane source terms.

Keeping the flux term for the moment, we can substitute Eq. (4.72) into (4.39) and write

$$\begin{aligned}
\Lambda^{\text{class}} &= \frac{1}{4v\mathcal{V}}\left(S^{(3)}_{\text{DBI}} + S^{(3)}_{\text{CS}}\right) + \frac{1}{4v\mathcal{V}}\int \mathcal{F}(-1) \\
&= \frac{1}{4v\mathcal{V}}\,\mu_3 \int \left(-\tilde{\star}_4 \mathrm{e}^{4A} - C_4^{\text{ext}}\right) \wedge \sigma(\delta_6^{(\overline{\text{D3}})}) + \frac{1}{16v\mathcal{V}}\,\mu_3 \int \left(\tilde{\star}_4 \mathrm{e}^{4A} - C_4^{\text{ext}}\right) \wedge \sigma(\delta_6^{(\text{O3})}) \\
&\quad - \frac{1}{4v\mathcal{V}} \int \tilde{\star}_4 1 \wedge H^b \wedge X_3,
\end{aligned} \tag{4.73}$$

where we have spelled out the contributions of the localized sources. Note that the O3-plane charge is $\frac{1}{4}$ of the $\overline{\text{D3}}$-brane charge μ_3, where $\mu_3 > 0$ in our conventions. Evaluating the above equation, we find that the total cosmological constant (4.68) is given by

$$\Lambda = -\frac{1}{4\mathcal{V}}\, N_{\overline{\text{D3}}}\,\mu_3\Big(\mathrm{e}^{4A_0} + \alpha_0\Big) + \frac{1}{16\mathcal{V}}\, N_{\text{O3}}\,\mu_3\Big(\mathrm{e}^{4A_*} - \alpha_*\Big) - \frac{1}{4\mathcal{V}} \int_{\mathcal{M}^{(6)}} H^b \wedge X_3 + \Lambda^{\text{np}}, \tag{4.74}$$

where A_0, α_0 and A_*, α_* denote the values of A, α at the positions of the $\overline{\text{D3}}$-branes and O3-planes, respectively.

4.4.2 The argument

Our goal is now to evaluate (4.74) and relate it to the near tip behavior of the energy density of the H flux. In order to do so, we make the following assumptions.

1. **Topological flux.** In the region of the conifold, F_3 carries a non-trivial topological flux along the directions of a 3-cycle called the A cycle, H carries a topological flux along the directions of the dual 3-cycle called the B cycle, and all other components of H and F_3 are exact. This assumption is due to the fact that the deformed conifold is topologically a cone over $S^2 \times S^3$, where the deformation has the effect of replacing the singular apex of the conifold by a finite S^3 (see e.g. [161, 171]). The deformed conifold therefore has a non-trivial compact 3-cycle along the S^3 (the A cycle) and a dual, non-compact 3-cycle (the B cycle). We will assume that also in our compact setting the relevant cycles threaded by topological flux are the A cycle and the B cycle, at least in the region of the conifold. Following the literature [158], we then place F_3 flux along the A cycle and H flux along the B cycle. On general compact manifolds, there may of course exist additional cycles that are threaded by flux. We will assume, however, that such additional topologically non-trivial terms in F_3 and H only become relevant deep in the UV, i.e., far away from the anti-D3-brane.

[13] To be precise, one finds that the integrated dilaton equation implies $-\int H^b \wedge \left[\mathrm{e}^{-\phi} \star_{10} H + F_3 \wedge C_4^{\text{ext}}\right] + \int F_7^b \wedge F_3 = 0$ in absence of sources with $p \neq 3$, such that $\int \mathcal{F}(c) = -\int \tilde{\star}_4 1 \wedge H^b \wedge X_3$ actually holds for any choice of c. This is consistent with the fact that also the source part of (4.39) is independent of c for $p = 3$. Thus the value of Λ^{class} is uniquely determined by (4.39) as it should be.

2. **IR boundary conditions.** The $\overline{\text{D3}}$-brane locally deforms the geometry as it would do in flat space. This implies in particular that the warp factor goes to zero in the vicinity of the $\overline{\text{D3}}$-brane as it usually does,
$$e^{2A} \to 0. \tag{4.75}$$
It also implies that we can locally approximate the internal geometry by
$$g_{mn} \approx e^{-2A}\tilde{g}_{mn} \tag{4.76}$$
at leading order in an expansion around the distance r to the brane, with $\tilde{g}_{mn}$ regular (in suitable coordinates).

   This is a standard assumption discussed recently e.g. in [59, 60] for the case of partially smeared $\overline{\text{D3}}$-branes. In an analogous setting, it was verified explicitly in [65] for the toy model with $\overline{\text{D6}}$-branes discussed in Sec. 4.3.2, where both the warp factor and the internal metric indeed diverge exactly as they would do in the corresponding flat space solution [165] at leading order in the distance parameter r. Some progress has also been made for the $\overline{\text{D3}}$-branes considered here in [172] (see also [59,60] for an analogous discussion of partially smeared $\overline{\text{D3}}$-branes in the non-compact Klebanov-Strassler solution).

   In order that the unperturbed deformed conifold metric $\tilde{g}_{mn}$ shrinks smoothly at the tip, we furthermore expect that the energy density of F_3 along the A cycle contracted with $\tilde{g}_{mn}$ does not vanish at the tip:
$$e^{\phi}|\tilde{F}_3^A|^2 \neq 0, \tag{4.77}$$
where the superscript denotes the component of F_3 along the A cycle.[14] This is motivated by the fact that the energy density of F_3^A is non-vanishing and prevents the A cycle from collapsing at the tip of the deformed conifold before the perturbation by the $\overline{\text{D3}}$-branes [158]. Using the results of [60], one can verify that (4.77) indeed holds for the case of partially smeared $\overline{\text{D3}}$-branes.

3. **UV boundary conditions.** The boundary conditions for the O3-planes in the UV far away from the $\overline{\text{D3}}$-branes are approximately the standard BPS boundary conditions,
$$\alpha_* \approx e^{4A_*}, \tag{4.78}$$
up to small corrections such that the O3-plane term in (4.74) is negligible compared to the other terms. To justify this, recall that in the GKP setup without the $\overline{\text{D3}}$-branes this is the usual BPS behavior that does not lead to a contribution to the cosmological constant. When a large flux background with a large number of O3-planes of this type is then perturbed by a small number of $\overline{\text{D3}}$-branes at the tip of a warped throat, the $\overline{\text{D3}}$-branes will give a small direct contribution to the cosmological constant due to their tree level brane action (see below). One might however wonder whether the $\overline{\text{D3}}$-brane backreaction on the geometry and the fields could also distort the relation (4.78) near the O3-planes, such that now also the O3-planes would contribute significantly to the vacuum energy. However, this backreaction effect would be of higher order in the small perturbation from the redshifted $\overline{\text{D3}}$-branes and should thus be negligible compared to the direct contribution from the $\overline{\text{D3}}$-brane source terms. This is analogous to the usual assumption of BPS asymptotics in the UV imposed in non-compact treatments of brane backreaction (e.g. [59,67]). It would be an interesting extension to explicitly compute the boundary conditions at the O-planes, e.g. following the analysis in [65].

[14] This is not to be confused with the notation of [58,60], where the superscript in F_3^A is an index running over all components of F_3.

Similarly, we also assume that the three-form field strengths approach their unperturbed values and thus become ISD in the UV far away from the $\overline{\text{D3}}$-branes, which implies

$$X_3^{\text{UV}} \approx 0, \tag{4.79}$$

again up to corrections that are negligible in (4.74). One might again wonder whether a small deviation from the ISD condition in the UV due to the anti-brane backreaction might be relevant for the value of the cosmological constant. As discussed above, however, it would be very surprising if the effect of such a deviation far away from the $\overline{\text{D3}}$-branes would not be negligible compared to their direct effect in the IR, so that we will adopt (4.79) as a reasonable assumption.

4. **Non-perturbative corrections.** Non-perturbative corrections to the effective potential (due to, e.g., Euclidean D3-branes or gaugino condensation on D7-branes) are captured by adding a *negative*[15] term to the overall cosmological constant, i.e.

$$\Lambda = \Lambda^{\text{class}} - |\Lambda^{\text{np}}|. \tag{4.80}$$

This assumption consists in fact of two parts: The first is that the non-perturbative effect gives, by itself, rise to a negative contribution to the vacuum energy, and the second is that it does not significantly change the classical contributions. These assumptions are implicit in the construction of [41], where the non-perturbative effects first make the vanishing cosmological constant of the GKP setup negative without significantly changing the classical background fluxes or the vevs and masses of the moduli that are stabilized by these fluxes (the complex structure moduli and the dilaton). Moreover, the subsequent de Sitter uplift due to $\overline{\text{D3}}$-branes is assumed to happen through their classical source terms only and does in turn not significantly change the vevs and masses of the moduli that are stabilized by the non-perturbative effects (the Kähler moduli). There has also been some progress in describing the above effects from an explicit 10D point of view [48–51]. In [51] it was argued that a non-vanishing gaugino bilinear $\langle\bar{\lambda}\lambda\rangle$ on D7-branes indeed leads to a negative contribution to the 4D spacetime curvature proportional to $|\langle\bar{\lambda}\lambda\rangle|^2$. On the other hand, the backreaction of this on the classical contribution Λ^{class} to the vacuum energy would be only a higher order effect. Similar properties are expected for the non-perturbative corrections due to Euclidean D3-brane instantons.

5. **Cosmological constant.** The presence of the $\overline{\text{D3}}$-branes uplifts the solution to a metastable de Sitter vacuum such that the total cosmological constant of the solution is positive,

$$\Lambda > 0, \tag{4.81}$$

as proposed in [41].

If one makes the above assumptions 1. - 5., our ansatz (4.74) for the cosmological constant drastically simplifies.

Let us at first discuss the flux term in (4.74). Since X_3 is closed by definition, we can make the ansatz

$$X_3 = \beta\omega_3^A + \mathrm{d}\omega_2 \tag{4.82}$$

in the conifold region. Here β is an unknown function of the internal coordinates, ω_2 is a 2-form, and ω_3^A is the harmonic 3-form along the A cycle satisfying $\mathrm{d}\omega_3^A = 0$. We have split X_3 into a

[15] A slightly different assumption was made in [172], but there it was shown that the overall conclusion, that the energy density of H diverges, is unaffected.

part, $\beta\omega_3^A$, along the A cycle, which can in general be non-exact, and a part, $\mathrm{d}\omega_2$, that is not necessarily along the A cycle and has to be exact.[16] Using $\mathrm{d}X_3 = \mathrm{d}\omega_3^A = 0$, we find from (4.82) that

$$\mathrm{d}\beta \wedge \omega_3^A = 0, \tag{4.83}$$

which implies that β is only a function of the coordinates parametrizing the S^3 but constant over the remaining directions. We can therefore set $\beta = \beta^{\mathrm{UV}} = 0$ without loss of generality, where β^{UV} denotes the value of β in the UV region of the warped throat far away from the $\overline{\text{D3}}$-branes.

The flux term in (4.74) then simplifies as follows. Since, under assumption 1., H only carries a flux along the B cycle in the conifold region, we find $H^b \wedge X_3 = H^b \wedge (\beta\omega_3^A + \mathrm{d}\omega_2) = H^b \wedge \beta^{\mathrm{UV}}\omega_3^A - \mathrm{d}(H^b \wedge \omega_2)$. We can therefore write

$$\int_{\mathcal{M}^{(6)}} H^b \wedge X_3 = \int_{\mathcal{M}^{(6)}} H^b \wedge X_3^{\mathrm{UV}} = 0 \tag{4.84}$$

such that the integral is completely determined by the units of H flux present in the compactification and the UV boundary conditions for the three-form field strengths but independent of the IR physics close to the $\overline{\text{D3}}$-branes.

Using Eq. (4.84) together with assumptions 2.- 4., we find that (4.74) reduces to

$$\Lambda \approx -\frac{1}{4\mathcal{V}}\, N_{\overline{\text{D3}}}\, \mu_3\, \alpha_0 - |\Lambda^{\mathrm{np}}|, \tag{4.85}$$

up to negligible corrections. From assumption 5. it then follows that

$$-\frac{1}{4\mathcal{V}}\, N_{\overline{\text{D3}}}\, \mu_3\, \alpha_0 > |\Lambda^{\mathrm{np}}|, \tag{4.86}$$

which implies that α_0 must be finite and negative.[17]

It is straightforward to see that this yields a singular energy density of the H flux in the region near the $\overline{\text{D3}}$-branes. As argued above, we can locally approximate the internal metric as $g_{mn} \approx \mathrm{e}^{-2A}\tilde{g}_{mn}$, where $\tilde{g}_{mn}$ is regular. Using (4.69), we can then write

$$\mathrm{e}^{-\phi}|H|^2 = \mathrm{e}^{\phi-8A}|\alpha F_3 + X_3|^2 \geq \alpha^2 \mathrm{e}^{-8A}\mathrm{e}^{\phi}|F_3^A|^2 \approx \alpha^2 \mathrm{e}^{-2A}\mathrm{e}^{\phi}|\tilde{F}_3^A|^2 \tag{4.87}$$

in the near brane region, where we have assumed that the component of X_3 along F_3^A vanishes[18]. Since $e^{\phi}|\tilde{F}_3^A|^2$ is expected to be non-zero at the tip of the conifold, it then follows from (4.75) and $\alpha_0 \neq 0$ that the energy density of the H flux at least diverges like the inverse of the warp factor,

$$\mathrm{e}^{-\phi}|H|^2 \sim \mathrm{e}^{-2A}. \tag{4.88}$$

Assuming a regular dilaton[19], the dilaton equation furthermore implies that the divergence in the energy density of H must be canceled by a divergent term in the energy density of F_3. We

[16] Note that, assuming the presence of F_3 flux along the A cycle, X_3 is not allowed to have a non-exact component along the B cycle as follows from the F_1 equation $\mathrm{e}^{-\phi}H \wedge \star_{10} F_3 = 0$ and the ansatz for H stated in (4.69).

[17] Note that α must change its sign somewhere in between the BPS region around the O3-planes (where $\alpha \approx \mathrm{e}^{4A}$) and the tip of the throat (where $\alpha < 0$). In the toy model discussed in [64], a similar constraint was used to formulate a topological no-go theorem, which is rederived in our framework in Sec. 4.3.2.

[18] In principle X_3 could cancel αF_3 at the tip such that no singularity occurs. The equations of motion then constrain the cancellation such that H must vanishes at the tip. The dilaton equation together with the assumption that F_3 does not vanish, then implies that the dilaton cannot be constant close to the brane. This is significantly different from what happens close to D3-branes in flat space [131].

[19] If the dilaton diverges at the brane even though it does not directly couple to it, $\mathrm{e}^{-\phi}|H|^2$ would still diverge, but the dilaton equation would not necessarily imply that $\mathrm{e}^{\phi}|F_3|^2$ also diverges.

thus find that the energy densities of H and F_3 diverge at least as[20]

$$\mathrm{e}^{-\phi}|H|^2 \sim \mathrm{e}^{-2A}, \qquad \mathrm{e}^{\phi}|F_3|^2 \sim \mathrm{e}^{-2A}. \tag{4.89}$$

Note that, due to its global nature, the argument is independent of most details of the bulk dynamics and does therefore not require simplifications such as a partial smearing of the branes or a linearization of the equations of motion. Under the assumptions discussed above, it holds for fully localized branes that backreact on the full non-linear equations of motion.

4.5 Discussion

We have shown how the 10D equations of motion for classical type II supergravity can be combined to give a surprisingly simple expression for the cosmological constant in terms of the classical near source behavior of the supergravity fields and a contribution from topologically non-trivial background fluxes. The derivation relies on no specific assumptions on the compactification manifold, but it holds only for maximally symmetric spacetimes of dimension four or more. In simple examples, the flux contribution can be chosen to be zero, and the expression reduces to contributions that have support only on localized sources. This extends the recent work [156] to general brane and flux setups. We checked our result against some well understood examples of flux compactifications and found agreement with all expectations. We specified the assumptions that are required to apply our result also to de Sitter uplifts from $\overline{\mathrm{D3}}$-branes in warped throats and showed that this would then indicate the presence of a singular H and F_3 energy density at the $\overline{\mathrm{D3}}$-brane similar to what has been reported in recent studies of the same setup [53, 54, 56, 57, 59]. Although our analysis does not clarify the physical meaning of this singularity (see [61,63,66] for a recent conjecture), it indicates that it is unlikely a mere artifact of approximations such as partial smearing or linearized field equations, which we do not use.

It should be interesting to apply our general result also to other aspects of string compactifications.

[20] Evaluating this equation for the case of partially smeared $\overline{\mathrm{D3}}$-branes, we recover the result of [60], where it was shown that $\mathrm{e}^{2A} \sim \tau^{1/2}$ and $\mathrm{e}^{-\phi}|H|^2 \sim \mathrm{e}^{\phi}|F_3|^2 \sim \tau^{-1/2}$ near the tip of the conifold and τ is the radial coordinate transverse to the branes in the conventions of [60].

Chapter 5

Exotic five-branes

As we have seen in the previous chapter, D-branes play a central role in flux compactifications for model building and phenomenology. Moreover, they are dynamical objects with tension inversely proportional to the string coupling and hence provide an excellent window into non-perturbative aspects of string theory. Apart from D-branes, the type II superstring theories (as well as the heterotic strings) contain Neveu-Schwarz 5-branes (NS5). These extended objects couple magnetically to the NSNS gauge potential and as such they are the magnetic cousins of Fundamental strings (F1). At weak string coupling they are heavier than D-branes, in the sense that their tension is inversely proportional to the second power of the string coupling. Other standard extended objects are the Kaluza-Klein Monopoles (KKM), which couple to a Kaluza-Klein gauge field. All these objects have been studied extensively over the past years.

The second wide window to non-perturbative string physics was opened by the discovery of string dualities [173]. Moreover, it was soon realized that the interplay between dualities and branes leads to a whole new family of extended objects, with D-branes, NS5-branes and KKM being just a fraction of it [174–177]. These new BPS-objects have codimension 2 and are obtained as orbits of the U-duality group. Some of them are even heavier than the NS5-brane, exhibiting a tension inversely proportional to the third or even fourth power of the string coupling. For a long time these branes were not much explored and remained in the shadow of more standard branes. Recent studies, however, suggest that such exotic branes have interesting properties and deserve more attention [116,120–123,178–182]. The property of exotic branes that we will be mainly concerned with in this chapter is that they induce non-trivial monodromies around them that generate what is often termed non-geometry[1] [116, 120].

Non-geometry is another property of string theory whose origin is based on dualities. It was realized many years ago that performing T-dualities on known string backgrounds often leads to backgrounds which are not globally well-defined in terms of conventional geometric quantities [95]. Here we would like to mention that one of the popular approaches to a better understanding of such setups is based on an attempt to geometrize the apparent non-geometric background in terms of a higher dimensional geometry [184]. This method relies on the so-called doubled formalism, where an additional set of coordinates dual to the standard geometric ones is introduced. Such dual coordinates may be thought of as auxilliary (as in the case of twisted doubled tori [185,186]) or as fundamental, dynamical ones (as in double field theory [105,106] [2]). Another approach to the issue of non-geometry uses modern techniques inspired by generalized complex geometry [110, 111], where the structure group of the combined tangent and cotangent bundle coincides with the T-duality group. The set of geometric operations on the NSNS field content of the background, i.e. diffeomorphisms and gauge B-transformations, is extended

[1]See also [183] for a discussion of 5-brane sources with nonstandard monodromies in the heterotic string.

[2]For some recent reviews and a more complete list of references, we refer to Refs. [107–109].

to include β transformations, associated to an antisymmetric 2-vector field which naturally appears in this extended formulation [162, 187–189]. Simply stated, this essentially results in the implementation of T-dualities as geometric operations on the fields when they are patched after traversing a non-contractible loop in target space.

In the present chapter, which is based on [124] we study some aspects of exotic five-branes, in particular their dynamics, their couplings and their relations to non-geometry by deriving their effective worldvolume actions. The effective actions of D-branes are as we have seen the well-known Dirac-Born-Infeld (DBI) action [128, 129] and the Wess-Zumino (WZ) action, which describe, respectively, the response of the brane to the NSNS and the RR sector of a supergravity background. Moreover, it implements T-duality in the sense that the T-dual action of a Dp-brane indeed gives the same action for a D($p\pm1$)-brane, depending on the direction that T-duality is performed. Similar worldvolume actions exist for NS5-branes and KKMs [190, 191]. Moreover, the authors of Ref. [192] determined the worldvolume actions for some exotic 6- and 7-branes. In this chapter, following an approach similar to Ref. [191], we determine the actions for the two exotic five-branes of the type IIB superstring. The first is called the 5^2_2-brane[3] which is related via two T-dualities to the type IIB NS5-brane[4] (or, equivalently, via one T-duality to the type IIA KKM), while the second brane is obtained from the 5^2_2-brane upon S-duality and denoted as 5^2_3.

Although the worldvolume DBI actions may be determined in a rather straightforward way, the issue of determining the WZ couplings in the case of exotic branes is less trivial. These couplings are well known for Dp-branes and were derived in [191] and [116, 120] for NS5-branes. We will see that it is worth revisiting the NS5 couplings. More importantly, using the appropriate duality rules, we will determine the gauge potentials which couple to the exotic five-branes. It turns out that these potentials are magnetic duals of the Kalb-Ramond field (for the 5^2_2-brane) and the RR 2-form (for the 5^2_3-brane), albeit not the standard magnetic duals to which the D5- and NS5-branes couple. Instead they can naturally be interpreted as higher rank forms that also carry vector indices. The role of non-standard duals to standard gauge potentials in the study of exotic branes was pointed out already in Ref. [121, 178, 179] from an alternative point of view. Our results support and clarify this role.

Having written down the worldvolume actions for the two exotic five-branes, mutually related by S-duality, we discuss and clarify their relation to non-geometry [123]. In particular we argue that the first brane acts as a source for non-geometric Q flux, as expected from its interpretation as a T-fold in the supergravity picture. This result is based on a rewriting of the type IIB action in terms of suitable variables, similar to the one that was performed in Ref. [113]. This rewriting allows one to consider the bulk and worldvolume actions on equal footing and to write down the corresponding modified Bianchi identity [123]. Another very interesting result is obtained for the second five-brane, which is related to the previous one by S-duality. This brane acts as a source of what appears as some sort of non-geometric RR flux, a situation which has not been widely discussed in the literature (see however Refs. [197, 198]). We discuss this RR non-geometry and argue that it is a very reasonable outcome, which has been slightly overlooked due to focus on T-duality and not S-duality which exchanges NSNS with RR potentials.

[3] The notation is explained in Sec. 5.1.

[4] Note that the NS5-brane breaks the circle isometry relevant for T-duality unless it is smeared [193, 194]. Nevertheless T-duality to the KKM remains valid also for the localized NS5-brane due to world sheet instanton corrections that lead to a localization of the KKM in winding space [195]. Further understanding emerges from the treatment in double field theory [196], also for analogous issue regarding the 5^2_2-brane [181, 182].

5.1 Preliminaries on dualities and branes

In this chapter we will focus on the type II superstring theories. We have seen that these contain a variety of Dp-branes, with $p = 0, 2, 4, 6, 8$ for the type IIA and $p = -1, 1, 3, 5, 7$ for the type IIB [5]. Dp-branes couple to the corresponding RR gauge potentials of these theories. In addition, both theories contain NS5-branes, which couple magnetically to the Kalb-Ramond field of the common sector of the two theories, as well as KK monopoles, which couple to abelian KK gauge fields arising from dimensional reduction.

The type II theories are exchanged under the action of T-duality, whereas the type IIB theory also features a strong-weak self-duality (S-duality). These stringy symmetries lead to new extended states, which were discovered and classified in Refs. [174–176]. These were revisited recently in Ref. [116, 120, 123], where it was shown that they are related to non-geometric backgrounds. In this chapter, we study the dynamics and supergravity couplings of such branes by constructing their effective action and further clarify their connections to non-geometry.

While a generic treatment of all these states would certainly be very interesting, it might also obscure some of the details and the differences between the different types of branes. We will therefore make two restrictions in this chapter. We focus (a) on the type IIB superstring theory, mainly because it also involves S-duality which will prove to be very interesting and leads to cases not much studied in the literature, and (b) on five-branes of this theory. It will turn out that these branes are sufficient to study the properties that we are interested in and to reach a self-contained set of results.

The five-brane states of the type IIB superstring are the D5-brane (5_1), the NS5-brane (5_2), the KKM (5_2^1) and two exotic states, the 5_2^2-brane and the 5_3^2-brane. We use the notation of Ref. [116, 120], where the main number denotes the amount of worldvolume directions (which is always 5 in this chapter, unless otherwise stated), the lower index denotes the power of the inverse string coupling in the tension of the object and the upper index denotes the number of "special (NUT) transverse directions", on which the mass of the brane depends quadratically. Therefore we observe that this set of objects contains a lot of diversity both in their "non-perturbativity" (the power of inverse string coupling is 1,2 or 3)[6] and in the amount of special transverse directions (0, 1 or 2)[7].

Let us now discuss how these branes are linked to each other by dualities. We begin with the D5- and the NS5-brane, which transform into each other under S-duality, as they form magnetic sources for the $SL(2;\mathbf{Z})$ doublet of the RR potential C_2 and the NSNS potential B. Next, the NS5-brane is linked to the KKM by two T-dualities, one along a transverse and one along a worldvolume direction (the former leads to the type IIA KKM, which T-dualizes to the type IIB KKM under the latter). Moreover, under two transverse T-dualities the NS5 is linked to the 5_2^2-brane. The latter can also be obtained from the KKM under two T-dualities, one along a worldvolume and one along a transverse direction (the intermediate type IIA brane in this case is also an exotic 5_2^2 state). Finally, the 5_3^2-brane is obtained from the type IIB 5_2^2 upon S-duality. For completeness, let us mention that the IIB KKM is self-dual under S-duality. What we explained with words is depicted in the accompanying Figure 1.

Since we plan to discuss connections of the exotic branes to non-geometry, it is suggestive to make contact with the corresponding flux chain. The standard T-duality chain of fluxes can be written as

$$H_{abc} \overset{T_a}{\longleftrightarrow} f^a{}_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}{}_c \overset{T_c}{\longleftrightarrow} R^{abc}, \tag{5.1}$$

[5] Let us recall that the D8-brane is special and related to a massive deformation of type IIA supergravity. Moreover, the D7-brane is also special since it is not asymptotically flat [199].

[6] It should be mentioned that there are also branes with power 4.

[7] More special directions, up to seven, appear as one considers lower dimensions.

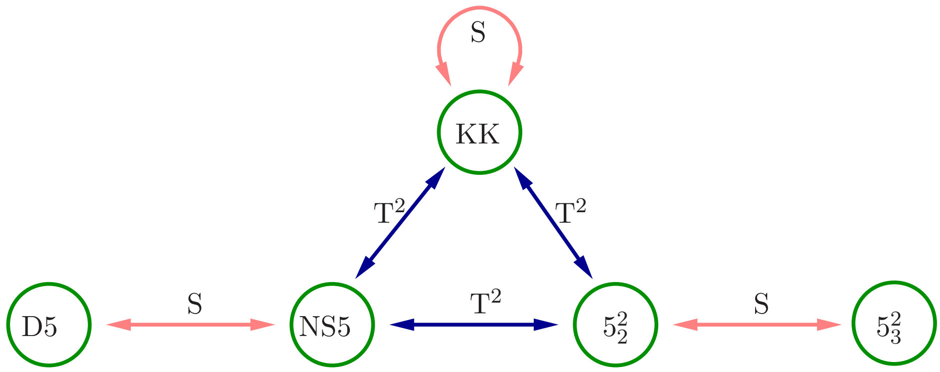
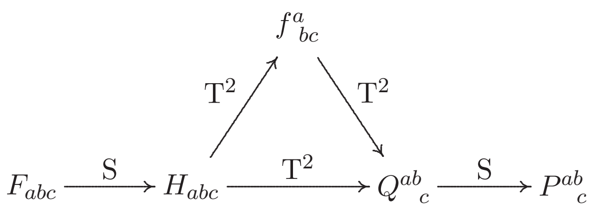


Figure 5.1: The duality chain of five-branes in type IIB string theory. In the figure S denotes S-duality and T^2 denotes two T-dualities along the two directions of a torus.

where the leftmost entry refers to a NSNS 3-form flux, which T-dualizes to a geometric flux (nilmanifold), while the other two entries, are commonly called non-geometric fluxes. In relation to the branes that we discuss here, the corresponding chain of fluxes reads as follows, where some of the T-dualities are also along directions without flux,



Some remarks are in order here. First, the chain we discuss involves also S-dualities and therefore RR fluxes appear as well, in particular F_3. This is not common in most discussions of non-geometric fluxes but it is important here because the rightmost entry of the chain can be viewed as a non-geometric RR flux, which we denoted by P [197, 198]. We will be more specific on this point later on. Furthermore, it should be noted that the chain related to the IIB five-branes does not contain an entry associated to the so-called R flux. We will have nothing more to say on this point, apart from some comments in the final discussion.

It is also instructive to remember the supergravity solutions describing the above standard and exotic branes. This will assist our discussion in Sec. 5.4. The first three, D5, NS5 and KKM, may be found in standard textbooks (see e.g. Ref. [200], chapter 18). The solutions for 5^2_2 and 5^2_3 were written down in Ref. [201] as generalized KKMs. These data appear in the following table[8].

[8] The last two rows of this table should be considered with some caution. They are local expressions which do not make sense globally in standard supergravity formulation. More details on this will be given in Sec. 5.4.

IIB Brane	Metric ds^2 (string frame)	Dilaton $e^{2\phi}$	Flux
D5 (5_1)	$H^{-\frac{1}{2}}dx_{\parallel}^2 + H^{\frac{1}{2}}dx_{\perp}^2$	H^{-1}	F_3
NS5 (5_2)	$dx_{\parallel}^2 + H dx_{\perp}^2$	H	H_3
KKM (5_2^1)	$dx_{\parallel}^2 + H dx_{\perp}^2 + H^{-1}dx_{\odot}^2$	1	f_1^2
5_2^2	$dx_{\parallel}^2 + H dx_{\perp}^2 + HK^{-1}dx_{\odot}^2$	HK^{-1}	Q_2^1
5_3^2	$(HK^{-1})^{-\frac{1}{2}}dx_{\parallel}^2 + (HK^{-1})^{\frac{1}{2}}(dx_{\perp}^2 + dx_{\odot}^2)$	$(HK^{-1})^{-1}$	P_2^1

In obvious notation, $x_{\parallel}$ denotes the worldvolume directions of the brane solution, while $x_{\perp}$ are the ordinary transverse directions. Moreover, $x_{\odot}$ denote special transverse directions. H is a harmonic function, different for each solution. Its explicit form may be found in the textbook [200]. Furthermore, K appearing in the 5_2^2 and 5_3^2 solutions, is another function of the form

$$K = H^2 + \left(\frac{R_{\odot}^2}{2\pi\alpha'}\theta\right)^2 .$$

Note that these two branes are of co-dimension two[9], with θ being the polar angle of the transverse directions $x_{\perp}$. Regarding the flux column, the first three rows are obvious. The fact that we associated a Q_2^1 flux to the fourth row was recently discussed to some extent in Refs. [123, 202] and will be revisited below. The P_2^1 in the fifth row is currently just a label but it will be explained in Sec. 5.4.

The worldvolume actions of all but the exotic five-branes with two special transverse directions were previously studied in [191].

5.2 The NS5-brane

In this section we revisit the worldvolume actions for the type IIB NS5-brane which were derived in [191] (see also [203] for a different approach to the type IIA NS5-brane). For the reader's convenience, we will rederive these actions.

Let us begin our discussion with the well-known DBI action of the D5-brane (see Sec. 1.4),

$$S_{\text{DBI,D5}} = -T_{\text{D5}} \int_{\mathcal{M}_{\text{D5}}} d^6\sigma\, e^{-\phi} \sqrt{-\det\left(G_{ij} + B_{ij} + \check{F}_{ij}\right)}, \tag{5.2}$$

where the indices i, j run along the six directions of the worldvolume $\mathcal{M}_{\text{D5}}$. The physical tension in string frame of the D5-brane is

$$\tau_{\text{D5}} = T_{\text{D5}} g_s^{-1},$$

being inversely proportional to the string coupling as for any Dp-brane. Here and in the following we set $2\pi\alpha' = 1$ since this dimensionful factor (with dimensions of $(\text{mass})^{-2}$) can always be reinserted by dimensional analysis. Moreover, let us recall that

$$G_{ij} = \partial_i X^M \partial_j X^N G_{MN} \quad \text{and} \quad B_{ij} = \partial_i X^M \partial_j X^N B_{MN}$$

[9]With co-dimension we mean the number of the ordinary transverse directions $x_{\perp}$

are the pullbacks of the spacetime fields G and B by the 10D embedding coordinates X^M of the brane. Finally,

$$\check{F}_2 = \mathrm{d}\check{\mathcal{A}}_1, \tag{5.3}$$

is the field strength of the worldvolume abelian gauge field living on the brane.

The D5-brane is also charged under the RR fields, in particular, it couples electrically to the RR gauge potential C_6 (or magnetically to the RR gauge potential C_2). Its couplings to C_6 and lower degree forms are determined by the Wess-Zumino action

$$S_{\mathrm{WZ,D5}} = \mu_{\mathrm{D5}} \int_{\mathcal{M}_{\mathrm{D5}}} \left\langle \mathrm{e}^{-\check{\mathcal{F}}} \wedge \mathbf{C} \right\rangle_6 , \tag{5.4}$$

where the bulk fields appearing in brane actions should always be interpreted as pullbacks of the respective 10D fields to the brane worldvolume (here denoted by $\mathcal{M}_{\mathrm{D5}}$). In this chapter we denote the charge of the D5 brane by μ_{D5} to distinguish it from other five-branes. This action is obtained as the gauge invariant completion of the coupling to the RR potential C_6. In particular, $\check{\mathcal{F}}_2$ is defined as

$$\check{\mathcal{F}}_2 = B + \mathrm{d}\check{\mathcal{A}}_1, \tag{5.5}$$

with the gauge transformation rules

$$\delta B = \mathrm{d}\Lambda_1, \qquad \delta\check{\mathcal{A}}_1 = -\Lambda_1 \quad \Rightarrow \quad \delta\check{\mathcal{F}}_2 = 0. \tag{5.6}$$

Which makes the action gauge invariant under the Kalb-Ramond gauge transformation.

As we already mentioned, the D5- and NS5-branes are S-dual to one another. The relevant S-duality rules can be simply expressed as

$$\tau \overset{S}{\longmapsto} -\frac{1}{\tau}, \quad C_2 \overset{S}{\longmapsto} B, \quad B \overset{S}{\longmapsto} -C_2 \quad \text{and} \quad G \overset{S}{\longmapsto} |\tau| G, \tag{5.7}$$

where $\tau = C_0 + i\mathrm{e}^{-\phi}$ is the type IIB axiodilaton.

According to the above duality rules, the NS5 DBI action is

$$S_{\mathrm{DBI,NS5}} = -T_{\mathrm{NS5}} \int_{\mathcal{M}_{\mathrm{NS5}}} \mathrm{d}^6\sigma \mathrm{e}^{-\phi} |\tau| \sqrt{-\det\Big(G_{ij} - |\tau|^{-1}\mathcal{F}_{ij}\Big)} \tag{5.8}$$

with

$$\mathcal{F}_2 = C_2 + \mathrm{d}\mathcal{A}_1, \tag{5.9}$$

where C_{ij} is the pullback of the corresponding supergravity potential and $\mathcal{A}_1$ is the worldvolume gauge field, defined such that $\mathcal{F}_2$ is gauge invariant. Under S-duality $\check{\mathcal{F}}_2$ transforms as $\check{\mathcal{F}}_2 \overset{S}{\longmapsto} -\mathcal{F}_2$. Equivalently, we can write

$$S_{\mathrm{DBI,NS5}} = -T_{\mathrm{NS5}} \int \mathrm{d}^6\sigma \mathrm{e}^{-2\phi} \sqrt{1 + e^{2\phi}C_0^2} \sqrt{-\det\Bigg(G_{ij} - \frac{e^{\phi}}{\sqrt{1+e^{2\phi}C_0^2}}\mathcal{F}_{ij}\Bigg)}. \tag{5.10}$$

The latter expression makes the non-perturbative nature of the NS5-brane more transparent, as the scaling of the physical tension with the inverse square of the string coupling becomes manifest,

$$\tau_{\mathrm{NS5}} \propto g_s^{-2}.$$

The WZ couplings of the NS5-brane can be found as before by constructing the fully gauge invariant completion of the coupling

$$\mu_{\mathrm{NS5}} \int_{\mathcal{M}_{\mathrm{NS5}}} B_6,$$

where B_6 is the magnetic dual of the Kalb-Ramond gauge potential (see Eq. (D.15)). In order to carry out this task, we need to know the gauge transformation of B_6. This is determined in App. D, and we repeat here the result,

$$\delta B_6 = \mathrm{d}\Lambda_5 + \mathrm{d}\lambda_3 \wedge C_2 + \mathrm{d}\lambda_1 \wedge B \wedge C_2,$$

where Λ_5 is a 5-form gauge parameter, and λ_1, λ_3 are the 1- and 3-form gauge parameters associated with the gauge transformations of C_2 and C_4, respectively. This dictates the couplings that have to be added in the WZ action in order to render it gauge invariant. This action may be written in polyform notation as before, defining the new polyform

$$\mathbf{B} = \frac{C_0}{|\tau|^2} - B - (C_4 - C_2 \wedge B) + \left(B_6 - \frac{1}{2} B \wedge C_2 \wedge C_2\right). \tag{5.11}$$

We can now write

$$S_{\mathrm{WZ,NS5}} = \mu_{\mathrm{NS5}} \int_{\mathcal{M}_{\mathrm{D5}}} \left\langle e^{-\mathcal{F}} \wedge \mathbf{B} \right\rangle_6. \tag{5.12}$$

This action can be also obtained very easily by directly applying the S-duality rules to the action (5.4) (see App. D). Indeed, $\mathbf{B}$ is the (negative) S-dual of the polyform $\mathbf{C}$. This is the approach followed in Ref. [191]. The approach we employed here was used in Ref. [116, 120], albeit with different conventions for the gauge potentials (see App. D).

5.2.1 Modified Bianchi identities.

The D5- and NS5-branes act as sources for RR and NSNS 3-form field strengths, respectively. Indeed, in their presence the corresponding Bianchi identities are modified, similar to what happens in standard electrodynamics in the presence of a magnetic source. For the NS5-brane the relevant terms are the kinetic term for the Kalb-Ramond potential and the corresponding leading WZ coupling, i.e.

$$S_{B_6} = -\int_{10} \tfrac{1}{2} \mathrm{e}^{2\phi} \star_{10} |H_7|^2 + \mu_{NS5} \int_{10} \delta_4 \wedge B_6. \tag{5.13}$$

The action is written in terms of the dual field strength H_7, and the worldvolume term is lifted to ten dimensions by a 4-form δ_4 with support on the worldvolume (see Sec. 1.4). The variation with respect to B_6 yields the modified Bianchi identity for the NSNS 3-form,

$$\mathrm{d}H_3 = j_4^{NS5}, \tag{5.14}$$

where $j_4^{NS5} = -\mu_{NS5}\delta_4$. This shows that the NS5-brane is a localized source of NSNS flux.

5.3 DBI action of exotic five-branes

Our aim here is to determine the analog of the DBI action of the 5_2^2 brane of the type IIB superstring, which is the double T-dual of the NS5 brane along two of its transverse dimensions, and of the 5_3^2 brane, which is the S-dual of the 5_2^2, as explained in Sec. 5.1. The WZ couplings of these exotic branes are discussed in the next section.

5.3.1 T-duality rules

In the following, we consider a 5^2_2-brane that is obtained by two T-dualities along two transverse directions of an NS5-brane. The worldvolumes of both branes are taken to be along the directions 034567, and we perform the two T-dualities along the directions 89, which form a two-torus. The resulting 5^2_2-brane then has the transverse directions 1289 denoted by $r\theta yz$, with yz being the special transverse directions analogous to the one special NUT like direction of the KKM.

We use the following KK ansatz for the metric[10],

$$\mathrm{d}s^2 = \hat{G}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + G_{mn}(\mathrm{d}x^m + A^m)(\mathrm{d}x^n + A^n),$$

where $A^m = A^m_\mu \mathrm{d}x^\mu$ are the two KK one forms. The indices m, n run over the directions yz of the compactified two-torus, μ, ν run over the rest, and we use capital indices M, N to denote ten-dimensional directions. Likewise we decompose the NSNS 2-form B as

$$B = \tfrac{1}{2}\left[\hat{B}_{\mu\nu} + A^m_\mu B_{m\nu}\right]\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu + B_{m\nu}\mathrm{d}x^m \wedge \mathrm{d}x^\nu + \tfrac{1}{2}B_{mn}\mathrm{d}x^m \wedge \mathrm{d}x^n.$$

In the following, it will often be useful to use the non-coordinate basis $(\mathrm{d}x^\mu, \eta^m)$ with $\eta^m = \mathrm{d}x^m + A^m$. In this basis the metric takes the form

$$\mathrm{d}s^2 = \hat{G}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + G_{mn}\eta^m \eta^n, \tag{5.15}$$

so that the determinant of the metric factorizes conveniently as

$$\sqrt{-\det(G_{MN})} = \sqrt{-\det(\hat{G}_{\mu\nu})}\sqrt{\det(G_{mn})}. \tag{5.16}$$

To rewrite the Kalb-Ramond field in the basis $(\mathrm{d}x^\mu, \eta^m)$, it is useful to introduce the 1-form

$$\theta_m = \theta_{m\mu}\mathrm{d}x^\mu \quad \text{and} \quad \theta_{m\mu} = B_{m\mu} - B_{mn}A^n_\mu, \tag{5.17}$$

which is motivated by a simple transformation rule under T-duality (see below). We can now write B as (cf. [204]),

$$B = \tfrac{1}{2}\hat{B}_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu + \tfrac{1}{2}B_{mn}\eta^m \wedge \eta^n + (\eta^m - \tfrac{1}{2}A^m) \wedge \theta_m. \tag{5.18}$$

The T-duality rules for the NSNS background fields under the combined T-dualities along the directions y and z are

$$\mathrm{e}^\phi \xmapsto{yz} \frac{\mathrm{e}^\phi}{\sqrt{\det(G_{mn} + B_{mn})}}, \tag{5.19}$$

$$G_{mn} \xmapsto{yz} \tilde{G}^{mn} = \frac{\det(G_{kl})}{\det(G_{kl} + B_{kl})}G^{mn}, \tag{5.20}$$

$$A^m_\mu \xmapsto{yz} \theta_{m\mu}, \tag{5.21}$$

$$\hat{G}_{\mu\nu} \xmapsto{yz} \hat{G}_{\mu\nu}, \tag{5.22}$$

$$B_{mn} \xmapsto{yz} \tilde{B}^{mn} = \frac{\det(B_{kl})}{\det(G_{kl} + B_{kl})}(B^{-1})^{mn}, \tag{5.23}$$

$$\theta_{m\mu} \xmapsto{yz} A^m_\mu, \tag{5.24}$$

$$\hat{B}_{\mu\nu} \xmapsto{yz} \hat{B}_{\mu\nu}. \tag{5.25}$$

[10] In this chapter we use hatted symbols for quantities that are invariant under T-duality.

This directly implies that the combination

$$\sqrt{\det(G_{mn})}\ \mathrm{e}^{-2\phi} = \mathrm{e}^{-2\hat{\phi}} \tag{5.26}$$

is T-duality invariant.

The T-duality rules for the RR potentials have to be determined as well. To this end, it turns out that it is also useful to work in the $(\mathrm{d}x^\mu, \eta^m)$ basis. Specifically, we define the following components,

$$\begin{aligned} C_0 &= \zeta_0, \\ C_2 &= \tfrac{1}{2}\zeta_{\mu\nu}\mathrm{d}x^{\mu\nu} + \zeta_{\mu m}\mathrm{d}x^\mu \wedge \eta^m + \tfrac{1}{2}\zeta_{mn}\eta^m \wedge \eta^n, \\ C_4 &= \tfrac{1}{24}\zeta_{\mu\nu\rho\sigma}\mathrm{d}x^{\mu\nu\rho\sigma} + \tfrac{1}{6}\zeta_{\mu\nu\rho s}\mathrm{d}x^{\mu\nu\rho} \wedge \eta^s + \tfrac{1}{4}\zeta_{\mu\nu rs}\mathrm{d}x^{\mu\nu} \wedge \eta^r \wedge \eta^s, \end{aligned}$$

which are the components of the RR potentials in this basis. The usual shorthand notation $dx^{i_1 i_2 \ldots i_p} = dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p}$ is hereby employed. Then the following duality rules for these forms are obtained as,

$$\begin{aligned} \zeta_0 &\overset{yz}{\longmapsto} \zeta_{yz} - B_{yz}\zeta_0, \\ \zeta_{mn} &\overset{yz}{\longmapsto} -\epsilon^{mn}\zeta_0 + \tilde{B}^{mn}(\zeta_{yz} - B_{yz}\zeta_0), \\ \zeta_{\mu n} &\overset{yz}{\longmapsto} \epsilon^{nm}\zeta_{\mu m}, \\ \zeta_{\mu\nu} &\overset{yz}{\longmapsto} \zeta_{\mu\nu yz} - \zeta_{\mu\nu}B_{yz}. \end{aligned}$$

The completely antisymetric tensor ϵ^{mn} appearing in these expressions is defined to be such that $\epsilon^{yz} = 1$.

Finally we give the duality rules of the worldvolume scalars and gauge potentials relevant for the DBI actions. The degrees of freedom of each brane are given in table 5.1. The T-duality rule is simply

$$X^\nu \overset{yz}{\longmapsto} X^\nu, \tag{5.27}$$

$$X^m \overset{yz}{\longmapsto} \tilde{X}_m, \tag{5.28}$$

$$\mathcal{A}_1 \overset{yz}{\longmapsto} \tilde{\mathcal{A}}_1. \tag{5.29}$$

Brane	Worldvolume scalars	Worldvolume gauge fields	Degrees of freedom
D5	$X^N = (X^\nu, X^n)$	$\check{\mathcal{A}}_1$	$4+4$
NS5	$X^N = (X^\nu, X^n)$	$\mathcal{A}_1$	$4+4$
5^2_2	$X^\nu, \tilde{X}_m$	$\tilde{\mathcal{A}}_1$	$2+2+4$
5^2_3	X^ν, X'_m	$\mathcal{A}'_1$	$2+2+4$

Table 5.1: The degrees of freedom of the branes discussed in this paper. In all cases the worldvolume gauge potential carries 4 degrees of freedom and the worldvolume scalars carry 4 degrees of freedom.

5.3.2 The 5^2_2 DBI action.

Using the above T-duality rules, we can calculate the DBI action of the 5^2_2-brane. The key benefit of having written the metric and the forms in the $(\mathrm{d}x^\mu, \eta^m)$ basis is that the components of the fields transform nicely under T-duality. Furthermore, the pullback of η^m transforms directly to a simple set of 1-forms $\tilde{\eta}_m$, under T-duality,

$$\eta^m_M \partial_i X^M = \eta^m_i \stackrel{yz}{\longmapsto} \tilde{\eta}_{im} = \partial_i \tilde{X}_m + \theta_{m\mu}\partial_i X^\mu,$$

where $\tilde{X}_m$ denotes the T-dual of the corresponding worldvolume field.

At this stage the pullback of each field appearing in the NS5 worldvolume action can be easily transformed. The pullback of the metric takes the form

$$\begin{aligned} G_{ij} &= \hat{G}_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + G_{mn}\eta^m_i \eta^n_j \\ &\stackrel{yz}{\longmapsto} \hat{G}_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + \tilde{G}^{mn}\tilde{\eta}_{im}\tilde{\eta}_{jn}. \end{aligned}$$

Moreover, the pullback of the 2-form is

$$\begin{aligned} C_{ij} &= \zeta_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + 2\zeta_{\mu n}\partial_{[i} X^\mu \eta^n_{j]} + \zeta_{mn}\eta^m_i \eta^n_j \\ &\stackrel{yz}{\longmapsto} \left(\zeta_{\mu\nu yz} - \zeta_{\mu\nu}B_{yz}\right)\partial_i X^\mu \partial_j X^\nu + 2\epsilon^{nm}\zeta_{\mu m}\partial_{[i}X^\mu \tilde{\eta}_{j]n} \\ &\quad + \Big(-\epsilon^{mn}\zeta_0 + \tilde{B}^{mn}(\zeta_{yz} - B_{yz}\zeta_0)\Big)\tilde{\eta}_{im}\tilde{\eta}_{jn}. \end{aligned}$$

We can now define the gauge invariant 2-form

$$\begin{aligned} \tilde{\mathcal{F}}_{ij} &= 2\partial_{[i}\tilde{\mathcal{A}}_{j]} + \left(\zeta_{\mu\nu yz} - \zeta_{\mu\nu}B_{yz}\right)\partial_i X^\mu \partial_j X^\nu + 2\epsilon^{nm}\zeta_{\mu m}\partial_{[i}X^\mu \tilde{\eta}_{j]n} \\ &\quad + \Big(-\epsilon^{mn}\zeta_0 + \tilde{B}^{mn}(\zeta_{yz} - B_{yz}\zeta_0)\Big)\tilde{\eta}_{im}\tilde{\eta}_{jn}, \end{aligned}$$

which is the T-dual of $\mathcal{F}_{ij}$, defined in Eq. (5.9). Finally, the axiodilaton τ transforms to the dual modulus

$$\tilde{\tau} = (C_{yz} - B_{yz}C_0) + i\sqrt{\det(E_{mn})}\mathrm{e}^{-\phi},$$

where $E_{mn} = G_{mn} + B_{mn}$ as usual. The full DBI action for the 5^2_2 can now be written down and acquires the form

$$\begin{aligned} S_{\mathrm{DBI},5^2_2} &= -T_{5^2_2}\int_{\mathcal{M}_{5^2_2}} \mathrm{d}^6\sigma\ \mathrm{e}^{-\phi}|\tilde{\tau}|\sqrt{\det(E_{kl})} \\ &\quad \times\sqrt{-\det(\hat{G}_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + \tilde{G}^{mn}\tilde{\eta}_{im}\tilde{\eta}_{jn} - |\tilde{\tau}|^{-1}\tilde{\mathcal{F}}_{ij})}. \end{aligned} \tag{5.30}$$

By noticing that $\tilde{\tau}$ is proportional to $\mathrm{e}^{-\phi}$ we see that the above action has the expected g_s^{-2} dependence.

5.3.3 S-duality and the 5^2_3 DBI action.

Using the S-duality rules (5.7) we can also write down the DBI action for the 5^2_3-brane. We define

$$\begin{aligned} G'^{mn} &= \frac{\det(|\tau|G_{kl})|\tau|^{-1}G^{mn}}{\det(|\tau|G_{kl} - C_{kl})}, \\ C'^{mn} &= \frac{\det(C_{kl})}{\det(|\tau|G_{kl} - C_{kl})}(C^{-1})^{mn}, \end{aligned} \tag{5.31}$$

which are the symmetric and antisymmetric part, respectively, of the inverse of $|\tau|G_{mn} - C_{mn}$ to which E_{mn} transforms. We then find

$$\tilde{\tau} \xmapsto{S} \tau' = B_{yz} - |\tau|^{-2} C_{yz} C_0 + i\sqrt{\det(|\tau|G_{mn} - C_{mn})}|\tau|^{-2}\mathrm{e}^{-\phi}.$$

Furthermore

$$\begin{aligned} \tilde{\mathcal{F}}_{ij} \quad \xmapsto{S} \quad & \mathcal{F}'_{ij} = 2\partial_{[i}\mathcal{A}'_{j]} + (\zeta_{\mu\nu yz} - \zeta_{\mu\nu}B_{yz} + 2\epsilon^{mn}\zeta_{\mu m}\theta_{\nu n})\,\partial_i X^\mu \partial_j X^\nu \\ & +2\epsilon^{nm}\theta_{\mu m}\partial_{[i}X^\mu \eta'_{j]n} + \left[\epsilon^{mn}|\tau|^{-2}C_0 + C'^{mn}(B_{yz} - |\tau|^{-2}C_{yz}C_0\right]\eta'_{im}\eta'_{jn}, \end{aligned}$$

where

$$\eta'_{in} = \partial_i X'_m - \zeta_{m\mu}\partial_i X^\mu.$$

We can now write down the DBI action for the 5^2_3

$$\begin{aligned} S_{\mathrm{DBI},5^2_3} \quad = \quad & -T_{5^2_3}\int \mathrm{d}^6\sigma\ \mathrm{e}^{-\phi}|\tau|^{-2}|\tau'|\sqrt{\det(|\tau|G_{kl} - C_{kl})} \\ & \times\sqrt{-\det(\hat{G}_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + |\tau|^{-1}\tilde{G}'^{mn}\eta'_{im}\eta'_{jn} - |\tau'|^{-1}\mathcal{F}'_{ij})}. \end{aligned} \tag{5.32}$$

Here we can again extract the g_s dependence of each factor and see that this action scales as g_s^{-3} as expected.

5.4 WZ actions of exotic five-branes and non-geometry

Having discussed the DBI action of the 5^2_2- and 5^2_3-branes, we now turn to their WZ actions. The WZ couplings we derive will make the interpretation of these branes as sources for non-geometric fluxes very transparent, complementing the effective supergravity analysis of [116, 120, 123]. Moreover, they will clarify the meaning of the mixed symmetry forms that occur in group theoretical considerations [121, 178, 179].

5.4.1 WZ actions of exotic five-branes

Writing down the WZ action of the exotic brane involves the T-dualization of B_6, the magnetic dual of B, that occurs in the WZ action of the NS5-brane. Let us recall that B_6 is defined by the non-local expression

$$H_7 = \mathrm{d}B_6 + \cdots = \mathrm{e}^{-2\phi}\star_{10}\mathrm{d}B, \tag{5.33}$$

where the dots represent the RR terms necessary for the consistent definition of B_6. They are given explicitly in App. D (Eq. (D.15)) and lead to the non-trivial Bianchi identity,

$$\mathrm{d}H_7 = F_3 \wedge F_5 - F_1 \wedge F_7,$$

which means that H_7 can be set equal to $\mathrm{d}B_6$ only under the assumption that the RR fields vanish. Let us for the moment employ this simplification and neglect the RR terms; i.e. we use $H_7 = \mathrm{d}B_6$, keeping in mind that the expressions below are subject to corrections from the RR sector. Starting from the NS5-brane top form coupling to B_6 it is clear that the 5^2_2-brane couples to a magnetic dual of the double T-dual of B. We have seen in Eq. (5.23-5.25) that the T-dual of B is a combination of fields from the NSNS sector, namely the B itself, the KK 1-forms A^m_μ and the internal metric G_{mn}. Since T-duality mixes fields from the NSNS sector only, this means that B_6 maps under double T-duality to *some* magnetic dual of the above mentioned fields from the NSNS sector. We will see that this magnetic dual can be naturally described in terms of an 8-form B^2_8 with two vector indices (see also [121, 178, 179]).

5.4.2 The KK monopole

As a warm up, let us begin by considering a single T-duality of B_6 which should give the top form coupling to the type IIA KKM. The full WZ action for the KKM was worked out in detail in [190, 191], but here we are only interested in the top form coupling[11]. Consider the metric ansatz in Eq. (5.15) where the internal indices $m, n, \ldots$ only take a single value, $m = z$, and the B field ansatz (cf. (5.18)) is

$$B = \hat{B} + \eta^z \wedge \theta_z - \frac{1}{2} A^z \wedge \theta_z,$$

where $\hat{B} = \frac{1}{2}\hat{B}_{\mu\nu} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$. Since there is only one internal direction, z, the B_{mn} of Eq. (5.17) vanishes and $\theta_{z\mu} = B_{z\mu}$. In order to obtain the magnetic dual B_6, we first determine the field strength $H = \mathrm{d}B$,

$$H = \mathrm{d}\hat{B} + \frac{1}{2}\left(\mathrm{d}A^z \wedge \theta_z + A^z \wedge \mathrm{d}\theta_z\right) - \eta^z \wedge \mathrm{d}\theta_z = \hat{H} - \eta^z \wedge \mathrm{d}\theta_z, \tag{5.34}$$

where we have defined the manifestly T-duality invariant 3-form $\hat{H}$ and used $\mathrm{d}\eta^z = \mathrm{d}A^z$. Note that, because of the assumed isometry in the z direction, none of the above differentials on the right hand side of Eq. (5.34) have legs in the η^z direction. With the current ansatz (5.15) for the metric the Hodge star factorizes and can be written in terms of $\hat{\star}_9$, the Hodge operator associated with the reduced metric $\hat{G}_{\mu\nu}$ (cf. (5.15)). Then

$$\begin{aligned} \mathrm{d}B_6 &= \mathrm{e}^{-2\phi} \star_{10} \mathrm{d}B \\ &= \mathrm{e}^{-2\phi}\sqrt{G_{zz}}(\hat{\star}_9\, \hat{H}) \wedge \eta^z + \mathrm{e}^{-2\phi}\sqrt{G^{zz}}\hat{\star}_9\, \mathrm{d}\theta_z. \end{aligned} \tag{5.35}$$

At this stage it is beneficial to define the projection operators

$$P_z\omega_p = \mathrm{d}z \wedge \iota_z\omega_p \quad \text{and} \quad \tilde{P}_z\omega_p = \eta^z \wedge \iota_z\omega_p,$$

where ω_p is any p-form. They are related through $\tilde{P}_z\omega_p = P_z\omega_p + A^z \wedge \iota_z\omega_p$. Here we have defined the contraction with the Killing vector ∂_z by ι_z[12]. With the assumed Killing isometry we can make use of the fact that $\mathrm{d}\iota_z\omega_p + \iota_z\mathrm{d}\omega_p = \mathcal{L}_z\omega_p = 0$ to show that $\mathrm{d}P_z\omega_p = P_z\mathrm{d}\omega_p$. Now we can decompose Eq. (5.35)

$$(1 - \tilde{P}_z)\mathrm{d}B_6 = \mathrm{e}^{-2\phi}\sqrt{G^{zz}}\, \hat{\star}_9\, \mathrm{d}\theta_z, \tag{5.36}$$

$$\mathrm{d}\iota_z B_6 = -\iota_z \mathrm{d}B_6 = -\mathrm{e}^{-2\phi}\sqrt{G_{zz}}\hat{\star}_9\, \hat{H}. \tag{5.37}$$

We can now easily find the T-dual of $\iota_z \mathrm{d}B_6$. Using the fact that the combination $\mathrm{e}^{-2\phi}\sqrt{G_{zz}}$ is T-duality invariant, everything on the right hand side in Eq. (5.37) is T-duality invariant, and we conclude that

$$\iota_z B_6 \stackrel{z}{\longmapsto} \iota_z B_6. \tag{5.38}$$

We are interested in the components of B_6 that couple to a NS5-brane transverse to the z coordinate. This is the combination

$$(1 - P_z)B_6 = (1 - \tilde{P}_z)B_6 + A^z \wedge \iota_z B_6.$$

[11] Higher derivative WZ couplings of D-branes and their T-duals were discussed in [170].

[12] Our conventions are such that if a form ω_p is decomposed as $\omega_p = \hat{\omega}_p + \eta^z \wedge \psi_{p-1}$ then $\iota_z\omega_p = \psi_{p-1}$.

In order to determine the top form coupling to the KKM we need to T-dualize this, which can be done using (5.36) and (5.37). We calculate

$$\begin{aligned}
\mathrm{d}(1-P_z)B_6 &= (1-P_z)\mathrm{d}B_6 \\
&= (1-\tilde{P}_z)\mathrm{d}B_6 + A^z \wedge (\iota_z \mathrm{d}B_6) \\
&= \mathrm{e}^{-2\phi}\sqrt{G^{zz}}\ \hat{\star}_9\ \mathrm{d}\theta_z - A^z \wedge \mathrm{d}(\iota_z B_6) \\
&\xmapsto{z} \mathrm{e}^{-2\phi}\sqrt{G^3_{zz}}\ \hat{\star}_9\ \mathrm{d}A^z - \theta_z \wedge \mathrm{d}(\iota_z B_6) \\
&= \mathrm{d}(\iota_z A^z_7),
\end{aligned} \tag{5.39}$$

where we have used the T-duality rules derived before. In the last step we have expressed the result in terms of the magnetic dual of A^z. In the nine-dimensional theory this is a 6-form that is guaranteed to be exact by the equations of motion in the reduced theory. This is clear since the reduced theory is $O(1,1)$ invariant and the expression we start with, before performing the T-duality (5.39), is exact. From a ten-dimensional perspective this 6-form can naturally be interpreted as a contraction of a 7-form A^z_7, which is the ten-dimensional magnetic dual of the KK 1-form A^z. We discuss the reduced theory and magnetic duals in more detail in App. E. We then get the T-duality rule

$$(1-P_z)B_6 \xmapsto{z} \iota_z A^z_7, \tag{5.40}$$

which is valid up to closed forms (and RR corrections). This rule agrees with the results of [191], although a different but equivalent definition of A^z_7 was used.

5.4.3 The 5^2_2-brane

Let us now return to the 5^2_2-brane. After having determined the T-duality rules for B_6 in the case of a single T-duality, the next step is a straightforward generalization. The B field ansatz becomes (5.18)

$$B = \hat{B} + \eta^m \wedge \theta_m - \frac{1}{2}A^m \wedge \theta_m + \frac{1}{2}B_{mn}\eta^m \wedge \eta^n.$$

The field strength of B is then

$$H = \hat{H} - H_m \wedge \eta^m + \frac{1}{2}\mathrm{d}B_{mn} \wedge \eta^m \wedge \eta^n, \tag{5.41}$$

where

$$\hat{H} := \mathrm{d}\hat{B} + \frac{1}{2}\left(\mathrm{d}A^m \wedge \theta_m + A^m \wedge \mathrm{d}\theta_m\right), \tag{5.42}$$

$$H_m := \mathrm{d}\theta_m + B_{mn}\mathrm{d}A^n. \tag{5.43}$$

The magnetic dual can be expressed in terms of the fields of Eqs. (5.42,5.43) as

$$\begin{aligned}
\mathrm{d}B_6 &= \mathrm{e}^{-2\phi}\ \star_{10}\ \mathrm{d}B \\
&= \mathrm{e}^{-2\phi}\sqrt{\det(G_{kl})}\ (\hat{\star}_8\hat{H}) \wedge \eta^y \wedge \eta^z \\
&\quad + \mathrm{e}^{-2\phi}\sqrt{\det(G^{kl})}\ \hat{\star}_8 H_m \wedge \epsilon^{mr}G_{rs}\eta^s \\
&\quad + \mathrm{e}^{-2\phi}\sqrt{\det(G^{kl})}\ \hat{\star}_8\ \mathrm{d}B_{yz}.
\end{aligned}$$

We immediately see that under double T-duality,

$$\iota_y\iota_z B_6 \xmapsto{yz} \iota_y\iota_z B_6.$$

Since we start with an NS5-brane that does not wrap the yz directions, the component of B_6 that couples to the brane is $(1-P_z)(1-P_y)B_6$. The correponding field strength is

$$\begin{aligned} \mathrm{d}(1-P_z)(1-P_y)B_6 &= \mathrm{e}^{-2\phi}\sqrt{\det(G_{kl})}\Big[\det(G^{kl})\left(\hat{\star}_8\mathrm{d}B_{yz}+\hat{\star}_8 H_m\wedge\epsilon^{mr}G_{rs}A^s\right) \\ &\quad +\hat{\star}_8\hat{H}\wedge A^y\wedge A^z\Big], \end{aligned}$$

which dualizes under double T-duality to

$$\mathrm{e}^{-2\phi}\sqrt{\det(G_{kl})}\left[\det(G_{np}+B_{np})\left(\hat{\star}_8\mathrm{d}\tilde{B}^{yz}-\hat{\star}_8\tilde{H}^m\wedge\epsilon_{mr}\tilde{G}^{rs}\theta_s\right)+\hat{\star}_8\hat{H}\wedge\theta_y\wedge\theta_z\right], \tag{5.44}$$

where, as before,

$$\tilde{B}^{mn}=\frac{\det(B_{kl})}{\det(G_{kl}+B_{kl})}(B^{-1})^{mn},$$

and

$$\tilde{H}^m=\mathrm{d}A^m+\tilde{B}^{mn}\mathrm{d}\theta_n.$$

The indices on $\tilde{B}^{mn}$ are lifted because of its relation to B^{-1}. In this sense, $\tilde{B}$ should not be interpreted as a 2-form, but as a 2-vector valued scalar, with a field strength $\mathrm{d}\tilde{B}$ which is a 2-vector valued 1-form. The form that couples to the 5^2_2-brane in Eq. (5.44) is a magnetic dual of this 2-vector valued scalar which is also 2-vector valued and has a natural ten-dimensional origin which we call B^2_8 for now. The superscript 2 indicates the number of vector indices. We therefore find the T-duality rule

$$(1-P_z)(1-P_y)B_6\overset{yz}{\longmapsto}\iota_y\iota_z B_8^{yz}, \tag{5.45}$$

where

$$\begin{aligned} \mathrm{d}\iota_y\iota_z B_8^{yz} &= \mathrm{e}^{-2\phi}\sqrt{\det(G_{kl})}\left[\det(G_{np}+B_{np})\left(\hat{\star}_8\mathrm{d}\tilde{B}^{yz}-\hat{\star}_8\tilde{H}^m\wedge\epsilon_{mr}\tilde{G}^{rs}\theta_s\right)\right. \\ &\quad \left.+\hat{\star}_8\hat{H}\wedge\theta_y\wedge\theta_z\right]. \end{aligned} \tag{5.46}$$

The reason we can be certain that the right hand side of Eq. (5.46) is closed is that it is T-dual to a closed expression. Just as for the KKM, this is a result of the equations of motion in the reduced theory. Since the reduced theory is $O(2,2)$ invariant, the equations of motion must also be satisfied in the dual frame and therefore the derivative of the expression on the right hand side of (5.46) must also vanish due to the equations of motion. This may also be verified explicitly using the equations of motion in the reduced theory but the calculation is not very illuminating so we don't perform it here. The rule (5.45) determines the top form coupling to the 5^2_2-brane.

This result was predicted in [121,178,179] using different methods. Decomposing the adjoint representation of the U-duality group in terms of representations of the T-duality group the authors of [121,178,179] found more degrees of freedom than could be accounted for by just considering the metric, NSNS 2-form, the RR sector and their magnetic duals. They concluded that more than one type of magnetic dual exists for almost all fields, and this is precisely what we observe here. In [121,178,179] these different types of magnetic duals were explained with the use of mixed symmetry forms [205]. A mixed symmetry form [205–207] (see also Ref. [208] for a related review in a different context) is a tensor with two or more sets of antisymmetrized indices. For example a mixed symmetry form $B_{8,2}$ has at the same time 8 standard form indices and 2 *additional* form indices, i.e. the first 8 indices of $B_{8,2}$ are completely antisymmetric and the last two are also antisymmetric. According to [121,178,179] the brane 5^2_2 should couple to the form $B_{8,2}$. Using the Buscher rules we have shown that the 5^2_2 indeed couples to a mixed symmetry form but the two additional indices should not be thought of as form indices but appear more naturally as vector indices. For this reason our notation includes superscripts in order to emphasize that these are indeed vector indices. It should be noted though that the 10D interpretation only makes sense in the presence of two isometry directions upon which no field depends.

Using S-duality we can determine the top form coupling for the 5^2_3-brane, which again will be written in terms of a contracted 8-form with two internal vector indices,

$$\iota_y \iota_z C_8^{yz}. \tag{5.47}$$

The definition of C_8^2 is found by S-dualizing Eq. (5.46). At the linearized level this is

$$\mathrm{d}\iota_y\iota_z C_8^{yz} = \frac{\mathrm{e}^{-2\phi}}{|\tau|^3}\sqrt{\det(G_{kl})}\,\det(|\tau|G_{np} - C_{np})\,\hat{\star}_8\,\mathrm{d}\tilde{C}^{yz}.$$

We do not include more terms in this expression, since it is very sensitive to the RR sector, which we truncated away before S-duality.

5.4.4 Modified Bianchi identity

In order to determine the modified Bianchi identities as a result of having a 5^2_2 source, one considers the type IIB supergravity action in addition to the 5^2_2 coupling

$$S = S_{\text{NSNS}} + \mu_{5^2_2}\int \iota_y\iota_z B_8^{yz}, \tag{5.48}$$

where the latter integral is six-dimensional over the worldvolume of the brane and where we have only included the NSNS action

$$S_{\text{NSNS}} = \int \mathrm{d}^{10}x\ \mathrm{e}^{-2\phi}\star_{10}\Big(R + 4|\mathrm{d}\phi|^2 - \tfrac{1}{2}|H|^2\Big), \tag{5.49}$$

since we are interested in the coupling to the NSNS sector.

Varying the combined action S is not straightforward, since they are essentially written in terms of different variables. The first step is to rewrite the NSNS action in terms of B_8^2, which turns out to be possible upon performing a redefinition of the NSNS fields inspired by generalized complex geometry [110, 111]. Let us recall that the above background fields may be collected in the so-called generalized metric,

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}, \tag{5.50}$$

where G and B are the 10D NSNS metric and 2-form respectively. However, in generalized geometry this is not the most general parametrization of the generalized metric. Indeed, a more general parametrization was used for example in Ref. [209] and reads as

$$\mathcal{H} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} + g\beta \\ -g^{-1}B - \beta g & g^{-1} - \beta g\beta \end{pmatrix}. \tag{5.51}$$

In the latter expressions $\beta = \beta^{mn}\partial_m \wedge \partial_n$ is a 2-vector, which appears naturally in generalized geometry, and g and B are a pair of different metric and 2-form than those that appear in Eq.

(5.50). This expression is the most general under the assumption[13] $B\beta = \beta B = 0$. In fact, for the case under study, it is enough to set $B = 0$ and determine the background in terms of g and β only [14]. Inspired by this reparametrisation let us write

$$G = (g^{-1} - \beta g \beta)^{-1}, \qquad B = (g^{-1}(\beta)^{-1} g^{-1} - \beta)^{-1}. \tag{5.52}$$

From this we calculate [113]

$$E^{-1} = (G + B)^{-1} = g^{-1} - \beta,$$

which shows that the components of the field $\tilde{B}$, defined in Eq. (5.23) are identical to the components of the 2-vector β along the isometry directions,

$$\tilde{B}^{mn} = \beta^{mn}.$$

As discussed above β can be thought of as a 2-vector valued scalar[15] with field strength

$$Q_1^2 = \mathrm{d}\beta, \tag{5.53}$$

a 2-vector valued 1-form. The magnetic dual of β is a 2-vector valued 8-form, i.e. it is essentially the mixed symmetry field B_8^2, at least at the linearized level. Hence we denote B_8^2 as β_8^2 from now on. The latter is defined as

$$Q_9^2 = \mathrm{d}\beta_8^2 = \star_{10} \mathrm{e}^{-2\phi} \mathrm{d}\beta. \tag{5.54}$$

This equation serves also as a definition of the field strength Q_9^2.

As for the rewriting of the NSNS action, this was essentially done in Ref. [113] (further approaches include [114, 115]). The effective action is written in terms of the metric g in the β parametrization of the generalized metric $\mathcal{H}$, the corresponding modified dilaton $\tilde{\phi}$ and the field strength (5.53) of the two-vector,

$$S_{\text{NSNS}} = \int \mathrm{d}^{10}x \mathrm{e}^{-2\tilde{\phi}} \star_{10} \left(\tilde{R} + 4|\mathrm{d}\tilde{\phi}|^2 - \tfrac{1}{12} Q_M^{NR} Q^M_{NR} \right) + \int \mathrm{d}(\cdots). \tag{5.55}$$

The last term of this action is a total derivative which does not modify the equations of motion. Performing this variation, the modified Bianchi identity for Q_1^2 is determined to be[16]

$$\mathrm{d}(Q_1^{MN} \wedge g_{My} \mathrm{d}y \wedge g_{Nz} \mathrm{d}z) = j_4^{5_2^2}, \tag{5.56}$$

where $j_4^{5_2^2} = \mu_{5_2^2} \delta_4$.

It is directly observed that the presence of a 5_2^2 source turns on components of the field strength Q. This should be interpreted as a non-geometric flux in accord with the arguments of Ref. [113]. Indeed, the whole point of rewriting the NSNS action in terms of the new variables is that, unlike the standard bulk action, it is well-defined for non-geometric backgrounds. This

[13] We could easily raise this assumption but this is not essential for our present purposes. The parametrization without this assumption appears for example in Ref. [209].

[14] This was done in Ref. [113] and leads to well-defined fields for some simple cases like the one we discuss in this section. However, as pointed out in Ref. [210], it can lead to globally ill-defined metric g for more elaborate cases. In general there is some freedom in parametrizing the generalized metric in terms of g, B and β.

[15] When referring to β we will use 2-vector and 2-vector valued scalar interchangeably.

[16] Recently a slightly different Bianchi identity for the 5_2^2 brane was proposed in [211] which differs from our expression. In [211] the 5_2^2 was treated as a seven-brane in ten dimensions while we treat it as a five-brane as suggested by the analysis of [179]. In an appendix of [211] the authors compared our expression (5.58) to what they found by smearing, and found a disagreement only at the non-linear level.

is expected in view of the fact that the standard NSNS action is not duality invariant. Then the interpretation is that the legitimate form of the bulk action depends on the duality frame of the background, i.e. the standard action is valid for geometric backgrounds and the rewritten one for non-geometric ones. Note that such a correspondence was also observed in the framework of matrix model compactifications in Ref. [212].

Finally, concerning the 5^2_3-brane, a set of S-dual statements hold. The role of $\tilde{B}$ is now played by $\tilde{C}$, also a 2-vector valued scalar with field strength

$$P_1^2 = \mathrm{d}\tilde{C}. \tag{5.57}$$

Although a rewriting of the supergravity action for the RR sector is not known and generalised complex geometry is not sufficient to account for this, it is anticipated from S-duality that the corresponding Bianchi identity is

$$\mathrm{d}(P_1^{MN} \wedge g_{My}\mathrm{d}y \wedge g_{Nz}\mathrm{d}z) = j_4^{5^2_3}. \tag{5.58}$$

We will argue in Sec. 5.4.6 that this situation can be understood in the framework of extended generalised geometry. However, it should be kept in mind that a clear justification of this equation would be possible only after a successful reformulation of IIB supergravity in terms of the suitable variables. This is a task which deserves separate treatment and we will not perform it in this thesis.

5.4.5 Relation to non-geometry

The relation between exotic branes and non-geometric backgrounds was first suggested in Ref. [116,120,123], and can be understood as follows. From a lower dimensional viewpoint (e.g. in three dimensions), supergravity scalar moduli undergo monodromies when they run around exotic point particle states. These monodromies are essentially elements of the lower dimensional U-duality group (e.g. $E_{8,8}(\mathbf{Z})$ in three dimensions) which arises upon toroidal dimensional reduction of the higher dimensional theory. However, from a higher dimensional viewpoint these moduli are components of the supergravity fields (metric, NSNS and RR gauge potentials) and exotic states are co-dimension-2 extended objects. Then the lower dimensional monodromies are seen by a "higher dimensional observer" as a multivaluededness of the supergravity background. This phenomenon is one manifestation of what is usually termed non-geometry. The terminology stems from the fact that the background fields cannot be patched locally by standard geometric transition functions (diffeomorphisms for the metric, gauge transformations for the gauge potentials). In that sense they are globally ill-defined. In the brane picture, the background is mapped to a U-dual version when going around an exotic brane.

Given that non-geometry was first discussed in the context of flux compactifications, it is useful to examine the relation between the compactification picture and the brane picture. This is possible due to the fact that branes may also be considered as supergravity solutions. This is of course true for Dp-branes and NS5-branes, as well as for KKMs. Apart from these solutions, there also exist generalised KKMs [201]. The background fields for the 5^2_2-brane are given by

$$\begin{aligned} \mathrm{d}s^2 &= H(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) + HK^{-1}(\mathrm{d}y^2 + \mathrm{d}z^2) + (\mathrm{d}x^{034567})^2, \\ e^{2\phi} &= HK^{-1}, \\ B &= -K^{-1}\theta\sigma\mathrm{d}y \wedge \mathrm{d}z, \end{aligned} \tag{5.59}$$

where

$$K = H^2 + (\sigma\theta)^2, \qquad \sigma = R_yR_z,$$

H is a harmonic function and R_y, R_z are the radii of the two circles of a 2-torus. Clearly, the brane extends along 034567, the transverse corrdinates yz correspond to a 2-torus and we use polar coordinates in the directions 12 in accord with Ref. [116, 120]. The multivaluededness of this solution is manifest. Indeed, as $\theta \to \theta + 2\pi$ all fields, the metric (its components in the yz directions), the dilaton and the NSNS potential (again the yz components) are not globally well-defined up to diffeomorphisms or gauge transformations. For example, since

$$B_{yz}(r,\theta) = -\frac{\theta\sigma}{H^2 + (\theta\sigma)^2},$$

we readily obtain

$$B_{yz}(r,\theta + 2\pi) = -\frac{\theta\sigma + 2\pi\sigma}{H^2 + (\theta\sigma + 2\pi\sigma)^2} \neq B_{yz}(r,\theta) + \delta B_{yz}, \tag{5.60}$$

for any gauge transformation δB. The form of the NSNS potential is actually very suggestive. It is a more elaborate version of the usual illustrative example of a non-geometric background, where one starts with a 3-dimensional torus penetrated by NSNS flux. When two T-dualities are performed, the background fields cease to be globally well-defined. The result is what is usually termed a T-fold, because in order to patch fields on the manifold one has to use T-dualites as transition functions.

Using the background fields appearing in Eq. (5.59), it is found that the yz components of the generalized metric are given explicitly by the following 4×4 matrix,

$$\mathcal{H}_{5^2_2,(yz)} = \frac{1}{H}\begin{pmatrix} 1 & 0 & 0 & -\sigma\theta \\ 0 & 1 & \sigma\theta & 0 \\ 0 & \sigma\theta & K & 0 \\ -\sigma\theta & 0 & 0 & K \end{pmatrix}, \tag{5.61}$$

where the parametrization (5.50) was used. Note that there is nothing with a non-geometric flavour in the entries of this matrix. Non-geometry becomes apparent when the B field components are written down. However, the same generalized metric can be considered in the alternative parametrization of Eq. (5.51). In that case, the result is (see also Ref. [202]),

$$\mathrm{d}s^2_{yz} = H^{-1}(\mathrm{d}y^2 + \mathrm{d}z^2), \tag{5.62}$$

$$\beta = -\sigma\theta\partial_y \wedge \partial_z \quad \Rightarrow \quad \beta^{yz} = -\sigma\theta, \tag{5.63}$$

where we exhibit only the components in the yz directions, since the rest remains unaffected. Also, in this case $B = 0$. In generalized geometric terms this is a perfectly geometric expression under diffeomorphisms and β gauge transformations. As such, this is understood as a geometrization of the apparently non-geometric background. This geometrization is based on the fact that in generalized geometry the structure group of the generalized bundle coincides with the T-duality group. Therefore T-dualities appear on equal footing with diffeomorphisms and gauge transformations of B in this framework. From Eq. (5.53) it is found that the standard derivative of the β components is

$$Q_\theta{}^{yz} = \partial_\theta \beta^{yz}, \tag{5.64}$$

which gives the Q flux, in accord with [113].

In order to relate this solution to our previous discussion on the couplings, let us calculate the 8-form β^2_8 for this solution. The relevant equation is (5.54), and it is important to note that in the present case the RR-corrections of that equation are absent anyway, since all RR forms vanish for this background. A direct computation gives

$$\beta_8^{yz} = H\mathrm{d}x^{034567} \wedge \mathrm{d}y \wedge \mathrm{d}z. \tag{5.65}$$

Accordingly, we obtain

$$\iota_y \iota_z \beta_8^{yz} = H \mathrm{d}x^{034567}. \tag{5.66}$$

It is directly observed that this field is globally well-defined, much like its dual β. The following remarks are in order here. First of all, we observe that the expressions for β and β_8 match with the corresponding ones for B and B_6 of the NS5 solution,

$$\begin{aligned} B^{\mathrm{NS5}} &= \theta\sigma \mathrm{d}y \wedge \mathrm{d}z, \\ B_6^{\mathrm{NS5}} &= H \mathrm{d}x^{034567}. \end{aligned}$$

Secondly, we would like to stress that it does not make sense to calculate the magnetic dual, B_6, of the non-geometric B-field. Remember that B_6 is related to B with a non-local expression and therefore one might get unexpected results for B_6 when B is globally ill-defined. For example, the authors of Ref. [116, 120] compute the standard dual B_6 of B for the 5_2^2-brane and they find it ill-behaved. We have shown here that this situation is not encountered when the proper field variables, β and its magnetic dual, are used.

Let us briefly mention that a second way to understand the above issues goes through the doubled formalism. This suggests the introduction of a dual set of coordinates, say $\check{x}_M$ (which may be understood as the Fourier duals for the winding numbers of the string in the same way that standard coordinates are the Fourier duals of its momenta). T-duality exchanges coordinates and dual coordinates much like it does for momenta and windings. From this "generalized T-duality" point of view there are no conventional Buscher rules. The 5_2^2 solution, as double T-dual along y and z of the NS5, is just

$$\mathrm{d}\check{s}^2{}_{yz} = H\Big(\mathrm{d}\check{y}^2 + \mathrm{d}\check{z}^2\Big), \tag{5.67}$$

$$\check{B} = -\sigma\theta \mathrm{d}\check{y} \wedge \mathrm{d}\check{z}. \tag{5.68}$$

These expressions are well-defined in the appropriate polarization of the doubled set of coordinates in the language of Ref. [185]. We are not going to elaborate further on the treatment of the brane worldvolume actions in this framework in this thesis. However, it is useful to point out that the Eqs. (5.63) and (5.68) show the correspondence between the two approaches. When a globally ill-defined background is encountered one has to either transform to the bivector parametrization in generalized geometry or to the 2-form in dual coordinates in the doubled formalism. The components in both cases are exactly equal.

5.4.6 S-duality and RR non-geometry

Up to now we focused our attention to the 5_2^2-brane and discussed its relation to NSNS non-geometry and the Q flux. It is very interesting to note that S-duality mediates non-geometry to the RR sector as well. Indeed, the 5_3^2-brane is S-dual to the 5_2^2 one, which is a T-fold as discussed above. What is then the 5_3^2-brane? In order to answer this question let us write down explicitly the corresponding supergravity solution [201]. It reads as

$$\begin{aligned} \mathrm{d}s^2 &= (HK^{-1})^{\frac{1}{2}}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) + (HK^{-1})^{\frac{1}{2}}(\mathrm{d}y^2 + \mathrm{d}z^2) + (HK^{-1})^{-\frac{1}{2}}(\mathrm{d}x^{034567})^2, \\ e^{2\phi} &= (HK^{-1})^{-1}, \\ C_2 &= -K^{-1}\theta\sigma \mathrm{d}y \wedge \mathrm{d}z. \end{aligned} \tag{5.69}$$

It is immediately observed that this is also a non-geometric background. The main focus in this case should be on the fact that a globally ill-defined RR gauge potential is encountered. This fact barely needs any further explanation; it is just the S-dual version of the Q flux non-geometry. The B is exchanged with C_2 and now for the latter holds that

$$C_{yz}(r,\theta+2\pi) \neq C_{yz}(r,\theta) + \delta C_{yz}, \tag{5.70}$$

where a gauge transformation for C_2 is

$$\delta C_2 = \mathrm{d}\lambda_1.$$

The global issues of this solution are expected to be resolved in a similar way as previously. However, one has to implement the RR sector into the discussion and therefore generalized geometry is not sufficient anymore. Extensions of generalized geometry to account for the RR sector and the full U-duality group were suggested in Refs. [213, 214] (see also [115, 215–223]). In this exceptional (or extended) generalized geometry setting, the structure group of the generalized bundle is extended from the T-duality group to the U-duality one. In such terms, we have to consider a new 2-vector

$$\gamma = \gamma^{mn}\partial_m \wedge \partial_n$$

and the associated γ transformations. Such quantities were considered in Ref. [197] and studied further in Ref. [198] using group theoretical techniques. In the context of what we have considered in this chapter, the components of this 2-vector are equal to the ones of $\tilde{C}$, defined in (5.31) and being the S-dual of $\tilde{B}$, i.e.

$$\gamma^{mn} = \tilde{C}^{mn}.$$

The flux associated to this solution is given by the derivative of γ with respect to θ and we denote it by P, as in Refs. [197, 198],

$$P_\theta{}^{yz} = \partial_\theta \gamma^{yz}. \tag{5.71}$$

This is the RR analogue of the Q flux. They have the same index structure and number of components. As such, P is to F_3 flux what Q is to H_3 flux. This essentially augments the chain of fluxes as

$$F_{abc} \xleftrightarrow{S} H_{abc} \xleftrightarrow{T_a} f^a{}_{bc} \xleftrightarrow{T_b} Q^{ab}{}_c \xleftrightarrow{S} P^{ab}{}_c. \tag{5.72}$$

In order to avoid confusion, let us note that the middle entry f refers here to the type IIA theory.

The above discussion shows that the 5^2_3-brane is a U-fold associated to structures that appeared before in the context of exceptional generalised geometry.

5.5 Discussion

Exotic branes are very interesting BPS states of string theory originating from string dualities. They are generically heavier than the standard Dp-branes, exhibiting a wide range of power dependencies on the inverse string coupling as well as a number of NUT type transverse directions. What is more, they induce non-trivial monodromies around them, an effect that leads to interesting consequences; in particular they correspond to some of the so-called non-geometric string backgrounds. In the present chapter we studied some aspects of the physics of such exotic branes. Focusing on the type IIB superstring and in particular on the five-branes of this theory, we first revisited the worldvolume actions of the standard five-branes (the NS5-brane and the Kaluza-Klein monopole) and expressed them in a compact form. Subsequently, we derived and applied the appropriate set of duality rules (including T and S-duality) with the aim of determining the analogs of the DBI action for exotic five-branes. These actions appear in Sec. 5.3 and they can be useful in a further study of the dynamics of such branes. A very interesting question concerns the couplings of these branes and their role as sources in the type IIB theory. In Sec. 5.4 we clarified these couplings up to contributions from the RR sector. In particular, we showed that one of the exotic five-branes, the 5^2_2, couples to an exotic magnetic dual of the Kalb-Ramond field, in accord with previous expectations. Together with a rewriting of the bulk NSNS action, we utilized this coupling to derive a modified Bianchi identity. This leads to the result that the 5^2_2-brane is a source of non-geometric Q flux. The analogous treatment for the second exotic five-brane of type IIB leads to its coupling to an exotic dual of the RR 2-form and a corresponding modified Bianchi identity. The latter renders the 5^2_3-brane a source of non-geometric RR flux, which we call P. Finally, we examined the connection among the above brane picture and the more familiar picture of flux compactifications. As advocated before in the literature, exotic branes correspond to U-folds, the latter being spaces which allow patching of fields with U-duality transformations. We examined in some detail this relation from the point of view of generalized complex geometry for the 5^2_2-brane. Moreover, we discussed a similar treatment within a broader context of extended generalized geometry for its S-dual 5^2_3-brane and advocated for a relation of the latter to non-geometric RR flux. The relation between the brane picture and the flux compactification picture may be summarized in the following diagram,

$$
\begin{array}{ccccccccc}
F_{abc} & \xleftrightarrow{S} & H_{abc} & \xleftrightarrow{T_a} & f^a{}_{bc} & \xleftrightarrow{T_b} & Q^{ab}{}_c & \xleftrightarrow{S} & P^{ab}{}_c \\
\updownarrow & & \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\
\text{D5} & \xleftrightarrow{S} & \text{NS5} & \xleftrightarrow{T} & \text{KKM} & \xleftrightarrow{T} & 5^2_2 & \xleftrightarrow{S} & 5^2_3
\end{array}
$$

The upper row depicts the compactification picture with the associated fluxes in each duality frame. The lower row depicts the five-branes that we studied and their relation through dualities. Most importantly, the vertical arrows connect the corresponding entries of each row in the sense that each brane is a source of each type of flux.

Limitations. There exist a number of limitations of our results that should be well kept in mind. First of all, the gauge completion of the leading coupling in the WZ action for the exotic branes is not a trivial task. In order to perform it, one should know how the corresponding mixed symmetry fields appear in the supergravity actions. Although we took some steps in this direction, one would need a full reformulation of the standard type II supergravities in terms of these fields, which is still lacking. Secondly, the exotic branes that we considered in this chapter do not have finite energy density as single objects in flat space. This is not surprising in view of similar features associated to more standard branes. In particular, the D7-brane (also a co-dimension-2 object) exhibits the same behaviour [199]. In addition, branes such as the KKM have special NUT-like direction that require isometry and cannot be made non-compact. Exotic branes essentially carry both problematic aspects as single objects, being at the same time co-dimension-2 and having several special directions as well. Moreover, it should be mentioned that our derivations reside fully on duality rules. In this sense the worldvolume actions for all but the Dp-branes cannot be directly associated to a worldsheet computation and therefore corrections are harder to determine. Finally, we would like to remind that branes associated to the so-called R flux were not considered here (see Ref. [123] for a discussion on a possible description of such branes). Although it remains unclear to us how to treat such branes with the methods used in this chapter, the experience from flux compactifications points to co-dimension-1 branes (recall that D8 is co-dimension-1 too). These “domain wall branes” were classified in Ref. [224] and it would be interesting to examine potential relations to R-type non-geometric fluxes.

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Appendix A

Conformal transformations and compactification

Let G be a D dimensional metric of a semi-Riemannian manifold M and let ∇ be the associated Levi-Civita connection with the Christoffel symbols

$$\Gamma^M_{NR} = G^{MS}\left(\partial_{(R}G_{N)S} - \frac{1}{2}\partial_S G_{MN}\right).$$

The Riemann curvature tensor is given by

$$R^M{}_{NRS} = 2\partial_{[R}\Gamma^M_{S]N} + 2\Gamma^M_{L[R}\Gamma^L_{S]N}.$$

We can calculate the effects of conformal transformation on these expressions. We write

$$G = \mathrm{e}^{2f}\tilde{G}$$

for a conformal factor f. Then

$$\Gamma^M_{NR} = \tilde{\Gamma}^M_{NR} + 2\delta^M_{(N}\tilde{\nabla}_{R)}f - \tilde{G}_{NR}\tilde{\nabla}^M f$$

and

$$R^M{}_{NRS} = \tilde{R}^M{}_{NRS} + \delta^M_{[S}S_{R]N} - \tilde{G}_{N[S}S^M_{R]},$$

where

$$S_{MN} = 2\tilde{\nabla}_M\tilde{\nabla}_N f - 2\tilde{\nabla}_M f\tilde{\nabla}_N f + \tilde{G}_{MN}(\tilde{\nabla}f)^2.$$

The Ricci scalar takes the form

$$R = \mathrm{e}^{-2f}\left\{\tilde{R} + (1-D)S^M_M\right\} = \mathrm{e}^{-2f}\left\{\tilde{R} + (1-D)\left[2\tilde{\nabla}^2 f + (D-2)(\tilde{\nabla}f)^2\right]\right\}.$$

We will now consider the metric

$$\mathrm{d}s^2 = G_{MN}(X)\mathrm{d}X^M\mathrm{d}X^N = \mathrm{e}^{2f(y)}\tilde{g}_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + \tilde{G}_{mn}(y)\mathrm{d}y^m\mathrm{d}y^n$$

where greek letters $\mu, \nu, \ldots$ run from 0 to $d-1$, where d is the number of external space-time dimensions, and latin letters $m, n, \ldots$ run from d to $D-1$. This yields the non-zero components of the curvature tensor

$$\begin{aligned}
R_{\mu\nu\rho\sigma} &= \mathrm{e}^{2f}\tilde{R}_{\mu\nu\rho\sigma} - 2\mathrm{e}^{4f}\tilde{g}_{\mu[\rho}\tilde{g}_{\sigma]\nu}(\tilde{\nabla}f)^2, \\
R_{mnrs} &= \tilde{R}_{mnrs}, \\
R_{\mu n\rho s} &= -\mathrm{e}^{2f}\tilde{g}_{\mu\rho}(\tilde{\nabla}_s\tilde{\nabla}_n f + \tilde{\nabla}_s f\tilde{\nabla}_n f).
\end{aligned}$$

The curvature scalar is determined to be

$$R_D = \mathrm{e}^{-2f}\tilde{R}_d + \tilde{R}_{D-d} - d(d+1)(\tilde{\nabla}f)^2 - 2d\tilde{\nabla}^2 f \tag{A.1}$$

We now include the components of the three-form H as torsion in the D dimensional theory, we construct the torsionful Christoffel symbols, schematically

$$\Gamma_+ = \Gamma - \frac{1}{2}H$$

where one index of H must be raised with the metric G^{MN}. If we assume that the non-zero components of H have legs in the internal direction and furthermore that they only depend on the internal coordinates y, we can write down the non-zero components of the curvature tensor R^+

$$\begin{aligned}
R^+_{\mu\nu\rho\sigma} &= \mathrm{e}^{2f}\tilde{R}_{\mu\nu\rho\sigma} - 2\mathrm{e}^{4f}\tilde{g}_{\mu[\rho}\tilde{g}_{\sigma]\nu}(\tilde{\nabla}f)^2,\\
R^+_{mnrs} &= \tilde{R}^+_{mnrs},\\
R^+_{\mu n\rho s} &= -\mathrm{e}^{2f}\tilde{g}_{\mu\rho}(\tilde{\nabla}^+_s\nabla_n f + \tilde{\nabla}_s f\tilde{\nabla}_n f) = -\mathrm{e}^{2f}\tilde{g}_{\mu\rho}(\tilde{\nabla}_s\tilde{\nabla}_n f - H_{msn}\tilde{\nabla}^m f + \tilde{\nabla}_s f\tilde{\nabla}_n f).
\end{aligned}$$

We are interested in $\mathrm{tr}|R^+|^2 = \frac{1}{2}R^+_{MNRS}R^{+MNRS}$ which appears in the first α' correction of the low energy effective action for the heterotic string 2.4, using the above we find

$$\begin{aligned}
\mathrm{tr}|R^+| &= \mathrm{e}^{-4f}\mathrm{tr}|\tilde{R}|^2_d - 2\mathrm{e}^{-2f}\tilde{R}_d(\tilde{\nabla}f)^2 + d(d-1)\left[(\tilde{\nabla}f)^2\right]^2 + \mathrm{tr}|\tilde{R}^+|^2_{D-d}\\
&+ 4d(\tilde{\nabla}_s\tilde{\nabla}_n f - H_{msn}\tilde{\nabla}^m f + \tilde{\nabla}_s f\tilde{\nabla}_n f)(\tilde{\nabla}^s\tilde{\nabla}^n f - H^{msn}\tilde{\nabla}_m f + \tilde{\nabla}^s f\tilde{\nabla}^n f)\\
&= \mathrm{e}^{-4f}\mathrm{tr}|\tilde{R}|^2_d - 2\mathrm{e}^{-2f}\tilde{R}_d(\tilde{\nabla}f)^2 + |\tilde{R}^+_{D-d}|^2,
\end{aligned} \tag{A.2}$$

where we have separated off the terms in the tilded frame that only depend on fields with internal indices.

Let us now use these formulae for the case of interest for the 4D effective field theory of the heterotic string (cf. Sec. 2.1)

$$\mathrm{d}s^2 = \frac{\mathrm{e}^{2A}}{\mathcal{V}\tau^2}\tilde{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}s^2_6. \tag{A.3}$$

Using the above formulae (A.1,A.2) we find

$$R = \mathcal{V}\tau^2\mathrm{e}^{-2A}\tilde{R}_4 + \tilde{R}_6 - 20(\partial A)^2 - 8\nabla^2 A, \tag{A.4}$$

$$\mathrm{tr}|R^+|^2 = \mathcal{V}^2\tau^4\mathrm{e}^{-4A}\mathrm{tr}|\tilde{R}|^2_4 - 2\mathcal{V}\tau^2\mathrm{e}^{-2A}\tilde{R}_4(\partial A)^2 + |R^+_6|^2. \tag{A.5}$$

As before, we introduced the shorthand notation $|R^+_6|^2 = 12\left[(\partial A)^2\right]^2 + 4|R^+_{\mu n\rho s}|^2 + |R^+_{mnrs}|^2$ to subsume all terms which in the tilded frame only depend on internal fields.

Appendix B

Leading order constraints on heterotic supergravity

In [149], it was suggested that heterotic supergravity with leading α'-corrections could have solutions with a cosmological constant of the form

$$\Lambda = -\alpha' C + \mathcal{O}(\alpha'^2), \tag{B.1}$$

where C is a non-negative constant given by

$$C = \frac{1}{2\mathcal{V}'}\int \mathrm{d}^6 y\sqrt{\tilde{g}_6}\,\mathrm{e}^{6A-\frac{\phi}{2}}\left\{3\left[(\partial\omega)^2\right]^2 + 2|(\partial_m\omega)(\partial_n\omega) - \tilde{\nabla}_m\partial_n\omega - \tilde{g}_{mn}(\partial\omega)^2|^2 \right.$$
$$\left. + \frac{1}{2}\mathrm{e}^{-4\omega}|H_{mn}{}^{l}\partial_l\omega|^2\right\} \tag{B.2}$$

with $\mathcal{V}' = \int \mathrm{d}^6 y\sqrt{\tilde{g}_6}\,\mathrm{e}^{8A}$ and $\omega = A + \frac{\phi}{4}$. We will now show explicitly, using arguments similar to [33, 225, 226], that all terms in C vanish up to higher order α'-corrections due to the leading order equations of motion. The result of [149] is therefore not in conflict with the argument given in Sec. 2.1.1.

To omit confusion, we will stick to the metric conventions of [149] in this appendix, which differ from those used in the main text of our thesis. The unwarped metric is then defined as $\tilde{g}_{MN} = \mathrm{e}^{-2A} g_{MN}$, where g_{MN} is the usual ten-dimensional Einstein frame metric. In the following, terms are always contracted with the Einstein frame metric, except for tilded objects and all terms in (B.2), which are contracted with the unwarped metric $\tilde{g}_{MN}$.

The leading order dilaton equation in Einstein frame reads

$$\nabla_M\partial^M\phi + \frac{1}{2}\mathrm{e}^{-\phi}|H|^2 = \mathcal{O}(\alpha'). \tag{B.3}$$

Assuming that the dilaton only depends on the internal coordinates, we can write $\nabla_M\partial^M\phi = \mathrm{e}^{-10A}\tilde{\nabla}_m \mathrm{e}^{8A}\tilde{g}^{mn}\partial_n\phi$ and integrate over internal space to find

$$\frac{1}{2}\int \mathrm{d}^6 y\sqrt{\tilde{g}_6}\,\mathrm{e}^{10A-\phi}|H|^2 = \mathcal{O}(\alpha') \tag{B.4}$$

and hence

$$\mathrm{e}^{10A-\phi}|H|^2 = \mathcal{O}(\alpha'). \tag{B.5}$$

The traced internal and space-time components of the leading order Einstein equation then read

$$-R_4 - 2R_6 + (\partial\phi)^2 = \mathcal{O}(\alpha'), \qquad -3R_4 - 2R_6 + (\partial\phi)^2 = \mathcal{O}(\alpha'). \tag{B.6}$$

Combining the two equations and rewriting R_4 in terms of the unwarped metric yields

$$R_4 = \mathrm{e}^{-2A}\tilde{R}_4 - \frac{1}{2}\mathrm{e}^{-10A}\tilde{\nabla}^2\mathrm{e}^{8A} = \mathcal{O}(\alpha'). \tag{B.7}$$

We can now integrate over internal space to find $\mathrm{e}^{8A}\tilde{R}_4 = \mathcal{O}(\alpha')$ which with (B.7) implies that $\tilde{\nabla}^2\mathrm{e}^{8A} = \mathcal{O}(\alpha')$. Hence the warp factor is a constant up to α'-corrections. The dilaton equation (B.3) then reduces to $\mathrm{e}^{-2A}\tilde{\nabla}^2\phi = \mathcal{O}(\alpha')$ and therefore also ϕ is a constant up to α'-corrections.

We have thus shown that two-derivative terms involving the warp factor or the dilaton are at least of order $\mathcal{O}(\alpha')$, which implies that the four-derivative terms appearing in (B.2) are of even higher order. It follows that $C = \mathcal{O}(\alpha')$, and hence (B.1) yields

$$\Lambda = \mathcal{O}(\alpha'^2). \tag{B.8}$$

Appendix C

Alternative derivation of Eq. (4.39)

Here we present an alternative derivation of our main result (4.39), which only uses the equations of motion. We first consider the Bianchi identity (1.8) for the internal RR field strength F^{int}_{8-p} and multiply by $\sigma(C^{\text{ext}}_{p+1})$,

$$\begin{aligned}
0 &= -\sigma(C^{\text{ext}}_{p+1}) \wedge \left\langle \mathrm{d}_{-H}\mathbf{F}^{\text{int}} + \mathbf{j} \right\rangle_{9-p} \\
&= -\kappa \mathrm{d}\left[\sigma(C^{\text{ext}}_{p+1}) \wedge F^{\text{int}}_{8-p}\right] + \langle \sigma(\mathrm{d}_{-H}\mathbf{C}^{\text{ext}})\rangle_{p+2} \wedge F^{\text{int}}_{8-p} + \sigma(H \wedge C^{\text{ext}}_{p-1}) \wedge F^{\text{int}}_{8-p} \\
&\quad +\sigma(C^{\text{ext}}_{p+1}) \wedge H \wedge F^{\text{int}}_{6-p} - \sigma(C^{\text{ext}}_{p+1}) \wedge j_{9-p} \\
&= -\kappa \mathrm{d}\left[\sigma(C^{\text{ext}}_{p+1}) \wedge F^{\text{int}}_{8-p}\right] + \sigma \left\langle \mathbf{F}^{\text{ext}} - \mathrm{e}^{B} \wedge \mathbf{F}^{b} \right\rangle_{p+2} \wedge F^{\text{int}}_{8-p} - \sigma(F^{\text{int}}_{8-p}) \wedge H \wedge C^{\text{ext}}_{p-1} \\
&\quad +\sigma(F^{\text{int}}_{6-p}) \wedge H \wedge C^{\text{ext}}_{p+1} + \sigma(j_{9-p}) \wedge C^{\text{ext}}_{p+1} \\
&= -\kappa \mathrm{d}\left[\sigma(C^{\text{ext}}_{p+1}) \wedge F^{\text{int}}_{8-p}\right] - \mathrm{e}^{(p-3)\phi/2} \star_{10} |F^{\text{int}}_{8-p}|^2 + \sigma(F^{\text{int}}_{8-p}) \wedge \left\langle \mathrm{e}^{B} \wedge \mathbf{F}^{b} \right\rangle_{p+2} \\
&\quad +\kappa H \wedge \sigma(F^{\text{int}}_{8-p}) \wedge C^{\text{ext}}_{p-1} - \kappa H \wedge \sigma(F^{\text{int}}_{6-p}) \wedge C^{\text{ext}}_{p+1} - \sigma(j_{9-p}) \wedge C^{\text{ext}}_{p+1}.
\end{aligned} \tag{C.1}$$

Here we have introduced the constant κ which equals $+1$ for type IIA and -1 for type IIB supergravity. Multiplying H equation of motion (4.12) by B yields

$$\begin{aligned}
0 &= 2B \wedge \mathrm{d}\left(\mathrm{e}^{-\phi} \star_{10} H\right) + \langle B \wedge \sigma(\mathbf{F}) \wedge \mathbf{F}\rangle_{10} \\
&= 2B \wedge \mathrm{d}\left(\mathrm{e}^{-\phi} \star_{10} H\right) + 2\left\langle B \wedge \sigma(\mathbf{F}^{\text{int}}) \wedge \mathbf{F}^{\text{ext}} \right\rangle_{10} \\
&= 2\mathrm{d}\left\langle \mathrm{e}^{-\phi} B \wedge \star_{10} H - \kappa B \wedge \sigma(\mathbf{F}^{\text{int}}) \wedge \mathbf{C}^{\text{ext}} \right\rangle_{9} - 2(H - H^{b}) \wedge \left(\mathrm{e}^{-\phi} \star_{10} H\right) \\
&\quad -2\left\langle -\kappa \mathrm{d}_{H}(B \wedge \sigma(\mathbf{F}^{\text{int}})) \wedge \mathbf{C}^{\text{ext}} + B \wedge \sigma(\mathbf{F}^{\text{int}}) \wedge \mathrm{e}^{B} \wedge \mathbf{F}^{b} \right\rangle_{10} \\
&= 2\mathrm{d}\left\langle \mathrm{e}^{-\phi} B \wedge \star_{10} H - \kappa B \wedge \sigma(\mathbf{F}^{\text{int}}) \wedge \mathbf{C}^{\text{ext}} \right\rangle_{9} - 2(H - H^{b}) \wedge \left(\mathrm{e}^{-\phi} \star_{10} H - \kappa \left\langle \sigma(\mathbf{F}^{\text{int}}) \wedge \mathbf{C}^{\text{ext}} \right\rangle_{7}\right) \\
&\quad -2\left\langle -B \wedge \sigma(\mathbf{j}) \wedge \mathbf{C}^{\text{ext}} + B \wedge \sigma(\mathbf{F}^{\text{int}}) \wedge \mathrm{e}^{B} \wedge \mathbf{F}^{b} \right\rangle_{10}.
\end{aligned} \tag{C.2}$$

Notice in above equation that F^{int}_{6} never appears since $\mathbf{F}^{\text{int}}$ is everywhere multiplied by either B or H, which must both be purely internal in a maximally symmetric compactification to $d \geq 4$ dimensions. We now take the combination $(1 + (p-3)c/2)$ times (C.1) plus $c/2$ times (C.2) and sum over p. Substituting the definition of $\mathbf{j}$ from Sec. 1.4, this yields

$$\begin{aligned} 0 &= \sum_{3\le p}\left(1+\frac{p-3}{2}c\right)\left\{-\mathrm{e}^{(p-3)\phi/2}\star_{10}|F_{8-p}^{\mathrm{int}}|^2-\sigma(j_{9-p})\wedge C_{p+1}^{\mathrm{ext}}\right\} \\ &\quad +c\left\langle \mathrm{e}^{-\phi}\star_{10}|H|^2+B\wedge\sigma(\mathbf{j})\wedge\mathbf{C}^{\mathrm{ext}}\right\rangle_{10}-\Sigma(c)+\text{total derivatives} \\ &= \sum_{3\le p}\left(1+\frac{p-3}{2}c\right)\left(-\mathrm{e}^{(p-3)\phi/2}\star_{10}|F_{8-p}^{\mathrm{int}}|^2-\mathcal{L}_{\mathrm{WZ}}^{(p)}\right) \\ &\quad +c\,\mathrm{e}^{-\phi}\star_{10}|H|^2-\Sigma(c)+\text{total derivatives}, \end{aligned} \tag{C.3}$$

where c is a free parameter. We also introduced the shorthand

$$\begin{aligned} \Sigma(c) &= -\sum_{2\le p}\left(1+\frac{p-3}{2}c\right)\langle \mathrm{e}^B\wedge\sigma(\mathbf{F}^{\mathrm{int}})\rangle_{8-p}\wedge F_{p+2}^b+\left(1-\frac{1}{2}c\right)\sigma(F_6^{\mathrm{int}})\wedge F_4^b \\ &\quad -cH^b\wedge\left(\mathrm{e}^{-\phi}\star H-\kappa\langle\sigma(\mathbf{F}^{\mathrm{int}})\wedge\mathbf{C}^{\mathrm{ext}}\rangle_7\right), \end{aligned} \tag{C.4}$$

where we have combined all terms that depend on background fluxes to simplify our notation.

The trace of the external components of the (trace-reversed) Einstein equation reads

$$\frac{4}{d}R_d=-\frac{1}{2}\mathrm{e}^{-\phi}|H|^2+\sum_{3\le p}\frac{p-7}{4}\left(\mathrm{e}^{(p-3)\phi/2}|F_{8-p}^{\mathrm{int}}|^2\pm\mu_p\mathrm{e}^{(p-3)\phi/4}\delta(\Sigma)\right)+\frac{5}{4}\mathrm{e}^{\phi/2}|F_4^{\mathrm{ext}}|^2, \tag{C.5}$$

where the upper sign is for D-branes and the lower sign for O-planes and we have used $|F_5^{\mathrm{ext}}|^2=-|F_5^{\mathrm{int}}|^2$ to rewrite the space-time-filling part of $|F_5|^2$. Note that space-time-filling F_4 flux can only be present for $d=4$ in type IIA supergravity, while F_5 flux can be present for $d=4$ or $d=5$ in type IIB supergravity.

The dilaton equation takes the form

$$0=-\nabla^2\phi-\frac{1}{2}\mathrm{e}^{-\phi}|H|^2+\sum_{3\le p}\frac{p-3}{4}\left(\mathrm{e}^{(p-3)\phi/2}|F_{8-p}^{\mathrm{int}}|^2\pm\mu_p\mathrm{e}^{(p-3)\phi/4}\delta(\Sigma)\right)+\frac{1}{4}\mathrm{e}^{\phi/2}|F_4^{\mathrm{ext}}|^2. \tag{C.6}$$

Combining (C.5) and (C.6), we find

$$\begin{aligned} \frac{4}{d}R_d &= c\,\mathrm{e}^{-\phi}|H|^2+\sum_{3\le p}\left(1+\frac{p-3}{2}c\right)\left(-\mathrm{e}^{(p-3)\phi/2}|F_{8-p}^{\mathrm{int}}|^2\mp\mu_p\mathrm{e}^{(p-3)\phi/4}\delta(\Sigma)\right) \\ &\quad +\left(1-\frac{c}{2}\right)\mathrm{e}^{\phi/2}|F_4^{\mathrm{ext}}|^2+\text{total derivatives}. \end{aligned} \tag{C.7}$$

Finally, we can combine (C.7) with (C.3) to get

$$\begin{aligned} \frac{4}{d}\star_{10}R_d &= \sum_{3\le p}\left(1+\frac{p-3}{2}c\right)\left(\mp\star_{10}\mu_p\,\mathrm{e}^{(p-3)\phi/4}\delta(\Sigma)+\mathcal{L}_{\mathrm{WZ}}^{(p)}\right)+\mathcal{F}(c) \\ &\quad +\text{total derivatives}, \end{aligned} \tag{C.8}$$

where we defined

$$\mathcal{F}(c)=\Sigma(c)-\left(1-\frac{c}{2}\right)\sigma(F_6^{\mathrm{int}})\wedge F_4^b \tag{C.9}$$

and used $\mathrm{e}^{\phi/2}\star_{10}F_4^{\mathrm{ext}}=-\sigma(F_6^{\mathrm{int}})$, which follows from the duality relations (4.4). Integrating over ten-dimensional space and using (4.9), we get rid of all total derivative terms and find

$$\frac{8v\mathcal{V}}{d-2}\Lambda=\sum_p\left(1+\frac{p-3}{2}c\right)\left[S_{\mathrm{DBI}}^{(p)}+S_{\mathrm{WZ}}^{(p)}\right]+\int\mathcal{F}(c), \tag{C.10}$$

with the volume factors v and $\mathcal{V}$ defined as in (4.11).

Appendix D

Gauge transformations and S-duality in type IIB

Let us consider type IIB supergravity. It contains the Kalb-Ramond potential B with field strength

$$dH = 0 \Rightarrow H = dB \tag{D.1}$$

and the Ramond-Ramond potential C_0, C_2, C_4, C_6, C_8 conveniently packed in $\mathbf{C}$. The equations of the theory may be written as Bianchi identities. In the conventions we use, the equations for the field strength take the form

$$\begin{aligned} d\mathbf{F} &= H \wedge \mathbf{F}, \\ dH_7 &= -\frac{1}{2} \langle \sigma(\mathbf{F}) \wedge \mathbf{F} \rangle_8 = -F_1 \wedge F_7 + F_3 \wedge F_5. \end{aligned}$$

In addition, these field strengths must be gauge invariant. This property allows us to determine the gauge transformation rules of the corresponding gauge potentials. In the simplest case of the Kalb-Ramond field, gauge invariance gives

$$\delta H = 0 \Rightarrow \delta B = d\Lambda_1, \tag{D.2}$$

where Λ_1 is the gauge transformation parameter and it is an 1-form. In general, we denote gauge parameters as Λ_p and λ_p, where the capital ones are reserved for B and its magnetic dual and lower case ones are used for the RR potentials. The index p declares the degree of the form.

The field strength of C_0 is

$$F_1 = dC_0 \tag{D.3}$$

and gauge invariance gives

$$\delta F_1 = 0 \Rightarrow \delta C_0 = 0. \tag{D.4}$$

The equation for the field strength F_3 is solved by

$$F_3 = dC_2 - C_0 H, \tag{D.5}$$

and therefore

$$\delta F_3 = 0 \Rightarrow d(\delta C_2) = 0 \Rightarrow \delta C_2 = d\lambda_1. \tag{D.6}$$

Moreover, the equation for F_5 may be solved as

$$F_5 = dC_4 - C_2 \wedge H, \tag{D.7}$$

which means that

$$\delta F_5 = 0 \Rightarrow \mathrm{d}(\delta C_4) - \mathrm{d}\lambda_1 \wedge H = 0 \Rightarrow \delta C_4 = \mathrm{d}\lambda_3 + \mathrm{d}\lambda_1 \wedge B. \tag{D.8}$$

Of course, there is no unique way to solve the corresponding identity. We could have solved it instead as

$$F_5 = \mathrm{d}C_4' - \tfrac{1}{2}C_2 \wedge H + \tfrac{1}{2}B \wedge \mathrm{d}C_2. \tag{D.9}$$

This is for example the definition used in Ref. [21]. In this case we obtain

$$\delta F_5 = 0 \Rightarrow \delta C_4' = d\lambda_3 + \tfrac{1}{2}\mathrm{d}\lambda_1 \wedge B - \tfrac{1}{2}\mathrm{d}\Lambda_1 \wedge C_2. \tag{D.10}$$

This is in agreement with the conventions of Ref. [191] and moreover it has a flavour of S-duality, unlike the previous expression. Of course, the choices are equivalent, since they both solve the same Bianchi identity. In this thesis we are going to use the former choice.

Finally, we need the gauge transformation of C_6, which is the form that couples (electrically) to the D5-brane. Its curvature may be defined as

$$F_7 = \mathrm{d}C_6 - C_4 \wedge H \tag{D.11}$$

or

$$F_7 = \mathrm{d}C_6' - C_4' \wedge H + \tfrac{1}{4}B \wedge B \wedge \mathrm{d}C_2, \tag{D.12}$$

which are both consistent with the corresponding Bianchi identity, the latter being consistent when F_5 is defined through the expression (D.9). As before, we are using the former expression. This leads to the following gauge transformation,

$$\delta C_6 = \mathrm{d}\lambda_5 + \mathrm{d}\lambda_3 \wedge B + \frac{1}{2}\mathrm{d}\lambda_1 \wedge B \wedge B. \tag{D.13}$$

What remains is the gauge transformation of the field B_6, the magnetic dual of the Kalb-Ramond potential B. This transformation is necessary in several instances in the main text. It can be determined from the equation of motion for H in the type IIB superstring, which translates into a Bianchi identity for H_7

$$\mathrm{d}H_7 + F_1 \wedge F_7 - F_3 \wedge F_5 = 0. \tag{D.14}$$

This equation is solved by the following field strength,

$$H_7 = \mathrm{d}B_6 - C_4 \wedge \mathrm{d}C_2 - \tfrac{1}{2}C_2 \wedge C_2 \wedge H - C_0 F_7. \tag{D.15}$$

Subsequently, imposing the gauge invariance of H_7,

$$\delta H_7 = 0,$$

we determine the gauge transformation of the B_6 to be

$$\delta B_6 = \mathrm{d}\Lambda_5 + \mathrm{d}\lambda_3 \wedge C_2 + \mathrm{d}\lambda_1 \wedge B \wedge C_2, \tag{D.16}$$

where Λ_5 is a 5-form gauge parameter.

For completeness we list here the S-duality rules using the above conventions, however we omit the S-duality relation for C_8 since we do not use it in this thesis. These relations read as follows,

$$
\begin{aligned}
G &\stackrel{S}{\longmapsto} |\tau| G, \\
\tau &\stackrel{S}{\longmapsto} -\frac{1}{\tau}, \\
C_2 &\stackrel{S}{\longmapsto} B, \\
B &\stackrel{S}{\longmapsto} -C_2, \\
C_4 &\stackrel{S}{\longmapsto} C_4 - C_2 \wedge B, \\
C_6 &\stackrel{S}{\longmapsto} -B_6 + \frac{1}{2} B \wedge C_2 \wedge C_2, \\
B_6 &\stackrel{S}{\longmapsto} C_6 - \frac{1}{2} C_2 \wedge B \wedge B,
\end{aligned}
\tag{D.17}
$$

where $\tau = C_0 + i\mathrm{e}^{-\phi}$ is the type IIB axio-dilaton. Using these rules we can easily write down the mapping of the polyform $\mathcal{C}$ under S-duality,

$$
\mathbf{C} \stackrel{S}{\longmapsto} -\mathbf{B} = -\frac{C_0}{|\tau|^2} + B + (C_4 - C_2 \wedge B) + \left(-B_6 + \frac{1}{2} B \wedge C_2 \wedge C_2\right) + \tilde{C}_8
$$

where $\tilde{C}_8$ is the S-dual of C_8. This should be inserted in to Eq. (5.12) to give the correct WZ terms for the NS5-brane. One can then directly check the gauge invariance of Eq. (5.12) using the gauge transformations (D.2,D.4,D.6,D.8,D.16).

Appendix E

Reduced type II action and magnetic duals

For the ansatz (5.15,5.18)

$$\begin{aligned} ds^2 &= \hat{G}_{\mu\nu}dx^\mu dx^\nu + G_{mn}\eta^m\eta^n, \\ B &= \hat{B} + (\eta^m - \frac{1}{2}A^m)\wedge\theta_m + \frac{1}{2}B_{mn}\eta^m\wedge\eta^n, \\ e^{2\phi} &= \sqrt{\det(G_{mn})}\, e^{2\hat{\phi}}, \end{aligned} \tag{E.1}$$

the NSNS action

$$S_{\rm NSNS} = \int e^{-2\phi} \star_{10}\left(R + 4|d\phi|^2 - \frac{1}{2}|H|^2\right)$$

reduces to [204, 227, 228]

$$\begin{aligned} S_{\rm NSNS} &= {\rm Vol}_{10-d}\int e^{-2\hat{\phi}}\Big(\hat{\star}_d\,\hat{R} + 4d\hat{\phi}\wedge\hat{\star}_d\,d\hat{\phi} - \frac{1}{2}\hat{H}\wedge\hat{\star}_d\,\hat{H} \\ &\quad + \frac{1}{8}{\rm tr}(dM\wedge\hat{\star}_d\,dM^{-1}) - \frac{1}{2}M_{IJ}d\mathcal{A}^I\wedge\hat{\star}_d\,d\mathcal{A}^J\Big) \end{aligned} \tag{E.2}$$

where d denotes the dimension of the space which we reduce to,

$$M_{IJ} = \begin{pmatrix} G_{mn} - B_{mm'}G^{m'n'}B_{n'n} & B_{mm'}G^{m'n} \\ -G^{mm'}B_{m'n} & G^{mn} \end{pmatrix}$$

is the scalar moduli matrix, I, J are $O(d,d)$ indices and

$$\mathcal{A}^I = \begin{pmatrix} A^m \\ -\theta_m \end{pmatrix}.$$

Then $\hat{H}$ can be neatly expressed as

$$\hat{H} = d\hat{B} - \frac{1}{2}L_{IJ}\mathcal{A}^I\wedge d\mathcal{A}^J, \qquad L_{IJ} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

We have also used simple decomposition of the Hodge star, for example when $d = 9$ we get for a p-form $\omega_p = \bar{\omega}_p + \omega_{p-1}\wedge\eta^z$ (we let z label the isometry direction, the 9-th coordinate,)

$$\begin{aligned} \star\,\bar{\omega}_p &= \sqrt{\det(G_{mn})}\;\hat{\star}\bar{\omega}_p\wedge\eta^z \\ \star(\omega_{p-1}\wedge\eta^z) &= (-1)^p\sqrt{\det(G_{mn})}\;G^{zz}\;\hat{\star}\omega_{p-1}, \end{aligned}$$

where $\hat{\star}$ is the reduced Hodge star operator associated with the metric $\hat{G}$.

Variation of the action (E.2) with respect to $\mathcal{A}$ gives the equations of motion

$$\mathrm{d}\left(\mathrm{e}^{-2\hat{\phi}} M_{IJ}\ \hat{\star}_d\ \mathrm{d}\mathcal{A}^I\right) = -L_{IJ}\mathrm{d}\mathcal{A}^I \wedge \mathrm{e}^{-2\hat{\phi}}\ \hat{\star}_d\ \hat{H}. \tag{E.3}$$

Here we have used the $\hat{B}$ equation of motion

$$\mathrm{d}\left(\mathrm{e}^{-2\hat{\phi}}\hat{\star}_d\hat{H}\right) = 0. \tag{E.4}$$

The magnetic dual of $\mathcal{A}$ can now be consistently defined as

$$\mathrm{d}\mathcal{A}_{J,7-d} = \mathrm{e}^{-2\hat{\phi}} M_{IJ}\ \hat{\star}_d\ \mathrm{d}\mathcal{A}^I + L_{IJ}\mathcal{A}^I \wedge \mathrm{e}^{-2\hat{\phi}}\ \hat{\star}_d\ \hat{H}. \tag{E.5}$$

We are interested in the nine dimensional case where we encounter the magnetic dual of A^m which couples to the KKM. In that case M_{IJ} takes a diagonal form and we get

$$\begin{aligned} \mathrm{d}A_6^z &= \mathrm{e}^{-2\hat{\phi}} G_{zz}\ \hat{\star}_9\ \mathrm{d}A^z - \theta_z \wedge \iota_z \mathrm{d}B_6, \\ \mathrm{d}\theta_{z,6} &= \mathrm{e}^{-2\hat{\phi}} G^{zz}\ \hat{\star}_9\ \mathrm{d}\theta_z - A^z \wedge \iota_z \mathrm{d}B_6, \end{aligned}$$

where B_6 is the ten-dimensional magnetic dual of B. We can directly relate the expression for $\mathrm{d}\theta_{z,6}$ to the ten-dimensional field B_6

$$\mathrm{d}\theta_{z,6} = \mathrm{d}(1 - P_z)B_6.$$

The expression for $\mathrm{d}A_6^z$ should also be related directly to a ten-dimensional field, but this will not be a 6-form but the 7-form A_7^z which is the ten-dimensional magnetic dual of A^z,

$$\mathrm{d}A_6^z = \mathrm{d}\iota_z A_7^z. \tag{E.6}$$

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