Essays on Collateralized Debt Obligations and Credit Default Swaps:
Dynamic Correlation Modeling, Measuring Systematic Risk, and Cross-sectional Pricing of Common Risks

Von der Wirtschaftswissenschaftlichen Fakultät der
Gottfried Wilhelm Leibniz Universität Hannover
zur Erlangung des akademischen Grades

Doktor der Wirtschaftswissenschaften
- Doctor rerum politicarum -

genehmigte Dissertation
von

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geboren am 01.06.1979 in Hildesheim

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Tag der Promotion: 14.05.2013

Abstract

Measuring and anticipating systematic risk in credit markets appropriately are currently the core challenges for many researchers, investors and regulatory authorities around the world, especially with regard to structured finance products such as collateralized debt obligations (CDO). The development of a comprehensive framework for evaluating the risk characteristics of CDOs due to systematic risk may also allow to identify and to measure pricing impacts of systematic risk on both structured securities and credit default swaps (CDS). In credit markets, CDS contracts represent the most common credit derivatives and they also constitute the collateral in synthetic asset securitizations.

This cumulative thesis confirms the relevance of structured finance products such as asset securitizations in financial markets and provides an analytical framework for obtaining detailed insights into their complex risk characteristics due to systematic risk. With regard to systematic influences, this analytical framework allows a closed-form comparison of comparably rated bonds and securitized tranches in terms of their default risk and related losses. The analytical results are backed by several Monte Carlo simulations. Due to the higher exposure to systematic risk, securitized tranches react much more sensitive to changes in the macroeconomic climate – as source for systematic risk – than corporate bonds, particularly in economic downturns. Based on the core characteristics of asset securitizations namely pooling and tranching, effects of both risk diversification and risk concentration are examined indicating the product-specific risk profiles. While idiosyncratic risk and sectoral risk may be diversified in securitizations, pooling and tranching also lead to the concentration of systematic risk. The corresponding effect sizes strongly depend on tranche seniorities. The results also indicate that classical credit ratings are insufficient metrics for measuring the entire risk of structured securities, particularly with respect to high-rated, e.g., AAA-rated, tranches.

Correlations, which usually describe the dependency structures within credit-risky portfolios, are identified as the main drivers for credit portfolio risk and constitute a major element in pricing portfolio credit derivatives such as synthetic single-tranche CDO swaps (STCDO). In the standard single-factor Gaussian copula model for pricing STCDOs, benefits of historical asset correlations gained from stock market returns are strongly limited for pricing securitized tranches of the 5-year iTraxx Europe credit index. Two alternative spread-dependent correlation skew models are proposed to model and to forecast implied correlations of iTraxx Europe index tranches from 2005 to 2008. The applied panel regressions show that the proposed dynamic mixed-effects regression correlation model (MERM) reaches highest forecast accuracy by accounting for i) random time-specific effects and ii) tranche-specific fixed effects on implied tranche correlations. The model-based forecast accuracy is measured in terms of root mean squared forecast errors. The empirical findings also indicate the presence of a systematic risk factor influencing all spreads of the index tranches simultaneously.

Quoted CDS spreads from 2004 to 2010 of 339 U.S. entities divided across ten economic sectors are used in order to examine whether common risk factors are priced in the cross-section of CDS spreads. By using two-pass regressions, the credit market climate, the cross-market correlation and the market volatility are identified as systematic risk factors simultaneously affecting the cross-section of CDS spreads, particularly in times of financial distress and even in the presence of idiosyncratic or firm-specific risk factors such as credit ratings, liquidity and leverage. Since swap contracts not only exhibit different sensitivities to systematic risk by rating class, but also within each rating class, the need for appropriate systematic risk measures is underlined. The proposed basic set of risk factors explains about 83% of the CDS spreads prior to the global financial crisis and about 90% during the crisis. The applied approach allows to identify contract-specific sensitivities to systematic risk and to calculate related premium payments. It may also facilitate the development of a risk-adjusted valuation framework for CDOs, particularly with respect to systematic risk.

The findings of this thesis are addressed to several interest groups, e.g, other researchers in the field of credit risk or derivatives, investors dealing with securitized tranches or swap contracts, risk managers in banks or insurance companies engaged in the management of credit risk, and regulatory authorities developing capital rules for risk-adjusted capital requirements of financial institutions.

Keywords:
Systematic Risk, Collateralized Debt Obligation, Credit Default Swap

Zusammenfassung

Die Entwicklung geeigneter Verfahren zur Messung und zur vorausschauenden Berücksichtigung systematischer Risiken in Kreditmärkten stellt gegenwärtig weltweit eine der zentralen Herausforderungen von vielen Wissenschaftlern, Investoren und regulatorischen Aufsichtsinstanzen dar. Dies gilt insbesondere mit Blick auf strukturierte Produkte wie Collateralized Debt Obligations (CDO). Die Entwicklung eines umfassenden Bewertungsrahmens zur Beurteilung der Risikocharakteristika von CDOs hinsichtlich systematischer Risiken kann zudem erste Anhaltspunkte dafür liefern, welchen Bepreisungseinfluss systematische Risiken auf Verbriefungstransaktionen und auf klassische Kreditderivate wie Credit Default Swaps (CDS) ausüben. CDS Kontrakte stellen die wesentlichen Kreditderivate auf globalen Kreditmärkten dar und fungieren in synthetischen Verbriefungstransaktionen als Referenzaktiva.

Im Rahmen dieser kumulierten Dissertation wird die Relevanz von Verbriefungstransaktionen auf Finanzmärkten bestätigt. Zudem wird ein analytisches Modell vorgestellt, das detaillierte Einblicke in die komplexen Risikocharakteristika von CDOs liefert, die sich speziell aufgrund systematischer Risiken ergeben. Dieser analytische Modellrahmen liefert ferner einen 'geschlossenen' Lösungsansatz, um den Einfluss systematischer Risiken auf produkt-spezifische Ausfallrisiken und damit verbundene Verluste zu quantifizieren. Zusätzliche Monte Carlo Simulationen stützen die erzielten analytischen Ergebnisse. Da verbriefte Tranchen systematischen Risiken in einem höheren Maße ausgesetzt sind, reagieren sie sehr viel sensitiver auf makroökonomische Veränderungen als Unternehmensanleihen mit vergleichbarem Ausgangsrating, insbesondere in wirtschaftlichen Abschwungsphasen. Anzumerken bleibt, dass makroökonomische Entwicklungen im Allgemeinen als Quelle systematischen Risikos angesehen werden. 'Poolen' und 'Tranchieren' stellen die wesentlichen Prozesse in Verbriefungstransaktionen dar, durch die sowohl Risikodiversifikations- als auch Risikokonzentrationseffekte erreicht werden. Beide Effekte bestimmen letztlich die Risikoprofile verbriefter Tranchen, wobei die beobachteten Effektstärken von der jeweiligen Tranchenseniorität abhängen. Die erzielten Ergebnisse lassen zudem vermuten, dass klassische Kreditratings unzureichende Meßverfahren für das produktbezogene Ausfallrisiko von strukturierten Verbriefungen darstellen, insbesondere in Bezug auf Tranchen, die ein besonders gutes Rating, bspw. ein AAA-Rating, aufweisen.

Korrelationen, die beispielsweise Abhängigkeitsstrukturen in ausfallrisikobehafteten Portfolien zum Ausdruck bringen, werden als Haupttreiber für das Kreditportfoliorisiko identifiziert und stellen daher einen wesentlichen Faktor zur Bepreisung von Kreditportfolioderivaten wie synthetischen Einzeltranchen-CDOs (STCDO) dar. Im standard Einfaktormodell mit Gauß-Copula Spezifikation liefern historische Asset-Korrelationen, die aus Aktienkursrenditen abgeleitet werden, nur eingeschränkt brauchbare Bepreisungsinformationen für verbriefte Tranchen des iTraxx Europe Kreditindex mit fünfjähriger Laufzeit. Daher werden zwei alternative Korrelationsmodelle zur Bestimmung und Prognose impliziter Korrelationen der untersuchten iTraxx Europe Indextranchen von 2005 bis 2008 vorgeschlagen, die spread-abhängige Variationen der tranchen-spezifischen impliziten Korrelationen berücksichtigen. Basierend auf den durchgeführten Panelregressionen (Paneldatenanalyse), erreicht das vorgeschlagenen dynamische Regressionsmodell MERM unter Berücksichtigung von zufälligen Zeiteffekten und tranchen-fixen Effekten die höchste Prognosegüte. Dabei wird die modelbezogene Prognosegüte durch die Wurzel aus der mittleren quadratischen Prognoseabweichung ausgedrückt. Die empirischen Ergebnisse deuten ebenfalls darauf hin, dass ein tranchen-übergreifender – also 'systematischer' – Risikofaktor zeitgleich die Risikoprämien aller Indextranchen beeinflusst.

CDS Prämienquotierungen der Jahre 2004 bis 2010 von 339 U.S. amerikanischen Unternehmen, die sich über insgesamt zehn Wirtschaftssektoren erstrecken, werden genutzt, um zu untersuchen, ob schuldnerübergreifende Risikofaktoren im Querschnitt der vorhandenen CDS Spreads bepreist werden. Auf Basis eines zweistufigen Regressionsverfahrens, werden das allgemeine Kreditmarktumfeld, die Kreuzmarktkorrelation und die Marktvolatilität als systematische Risikofaktoren identifiziert, die zeitgleich sämtliche Risikoprämien im Querschnitt der untersuchten Swap Kontrakte beeinflussen, insbesondere in finanzmarktbezogenen Krisenzeiten. Dieser Bepreisungseinfluss systematischer Risikofaktoren bleibt auch dann noch bestehen, wenn für idiosynkratische oder unternehmens-spezifische Risikofaktoren wie Kreditratings, Liquidität und den Verschuldungsgrad kontrolliert wird. Da die Sensitivität der untersuchten CDS Kontrakte gegenüber systematischen Risiken nicht nur kontraktspezifisch variiert, sondern auch innerhalb der betrachteten Ratingklassen deutlich schwankt, wird der Bedarf nach geeigneten Meßverfahren für systematische Risiken besonders deutlich. In der Zeit vor dem Einsetzen der letzten globalen Finanzkrise erklären die vorgeschlagenen Risikofaktoren des Basismodells ungefähr 83% der CDS Spreads, während sie im Krisenzeitraum ungefähr 90% der entsprechenden Risikoprämien erklären. Insgesamt ermöglicht der implementierte Regressionsansatz zum einen eine Identifikation kontraktspezifischer Sensitivitäten hinsichtlich systematischer Risiken und zum anderen die Berechnung entsprechender Risikoprämien. Zudem liefert er erste Erkenntnisse zur Entwicklung eines risikoadjustierten Bewertungsrahmens für CDOs, insbesondere mit Blick auf die Berücksichtigung systematischer Risiken.

Die Arbeitsinhalte und -ergebnisse dieser Dissertation richten sich an verschiedene Interessengruppen: beispielsweise an andere Wissenschaftler im Kreditrisiko- oder Derivatebereich, an Investoren, die mit strukturierten Verbriefungen oder CDS-Kontrakten handeln, an Risikomanager in Banken oder Versicherungsgesellschaften, die mit dem Management von Kreditrisken betraut sind, aber auch an regulatorische Aufsichtsinstanzen, die risikoadjustierte Eigenkapitalvorschriften für Banken entwickeln und festsetzen.

Schlagwörter:
Systematisches Risko, Collateralized Debt Obligation, Credit Default Swap

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Chapter 1

Introduction

1.1 Credit Risk and Structured Finance Securities

Many practitioners and researchers consider Collateralized Debt Obligations (CDO) as one of the most important financial innovations of the recent decades (see Hull & White 2008, Longstaff 2010). However, these structured financial instruments have often been controversially discussed, since they were also identified as a major source of credit losses in the recent credit crisis (see Longstaff 2010). The developments in structured finance markets show both a sharp rise in the demand for these structured instruments up to the year 2007 and a dramatic fall of their issuance volume in the aftermath, which began with the onset of the Global Financial Crisis (GFC). Since 2010, the market developments have shown that asset securitization has been rediscovered as valuable tool for managing credit risk.[1]

Dealing with risk is the day-to-day business of financial institutions in globalized financial markets (Bruyère et al. 2006).[2] Thereby, risk is commonly divided into market risk, operational risk, liquidity risk and credit risk (BIS 2006, 2010*a*). Particularly credit risk gained increasingly in importance during the last two decades and encouraged not only the emergence of credit deriva-

[1] Credit risk can be seen as default risk or the risk of a decrease in the market value of a liability due to changes in the obligor's credit quality (Duffie & Singleton 2003). The management of credit risk may refer to buy-and-hold strategies and to transferring or hedging credit risk in general.

[2] For further information about the set of activities related to the risk management of financial institutions see Duffie & Singleton (2003) and Hull (2007).

tives in general, but also the design of more complex securitization structures.[3] The growing attention of numerous researchers, market participants, and regulatory authorities around the world was caused by several waves of corporate bankruptcies, for example, in Germany during the nineties (e.g., Herstatt-Bank, Bankhaus Fischer, and SchmidtBank), and during the last financial crisis.[4] In this context, the need for more transparent supervision mechanisms for credit risk has triggered the introduction of a revised capital adequacy framework – known as Basel II – that became effective in January 2007 (BIS 2006). Due to the recent financial turmoil, the Basel Committee on Banking Supervision tightened these seemingly insufficient regulatory requirements in another revision called Basel III. The Basel III framework particularly focuses on the regulatory treatment of asset securitizations and re-securitizations in order to reestablish and maintain stability in global financial markets (BIS 2009*b*, 2011).

The GFC has also shown that unexpected high default rates have caused credit losses on global credit markets, which were far above investors' expectations, even if those expectations primarily relied on classical credit ratings (Moody's 2011*b*).[5] Thereby, default rates and related losses were only one channel, which has shown the wide impacts of credit risk.[6] Normally, an excellent credit rating, e.g., 'Aaa', provided by credit-rating agencies (CRA) such as Moody's, Standard & Poor's (S&P) and Fitch, suggests the highest creditworthiness of the rated borrower. However, in the last few years particularly high-rated financial instruments were strongly affected by macroeconomic shocks – or more generally by systematic risk. Thus, market participants who were invested in these 'low risk' products have been surprised by high default rates.

This effect was even stronger for credit derivatives than for corporate debt claims such as classical bonds. Especially, structured financial instruments

[3] While in 2003 the annual survey of the Center for the Study of Financial Innovation (CSFI) ('Banking Banana Skins 2003') reported that derivatives and credit risk were the main risks for the banking community, the annual survey of 2012 identified macroeconomic risk and credit risk as the top risks in banking (CSFI 2003, 2012).

[4] A well-known example is the bankruptcy of Lehman Brothers Inc. on September 15^{th}, 2008. This default event was a major reason for distrust in the banking community which has caused liquidity troubles within the interbank market in the aftermath.

[5] Typically, a rating refers to the obligor's creditworthiness and thus expresses an opinion about his ability to fulfill contractual interest and liquidation payments (liabilities).

[6] Other channels, for example, were governmental, regulatory and bank internal adjustments of credit risk policies.

like CDOs relying on entire debt portfolios or Credit Default Swap (CDS) baskets leveraged such systematic risk through pooling and tranching. Thus, empirical observations from 2007 to 2010 have shown that the physical default rates of structured securities were multiples of those related to comparably rated bonds (Moody's 2011*b*,*a*). The risk characteristics of securitizations – even in terms of financial distress – may be the reason why the market for structured securities rapidly broke down in these years, although the demand for asset securitizations was impressive in the decade prior to the GFC.

Simultaneously, credit spreads for corporate bonds and debt-related instruments such as CDS or securitized tranches increased across all rated products, but the effects on high-rated instruments were disproportionally intense. Risk premiums for 'Aaa'-rated bonds or tranches increased much more rapidly than those for lower-rated credit assets. In corporate bond markets, this phenomenon is known as the 'credit spread puzzle' dealing with the mismatch between prices for the product's physical default risk and the risk neutral valuation of the product's total risk (compare Amato & Remolona 2003, Hui 2010). Therefore, solving this puzzle implies having to look beyond the product's physical default risk indicated by its rating in order to identify further pricing components compensating for, e.g., related liquidity risk, counter-party risk and systematic risk. With respect to systematic risk, recent studies come to the conclusion that ratings do not appropriately reflect this kind of risk, especially in terms of securitizations (see, e.g., Rösch & Scheule 2009, 2010). Thus, Coval et al. (2009*b*) state that particularly market participants investing in asset securitizations should claim premiums beyond the products's physical default probabilities compensating for impacts of systematic risk.

Recent empirical studies address several determinants of credit spreads with respect to corporate bond markets (among others Collin-Dufresne et al. 2001, De Jong & Driessen 2006, Hui 2010, Iannotta & Pennacchi 2011, Giesecke, Longstaff, Schaefer & Strebulaev 2011, Friewald et al. 2012) in order to decompose observed credit spreads into their major pricing elements. These studies often suspect common risk factors as the main drivers for pricing credit risk of corporate debt claims. Besides bond spreads, the analysis of systematic risk factors in pricing CDS contracts seems to be of special interest for several reasons: firstly, single-name CDS represent a substantial section of the credit derivatives market (BBA 2006, SIFMA 2012*c*). Secondly, several types of securitizations often involve swap contracts. Thirdly, baskets of single-name CDS contracts mainly represent the collateral of synthetic CDOs which has been increased in their market relevance since 2002 (BBA 2006, SIFMA 2012*c*).[7] In fact, only a few studies examine how systematic risk is affecting swap premiums (Amato 2005, Blanco et al. 2005, Ericsson et al. 2009, Gala et al. 2010, Gandhi et al. 2012).

Thus far, the recent financial literature and also many public discussions underline the relevance of that topic and show that measuring and anticipating systematic risk are currently the core challenges for researchers, investors and regulatory authorities around the world – not only with regard to structured products. Indeed, the recent literature shows a lack of a comprehensive framework for evaluating the risk characteristics of structured products due to systematic risk, which may also allow to identify and to measure pricing impacts of systematic risk.

Despite the critical discussion of securitizations, a couple of reasons support the commitment of structured securities.[8] At least these benefits underline the need for further empirical research investigating the complex risk profiles of such asset securitizations in terms of default risk and related losses, which determine their 'natural' behavior due to systematic risk. Furthermore, scientific efforts are gaining in importance even if the discussion of such products is becoming more and more emotional and backed less by empirical evidence, as reported in FCIC (2011).

This cumulative thesis confirms the relevance of structured products in financial markets and provides an analytical framework for obtaining detailed insights into the risk characteristics of structured products due to systematic risk. The analytical models as well as Monte Carlo (MC) simulations are applied to quantify the impacts of systematic risk on default rates and related losses of securitized tranches. The suggested model setup additionally allows to spotlight the effects of both risk diversification and concentration in securitizations. The major limitations of CRA ratings are also indicated in this framework. Particularly with respect to systematic risk, the empirical studies presented in this thesis firstly suggest that tranche spreads of the 5-year iTraxx Europe credit index may jointly be driven by a systematic component

[7] Compare Laurent & Gregory (2005) for a detailed analysis of CDS baskets.
[8] Compare Chapter 2 and Rajan et al. (2007) for an introductory overview.

and secondly that systematic risk is also priced in U.S. CDS spreads.[9] Related to the CDS-spread study, some measures for systematic risk are identified and empirically tested for their cross-sectional pricing impact on CDS spreads in order to quantify systematic risk premiums, even after controlling for essential idiosyncratic risk factors such as rating information, firm leverage or market capitalization. The results also show that the risk premium linked to credit ratings does not sufficiently compensate for contract-specific systematic risk exposures. Thus, market participants may claim a separate risk premium for facing systematic risk.

In conclusion, this thesis targets

- a better understanding of 'complex' securitizations, their functionality and relevance in financial markets,
- the application of dynamic implied correlation concepts for pricing single-tranche CDO swaps,
- the risk characteristics of structured products due to systematic risk, and
- common determinants of credit default swap spreads to quantify the pricing impacts of systematic risk on such credit derivatives.

The empirical findings related to the latter objective may facilitate the development of a risk-adjusted valuation framework for CDOs, particularly with respect to systematic risk. Thus, the findings are addressed to several groups, e.g, other researchers in the field of credit risk or derivatives, investors dealing with securitized tranches or swap contracts, risk managers in banks or insurance companies engaged in the management of credit risk, and regulatory authorities developing capital rules for risk-adjusted capital requirements. Eventually, this work may help to return more confidence to structured finance instruments for a 'healthy' or rather sustainable handling of these products in order to increase the transparency and stability in global financial markets.

[9] Nowadays, such swap contracts constitute not only the most issued and liquid credit derivatives, but also the major elements of synthetic asset securitizations, such as synthetic CDOs. For a good description of the 5-year iTraxx Europe as one of the most popular credit indices and an example for such synthetic CDOs see www.iTraxx.com.

1.2 Outline and Contributions

The remainder of this thesis proceeds as follows. Chapter 2 examines why especially CDOs – one major class of Asset-backed Securities (ABS) – have been widely seen as a popular tool for managing credit risk prior to the GFC.[10] For the time period from 2000 to 2011 developments of the global CDO market are examined by year in terms of their issuance, global outstanding, collateral and purpose. The developments in structured financial markets show the sharp rise of this security class until the year 2007 and their dramatic fall triggered by the beginning of the GFC. The recent market developments also indicate that their popularity has again increased since 2010. Furthermore, the analysis shows that rating-based default rates reported by Moody's indicate limitations of current rating metrics, especially in macroeconomic downturns. Eventually, the market developments may reflect the influences of systematic risk on structured securities, particularly caused by changes in the global macroeconomic climate as a source for systematic risk.

Chapter 3 reviews the existing paradigms for modeling default risk of a single borrower: structural models and intensity models. Factor models for credit risk are introduced as special case of intensity models and a unifying synthesis of the models is provided. Based on these preliminaries, the models are extended to portfolio credit risk, which is essential for valuation purposes of asset securitizations. In this context, the single-factor Gaussian copula model is specified, which represents the market standard model for pricing CDOs and single-tranche CDO swaps (STCDO), see Hull & White (2008) and Finger (2009). Additionally, the copula approach is briefly presented as flexible framework for modeling joint default times of borrowers in a credit portfolio. Common correlation concepts such as asset and default correlation are introduced in order to reflect dependency structures between borrowers. Correlations are identified as major determinants of credit portfolio risk and thus also constitute important parameters for pricing structured securities.

The empirical study in Chapter 4 contrasts several correlation approaches and confirms the limitations of historical asset correlations in the standard

[10] The database of the empirical analysis 'Developments in Structured Finance Markets' was provided by the Securities Industry and Financial Markets Association (SIFMA) and refers to U.S. ABS, global CDOs and European securitizations from January 1996 to December 2011. The respective impairment and rating database was provided by Moody's.

single-factor Gaussian copula model for pricing STCDOs.[11] Compound and base correlations, as two popular concepts of implied correlations, are introduced in order to overcome the pricing limitations of a single-correlation approach. Within a dynamic panel regression framework, two alternative spread-dependent correlation skew models are proposed to model and forecast implied correlations of tranches referring to 'on the run' series of the 5-year iTraxx Europe credit index. Thereby, random effects are incorporated in order to account for unobservable time-specific effects on implied tranche correlations. The proposed dynamic mixed-effects regression correlation model (MERM) is checked for its forecast accuracy in comparison to a dynamic asset correlation model and a fixed-effects regression correlation model (FERM). The empirical findings suggest that historical asset correlations gained from stock market returns – as proposed in the financial literature several times – are insufficiently reflecting the dependency structure across single-name CDS in the credit index. This leads to a mismatch between tranche-specific model spreads and market spreads. Indeed, the highest forecast accuracy measured in terms of root mean squared forecast errors (RMSFE) is reached by applying the proposed MERM. Since each regression model refers to three different sample periods, the prediction power of all three models is also checked under varying economic climates: in times of financial distress (during the GFC), in moderate market conditions (pre-crisis) and for the entire period from August 2005 to September 2008. Eventually, the empirical findings also hold for different macroeconomic conditions and indicate the presence of a common risk factor influencing all tranche spreads simultaneously.

In Chapter 5, effects of systematic risk on asset securitizations are explicitly addressed.[12] The provided analytical framework refers to a basic model extension of the standard single-factor Gaussian copula model (see Gordy 2003) and allows a closed-form comparison of comparably rated bonds and tranches. The comparison provides insights into product-specific default risks, related losses

[11] The empirical study 'Dynamic Implied Correlation Modeling and Forecasting in Structured Finance' refers to daily index and tranche spreads of the 5-year iTraxx Europe credit index and its securitized tranches from August 2005 to September 2008. The index tranches refer to a basket of the 125 most liquid and equally weighted single-name CDS on European entities. The spread database was provided by Markit, contains 4,494 spread notations in total, and covers six 'on the run' series of the credit index.

[12] In the analytical study 'Systematic Risk Sensitivity of Structured Financial Products' the natural behavior of asset securitization due to systematic risk is demonstrated. In several case studies based on Monte Carlo simulations, the basic assumptions of the analytical model are stepwise relaxed to account for a more 'realistic' model setup.

and sensitivities to systematic risk. Even if both products share the same (unconditional) probability of default in the model setup, they exhibit greatly different risk profiles with respect to systematic risk. Furthermore, these findings show that tranches are much more sensitive to systematic risk than corporate bonds due to their increased exposure to systematic risk. The risk characteristics of securitized tranches are strongly dependent on the subordination level. A MC approach also demonstrates the effects of pooling and tranching and additionally facilitates the investigation of both risk diversification and concentration effects. While bonds are typically exposed to idiosyncratic, sectoral and macroeconomic risk, securitizations allow the diversification of both idiosyncratic and sectoral risk, but also lead to the concentration of systematic risk exposures. The effects may be even more severe for tranches of high seniority. Eventually, the higher concentration of systematic risk exposures in structured products may be responsible for the dramatic increase of impairments in economic downturns as it was observable during the GFC (compare Chapter 2). However, the analytical as well as the MC approach indicate that classical ratings are insufficient metrics for measuring risks of structured securities, particularly with respect to high-rated tranches. Overall, the model-based outcome corresponds to the empirical findings in Chapter 2, where historical impairments of securitizations are reported by rating.

In Chapter 6, determinants of CDS spread changes are investigated based on a comprehensive spread data set.[13] Several macroeconomic and financial variables are proposed to explain the cross-section of CDS spreads. The proxies for these common risk variables are applied in a two-pass regression approach in order to examine factor-specific pricing contributions (compare Fama & MacBeth 1973).

In the first pass, the contract-specific sensitivities to systematic risk are evaluated based on proxies for the *Cross-market Correlation*, the *Market Volatility*, the *Credit Market Climate*, the *Slope of the Term Structure* and the *Spot Rate*. Since swap contracts not only show different sensitivities to systematic risk by rating class, but also within each rating class, the need for systematic risk measures is underlined. In the second pass, these systematic risk sensitivities are tested for their explanatory power in pricing swap contracts cross-sectionally,

[13] The empirical study 'Valuation of Systematic Risk in the Cross-section of Credit Default Swap Spreads' relies on a CDS spread database of 339 U.S. firms divided across ten economic sectors from January 2004 to December 2010. The spread database was provided by Markit and contains 124,413 weekly spread observations in total.

even after controlling for several idiosyncratic risk factors such as the firm's *Rating*, *Leverage Ratio*, *Market Capitalization* and the contract-specific *Swap Liquidity*. Even though credit ratings may determine the general CDS spread level, these spread levels seem to depend on the macroeconomic climate and, therefore, vary over time. Hence, two-pass regressions are conducted for two different time intervals (subsampling): while the first sample refers to moderate economic conditions prior to the GFC, the second sample covers the period of financial distress during the GFC. The empirical findings demonstrate that the premium for the firm's physical default risk is not sufficiently compensating for systematic risk. In addition to the firm's physical default risk and other idiosyncratic risk premiums, particularly the *Credit Market Climate*, the *Cross-market Correlation* and *Market Volatility* are cross-sectionally rewarded in CDS spreads. In contrast to equity markets, the Fama-French factors exhibit rather limited explanatory power for the pricing of swap contracts. Furthermore, the main findings are quite robust for different proxies of the interest term structure. Thus, the empirical results hold, even if swap rates are incorporated instead of Treasury bills. A principal component analysis suggests the presence of an additional common risk factor, which is not explicitly addressed, but significantly priced in the cross-section of swap contracts during the GFC. Overall, this study contributes a framework for identifying contract-specific sensitivities to systematic risk and allows to quantify the factor-specific pricing contributions in the cross-section of CDS spreads. Chapter 7 concludes and provides a brief outlook for suggested research.

Chapter 2

Developments in Structured Finance Markets

The content of this chapter will be published as Löhr S. (2013) 'Developments in Structured Financial Markets' in Rösch, D., and Scheule, H. (eds.): 'Credit Securities and Derivatives: Challenges for the Global Markets', Wiley.

2.1 Impairments of Asset-backed Securities and Outstanding Ratings

In 2010 the number of impaired Asset-backed Securities (ABS) fell for the first time in five years to 8,071 from 14,242 in 2009 (Moody's 2011*b*). By contrast, there were only 106 ABS impairments reported for 2006 which was several months before the Global Financial Crisis (GFC) began in June 2007.[14]

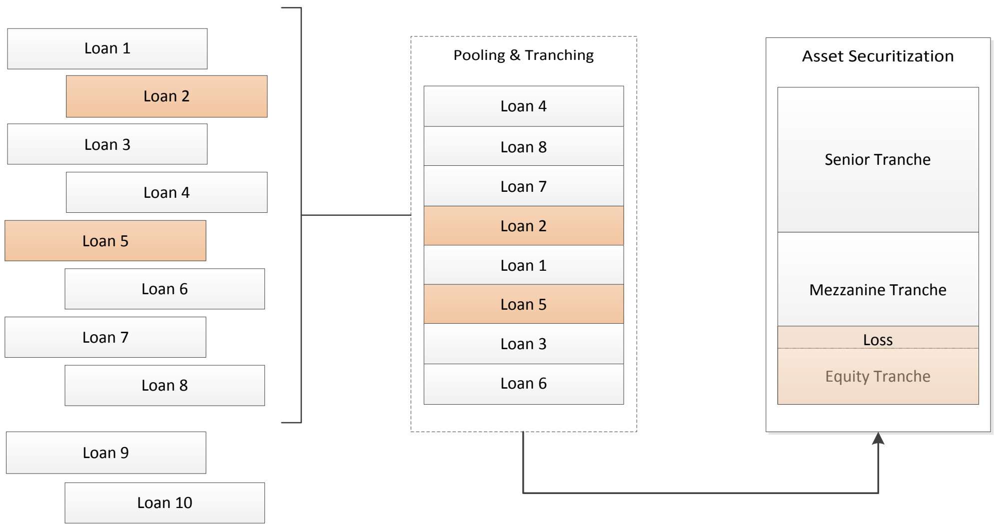
Before analyzing the impairments by year, as well as further market developments, the major ABS structures and its functionality are briefly discussed: the U.S. Securities and Exchange Commission (SEC 2004) defines ABS as financial securities "that are backed by a discrete pool of self-liquidating financial assets." The SEC (2004) further defines asset-backed securitization in its regulation rules as "a financing technique in which financial assets [...] are pooled and converted into instruments that may be offered and sold in the capital markets. In a basic securitization structure, an entity, often a financial institution and commonly known as a 'sponsor' originates or otherwise acquires a pool of financial assets, such as mortgage loans [...]. It then sells

[14] For a chronology of the GFC see BIS (2009*a*).

the financial assets [...] to a specially created investment vehicle that issues [...] asset-backed securities. Payment on the asset-backed securities depends primarily on the cash flows generated by the assets in the underlying pool and other rights designed to assure timely payment, such as liquidity facilities, guarantees or other features generally known as credit enhancements."[15]

Based on this definition Figure 2.1 summarizes the functionality of a simple asset securitization focusing on the loss flow in such a structure. Corresponding to the SEC's definition of ABS, the underlying asset pool, which is also called the collateral, typically consists of debt assets that are unable to be traded individually. These debt assets are represented by single-name *Loan 1* to *10* on the left hand side of Figure 2.1. Furthermore, *Loan 1* to *Loan 8* (in any order) constitute the specified pool of loans (collateral), which is tranched afterwards.

Figure 2.1: Illustration of the Loss-flow in a Simple Asset Securitization



Notes: This figure shows the loss flow in a simple asset securitization. Other elements such as premium flows, issuance and rating structures and involved participants are omitted for simplicity.

Through pooling and tranching – as main characteristics of securitizations – the original debt claims are converted to tradeable financial instruments (tranches) that may be sold to external investors in accordance to their in-

[15] In § 364 of the Standard Financial Accounting Standards No. 140 securitizations are similarly defined as "the process by which financial assets are transformed into securities." (FASB 2000).

dividual risk-return profile.[16] The investor's risk-return profile is determined by his internal willingness to face risk related to the respective security. Depending on the instrument's inherent risk, an investor may expect a premium that compensates him for bearing this risk.[17] The Capital Asset Pricing Model (CAPM), for example, is the most popular factor model for pricing the risk of assets (compare Sharpe 1964). How to calculate credit risk premiums of tranches is discribed in Chapter 3, where the standard single-factor Gaussian copula model is introduced for pricing single-tranche CDO swaps.

The asset securitization in Figure 2.1 consists of three tranches representing generic tranche types: the equity, mezzanine and senior tranche. Often, the originator (sponsor) partly retains the issued securitization to signal the credit quality of underlying debt claims.[18] The retained part of the securitization is mostly the equity tranche, which is also called first loss piece, since it covers first losses in the collateral (see Renault 2007).

If losses in the collateral exceed the size of the equity tranche, measured in its nominal or in percentage of the total portfolio loss, then the next tranche of higher seniority suffers from defaults in the collateral and so on. Referring to Figure 2.1, the cumulated losses of *Loan 2* and *Loan 5* exceed the thickness of the equity tranche and thus also hit the mezzanine tranche. Eventually, both tranches are impaired, the equity tranche completely and the mezzanine tranche in parts. Thus, investors of both tranches suffer from losses in the collateral: while the nominal of the equity-tranche investor has been entirely eliminated, the exceeding losses are covered by the investor who holds the mezzanine tranche. Consequently, the nominal of senior tranche holders remains unaffected. Hence, according to this subordination principle, subordinated tranches provide loss buffers for more senior tranches. Thereby, tranche losses are generally restricted to the nominal or principal of the respective tranche (thickness). Thus, the risk profiles of securitized tranches may clearly differ from each other in terms of default risk and related losses strongly depend-

[16] Note that risk is defined here as uncertainty measured in terms of the standard deviation of expectations, e.g., referring to expected returns or expected losses (compare Modigliani & Pogue 1974). Indeed, other definitions of risk are available.

[17] Under the assumption that risk averse investors attempt to maximize their expected returns according to their individually acceptable levels of risk – which is one of the most important capital market theories – there should exist a relationship between expected return and risk (compare Modigliani & Pogue 1974).

[18] Examples for such originators are banks, monoline insurers, reinsurers, and pensions funds (compare Rajan et al. 2007).

ing on i) the risk characteristics of the collateral and ii) the seniority of the tranche.

Finally, each security funds a fraction of the underlying pool and transfers the related risk to the investors, such as banks, insurance companies, hedge funds, investment banks. In turn, tranche investors receive a premium payment which is periodically paid out, e.g., quarterly, and which is a compensation for the default risk. Thus, related cash flows can be distinguished by their payment directions into the premium leg (pass-through structure) and the protection leg.[19] The premium leg contains the investors' risk premium paid by the issuer. The risk premium is mostly raised from cash flows generated by the collateral through interest and/or liquidation payments. Premium payments also follow the subordination principle: hence, the premium claims of senior tranche investors are firstly served stepwise followed by claims of investors who purchased subordinated tranches (waterfall principle). Since premium payments strongly depend on the risk-profile of securitized tranches, the premiums for the equity tranches are generally much higher than the respective ones of more senior tranches. The protection leg (contingent payments) has to be paid from investors to its counterparts in terms of a default event within the collateral to compensate for occurred losses. In general, the definition of a default event may vary. However, in standard securitizations such default events are triggered by delayed or failed interest payments and liquidation.[20]

In order to achieve an appropriate risk profile of the entire credit exposure, both the originator as well as the contract counterparties may engage in asset securitizations. Required customization as well as optimization of the counterparts' credit portfolio risk can easily be executed with credit derivatives involving so-called bespoke, or customized, CDO tranches (Rajan et al. 2007). Bespoke securitizations are often generated in cooperation with rating agencies such as Moody's, S&P and Fitch. For example, a single investor announces his individual risk-return preference, e.g., expressed by a desired tranche rating in line with a risk-adjusted premium claim, to the issuer and the cooperating rating agency. In the following, the issuer defines both the collateral and the tranche sizes as well as the subordination in order to meet

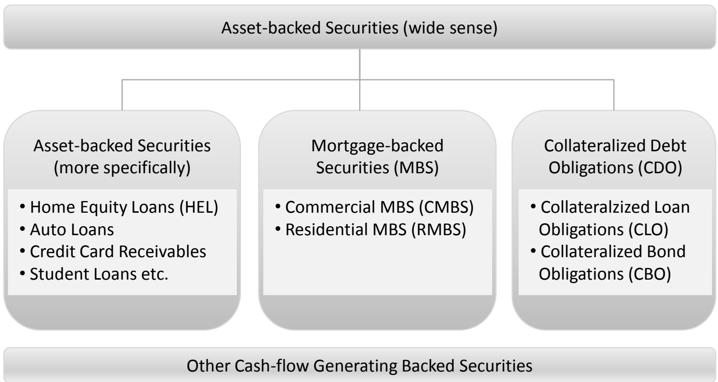
[19] Both payment legs play a crucial role for further valuation purposes referring to structured securities, compare Chapters 3 and 6.

[20] Moody's, one of the leading rating agencies worldwide, for example, defines an interest impairment, that is also an default event, as an interest shortfall continuing for 12 months or more (see Moody's 2011*b*).

the rating agency's requirements to achieve the target rating.[21] Afterwards, the individually securitized tranche is purchased by the investor who adds diversity to his portfolio. Besides bespoke asset securitization, the ABS market offers a variety of business opportunities for global rating agencies, which is also indicated by a rapidly increasing number of tranche ratings, as shown later on.

To alleviate the following market analysis of structured securities, Figure 2.2 provides an overview of the major ABS structures.

Figure 2.2: Major Asset-backed Security Classes



Notes: This figure summarizes the three major classes of Asset-backed Securities and its sub-classes.

In general, ABS may be seen as hypernym for all asset backed securities (wide sense), but more specifically ABS are for themselves seen as financial securities backed by, e.g., home equity loans (HEL), auto loans, leases, credit card receivables, student loans, aircraft leases etc. Other sub-classes of ABS are Mortgage Backed Securities (MBS) and Collateralized Debt Obligations (CDO). MBS can further be separated into Commercial MBS and Residential MBS.[22] Collateralized Loan Obligations (CLO) as well as Collateralized Bond Obligations (CBO) represent sub-categories of CDOs. Hence, depending on the underlying collateral and its characteristics ABS structures may be further sub-classified.

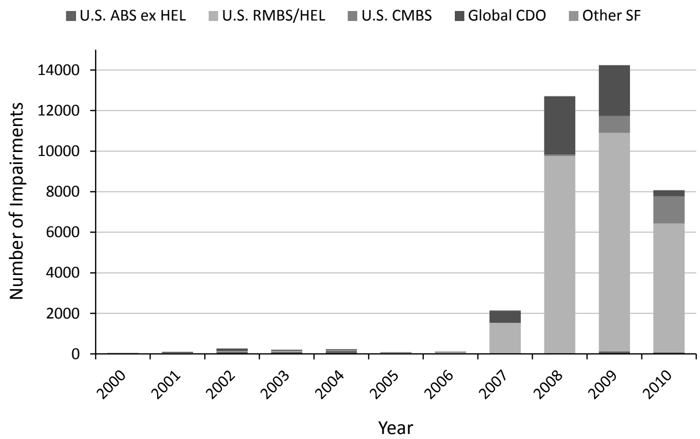
Figure 2.3 underlines the development of annual impairments of structured finance securities (x-axis) from 2000 to 2010 (y-axis) for major Asset-backed

[21] The Moody's long-term ordinal rating scale for bonds and structured finance, for example, reaches from 'Aaa' (highest creditworthiness) to 'C' (lowest creditworthiness) embedding 21 categories (grades) (see Moody's 2009*b*).

[22] Agency MBS are securities issued or guaranteed by government-sponsored enterprises such as Fannie Mae or Freddie Mac representing a major category of MBS in the U.S. (see SEC 2011).

Security (ABS) structures (Moody's 2011*b*).[23]

Figure 2.3: Total Impairments of Structured Securities



Notes: This figure shows the amount of impairments for major U.S. Asset-backed Security classes from 2000 to 2010. *Other SF* contains structured finance securities that are not categorized in the four major sectors (ABS, CDO, CMBS, and RMBS). Data Source: Moody's (2011*b*).

In contrast to the period from 2000 to 2006, where 1,064 cumulated impairments occurred, the number of impairments dramatically increased in the years 2007 to 2009 triggered by the Global Financial Crisis (GFC).[24] The events of the GFC come along with strongly increasing *credit spreads*[25] particularly on the credit derivative markets around the world, e.g., the markets for Credit Default Swaps (CDS) (see Chapter 6). Simultaneously, the credit risk premiums of portfolio credit derivatives related to popular credit indices such as the iTraxx Europe index families and the U.S. CDX index families also increased rapidly, as shown in Chapter 4 with respect to tranches of the 5Y iTraxx Europe credit index.[26]

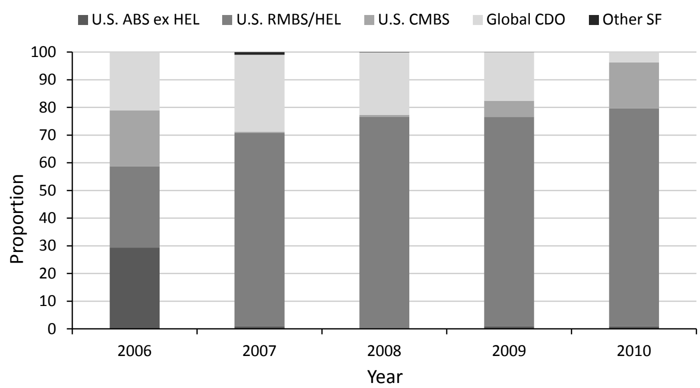
The sharp rise of ABS impairments in 2007 can be attributed to the U.S. housing crisis, which was spawned by nationwide U.S. housing price declines

[23] Note that Moody's definition of material impairments includes a downgrade to 'Ca' or 'C', which often occurs far in advance of any interest shortfall or principal write-down.
[24] For more detailed information on the chronology of the GFC compare BIS (2009*a*).
[25] A credit spread may simply be seen as premium compensating, e.g., investors, for the related default risk, see also Footnote 50 for a literature remark.
[26] These credit indices are baskets containing the 125 most liquid CDS contracts (equally weighted) either from U.S. entities having investment grade (IG) ratings (CDX) or from European entities having IG ratings (iTraxx). For further information to CDOs and credit indices see Chapter 4 and Longstaff & Rajan (2008).

combined with a sudden tightening of credit standards and rising interest rates (Moody's 2011*b*). Although, the tranche impairments in 2007 (2,153) were already as twice as high as the cumulated impairments observed over the previous six years, the total amount was rapidly increasing to 12,719 in 2008 which was almost six times higher than in 2007. With 14,242 the peak of impairments was reached in 2009 due to 13,618 principal write-downs (95.61%) and 624 interest shortfalls (4.39%).[27]

Based on total impairments by year shown in Figure 2.3, Figure 2.4 shows the fraction of impairments in percent for U.S. ABS excluding (ex) HEL, U.S. RMBS/HEL, U.S. CMBS, global CDO and other structured finance (SF) securities.

Figure 2.4: Impairments of Structured Securities by Sector



Notes: This figure shows the percentages of material impairments from 2006 to 2010 related to the four major asset-backed security classes (or sectors): *U.S. ABS ex Home Equity Loans* (HEL), *U.S. RMBS/HEL*, *U.S. CMBS*, *Global CDO*, and *Other SF*. *Other SF* contains structured finance securities that are not categorized in the four major sectors. Data Source: Moody's (2011*b*).

While the U.S. ABS ex HEL market exhibited the highest default frequency until 2007 with on average 36.6% over the years 2000 up to 2006, impairments are clearly dominated by U.S. RMBS/HEL since 2007. In contrast to 2006, where we observed almost balanced impairments across all major ABS classes (except other SF), especially the market for U.S. Residential Mortgage Backed Securities (RMBS), Home Equity Loans (HEL) and global Collateralized Debt Obligations (CDO) suffered from default events related to its borrowers in later years.

[27] In the previous 5 years, the proportion of principal write-downs on the total impairments was above 99% throughout. The number of interest shortfalls is generally small because most either can be cured (repaid) or become principal impairments.

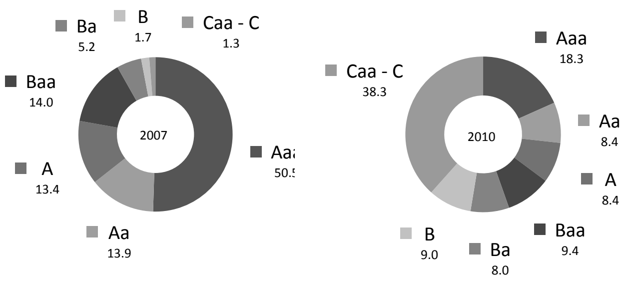
In 2007, 1,505 tranche defaults in the U.S. RMBS/HEL market account for more than 69.9% of the total, followed by impairments of global CDO tranches with a proportion of 27.7%. Thus, both securitization classes accounted for more than 97.6% of reported tranche defaults in the first year of the GFC.

In the following years, there was again a slight shift in the proportion of sectoral impairments: up to the impairment peak in 2009 the U.S. RMBS/HEL proportion of tranche defaults increased to 75.6% (10,774), while the respective proportion of global CDOs clearly decreased to 17.5%. But despite this decrease the absolute number of 2,496 impairments was relatively high and still higher than the overall impairments in 2007.

In 2010, U.S. RMBS/HEL accounted for 78.7%, U.S. Commercial MBS for 16.6% and global CDOs for 3.8% of the 8,071 reported tranche defaults. Thereby, the new impairments of U.S. CMBS increased by 59% from 839 to 1,337. However, we observe a decrease of almost 47% in new impairments in comparison to the previous year in total across the reported ABS classes. The decrease of impairments from absolute 2,496 to 304 (about 88%) is even higher in the global CDO market.

Figure 2.5 shows the proportion of outstanding ratings across the reported ABS classes for January 2007 and 2010 (compare Moody's 2008, 2011*b*).

Figure 2.5: Comparison of Outstanding Ratings



Notes: This figure shows the percentage of outstanding ratings for January 2007 (total: 86,671) and January 2010 (total: 94,326). The ratings refer to the entire reported Asset-backed security market, particularly to *U.S. ABS ex HEL*, *U.S. RMBS/HEL*, *U.S. CMBS*, *Global CDO* and *Other SF*. Other SF contains structured finance securities that are not categorized in the four major sectors. Data Source: Moody's (2008, 2011*b*).

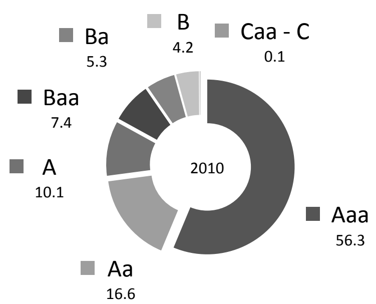
Moody's (2011*b*) reports that the number of new ratings by closing year exponentially increased from 1993 to 2006 and reached its peak in 2006 with over 27,000 ratings. During the turmoils of the global financial markets, the number of new ratings decreased strongly and fell below 2,500 in 2010 which is the second lowest level since 1993 (compare Moody's 2011*b*). Consequently, the number of outstanding ratings moderately declines for the second year in a row across all ABS classes.

In 2010 – analogous to 2007 (reported in parentheses) – the amount of ratings for structured securities backed by U.S. RMBS/HEL approximately accounted for 61.4% (60.1%) of the outstanding ratings and was thus leading, followed by U.S. CMBS ratings approximately accounting for 9.3% (9.8%) and by global CDO ratings approximately accounting for 13.4% (13.4%).

While the distribution of outstanding ratings was heavily skewed in the beginning of 2007 towards Investment Grade (IG) ratings, the respective distribution for 2010 was not: with over 50% 'Aaa'-rated tranches the IG ratings making up 91.8% of all asset-backed security ratings in 2007. In contrast, approximately 55.3% of all structured ratings were below the IG rating in the beginning of 2010. The proportion of tranches in the 'Aaa' category experienced a decline of over 36%, while the amount of non-IG rated tranches was about 6.7 times higher than in 2007, despite the numerous impairments in the previous years.

Figure 2.6 shows the distribution of material impairments by original rating of structured securities in 2010.

Figure 2.6: Impairments of Asset-backed Securities by Original Rating (in %)



Notes: This figure shows the distribution of material impairments, defined as interest shortfalls or principal write-downs, by the original rating of Asset-backed Securities for the year 2010 in percent. Data Source: Moody's (2011*b*).

As already indicated, in 2010 most tranche impairments occurred in rating category 'Aaa' which is expected to contain the most secure tranches in terms of default risk. Interestingly, tranches of category 'Caa' to 'C' exhibiting the lowest creditworthiness represent the smallest group of defaulted securities. Overall, 90.4% of impaired tranches were labeled with an IG rating, which underlines the shortcomings of current rating metrics.

These rating-based descriptives lead to two major results: firstly, the distress on the global financial markets arrived with strongly increasing material impairments across all securitized asset classes. From an economic perspective a dramatic increase of physical defaults was observed across various financial instruments, e.g., bonds, loans, leases, structured products, even during the economic downturn, either caused by principal write-downs or continuing interest shortfalls. However, structured financial instruments seem to have been particularly affected by the financial turmoil, since its investors were faced with unexpected high default frequencies/rates, even though they invested in 'Aaa'-rated securities.

Under the assumption that global economic movements are caused by unobservable systematic risk which affects all economic sectors simultaneously, one may conclude that particularly structured finance instruments are exposed to systematic risk due to pooling and tranching (compare also Chapter 5). The impacts of systematic risk are especially observable in economic downturns since this economic distress becomes manifest in an increase of impairments (downside risk). On the other hand, the systematic upside risk is rather negligible since default events are rarely triggered by economic upturns.

Eventually, these market developments indicate not only that structured finance products exhibit a higher sensitivity to systematic risk than other financial instruments such as classical bonds, but also that the systematic risk sensitivity is monotonically increasing with the tranche seniority.[28]

Despite the absence of an exact knowledge about the established rating methodologies, one may secondly conclude that ratings are not appropriately measuring default risk of structured securities at all. Rather, they seem to underestimate risk characteristics of structured financial products, especially in times of market crisis, as could be deduced by market participants from the recent GFC. Since particularly investors in structured products bore unexpected high default rates and also suffered from related severe losses due to the numerous tranche impairments, one may suspect that agency ratings do not reflect appropriately the risk characteristics of structured financial products – neither in terms of default risk nor in terms of losses caused by such impairments.

In conclusion, it is suggested that current rating metrics do not account appropriately for systematic risks inherent in Asset-backed Securities since

[28] Chapter 5 demonstrates that pooling and tranching in asset securitizations lead to concentration effects of systematic risk exposures, which are higher in tranches of high seniority.

they obviously underestimate cyclical influences which affect impairments.

2.2 Issuance of Asset-backed Securities and Outstanding Volume

One way to examine the economic relevance and popularity of asset securitizations in global financial markets is to analyze recorded issuance activities and outstanding volumes of these securities.

Starting with a description of the developments on current structured finance markets, Figure 2.7 compares the market issuance from 1996 to 2010 concerning asset- and mortgage-backed securities, which represent two major ABS classes (see Figure 2.2). The upper chart refers to ABS and compares the issuance volume related to the U.S. and Europe.[29] In the lower chart, the security issuance is analogously compared with respect to MBS. The issuance volume is denoted in USD billions (bn) on the y-axes from 1996 to 2010 (x-axes).

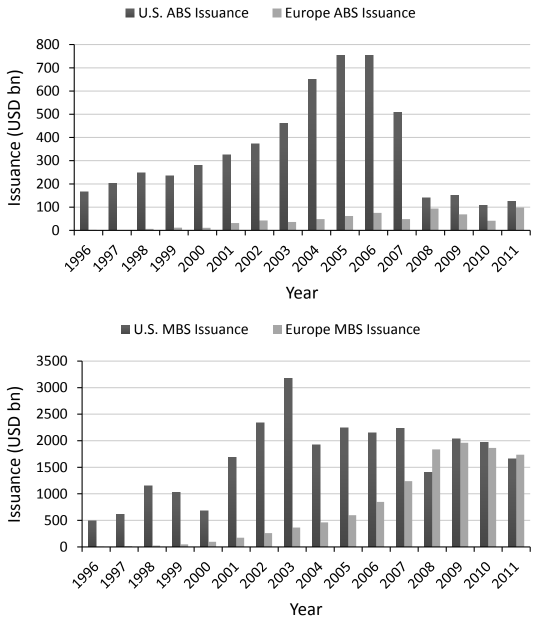
Similarly to the developments of tranche impairments shown in Figure 2.3, the U.S. ABS issuance increased more than 19 years in row and reached its all-time high in 2006 with over 753 USD bn (upper chart).[30] In the following years it fell dramatically, reaching a low of 107 USD bn in 2010. From 2010 to 2011, the issuance increased about 16% to more than 124 USD bn. This was the first reported increase after four weak years. The increase in volume comes along with the decrease in new material impairments which was described earlier.

Even though first European securitizations were already recorded in 1987, the total market for ABS was comparably less developed in Europe until 1997. While the U.S. ABS issuance was at 202 USD bn, its European pendant solely denoted at 1.08 USD bn. However, from 1997 to 2011 the developments in terms of absolute growth were still dominated by the U.S. markets, but the relative growth rates indicate the increasing importance of the European mar-

[29] According to the Securities Industry and Financial Markets Association "European securities are defined as securitizations with collateral predominantly from the European continent, including Turkey, Kazakhstan, the Russian Federation, and Iceland." (SIFMA 2012*a*).

[30] The collateral assets of U.S. ABS refer to auto loans and leases, credit card receivables, equipment, home equity loans, manufactured housing, student loans and other asset categories that do not fit any other categories. The European ABS refer to auto loans, consumer loans, credit card receivables, leases and others in the sense of above. For more details compare SIFMA (2012*d*,*a*).

Figure 2.7: Comparison of ABS and MBS Issuance



Notes: This figure compares the ABS issuance (upper chart) and MBS issuance (lower chart) with respect to the U.S. and Europe from 1996 to 2011 in USD billions (bn). Data Source: SIFMA (2012*a*,*d*).

ket segment: despite a moderate break-down in 2007 due to the turmoil in global financial markets, the European security issuance increased to more than 98 USD bn, which is more than 90 times as high as the volume in 1997. Due to the increasing demand for European ABS structures both ABS markets (U.S. and Europe) exhibit a comparable level in terms of issuance volume. This also underlines the emergence of Europe as one of the major markets for structured securities.

Similar developments may be observed with regard to MBS markets (lower chart). Until 2010 the U.S. MBS issuance was dominating the respective European one in absolute pattern. Interestingly, after the U.S. issuance peak in 2003 (over 3,179 USD bn) the volume declined to 1,924 USD bn in 2004, but varies around 2,000 USD bn with the exception of 2008. In 2008, the issuance fell to 1,403 USD bn, which was about 37% less volume than in the previous year. Thus, in contrast to the ABS markets one may conclude that the U.S. MBS markets experienced a relatively strong issuance of structured securities despite the turmoil on the global financial markets.

Similar to European ABS, European MBS have increased since 1987. From 1987 to 1996 the issuance increased moderately from 1.0 USD bn to about 9.76 USD bn. In the following years, the demand for European MBS has also increased and the issuance denoted an all-time high in 2009 with over 1,961 USD bn, which is more than 200 times higher than the issuance in 1996. Nevertheless, the absolute volume was slightly lower than the respective one in U.S. markets. This has changed in the years 2009 to 2011: while the U.S. Market issuance has decreased about 18.7% the European issuance has declined only about 11.5%. Thus, in 2011, the absolute U.S. issuance volume was 1,660 USD bn while the European was 1,736 U.S. bn, which was historically the first time that the U.S. MBS issuance was below the European one.

2.3 Global CDO Issuance and Outstanding Volume

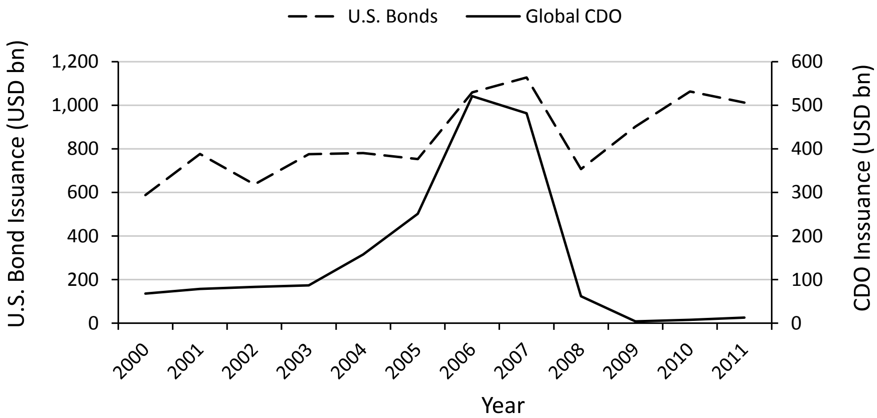
Since many practitioners and researchers widely view CDOs as one of the most important financial innovations of the past decade and identify CDOs as a major source for credit losses in the recent credit crisis (see, e.g., Longstaff 2010), the remainder of this chapter focuses on CDOs as heavily and most critical discussed ABS class. In order to underline their special role on global financial markets, both the global CDO issuance and CDO outstandings are addressed from several perspectives.

Initially, North America and Europe were the main markets for credit derivatives such as credit default swaps and CDOs as well. Recently trading activities have begun in Asia, Japan and a number of emerging markets (Rajan et al. 2007). Although, the list of participants has grown, banks are major market participants next to others such as hedge funds, monoline insurers, reinsurers, pensions funds, mutual funds and corporations. Nowadays, most market participants are buyers as well as sellers of default protection (Rajan et al. 2007).

Figure 2.8 compares the global CDO issuance (black line) with the U.S. bond issuance (dashed line). The primary y-axis denotes the U.S. bond issuance and the secondary y-axis shows the global CDO issuance in USD billions (bn) from 2000 to 2011 (x-axis).

After four years of moderate CDO issuance growth from 67.99 USD bn in 2000 to 86.63 USD bn in 2003, the issuance growth rate was strongly increasing over the next three years. This led to a peak in 2006 that is marked by an issuance of more than 520 USD bn. During these three years, the market

Figure 2.8: The Global CDO and Bond Issuance from 2000 to 2011



Notes: This figure shows the global CDO issuance (secondary y-axis) from 2000 to 2011. Respective developments in the U.S. bond market are also denoted (primary y-axis). Data Source: SIFMA (2012*a*).

demand for global CDOs rapidly increased and was six times higher than in 2003. In line with the distress on financial markets, the global issuance fell for three years in a row and reached its recorded all-time low at 4.3 USD bn in 2009. Since 2010 the volume is again raising and denoted at almost 13 USD bn in 2011.[31]

While varying around approximately 800 USD bn between 2000 and 2005, the issuance on U.S. bond markets clearly increased in 2006 to 1,058 USD bn from 752 USD bn in 2005. In contrast to the CDO markets, the bond issuance grew also in 2007 up to 1,127 USD bn (almost 6.5%). Further, the volume was just declining in 2008 to 707 USD bn due to the crisis. Thus, the issuance only fell approximately back to the level established between 2000 and 2005. Additionally, the U.S. bond issuance rebounded fast to over 1,000 USD bn in 2010, which is close to the former peak in 2007.

By contrast, the global CDO issuance broke down heavily during the GFC and only started recovering in 2011, while the U.S. bond market was seemingly much less affected by the recent turmoil. Additionally, the default rates were much lower in this period on corporate bond markets than for comparably rated asset securitizations (compare Moody's 2010*a*,*c*).

These market developments suggest that corporate bonds and structured finance securities vary in their risk characteristics, and they also indicate differences in the instruments' sensitivities to systematic risk, which are explicitly

[31] Note that unfunded synthetic CDO tranches are not included in this data set (compare SIFMA 2012*a*).

addressed in Chapter 5.

Before current market developments for the global CDO issuance are analyzed in detail, the most common types of CDOs are briefly contrasted. As sub-class of Asset-backed Securities in general, CDOs may further be categorized, among others, by its type, purpose, collateral, and domination. Regarding the type of CDOs, one may distinguish between *Cash Flow*, *Synthetic*, *Hybrid* and *Market Value CDOs*. Generally, *Cash Flow*, *Synthetic*, *Hybrid* and *Market Value CDOs* refer to the source of funds related to the securitization. In a *Cash Flow CDO*, a portfolio of individual debt asset such as loans, bonds (high yield or IG bonds), other ABS or MBS etc. is physically acquired at launch of the deal and securitized afterwards. Although there is only little change on the asset side during the securitization's term, the focus lies on the management of the collateral in order to maintain a pre-specified credit quality of the underlying assets, particularly when credit impairments occur so that single-name asset must be replaced (compare Batchvarov 2007).[32]

Typically, a special purpose entity (SPE) is involved – in Europe often called a special purpose vehicle – that is especially designed to acquire the collateral of the securitization and issues bonds to investors for cash used to purchase the underlying *Cash Pool* (compare SEC 2005).[33] In this way, the originator legally conveys the ownership of the debt assets to the SPE (true sale transaction) and is thus isolated from the financial risks of the entire securitization, e.g., credit risk and market risk. Thus, an SPE can be characterized by its narrow as well as temporary objectives. This mechanism of course may create a number of moral hazard risks since the originator is aware that he may not suffer any credit losses on the loans he makes because they will be sold as repackaged CDOs (compare Longstaff 2010).

Eventually, the issuer engaged in such off-balance sheet transactions looks less leveraged and may be permitted to borrow money on capital markets at cheaper interest rates. Further, the originator raises liquidity increasing his financial flexibility through the sale of receivables (off-balance sheet financing).

By isolating inherent credit risks and transferring it to external investors,

[32] This is often handled by an employed CDO manager, who also selects the initial asset pool (collateral).

[33] SPE are often located in countries with lax taxation, e.g., cayman islands. However, issues concerning the taxation or accounting standards of SPEs are, among others, no central topic of this exercise and are therefore omitted. For further information see, for example, SEC (2005).

the financial institution may also optimize its credit portfolio risk. For example, through the sale of sectoral concentrated debt assets such as ship financing and auto loans, concentration risk may be reduced (diversification effects). In consequence, the issuing bank may achieve a release of required regulatory capital. The reduction of regulatory capital requirements is among others, a frequent reason for issuing *Balance Sheet CDOs*. By contrast, if the sale of receivables is recognized on the (consolidated) balance sheet of the originator then this securitization is called an on-balance sheet transaction (on-balance sheet financing).

Thus, additional to enhancing liquidity, facilitating lower-cost funding there are further reasons for the originator to engage in these securitized transactions such as managing risk, e.g., by diversifying credit portfolio risk, trading various components of credit risk and separating legal from beneficial ownership.[34] On the other hand, investors are also able to customize the exposures they want to hold in their portfolios (compare FCIC 2011).[35] Once credit risk or specified elements of credit risk have been separated – such as default risk and related losses, spread volatility, counterparty risk and correlation risk – market participants can choose which ones they want to hold or to hedge. Thereby, derivative contracts are naturally two-sided and thus allow long and short positions to be taken on each element of credit risk (Rajan et al. 2007). For example, investors can use structured securities to get access to products whose spread would otherwise be either too high or too low for their needs (Rajan et al. 2007). An investor looking for 'A'-rated risk can either purchase a junior tranche (note) backed by 'AAA'-collateral or, instead, invest in a senior tranche that is backed by 'B'-rated assets in the structured credit market. Eventually, structured finance securities may fulfill numerous useful functions, as briefly described in this chapter.

In contrast to *Cash Flow CDOs*, *Synthetic CDOs* do not involve cash assets, but take on credit exposures through embedding credit default swaps (CDS) or baskets of CDS (compare Longstaff & Rajan 2008). A CDS is a credit derivative that is linked to a specified credit risky asset or basket of assets (reference asset or underlying). In a CDS contract, the protection seller, e.g., an exter-

[34] For example, investors may use default swaps to add names to their portfolios in order to diversify their exposures away from large and concentrated holdings in, e.g., plain-vanilla credit, interest-rate product classes etc. (Rajan et al. 2007).

[35] An useful illustration of the practical importance of structured products can be found in Rajan et al. (2007).

nal investor, offers protection against the default risk of the underlying, and compensates the protection buyer for losses related to the reference asset in terms of an default event such as an interest shortfall or principal impairment (similar to an insurance contract). In turn, the protection buyer, e.g., a bank, owning the underlying, periodically pays a risk premium (spread) to the investor – usually on the outstanding nominal – for taking the default risk.[36] Most popular examples for standardized *Synthetic CDOs* are credit indices such as the iTraxx Europe and the CDX index families.[37] In such *Synthetic CDOs*, CDS contracts are used to synthetically replicate a *Cash Flow CDO*.

Since many CDOs actually take on credit risk through both cash assets and CDS, the boundary between *Cash Flow CDOs* and *Synthetic CDOs* is often blurred. To avoid mis-specifications, these structures can be summarized below *Hybrid CDOs* (intermediate securities).[38] Apart from other involved parties, e.g., an underwriter, a special purpose entity, a credit rating agency (CRA) and a trustee, Figure 2.9 points out that *Hybrid CDOs* involving CDS contracts may lead to complex securitization structures.

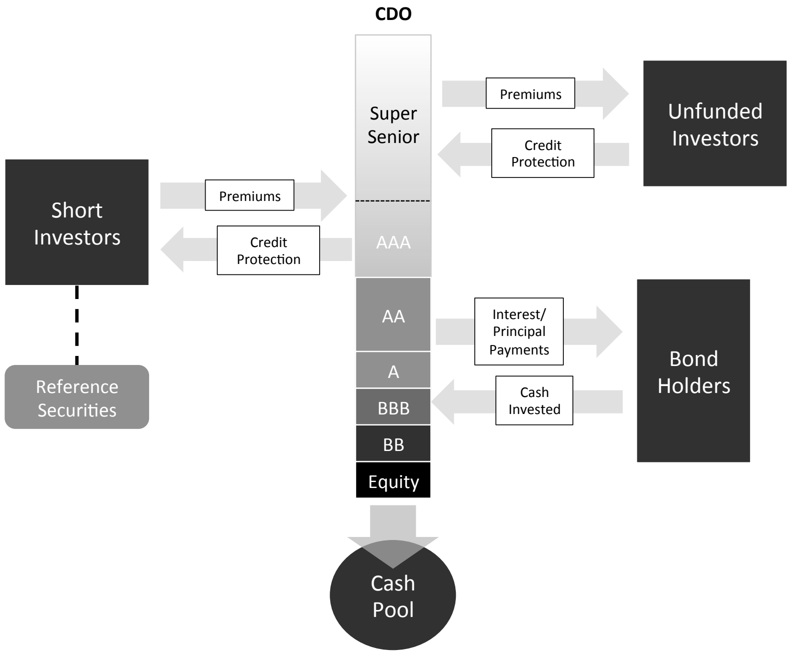
The invested capital is typically provided by *Bond Holders* acquiring securitized tranches of the CDO. Due to the credit quality of tranches which is indicated through its CRA rating, e.g., 'BB', 'BBB', 'A', *Bond Holders* obtain interest payments throughout the tranches' maturity – periodically on the remaining nominal – and the residual principal at maturity. Eventually, the size of principal re-payment depends on the losses of the respective tranche until maturity. If the principal covering for tranche losses expires, the investor of this tranche neither receive any principal payback nor any interest payments.

In contrast to funded investors, here the *Bond Holders*, unfunded investors typically buy the most senior tranche and are effectively engaged in a CDS with the 'CDO'. Thus, such investors offer credit protection to losses occurring in the super senior tranche and receive in turn premiums (also periodically) for facing the default risk of that tranche.

Additional funds are generated by *Short Investors*, who also enter into CDS contracts with the 'CDO'. They demand credit protection related to own *Reference Securities* such as other ABS, which are independent on the major CDO (compare Figure 2.9). Offering credit protection for losses in these *Reference*

[36] For more information, also for CDS markets compare Gandhi et al. (2012).
[37] See Footnote 26, and compare www.markit.com for detailed information.
[38] Other illustrated examples for *Hybrid CDOs* and more detailed information are either given in Rajan et al. (2007) or in Jobst (2007).

Figure 2.9: Complexity of Hybrid CDOs: An Example



Notes: This figure shows an example for a complex hybrid CDO structure, similar to FCIC (2011).

Securities the 'CDO' receives CDS premiums (funds). But, if losses occur within the external collateral (*Reference Securities*), the 'CDO' has to provide protection payments to the *Short Investors*.[39] By embedding CDS contracts in securitizations, a possibility for market participants was created to bet for or against the performance of these securities. Generally, through offering synthetic CDOs the demand for this kind of betting heavily increased and added liquidity to the market, which is sometimes referred to as social utility (FCIC 2011).

Funds generated through premiums by the *Short Investors* can be retained on cash reserve accounts and used to cover losses within the most senior tranches.[40] If funds are not enough to cover losses, e.g., in the super senior tranche, then *Unfunded Investors* in this tranche have to cover the remaining

[39] Other instruments that are used to protect investors against losses are, among others, credit enhancements such as over-collateralization of the assets sold, cash reserve accounts and guarantees (compare SEC 2005).

[40] Generally, a cash reserve account is a form of credit protection funded from a portion of proceeds from the securitization transaction. Principal losses and/or interest shortfalls are first covered by that reserve up to the amount funded in such account. Thus, a cash reserve provides a form of credit enhancement to the third-party investors of the securitization, e.g., *Unfunded Investors*.

losses according to signed swap contracts.

Underwriters of securitizations such as investment banks raise investment capital from investors on behalf of the securitization's issuer, e.g., the originating bank. In collaboration with a hired asset manager of the CDO – or asset management firm – that selects the collateral, the underwriter structures the securitization related to the tranche's thickness, subordination and credit quality in order to meet desired requirements of involved parties such as the originator himself, investors or credit rating agencies.[41] Thus, the underwriter acts as intermediary between the issuer and potential investors on the financial markets.

For i) bearing the market risk while holding issued securities on its own books until all securitized tranches are completely sold to market participants, ii) shorting many of these deals and iii) providing sale channels facilitating transactions between buyer and seller of credit default swap protection the underwriter is rewarded with a compensating fee. This fee may range from 0.5% to 1.50% of the total deals (see FCIC 2011). Further proceeds often result from an exclusive sale agreement on the securities. Indeed, the originator is insulated from the market risk related to the entire issuance of the securitization on capital markets at a sufficient price.

Generally, CDO tranches may find their way into several asset securitizations which boosts the complexity of the overall mosaic capturing all cross-links and other dependencies related to structured finance products (compare FCIC 2011). Often single mortgage-backed securities are referenced multiple times in *Synthetic CDOs*. As long as the reference securities perform well, investors betting that the tranche would fail (short investors) would make regular payments to the protection sellers. If the reference securities default, then the long investors would make large payments to the protection buyer (short investor). For example: if the reference securities, e.g., bonds, are worth 10 USD million and there are bets placed through CDS contracts on that securities worth 50 USD million, then on the basis of the performance of 10 USD million in bonds, more than 60 USD million could potentially change hands.

Due to the structure of such synthetic CDOs, losses from the bursting of the housing bubble were multiplied exponentially during the GFC through *Synthetic CDOs* by magnifying the overall risk (compare FCIC 2011).

[41] Note that an experienced CDO manager is crucial for both the construction and maintenance of the collateral.

However, investors often relied on the rating agencies' opinions rather than conduct their own credit analysis. Thus, it was a great business for rating agencies such as Moodys, S&P and Fitch since they were paid according to the size of each deal.[42] In providing credit ratings, the agencies were faced to two key challenges: firstly, estimating the probability of default for the MBS purchased by the CDO or its synthetic equivalent and, second, gauging the correlation between defaults measuring the dependency between security defaults at the same time. To estimate the default probability, Moody's relied almost exclusively on its own ratings of the mortgage-backed securities purchased by the CDO (FCIC 2011). The rating agencies did rarely 'look through' the securities to the underlying, e.g., subprime mortgages, which led to problems for Moody's and investors (FCIC 2011). Necessary assessments may be even more difficult in complex CDO structures. On the other hand, the increased complexity of structured products also allowed rating agencies to increase their proceeds since it was even harder for investors to provide their own due diligence. This led to situations in which investors relied more heavily on ratings than for other types of rated financial instruments such as corporate bonds (FCIC 2011).

Market Value CDOs represent another typ of CDOs. *Market Value CDOs* can be characterized by frequent trading activities in order to maintain a specified ratio of the collateral's market value to the structure's obligations.[43] Typically, the collateral must be liquidated, either in part or in whole, if the specified ratio falls below a specified threshold (compare Moody's 1998). Revenues from liquidated collateral are used to reduce the liabilities to tranche investors until the specified ratio is again fulfilled (re-balancing). Overall, *Market Value CDOs* tend to offer a variety of useful applications, even in structures of unpredictable cash flows, such as distressed debt (compare Moody's 1998).

Referring to the purpose of a CDO, another couple of sub-categories is represented by *Arbitrage CDOs* and *Balance Sheet CDOs*. In an *Arbitrage CDO*, whether cash, synthetic or hybrid, the respective arranger undertakes transactions that are mainly targeted at the spread differences between relatively high yielding pool assets (spreads on loans or CDS) and lower yielding CDO liabilities (spreads on CDO notes), compare Renault (2007). Thus, assets of

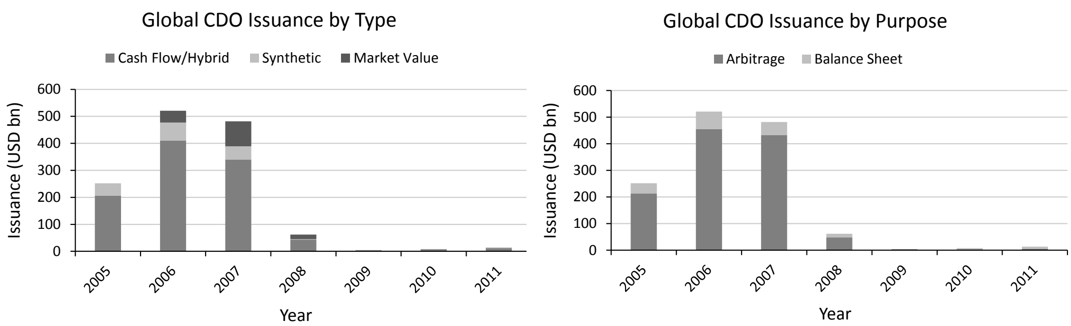
[42] Moody's set for a 'standard' CDO 500,000 USD and as much as 850,000 USD for a 'complex' CDO in 2006 and 2007, see FCIC (2011).

[43] The obligation of a CDO is the sum of amortized principal and accrued interest, that has to be paid to investors of tranches until maturity.

an *Arbitrage CDO* are particularly purchased for arbitrage transactions rather than holding these assets on the originator's book. Note that all sub-categories do not obviate each other. Thus, *Arbitrage CDOs* may refer to *Cash Flow*, *Synthetic*, *Hybrid*, and *Market Value CDOs*.

Based on the global CDO issuance by year shown in Figure 2.8, Figure 2.10 shows the distribution of the global CDO issuance (y-axes) by type and purpose for the period 2005 to 2011 (x-axes) denoted in USD billions (bn). Thereby, the left chart refers to several securitization types and the right chart to its purposes.[44]

Figure 2.10: Comparison of Global CDO Issuance by Type and Purpose



Notes: This figure shows the global CDO issuance from 2005 to 2011 by type divided into *Cash Flow/Hybrid CDOs*, *Synthetic* and *Market Value CDOs* (left chart), and also by purpose (right chart) distinguished between *Arbitrage* and *Balance Sheet CDOs*. Data Source: SIFMA (2012*b*).

The percentage of *Cash Flow/Hybrid CDOs* dominated the issuance volume in each year from 2005 to 2011 (left chart). Despite a decline from 82% in 2005 to 59% in 2009, in 2010 almost each issuance was of this type (99.5%). Further, its proportion remained relatively stable at 70% in 2007 and 2008.

While in 2005 and 2006 *Synthetic CDOs* represented the second major issuance type with 17.7% and 12.8%, respectively, the proportion shifted in 2007 as *Market Value CDOs* accounted for 19.3% and *Synthetic CDOs* only for 10.1%. Until 2009, this gap widened to 35.1% (*Market Value*) versus 5.9% (*Synthetic*), so that the global issuance of *Market Value CDOs* accounted for more than one third of the total volume. However, in 2010 only 11% was attributed to *Market Value CDOs*.

[44] With respect to the data source, in this analysis only funded *Synthetic CDOs* are considered. Funded tranches require the deposit of cash to an SPV account at the inception of the deal to collateralize the SPE's potential swap obligations in the transaction (compare SIFMA 2012*a*).

Until 2009, the global CDO issuance by purpose was clearly dominated by *Arbitrage CDOs*. From 2005 to 2009 this kind of CDO issuance accounted for at least 77%. In 2010 and 2011, the demand for *Balance Sheet CDOs* strongly increased from 20.6% in 2009 to 57.9% and 58.8%, respectively. Thus, in these years the issuance market was slightly dominated by *Balance Sheet CDOs*, which underlines the rising attractiveness of these securitizations.

Next, Figure 2.11 shows the global CDO issuance with respect to the collateral (upper chart), and the denomination (lower chart) for 2005 to 2011 (x-axes) in USD billions (bn). The respective issuance volume is denoted on the y-axes. The total issuance by year corresponds to the plotted ones in the previous Figures 2.8 and 2.10. The issuance by collateral shown in the upper chart is distinguished in *High Yield* and *Investment Grade Bonds*, *High Yield Loans*, *Mixed Collateral*, *Structured Finance* and *Other*.

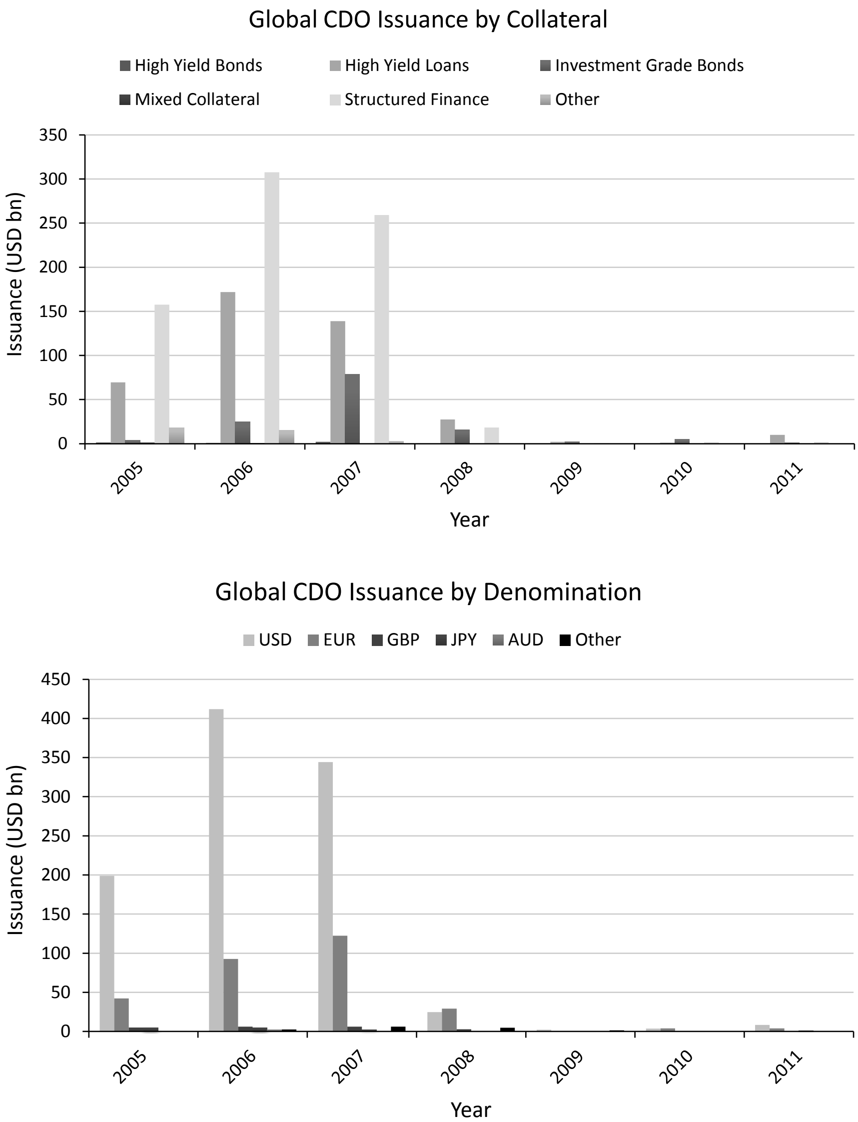
Investment Grade Bonds are defined as bonds that are rated by authorized credit rating agencies with an investment grade rating being equal or above 'Baa3' ('BBB') in terms of the Moody's (S&P) rating scale. On the other hand, bonds that are rated below the investment grade are defined as *High Yield Bonds*. With respect to the data, *High Yield Loans* are defined as debt assets of borrowers with senior unsecured debt ratings that are at financial close below Moody's 'Baa3' or S&P's 'BBB' (SIFMA 2012*b*).

Structured Finance collateral includes underlying assets such as RMBS, CMBS, ABS, CMO, CDO, CDS, and other securitized or structured products (SIFMA 2012*b*). In category *Other* SIFMA (2012*a*) summarizes collateral such as funds, insurance receivables, cash, and assets that are not captured by the other categories noted above. Further, a CDO that has 51% or more of a single collateral type, is included in this bucket, otherwise in *Mixed Collateral*.

With an absolute issuance of 746.94 USD bn from 2005 to 2011 (relative 55.72%) structured finance (SF) securities represent the major collateral related to the total global CDO issuance of 1,340 USD bn. This collateral type was followed by high yield loans with 420.85 USD bn (relative 31.4%) and IG bonds with 131.08 USD bn (relative 9.8%).

In 2006, the percentage of SF was 59.1% with absolute 307 USD bn, which was more than 12.3 times higher than the IG collateral and 1.79 times as high as the high-yield loan issuance (24.86 USD bn, and 171.9 USD bn compared to 520.64 USD bn in total). Up to 2009 the SF as well as the HYL issuance was rapidly decreasing: while the SF accounted for 7.64% (absolute 0.33 USD bn),

Figure 2.11: Comparison of Global CDO Issuance by Collateral and Denomination



Notes: This figure shows the global CDO issuance from 2005 to 2011 by its collateral in the upper chart and by its denomination in the lower one. The collateral is distinguished in *High Yield* and *Investment Grade Bonds*, *High Yield Loans*, *Mixed Collateral*, *Structured Finance* and *Other*. The category *Other* refers to collateral assets such as funds, insurance receivables, cash, and assets that are not captured by the other categories noted above. With respect to the lower chart, *Other* refers to currencies other than USD, EUR, GBP, JPY and AUD. Data Source: SIFMA (2012*a*).

HYL accounted for 46.9% (absolute 2 USD bn) of the annual issuance of 4.33 USD bn. Interestingly, that was less than 1% of the total issuance in 2006.

After four years in a row, the issuance slightly increased in 2010 for the first time in both collateral groups. Finally, the demand for HYL recovered faster since the percentage was 76.9% with 10.01 USD bn in 2011, and thus 8.74 times higher than in 2010. By contrast, the SF issuance increased by 14.1% compared to 2010 to 1.98 USD bn in 2011.

In fact, the percentage of IG bonds (IGB) strongly increased from 2005 to 2010 from 1.5% to 62.6%. Additionally, the demand for IGB was with 78.51 USD bn more than 3 times higher than in the year before (24.86 USD bn). In 2010, the IGB collateral was more than 4 times higher than the HYL and 2.8 times higher than the SF collateral. These market developments indicate that the confidence in structured markets massively declined, but they also show that the investors' confidence has slightly returned since 2010. Thus, the market demand for more secured products like IG bonds seems to increase again and HYL are preferred over the structured collateral.

Additionally, most of the global CDO securities are either denoted in U.S. dollars or in Euros and accounted together for at least 96% of the global issuance in 2005 to 2007. Thereby, the USD issuance was throughout around 75%. With 29.2 USD bn (relative 47.12 %) the EUR issuance was in 2008 the first time above the USD notations since 2000. In 2011, around two thirds (65.1%) of the issuance was denoted in USD and about one third in EUR (30.1%).

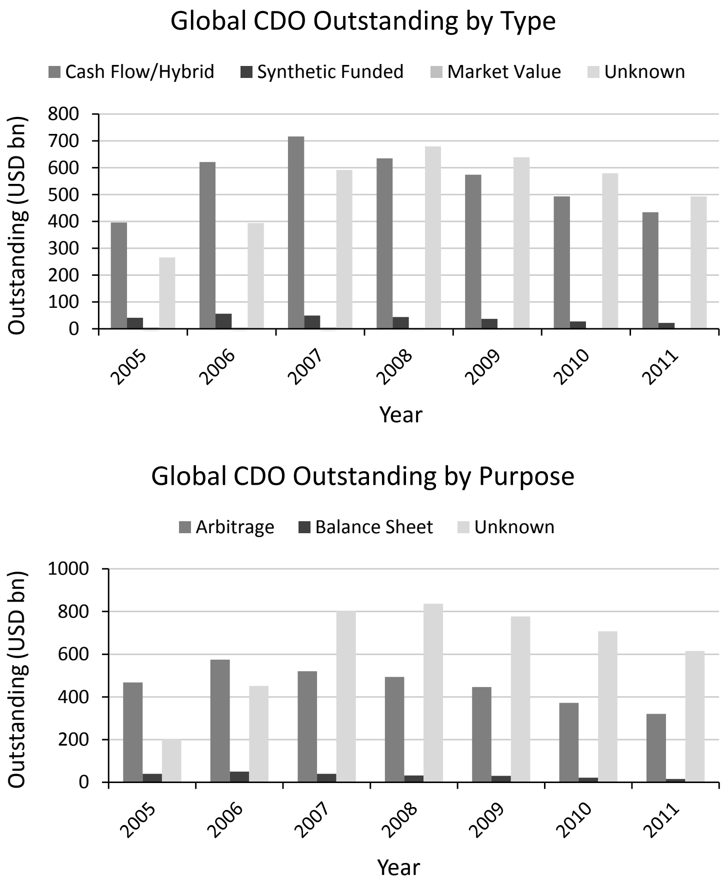
Intuitively, another important indicator for current market developments is the outstanding of global CDOs. For this reason, Figure 2.12 reports the global CDO outstanding in USD bn (y-axis) by type and purpose from 2005 to 2011 (x-axis).

From 1995 to 2007, the outstandings of global CDOs rapidly grew from 1.39 USD bn to more than 1,363 USD bn. After its peak in 2007, these outstandings declined for four years in a row to approximately 951 USD bn.[45]

From 2005 to 2007, *Cash Flow and Hybrid CDOs* dominated the global outstandings. All CDO outstandings that could not be captured by other categories are summarized below *Unknown* (see SIFMA 2012*b*). From 2008 to

[45] Note that source data for outstanding global CDOs are not the same for global CDO issuance. Due to differences in underlying data, contents are not directly comparable. Fore more details compare SIFMA (2012*b*).

Figure 2.12: Comparison of Global CDO Outstanding by Type and Purpose



Notes: This figure shows the outstanding of global CDOs from 2005 to 2011 by type (*Cash Flow/Hybrid*, *Synthetic* and *Market Value CDOs*) in the upper chart and by purpose (*Arbitrage* and *Balance Sheet CDOs*) in the lower chart. All CDO structures that may not be allocated in any of the other categories are included in category *Unknown*. Data Source: SIFMA (2012*b*).

2011, around 50% of the entire outstandings were attributed to that category.

In 2011, *Cash Flow and Hybrid CDOs* accounted with 434.8 USD bn for approximately 45.7%, while solely 2.3% could be attributed to *Synthetic Funded CDOs* (absolute 21.9 USD bn). In contrast to 2006, where the attributed volume was with 56.2 USD bn at an all-time high accounting for 5.2% of the total outstanding, the *Synthetic Funded CDO* outstandings denoted at an all-time low in 2011 due to a decline for 7 years in a row.

In 2005, the outstanding volume of *Market Value CDOs* accounted for solely 0.8% of the global outstanding with 6 USD bn. Since 2005, the absolute as well as the relative outstandings of *Market Value CDOs* decreased and reached in 2011 an all-time low of 1.1 USD bn and 0.1%, respectively.

Overall, neither *Synthetic Funded CDOs* nor *Market Value CDOs* seem to play a major role on global CDO markets. Instead, at least *Cash Flow and Hybrid CDOs* are identified as one of the most important securities on current markets accounting for almost 50% of the securitized market volume.[46]

Related to the purpose of global CDOs, *Arbitrage CDOs* were dominating *Balance Sheet CDOs* in terms of global outstandings from 1999 to 2005. In 2006, *Arbitrage CDOs* reached its all-time high with over 574 USD bn or 53.4% of total outstandings. In line with the turmoil on financial markets, the volume declined to 320 USD bn in 2011, which denoted 33.7% of the annual outstandings. Simultaneously, the proportion of *Balance Sheet CDOs* also continuously declined from 5.63% in 2005 (absolute about 40 USD bn) to 1.64% in 2011 with an absolute volume of about 15.61 USD bn.

However, *Unknown CDOs* which are not attributable to one of both standard categories play a major role and account for 28.45% of the outstandings (201.8 USD bn) in 2005. Their proportion continuously increased to 64.66% of the total outstandings with an absolute volume of 615 USD bn in 2011.

2.4 Concluding Remarks

- Structured finance securities such as Collateralized Debt Obligations facilitate the isolation of credit risk and its transfer to external investors.
- Numerous useful applications of structured products made these financial instruments the most popular tools in the last decade.
- During the GFC the default rates of structured securities increased dramatically, even those of ‘Aaa’-rated tranches.
- More than 90% of the reported impairments referred to IG-rated securities indicating the shortcomings of the current rating metrics.
- Seemingly, securitizations exhibit specific risk characteristics not sufficiently reflected by credit ratings due to their high sensitivity to systematic risk.
- Within the financial turbulences, impairments were concentrated in the market for RMBS, HEL and CDOs.

[46] Note that results of this exercise strongly depend on the quality of the data.

- Dramatically affected was the demand for U.S. ABS, global CDOs, but less strongly the demand on U.S. MBS markets.
- Indeed, the European ABS issuance tended to move sideways on a relatively low level, while the MBS issuance clearly increased, even in the GFC.
- In 2011, the European ABS issuance reached the U.S. level with almost 98 USD bn, while the MBS issuance was above the U.S. one for the first time.
- Related to the global CDO issuance an all-time low was reached in 2009 with less than 1% of the volume in 2006.
- After four years in a row the global CDO issuance increased for the first time in 2010 with changed major collateral types.
- In fact, since 2010 more secured collateral such as IG bonds, and High-yield loans are preferred over structured collateral.
- Due to the numerous tranche impairments as well as the decreased issuance, the global CDO outstanding was also slightly decreasing from 2008 to 2011.
- Since 2010 when the number of tranche impairments fell for the first time in five years, the confidence in structured securities has begun to return slightly.

Chapter 3

Approaches to Credit Risk for Valuing Structured Finance Securities

3.1 Introduction

Longstaff (2010), among others, states that structured finance securities are the most important financial innovations of the last decade. Particularly, the demand for Collateralized Debt Obligations (CDO) increased rapidly from 2001 to 2007 due to several reasons outlined in Chapter 2. In the aftermath, especially the U.S. market for structured securities was heavily shocked by the turmoil in the global financial markets. Due to decreasing numbers of impairments, confidence in structured products has returned slightly since 2010 and thus the issuance volume of Asset-backed Securities is again increasing.[47] In consequence, practitioners and researchers are again focusing on appropriate valuation concepts for these products.

Although the limitations of the Black & Scholes (1973) approach have been broadly discussed by market participants and researchers as well, it has become the market standard pricing model on equity and option markets (De Servigny 2007). Comparably developments are observable on credit markets, where the basic single-factor Gaussian copula model has become the most established setup for the valuation of CDOs (see Hull & White 2006, Finger 2009, Rosen & Saunders 2009). Even though related valuation techniques are not entirely

[47] Compare Chapter 2 for more detailed analyses of structured finance markets.

satisfying, they certainly facilitate the fast expansion of the market for such credit-linked instruments (De Servigny 2007).

In general, a CDO refers to a reference pool of credit risky assets that is also called collateral or underlying. Thus, the risk profiles of securitized tranches are determined by the risk characteristics of its collateral.[48] In turn, the credit risk profile of the reference pool is determined by i) the risk characteristics of its single debt assets and by ii) pool-specific dependency structures across embedded debt assets. For this reason, approaches to credit risk of single borrowers are presented before portfolio credit risk is addressed.[49] Note that financial institutions may be faced with the measurement of default risk related to non-traded financial instruments. In this case, they can apply the credit risk approaches to model the 'physical' default risk of these instruments using natural probability measures in order to quantify, e.g., the required economic capital or risk charges (compare Bluhm et al. 2003). By contrast, if market prices, e.g., *credit spreads*[50] on corporate bonds or Credit Default Swaps (CDS), are used to derive the (implied) default risk then *risk-neutral probability measures* – or rather *equivalent martingale probability measures* – are applied due to the pricing theory of financial assets (see Jarrow & Turnbull 2000).[51] In context of portfolio credit risk, common correlation concepts are described for modeling dependency structures within credit portfolios. Although 'true' correlations are not observable, they are crucial determinants for the risk profile of a credit portfolio (see Schönbucher 2001). Eventually, the risk profile of a credit portfolio – summarizing the portfolio's risk characteristics – causes the portfolio's loss distribution. Since the valuation process of securitizations is strongly dependent on the portfolio's loss distribution (Longstaff & Rajan 2008), also correlation concepts become central for valuing securitizations.[52]

The remainder of this chapter is structured as follows: after a brief introduction to the risk-neutral valuation of debt assets, *structural*, *intensity* and

[48] Recall that the risk profile of a financial instrument reflects its risk characteristics in terms of default risk and related losses from default.

[49] Current bottom-up and top-down approaches for modeling portfolio credit risk are differentiated later on.

[50] According to Bluhm et al. (2003), a credit spread is the difference between the yield on a particular debt security and the risk-free yield. Yields on government bonds are often assumed as benchmark proxy for the risk-free interest rates.

[51] In the next section, the risk-neutral valuation of credit-linked securities and the differences between 'physical' and risk-neutral default probabilities are briefly described.

[52] Recall that securitized tranches are sold to investors who offer credit protection for losses within the credit portfolio affecting these tranches.

factor models are introduced in Section 3.2, as the main approaches to credit risk of single borrowers. Based on these approaches, a general framework for modeling portfolio credit risk is presented in Section 3.3. In this context, the basic single-factor Gaussian copula model is classified. Relying on the Gaussian model specification, a simple valuation framework for single-tranche CDO swaps (STCDO) is also introduced. Moreover, the most common correlation concepts represented by default correlations and asset correlations are briefly described. These correlation concepts are not only crucial elements for measuring and modeling the risk of credit portfolios, but also for the valuation of CDOs (see Hull & White 2008). Thus, this chapter particularly provides theoretical preliminaries for the Chapters 4 and 5.

3.2 Approaches for Modeling and Measuring Credit Risk

3.2.1 An Introduction to Physical Default Risk and Risk-neutral Valuation

The main approaches to credit risk, which will be presented in the following sections, primarily address the physical default risk of obligors.[53] In this context, the 3-tuple $(\Omega, \mathcal{F}, \mathbb{P})$ denotes the underlying probability space for modeling the physical default risk. This probability space consists of a sample space Ω, a σ-Algebra $\mathcal{F} \subset \mathcal{P}(\Omega)$ representing a nonempty collection of countable subsets of the power set $\mathcal{P}(\Omega)$, and the probability measure function $\mathbb{P}$.[54] Thereby, Ω contains the set of all possible outcomes, the elements of $\mathcal{F}$ describe the measurable events of the respective model, e.g., the obligor's default time, commonly identified with all information available, and $\mathbb{P}$ assigns the physical or real-world probabilities to the measurable random events (Bluhm et al. 2003). The risk measure $\mathbb{P}$ can be related to actual default rates, which are derived from historical default information reported by rating agencies such as Moody's and Standard & Poor's (see Hull 2007). Then, the assigned default probabilities can be used to estimate the economic capital and risk charges of

[53] The terms 'obligor', 'firm' and 'borrower' are used interchangeably.

[54] For more technical details describing, for example, the element properties of the 3-tuple compare Harrison & Pliska (1981) and Martin et al. (2006).

a single borrower or a credit portfolio (compare Bluhm et al. 2003).[55]

When estimating the default risk and valuing credit-related financial instruments based on financial market information such as observable credit spreads (pricing perspective), the probability measure function refers to the risk-neutral valuation framework (compare Bluhm et al. 2003). Then, the probability space is denoted by $(\Omega, \mathcal{F}, \mathbb{Q})$, where $\mathbb{Q}$ describes the probability measure function assigning risk-neutral probabilities to the measurable events in $\mathcal{F}$. The probability measure $\mathbb{Q}$ is called equivalent martingale measure on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ due to the equivalence of $\mathbb{P}$ and $\mathbb{Q}$.[56]

Mostly, the valuation of financial instruments such as derivatives is based on the fundamental assumption that there are no arbitrage opportunities on financial markets (Hull 2009). No arbitrage implies that the cost of synthetic instruments (financial replication) must equal the value of the traded (replicated) instrument. In the absence of arbitrage, there exist unique martingale probabilities – termed *risk-neutral probabilities* or rather *equivalent martingale probabilities* – that can be used to price financial instruments such as options, futures and credit derivatives (compare Jarrow & Turnbull 2000).[57] It can be shown that the resulting prices are valid not only in the risk-neutral world, but in the real-world as well (compare Bluhm et al. 2003, Hull 2009).

The risk-neutral valuation is a general principle, e.g., in the option pricing theory (Jarrow & Turnbull 2000), which "states that it is valid to assume the world is risk-neutral when pricing options" (Bluhm et al. 2003). In a risk-neutral world all individuals i) are indifferent to risk, ii) require no compensation for risk and iii) the expected return on all securities is the risk-free interest rate, see Bluhm et al. (2003) and Hull (2009). Hence, the risk-neutral individuals make their investment decisions only on the basis of expected values and they do not consider the dispersion of distributions (Jarrow & Turnbull 2000).

Equation (3.1) exemplarily shows the core of risk-neutral pricing under risk-

[55] General limitations of ratings for extracting the products' credit curve representing the time-dependent default risk are discussed, e.g., in Li (2000). Shortcomings of ratings quantifying the inherent credit risk of structured financial instruments in terms of default risk and related losses are explicitly addressed in Chapter 5.

[56] For a good introduction to probability spaces and definitions, martingales and equivalent martingale probabilities see, e.g., Harrison & Pliska (1981) and Martin et al. (2006). For more detailed information on martingales and stochastic integrals see also Rogers & Williams (2000).

[57] For a general proof see Harrison & Kreps (1979).

neutral probabilities. Under the risk-neutral probability measure $\mathbb{Q}$, the price of a financial instrument $A(t)$ relative to the value of the *money market account*[58] $W(t)$ at time t follows a martingale

$$\frac{A(t)}{W(t)} = \mathbb{E}^{\mathbb{Q}}\left[\frac{A(t+1)}{W(t+1)}\right] \tag{3.1}$$

with $t \in \{0, 1, 2, ..., T\}$, $W(0) = 1$ (by definition) and corresponding expectations $\mathbb{E}^{\mathbb{Q}}(\cdot)$, compare Jarrow & Turnbull (2000).[59] Then, the present value of a financial instrument $A(0)$ is determined by the expected value of the instrument's future cash flows discounted at the time-specific risk-free rates of interest (Jarrow & Turnbull 2000).

"In the credit risk context, risk-neutrality is achieved by calibrating the default probabilities of individual credits [or credit portfolios] with the market-implied probabilities drawn from bonds or credit default swap spreads" (Bluhm et al. 2003). Under risk-neutrality investors should exactly be compensated for expected losses due to a possible default. But in a real world scenario, investors should intuitively be concerned about default risk. Due to the investors' aversion to bear more risk, they may demand an additional risk premium and the pricing of financial instruments should somehow account for this risk aversion (Bluhm et al. 2003). This means that financial instruments are priced as though they were a break-even trade for investors who are not risk adverse but assume a higher probability of default than the physical one. The assumed probability is called risk-neutral probability of default.[60] The difference between the risk-neutral and the physical probability reflects the risk premium required by market participants for taking risk (Bluhm et al. 2003).

[58] On the money market account M, money can be invested in the risk-less asset. At the start, the account's value equals one, $W(0) = 1$. Assume that R_0^f is the time-dependent one-period-ahead return from investing one unit in the risk-less asset at date 0, then $W(1) = W(0) \cdot R_0^f$ and the corresponding risk-less discount factor Q equals $1/R_0^f$. Thus, the value of the investment at date $t+1$ is $W(t+1) = W(t) \cdot R_t^f$, compare Jarrow & Turnbull (2000). Depending on the model framework, the return on the money market account can be easily defined in both a continuous-time or a discrete-time setting (see, e.g., Martin et al. 2006).

[59] According to Harrison & Pliska (1981), an adapted positive process $K = \{k_t; 0 \leq t \leq T\}$, which is right continuous with left limits such that K_t is integrable, is a martingale and $\mathbb{E}(K_t) = k_0$. Thus, martingales can be associated with "fair" gambles.

[60] Since market participants cannot really be assumed as risk-neutral, this terminology is somehow misleading, see Jarrow & Turnbull (2000).

3.2.2 Structural Models

The single-factor Gaussian copula model, firstly applied to credit risky portfolios by Vasicek (1987), has become the industry standard model for the pricing of securitized tranches (Hull & White 2008). Vasicek's approach is based on a structural model framework introduced by Black & Scholes (1973) and Merton (1974, 1977). Within this economically grounded framework, the firm's liabilities are assumed to be contingent claims on the firm's assets. Merton (1974) describes that the physical asset value $G_{i,t}$ of firm $i \in \{1, ..., I\}$ at any point in time t follows a continuous-time random walk (geometric Brownian motion)

$$\frac{dG_{i,t}}{G_{i,t}} = \mu_i \; dt + \sigma_i \; dW_{i,t}, \quad G_{i,0} > 0 \tag{3.2}$$

with exogenously specified expected rate of return $\mu_i \in \mathbb{R}$ and volatility $\sigma_i > 0$, where $W_{i,t}$ is a standard Brownian motion (standard Gauss-Wiener process).[61]

According to Merton (1974), the default of firm i occurs if the asset value given from Itô's Lemma by

$$G_{i,t} = G_{i,0} \cdot \; \exp\Big(\big(\mu_i - 0.5 \cdot \sigma_i^2\big) \cdot t + \sigma_i \; W_{i,t}\Big) \tag{3.3}$$

undergoes the principal value P_i of the zero bond at maturity t so that prespecified claims on these assets can not be served at maturity (principal shortfall).[62]

Let T_i denote a random variable for the discrete default time of firm i

$$T_i = \begin{cases} t & \text{if} \quad G_{i,t} < P_i, \\ \infty & \text{else} \end{cases} \tag{3.4}$$

then the physical probability of default $\pi_{i,t}$ is

$$\pi_{i,t} = \mathbb{P}\left(T_i = t\right) = \mathbb{P}\left(G_{i,t} < P_i\right). \tag{3.5}$$

According to Equation (3.2), the natural logarithm of the asset value growth

[61] In Merton's original model the liabilities are represented by a single zero-coupon bond with a 1-year maturity. Thus, the default threshold is exogenously set to the bond's principal and the default event can only occur at the bond's maturity (Merton 1974).

[62] Itô's lemma is a crucial tool for dealing with stochastic differential equations, compare Franke et al. (2011) and Hull (2009).

$U_{i,t}$ is normally distributed (Hull 2009)

$$U_{i,t} = \ln\left(\frac{G_{i,t}}{G_{i,0}}\right) = \ln G_{i,t} - \ln\ G_{i,0} \sim \mathcal{N}\left(\left(\mu_i - 0.5 \cdot \sigma_i^2\right) \cdot t, \sigma_i^2 \cdot t\right) \tag{3.6}$$

and thus the normalized asset return $Z_{i,t}$ of firm i is given by

$$Z_{i,t} = \frac{U_{i,t} - (\mu_i - 0.5 \cdot \sigma_i^2) \cdot t}{\sigma_i \cdot \sqrt{t}}. \tag{3.7}$$

Under consideration of the initial leverage ratio $\frac{P_i}{G_{i,0}}$, the normalized default threshold $c_{i,t}$ is

$$c_{i,t} = \frac{\ln\left(\frac{P_i}{G_{i,0}}\right) - (\mu_i - 0.5 \cdot \sigma_i^2) \cdot t}{\sigma_i \cdot \sqrt{t}}. \tag{3.8}$$

Based on Equations (3.5), (3.7) and (3.8), the physical probability of default $\pi_{i,t}$ is given by

$$\begin{aligned}\pi_{i,t} &= \mathbb{P}\left(G_{i,t} < P_i\right) = \mathbb{P}\left[\ln\left(\frac{G_{i,t}}{G_{i,0}}\right) < \ln\left(\frac{P_i}{G_{i,0}}\right)\right] = \mathbb{P}\left(Z_{i,t} < c_{i,t}\right) \\ &= \Phi\left(c_{i,t}\right)\end{aligned} \tag{3.9}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution (compare Leland 2004).

At maturity t, the bondholders receive their principal in full if $G_{i,t} > P_i$ and hence no default occurred. In case of $G_{i,t} < P_i$, the bondholders liquidate the firm's assets and receive solely the recovery rate. Thus, they suffer from losses on the zero-bond, that are in value $P_i - G_{i,t}$.

From the pricing perspective, this structural model setup provides an attractive approach for the valuation of credit risk, since the credit risky debt claim (zero-bond) can be replicated by a long position in a risk-free zero-bond with maturity t plus a short position in a put option sold to equity holders. The put option allows the equity holders at maturity t to put the firm at the strike price. Several authors extended this standard approach in the following years. For example, Black & Cox (1976) and Longstaff & Schwartz (1995) developed the so-called first-passage model by incorporating stochastic default times before maturity. Another extension refers to stochastic risk-free interest rates as proposed by Kim et al. (1993), Longstaff & Schwartz (1995) or Barnhill

& Maxwell (2002).[63] However, in most structural models the credit spreads for short maturities tend to zero which is often not compatible with empirical observations (compare Uhrig-Homburg 2002).[64] While structural models have the advantage of sound economic underpinning, the disadvantage is that they are difficult to calibrate to market data, e.g., credit spreads, and usually have to be implemented via Monte Carlo simulations (Hull & White 2008).

3.2.3 Intensity Models

Another approach to model default times is represented by so-called intensity models. Firstly introduced by Jarrow & Turnbull (1995) and Artzner & Delbaen (1995), a stochastic process is applied to model the default time of an obligor, which consequently comes randomly (Uhrig-Homburg 2002, Giesecke 2004).[65] Since intensity models allow to model default times for any time horizons, this model type enables short-term defaults (Li 2000). Thus, intensity models can be applied to provide short-term credit spreads, which are empirically observable, and eases model fits on empirical data. In contrast to structural models, the causality of the default event is not explicitly modeled along economic theory. Due to this reduction of complexity, intensity based models are often called *reduced form* models, which is instantly plausible (compare Duffie & Singleton 1999). Generally, default events are triggered by surprising jump processes which are rather formally described.

In this context, T_i denotes a nonnegative, continuous random variable describing the random default time of obligor $i \in \{i = 1, ..., I\}$. The random

[63] Note that the expected rate of return $\mu_{i,t}$ in Equation (3.2) depends on the risk preferences of investors. The higher the investor's level of risk aversion the higher $\mu_{i,t}$ of firm i will be (Hull 2009). Because $\mu_{i,t}$ drops out in the derivation of the Black-Scholes-Merton differential equation, the differential equation is independent of risk preferences and any set of risk preferences can be used in this framework (compare Hull 2009). In particular, the assumption that all investors are risk-neutral. In a risk-neutral world, the expected return on firm i equals the risk-free interest rate r_t ($\mu_{i,t} = r_t$) and r_t can be modeled stochastically (Hull 2009).

[64] In continuous-time structural models that are based on a diffusion process for the asset value, credit spreads decline to zero as the maturity goes to zero (Uhrig-Homburg 2002). Since basic Merton-type (or structural) models ignore the possibility of short-term or early defaults (before the product's maturity), they are mainly used by the financial industry to estimate 1-year default probabilities. First-passage-time models can be used to estimate default probabilities over any time horizons (Zhou 2001). By extending the structural approach, for example, Zhou (2001) develops such a first-passage-time model for credit portfolios.

[65] While the default time becomes predictable in structural models, it purely is a random event (prescribed exogenously) in intensity models (Uhrig-Homburg 2002).

default time is modeled via the stochastic default indicator $D_{i,t}$, where

$$D_{i,t} = \mathbb{1}_{\{T_i \leq t\}} = \begin{cases} 1 & \text{if} \quad T_i \leq t, \\ 0 & \text{else} \end{cases} \tag{3.10}$$

is a point process with one jump of size one at default (see Giesecke 2004).

If $f_i(t)$ denotes the density and $F_i(t)$ the distribution function of random default time T_i, then

$$F_i(t) = \mathbb{P}(T_i \leq t) = \mathbb{P}(D_{i,t} = 1) = \int_0^t f_i(u)du, \tag{3.11}$$

and from Equation (3.11) follows the corresponding survivor function with

$$S_i(t) = \mathbb{P}(T_i > t) = 1 - F_i(t), \tag{3.12}$$

see Meeker & Escobar (1998). The hazard rate $h_i(t)$ is defined as the marginal conditional probability of default in the time interval $(t, t+\Delta t]$ given the obligor's survival to time t with

$$\begin{aligned} h_i(t) &= \lim_{\Delta t \to 0, \Delta t > 0} \frac{1}{\Delta t} \mathbb{P}(t < T_i \leq t + \Delta t \mid T_i > t) \\ &= \frac{f_i(t)}{S_i(t)} = \frac{f_i(t)}{1 - F_i(t)}, \end{aligned} \tag{3.13}$$

and thus it describes the immediate default risk of obligor i being 'alive', i.e., not defaulted, at time t (compare Meeker & Escobar 1998, Li 2000).

The hazard rate $h_i(t)$ or instantaneous failure rate may also be interpreted as default intensity and thus hazard models are often called *intensity* models as well. The cumulative hazard rate follows from

$$\begin{aligned} H_i(t) &= \int_0^t h_i(u)du = \int_0^t \frac{f_i(u)}{1 - F_i(u)} du \\ &= -\ln\left[1 - F_i(t)\right] = -\ln S_i(t) \end{aligned} \tag{3.14}$$

and therefore

$$F_i(t) = 1 - \exp\left(-\int_0^t h_i(u)du\right) \tag{3.15}$$

and

$$f_i(t) = h_i(t) \cdot \exp\left(-\int_0^t h_i(u)du\right), \tag{3.16}$$

compare Meeker & Escobar (1998) and Li (2000). In practice, default intensities are often extracted from the credit spread curves of traded financial instruments such as CDS or corporate bonds (Martin et al. 2006). In these cases, the default intensities refer to the risk-neutral probability measure $\mathbb{Q}$ (compare Section 3.2.1).[66] Eventually, the setup of an intensity model may refer to several sources such as ratings (compare Das & Tufano 1996, Jarrow et al. 1997), stock prices (see Madan & Unal 1998) and other state variables (compare Lando 1998, Duffie & Singleton 1999) to account for, e.g., time-variant default intensities reflected by market data.[67]

In contrast, Jarrow & Turnbull (1995) assume a constant intensity under the real-world probability measure. Under a constant default intensity $h_i(t) = h_i,\ t \geq 0, h_i > 0$, the default time T_i is exponentially distributed with distribution function

$$F_i(t) = 1 - \exp(-h_i \cdot t) \tag{3.17}$$

and density

$$f_i(t) = h_i \cdot \exp(-h_i \cdot t), \tag{3.18}$$

[66] If calibrated on market data referring to the risk-neutral valuation (no arbitrage) the default indicator $D_{i,t}$ (submartingale) has to be transformed since the default process has an upward trend (compare Uhrig-Homburg 2002, Giesecke 2004). The transformation of $D_{i,t}$ into a martingale follows the Doob-Meyer decomposition. For a throughout description of this decomposition compare Dellacherie & Meyer (1978).

[67] Compare Uhrig-Homburg (2002) for an insightful overview referring to the applications of reduced-form models and structural models as well.

see Li (2000).[68]

In practice, the parameter specifications of structural models are often not practicable. In such cases, it may be more convenient for practitioners to estimate the parameters of an intensity model (Uhrig-Homburg 2002). With respect to empirical applications, it is convenient to model discrete-time defaults and to divide the time line $(0, \infty)$ into $k+1$ time intervals

$$(p_0, p_1], (p_1, p_2], ..., (p_{k-1}, p_k], (p_k, p_{k+1}), \tag{3.19}$$

where p_0 usually equals 0 and $p_{k+1} = \infty$ (compare Meeker & Escobar 1998).[69] Note that the time intervals need not be of equal length. The last interval is of infinite length. Thereby, the time interval $t \in \{1, 2, ..., k\}$ is denoted by $(p_{t-1}, p_t]$ and describes, for example, a day or a year. Thus, T_i becomes an integer random variable and $T_i = t$ indicates that the obligor's default has occurred during the interval $(p_{t-1}, p_t]$.

The unconditional discrete-time probability of default $\pi^d_{i,t}$ states that obligor i defaults in interval t without any conditions on its survival and is defined by

$$\pi^d_{i,t} = \mathbb{P}(T_i = t) = \mathbb{P}(p_{t-1} < T_i \leq p_t) = F_i(p_t) - F_i(p_{t-1}) \tag{3.20}$$

with $\pi^d_{i,t} \geq 0$, $\sum_{j=1}^{k+1} \pi^d_{i,j} = 1$, and

$$S^d_{i,t} = \mathbb{P}(T_i > t) = S^d_i(p_t) = \mathbb{P}(T_i > p_t) = 1 - F_i(p_t) \tag{3.21}$$

denotes the discrete-time survivor function evaluated at time interval t with $S^d_i(p_0) = 1$ (compare Meeker & Escobar 1998).

Analogous to Equation (3.13), the discrete-time hazard rate $h^d_{i,t} \in [0, 1]$ can then be defined by

$$\begin{aligned} h^d_{i,t} = h^d_i(p_t) &= \mathbb{P}(p_{t-1} < T_i \leq p_t | T_i > p_{t-1}) \\ &= \frac{F_i(p_t) - F_i(p_{t-1})}{1 - F_i(p_{t-1})} = \frac{\pi^d_{i,t}}{S^d_i(p_{t-1})} \end{aligned} \tag{3.22}$$

indicating that the default of obligor i occurs in period t conditional on the

[68] The assumption of a constant default intensity implies that the credit quality of a bond does not change up to the default event. Changing credit qualities can be considered by intensity models that incorporate time-variant default intensities (see Uhrig-Homburg 2002).

[69] Note that failure-time data are always discrete (Meeker & Escobar 1998).

obligor's survival until the beginning of period t (see Meeker & Escobar 1998).

The physical probability of default $\pi_{i,t}$ denoted in Equation (3.5) is also conditional on the fact that obligor i has not defaulted before. Thus, the discrete-time hazard rate $h_{i,t}^d$ for time period t equals $\pi_{i,t}$. Due to this discrete-time setting, $\pi_{i,t}$ is rather called probability of default than default intensity (see Rösch 2004).

The survivor function is denoted by

$$S_{i,t}^d = \prod_{j=1}^{t} (1 - h_{i,j}^d) \tag{3.23}$$

and the cumulative distribution function of T_i, evaluated at t, can be expressed as

$$\begin{aligned} F_{i,t}^d &= 1 - \prod_{j=1}^{t} (1 - h_{i,j}^d) \\ &= \sum_{j=1}^{t} \pi_{i,j}^d \end{aligned} \tag{3.24}$$

with $F_{i,t}^d = F_i(p_t)$, compare Meeker & Escobar (1998).

Although default times are not endogenously modeled in the classical model setup, intensity models do not generally obviate economic intuitions. Among others, Duffie & Gârleanu (2001), Rösch (2004), Longstaff & Rajan (2008), and Hull & White (2008) show that several sources of default risk approximated by, e.g., firm-specific fundamentals or macroeconomic variables, can be easily embedded into such intensity approaches through the application of a factor model setup which is described in the following section.

3.2.4 Factor Models

The introduction of the first well-known factor models dates back to the early beginnings of the capital market theory. Two of the most popular factor models are the Capital Asset Pricing Model introduced by Sharpe (1964) and the Arbitrage Pricing Theory introduced by Ross (1976).

Factor models for credit risk can be regarded as a special case of intensity models and may be established in either a continuous-time or a discrete-time framework (Rösch 2004). Generally, the description of factor models arises from the separation of embedded risk factors and the incorporation of systematic risk. Hence, factor models are typically characterized by the distinction between firm-specific (idiosyncratic) risk and systematic risk, where the latter is typically assumed to affect all firms in the model setup (see Bluhm et al. 2003).[70] The dependence between individual default events of firms is driven by systematic risk which is approximated by one or more unobservable systematic risk factors (Schönbucher 2001). Thus, factor models are a well established technique for identifying common drivers of correlated defaults and for reducing the computational effort related to the calculation of correlated defaults and losses (compare Bluhm et al. 2003). They provide a practicable framework to interpret default correlations in terms of economic variables and also allow to explain higher default rates in economic downturns in a sound manner (Schönbucher 2001, Bluhm et al. 2003).

In a one-factor or single-factor model, the default events of firms are independent from each other conditional on a realization of the systematic risk factor Y_t.[71] In this framework, the default indicator $D_{i,t}$ conditional on a realization of $Y_t = y_t$ is a Bernoulli random variable with

$$D_{i,t}|Y_t = y_t \sim \text{Ber}\left[\mathbb{E}^{\mathbb{P}}(D_{i,t}|Y_t = y_t)\right] \tag{3.25}$$

for all $i \in \{1, ..., I\}$ and $t \in \{1, ..., T\}$, compare Bluhm et al. (2003).[72] Thereby, the conditional expectations of $D_{i,t}|Y_t = y_t$ are given by

$$\mathbb{E}^{\mathbb{P}}(D_{i,t}|Y_t = y_t) = \pi_{i,t}(y_t) \tag{3.26}$$

and the variance by

$$\text{Var}^{\mathbb{P}}(D_{i,t}|Y_t = y_t) = \pi_{i,t}(y_t) \cdot [1 - \pi_{i,t}(y_t)], \tag{3.27}$$

compare Martin et al. (2006). Thus, the default indicator $D_{i,t}|Y_t = y_t$ equals one with probability $\pi_{i,t}(y_t)$, and zero with probability $1 - \pi_{i,t}(y_t)$.

[70] Note that the included risk factors can also represent other non-firm-specific risks such as sectoral or industry risk and country-specific risk (see Bluhm et al. 2003).

[71] In Chapter 5, an analytical study is presented in which the systematic risk factor of the simple one-factor approach is decomposed into a linear combination of two risk factors, similar to Gordy (2003). Analogously, the single-risk factor can linearly be composed of multiple risk factors, see Koyluoglu & Hickman (1998).

[72] For a good description of Bernoulli-type random variables compare also Giesecke (2004), Martin et al. (2006) or Franke et al. (2011).

While the density of the systematic risk factor Y_t is denoted by $f(y_t)$, expectations of the conditional default probability lead to the unconditional probability of default $\pi_{i,t}$ with

$$\begin{aligned}\pi_{i,t} &= \mathbb{E}^{\mathbb{P}}(D_{i,t}) = \mathbb{E}^{\mathbb{P}}\left[\mathbb{E}^{\mathbb{P}}(D_{i,t}|y_t)\right] = \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y)\right] \\ &= \int_{-\infty}^{+\infty} \pi_{i,t}(y_t) \cdot f(y_t)\; dy_t, \qquad (3.28)\end{aligned}$$

compare Martin et al. (2006). Note that the default probability $\pi_{i,t}$ is unconditional on Y_t, but conditional on the fact that obligor i has not defaulted before t (compare Section 3.2.3.). In the following, the terms *conditional* and *unconditional* are only related to the systematic risk factor Y_t.

According to Bluhm et al. (2003), the unconditional variance is given by

$$\begin{aligned}\mathrm{Var}^{\mathbb{P}}(D_{i,t}) &= \mathrm{Var}^{\mathbb{P}}\left[\mathbb{E}^{\mathbb{P}}\left(D_{i,t}|y_t\right)\right] + \mathbb{E}\left[\mathrm{Var}^{\mathbb{P}}\left(D_{i,t}|y_t\right)\right] \\ &= \mathrm{Var}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right] + \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\cdot(1-\pi_{i,t}(Y_t))\right] \\ &= \mathrm{Var}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right] + \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right] - \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)^2\right] \\ &= \mathrm{Var}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right] + \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right] - \left[\mathrm{Var}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right] + \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right]^2\right] \\ &= \pi_{i,t}\cdot(1-\pi_{i,t}). \qquad (3.29)\end{aligned}$$

As an alternative to the intensity-based view, the representation of a factor model can be based on the structural approach. Recall that in a structural setup, the default of obligor i occurs at time t when the latent asset return $Z_{i,t}$ falls below some critical threshold $c_{i,t}$, see Equation (3.9). In a factor-model specification, the default threshold $c_{i,t}$ may also vary across borrowers i and over time t. The latent asset return $Z_{i,t}$ is then specified as a function of a common systematic risk factor Y_t and a borrower-specific (idiosyncratic) risk factor $E_{i,t}$

$$Z_{i,t} = Z(Y_t, E_{i,t}) \qquad (3.30)$$

with $i \in \{1, ..., I\}$ and $t \in \{1, ..., T\}$. Even though factor models are jointly characterized by common systematic risk factors, they may differ in terms of their default specification, the amount of incorporated risk factors and the factors' distributional assumptions.[73]

The presented three approaches to credit risk may primarily be applied to model the default risk of a single borrower. These approaches may also represent the basic elements for evaluating portfolio credit risk, which is generally related to a portfolio of debt assets (loans, bonds etc.) referring to numerous borrowers. Measuring credit portfolio risk appropriately is also essential for the valuation of asset securitizations since structured securities are basically related to portfolios or baskets of debt claims (see, e.g., Schönbucher 2001).

As already indicated in Section 3.2.3, intensity-based models are often preferred due to their convenient calibration process on empirical data. In this context, both presented approaches to credit risk – intensity-based and structural-based – can be easily translated to each other (see, e.g., Li 2000), then

$$\mathbb{P}(T_i \leq t) \Leftrightarrow \mathbb{P}(Z_{i,t} < c_{i,t}). \tag{3.31}$$

Referring to Equivalence (3.31) and Equations (3.15) and (3.17), it follows under the Gaussian specification of $Z_{i,t} \sim \mathcal{N}(0,1)$ in terms of a time-variant default intensity

$$\Phi(c_{i,t}) = 1 - \exp\left(-\int\limits_0^t h_i(u)du\right) \tag{3.32}$$

or correspondingly

$$c_{i,t} = \Phi^{-1}\left(1 - \exp\left(-\int\limits_0^t h_i(u)du\right)\right), \tag{3.33}$$

and in terms of a time-constant default intensity

$$c_{i,t} = \Phi^{-1}\left(1 - \exp(-h_i \cdot t)\right), \tag{3.34}$$

[73] Relying on a single-factor structure, Hamerle & Rösch (2006) compare three popular model specifications for portfolio credit risk in terms of 'model risk'. Based on this earlier work, Hamerle et al. (2011) use a multi-factor credit risk model with observable macroeconomic and latent variables to explain individual default risk. Thus, observable variables are incorporated to economically 'explain' unobservable risk factors.

compare Martin et al. (2006).[74] Φ^{-1} denotes the inverse of the standard normal distribution function Φ.

Particularly, the relation in Equivalence (3.31) is used in many empirical studies which examine the pricing of classic CDOs or STCDOs (see Laurent & Gregory 2005, Hull & White 2004, Finger 2009), as it will be further described in the following sections. Especially for valuing structured securities, default thresholds are often extracted from market data under the assumption that default intensities are constant over the products' maturity (compare Hull & White 2004, 2006).[75]

In the next section, the most established valuation framework for structured securities is presented in the context of credit portfolio risk. Thereby, basic concepts accounting for dependency structures within credit portfolios are described since these dependency structures represent one of the most crucial risk parameters determining the portfolio's loss distribution (compare Schönbucher 2001, Bluhm et al. 2003). Recall that the loss distribution reflects the credit risk inherent in such a credit portfolio and is therefore determining the risk characteristics of securitized tranches.

3.3 Portfolio Credit Risk and the Valuation of STCDOs

3.3.1 The Single-factor Gaussian Copula Model

Real-world scenarios such as global economic downturns show that sometimes the number of borrower defaults is jointly increasing in and across several economic sectors (see, e.g., Bluhm et al. 2003). Since a credit portfolio typically consists of debt assets from numerous borrowers, correlated borrower defaults are an important aspect for measuring portfolio credit risk (Schönbucher 2001). Apart from global economic downturns, joint borrower defaults may generally indicate the existence of dependency structures across obligors in a credit portfolio which may be caused by i) direct links between obligors referring to, e.g., credit guarantees, or other contractual relationships, and ii)

[74] Note that the time horizon t, $t > 0$, has to be fixed for the model calibration, since classic Merton-type models refer to a specified point in time, e.g., one year or the products maturity (compare Section 3.2.2).

[75] An empirical application for valuing STCDOs based on these preliminaries is presented in Chapter 4.

indirect links such as sectoral influences (demand or price shocks), or general states of an industry (compare Schönbucher 2001). Particularly, for the consideration of dependency structures between borrowers in a credit portfolio common risk factors become central (compare Section 3.2.4). In this context, the Gaussian single-factor model is the most popular approach for measuring credit portfolio risk and it also represents the market standard model for pricing portfolio credit derivatives such as STCDOs (see Hull & White 2006, Finger 2009).

Firstly applied by Vasicek (1987) to aggregate credit risk of credit portfolios and further analyzed by Gordy (2000, 2003), Laurent & Gregory (2005), Rösch & Winterfeldt (2008), Bade et al. (2011), Rösch & Scheule (2012), the Gaussian single-factor model constitutes the major element of the regulatory capital formula under the Basel II Capital Accord to calculate risk-weighted capital requirements (BIS 2005).[76]

Based on Merton's (1974) structural approach (see Section 3.2.2), obligor i defaults on his bond if the return on his assets $Z_{i,t}$ undergoes a critical threshold $c_{i,t}$ at time t, then

$$D_{i,t} = 1 \Leftrightarrow Z_{i,t} < c_{i,t}. \tag{3.35}$$

In a factor-model setup, the asset return $Z_{i,t}$ is driven by a common risk factor $Y_t \sim \mathcal{N}(0,1)$ which represents, e.g., macroeconomic influences jointly affecting all obligors $i \in \{1, ..., I\}$ in an economy, and an individual risk component $E_{i,t} \sim \mathcal{N}(0,1)$ only affecting borrower i (idiosyncratic risk). Y_t and $E_{i,t}$ are independent and identically distributed (i.i.d). $\rho \in [0,1]$ determines the exposure to systematic risk, which is equal for each obligor in this simple specification.[77] Then, the latent asset return $Z_{i,t}$ at time $t \in \{1, ..., T\}$ is modeled by

$$Z_{i,t} = \sqrt{\rho} \cdot Y_t + \sqrt{1-\rho} \cdot E_{i,t}, \tag{3.36}$$

and also standard normal distributed with $Z_{i,t} \sim \mathcal{N}(0,1)$. Conditional on a realization of the common risk factor Y_t, the asset returns of borrowers are i.i.d. random variables due to the independence of idiosyncratic risk factors.

[76] In the Basel II framework for regulatory capital requirements, this model is also called the Asymptotic Single Risk Factor (ASRF) model (BIS 2005).

[77] For ease of exposition only, the weighting factor ρ is assumed to be time-constant until maturity with $\rho_t = \rho$. Thus, index t is not carried.

The conditional probability of default follows from Equation (3.36) with

$$\begin{aligned}\pi_{i,t}(y_t) = \mathbb{P}(Z_{i,t} < c_{i,t}|y_t) &= \mathbb{P}\left(E_{i,t} < \frac{c_{i,t} - \sqrt{\rho}\cdot y_t}{\sqrt{1-\rho}}\right)\\ &= \Phi\left(\frac{c_{i,t} - \sqrt{\rho}\cdot y_t}{\sqrt{1-\rho}}\right), \qquad (3.37)\end{aligned}$$

see, e.g., Bluhm et al. (2003). Based on the following identity

$$\int\limits_{-\infty}^{+\infty} \Phi(\alpha y + \beta)\cdot\varphi(y)dy = \Phi\left(\frac{\beta}{\sqrt{1+\alpha^2}}\right) \qquad (3.38)$$

with

$$\alpha = -\frac{\sqrt{\rho}}{\sqrt{1-\rho}} \quad \text{and} \quad \beta = \frac{c_{i,t}}{\sqrt{1-\rho}},$$

and related to Equations (3.37) and (3.28) the unconditional probability of default is given as expectations over all realizations of the common factor Y_t

$$\begin{aligned}\pi_{i,t} &= \mathbb{E}^{\mathbb{P}}\left[\mathbb{E}^{\mathbb{P}}(D_{i,t}|y_t)\right] = \mathbb{E}^{\mathbb{P}}\left[\pi_{i,t}(Y_t)\right]\\ &= \int\limits_{-\infty}^{\infty} \Phi\left(\frac{c_{i,t} - \sqrt{\rho}\cdot y_t}{\sqrt{1-\rho}}\right)\cdot\varphi(y_t)\ dy_t\\ &= \Phi(c_{i,t}), \qquad (3.39)\end{aligned}$$

where $\varphi(\cdot)$ denotes the Gaussian density function (compare Martin et al. 2006).

Due to i) the independence of systematic and idiosyncratic risk factors and ii) the standard normal distributed asset returns $Z_{i,t} \sim \mathcal{N}(0,1)$, the asset correlation $\rho_{i,j}$ is identical across all pairs of borrowers in this model setup, which is given by

$$\begin{aligned}\rho_{i,j} = \mathrm{Corr}^{\mathbb{P}}(Z_{i,t}, Z_{j,t}) &= \frac{\mathbb{E}^{\mathbb{P}}(Z_{i,t}\cdot Z_{j,t}) - \mathbb{E}^{\mathbb{P}}(Z_{i,t})\cdot\mathbb{E}^{\mathbb{P}}(Z_{j,t})}{\sqrt{\mathrm{Var}^{\mathbb{P}}(Z_{i,t})}\cdot\sqrt{\mathrm{Var}^{\mathbb{P}}(Z_{j,t})}}\\ &= \rho\cdot\mathbb{E}^{\mathbb{P}}\left(Y_t^2\right)\\ &= \rho\cdot\left[\mathbb{E}^{\mathbb{P}}\left(Y_t\right)\cdot\mathbb{E}^{\mathbb{P}}\left(Y_t\right) + \mathrm{Cov}^{\mathbb{P}}\left(Y_t, Y_t\right)\right]\\ &= \rho \qquad (3.40)\end{aligned}$$

for all $i \neq j$.[78]

In Vasicek's basic model specification, the credit portfolio consists of homogeneous loans, i.e. each loan exhibits the same risk characteristics in terms of default risk and related losses, and the credit portfolio is additionally assumed to be infinitely granular. Furthermore, the borrowers' asset returns are correlated with coefficient ρ for any two borrowers, as shown in Equation (3.40).[79] This leads to a large homogeneous credit portfolio (LHP) which is solely exposed to systematic risk since idiosyncratic risks are fully diversified, see Gordy (2003).[80] Then, the density of the percentage loss L_t on the portfolio at time t is given by

$$v(l_t) = \sqrt{\frac{1-\rho}{\rho}} \cdot \exp\left(-\frac{\left(\sqrt{1-\rho}\cdot\Phi^{-1}(l_t) - \Phi^{-1}(\pi_t)\right)^2}{2\cdot\rho} + \frac{\left(\Phi^{-1}(l_t)\right)^2}{2} \right). \tag{3.41}$$

π_t describes the default probability of the LHP, which equals the homogeneous default probability $\pi_{i,t}$ of a single loan in the credit portfolio ($\pi_{1,t} = ... = \pi_{I,t} = \pi_t$ with $I \to \infty$), and due to the law of large numbers also the expected default rate of the LHP (Vasicek 1987, Bluhm et al. 2003). According to Vasicek (1987, 1991), the cumulative probability that the percentage loss of the LHP does not exceed $L_t \in [0,1]$ follows from the cumulative distribution function

$$V(l_t) = \Phi\left(\frac{\sqrt{1-\rho}\cdot\Phi^{-1}(l_t) - \Phi^{-1}(\pi_t)}{\sqrt{\rho}} \right). \tag{3.42}$$

According to BIS (2005) and Franke et al. (2011), the expected loss $\mathbb{E}^{\mathbb{P}}(L_{i,t})$ of a single borrower i at time t is a random variable, which is determined by

[78] Note that asset correlations may generally vary among borrowers in factor models, but in this model specification, the asset correlation ρ is homogeneous and ranges between 0 and 1 due to the factor-model assumptions (see also Bluhm et al. 2003). For a good description of a generalized framework to calculate asset correlations in factor models compare Bluhm et al. (2003). Furthermore, homogeneous asset correlations can easily be relaxed in an expanded factor-model approach, as shown in Chapter 5, where several cross-correlation concepts, e.g., inter-sectoral asset correlations, are applied to account for different asset correlations among borrowers.

[79] Limitations of such a model setup are discussed in the context of the empirical applications in Chapters 4 and 5.

[80] If the number of credit risky assets in a portfolio is large and if the credit exposures of these assets are relatively small (granularity), then idiosyncratic risks – which are generally associated with individual exposures – tend to cancel out one another (BIS 2005). Eventually, only systematic risks have a material effect on portfolio losses and thus may be compensated through a respective risk premium.

the exposure at default $EAD_{i,t}$, the loss given default $LGD_{i,t} = 1 - R_{i,t}$ with recovery rate $R_{i,t}$, and the probability of default $\pi_{i,t} = \mathbb{E}^{\mathbb{P}}(D_{i,t})$ representing the expectations of the default indicator $D_{i,t}$:[81]

$$\mathbb{E}(L_{i,t}) = EAD_{i,t} \cdot LGD_{i,t} \cdot \pi_{i,t} \tag{3.43}$$

where $L_{i,t}$ describes the loss of obligor i at time t

$$L_{i,t} = EAD_{i,t} \cdot LGD_{i,t} \cdot D_{i,t}. \tag{3.44}$$

Based on the additivity of expectations (Bluhm et al. 2003), the expected loss $\mathbb{E}^{\mathbb{P}}(L_{P,t})$ of a credit portfolio P is given by

$$\mathbb{E}^{\mathbb{P}}(L_{P,t}) = \sum_{i=1}^{I} EAD_{i,t} \cdot LGD_{i,t} \cdot \pi_{i,t}, \tag{3.45}$$

where the credit portfolio P contains debt claims of borrowers i, $i \in \{1, ..., I\}$.

Let the weight $\omega_{i,t}$ of each credit exposure $EAD_{i,t}$ in the entire credit portfolio be defined by

$$\omega_{i,t} = \frac{EAD_{i,t}}{\sum\limits_{j=1}^{I} EAD_{j,t}}. \tag{3.46}$$

Then, the loss rate of the portfolio L_t is a random variable given by

$$L_t = \sum_{i=1}^{I} \omega_{i,t} \cdot LGD_{i,t} \cdot D_{i,t}, \tag{3.47}$$

and the expected loss rate follows from

$$\mathbb{E}^{\mathbb{P}}(L_t) = \sum_{i=1}^{I} \omega_{i,t} \cdot LGD_{i,t} \cdot \pi_{i,t}. \tag{3.48}$$

According to Martin et al. (2006), the conditional expected loss rate of a

[81] Although $EAD_{i,t}$ and $R_{i,t}$ may generally be stochastic, both measures are regarded as known parameters (deterministic) for ease of expositions.

credit portfolio is obtained by

$$\mathbb{E}^{\mathbb{P}}(L_t|y_t) = \sum_{i=1}^{I} \omega_{i,t} \cdot LGD_{i,t} \cdot \pi_{i,t}(y_t). \tag{3.49}$$

Let $\pi_{1,t} = ... = \pi_{I,t} = \pi_t$, $LGD_{1,t} = ... = LGD_{I,t} = 1$ and $EAD_{1,t} = ... = EAD_{I,t} > 0$ with $I \to \infty$, reflecting the assumptions of the LHP, then

$$\mathbb{E}^{\mathbb{P}}(L_t|y_t) = \pi_t(y_t), \tag{3.50}$$

and the unconditional expected loss rate of the portfolio is

$$\mathbb{E}^{\mathbb{P}}(L_t) = \mathbb{E}^{\mathbb{P}}\left[\mathbb{E}^{\mathbb{P}}(L_t|y_t)\right] = \pi_t \tag{3.51}$$

where π_t equals the unconditional default probability of a single debt asset within the homogeneous credit portfolio (Vasicek 1987). Under these assumptions and based on Equation (3.36), the expected loss rate of the portfolio L_t conditional on a realization of $Y_t = y_t$ is given by

$$\mathbb{E}^{\mathbb{P}}(L_t|Y_t = y_t) = \Phi\left(\frac{\Phi^{-1}(\pi_t) - \sqrt{\rho} \cdot y_t}{\sqrt{1-\rho}}\right). \tag{3.52}$$

The variance of the conditional loss rate in a homogeneous portfolio is

$$\begin{aligned}\mathrm{Var}^{\mathbb{P}}(L_t|Y_t = y_t) &= \sum_{i=1}^{I} (\omega_{i,t})^2 \cdot \ \mathrm{Var}^{\mathbb{P}}\left(D_{i,t} = 1|Y_t = y_t\right) \\ &= \sum_{i=1}^{I} (\omega_{i,t})^2 \cdot \pi_t(Y_t) \cdot [1 - \pi_t(Y_t)],\end{aligned} \tag{3.53}$$

where $\mathrm{Var}^{\mathbb{P}}(L_t|Y_t = y_t) \to 0$ if $I \to \infty$, compare Martin et al. (2006).[82] Furthermore, Martin et al. (2006) show that the variance of portfolio losses is

[82] For the additivity of variances it is sufficient that the involved random variables are pairwise uncorrelated and integrable (Bluhm et al. 2003). In this thesis, the independence of involved random variables is assumed only for ease of this exercise. For example, Pykhtin (2003) provides a model approach considering dependency structures between recovery rates and default probabilities. This model is extended and empirically tested by Bade et al. (2011).

given by

$$\begin{aligned}
\mathrm{Var}^{\mathbb{P}}(L_{P,t}) &= \sum_{i,j=1}^{I} \mathrm{Cov}^{\mathbb{P}}(L_{i,t}, L_{j,t}) \\
&= \sum_{i,j=1}^{I} EAD_{i,t} \cdot LGD_{i,t} \cdot \mathrm{Cov}^{\mathbb{P}}(D_{i,t}, D_{j,t}) \\
&= \sum_{i=1}^{I} EAD_{i,t} \cdot LGD_{i,t} \cdot \mathrm{Var}^{\mathbb{P}}(D_{i,t}) + \sum_{i,j;i\neq j}^{I} \varrho_{i,j,t} \cdot \sigma_{i,t} \cdot \sigma_{j,t} \qquad (3.54)
\end{aligned}$$

with $\varrho_{i,j,t}$ denoting the default correlation between obligor i and j at time t

$$\begin{aligned}
\varrho_{i,j,t} = \mathrm{Corr}^{\mathbb{P}}(D_{i,t}, D_{j,t}) &= \frac{\mathrm{Cov}^{\mathbb{P}}(D_{i,t}, D_{j,t})}{\sqrt{\mathrm{Var}^{\mathbb{P}}(D_{i,t}) \cdot \mathrm{Var}^{\mathbb{P}}(D_{j,t})}} \\
&= \frac{\mathbb{E}^{\mathbb{P}}(D_{i,t} \cdot D_{j,t}) - \pi_{i,t} \cdot \pi_{j,t}}{\sqrt{\pi_{i,t} \cdot (1 - \pi_{i,t}) \cdot \pi_{j,t} \cdot (1 - \pi_{j,t})}}, \qquad (3.55)
\end{aligned}$$

and

$$\sigma_{x,t} = EAD_{x,t} \cdot LGD_{x,t} \cdot \sqrt{\mathrm{Var}^{\mathbb{P}}(D_{x,t})}, \qquad (3.56)$$

where $\mathrm{Var}^{\mathbb{P}}(D_{x,t}) = \pi_{x,t} \cdot (1 - \pi_{x,t})$, $x \in \{i, j\}$. The standard deviation $\sqrt{\mathrm{Var}^{\mathbb{P}}(L_{P,t})}$ measures the magnitude of the deviation of portfolio losses from the expected portfolio loss. As deviation from expectations, the standard deviation may be seen as measure for the inherent credit risk of a credit portfolio. Thus, it is commonly used to quantify unexpected losses $UL_{P,t}$ of a credit portfolio P at time t

$$UL_{P,t} = \sqrt{Var^{\mathbb{P}}(L_{P,t})}, \qquad (3.57)$$

see Bluhm et al. (2003) and Martin et al. (2006). As Equation (3.54) shows, default dependencies play a fundamental role for quantifying portfolio credit risk and they represent main drivers for credit risk (Bluhm et al. 2003). For this reason, the next section briefly introduces the copula approach for modeling joint default times of borrowers in a credit portfolio, which is also widely used for quantifying default correlations.

3.3.2 Modeling Dependency Structures: The Copula Approach

Default probabilities and dependency structures such as default and asset correlations play a crucial role in commercial credit risk models, even in the Basel II Capital Accord (see Hamerle et al. 2003, Duellmann et al. 2010). Consequently, correlations – in particular default correlations – are also critical for the valuation of credit derivatives such as CDS or CDOs, which are regarded as the most popular portfolio credit derivatives (see Hull & White 2008).

Valuing asset securitizations usually refers to the valuation of cash flows linked to a basket of reference assets such as loans, bonds or CDS. These cash flows strongly depend on the ensemble of asset-specific default times T_i with $i \in \{1, ..., I\}$ that can be denoted in the random vector $\mathcal{T} = (T_1, ..., T_I)$. In order to determine the multivariate distribution function F of random default times T_i in such a random vector

$$F(t_1, ..., t_I) = \mathbb{P}(T_1 \leq t_1, ..., T_I \leq t_I) \tag{3.58}$$

Li (2000), Frey et al. (2001), Bluhm et al. (2003) and Hamerle & Rösch (2005) among others transferred a copula framework to credit risk. Particularly for the valuation of CDOs, Li (2000) firstly proposed some common copula functions, e.g., the Gaussian Copula and Archimedian Copulae, to compute the default time of defaultable instruments and the pairwise correlation of default times.

The copula approach offers a practicable and flexible framework to examine the behavior of multivariate distributions, particularly with respect to portfolio credit risk (Li 2000). Generally, a copula function is a function that links univariate marginal distributions of random variates to a joint multivariate distribution function (see Sklar 1959, 1973). Sklar (1973) shows that a copula function K can be specified for each joint multivariate distribution function. Let $U_1, ..., U_I$ be I uniform distributed random variables then the joint distribution function K

$$K(u_1, ..., u_I) = \mathbb{P}(U_1 \leq u_1, ..., U_I \leq u_I) \tag{3.59}$$

is called a copula function (see Li 2000). Next, let $X_1, ..., X_I$ be I continuous random variates with one-dimensional marginal distribution functions $F_1(x_1), ..., F_I(x_I)$, where F denotes the joint distribution function of $x_i, ..., x_I$.

Then, the copula function K (I-dimensional) with marginals $F_1(x_1), ..., F_I(x_I)$ can be defined by

$$\begin{aligned} K(F_1(x_1), ..., F_I(x_I)) &= \mathbb{P}(U_1 \leq F_1(x_1), ..., U_I \leq F_I(x_I)) \\ &= \mathbb{P}(F_1^{-1}(U_1) \leq x_1, ..., F_I^{-1}(U_I) \leq x_I) \\ &= \mathbb{P}(X_1 \leq x_1, ..., X_I \leq x_I) \\ &= F(x_1, ..., x_I), \end{aligned} \tag{3.60}$$

compare Sklar (1973) and Li (2000).[83] Furthermore, Sklar (1973) shows that a copula function K is unique if each of its marginal distribution functions is continuous. According to Sklar (1973), an I-dimensional copula function $K(u_1, ..., u_I)$ satisfies, for example, the conditions that for each $i \leq I$ and all $u_i \in [0, 1]$

$$K(1, ..., 1, u_i, 1, ..., 1) = u_i. \tag{3.61}$$

and if $u_i = 0$ then

$$K(u_1, ..., u_I) = 0 \tag{3.62}$$

for any $i \leq I$.[84]

The most popular copula specification is the Gaussian copula K_G related to Gaussian marginals. This specification is used in credit risk models such as JP Morgan‘s CreditMetrics (see Gupton et al. 1997), KMV‘s Portfolio Manager (see Crosbie & Bohn 2002) and the Basel II capital weight function (compare Hamerle & Rösch 2005, Rösch 2010, Franke et al. 2011), and also represents the standard copula for pricing STCDOs referring to synthetic CDOs (Hull & White 2008, Finger 2009).[85]

Based on the Gaussian specification and referring to Equation (3.58), it is assumed that i) each credit asset i in the portfolio exhibits a hazard function for its default time T_i, and ii) the distribution of T_i is $F_i(t)$, then the joint

[83] For more technical details see Sklar (1973).

[84] The conditions satisfied by an I-dimensional copula function $K(u_1, ..., u_I)$ are described in detail in Sklar (1973).

[85] For a good distinction between the industry credit risk models compare Bluhm et al. (2003).

distribution of the default times is given by

$$\begin{aligned} F(t_1, ..., t_I) &= K_G(F_1(t_1), ..., F_I(t_I)) \\ &= \Phi_I\left(\Phi^{-1}(F_1(t_1)), ..., \Phi^{-1}(F_n(t_n)); \Sigma\right), \end{aligned} \tag{3.63}$$

(compare Li 2000), where Φ_I is the I-dimensional standard normal cumulative distribution function with correlation matrix $\Sigma \in \mathbb{R}^{I \times I}$

$$\Sigma = \begin{pmatrix} 1 & \gamma_{1,2} & \cdots & \gamma_{1,I} \\ \gamma_{2,1} & 1 & & \vdots \\ \vdots & & \ddots & \gamma_{I-1,I} \\ \gamma_{I,1} & \cdots & \gamma_{I,I-1} & 1 \end{pmatrix} \tag{3.64}$$

containing the correlation coefficients $\gamma_{i,j}$ with $i, j \in \{1, ..., I\}$ and $\gamma_{i,j} \in [-1, 1]$ for $i \neq j$. Thus, the correlation matrix Σ describes the dependency structure between the asset-specific default times.

Referring to the single-factor setup, the probability for a joint default of two borrowers – borrower i and borrower j – at time t conditional on a realization of $Y_t = y_t$ is given by

$$\mathbb{P}(D_{i,t} = 1, D_{j,t} = 1|y_t) = \mathbb{E}^{\mathbb{P}}(D_{i,t} \cdot D_{j,t}|y_t) = \pi_{i,t}(y_t) \cdot \pi_{j,t}(y_t). \tag{3.65}$$

Equation (3.65) shows that the joint probability of default conditional on a realization of the systematic risk factor $Y_t = y_t$ is the product of the marginal conditional default probabilities due to the independence of borrower-specific risk factors leading to independent asset returns.

Analogous to Equation (3.28), the unconditional joint probability of default is given as expectations over all realizations of Y_t

$$\begin{aligned} \mathbb{P}(D_{i,t} = 1, D_{j,t} = 1) = \mathbb{E}^{\mathbb{P}}(D_{i,t} \cdot D_{j,t}) &= \mathbb{E}^{\mathbb{P}}\left[\mathbb{E}^{\mathbb{P}}(D_{i,t} \cdot D_{j,t}|y_t)\right] \\ &= \int_{-\infty}^{+\infty} \pi_{i,t}(y_t) \cdot \pi_{j,t}(y_t) \cdot \varphi(y_t)\ dy_t. \end{aligned} \tag{3.66}$$

In order to calculate the unconditional joint probability of default in the Gaussian specification, the bivariate Gaussian copula K_G given by

$$K_G(u_{1,t}, u_{2,t}) = \int\limits_{-\infty}^{\Phi^{-1}(u_{1,t})} \int\limits_{-\infty}^{\Phi^{-1}(u_{2,t})} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{-(v_1^2 - 2\rho v_1 v_2 + v_2^2)}{2(1-\rho^2)}\right) dv_1\, dv_2$$
$$= \Phi_2\left(\Phi^{-1}(u_{1,t}), \Phi^{-1}(u_{2,t}); \Sigma\right) \tag{3.67}$$

is used, see Martin et al. (2006). Φ_2 denotes the bivariate Gaussian cumulative distribution function and $\Sigma \in \mathbb{R}^{2\times 2}$ describes the correlation matrix reflecting the dependency structure between the random variates.[86] Analogous to Equation (3.39), the unconditional joint probability is then

$$\begin{aligned} \mathbb{P}(D_{i,t} = 1, D_{j,t} = 1) &= \mathbb{E}^{\mathbb{P}}(D_{i,t} \cdot D_{j,t}) \\ &= \int_{-\infty}^{\infty} \Phi\left(\frac{c_{i,t} - \sqrt{\rho}\cdot y_t}{\sqrt{1-\rho}}\right) \cdot \Phi\left(\frac{c_{j,t} - \sqrt{\rho}\cdot y_t}{\sqrt{1-\rho}}\right) \cdot \varphi(y_t)\, dy_t \\ &= \Phi_2\left(c_{i,t}, c_{j,t}; \Sigma\right) \end{aligned} \tag{3.68}$$

with correlation matrix Σ

$$\Sigma = \begin{pmatrix} 1 & \rho_{1,2} \\ \rho_{2,1} & 1 \end{pmatrix} \tag{3.69}$$

containing the asset correlations $\rho_{i,j}$ with $i, j \in \{1, 2\}$, $\rho_{i,j} \in [0, 1]$ for $i \neq j$, compare Martin et al. (2006).

Through the inclusion of a common risk factor that simultaneously affects all borrowers in a sector or an economy, the borrower defaults are correlated in such a factor-model setup (compare Section 3.2.4). In the Gaussian specification and referring to Equation (3.55), the joint correlation of borrower defaults

[86] For more details on the bivariate standard normal (Gaussian) cumulative distribution function see Li (2000) and Giesecke (2004).

can be obtained by

$$\begin{aligned}\mathrm{Corr}^{\mathbb{P}}(D_{i,t}, D_{j,t}) &= \frac{\mathbb{E}^{\mathbb{P}}(D_{i,t} \cdot D_{j,t}) - \mathbb{E}^{\mathbb{P}}(D_{i,t}) \cdot \mathbb{E}^{\mathbb{P}}(D_{j,t})}{\sqrt{\mathrm{Var}^{\mathbb{P}}(D_{i,t}) \cdot \mathrm{Var}^{\mathbb{P}}(D_{j,t})}} \\ &= \frac{\int_{-\infty}^{+\infty} \pi_{i,t}(y_t) \cdot \pi_{j,t}(y_t) \cdot \varphi(y_t)\ dy_t - \pi_{i,t} \cdot \pi_{j,t}}{\sqrt{\pi_{i,t} \cdot (1 - \pi_{i,t}) \cdot \pi_{j,t} \cdot (1 - \pi_{j,t})}} \\ &= \frac{\Phi_2(\Phi^{-1}(\pi_{i,t}), \Phi^{-1}(\pi_{j,t}); \Sigma) - \pi_{i,t} \cdot \pi_{j,t}}{\sqrt{\pi_{i,t} \cdot (1 - \pi_{i,t}) \cdot \pi_{j,t} \cdot (1 - \pi_{j,t})}}. \end{aligned} \tag{3.70}$$

Equation (3.70) also shows that the joint default probability generally increases with an increasing default correlation. However, in factor models the correlation of defaults arises from correlations in the latent asset returns. If the latent asset returns are independent, i.e., uncorrelated with $\rho_{i,j} = 0\ \forall\ i \neq j$ and $i, j \in \{1, ..., I\}$, the joint default probability is reduced to the product of the obligors' marginal default probabilities. Economically, this may be the case if one borrower – either borrower i or j – is solely exposed to idiosyncratic risk.

Based on Equation (3.40), the asset correlation $\rho_{i,j}$ is identical across all pairs of obligors in the simple LHP specification. Modeling these dependency structures more realistically (or rather less restrictively), allows to examine effects of diversification in multi-sectoral credit portfolios and asset securitizations.[87] Eventually, limitations of the LHP can be cured by considering heterogenous (or even individual) obligor structures, e.g., sector-specific cross-correlations and individual default probabilities.

As it is pointed out, asset correlations represent a substantial driver of portfolio credit risk. Unfortunately, neither asset returns nor the respective asset correlations are explicitly observable. Therefore, some recent works examine several approaches to parameterize these important correlation parameters. For example, Duellmann et al. (2010) suggest to estimate correlation parameters on corresponding stock returns rather than on default rates.[88] Such cross-correlation estimates may then be incorporated into the correlation-coefficient

[87] Chapter 5 addresses effects of sectoral diversification and systematic risk concentration in asset securitizations by a Monte Carlo simulation approach.

[88] Recall that asset correlations may generally vary across obligors and over time, which is in contrast to the simplifying assumption of time-constant and identical asset correlations across obligors.

matrix in Equation (3.64) in order to calculate obligor defaults and corresponding default correlations. Other authors, e.g., Hamerle & Rösch (2005), examine the sensitivity of several copula approaches to misspecifications via a maximum likelihood framework. For estimating default probabilities and correlations on empirical default data they suggest the Gaussian copula approach rather than other examined copula approaches.[89]

3.3.3 Valuing Single-tranche CDO Swaps

For valuing single-tranche CDO swaps (STCDO), a precise knowledge of expected tranche losses is essential. These expected tranche losses are triggered by the portfolio's credit risk and determined by the cumulative loss distribution of the underlying credit risky portfolio.

In order to determine portfolio loss distributions, there are generally two alternative approaches to setting up intensity-based models for portfolio credit risk: the so-called bottom-up and top-down approaches. In a bottom-up approach, the default times of individual obligors are modeled and afterwards aggregated to obtain the loss distribution of the entire credit portfolio. Corresponding approaches to estimate portfolio loss distributions (bottom-up) are provided by Li (2000) and Duffie & Gârleanu (2001).[90] Beyond the single-factor Gaussian copula specification (see Li 2000), Duffie & Gârleanu (2001) develop a multi-factor affine jump-diffusion model to address portfolio credit risk.[91] As long as the number of debt asset is small, e.g., with regard to CDS portfolios (or baskets) of 10 assets, Li's (2000) approach is commonly used for pricing the related credit risk based on comprehensive Monte Carlo simulations, which are applied in order to specify the joint distribution of default times (Martin et al. 2006).[92] With an increasing number of individual credit

[89] In this context, parameter (estimation) risk and model risk in measuring portfolio credit risk is addressed by Löffler (2003), Hamerle & Rösch (2006), Heitfield (2009) and Tarashev (2010).

[90] Other reduced-form approaches to credit derivatives can be found in Graziano & Rogers (2009), where defaults of different names are driven by a common continuous-time Markov process, in Joshi & Stacey (2006) who provide an intensity-gamma model to value portfolio credit derivatives, and in Chapovsky et al. (2007) who apply a stochastic intensity model for pricing exotic structured credit derivatives.

[91] Within this approach the default time of an obligor is due to three risk factors: one issuer-specific, one common to all issuers in a specific sector and at least one common to all issuers across all sectors (global).

[92] Default events can be generated by bootstrapping the multivariate distribution of default times from empirical data such as CDS notations.

assets, bottom-up approaches may become less applicable since the computational resources for the simulation of a sufficient frequency of default times are limited.

In contrast to bottom-up approaches, the evolution of the losses on a credit risky portfolio is sometimes modeled top-down, where only the aggregated loss process of the entire credit portfolio is modeled, but not explicitly the behavior of single obligors. Within such a top-down approach, Longstaff & Rajan (2008) provide a multi-factor portfolio credit model for pricing CDOs which is motivated by the idea of Duffie & Gârleanu (2001): similar to Duffie & Gârleanu (2001), defaults which cause losses in the credit portfolio follow a jump process which is due to an idiosyncratic (firm-specific) component, a common sector-specific risk factor and a common systematic risk factor affecting all firms economywide.[93] Top-down models exhibit advantages when individual obligors in the portfolio are not dominating the others – or in other words – if the credit exposure of each individual obligor is small compared to the reference credit portfolio.[94] The gain in flexibility and tractability in such top-down approaches outweigh the limitations, which are due to assumed simplifications related to the borrower structure. This may be a reason for the increasing attractiveness of top-down models, even with regard to the strong growth of the markets for STCDOs (Ehlers & Schönbucher 2009).

In the following, the single-factor Gaussian Copula specification is applied for valuing STCDOs, as also proposed by, e.g., Hull & White (2008) and Finger (2009).[95] Recall that L_t denotes the percentage loss of a credit portfolio, as defined in Equation (3.47). Then, the market participants invested in tranche $T_{[A,D)}$ suffer from losses on the portfolio's debt assets if the loss rate L_t exceeds the lower attachment point $A \in [0, 1)$ of the tranche. The upper attachment point $D \in (A, 1]$ of tranche $T_{[A,D)}$ is also called detachment point and represents the tranche's upper bound. The default event of a tranche occurs if the loss rate of the underlying credit portfolio exceeds the lower bound (attachment

[93] Other top-down approaches may be found in Ehlers & Schönbucher (2009), where the evolution of the loss distribution is modeled as a Markov chain, and in Giesecke, Goldberg & Ding (2011), who propose a 'random thinning' approach in order to decompose the portfolio-level default intensity into the sum of the constituent intensities.

[94] This is the case, for example, in retail portfolios or with respect to standard credit indices such as the CDX or iTraxx index families.

[95] Note that many other copula approaches exist such as the t-copula, the Clayton copula, the Archimedian copulae and the Marshall Olkin copula which can be applied to the valuation of CDOs. In some cases, these models may provide a much better fit to market data than the Gaussian copula model (Hull & White 2008).

point A) for the first time. The tranche and thus investors suffer from losses as long as the loss rate is below the upper bound D. Therefore, tranche-specific losses are restricted to the tranche size, which is the difference of detachment point D and attachment point A. Consequently, the tranche loss at time t is given by

$$L_t^{T_{[A,D)}} = \min\left(\max\left(0, L_t - A\right), D - A\right). \tag{3.71}$$

The boundaries of a tranche (A and D) determine the tranche seniority within an asset securitization. Depending on the risk characteristics of the underlying credit portfolio, which determine the portfolio's loss distribution, the risk profile of a securitized tranche varies by seniority. In practice, a specific risk profile of a tranche is sometimes proposed (or claimed) by investors or credit-rating agencies in order to meet the investor's risk appetite or the CRA rating requirements, respectively (compare Section 2.3). To fulfill the specified risk profile, the tranche boundaries can be calibrated on a given portfolio loss distribution individually.[96]

According to Martin et al. (2006), the loss rate of a specified tranche $L_t^{T_{[A,D)}}$ with attachment point A and detachment point D follows from

$$L_t^{T_{[A,D)}} = \frac{1}{D-A}\left[(L_t - A)^+ - (L_t - D)^+\right] \tag{3.72}$$

with $(\cdot)^+ = \max(\cdot, 0)$ and expectations

$$\mathbb{E}^{\mathbb{P}}\left(L_t^{T_{[A,D)}}\right) = \frac{1}{D-A}\left[\mathbb{E}^{\mathbb{P}}\left((L_t - A)^+\right) - \mathbb{E}^{\mathbb{P}}\left((L_t - D)^+\right)\right]. \tag{3.73}$$

In this context, tranche losses can be seen as a call spread option on the loss rate L_t (Longstaff & Rajan 2008). Thus, the calculation of expected tranche losses may follow the classic valuation procedure of plain vanilla call options on the firm's equity, as presented in Martin et al. (2006): although the pool recovery rate R_t may generally vary between $[0, 1)$, the proof can be reduced to the special case of $R_t = 0$ since

$$\mathbb{E}^{\mathbb{P}}\left((L_t - A)^+\right) = (1 - R_t) \cdot \mathbb{E}_0^{\mathbb{P}}\left(\left(L_t - \frac{A}{1-R_t}\right)^+\right) \tag{3.74}$$

96 An illustrative analytical application is provided in Chapter 5, where the systematic risk sensitivity of structured products is examined.

is valid with $\mathbb{E}_0^{\mathbb{P}}(\cdot)$ describing expectations under a recovery rate $R_t = 0$. Due to the linearity of $\mathbb{E}(\cdot)$, only expectations of $\mathbb{E}_0^{\mathbb{P}}\left((L_t - A)^+\right)$ are calculated. Using integration by parts, it follows that

$$\begin{aligned}
\mathbb{E}_0^{\mathbb{P}}\left((L_t - A)^+\right) &= \int_0^1 (l_t - A) \cdot \mathbb{1}_{\{l_t \geq A\}} \cdot g_t(l_t) dl_t \\
&= \int_A^1 (l_t - A) \cdot g_t(l_t) dl_t \\
&= \int_A^1 l_t \cdot g_t(l_t) dl_t - A \cdot \int_A^1 g_t(l_t) dl_t \\
&= \left[l_t \cdot G_t(l_t)\right]_A^1 - \int_A^1 1 \cdot G_t(l_t) dl_t - \left[A \cdot G_t(l_t)\right]_A^1 \\
&= 1 - A - \int_A^1 \mathbb{P}(L_t \leq l_t) dl_t, \qquad (3.75)
\end{aligned}$$

where $g_t(l_t)$ denotes the density function of the portfolio's loss rate with $l_t \in [0, 1]$ and $G_t(l_t) = \mathbb{P}(L_t \leq l_t)$ describes the respective cumulative distribution function. Equation (3.75) shows that the calculation method for the expected loss rate of a tranche is independent from specific distributional assumptions and holds for every cumulative distribution function $G_t(l_t)$.

Next, the expected loss rate of tranche $T_{[A,D)}$ is calculated with respect to the presented single-factor Gaussian copula specification referring to a LHP. Related to Vasicek's cumulative distribution function, see Equation (3.42), it follows that

$$\begin{aligned}
\mathbb{P}(L_t \leq l_t) &= \Phi\left(\frac{\sqrt{1-\rho} \cdot \Phi^{-1}(l_t) - \Phi^{-1}(\pi_t)}{\sqrt{\rho}}\right) \\
&= 1 - \Phi\left(\frac{\Phi^{-1}(\pi_t) - \sqrt{1-\rho} \cdot \Phi^{-1}(l_t)}{\sqrt{\rho}}\right) \qquad (3.76)
\end{aligned}$$

with π_t denoting the default probability of the LHP, compare Section 3.3.1.

Furthermore, the identity provided by Andersen & Sidenius (2005*a*) is used

$$\int_{-\infty}^{\gamma} \Phi(\alpha \cdot x + \beta) \cdot \varphi(x) dx = \Phi_2 \left(\frac{\beta}{\sqrt{1+\alpha^2}}, \gamma; \Sigma \right), \tag{3.77}$$

where the correlation matrix Σ denotes the underlying dependency structure

$$\Sigma = \begin{pmatrix} 1 & \frac{-\alpha}{\sqrt{1+\alpha^2}} \\ \frac{-\alpha}{\sqrt{1+\alpha^2}} & 1 \end{pmatrix}. \tag{3.78}$$

Then, $\mathbb{E}_0^{\mathbb{P}}\left((L_t - A)^+\right)$ is obtained by inserting the results of Equation (3.76) into Equation (3.75) and applying the identity of Equation (3.77). Firstly, this leads to

$$\mathbb{E}_0^{\mathbb{P}}\left((L_t - A)^+\right) = \int_{A}^{1} \Phi\left(\frac{\Phi^{-1}(\pi_t) - \sqrt{1-\rho} \cdot \Phi^{-1}(l_t)}{\sqrt{\rho}} \right).$$

By substituting $x = -\Phi^{-1}(l_t)$ and defining

$$\alpha = \frac{\sqrt{1-\rho}}{\sqrt{\rho}}, \quad \beta = \frac{\Phi^{-1}(\pi_t)}{\sqrt{\rho}} \quad \text{and} \quad \gamma = \Phi^{-1}(A),$$

it follows secondly that

$$\begin{aligned}
\mathbb{E}_0^{\mathbb{P}}\left((L_t - A)^+\right) &= \int_{-\infty}^{-\Phi^{-1}(A)} \Phi\left(\frac{\sqrt{1-\rho}}{\sqrt{\rho}} \cdot y + \frac{\Phi^{-1}(\pi_t)}{\sqrt{\rho}} \right) \varphi(y)\, dy \\
&= \Phi_2 \left(\frac{\frac{\Phi^{-1}(\pi_t)}{\sqrt{\rho}}}{\sqrt{1+\left(\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\right)^2}}, -\Phi^{-1}(A); -\frac{\frac{\sqrt{1-\rho}}{\sqrt{\rho}}}{\sqrt{1+\left(\frac{\sqrt{1-\rho}}{\sqrt{\rho}}\right)^2}} \right) \\
&= \Phi_2\left(-\Phi^{-1}(A), \Phi^{-1}(\pi_t); \Sigma\right)
\end{aligned} \tag{3.79}$$

with $\Phi_2(x, y; \Sigma) = \Phi_2(y, x; \Sigma)$ due to the properties of Gaussian functions and with correlation matrix Σ

$$\Sigma = \begin{pmatrix} 1 & -\sqrt{1-\rho} \\ -\sqrt{1-\rho} & 1 \end{pmatrix}. \tag{3.80}$$

Eventually, it follows from the linearity of expectations that the expected loss of a tranche is given by

$$\mathbb{E}^{\mathbb{P}}\left(L_t^{T_{[A,D)}}\right) = \frac{1-R_t}{D-A}\left[\Phi_2\left(\kappa(A), c_t; \Sigma\right) - \Phi_2\left(\kappa(D), c_t; \Sigma\right)\right] \tag{3.81}$$

with

$$c_t = \Phi^{-1}(\pi_t), \qquad \text{and} \qquad \kappa(\chi) = -\Phi^{-1}\left(\frac{\chi}{1-R_t}\right)$$

for $\chi < 1 - R_t$ and $\chi \in \{A, D\}$.[97]

This basic single-factor framework is extended in Chapter 5 by a decomposition of the systematic risk factor Y_t into a sectoral and a super-systematic risk component. Analogous to the basic model, an analytical framework for quantifying default probabilities and expected losses is provided in order to evaluate the systematic risk sensitivity of structured products in comparison to the risk sensitivity of corporate bonds exhibiting comparable risk profiles.

An appropriate evaluation of expected tranche losses is particularly important for the calculation of STCDO spreads. Similar to credit derivatives such as classic CDS, the pricing of a STCDO implies evaluating the instrument's cash flows over its maturity M in order to calculate the fair spread of a tranche which is is defined by

$$S^{T_{[A,D)}}(0, M) = s^{T_{[A,D)}}. \tag{3.82}$$

The cash flows of a STCDO generally refer to the instrument's premium leg and protection leg. In single-tranche CDO swaps, a tranche investor generally acts as a protection seller, who covers losses within the collateral that affect the respective securitized tranche (protection leg). Until maturity, related protection payments are limited to the size of the tranche (thickness). For this provided credit protection the investor periodically receives a risk premium compensating for the related default risk. These premium payments constitute the premium leg.

In capital markets, the fair spread of a tranche refers to the present values of both legs and is paid to the investor on discrete payment dates t_j, $j \in \{1, ..., \kappa\}$, e.g., quarterly, with respect to the remaining face value. Under the risk-neutral

[97] From Equation (3.74) it follows that the expected loss equals 0 for $\chi \geq 1 - R$.

probability measure $\mathbb{Q}$, the present value of the premium leg $V_{\mathbb{Q}}^{T_{[A,D)}}$ is given by

$$V_{\mathbb{Q}}^{T_{[A,D)}} = s^{T_{[A,D)}} \cdot \eta \cdot \mathbb{E}^{\mathbb{Q}} \left[\sum_{j=1}^{\kappa} \Delta_j \cdot Q_{t_j} \cdot \left(1 - L_{t_j}^{T_{[A,D)}} \right) \right] \tag{3.83}$$

where η describes the face value (notional) of the tranche, Δ_j denotes the constant distance between the fixed payment dates t_j, Q_{t_j} denotes the time-specific discount factor depending on the term-structure of risk-less interest rates and $L_{t_j}^{T_{[A,D)}}$ denotes the tranche-specific loss rate at payment date t_j, compare Martin et al. (2006). $\mathbb{E}^{\mathbb{Q}}(\cdot)$ describes corresponding expectations under $\mathbb{Q}$.[98]

Furthermore, the present value of the protection leg $C_{\mathbb{Q}}^{T_{[A,D)}}$ (contingent payment) of tranche $T_{[A,D)}$ is defined by

$$C_{\mathbb{Q}}^{T_{[A,D)}} = \eta \cdot \sum_{j=1}^{\kappa} Q_{t_j} \cdot \left[\mathbb{E}^{\mathbb{Q}} \left(L_{t_j}^{T_{[A,D)}} \right) - \mathbb{E}^{\mathbb{Q}} \left(L_{t_{j-1}}^{T_{[A,D)}} \right) \right] \tag{3.84}$$

with regard to discrete payment dates t_j, $j \in \{1, ..., \kappa\}$. As already described, the evaluation of the tranche's expected loss rate $\mathbb{E}^{\mathbb{Q}} \left(L_{t_j}^{T_{[A,D)}} \right)$ at date t_j requires knowledge about the default intensity of the underlying credit portfolio. The most popular single-tranche CDO swaps refer to credit indices such as the iTraxx Europe families or its U.S. American counterparts the CDX families (compare Chapters 2 and 4). Each credit index represents a specified basket of the 125 most liquid CDS contracts (equally weighted), which are related to either European (iTraxx) or North American entities (CDX). Referring to such credit indices, the default intensity of the entire CDS basket can be obtained by bootstrapping the time-specific default intensities of its constitutes from the respective credit spread curves and generating the multivariate distribution of default times based on the individual default intensities (compare, e.g., Martin et al. 2006). Alternatively, the default intensity of a credit index may directly be extracted from index spread notations by using the so-called credit triangle, as presented in O'Kane (2008).[99]

Based on the risk-neutral probability measure $\mathbb{Q}$, the fair tranche spread of

[98] Only for ease of this exercise, a discrete-time model framework is presented for the valuation of STCDOs. In fact, it is quite possible to model fair tranche spreads in a continuous-time framework, as presented in Martin et al. (2006).

[99] In Chapter 4, the credit triangle is applied to daily index spread notations of the 5Y iTraxx Europe to approximate the default intensity of this credit index.

a tranche equals the mark-to-market value of a STCDO to zero. Thus, the fair tranche spread is obtained by equalizing Equations (3.83) and (3.84) and solving for $s^{T_{[A,D)}}$. This leads to

$$s^{T_{[A,D)}} = \frac{\sum_{j=1}^{\kappa} Q_{t_j} \left[\mathbb{E}^{\mathbb{Q}} \left(L_{t_j}^{T_{[A,D)}} \right) - \mathbb{E}^{\mathbb{Q}} \left(L_{t_{j-1}}^{T_{[A,D)}} \right) \right]}{\sum_{j=1}^{\kappa} \Delta_j \cdot Q_{t_j} \cdot \left[1 - \mathbb{E}^{\mathbb{Q}} \left(L_{t_j}^{T_{[A,D)}} \right) \right]} \tag{3.85}$$

under the assumption of fixed payment dates t_j with constant Δ_j and related to a given term-structure of risk-less interest rates determining the time-specific discount factors Q_{t_j}. The standard Gaussian copula valuation framework offers also scope for further model extension, as described above. With respect to STCDOs, Andersen & Sidenius (2005*b*), for example, suggest the incorporation of stochastic recovery rates and the consideration of random systematic factor loadings to allow for different correlation regimes when evaluating such synthetic CDOs. Two alternative correlation skew models providing a dynamic spread-dependent correlation specification for the standard Gaussian copula model are presented in Chapter 4. There, a dynamic panel regression approach is suggested to model and forecast implied correlations for pricing STCDOs on the 5Y iTraxx Europe credit index. Thereby, random effects are introduced to account for unobservable time-specific effects on implied tranche correlations. The spread forecasts based on the dynamic implied correlation models are compared to forecasts using historical correlations from asset returns. The empirical findings support the proposed dynamic mixed-effects regression correlation model (MERM) even during the Global Financial Crisis and indicate several implications for pricing and hedging credit derivatives.

Chapter 4

Dynamic Implied Correlation Modeling and Forecasting in Structured Finance

The content of this chapter refers to the working paper 'Dynamic Implied Correlation Modeling and Forecasting in Structured Finance' by Löhr, S., Mursajew, O., Rösch, D., and Scheule, H., 2012.

4.1 Introduction

The market volume of credit derivatives increased rapidly from \$180 billion in 1996 to over \$57 trillion in 2008 (BBA 2006, BIS 2010*b*). This growth rate highlights the importance of these new instruments in financial markets. Consequences of the Global Financial Crisis (GFC), e.g., the Lehman Brothers' bankruptcy in 2008, underline the challenge to aggregate individual risk contributions in the presence of correlations, which is essential for pricing credit derivatives such as collateralized debt obligations (CDOs). The GFC has especially shown that pooling and tranching within CDO structures amplify mistakes in the assessment of underlying asset default risks and correlations (compare Coval et al. 2009*b*).

The Gaussian copula model is market standard for pricing synthetic single-tranche CDO swaps (STCDO) in the finance industry (Hull & White 2006), either in a structural single-factor framework (compare Merton 1974, Vasicek 1987) or in respective default intensity models which were initially introduced by Li (2000). Increasing liquidity in synthetic CDO markets, particularly in that for standardized index tranches, reveals shortcomings of standard Gaussian copula models for pricing STCDOs. Consequences of these shortcomings appear mainly in observable differences between model spreads and quoted market spreads. Earlier literature on STCDOs shows that several extensions to the Gaussian copula approach are presented to overcome existing model limitations. Within these studies improvements in pricing STCDOs are reached by implementing different copula structures or relaxing standard correlation restrictions (compare Hull & White 2004, Laurent & Gregory 2005, Andersen & Sidenius 2005*a*, Longstaff & Rajan 2008).

In general, credit risks and their correlations determine the loss distributions of credit portfolios related to credit derivatives (Longstaff & Rajan 2008). However, 'true' correlations are not observable and thus they are unknown parameters. This underlines the need for appropriate correlation models in order to estimate expected tranche losses, which in turn determine fair tranche spreads. Accordingly, appropriate correlation forecasts are important parameters in pricing models of structured financial instruments.

In practice, several standard approaches are used to estimate implied correlations matching observable market spreads of credit derivatives (Hull & White 2006). Analogous to the Black-Scholes methodology to extract implied volatilities from option market prices, implied correlations can be extracted from CDO tranche prices (compare Ncube 1996, Finger 2009). Similar to implied volatilities, the spread-dependent implied correlations may also differ widely in asset securitizations (compare Hull & White 2006). Two popular standard approaches for modeling implied correlations are represented by *compound correlations* and *base correlations*. The concept of *base correlations* was introduced in 2004 by McGinty et al. (2004) in order to overcome the ambiguity of *compound correlations*.

This paper provides an econometric framework which extends existing literature on pricing credit derivatives. Three correlation estimation approaches are examined within this empirical spread analysis and evaluated with regard to their forecast performance. Firstly, we derive both *compound correlations* and *base correlations* as different types of implied correlations from quoted *iTraxx Europe* index tranche spreads. The 5-year (5Y) *iTraxx Europe* is one of the most popular credit default swap (CDS) indices representing a portfolio of 125 most liquid as well as equally weighted single-name CDS contracts.[100] Secondly, *base correlations* are modeled with two different dynamic regression correlation models: i) a dynamic fixed-effects regression correlation model (FERM) and ii) a dynamic mixed-effects regression correlation model (MERM). The dynamic FERM solely considers fixed-tranche effects. By implementing dummy variables and error components, we account for fixed tranche effects as well as random time effects in the dynamic MERM. In the following, our models are used to forecast daily tranche spreads. Thirdly, daily spreads are forecasted using a dynamic asset correlation approach. Within this approach dynamic *historical asset correlations* are derived from corresponding asset returns.[101]

Eventually, we compare the forecast performances of the provided models. We show that the implied correlation models are superior to the dynamic *historical asset correlation* approach in terms of the prediction error metrics. Thus, the results correspond to findings related to option markets, where implied volatility regression models outperform forecasts based on standard deviations of log-returns (compare Ncube 1996). We find that the accuracy of daily spread forecasts strongly depends on both correlation types in use and its underlying estimation approach. Overall, the forecast quality of our dynamic panel regression correlation models outperforms forecasts based on estimations referring to standard correlation approaches. Especially, the dynamic MERM accounting for random time effects as well as fixed tranche effects, seems to be highly relevant for pricing STCDOs. Last but not least, our empirical study provides useful implications for both hedging credit risk and pricing several kinds of credit derivatives, e.g., non-standardized CDO tranches and bespoke portfolios.

The remainder of the paper is organized as follows. In Section 5.2, we develop the theoretical pricing framework, introduce three different correlation approaches and demonstrate how to calculate the correlation types. We then provide both of our dynamic panel regression correlation models MERM and FERM considering the aforementioned effects in correlation modeling. In Section 4.3, our empirical results are presented. Section 6.4 concludes.

[100]Offered STCDOs are standardized with attachment points $A \in \{0\%, 3\%, 6\%, 9\%, 12\%\}$ and detachment points $D \in \{3\%, 6\%, 9\%, 12\%, 22\%\}$, respectively. The composition of the basket is fixed until maturity. For a detailed description of the *iTraxx Europe* family refer to www.iTraxx.com.

[101]Within our empirical analysis we consider log-returns of entities incorporated in the 5Y *iTraxx Europe* index and listed on stock exchanges as well.

4.2 Correlation Approaches and Dynamic Panel Regression Models

4.2.1 Valuation of Single-tranche CDO Swaps

A collateralized debt obligation (CDO) is a structured financial instrument that securitizes a specific portfolio of credit risky assets (collateral). Inherent credit risks of debt assets are transferred to external investors by repacking the original risk profile and offering risk-adjusted tranches to investors. Tranches are backed by the collateral. Eventually, the initial debt portfolio is bundled in tranches of different seniority. Subsequently, the seniority of tranches reflects the order in which losses within the collateral affect different tranches and thus tranche investors. Each tranche is defined by the percentage of losses in the collateral that it carries (for more detailed information compare Bluhm 2003). According to their individual risk-return preferences, investors purchase corresponding tranches.

In contrast to CDOs, synthetic CDOs refer to a basket of different single-name CDS contracts.[102] A well known standardized credit index of single-name CDS contracts is the 5Y *iTraxx Europe* credit index. Analogous to CDOs, external investors purchase risk-adjusted tranches from this CDS basket in accordance to their individual risk-return preferences. In its role as protection seller, each investor receives a periodic premium payment depending on the tranche-specific risk profile.[103] It is generally paid out quarterly. The *premium leg* contains all premium payments over the product's maturity. In turn, an investor suffers losses within the underlying CDS basket respective to specific tranche characteristics mentioned above. The resulting protection or fee payments are called *protection leg* and refer to cash flows that cover losses affecting the related tranche. Thus, the *protection leg* refers to cash flows paid out to the protection buyer in cases of default events causing losses within the underlying CDS basket.

The valuation of STCDOs implies calculating the fair spreads of each tranche. By definition a fair tranche spread equals the mark-to-market value of a STCDO

[102] See Chapter 2 for more detailed information on synthetic CDOs and their functionality.

[103] The premium payment depends on i) the tranche-specific spread varying by seniority and ii) the respective outstanding national amount. Tranche spreads reflect the compensated credit risk and are thus determined by the tranche-specific risk profile in terms of default risk and related losses.

to zero using risk-neutral valuation. Risk-neutral valuation is based on a risk-neutral martingale measure $\mathbb{Q}$ which is taken into consideration for all expectations in the following.

As we will see below, the determination of fair tranche spreads decisively relies on the cumulative loss distribution of the underlying credit portfolio. The industry-standard model for the valuation of STCDOs is the single-factor Gaussian copula model (Hull & White 2008, Finger 2009), which was firstly applied to portfolio credit risk by Vasicek (1987) and combined with default intensity models by Li (2000), Schönbucher (2003), Laurent & Gregory (2005) and Longstaff & Rajan (2008).

Tranche investors suffer losses at time t if the total portfolio loss L_t^P in percent of its notional exceeds the lower attachment point $A \in [0,1)$ of the respective tranche $T_{[A,D)}$. Occurring tranche losses $L_t^{T_{[A,D)}}$ at time t are restricted to the difference of the upper attachment point $D \in (0,1]$, and the lower attachment point A of tranche $T_{[A,D)}$. In terms of the total portfolio loss L_t^P, it follows for the tranche-specific losses:

$$L_t^{T_{[A,D)}} = \min\left(\max\left(0, L_t^P - A\right), D - A\right) \tag{4.1}$$

In order to calculate the present value PV of both the *premium leg* and the *protection leg* of a STCDO referring to tranche $T_{[A,D)}$ with maturity of five years ($M = 5$), we proceed as follows:[104]

Firstly, we define the fair STCDO premium

$$S^{T_{[A,D)}}(0, M) = s^{T_{[A,D)}}. \tag{4.2}$$

This premium is paid to investors at discrete payment dates t_j, $j \in \{1, ..., \kappa\}$, with respect to the remaining face value of tranche $T_{[A,D)}$.

Therefore, the present value of the *premium leg* $PV_{\text{prem}}^{T_{[A,D)}}$ is defined by

$$PV_{\text{prem}}^{T_{[A,D)}} = s^{T_{[A,D)}} \cdot \eta \cdot \mathbb{E}\left[\sum_{j=1}^{\kappa} \Delta_j \cdot Q_{t_j}\left(1 - L_{t_j}^{T_{[A,D)}}\right)\right] \tag{4.3}$$

where η denotes the face value (notional), Q_{t_j} describes the time-specific discount factor, and $\mathbb{E}(\cdot)$ corresponding expectations. Δ_j describes the constant

[104]This valuation exercise primarily refers to Martin et al. (2006) and is described in detail Section 3.3.3.

distance between fixed payment dates. Tranche-specific losses at time t_j are denoted by $L_{t_j}^{T_{[A,D)}}$.

Secondly, we calculate the present value of the *protection leg* $PV_{\text{prot}}^{T_{[A,D)}}$ with regard to discrete payment dates t_j, $j \in \{1, ..., \kappa\}$

$$PV_{\text{prot}}^{T_{[A,D)}} = \eta \sum_{j=1}^{\kappa} Q_{t_j} \left[\mathbb{E}\left(L_{t_j}^{T_{[A,D)}} \right) - \mathbb{E}\left(L_{t_{j-1}}^{T_{[A,D)}} \right) \right]. \tag{4.4}$$

Finally, we infer fair tranche spreads $s^{T_{[A,D)}}$ by equalizing Equations (4.3) and (4.4):

$$s^{T_{[A,D)}} = \frac{\sum_{j=1}^{\kappa} Q_{t_j} \left[\mathbb{E}\left(L_{t_j}^{T_{[A,D)}} \right) - \mathbb{E}\left(L_{t_{j-1}}^{T_{[A,D)}} \right) \right]}{\sum_{j=1}^{\kappa} \Delta_j \cdot Q_{t_j} \cdot \left[1 - \mathbb{E}\left(L_{t_j}^{T_{[A,D)}} \right) \right]}. \tag{4.5}$$

The valuation process shows that in cases of spread calculation a precise knowledge of expected tranche losses is essential. These expected tranche losses are determined by the cumulative loss distribution of the underlying credit risky portfolio. Approaches to estimate portfolio loss distributions are provided, for example, by Li (2000) and Duffie & Gârleanu (2001).

Within the framework of the single-factor Gaussian copula model, we can use asymptotic analytical approximation procedures to calculate expected tranche losses (compare Vasicek 1987):
The expected loss of tranche $T_{[A,D)}$ with $D \in (0,1]$ is analytically given by

$$\mathbb{E}\left(L^{T_{[A,D)}} \right) = \frac{1-R}{D-A} \left[\Phi_2 \left(\omega(A), c; -\sqrt{1-\rho} \right) - \Phi_2 \left(\omega(D), c; -\sqrt{1-\rho} \right) \right] \tag{4.6}$$

with $\omega(\chi) = -\Phi^{-1}\left(\frac{\chi}{1-R}\right)$ for $\chi \in \{A, D\}$ and $R \in [0,1)$ describing the recovery rate of the underlying credit portfolio (compare Kalemanova et al. 2007).[105] $\Phi_2(\cdot)$ denotes the cumulative distribution function (cdf) of the bivariate normal distribution and $\rho \in [0,1]$ its correlation parameter. Additionally, we

[105]A recovery rate of $R = 40\%$, for example, indicates that 40% of the contract's face value will be recovered in case of a default event. For a general analysis of basket default swaps, we refer to Laurent & Gregory (2005).

define the time-dependent default threshold c by

$$c = c(t_j) = \Phi^{-1}\left(1 - \exp\left(-\lambda \cdot \frac{t_j}{4}\right)\right). \tag{4.7}$$

In accordance to the so-called *credit triangle* (O'Kane 2008), we approximate λ by

$$\lambda = \frac{s^{\text{Trax}}}{1 - R^{\text{Trax}}} \tag{4.8}$$

with λ as time-constant default intensity of the 5Y *iTraxx Europe* (Trax) derived from the daily index spread s^{Trax} and the related index recovery rate $R^{\text{Trax}} = R$.[106]

From Equation (4.6) follows that the correlation parameter ρ affects decisively the expected tranche loss $L^{T_{[A,D)}}$ and thus the tranche spread $s^{T_{[A,D)}}$, compare Equation (4.5). We conclude that fair tranche spreads $s^{T_{[A,D)}}$ are highly sensitive to variations of the correlation ρ. Thus, ρ is a decisive factor in pricing STCDOs.

4.2.2 Correlation Approaches: Compound, Base and Asset Correlations

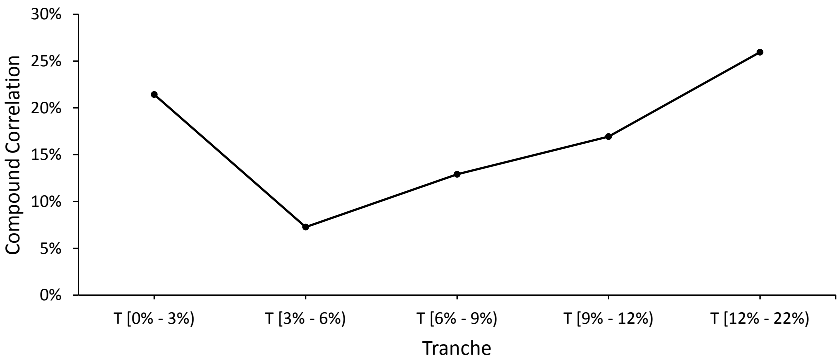
Increasing standardization of credit derivatives is a main driver for an enlargement of trading activities in the STCDO market (compare Finger 2009). In consequence, relatively high liquidity is achieved in these markets by sufficient supply and demand for structured finance products. This is in tandem with the rising popularity of credit indices such as the *iTraxx Europe* credit index family. Thus, observable market quotes for STCDOs reflect a market-specific view of correlations (compare Longstaff & Rajan 2008). In the following subsection, we introduce the main approaches for extracting three different correlation types from given market information:

[106]Note that on each day the default intensity of the 5Y *iTraxx Europe* is derived from its daily index spread notation. Thus, the implied default intensity may vary by day, but nevertheless it is assumed to be constant over the 5-year maturity of the index.

Compound Correlations

Within the concept of *compound correlations*, implied correlations are derived from quoted market spreads. This practice was inspired by the implied volatility approach from Black-Scholes in which implied volatilities are derived from option market prices.[107] The compound correlation approach can therefore be termed as a direct adaption of Black-Scholes implied volatilities to the STCDO market (compare Hull & White 2004). Basically, we obtain the respective compound correlation by inverting the introduced Gaussian copula model and matching model generated prices to market quoted spreads. In detail, we use a numeric inversion procedure referring to Equation (4.5) to infer the correlation parameter which produces a model spread equal to the market quoted spread. Usually, the time-varying compound correlation differs across tranches $T_{[A,D)}$. Additionally, there is obviously another similarity to implied volatilities called *correlation smile*. In the same way that a *volatility smile* is a function of the option's strike, the *correlation smile* is a function of the tranche-specific subordination level. Figure 4.1 shows a typical *correlation*

Figure 4.1: Compound Correlation Smile, 16^{th} of September 2005



Notes: This figure shows the *compound correlation smile* for tranche spreads of the 5Y *iTraxx Europe* on 16^{th} of September 2005. The x-axis denotes the subordination levels of tranches while the y-axis denotes the *compound correlations*. As we can see the correlation of tranche $T_{[0\%,3\%)}$ is higher than the correlation of tranche $T_{[3\%,6\%)}$. The remaining compound correlations increase with increasing tranche seniorities.

smile. While the compound correlations are denoted on the y-axis, the x-axis denotes the various subordination levels. The correlation of tranche $T_{[0\%,3\%)}$ is higher than the correlation of tranche $T_{[3\%,6\%)}$. The remaining compound correlations monotonically increase with increasing tranche seniorities. Thus,

[107]Implied volatilities provide a common benchmark for a comparison of options across maturities and strikes as well (Ncube 1996).

the plotted function implies that different tranches on the same underlying credit portfolio trade at various correlations, whereas the model to estimate the underlying portfolio loss distribution uses only a single parameter ρ to summarize the overall dependency structure of borrowers. This also means that for valuing STCDOs a flat correlation is not sufficient to model market spreads (Andersen & Sidenius 2005*b*).

One of the main shortcomings with compound correlations is related to the applied quadratic optimizing techniques whose solutions are not unique.[108] The observable ambiguity of *compound correlations* is often mentioned critically in the recent literature: according to the spread function in Equation (4.5) two different - but still plausible - *compound correlations* may lead to the same observable tranche spread. Such a lack of uniqueness makes interpretations of *compound correlations* much more difficult, even more so in CDO hedging (see Finger 2009). Non-monotonic correlations especially in tranches of middle seniority have a weakening effect on the applicability of this concept. For more details compare McGinty et al. (2004).

Base Correlations

In 2004, the *base correlation* approach was proposed by McGinty et al. (2004) to overcome limitations of *compound correlations*. Basically, *base correlations* are implied correlations as well. Analogous to *compound correlations*, they are derived from market spreads. In contrast to *compound correlations*, they are defined as implied correlations of virtual equity tranches.[109] These virtual equity tranches $T_{[0\%,D_i)}$ have the same lower attachment point as standard equity tranches, but differ in their detachment level D_i with level $i \in \{1, 2, 3, 4, 5\}$. The detachment points of these virtual equity tranches correspond to the upper attachment points of all other standard tranches, and thus $D_i \in \{3\%, 6\%, 9\%, 12\%, 22\%\}$. In general, the correlations of fictive tranches $T_{[0\%,D_i)}$ are received in line with the procedure introduced in Section 4.2.1 for the *compound correlations*. A methodical modification within the *base correlation* approach is a bootstrap process presented by JP Morgan. Simplified, it means to derive *base correlation* for higher levels i from the first equity tranche

[108] Solving for the correlation ρ means to minimize the sum of square errors of quoted market and model spreads which leads to the stated quadratic optimization problem.

[109] The equity tranche $T_{[0\%,3\%)}$ is also called first loss tranche.

$T_{[0\%,3\%)}$. The expected tranche loss $\mathbb{E}\left(L^{T_{[0\%,D_1)}}\right)$ is equal to the expected loss from the first observable equity tranche. Consequently, the *base correlation* for the first equity tranche $T_{[0\%,D_1)}$ equals its *compound correlation*. Remaining *base correlations* for higher detachment levels i are obtained by calculating the expected tranche losses $\mathbb{E}\left(L^{T_{[0\%,D_i)}}\right)$

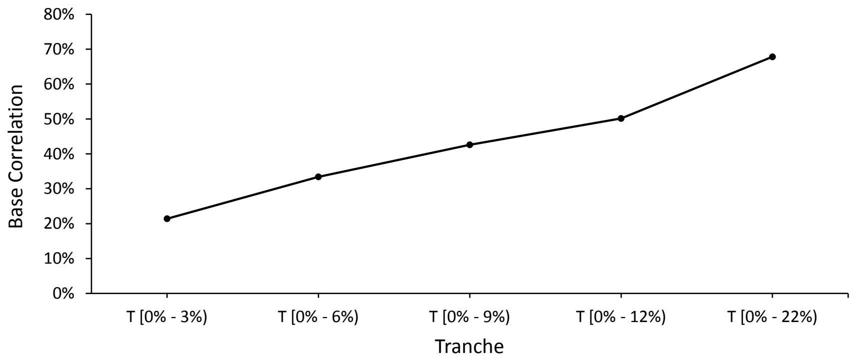
$$\mathbb{E}\left(L^{T_{[0\%,D_i)}}\right) = \mathbb{E}\left(L^{T_{[0\%,D_{i-1})}}\right) + \mathbb{E}\left(L^{T_{[D_{i-1},D_i)}}\right) \tag{4.9}$$

with $i \geq 1$ and $D_0 = 0\%$. $\mathbb{E}\left(L^{T_{[D_{i-1},D_i)}}\right)$ is calculated using the market spread $s^{T_{[D_{i-1},D_i)}}$.

Once expected losses for the sequence of first loss tranches $T_{[0\%,D_i)}$ are provided, we can solve for single *base correlations* for each tranche $T_{[0\%,D_i)}$ (compare Parcell & Wood 2007).[110]

Figure 4.2 reveals the monotonic function of both correlation and spread leading to a typical *base correlation skew*. In contrast to Figure 4.1, *base*

Figure 4.2: Base Correlation Skew, 16^{th} of September 2005



Notes: This figure shows the *base correlation skew* for tranche spreads of the 5Y *iTraxx Europe* on 16^{th} of September 2005. The x-axis denotes synthetic equity tranches $T_{[0\%,D_i)}$. The y-axis denotes *base correlations*. The *base correlations* increase monotonically with increasing detachment level $D_i \in \{3\%, 6\%, 9\%, 12\%, 22\%\}$.

correlations (y-axis) increase monotonically with increasing detachment points of the virtual equity tranches (x-axis). We also conclude that the concept of *base correlation* exploits the monotonicity of equity tranches to overcome the problem of non-uniqueness of *compound correlations*, which leads to a more meaningful skew.

The main benefit of this approach is that *base correlations* can be calculated for any virtual equity tranche $T_{[0\%,D_i)}$ with $D_i \in (0, 1]$. In this manner,

[110]For example: $T_{[0\%,D_2)} = T_{[0\%,6\%)}$.

base correlations can be used to value non-standardized CDOs with specific interpolation methods as provided by Parcell & Wood (2007).

The presented *base correlation* approach provides market participants with a simple measure of implied correlation inherent in quoted tranches. This measure leads to unique solutions and offers a reasonable valuation framework for non-standardized CDOs (Andersen & Sidenius 2005*a*, Hull & White 2006, Finger 2009).

Historical Asset Correlations

In the basic single-factor Gaussian copula framework, only a single correlation parameter ρ is needed to describe the overall dependency structure of borrowers in the underlying credit portfolio. After estimating the correlation parameter ρ and based on a specified default threshold c, the bivariate Gaussian copula model can be applied to calculate fair tranche spreads $s^{T_{[A,D)}}$ as denoted in Equations (4.5) and (4.6).[111]

One standard approach to directly approximate the dependency parameter ρ is inspired by fundamental assumptions in the Merton model. Within this framework the default event of a firm is endogenously modeled by assessing the firm's capital structure (compare Merton 1974). This idea can be extended to a credit portfolio. In order to derive joint default correlations of firms in the credit portfolio, the firms‘ asset returns and their dependency structures are taken into consideration. Hence, current market information is included in modeling dependency structures across borrowers. Other authors who address this kind of direct modeling are, for example, Lucas (1995), Gupton et al. (1997) and Zhou (2001).

For our empirical analysis, we modify this standard approach as follows: Firstly, we define several sample periods of our empirical study to account for different states of the global economy, as described in Section 4.3. Secondly, we investigate the log-asset returns of the firms which are included in 'on the run' series of the 5Y *iTraxx Europe* in order to calculate average asset correlations.[112] The daily average asset correlations – also called *historical asset*

[111] In the market standard model for pricing STCDOs, a single correlation parameter is sufficient for pricing securitized tranches. However, such a single correlation parameter implies a flat correlation structure, which is in contrast to observable implied correlation smiles and skews.

[112] Using *DataStream* provided by Thomson Reuters.

correlations – are dynamically calculated for every trading day t in the specified sample period, with $t \in \{1, ..., T\}$. Each *historical asset correlation* refers to a 250-day time window. Thus, the calculation of a *historical asset correlation* is based on the assets' last 250 trading days, which are approximately representing an one-year time horizon. The first window ends one day before the first spread forecast. In the following, the 250-day time window dynamically rolls through the specified forecast periods day by day. On each day, an average *historical asset correlation* is calculated which is then used for forecasting tranche spreads (one day ahead).[113] Eventually, we daily approximate the asset correlation ρ applied in Equation (4.6) by the average *historical asset correlations* among the index entities of 'on the run' series.

In contrast to forecasts based on implied correlations, forecasts with various types of *historical asset correlations* assume an identical correlation for each tranche $T_{[A,D)}$. Consequently, daily spread forecasts based on implied correlations consider tranche-specific correlation values, while the dynamic asset correlation model (ACM) does not. Despite these general limitations of single-correlation models, the dynamic ACM is used as a benchmark model due to its pricing popularity in the past.

4.2.3 Dynamic Panel Regression Approach for Base Correlations

Implied correlations are not constant, but change over time and between tranches. By inverting the spread formula in Equation (4.5) numerically, we get daily implied correlation parameters for each tranche $T_{[A,D)}$.[114] With respect to the benefits of *base correlations*, we focus in the following on this type of implied correlation. Thus, our panel data consists of 5 cross-sectional units (tranches $T_{[A,D)}$) which are observed over time $t \in \{1, ..., T\}$. Hence, panel data models can be estimated for the implied correlation parameter in order to reflect both cross-sectional and time characteristics. According to our model assumptions, the correlations range between 0 and 1, $\rho_t^i \in [0, 1]$. Thus, a probit transformation is performed in order to create a variable ranging between

[113]The specific length of different forecast periods is denoted in Table 4.2 of Section 4.3.2.
[114]In order to obtain implied correlations, we used the Quasi-Newton Method.

$[-\infty, +\infty]$. Further, i refers to synthetic tranches $T_{[0\%, D_i)}$

$$i = T_{[0\%, D_i)}. \tag{4.10}$$

Since we regard tranches $T_{[0\%, D_i)}$ of five different levels i, it follows that $i \in \{1, ..., 5\}$. Additionally, we involve the lagged correlation parameter ρ^i_{t-1} of tranche i as explanatory variable in the model to account for autocorrelation. $\bar{\beta}^i_0 = \bar{\beta}_0 + \alpha^i$ is the intercept for the i-th *base correlation*, $\bar{\beta}_0$ represents the 'mean' intercept (Ncube 1996). α^i is the difference between the individual intercept and the 'mean' intercept and thus accounting for time-constant differences among tranches in both of our proposed dynamic panel regression models. Therefore, fixed effects vary across tranches i depending on their seniority. Besides we assume that ρ^i_{t-1} is a convenient predictor for the *base correlation* ρ^i_t. For the i-th tranche the model is given by

$$\Phi^{-1}(\rho^i_t) = (\bar{\beta}_0 + \alpha^i) + \beta_1 \cdot \Phi^{-1}\left(\rho^i_{t-1}\right) + v_t + u^i_t, \tag{4.11}$$

with $t \in \{1, ..., T\}$. $\Phi^{-1}(\cdot)$ denotes the inverse of the standard normal distribution function and T describes the amount of days. β_1 describes the sensitivity with respect to the lagged correlation which is identical for all tranches i. To complete our model, we need to specify whether α^i and v_t, respectively, are stochastic or fixed. If a variable (α^i, v_t) is assumed stochastic, we require some distributional assumptions on the effects as well.

Firstly, we propose a mixed-effects regression correlation model (MERM) allowing for fixed and random effects.[115] In this manner, we assume that the parameter v_t describes an unobservable random effect accounting for any time-specific effect that is not included in the regression.

The residual e^i_t consists of two components:

$$e^i_t = v_t + u^i_t, \tag{4.12}$$

where $v_t \sim \mathcal{N}(0, \sigma^2_v)$ i.i.d., $u^i_t \sim \mathcal{N}(0, \sigma^2_u)$ i.i.d. and $\sigma^2_e = \sigma^2_v + \sigma^2_u$. While v_t describes an unobservable time effect, u^i_t is the remainder stochastic disturbance term varying in time and with tranche seniority.

[115] For a discussion of dummy-variable and error-component models refer to Hsiao (1986) and Baltagi (1995). Time-series and cross-section studies not controlling for heterogenous individuals run the risk of obtaining biased results, see, e.g., Baltagi (1995).

We assume the following residual covariance structure:

$$cov(e_t^i, e_s^j) = \begin{cases} \sigma_v^2 & \text{if} \quad i \neq j, t = s \\ \sigma_e^2 & \text{if} \quad i = j, t = s \\ 0 & \text{otherwise.} \end{cases} \tag{4.13}$$

The correlation structure is given by

$$\psi = corr(e_t^i, e_s^j) = \begin{cases} \sigma_v^2/\sigma_e^2 & \text{if} \quad i \neq j, t = s \\ 1 & \text{if} \quad i = j, t = s \\ 0 & \text{otherwise,} \end{cases} \tag{4.14}$$

and shows that the correlation between *base correlations* ρ_t^i is determined by the variance σ_v^2 of the random time effect v_t for a given time period. This intra-class correlation ψ

$$\psi = \frac{\sigma_v^2}{\sigma_u^2 + \sigma_v^2} \tag{4.15}$$

measures the extent of unobserved latent time-invariant variation relative to the total unobserved variation. Since i) all tranches are affected by the time-specific effect v_t in the same way and ii) the intra-class correlation measures the ratio of its variance (σ_v^2) to the total variance, we conclude that the intra-class correlation provides an indication of systematic risk influences on the correlation parameter ρ_t^i. Thus, we suggest that the higher (lower) the systematic risk influence, the higher (lower) the relation of σ_v^2 to σ_u^2 will be and also the intra-class correlation.

Secondly, we propose a fixed-effects regression model (FERM) allowing for fixed effects only. In this manner, we assume that α^i is a fixed tranche-specific effect and the residual e_t^i consists only of one component

$$e_t^i = u_t^i, \tag{4.16}$$

where $u_t^i \sim \mathcal{N}(0, \sigma_u^2)$ i.i.d. This leads to the following dummy-variable model for the i-th tranche

$$\Phi^{-1}(\rho_t^i) = (\bar{\beta}_0 + \alpha^i) + \beta_1 \cdot \Phi^{-1}(\rho_{t-1}^i) + u_t^i \tag{4.17}$$

with $t \in \{1, ..., T\}$. By expanding our FERM, it is easily possible to additionally account for fixed time-specific effects. But even if we are able to estimate these effects ex post, it is barely possible to forecast these effects ex ante. Thus, fixed time effects are not considered.

4.3 Empirical Analysis

4.3.1 Panel Data

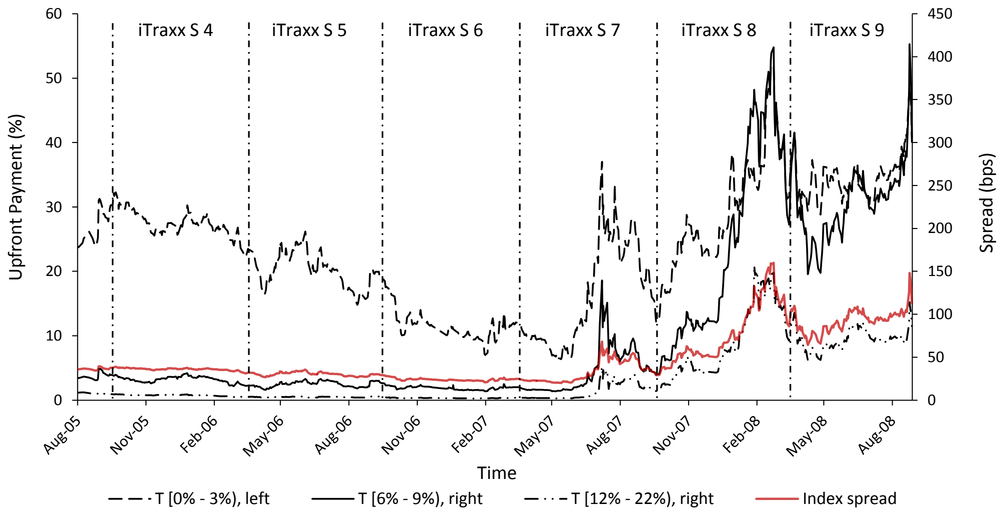
Our database contains daily spreads of both the 5Y *iTraxx Europe* index and its standardized tranches from August 2005 to September 2008.[116] Within our empirical study we focus on quoted market spreads referring to 'on the run' series of the 5Y *iTraxx Europe*.[117]

Figure 4.3 shows historical spreads of the 5Y *iTraxx Europe* index (red line) and the time series of several tranche spreads (black lines) from August 24^{th}, 2005 to September 19^{th}, 2008. The quoted spreads refer to several 'on the run' series. The runtime of each series is indicated by the dashed-dotted vertical lines and marked by *iTraxx S 4* to *iTraxx S 9*. The x-axis denotes the observation days. While the y-axis on the left hand side denotes the market upfront payment of the equity tranche $T_{[0\%,3\%)}$ in percent, the secondary y-axis on the right hand side denotes spreads in basis points (bps). From August 24^{th}, 2005 to June 18^{th}, 2007 we observe slightly decreasing index spreads from 39 bps to almost 20 bps. In contrast to the remaining time series of the 5Y *iTraxx Europe*, the index spread movements are moderate and at a relatively low level during this period. For this reason, we define our first sample from August 24^{th}, 2005 to June 18^{th}, 2007 (*Sample 1*). On June 18^{th}, 2007 it is reported for the first time, that Merrill Lynch seizes collateral from a Bear Stearns hedge fund invested heavily in subprime loans, which leads to strongly increasing credit spreads over the following days. Therefore, we define the 19^{th} of June as the beginning of our GFC sample (*Sample 2*). Several days after this announcement - at the end of July 2007 - the 5Y *iTraxx Europe* reaches its first peak at 68 bps. Despite loan interventions through the Federal Reserve Bank (New York) in March 2008 attempting to avert a sudden collapse of Bear

[116]All quoted market spreads are provided by *Markit*. For daily closing quotes we consider the mid of quoted bid/ask spreads.

[117]Every six months a new 'on the run' series is issued with a constant maturity of 5 years and a fixed basket of CDS.

Figure 4.3: Spreads of the Standardized 5Y iTraxx Europe from 2005 to 2008



Notes: The figure shows historical 5Y *iTraxx Europe* index spreads (red line) as well as several tranche spreads with regard to 'on the run' series S 4 to S 9 which are indicated by the dashed-doted vertical lines. The sample period is August 24^{th}, 2005 to September 19^{th}, 2008. During series 7, all spreads strongly increase for the first time in line with increasing spread volatility due to the beginning of the GFC.

Stearns, the company can not be saved and is sold to JP Morgan Chase later on. Within these market turbulences, the 5Y *iTraxx Europe* registers a new all time high of 160 bps on March 17^{th}, 2008, when JP Morgan Chase offers to acquire Bear Stearns. This peak is at least 8 times higher than the last peak in June 2007. The time series of the *iTraxx Europe* strongly reflects the chronology of the GFC.[118] In comparison to *Sample 1* (before the GFC), the observed 5Y *iTraxx Europe* index spreads are much more volatile as well as higher quoted throughout *Sample 2* (during the GFC). Lastly, we define our third sample as the entire observation period by merging *Sample 1* and *2* into *Sample 3*, in which we are not accounting explicitly for the GFC.

Corresponding to the index chart in Figure 4.3 all 5Y *iTraxx Europe* tranche spreads are slightly decreasing in *Sample 1* and strongly increasing after the 18^{th} of June 2007. Throughout *Sample 2* we observe i) relatively high spread volatilities and ii) an absolute increase in the tranche-specific spread levels. For example: while the standard deviation (STD) in bps of the mezzanine tranche $T_{[6\%,9\%)}$ is 18 times higher in *Sample 2* than in *Sample 1* ($STD_1^{T_{[6\%,9\%)}} = 5.78$ vs. $STD_2^{T_{[6\%,9\%)}} = 105.59$)[119], the mean of tranche spreads $s^{T_{[6\%,9\%)}}$ increases from 18.81 bps to 164.38 bps. In fact, the most senior tranche (dotted line)

[118]The chronology of the GFC is reported in more detail in BIS (2009*a*).
[119]Index 1 refers to *Sample 1* and index 2 to *Sample 2*.

seems to be much more affected by economic downturns like the GFC than other tranches. This may be indicated by the relation between average tranche spreads $\left[s_2^{T_{[A,D)}} / \, s_1^{T_{[A,D)}} \right]$. This relation is increasing with tranche seniority: regarding the most senior tranche we obtain a value of around 14, which means that the average tranche spread $s_2^{T_{[12\%,22\%)}}$ is 14 times higher in *Sample 2* than in *Sample 1*. For the upfront payment of the equity tranche (dashed line) and the mezzanine tranche $T_{[6\%,9\%)}$ (continuous black line) this relation is 1.54 and 8.74, respectively.

Table 4.1 provides additional summary statistics for index and tranche data referring to *Sample 3*.

Table 4.1: Summary Statistics of 5Y Itraxx Europe Index and Tranche Spreads

Tranche	Mean	STD	Min.	Max.	N
0% - 3%	2,321.84	953.41	636.07	5,219.11	749
3% - 6%	156.47	145.09	39.70	685.93	749
6% - 9%	81.36	100.11	10.25	414.69	749
9% - 12%	51.45	68.20	4.18	323.24	749
12% - 22%	25.93	34.56	1.72	156.79	749
iTraxx Index	48.42	30.42	20.09	160.00	749

Notes: This table provides summary statistics of market spreads in basis points for *Sample 3*. The results refer to merged 'on the run' time series. The sample period is August 24^{th}, 2005 to September 19^{th}, 2008. N denotes the amount of observations. STD describes the respective standard deviation of market spreads.

The entire sample contains 749 daily observations for i) the 5Y *iTraxx Europe* and ii) each of its securitized tranches. The mean spreads monotonically decrease with increasing tranche seniority. Thus, the highest (lowest) default risk is linked to the equity (senior) tranche which is indicated by the highest (lowest) mean spread of 2,321.84 bps (25.93 bps). In fact, the senior tranche exhibits the largest spread range: its maximum spread is 91 times higher than its minimum spread. Additionally, considering the relation between standard deviation and average spread ($STD/Mean$) we suggest that the sensitivity to systematic risk, e.g., in economic downturns, is increasing monotonically in tranche seniority due to the increasing systematic risk exposures (compare Chapter 5). Depending on the systematic risk exposures, credit risk premia of tranches may increase in economic downturns, and we suggest that this increase is even higher for high-seniority tranches.

Depending on the specified time periods (*Sample 1, 2* and *3*), the sample sizes of our empirical analysis vary: while *Sample 1* contains 431 daily observations for the credit index and each of its tranches (total quotes: 2,586), *Sample 2* includes 318 daily spreads for the *iTraxx* index and 1,509 daily tranche spreads (total quotes: 1,908). The entire period (*Sample 3*) refers to 749 trading days with 4,494 quoted spreads in total (index and tranche spreads).

4.3.2 Analysis of Panel Regressions

In order to obtain tranche-specific *base correlations* ρ_t^i with $i \in \{1, ..., 5\}$ – using the Quasi-Newton Method – we make commonly applied parameter assumptions referring to

- the recovery rate R^{Trax} of the 5Y *iTraxx Europe* index,
- the default intensity λ of the 5Y *iTraxx Europe* index, and
- the risk-less rate r, which is used to calculate the time-dependent discount factors Q_{t_j}.[120]

According to various authors such as Andersen & Sidenius (2005*b*), Laurent & Gregory (2005), Hull & White (2006) and *Markit*, we assume a constant recovery rate of 40% for investment grade names of the 5Y *iTraxx Europe* index which leads to $R = R^{Trax} = 40\%$.[121]

Similar to Hull & White (2004) and analogous to Equation (4.7), we assume a time-constant default intensity λ. λ approximates the risk-neutral default intensity of the 5Y *iTraxx Europe* which is implicitly given by the daily index spread notations, compare Equation (4.8). Thus, λ is time-variant by day, but constant over the index maturity.

Further, we follow other authors in the recent literature by assuming a flat term structure of risk-less interest rates (compare Hull & White 2004, Heitfield 2009). Based on *Sample 3*, we set the risk-less interest rate r at 2%, which is at the lower end of the related historical T-bill term-structure.

Since i) the market spread is given and ii) we implement an identical pricing model in our entire analysis, we make two observations: firstly, with respect to our dynamic regression models, parameter changes referring to, e.g., recovery or interest rates, lead mainly to shifts in the implied correlation level.

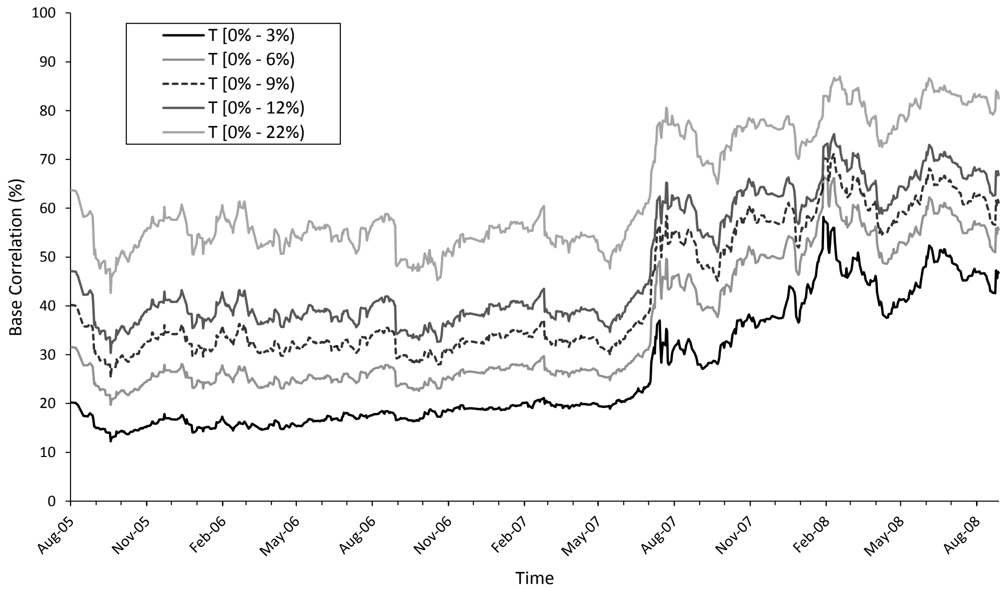
[120] Compare Equations (4.5), (4.7) and (4.8).
[121] Recall that the quoted market spreads are provided by *Markit*.

Consequently, solving for observable market spreads makes parameter settings somewhat less important, as shown in Appendix A.

Secondly, parameter settings may be more decisive in terms of our benchmark model: in this case the single correlation parameter approximated by the *historical asset correlation* directly determines the model spread. Thus, we may slightly amend the results of our benchmark model (ACM) by implementing a 'real' – or rather a more realistic – term structure. We provide some sensitivity analysis related to historical Treasury bills in Appendix A. But even if we consider differing assumptions, e.g., a term structure referring to Treasury bills, we only observe small benefits. Market spreads show that single-correlation models remain strongly limited in pricing all tranches simultaneously. However, often it is impossible to fit a single correlation on the tranches' market spreads - independent from specific parameter settings.

Next, we derive *base correlations* ρ_t^i from market quotes of the standardized 5Y *iTraxx Europe* tranches. Figure 4.4 shows *base correlation* curves for each tranche $T_{[0\%,D_i)}$ from August 24^{th}, 2005 to September 19^{th}, 2008. Throughout

Figure 4.4: Time Series of Base Correlations from 2005 to 2008



Notes: The figure shows time series of calculated *base correlations* $T_{[0\%,D_i)}$ from August 24^{th}, 2005 to September 19^{th}, 2008. The *Base correlations* monotonically increase with increasing detachment level and are strongly linked to each other. Similar to Figure 4.3, the *base correlations* are also strongly increasing in the beginning of the GFC and exhibit a higher volatility in the aftermath.

Sample 3 (x-axis), the skewed curves indicate that *base correlations* ρ_t^i (y-axis) are increasing monotonically with increasing detachment level i, which is also pointed out in Figure 4.2. This effect can be expected as long as spreads of index tranches are decreasing in line with increasing tranche seniority. In other words, the synthetic equity tranche $T_{[0\%,22\%)}$ exhibits the highest ρ_t^5, while tranche $T_{[12\%,22\%)}$ with highest seniority exposes the lowest tranche spread $s^{T_{[12\%,22\%)}}$ (compare Figure 4.3). In contrast, the equity tranche $T_{[0\%,3\%)}$ with lowest seniority exhibits the lowest base correlation ρ_t^1 as well as the highest tranche spread $s^{T_{[0\%,3\%)}}$ across all other securitized tranches. The plotted *base correlations* correspond also to findings related to Figure 4.3: as the *base correlations* ρ_t^i move moderately sideways in *Sample 1* the correlation levels increase simultaneously at the beginning of the GFC. During the GFC the tranche-specific *base correlations* are on average about 1.8 times higher than in *Sample 1*, while the average standard deviation of *base correlations* across all tranches is about 2.8 times higher than in *Sample 1*.

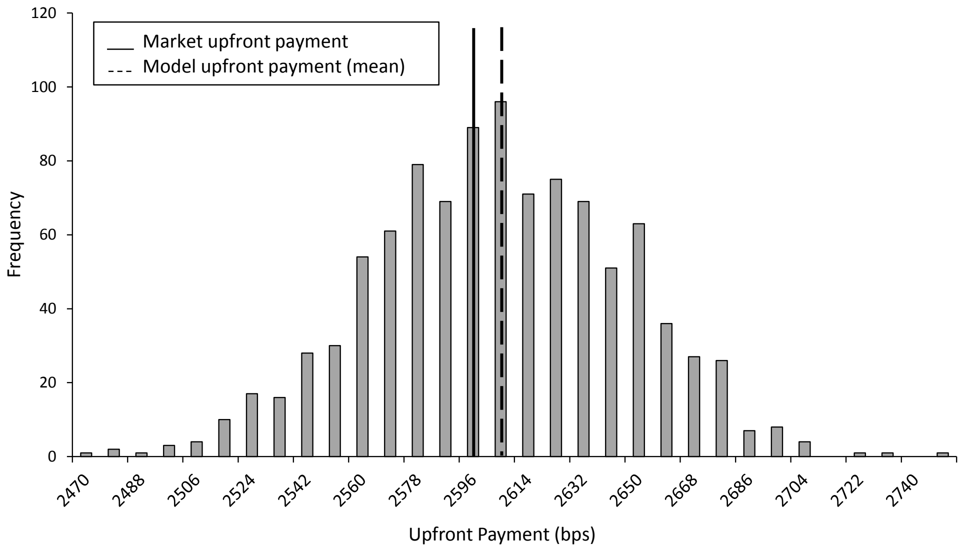
Our dynamic regression correlation models – Equations (4.11) and (4.17) – are tested for accuracy in pricing STCDOs against a benchmark measure using *historical asset correlations*. In order to validate all our proposed models, we examine their accuracy in matching quoted market spreads. Eventually, we test which correlation measure prices STCDOs more accurately by comparing root mean square forecast errors (RMSFE) of our model-based spread forecasts. In this respect, we compare the following three correlation models:

- the dynamic mixed-effects regression correlation model (MERM) which accounts for both random time-specific effects and fixed tranche effects,
- the dynamic fixed-effects regression correlation model (FERM) which accounts for fixed tranche effects, and
- the dynamic asset correlation model (ACM).

In the following, the regression and forecast methodology is briefly explained: regression windows (or calibration windows) that are fixed in their size (40-, 50- and 60-days) dynamically roll through *Sample 1, 2* and *3* incremental day by day.

Based on each regression window, we calibrate both of our prediction models MERM and FERM to forecast *base correlations* ρ_t^i one day ahead. Referring to the next day t (forecasting day), we aim not only to achieve point predictions for ρ_t^i, but also probability distributions of *base correlations*. In this respect, we use Monte Carlo simulations to compute daily samples of 1,000 observations for the time-specific effects v_t and the residual u_t^i. Then, we compute 1,000 one day ahead forecasts for *base correlations* ρ_t^i. Before the estimation window is shifted forward one day, this set of out-of-sample correlation forecasts is used in Equation (4.5) to compute a mean spread for each tranche $T_{[A,D)}$ denoting the respective spread forecast. This indirect spread forecasting technique allows us to derive useful descriptive statistics from the simulated spread distribution. Figure 4.5 displays such a spread distribution for a single forecast day in comparison to the real market upfront payment (UP). The x-axis denotes various spread classes and the y-axis denotes the respective frequency.

Figure 4.5: Distribution of Spread Forecasts for Tranche $T_{[0\%,3\%)}$ on December 2^{nd}, 2005



Notes: The figure shows a histogram of 1,000 simulated model upfront payments (UPs) for tranche $T_{[0\%,3\%)}$ on December 2^{nd}, 2005 (MERM). The mean of the simulated model UPs is denoted by the dashed line and the real observed market UP by the black line. For forecast purposes, the mean of the model UPs is taken into consideration.

The mean spread of these 1,000 spread realizations constitutes our UP forecast for the equity tranche $T_{[0\%,3\%)}$ (dashed line). The observed market UP is denoted by the black line. In this example, the difference between the market UP (2,590 bps) and the mean model spread (2,599 bps) is 9 bps which is less than 0.4 percent of the market UP. This result indicates the accuracy of our model forecasts.

Analogously, we predict spreads $s^{T_{[A,D)}}$ one day ahead for all tranches. As we receive new spread information every day, we focus methodically only on one day ahead forecasts. Forecasted tranche spreads $s^{T_{[A,D)}}$ are then compared to those forecasted with the *historical asset correlation* parameter. Results of all three forecast models are compared to market spreads. Since all our spread forecasts refer to the first day after each calibration window, our forecasts are out of sample forecasts or out of window forecasts. After the regression window is incrementally rolled forward one day, we recalibrate our prediction models, forecast *base correlations* and finally derive model-specific spread forecasts. This estimation and forecast exercise is repeated until all regression windows reach the end of each sample. For comparability, the forecast periods related to each window size (40-, 50- and 60-days) should correspond to each other. Therefore, they are identically defined across all window sizes within our samples. Table 4.2 summarizes the three forecast periods depending on the defined rolling regression windows.

Table 4.2: Sample-specific Forecast Periods of the Empirical Analysis

		Sample 1	Sample 2	Sample 3
Forecast period	Begin:	2^{nd} of Dec 05	12^{th} of Oct 07	2^{nd} of Dec 05
	End:	18^{th} of Jun 07	19^{th} of Sep 08	19^{th} of Sep 08
Spread forecasts		370 (1,850)	258 (1,290)	688 (3,440)

Notes: The table shows an overview of three different forecast periods which are identically defined across all regression windows. Values in brackets describe the total amount of tranche spread forecasts related to all of the five index tranches.

Depending on our forecast period, we estimate between 258 and 688 regressions of Equations (4.11) and (4.17) to get the same number of predicted *base correlations* ρ_t^i for each synthetic equity tranche $T_{[0\%,D_i)}$, which we also use to derive corresponding spread forecasts. Eventually, we forecast 1,850 model-specific tranche spreads in terms of *Sample 1*, 1,290 spreads in terms of *Sample 2* and 3,440 spreads in terms of *Sample 3*.

An example: the first forecast day in *Sample 1* is December 2^{nd}, 2005. In case of a 40-days regression window, we estimate our models on daily data observed from October 5^{th}, 2005 to December 1^{st}, 2005 (40 observation days). We analogously proceed for the remaining two regression windows. As a result, *Sample 1* contains three different starting dates for the rolling windows depending on the window size (40-, 50- or 60-days), but the first forecast day is jointly December 2^{nd}, 2005. For re-estimating the dynamic correlation models the fix-sized regression windows are incrementally rolled forward one day ahead. Thus, the second forecast day is jointly December 3^{rd}, 2005 and so on.

Our rolling regression analysis is applied to backtest all prediction models on historical data. In order to evaluate the model-specific forecast accuracy, we implement a root mean square forecast errors (RMSFE) metric referring to the difference between market spreads and model spreads. Finally, our RMSFE metric is i) window-specific, ii) model-specific and iii) sample-specific.

To ensure the comparability of our results, we calculate the RMSFEs according to

$$RMSFE_p = \sqrt{\frac{\sum_{t=1}^{\mathcal{T}} \left(1 - \frac{\hat{s}_{t,p}}{s_{t,p}}\right)^2}{\mathcal{T}}}. \tag{4.18}$$

Thereby, $p \in \{1, 2, 3\}$ indicates the related sample, $\hat{s}_{t,p}$ denotes the predicted tranche spread on day t, $s_{t,p}$ describes the corresponding market spread and $\mathcal{T}$ denotes the amount of sample-specific daily forecasts.

The descriptive statistics provided in Section 4.3.1 underline decisive differences in the behavior of tranche spreads within the entire observation period (*Sample 3*). By splitting off this entire period in subsamples, we test the forecasting accuracy of our models in i) times of market turbulences (*Sample 2*) and ii) under moderate market conditions (*Sample 1*). In this way, we control for various calibration periods, market conditions as well as different prediction models to increase the robustness of our findings.

4.3.3 Results

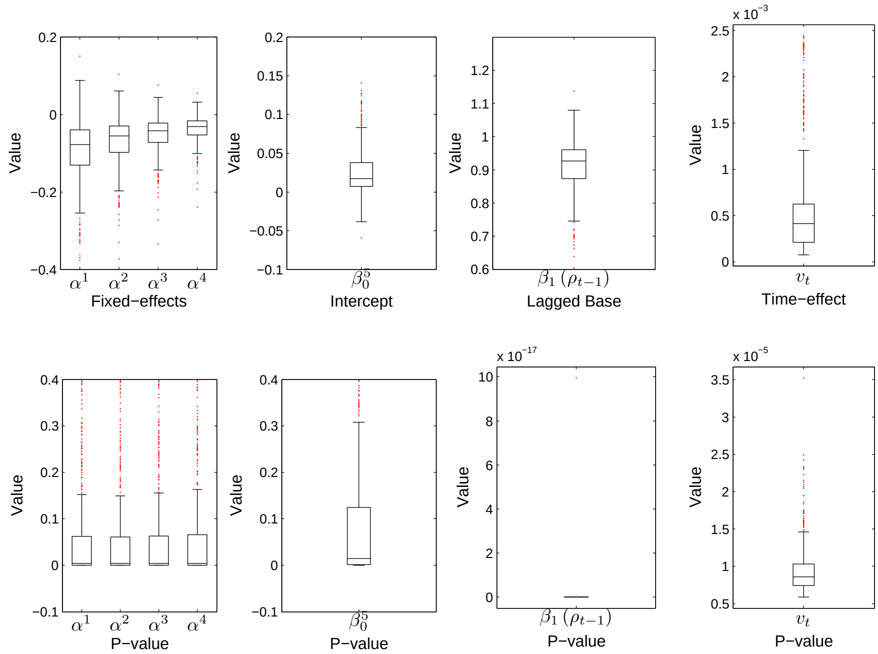
Firstly, we estimate our dynamic mixed-effects regression correlation model (MERM) as well as our dynamic fixed-effects regression correlation model (FERM) for the three samples (*Sample 1, 2* and *3*).[122]

Figure 4.6 illustrates the parameter estimates for our dynamic regression model MERM in Equation (4.11) based on 40-days estimation windows. The various boxplots refer to estimation results of 688 regressions within *Sample 3*. While the upper four boxplots describe the parameter estimates, the lower

[122]The parameters of the regression models in Equations (4.11) and (4.17), several variance components and the random time effects were estimated using the Maximum Likelihood method or rather the Restricted Maximum Likelihood method. This so-called *mixed* procedure is implemented in SAS.

boxplots summarize the corresponding p-values indicating the statistical significances of the estimates.

Figure 4.6: MERM Estimation Results



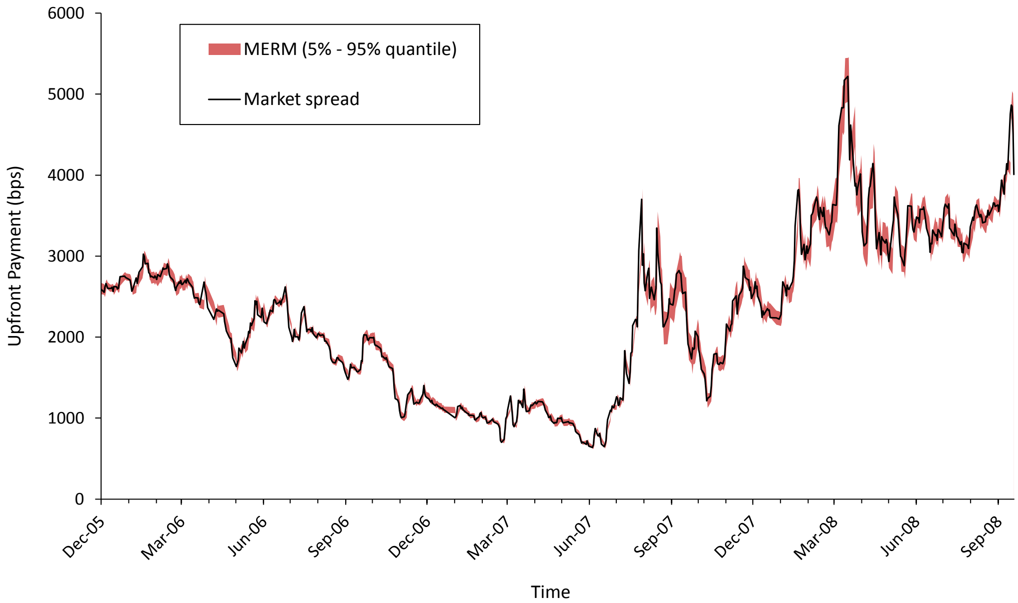
Notes: The figure shows various boxplots for *Sample 3* which summarize the estimation results of 688 regressions based on our MERM (40-days). The upper four boxplots refer to the distribution of tranche-specific fixed effects α^i, the intercept ($\bar{\beta_0}^5$) considering the reference group α^5, the coefficient β_1 of the lagged base correlation ρ_{t-1} and the random time effect v_t. The four lower boxplots show the distribution of the respective p-values.

Based on the reference intercept $\bar{\beta_0}^5$ composed of the 'mean' intercept $\bar{\beta_0}$ and α^5 (the fixed effect related to the senior tranche), the mean of tranche-specific fixed effects α^i is monotonically decreasing with decreasing tranche seniority. The fixed-effect estimates are on average statistically significant at the 9%-level. Thus, considering tranche-specific fixed effects should generally amend the spread forecast performance.

According to the upper boxplots, lagged base correlations ρ_{t-1} are highly influencing the endogenous variables which is also underlined by corresponding p-values (boxplot below): each coefficient β_1 (min: 0.37, max: 1.14) exhibits a p-value < 0.0001 throughout *Sample 3* indicating a high statistical significance of these estimates. Similar to lagged *base correlations*, random time-specific effects v_t significantly impact the endogenous variable with p-values below the 0.0035% quantile.

Secondly, we provide tranche spreads $s^{T_{[A,D)}}$ for each index tranche in accordance to our forecasted *base correlations* ρ_t^i. Figure 4.7 shows the time series of market upfront payments (black line) in comparison to forecasted UPs of tranche $T_{[0\%,3\%)}$. The red area refers to the 5% and 95% quantiles of the simulated model spreads under the applied MERM for *Sample 3* (x-axis). The y-axis denotes the UP in bps.

Figure 4.7: Forecast Performance with Base Correlations for Tranche $T_{[0\%,3\%)}$



Notes: This figure shows the market upfront payment (UP) for tranche $T_{[0\%,3\%)}$, the red area describes the forecast interval related to the 5% to 95% quantile of forecasted UPs. All forecasts are calculated with estimated *base correlations* (MERM, 40-days window). The red area surrounds the market upfront payment throughout the forecast horizon, which is from December 2^{nd}, 2005 to September 19^{th}, 2008.

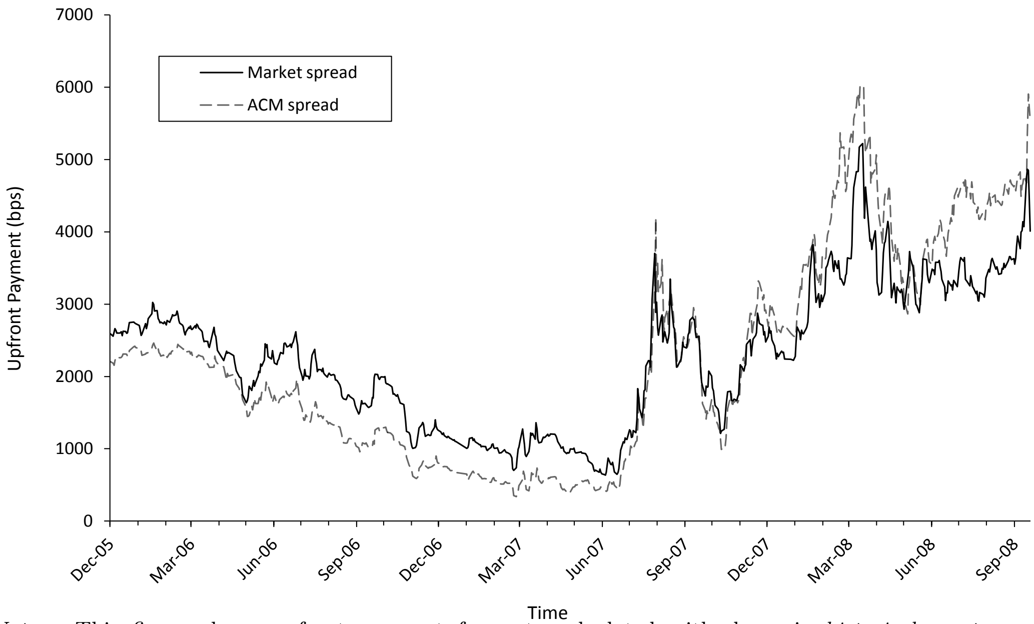
According to the plotted quantiles the predicted UPs of tranche $T_{[0\%,3\%)}$ continuously surround observed market UPs, even during the GFC.

In Figure 4.8, we also provide corresponding UPs (y-axis) using the dynamic *historical asset correlation* model (ACM) for *Sample 3* (x-axis). The dashed line shows the performance of UP forecasts related to the equity tranche.

In comparison to Figure 4.7, UP forecasts underestimate market spreads (black line) throughout *Sample 1*. Reasonable results are reached at the beginning of the GFC, but during the last months of *Sample 3* the ACM forecasts are clearly overestimating quoted market spreads. This leads to higher RMSFEs in *Sample 3* (see Table 4.3).

Referring to both the equity tranche and most senior tranche of the 5Y *iTraxx* Europe, we provide model-specific scatter plots in Figure 4.9. The scatter plots indicate the forecast accuracy of our dynamic regression correlations models in comparison to our ACM benchmark model in terms of the

Figure 4.8: Forecast Performance with Dynamic Asset Correlations for Tranche $T_{[0\%,3\%)}$



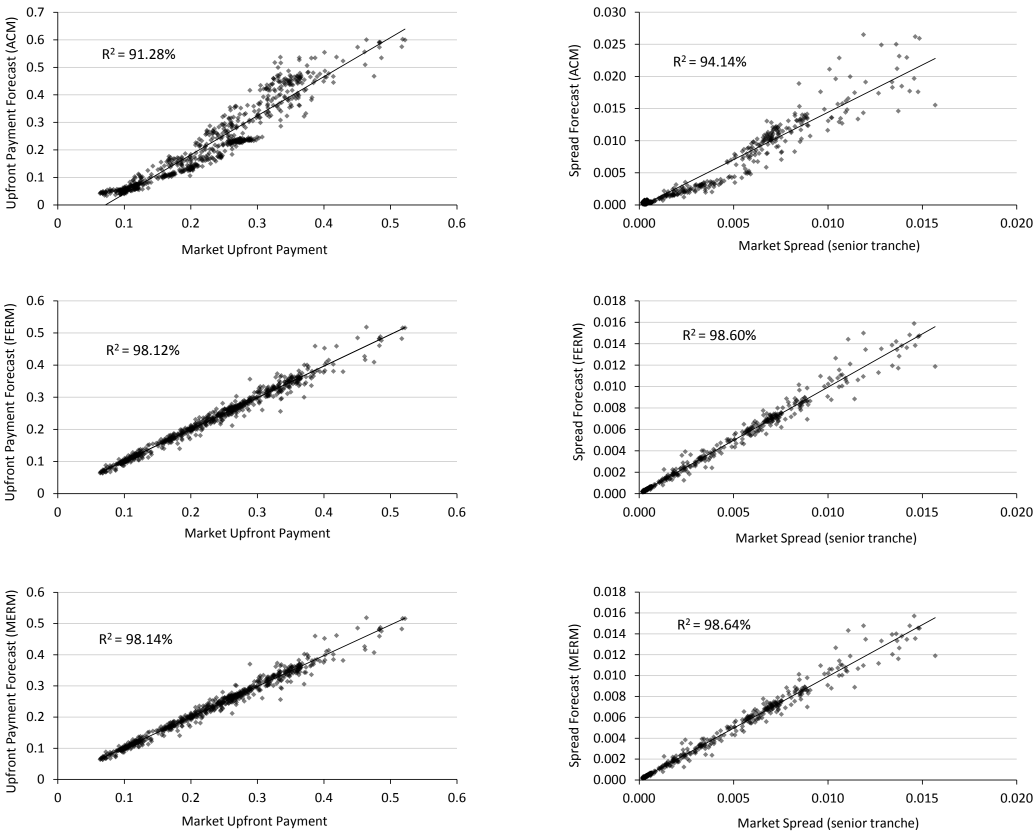
Notes: This figure shows upfront payment forecasts calculated with dynamic *historical asset correlations* versus real market upfront payments of tranche $T_{[0\%,3\%)}$. The forecasting horizon is December 2^{nd}, 2005 to September 19^{th}, 2008.

coefficient of determination (R^2). More comprehensive results - also related to other tranches - as well as applied test statistics are provided in Appendix B. In each chart, the observed market spreads (x-axes) are plotted against the respective model spreads (y-axes). The three charts on the left hand side of Figure 4.9 refer to the UPs of the equity tranche. The three charts on the right hand side provide results related to the senior tranche. Both upper charts focus on the ACM benchmark model, both charts in the middle refer to our FERM. The results of the two lower charts refer to the MERM.

All charts indicate that the spread forecasts scatter more during the GFC (*Sample 2*) since the spread deviations are higher at high spread levels. During *Sample 1*, where the spreads across all tranches are at the lowest level, the forecast error of each model is also lowest. Concerning the three left charts, the ACM provides the worst UP forecasts in terms of R^2 ($R^2_{ACM} = 91.28\%$). Therefore, both dynamic regression correlation models outperform our ACM: the R^2 of FERM is almost 98.12% and the one of MERM is 98.14%.

In comparison to UP forecasts, we obtain a higher accuracy in forecasting senior tranche spreads with each of the three models (three charts on the

Figure 4.9: Market Spreads versus Model Spread Forecasts



Notes: This figure shows several scatter plots which may indicate the forecast performance of our dynamic regression correlation models MERM (lower charts) and FERM (middle charts) in comparison to our benchmark model ACM (upper charts). In each chart, the real market spread is plotted against the model-specific spread forecast. The three charts on the left hand side show the forecast results referring to the market upfront payments, the charts on the right hand side analogously show the results related to spreads of the senior tranche. The forecasting horizon is December 2^{nd}, 2005 to September 19^{th}, 2008 (*Sample 3*).

right). Again, the forecast errors are highest in case of the benchmark model ($R^2_{ACM} = 94.14\%$). The lowest errors are provided by the MERM ($R^2_{MERM} = 98.64\%$), closely followed by those of the FERM ($R^2_{FERM} = 98.6\%$).

Over all, the R^2 metric shows that we reach the highest explanatory power for all tranches of the 5Y *iTraxx Europe* by considering both random time-effects and fixed tranche effects (MERM).

In order to obtain more detailed insights into the spread forecast accuracy, we compare all mentioned correlation models - FERM, MERM and ACM - by calculating the introduced RMSFE metric, compare Equation (4.18). Table 4.3 shows an overview of the RMSFEs related to the three correlation models for *Sample 1, 2* and *3* depending on the various regression windows.

Table 4.3: Root Mean Square Forecast Errors by Sample

Sample 1						
Model	Window (days)	Tranche 0% - 3%	3% - 6%	6% - 9%	9% - 12%	12% - 22%
MERM	40	0.0441	0.0390	0.0544	**0.0696**	**0.0661**
	50	**0.0440**	0.0388	0.0537	0.0714	0.0669
	60	0.0441	0.0387	**0.0531**	0.0697	0.0673
FERM	40	0.0442	0.0395	0.0552	0.0723	0.0676
	50	0.0441	0.0391	0.0551	0.0719	0.0696
	60	0.0443	**0.0385**	0.0554	0.0735	0.0675
ACM		0.3399	1.4351	1.9897	1.7799	0.7334

Sample 2						
Model	Window (days)	Tranche 0% - 3%	3% - 6%	6% - 9%	9% - 12%	12% - 22%
MERM	40	0.0510	**0.0641**	**0.0705**	0.0804	0.0810
	50	**0.0504**	0.0642	0.0706	**0.0804**	**0.0810**
	60	0.0508	0.0645	0.0711	0.0802	0.0811
FERM	40	0.0512	0.0647	0.0725	0.0826	0.0822
	50	0.0512	0.0652	0.0727	0.0824	0.0820
	60	0.0513	0.0656	0.0717	0.0850	0.0815
ACM		0.230	1.043	0.948	0.717	0.436

Sample 3						
Model	Window (days)	Tranche 0% - 3%	3% - 6%	6% - 9%	9% - 12%	12% - 22%
MERM	40	0.0534	0.0636	0.0783	**0.0922**	**0.0927**
	50	**0.0531**	**0.0636**	**0.0781**	0.0931	0.0932
	60	0.0533	0.0637	0.0781	0.0924	0.0934
FERM	40	0.0536	0.0638	0.0794	0.0946	0.0943
	50	0.0534	0.0639	0.0792	0.0953	0.0949
	60	0.0535	0.0637	0.0801	0.0959	0.0940
ACM		0.292	1.285	1.614	1.402	0.605

Notes: This table shows RMSFEs of tranche spread forecasts for each correlation model. MERM denotes the dynamic mixed-effects regression correlation model for the *base correlation* estimation. This model accounts for random time-specific effects and for fixed tranche-specific effects as well. The second *base correlation* model is the dynamic fixed-effects regression correlation model (FERM) accounting only for fixed tranche effects. ACM denotes the dynamic *historical asset correlation* model which is independent of different regression windows. Spread forecasts are made on the base of all three correlation models. Depending on i) the window size, ii) the respective tranche and iii) the specific sample the RMSFEs vary. The lowest RMSFEs are highlighted in bold.

In *Sample 1*, the dynamic ACM performs the worst across all tranches and regression windows. Additionally, the results confirm some theoretical findings related to our benchmark model: even though the ACM may provide valuable results for both the equity tranche and the senior tranche, its applicability seems to be strongly limited regarding the remaining tranches. Thus, our dynamic regression correlation models outperform the ACM clearly since we simultaneously obtain more valuable results for all securitized tranches. The results reveal further that the dynamic MERM provides overall the highest performance accuracy across all tranches and all regression windows related to *Sample 1* in terms of our RMSFE metric.

During the GFC, the performance of our dynamic correlation models is slightly worse, but still dominates our benchmark model. In contrast to *Sample 1*, we observe a decrease of forecast accuracy for both models MERM and FERM: across all tranches and regression windows the RMSFE are on average 21% higher. Furthermore, the results show that the forecasted ACM spreads lead to lower RMSFEs during the GFC. Thus, the forecast accuracy of the ACM strongly increases under these volatile market conditions. Averaging the improvements across all tranches, we observe a RMSFE decrease of almost 80% in *Sample 2* related to the ACM. Nevertheless, the dynamic MERM still outperforms all other model approaches.

Referring to both dynamic correlation models, we find further that in contrast to rather moderate economic climates (*Sample 1*) the RMSFEs are monotonically as well as disproportionately strongly increasing with the tranche seniority during the GFC. Thus, particularly in times of financial distress the spread forecast performance seems to decrease with increasing tranche seniority. We suggest that this may be due to the specific risk characteristics of high-seniority tranches related to systematic risk.

Our empirical study shows similar results for the remaining sample. Even in the entire time interval (*Sample 3*), the overall results show that the forecast performance of both dynamic regression correlation models is better than the respective forecasts with *historical asset correlations*. Across all tranches, *base correlation* estimates provide lower RMSFEs of spread forecasts. This underlines the superiority of our proposed dynamic regression correlation models, especially if we account for random time-specific and fixed tranche effects.[123]

We conclude that the dynamic ACM is inferior to both of the dynamic regression correlation models in terms of RMSFEs. Thus, the inclusion of correlation parameters gained from corresponding log-returns leads to spread forecasts which may widely differ from quoted market spreads. We confirm that the consideration of dependency structures reflected by historical asset returns is not sufficient for pricing STCDOs. Consequently, we assume that *historical asset correlations* incompletely reflect relevant market information for pricing and forecasting structured financial instruments. As we show, *base correlations* are much more suitable for relatively accurate tranche spread forecasts than *historical asset correlations*. In consequence, we assume that *base correlations* contain more relevant market information for a reliable STCDO pricing. However, our dynamic regression correlation models also indicate an underestimation of systematic risk which is i) especially affecting securitized tranches (see Coval et al. 2009*b*) and ii) potentially causing the disproportionate strong increase of RMSFEs related to high-seniority tranches during the GFC.

All our empirical findings show that the dynamic MERM is superior to all other models considered in this paper in terms of one-day-ahead forecast accuracy using a RMSFE metric. Our FERM outperforms the dynamic ACM as well and underlines the general superiority of dynamic regression correlation models, even during the GFC.

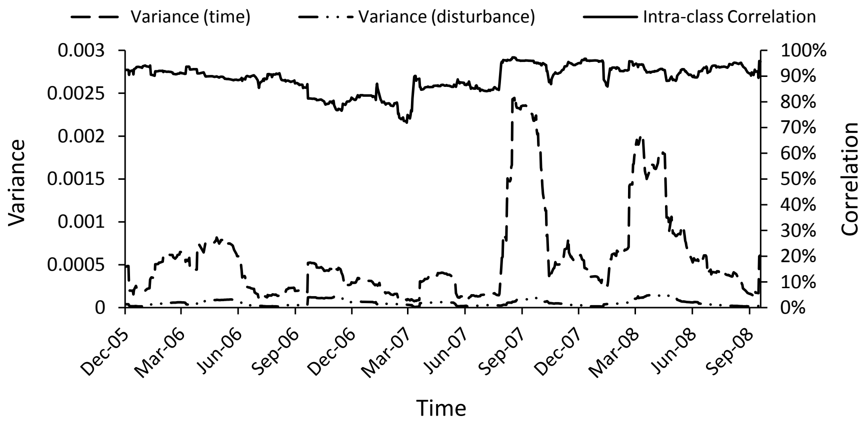
Based on the suggested RMSFE metric, we conclude that the inferiority of the FERM to the MERM reveals the existence of important unobservable time-specific effects. In order to improve the forecast accuracy indicated by decreasing RMSFEs, spread forecast models should account for both random time-specific effects and fixed tranche-specific effects.

Based on the intra-class correlation which exists across all tranches, we assume the presence of a systematic risk factor simultaneously affecting all tranches. Figure 4.10 shows both the variance components and the intra-class correlation curve for *Sample 3* (x-axis). The primary y-axis on the left hand side refers to the variances, the secondary y-axis on the right hand side denotes

[123]Since *Sample 3* refers to the entire period, it accounts also for 60 unconsidered days between *Sample 1* and *2* which are the calibration days of *Sample 2* with respect to the 60-days regression window. During these 60 days, we observe extremely high RMSFEs. Thus, *Sample 3* reveals relative worse RMSFEs compared to *Sample 2* (methodically caused).

the intra-class correlation values.

Figure 4.10: Variance Effects and Intra-class Correlation



Notes: This figure shows the time series (x-axis) of derived intra-class correlations (y-axis on the right) for *Sample 3* and both parts of the estimated variance (y-axis on the left): (1) The time-specific component v_t and (2) the disturbance u_t^i. The results refer to the dynamic panel regression correlation model accounting for mixed-effects (MERM) and a regression window of 40-days.

The intra-class correlation is extremely high throughout the entire sample and varies between 74% and 97%. This indicates that the variance of the time-specific effect explains a high ratio to the total variance of *base correlation* forecasts. This is also underlined by the remaining two functions in Figure 4.10: while the dashed line denotes the unobservable time-specific component σ_t of the total variance, the dotted line denotes the residual disturbance u_t of that variance (compare Section 4.2.3). We conclude that the time-specific component represents a main part of the total variance throughout *Sample 3*. This time effect simultaneously affects all tranches of the 5Y *iTraxx Europe* index, especially during the GFC, and also reaches statistical significance. Eventually, the time-specific effect is particularly striking in times of financial distress and can not be considered by our FERM. Based on the intra-class correlation which exists across all tranches, we suspect the presence of a systematic risk factor simultaneously affecting all tranches.

Overall, these results suggest the existence of systematic factors which are indicated by our MERM but not considered explicitly. In consequence, the intra-class correlation may be interpreted as an indicator of unconsidered systematic risk factors varying over time and simultaneously influencing all tranches.

Finally, we conclude that the dynamic FERM is superior to the dynamic ACM but not appropriately reducing RMSFEs of tranche spread forecasts. The decrease of RMSFEs related to the MERM suggests that the spread forecast performance may be improved by accounting for both random time effects and fixed tranche effects. In addition, further systematic risk factors may be identified through our dynamic MERM.

4.4 Summary

The empirical analysis underlines the importance of correlations in pricing models of structured finance instruments. We point out that 'true' correlation parameters are not observable, but constitute the main factors determining expected losses of single-tranche collateralized debt obligation swaps (STCDO). In turn, these expected tranche losses decisively determine respective tranche spreads. Consequently, spreads of STCDOs are highly sensitive to correlation changes. This constitutes the importance of appropriate correlation models for valuing and hedging STCDOs. The most common correlation approaches are introduced and evaluated. Inherent limitations of the *compound correlation* approach, especially the non-uniqueness of solutions, reduce its applicability. *Base correlations* are provided to overcome shortcomings of *compound correlations*. In terms of pricing and hedging STCDOs, the *base correlation* approach produces more reasonable solutions. Additionally, this simple measure also provides market participants with a practicable valuation framework for non-standardized CDOs.

Considering the benefits of *base correlations*, we develop two dynamic regression correlation models in order to forecast various *base correlations*. Our proposed dynamic fixed-effects regression correlation model (FERM) accounts only for fixed tranche-specific effects, whereas our proposed dynamic mixed-effects regression correlation model (MERM) additionally assumes that the time effect is varying stochastically. Through Monte Carlo simulations, we estimate daily STCDO spreads. Within the three samples, we measure the forecast accuracy of our models by calculating root mean square forecast errors (RMSFE) of forecasted STCDO spreads. Analogously, we forecast daily STCDO spreads with a dynamic asset correlation model (ACM). A comparison of the forecast accuracy suggests the superiority of our proposed dynamic regression correlation models in terms of the suggested RMSFE metric. Applying the MERM, we can account for both cross-sectional and time-series information in quoted market spreads related to the securitized tranches. The consideration of both existing effects is important for more accurate correlation forecasts, even during the Global Financial Crisis. The increased accuracy leads to an improvement of the overall spread forecast performance with several implications for financial institutions and regulatory authorities dealing with structured finance instruments.

The intra-class correlation reveals the existence of unconsidered systematic risk factors varying in time. Furthermore, it underlines the importance of applying our proposed dynamic MERM in order to account for such systematic time effects. By expanding the MERM to other relevant systematic risk factors, useful information can be derived in order to develop appropriate stress-tests for structured finance products. It also helps to quantify risk contributions of STCDOs to portfolio inherent credit risks, which is highly relevant for investors in securitized tranches.

Our findings are essential for pricing standardized and non-standardized structured finance instruments. Additionally, our results provide reasonable implications for hedging both STCDOs and other credit derivatives. Further research in other relevant explanatory variables is suggested to expand our proposed dynamic regression approach. In this way, forecast performance could be increased, especially in times of market turbulence.

The Global Financial Crisis has revealed that current credit risk and pricing models exhibit a low degree of transparency. Addressing the importance of appropriate approaches for modeling and forecasting correlations is one step that could be taken in order to improve the understanding of structured instruments and to return more transparency as well as confidence in credit related financial markets, institutions and instruments.

Chapter 5

An Analytical Approach for Systematic Risk Sensitivity of Structured Finance Products

The content of this chapter refers to the working paper 'An Analytical Approach for Systematic Risk Sensitivity of Structured Finance Products' by Löhr, S., Claußen, A., and Rösch, D., 2012.

5.1 Introduction

In 2007, the global market volume of collateralized debt obligations (CDOs) reached its historical maximum with over \$1.3 trillion USD and the global CDO market issuance denoted at \$481 billion USD (compare SIFMA 2012*b*). During the Global Financial Crisis (GFC) the global CDO issuance decreased rapidly and denoted at \$4 billion USD in 2009. Since 2010, the issuance volume is slightly increasing (see SIFMA 2012*b*). Contemporaneously, the 5-year cumulative impairment rates for 'A'-rated CDOs increased from 5% in 2005 to 57% in 2009, while respective impairment rates of 'A'-rated bonds increased only from 0.56% in 2005 to 0.81% in 2009 (compare Moody's 2006*a*,*b*, 2010*b*,*c*).[124]

In terms of impairment risk, there are obvious differences between risk characteristics of 'A'-bonds and 'A'-CDOs. Hence, several authors address the

[124]The 5-year cumulative impairment rates for CDOs and bonds are provided by the credit-rating agency (CRA) Moody's and they differ in a similar fashion for other rating classes.

higher systematic risk exposures in CDO structures due to the securitization of debt assets (see Krahnen & Wilde 2008, Coval et al. 2009*a*, Eckner 2009, Hamerle & Plank 2009, Rösch & Scheule 2010). With respect to systematic risk influences, some authors conclude that investors should demand far larger risk premia for holding structured claims than holding comparably rated corporate bonds (Coval et al. 2009*b*). Indeed, in the recent literature appears a lack of detailed systematic risk analysis elaborating the specific risk profile of structured finance products. Our main contribution is a comprehensive analysis of product-specific sensitivities to systematic risk based on both a simple analytical approach and several simulation studies considering more realistic product structures by implementing inter-sectoral diversification effects.

Based on the single-factor Gaussian copula model – firstly applied to portfolio credit risk by Vasicek (1987) – a simple model is applied to quantify risk characteristics of both securitized tranches and comparably rated corporate bonds. In particular, we account for product-specific exposures to systematic risk determining default risks and related losses. Within the expanded model framework, the single risk factor is split off into a i) super-systematic component and ii) a sector-specific factor (compare Gordy & Howells 2006, Pykthin & Dev 2002).[125] In this model setup, our approach provides an analytical method to measure product-specific sensitivities to the super-systematic risk factor. By developing conditional risk clusters (CRCs) related to historical default rates, we additionally account for cluster migration risk due to changes of the super-systematic factor. Thus, we capture downside risks of both tranches and straight bonds related to systematic risk. Our results show that all tranches of an asset securitization contain a higher exposure to systematic risk than comparably rated straight bonds due to pooling and tranching.

The contributions of the paper are as follows: firstly, we implement an analytical approach for modeling and measuring systematic risk of tranches created from pools of default risky securities. Secondly, we use the model to compare straight bonds with securitized tranches in terms of systematic risk exposures. Thirdly, we analytically derive product-specific risk sensitivities to changes in the global economy in terms of both default risk and losses from default. Fourthly, we demonstrate that the cluster migration risk is more se-

[125]Pykthin & Dev (2002) address generally credit risk in asset securitizations. Their results provide the foundation for the current Ratings Based Approach in Basel II referring to regulatory capital requirements for structured finance products.

vere for securitizations than for straight bonds. In comparison to straight bonds, the downside risk of tranches is much higher, especially during economic downturns. Our results suggest that risk-adjusted ratings are strongly recommended for securitizations in order to reflect the inherent behavior due to systematic risks, even if securitized debt claims are sectorally diversified.

The remainder of the paper is organized as follows: in Section 5.2, we introduce a theoretical framework to develop default probabilities based on the single-factor Gaussian copula model. In Section 5.3, we develop a model extension to measure systematic risk sensitivities of both straight bonds and securitized tranches. The analytical results are presented in Section 5.4. The outcome of our inter-sectoral simulation studies consecutively underlines these results. Finally, we examine systematic impacts on the product-specific cluster migration risk within our CRC approach. Section 5.5 concludes.

5.2 Risk Measures in the Single-factor Model

5.2.1 The Single-factor Model

In the financial industry, the single-factor Gaussian copula model is market standard for quoting synthetic CDOs (Hull & White 2006).The model was introduced by Vasicek (1987) with the intention to aggregate the credit risk of credit portfolios and was further analyzed by Gordy (2000, 2003). It also constitutes the core of the regulatory capital formula under the Basel II Capital Accord. In order to describe losses of bonds as well as the loss distribution of a credit portfolio, a structural approach developed by Merton (1974) is applied. According to Merton's basic model, a corporate borrower B defaults on his debt, which is assumed to consist of a single zero bond, when his asset return Z_t^B falls below a critical threshold c_t^B (e.g., a function of the nominal amount of debt) at time t.[126] The default probability π_t^B of borrower B at time t is defined with

$$\pi_t^B = \mathbb{P}(Z_t^B < c_t^B). \tag{5.1}$$

[126]In the remainder a corporate borrower B is also represented by bond B.

The default event is indicated by a default variable D_t^B:

$$D_t^B = \begin{cases} 1 & \text{if borrower } B \text{ defaults at time } t, \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

In this context, the asset return Z_t^B is modeled considering two different factors: i) a systematic – or common – risk factor Y_t jointly affecting all borrowers of the credit portfolio at time t, and ii) an independent idiosyncratic risk factor ε_t^B which is borrower-specific at time t. Both factors Y_t and ε_t^B are assumed to be independent and identically distributed (i.i.d.) with Y_t, $\varepsilon_t^B \sim \mathcal{N}(0,1)$. Thus,

$$Z_t^B = \sqrt{\rho} \cdot Y_t + \sqrt{1-\rho} \cdot \varepsilon_t^B \tag{5.3}$$

where $\rho \in [0,1]$ represents the correlation of the borrowers' asset returns. Despite that ρ may generally vary across borrowers and even across time it is identical for all borrowers due to our aforementioned assumptions.[127] Further, the so-called asset correlation ρ determines the weighting of the systematic factor simultaneously affecting all borrowers. Since Y_t and ε_t^B are i.i.d. normal with Y_t, $\varepsilon_t^B \sim \mathcal{N}(0,1)$, the random variable Z_t^B is also standard normal distributed with $Z_t^B \sim \mathcal{N}(0,1)$.[128] In the following, we focus our study on a single period and skip time index t.

5.2.2 Default Probability of a Tranche

A collateralized debt obligation (CDO) is a structured finance instrument that securitizes a portfolio of credit risky assets (collateral). The inherent credit risk of debt assets is transferred to external investors by repacking the original risk profile and selling risk-adjusted tranches of different seniority to investors. Offered tranches are backed by the collateral. The seniority reflects the order in which losses within the collateral affect different tranches and thus tranche investors. Each tranche is defined by the percentage of losses in the collateral that it carries.

For example: An investor of tranche $T_{[3\%;6\%)}$ gets a premium payment (pre-

[127] Compare Equation (3.40) in Section 3.3.1.

[128] Conditional on a realization of the common risk factor Y_t, the asset returns of borrowers are i.i.d. random variables due to the independence of idiosyncratic risk factors.

mium leg) by the protection buyer – the originator of securitization – for covering all losses in the underlying portfolio that occur within the tranche-specific boundaries. Here, these boundaries are determined by the attachment point $A = 3\%$ and the detachment point $D = 6\%$. Premium payments are generally paid out quarterly and they refer to respective outstanding notional amounts. In turn, the protection payments of investors (protection leg) are paid out to the protection buyer in cases of default events causing losses within the credit portfolio related to the tranche boundaries. Exceeding portfolio losses are carried by tranches of higher seniority.[129]

More generally spoken, tranche investors suffer losses if the total portfolio loss L in percent of its notional exceeds the attachment point A of tranche $T_{[A,D)}$. Occurring tranche losses $L^{T_{[A,D)}}$ are restricted to the difference between detachment point D and lower attachment point A of the tranche. In terms of total portfolio loss L, tranche-specific losses follow from:

$$L^{T_{[A,D)}} = \min\left(\max\left(0, L - A\right), D - A\right). \tag{5.4}$$

The tranche $T_{[A,D)}$ experiences a loss (and, therefore, an impairment or default) if the default rate in the portfolio exceeds the attachment point A.

For modeling tranche losses, we employ some simplifying assumptions. The credit portfolio is assumed to be infinitely granular and all borrowers or bonds, respectively, are homogeneous. This leads to a large homogeneous credit portfolio (LHP) in which idiosyncratic risks are fully diversified, see Gordy (2000, 2003). Furthermore, the default rate of the LHP follows the so-called 'Vasicek-distribution' with density

$$v(x) = \sqrt{\frac{1-\rho}{\rho}} \cdot \exp\left(-\frac{\left(\sqrt{1-\rho}\cdot\Phi^{-1}(x) - \Phi^{-1}(\pi)\right)^2}{2\cdot\rho} + \frac{\left(\Phi^{-1}(x)\right)^2}{2}\right) \tag{5.5}$$

and distribution function

$$V(x) = \Phi\left(\frac{\sqrt{1-\rho}\cdot\Phi^{-1}(x) - \Phi^{-1}(\pi)}{\sqrt{\rho}}\right). \tag{5.6}$$

Equation (5.6) denotes the cumulative probability that the percentage loss L of the LHP does not exceed $x \in [0,1]$, see Vasicek (1991). π describes the default probability of included bonds and thus the expected default rate of the

[129]For more detailed information compare Bluhm et al. (2003).

LHP. Φ^{-1} denotes the inverse of the standard normal distribution function Φ.

Then, the default probability $\pi^{T_{[A,D)}}$ of tranche $T_{[A,D)}$ is given by

$$\pi^{T_{[A,D)}} = 1 - V(A) = \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{1-\rho}\cdot\Phi^{-1}(A)}{\sqrt{\rho}}\right). \tag{5.7}$$

5.2.3 Product-specific Expected Losses

The expected loss $\mathbb{E}\left(L^B\right)$ of bond B is the product of its i) default probability π^B, ii) exposure at default EAD^B which is fixed at unity for tractability and iii) loss given default $LGD^B = \left(1 - R^B\right)$ with recovery rate R^B:

$$\begin{aligned} \mathbb{E}\left(L^B\right) &= \pi^B \cdot EAD^B \cdot \left(1 - R^B\right) \\ &= \left(1 - R^B\right) \cdot \pi^B. \end{aligned} \tag{5.8}$$

The expected loss $\mathbb{E}\left(L^{T_{[A,D)}}\right)$ of tranche $T_{[A,D)}$ depends on the cumulative losses in the underlying LHP. Related to the cumulative loss distribution, we calculate expected tranche losses by

$$\mathbb{E}\left(L^{T_{[A,D)}}\right) = \frac{1-R}{D-A}\left[\Phi_2\left(\kappa(A), c; \varrho\right) - \Phi_2\left(\kappa(D), c; \varrho\right)\right] \tag{5.9}$$

with

$$c = \Phi^{-1}(\pi), \qquad \varrho = -\sqrt{1-\rho} \qquad \text{and} \qquad \kappa(\chi) = -\Phi^{-1}\left(\frac{\chi}{1-R}\right)$$

for $\chi < 1 - R$ and $\chi \in \{A, D\}$, see Appendix D.[130] Φ_2 denotes the bivariate Gaussian cumulative distribution function. R is the recovery rate of the LHP which equals the recovery rate of homogeneous bonds in the portfolio.

[130]According to Appendix D, the expected loss equals 0 for $\chi \geq 1 - R$.

5.3 Measuring Systematic Risk Sensitivity: The Extended Model

5.3.1 Introduction of a Super-systematic Factor

In the following, we introduce an extended model to quantify impacts on both straight bonds B and securitized tranches related to super-systematic changes. We divide borrowers into sectors so that firms in the same sector share a common risk factor (compare Gordy & Howells 2006). If borrower B is in sector j with $j \in \{1, 2, ..., J\}$, his asset return Z^{B_j} is given by

$$Z^{B_j} = \sqrt{\rho} \cdot Y^j + \sqrt{1-\rho} \cdot \varepsilon^{B_j}. \tag{5.10}$$

The idiosyncratic risk component of borrower B in sector j is described by ε^{B_j} i.i.d. standard normal, while $\rho \in [0, 1]$ is jointly assumed for all borrowers B.

We decompose the sectoral risk factor Y^j of Equation (5.10) into both a super-systematic factor Y^* and a sectoral risk component U^j according to

$$Y^j = \sqrt{\delta} \cdot Y^* + \sqrt{1-\delta} \cdot U^j \tag{5.11}$$

where Y^* and U^j are i.i.d. standard normal. Y^* is a univariate factor representing the overall macroeconomy, U^j is a sector specific factor and $\delta \in [0, 1]$ determines the strength of dependence across sectors. δ is assumed to be constant across all sectors j for simplicity. Since all regarded random variables are i.i.d. with $Y^*, U^j, \varepsilon^{B_j} \sim \mathcal{N}(0, 1)$, the random variable Z^{B_j} is also standard normal distributed with $Z^{B_j} \sim \mathcal{N}(0, 1)$.

For further analytical analyses, we examine just a single sector, thus, we skip index j which leads to

$$Z^B = \sqrt{\rho \cdot \delta} \cdot Y^* + \sqrt{\rho - \rho \cdot \delta} \cdot U + \sqrt{1-\rho} \cdot \varepsilon^B. \tag{5.12}$$

In this way, the conditional default probability of bond B is linked to realizations of a super-systematic factor Y^* which is essential for further examinations.

5.3.2 Conditional Risk Measures of a Bond

We obtain the conditional default probability $\pi^B(y^*)$ of bond B depending on realizations of the super-systematic factor $Y^* = y^*$ with

$$\pi^B(y^*) = \mathbb{P}(D^B = 1|Y^* = y^*) = \Phi\left(\frac{\Phi^{-1}(\pi^B) - \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{1-\rho\cdot\delta}}\right). \tag{5.13}$$

It follows from Equation (5.8) that the conditional expected loss $\mathbb{E}\left(L^B|Y^* = y^*\right)$ of bond B in dependence on Y^* is given by

$$\mathbb{E}\left(L^B|Y^* = y^*\right) = (1-R^B)\cdot\Phi\left(\frac{\Phi^{-1}(\pi^B) - \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{1-\rho\cdot\delta}}\right). \tag{5.14}$$

5.3.3 Conditional Risk Measures of a Tranche

Referring to the cumulative loss distribution of the LHP with recovery rate $R \in [0,1)$, we provide the conditional default probability $\pi^{T_{[A,D)}}(y^*)$ of tranche $T_{[A,D)}$ related to Y^*, which is given by

$$\begin{aligned}\pi^{T_{[A,D)}}(y^*) &= \mathbb{P}\left(D^{T_{[A,D)}} = 1|Y^* = y^*\right)\\ &= \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{1-\rho}\cdot\Phi^{-1}\left(\frac{A}{1-R}\right) - \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{\rho-\rho\cdot\delta}}\right),\end{aligned} \tag{5.15}$$

see Appendix C.[131] Along this, we analytically quantify expected losses of tranche $T_{[A,D)}$ under consideration of Y^*, which is in line with the theoretical framework of the single-factor Gaussian copula model (see Appendix D):

$$\mathbb{E}\left(L^{T_{[A,D)}}|Y^* = y^*\right) = \frac{1-R}{D-A}\left[\Phi_2\left(\kappa(A),\tilde{c};\tilde{\varrho}\right) - \Phi_2\left(\kappa(D),\tilde{c};\tilde{\varrho}\right)\right] \tag{5.16}$$

with

$$\tilde{c} = \frac{\Phi^{-1}(\pi) - \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{1-\rho\cdot\delta}} \qquad \text{as well as} \qquad \tilde{\varrho} = \frac{-\sqrt{1-\rho}}{\sqrt{1-\rho\cdot\delta}}$$

and

$$\kappa(\chi) = -\Phi^{-1}\left(\frac{\chi}{1-R}\right) \quad \text{for} \quad \chi < 1-R\ ,\ \chi \in \{A, D\}.$$

[131]For $A < 1-R$, otherwise $\pi^{T_{[A,D)}}(y^*) = 0$.

Consider that the expected tranche loss $\mathbb{E}\left(L^{T_{[A,D)}}\right)$ follows in accordance to the law of iterated expectations from expectations $\mathbb{E}(\cdot)$ of $\mathbb{E}\left(L^{T_{[A,D)}}|Y^* = y^*\right)$:

$$\mathbb{E}\left(\mathbb{E}\left(L^{T_{[A,D)}}|Y^* = y^*\right)\right) = \mathbb{E}\left(L^{T_{[A,D)}}\right). \tag{5.17}$$

5.3.4 Risk-adjusted Attachment and Detachment Points

Next, we aim to compare the behavior of a specific bond B^* with that of a CDO tranche related to variations of Y^*. To ensure an appropriate comparison of both products, the risk profile of tranche $T_{[A^*,D^*)}$ should correspond with the risk profile of bond B^*. For this purpose, we focus on the following risk characteristics:

1. Corresponding probabilities of default (PDs)

$$\pi^{B^*} = \pi^{T_{[A^*,D^*)}}. \tag{5.18}$$

2. Corresponding expected losses (ELs)

$$\mathbb{E}\left(L^{B^*}\right) = \mathbb{E}\left(L^{T_{[A^*,D^*)}}\right). \tag{5.19}$$

With respect to a LHP recovery rate $R \in [0, 1)$, we analytically obtain attachment point A^* from Equations (5.7) and (5.18):

$$A^* = (1 - R) \cdot \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho} \cdot \Phi^{-1}\left(\pi^{B^*}\right)}{\sqrt{1-\rho}}\right). \tag{5.20}$$

By inserting this result into Equation (5.15), we obtain the conditional default probability (CPD) of tranche $T_{[A^*,D^*)}$ depending on realizations of Y^*:

$$\pi^{T_{[A^*,D^*)}}(y^*) = \Phi\left(\frac{\Phi^{-1}(\pi^{B^*}) - \sqrt{\delta} \cdot y^*}{\sqrt{1-\delta}}\right). \tag{5.21}$$

Equation (5.21) shows that under the aforementioned assumptions the CPD of tranche $T_{[A^*,D^*)}$ does not depend on the detachment point $D^* \in (A^*, 1]$.

After inserting (5.20) into Equation (5.19) and solving numerically, we obtain detachment point D^*. Eventually, the risk-adjusted tranche $T_{[A^*,D^*)}$ and bond B^* share the same risk characteristics in terms of PDs and ELs.

In order to calculate conditional expected tranche losses (CELs) depending on Y^*, we insert the tranche-specific attachment point A^* and detachment point D^* into Equation (5.16).

In terms of CPD changes, our analytical study relies on a comparison of results from Equations (5.13) and (5.21). Based on the calibration of A^* and D^*, the comparison of results related to Equations (5.14) and (5.16) shows the product-specific risk characteristics with respect to the CELs.

5.3.5 Product-specific Sensitivity to Systematic Risk

Concerning the sensitivities of both straight bonds and securitized tranches to systematic influences, we derive the partial derivatives of the product-specific i) conditional default probabilities and ii) conditional expected losses with respect to changes of Y^*. Firstly, we define the partial derivatives of a straight bond B^* with respect to Y^*:

1. Conditional default probability $\pi^{B^*}(y^*)$ of bond B^*

$$\begin{aligned}\xi_{CPD}^{B^*} &= \frac{\partial}{\partial y^*}\left(\pi^{B^*}(y^*)\right)\\ &= \frac{-\sqrt{\rho\cdot\delta}}{\sqrt{1-\rho\cdot\delta}}\cdot\varphi\left(\frac{\Phi^{-1}(\pi^{B^*})-\sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{1-\rho\cdot\delta}}\right).\end{aligned}\tag{5.22}$$

2. Conditional expected loss $\mathbb{E}\left(L^{B^*}(y^*)\right)$ of bond B^*

$$\begin{aligned}\xi_{CEL}^{B^*} &= \frac{\partial}{\partial y^*}\left(\mathbb{E}(L^{B^*}(y^*))\right)\\ &= \frac{-(1-R^{B^*})\cdot\sqrt{\rho\cdot\delta}}{\sqrt{1-\rho\cdot\delta}}\cdot\varphi\left(\frac{\Phi^{-1}(\pi^{B^*})-\sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{1-\rho\cdot\delta}}\right).\end{aligned}\tag{5.23}$$

Secondly, we provide the sensitivities of tranche $T_{[A^*,D^*)}$ with respect to Y^*:

1. Conditional default probability $\pi^{T_{[A^*,D^*)}}(y^*)$ of tranche $T_{[A^*,D^*)}$

$$\begin{aligned}\xi_{CPD}^{T_{[A^*,D^*)}}(y^*) &= \frac{\partial}{\partial y^*}\left(\pi^{T_{[A^*,D^*)}}(y^*)\right)\\ &= \frac{-\sqrt{\delta}}{\sqrt{1-\delta}}\cdot\varphi\left(\frac{\Phi^{-1}(\pi)-\sqrt{\delta}\cdot y^*}{\sqrt{1-\delta}}\right).\end{aligned}\tag{5.24}$$

2. Conditional expected loss $\mathbb{E}\left(L^{T_{[A^*,D^*)}}(y^*)\right)$ of tranche $T_{[A^*,D^*)}$

$$\xi_{CEL}^{T_{[A^*,D^*)}}(y^*) = \frac{\partial}{\partial y^*}\left(\mathbb{E}\left(L^{T_{[A^*,D^*)}}(y^*)\right)\right)$$
$$= \frac{1-R}{D^*-A^*} \cdot a \cdot \varphi(a \cdot y^* + b) \cdot \left[\tilde{\Phi}(A^*) - \tilde{\Phi}(D^*)\right] \quad (5.25)$$

with

$$\tilde{\Phi}(\chi) = \Phi\left(\kappa(\chi), \tilde{\rho} \cdot (a \cdot y^* + b), \sqrt{1-\tilde{\rho}^2}\right), \quad \kappa(\chi) = -\Phi^{-1}\left(\frac{\chi}{1-R}\right)$$

and

$$a = \frac{-\sqrt{\rho \cdot \delta}}{\sqrt{1-\rho \cdot \delta}}, \quad b = \frac{\Phi^{-1}(\pi)}{\sqrt{1-\rho \cdot \delta}}, \quad \widetilde{\rho} = \frac{-\sqrt{1-\rho}}{\sqrt{1-\rho \cdot \delta}} \quad \text{and} \quad \chi \in \{A^*, D^*\}.$$

The results of Equation (5.25) refer to linearity of the differential operator and the partial derivative of the bivariate Gaussian cumulative distribution function (see Appendix E).

5.4 Analysis of Systematic Risk Sensitivity

5.4.1 Product-specific Sensitivity of CPDs

In this section, we apply our extended model to examine CPD sensitivities of both securitized tranches $T_{[A^*,D^*)}$ and specific bonds B^* to a super-systematic risk factor Y^*. Before calculating sensitivities, we calibrate the attachment and detachment points of our tranche $T_{[A^*,D^*)}$ along the risk characteristics of a selected bond B^* (compare Section 5.3.4). Exemplarily, we set the default probability π^{B^*} of bond B^* to 0.324%. Additionally, we assume the default rate π of the LHP with 1.180%.

The default probabilities in Table 5.1 refer to the Moody's rating scale, see Moody's (2010*b*). Thus, our regarded bond B^* is linked to an '*Aa*3' rating, whereas the LHP exhibits a default rate corresponding with a '*Baa*1' rating.

Further, the recovery rate of the LHP as well as the recovery rate of bond B^* are fixed at $R = R^{B^*} = 50\%$.[132] Correlation parameters ρ and δ are assumed

[132] While the recovery rate of the LHP as well as the recovery rate of the reference bond B^* are set to 50% (deterministic), the tranche recovery rates remain stochastic due to general characteristics of securitizations.

Table 5.1: 5-year Cumulative Default Rates

Rating	Default rate (%)
Aaa	0.086
Aa1	0.141
Aa2	0.195
Aa3	0.324
A1	0.854
A2	0.746
A3	0.830
Baa1	1.180
Baa2	2.024
Baa3	3.081

Notes: The table shows average 5-year cumulative default rates for Moody's rated corporate bonds (annual cohorts from 1983 to 2009) in percent, by initial alphanumeric rating.

with $\rho = 0.25$ and $\delta = 0.25$, respectively. Inserting these values into Equation (5.20) leads to the risk-adjusted attachment point $A^* = 7.44\%$. Numerically solving Equation (5.19) we obtain the tranche-specific detachment point with $D^* = 11.10\%$.

In consequence, both the selected bond B^* and the tranche $T_{[7.44\%,11.10\%)}$ of the LHP are comparable in their risk characteristics:

1. The shared probability of default is 0.324%:

$$\pi^{B^*} = \pi^{T_{[7.44\%,11.10\%)}} = 0.324\%.$$

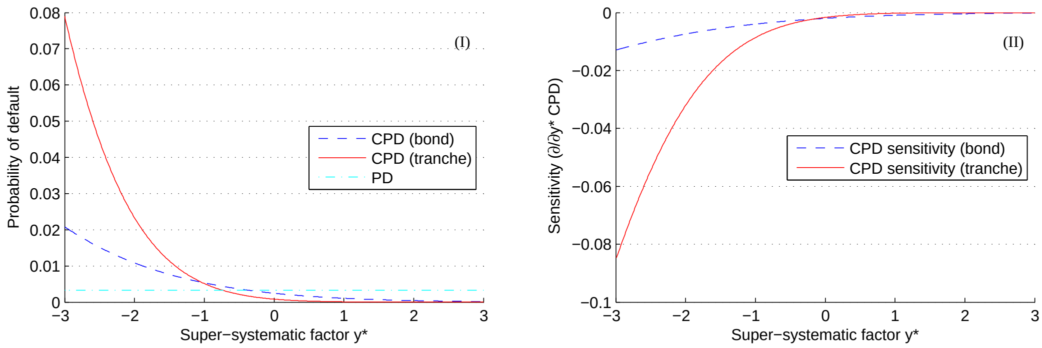
2. Both financial instruments face identical expected losses of 0.162%:

$$\mathbb{E}\left(L^{B^*}\right) = \mathbb{E}\left(L^{T_{[7.44\%,11.10\%)}}\right) = 0.162\%.$$

We proceed in a similar way for other default probabilities derived from the rating grades.

Figure 5.1 shows the impacts on product-specific conditional default probabilities caused by changes of the super-systematic risk factor Y^* (x-axes) with $Y^* \in [-3, 3]$. The left chart of Figure 5.1 denotes the PDs (x-axis) depending on Y^*. The right chart describes the respective sensitivities of CPDs (x-axis). Intuitively, the CPDs of both products are higher in economic downturns than in economic upturns. As we can see, the negative realization of $Y^* = -3$ representing the worse economic downturn is linked to the highest CPDs, while positive realizations referring to several degrees of economic upturns lead to lower CPDs. Thus, our model reflects economic intuition.

Figure 5.1: Sensitivities of CPDs to the Super-systematic Factor



Notes: The figure compares product-specific effects on conditional default probabilities (CPDs) related to Y^*. Results are based on a default probability $\pi^{B^*} = 0.324\%$ of bond B^*, a default rate $\pi = 1.180\%$ of the LHP, a recovery rate $R^{B^*} = 50\%$ of bond B^* , a LHP recovery rate $R = 50\%$. Correlation parameters are set to $\rho = 0.25$ and $\delta = 0.25$. The tranche seniority is determined with $A^* = 7.44\%$ and $D^* = 11.10\%$.

In general, CPDs of both products increase with decreasing realizations of Y^*. For further analyses, we exemplarily regard realizations of $Y^* \in \{-5, -3\}$ as proxies for extremely bad states of the global economy. By contrast, we assume a realization of $Y^* = 1$ as an (moderate) economic upturn. While the plotted functions in Figure 5.1 refer to financial products, which jointly exhibit a default probability of 0.324% ('Aa3'), the results presented in Table 5.2 provide more detailed insights into the relations between PDs and the product-specific CPDs related to both several rating grades and negative realizations of Y^*.

In relation to the underlying PD, the product-specific CPDs are impressively increasing with decreasing values of Y^*. While the CPD of bond B^* is 20 times higher in an economic downturn than its initial PD, the respective CPD of a comparable tranche is more than 123 times higher. In case of initially 'Aaa'-rated products, the respective CPD/PD ratio is 29.99 for bond B^* and 269.57 for tranche $T_{[10.54\%,14.52\%)}$, respectively. Therefore, CPDs of both financial instruments are increasing in economic downturns, but tranche-specific CPDs increase much more rapidly.

Despite both products being characterized by the same initial PD, their sensitivities to negative realizations of Y^* are quite different. According to Table 5.2, the CPD of an 'Aaa'-rated tranche $T_{[10.54\%,14.52\%)}$ is up to 9 times higher in comparison to the CPD of a comparably rated bond.

Particularly, in an extremely bad economic environment the growth rate

Table 5.2: Impacts of Economic Downturns on Probabilities of Default

$Y^* = -5$								
Rating	PD (%)	CPD (%)		CPD Sensitivity (%)		CPD Bond / PD	CPD Tranche / PD	CPD Tranche / CPD Bond
		Bond	Tranche	Bond	Tranche			
Aaa	0.086	2.579	23.183	-1.545	-17.571	29.993	269.566	8.988
Aa1	0.141	3.643	28.703	-2.057	-19.635	25.839	203.570	7.878
Aa2	0.195	4.553	32.789	-2.465	-20.828	23.350	168.150	7.201
Aa3	0.324	6.416	39.864	-3.235	-22.269	19.803	123.038	6.213
A2	0.746	11.064	53.026	-4.867	-22.972	14.830	71.080	4.793
Baa1	1.180	14.759	60.752	-5.947	-22.209	12.508	51.485	4.116
Baa2	2.024	20.468	69.880	-7.322	-20.140	10.113	34.526	3.414
Baa3	3.081	26.131	76.691	-8.390	-17.699	8.481	24.892	2.935
$Y^* = -3$								
Rating	PD (%)	CPD (%)		CPD Sensitivity (%)		CPD Bond / PD	CPD Tranche / PD	CPD Tranche / CPD Bond
		Bond	Tranche	Bond	Tranche			
Aaa	0.086	0.689	2.954	-0.495	-3.858	8.011	34.353	4.288
Aa1	0.141	1.044	4.301	-0.713	-5.250	7.406	30.504	4.119
Aa2	0.195	1.369	5.475	-0.901	-6.370	7.018	28.077	4.001
Aa3	0.324	2.082	7.904	-1.290	-8.471	6.425	24.395	3.797
A2	0.746	4.097	14.034	-2.264	-12.832	5.492	18.813	3.426
Baa1	1.180	5.900	18.893	-3.029	-15.573	5.000	16.011	3.202
Baa2	2.024	8.989	26.313	-4.182	-18.808	4.441	13.000	2.927
Baa3	3.081	12.390	33.506	-5.274	-21.009	4.021	10.875	2.704

Notes: The table provides a comparison of several risk measures referring to bonds B^* and tranches $T_{[A^*,D^*)}$, respectively, conditional on different realizations of $Y^* \in \{-5, -3\}$. Selected values of Y^* reflect different degrees of economic downturns.

of tranche-specific CPDs is up to 11 times higher (in case of an initial 'Aaa' rating) than sensitivities of corresponding bonds B^*. Thus, these financial instruments highly differ from each other not only in their absolute CPDs, but also in their sensitivities to Y^*.

Table 5.3 shows the impacts of economic upturns on the regarded risk measures. As we can see, the tranche-specific CPDs are much lower than the

Table 5.3: Impacts of Economic Upturns on Probabilities of Default

$Y^* = 1$								
Rating	PD (%)	CPD (%)		CPD Sensitivity (%)		CPD Bond / PD	CPD Tranche / PD	PD Tranche / CPD Bond
		Bond	Tranche	Bond	Tranche			
Aaa	0.086	0.024	0.001	-0.023	-0.003	0.275	0.016	0.057
Aa1	0.141	0.041	0.003	-0.038	-0.007	0.294	0.020	0.068
Aa2	0.195	0.060	0.005	-0.054	-0.011	0.308	0.024	0.077
Aa3	0.324	0.107	0.010	-0.092	-0.022	0.330	0.031	0.093
A2	0.746	0.278	0.035	-0.220	-0.073	0.373	0.047	0.126
Baa1	1.180	0.472	0.071	-0.353	-0.140	0.400	0.060	0.150
Baa2	2.024	0.879	0.162	-0.613	-0.300	0.434	0.080	0.185
Baa3	3.081	1.432	0.312	-0.937	-0.542	0.465	0.101	0.218

Notes: The table provides exemplarily a comparison of several risk measures referring to bonds B^* and tranches $T_{[A^*,D^*)}$, respectively, conditional on a positive realization of $Y^* = 1$ which reflects an economic upturn.

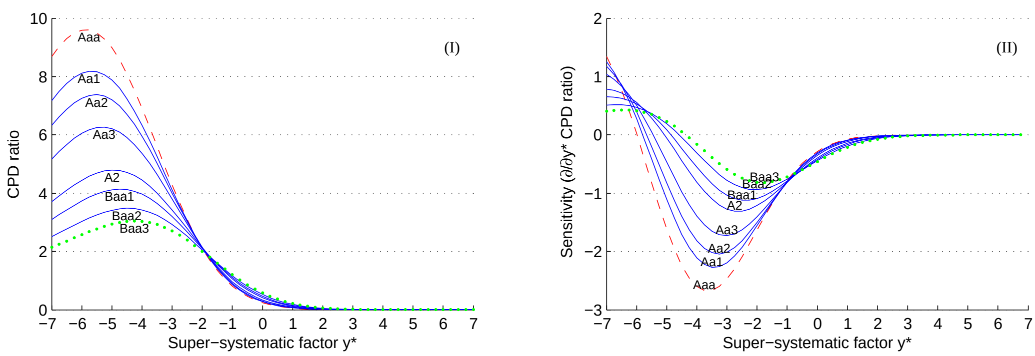
respective bond CPDs. The result is in line with our expectations since the product's PD is the expected CPD over all possible realizations of Y^*. This means that the tranche-specific CPD must fall below both the bond's CPD and the PD at some point (compare Figure 5.1). In other words, the CPD of tranche $T_{[A^*,D^*)}$ is decreasing more rapidly in comparison to the bonds CPD with respect to increasing values of Y^*. This may be a reason why the market volume of securitization increased strongly in times of a moderate economic environment (compare BBA 2006).

Following the principles of securitization, we theoretically link the introduced PDs to tranche seniorities in accordance to Table 5.3: on the one hand, we regard a tranche with PD of 0.086% as a tranche of highest seniority which is assessed with an 'Aaa' rating. On the other hand, we specify a tranche with a PD of 3.081% as equity tranche assessed with a 'Baa3' rating (lowest seniority).

From Table 5.2 we conclude that the exposures to systematic risk vary not only between bonds and tranches, but also across the different securitized tranches of the LHP. The ratio between CPD and PD reveals that tranche-specific exposures to systematic risk increase with raising tranche seniority.

This result is underlined in Figure 5.2 indicating the product-specific exposures to systematic risk. The left chart shows the *CPD ratios* (x-axis) depending on the super-systematic factor Y^* (x-axes) with $Y^* \in [-7, 7]$ for several rating grades ('Aaa' to 'Baa3'). In the right chart, corresponding sensitivities (y-axis) of the *CPD ratios* are provided. The plotted functions in the left

Figure 5.2: Indication of Product-based Exposures to Systematic Risk (CPD)



Notes: The figure points out systematic risk exposures of securitized tranches $T_{[A,D)}$ related to Y^*. The *CPD ratio* describes the relation between the product-based CPDs ($CPD^{T_{[A^*,D^*)}}/CPD^{B^*}$). The dashed line represents the tranche with the highest seniority ('Aaa' rating). In turn, the dotted line describes the tranche with the modest rating ('Baa3') and thus it can be regarded as the tranche of lowest seniority (equity tranche).

chart indicate that the tranche CPDs are more strongly affected by variations of Y^* than the CPDs of bonds. Further, this effect is highly correlated with the tranche seniority: the higher the seniority of a tranche, the stronger is the effect and vice versa. Finally, we observe this effect as long as the tranche-specific CPD is above the linked PD. After crossing the this PD threshold, we obtain a contrary effect.

Additionally, we conclude that the *CPD ratio* of 'Aaa'-rated products reveals not only the highest absolute values, but it also reacts most sensitively to negative realizations of Y^*, as indicated in the right chart. In comparison to 'Baa3'-rated bonds, 'Baa3'-rated tranches are more affected by negative economic developments. This effect is strengthened and accelerated with increasing seniority.

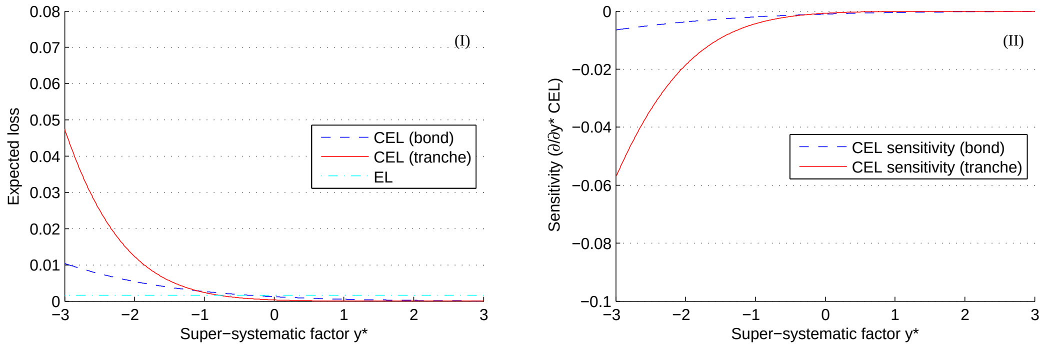
All these analytical findings underline that particularly securitized tranches of high credit quality exhibit a high degree of systematic risk in comparison to straight bonds of identical credit quality. Therefore, our model confirms not only the results of the aforementioned studies of systematic risk influences on structured financial products, it also completes theoretical findings with a simple analytical solution.

5.4.2 Product-specific Sensitivity of CELs

As defined in Equation (5.8), expected losses of straight bonds are determined by the bond's PD, EAD and LGD. Concerning expected tranche losses, the portfolio correlation parameters (ρ and δ) as well as the tranche-specific seniority are decisively important. Within our ongoing analysis, we regard impacts on product-specific expected losses with respect to realizations of Y^*. All other parameters remain deterministic and unchanged (ceteris paribus).

Figure 5.3 shows the sensitivities of product-specific expected losses to realizations of Y^* (x-axes) with $Y^* \in [-3, 3]$. In the left chart, impacts on the products' expected losses (y-axis) conditional on Y^* are denoted. The right chart shows the corresponding sensitivities (y-axis) to Y^*. In comparison to Section 5.4.1, we obtain similar product-specific results for the sensitivities of expected losses related to Y^*:
Conditional expected losses of both financial products increase in economic downturns. Analogous to the developments of CPDs, the sensitivity of the expected tranche loss $\mathrm{CEL}^{T_{[7.44\%,11.10\%)}}$ is much higher than the sensitivity of CEL^{B^*} (bond). Related to the introduced rating grades, Table 5.4 provides some insights into the relation between EL and the product-specific CEL de-

Figure 5.3: Sensitivities of CELs to Systematic Risk



Notes: The figure compares effects on conditional expected losses (CELs) of a bond B^* and a tranche $T_{[7.44\%,11.10\%)}$, respectively, related to a super-systematic risk factor Y^*. The results are based on an expected loss $EL^{B^*} = 0.162\% = EL^{T_{[7.44\%,11.10\%)}}$, a default probability $\pi = 1.180\%$ of the LHP, a recovery rate $R^{B^*} = 50\%$ of bond B^* , a LHP recovery rate $R = 50\%$. Correlation parameters are set to $\rho = 0.25$ and $\delta = 0.25$.

pending on several states of economic downturns.

Table 5.4: Impacts of Economic Downturns on Expected Losses

Rating	EL (%)	CEL (%) Bond	CEL (%) Tranche	CEL Sensitivity (%) Bond	CEL Sensitivity (%) Tranche	CEL Bond / EL	CEL Tranche / EL	CEL Tranche / CEL Bond
Y^* = -5								
Aaa	0.043	1.290	16.321	-0.773	-14.043	29.993	379.565	12.655
Aa1	0.071	1.822	20.573	-1.029	-16.218	25.839	291.810	11.293
Aa2	0.098	2.277	23.787	-1.233	-17.617	23.350	243.971	10.448
Aa3	0.162	3.208	29.528	-1.617	-19.649	19.803	182.273	9.204
A2	0.373	5.532	40.785	-2.434	-22.078	14.830	109.343	7.373
Baa1	0.590	7.380	47.819	-2.974	-22.635	12.508	81.050	6.480
Baa2	1.012	10.234	56.622	-3.661	-22.330	10.113	55.951	5.533
Baa3	1.541	13.065	63.661	-4.195	-21.277	8.481	41.325	4.872
Y^* = -3								
Aaa	0.043	0.344	1.683	-0.247	-2.374	8.011	39.143	4.886
Aa1	0.071	0.522	2.481	-0.356	-3.297	7.406	35.187	4.751
Aa2	0.097	0.684	3.184	-0.450	-4.057	7.018	32.652	4.653
Aa3	0.162	1.041	4.665	-0.645	-5.536	6.425	28.795	4.482
A2	0.373	2.048	8.517	-1.132	-8,833	5.492	22.833	4.158
Baa1	0.590	2.950	11.678	-1.515	-11.114	5.000	19.793	3.958
Baa2	1.012	4.495	16.661	-2.091	-14.129	4.441	16.463	3.707
Baa3	1.541	6.195	21.681	-2.637	-16.585	4.021	14.074	3.500

Notes: The table provides a comparison of several risk measures referring to bonds B^* and tranches $T_{[A^*,D^*)}$, respectively, conditional on different realizations of the super-systematic risk factor $Y^* \in \{-5, -3\}$. Selected values of Y^* reflect different degrees of economic downturns.

With regard to tranche $T_{[7.44\%,11.10\%)}$, the CEL is 182 times higher in economic downturns than the EL, while the CEL of bond B^* is about 20 times as high as the EL. For this reason the CEL of tranche $T_{[7.44\%,11.10\%)}$ is more than 9 times higher in comparison to the bond's CEL. The highest relative CEL impact follows for 'Aaa'-rated tranches. Related to a very bad state of the global economy, the tranche CEL is 12.66 times higher as the bond's CEL. Finally, all tranche-specific *CEL ratios* expand the corresponding *CPD ratios* across all rating classes and thus tranche seniorities. Even for a 'Baa3'-rated tranche the product's *CEL ratio* ($CEL^{T_{[A^*,D^*)}}/CEL^{B^*}$) is 1.66 times higher than the respective *CPD ratio* ($CPD^{T_{[A^*,D^*)}}/CPD^{B^*}$). These results confirm that the product-specific sensitivities of CELs to systematic changes are generally higher than the impacts on the CPDs. The effect increases in line with tranche seniority. Particularly, tranches of highest seniority reveal the highest sensitivity.

While Table 5.4 is dealing with economic downturns, Table 5.5 shows analogous impacts on CEL in an economic upturn.

Table 5.5: Impacts of Economic Upturns on Expected Losses

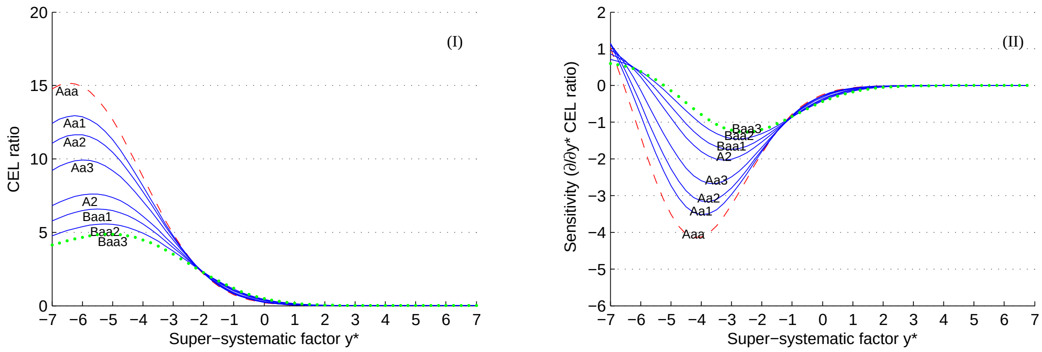
$Y^* = 1$								
Rating	EL (%)	CEL (%)		CEL Sensitivity (%)		$\frac{\text{CEL Bond}}{\text{EL}}$	$\frac{\text{CEL Tranche}}{\text{EL}}$	$\frac{\text{CEL Tranche}}{\text{CEL Bond}}$
		Bond	Tranche	Bond	Tranche			
Aaa	0.043	0.012	0.001	-0.011	-0.001	0.275	0.012	0.04
Aa1	0.071	0.021	0.001	-0.019	-0.003	0.294	0.015	0.052
Aa2	0.097	0.030	0.002	-0.027	-0.004	0.308	0.018	0.058
Aa3	0.162	0.054	0.004	-0.046	-0.009	0.330	0.023	0.070
A2	0.373	0.139	0.013	-0.110	-0.029	0.373	0.035	0.095
Baa1	0.590	0.236	0.026	-0.177	-0.055	0.400	0.045	0.112
Baa2	1.012	0.440	0.060	-0.307	-0.119	0.434	0.060	0.137
Baa3	1.541	0.716	0.115	-0.468	-0.215	0.465	0.075	0.161

Notes: The table provides exemplarily a comparison of several risk measures referring to bonds B^* and tranches $T_{[A^*,D^*)}$, respectively, conditional on a positive realization of the super-systematic risk factor $Y^* = 1$ which reflects an economic upturn.

Similar to the aforementioned findings, an economic upturn leads to reduced tranche-specific CEL as intuitively expected. In comparison to our CPD results, the CELs across all tranches (from a 'Baa3' rating up to an 'Aaa' rating) are decreasing more rapidly: the *CPD ratio* between a tranche with an 'Aaa' rating and a comparably rated bond B^* is about 0.06, while the respective *CEL ratio* is just 0.04 in value. 'Baa3'-rated instruments exhibit a product-specific *CPD ratio* of 0.22, while the *CEL ratio* is about 0.16. Thus, we suggest that the CPDs of both products decrease in economic upturns which leads to a disproportionately high reduction of CELs. We assume this effect particularly in highly rated tranches.

Figure 5.4 indicates the product-specific exposures to systematic risk. The left chart reflects the product-based *CEL ratios* (y-axis) depending on the super-systematic factor Y^* (x-axis) for several rating grades. The right chart shows the corresponding sensitivities of the *CEL ratios* (y-axis) to Y^* (x-axis).

Figure 5.4: Indication of Product-based Exposures to Systematic Risk (CEL)



Notes: The figure points out systematic risk exposures of securitized tranches $T_{[A,D)}$ related to Y^*. The *CEL ratio* describes the relation between product-based CELs ($CEL^{T_{[A^*,D^*)}}/CEL^{B^*}$). The dashed line represents the tranche with the highest seniority ('Aaa' rating). In turn, the dotted line describes the tranche with a modest rating ('Baa3') and thus it can be regarded as the tranche of lowest seniority (equity tranche).

Similar to our findings presented in Figure 5.2, we conclude from Figure 5.4 that the higher the seniority of a tranche, the higher is the ratio between tranche-specific CELs and bond-specific CELs. Comparing the right charts of Figure 5.2 and Figure 5.4, the sensitivity of *CEL ratios* is even higher in economic downturns than the respective sensitivity of *CPD ratios*. This effect is again strengthened and accelerated with increasing seniority.

The default probability of the LHP affects the tranche-specific impairment risk and the expected tranche losses, which vary over the business cycle.[133] In particular, default probabilities are lower in economic upturns than in economic downturns. However, credit rating agencies assess the credit quality of corporate bond issues and issuers 'through-the-cycle' and focus primarily on idiosyncratic risk characteristics (compare Löffler 2004, Heitfield 2005). Crucial macroeconomic ('point-in-time') information are explicitly not included, even though they may decisively affect the borrower's ability to fulfill his contractual payment obligations. By focusing on the risk characteristics *probability of default* or the *expected loss*, solely the first moment of the loss distribu-

[133] For this reason all these parameters can generally be modeled as random variables.

tion is considered (Fender et al. 2008). Other relevant risk characteristics are omitted. Especially, securitized tranches are affected decisively by systematic influences which can be traced back to the specific structure of securitization. Since idiosyncratic risks are mostly diversified within the underlying portfolio, pooling and tranching leads to a concentration of systematic risk exposures in securitized tranches. This effect may be boosted in tranches of high seniority. We conclude that the effects of securitization come along with an increased premium (spread) payment for senior tranches due to their higher exposure to systematic risk which is indicated by an increased sensitivity to systematic influences. Underestimated systematic risk exposures may lead to a dramatic mismatch between impairment expectations and occurred losses, which we could observe during the GFC.

5.4.3 Product-specific Downside Risk

In the following, we introduce conditional risk clusters (CRC) as a new scale to account for possible risk migrations. Similarly to the methodology of CRA ratings, the CRCs are linked to historical PDs provided by Moody's for corporate bonds. In contrast to CRA ratings, our CRCs provide rating-grade outlooks referring to expected realizations of Y^*. Similar to classical ratings, the CRCs are graded. Thus, our CRC scale allows us to quantify the product-specific downside risk with respect to Y^*.

Table 5.6 shows the complete CRC scale from CRC 1 (lowest PD) to CRC 8 (highest PD). In terms of PD, the latter CRC corresponds with the investment grade rating of the Moody's rating scale ('Baa3').

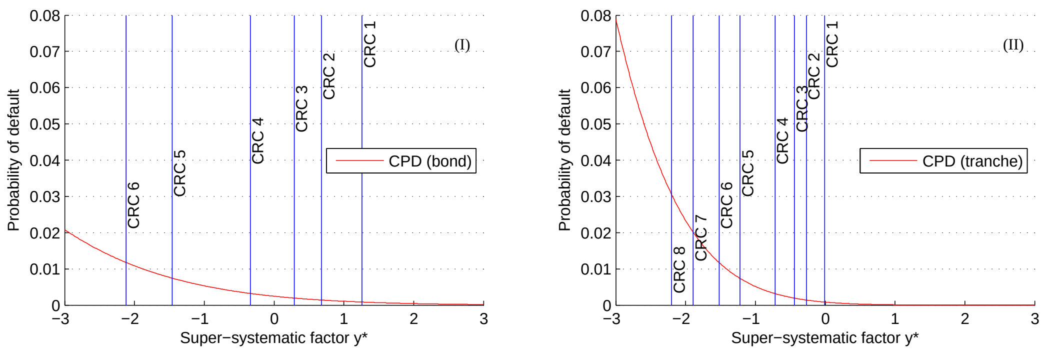
Table 5.6: Conditional Risk Clusters and Corresponding Default Rates

Conditional Risk Cluster (CRC)		Moodys Rating	
Cluster	Default Rate Interval (%)	Default Rate (%)	Rating
CRC 1	$\leq$ 0.086	0.086	Aaa
CRC 2	0.086 - 0.141	0.141	Aa1
CRC 3	0.141 - 0.195	0.195	Aa2
CRC 4	0.195 - 0.324	0.324	Aa3
CRC 5	0.324 - 0.746	0.746	A2
CRC 6	0.746 - 1.180	1.180	Baa1
CRC 7	1.180 - 2.024	2.024	Baa2
CRC 8	2.024 - 3.081	3.081	Baa3

Notes: The table defines conditional risk clusters (CRCs) related to historical average 5-year cumulative default rates from 1983 to 2009 provided by Moody's (2010). Default rate intervals describe the cluster-specific boundaries. Default rates above 3.081% refer to rating grades below the investment grade. Non-investment grade ratings are not considered.

Figure 5.5 provides product-specific cluster outlooks related to Y^* (x-axes). While the left chart refers to bond B^*, the right chart refers to tranche $T_{[7.44\%,11.10\%)}$. Respective cluster outlooks are linked to historical default rates in accordance to Table 5.6. The CRCs are indicated by vertical lines within both charts of Figure 5.5. The vertical cluster lines cross product-specific CPD functions at corresponding PDs (y-axes).

Figure 5.5: Impacts of Systematic Risk on Cluster Migrations



Notes: The figure provides product-specific cluster outlooks related to realizations of Y^*. Cluster outlooks are linked to historical default rates in accordance to Table 5.6. The results are presented with respect to i) a corporate bond B^* and ii) a tranche $T_{[7.44\%,11.10\%)}$. Both financial instruments exhibit the same default probability 0.324% which is linked to CRC 4. The default rate of the LHP is $\pi = 1.180\%$; the recovery rate of bond B^* is $R^{B^*} = 50\%$; the recovery rate of the LHP is $R = 50\%$. Correlation parameters are set to $\rho = 0.25$ and $\delta = 0.25$.

Both tranche $T_{[7.44\%,11.10\%)}$ and straight bond B^* exhibit identical risk characteristics. This leads to a joint rating of '*Aa3*' corresponding with our CRC 4 in terms of PD. Nevertheless, their inherent migration risk differs widely from each other with respect to economy changes. Particularly, the downside risk of tranche $T_{[7.44\%,11.10\%)}$ is much more sensitive to negative realizations of Y^* in comparison to the downside risk of straight bond B^*. Obviously, tranche $T_{[7.44\%,11.10\%)}$ reaches the denoted thresholds for cluster downward movements more rapidly than bond B^*. In other words, the global economy must be at worse states to cause a downward movement of bond B^* below the investment grade cluster (CRC 8). This indicates that a bond cluster is much more constant over time and exhibits a greater buffer to economy changes in terms of migration risk.

Table 5.7 gives an overview of migrations probabilities related to the conditional risk clusters (CRC).

Table 5.7: Migration Probabilities with respect to Systematic Risk

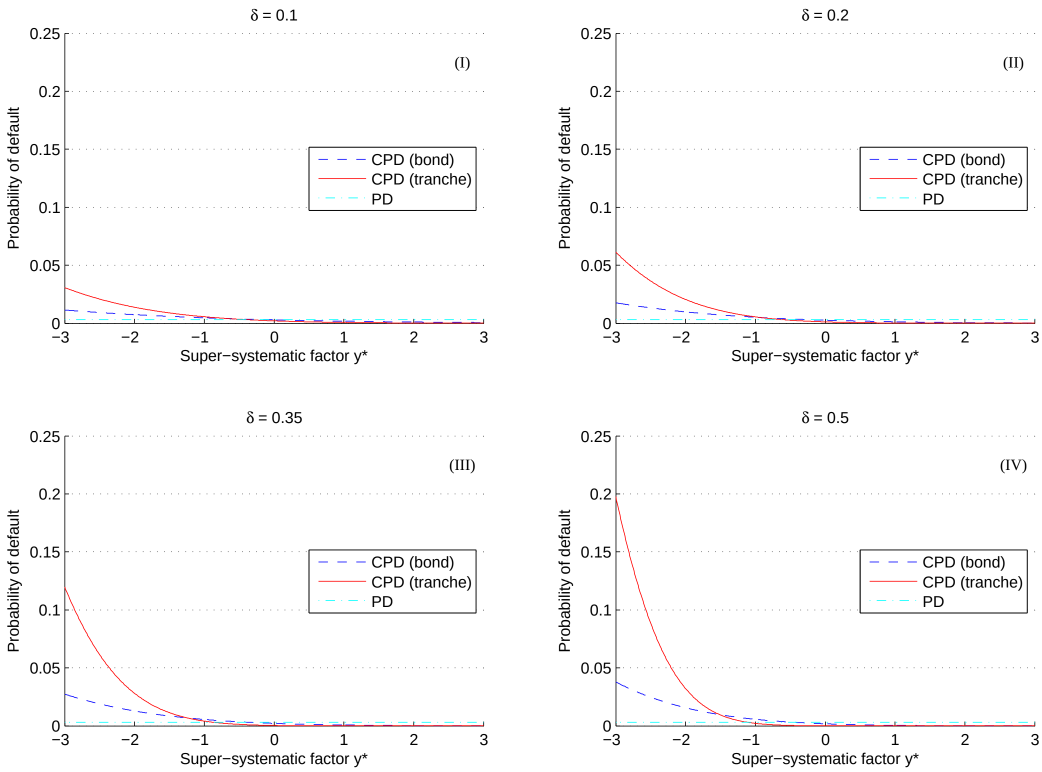
Conditional Risk Cluster (CRC)	Bond (%)	Tranche (%)
CRC 1	10.383	50.399
CRC 2	14.442	10.243
CRC 3	13.766	6.361
CRC 4	24.716	9.421
CRC 5	29.478	12.453
CRC 6	5.514	4.698
CRC 7	1.541	3.488
CRC 8	0.024	1.548
$\leq$ CRC 8	0.135	1.390

Notes: The table provides a migration measure due to realizations of Y^*. Product-specific percentages describe the probability to remain in the respective conditional risk cluster (CRC). The results are presented with respect to i) a corporate bond B^* and ii) the tranche $T_{[7.44\%,11.10\%)}$. Both financial instruments exhibit the same default probability of 0.324%. The default rate of the LHP is $\pi = 1.180\%$; the recovery rate of bond B^* is $R^{B^*} = 50\%$; the recovery rate of the LHP is $R = 50\%$. Correlation parameters are set to $\rho = 0.25$ and $\delta = 0.25$.

With a probability of 1.39% the tranche's cluster falls below the investment grade cluster, while the corresponding probability of a straight bond B^* is just 0.14%, which is more than 10 times lower. The probability to hold the initial cluster (CRC 4) is about 25% for bond B^* and only about 9% for tranche $T_{[7.44\%,11.10\%)}$. Further, the probability of bond B^* to prevent a downward movement of more than one cluster is almost 2.5 times higher (54.19%) than the respective probability of tranche $T_{[7.44\%,11.10\%)}$ (21.87%). This means that bond B^* reaches a CRC 4 or CRC 5 rating with a probability of 54.19%, while the respective tranche probability is at 21.87%. On the other hand, the probability of tranche $T_{[7.44\%,11.10\%)}$ is clearly higher for obtaining a cluster upgrade conditional on Y^* than for bond B^* (67.00% vs. 38.59%). Despite this result the downside risk remains striking and should be seriously taken into consideration by investors with respect to their investment decisions.

Generally, the presented analytical results are dependent on specific parameter assumptions. However, parameter variations lead to qualitatively similar results. Under consideration of several δ variations, Figure 5.6 exemplarily shows alternative CPD functions (y-axes) depending on Y^* (x-axes).

Figure 5.6: Systematic Risk Impacts depending on Variations of δ



Notes: The figure compares effects on product-specific probabilities of default (y-axes) related to several weightings $\delta \in \{0.1, 0.2, 0.35, 0.5\}$ of Y^* (x-axes). The results refer to identical default probabilities of bond B^* and tranche $T_{[7.44\%,11.10\%)}$ ($\pi^{B^*} = 0.324\% = \pi^{T_{[7.44\%,11.10\%)}}$), a default rate $\pi = 1.180\%$ of the LHP, a recovery rate $R^{B^*} = 50\%$ of bond B^*, and a recovery rate $R = 50\%$ of the LHP. While the correlation parameter δ varies, the correlation parameter ρ is set to $\rho = 0.25$.

We find that the reported impacts of economy changes hold for all δ values, $\delta \in \{0.1, 0.2, 0.35, 0.5\}$. Thus, product-specific PDs increase with rising values for δ and vice versa.

5.4.4 Systematic Risk Sensitivity due to Inter-sectoral Diversification

In the following, our analytical findings will be further amended by an additional multi-sector simulation study. While the analytical framework focuses solely on borrowers located in a single sector, the multi-sector approach accounts exemplarily for structured claims which refer to borrowers from different economic sectors such as the financial industry, consumer goods and industrials. This multi-sectoral perspective may provide more realistic insights into the behavior of structured finance instruments related to economy changes. In this way, we simultaneously check the robustness of our analytical findings.

Analogous to Section 5.3, we firstly quantify portfolio losses (case-wise) depending on the borrowers' sector-specific asset returns $Z_m^{B_{i,j}}$ through Monte Carlo (MC) simulations. Index m indicates the multi-sector approach. $B_{i,j}$ refers to a borrower i located in sector j. $Z_m^{B_{i,j}}$ is evaluated depending on i) a super-systematic risk factor Y^*, ii) a sectoral risk component U^j and iii) an idiosyncratic risk term $\varepsilon^{B_{i,j}}$.[134] This leads to

$$Z_m^{B_{i,j}} = \sqrt{\rho^j \cdot \delta^j} \cdot Y^* + \sqrt{\rho^j - \rho^j \cdot \delta^j} \cdot U^j + \sqrt{1 - \rho^j} \cdot \varepsilon^{B_{i,j}}. \tag{5.26}$$

Since all random variables are i.i.d. with Y^*, U^j, $\varepsilon^{B_{i,j}} \sim \mathcal{N}(0,1)$, $Z_m^{B_{i,j}}$ is also standard normal distributed with $Z_m^{B_{i,j}} \sim \mathcal{N}(0,1)$. While ρ^j describes the borrowers' asset correlation in sector j with $\rho^j \in [0,1]$, δ^j indicates the dependency of sector j on Y^*, compare Equation (5.11).

In order to examine the behavior of CDO tranches related to systematic risk influences under consideration of inter-sectoral diversification effects, we focus on the following two case studies:

In *Case 1*, we examine a portfolio containing debt claims related to 2000 homogeneous borrowers who are equally divided over four sectors denoted by $j \in \{1,2,3,4\}$. The asset correlation ρ^j among borrowers $B_{i,j}$ of sector j (intra-sectoral asset correlation) is the same in each sector j with $\rho^j = 0.25$. Each sector j contains 500 homogeneous borrowers and is identically affected by the super-systematic risk factor Y^*, which is indicated by $\delta^j = 0.25$ being constant across sectors.

The setting in *Case 2* corresponds to *Case 1*, except the sectoral influence

[134] Recall that Y^* is simultaneously affecting all borrowers across all sectors, U^j only affects all borrowers in sector j and $\varepsilon^{B_{i,j}}$ affects solely a single borrower i in sector j.

of the systematic risk factor Y^* and the intra-sectoral asset correlation ρ^j. In contrast to *Case 1*, the dependency on Y^* varies across sectors, which leads to sector-specific weights $\delta^j \in \{0.4, 0.5, 0.6, 0.7\}$. The intra-sectoral asset correlation ρ^j is homogeneous for all borrowers in a specific sector, but inhomogeneous across sectors with $\rho^j \in \{0.2, 0.3, 0.4, 0.5\}$.

Using the parameter settings of our analytical study further simulation results are computed referring to a pool of 500 homogeneous borrowers located in a single sector. Thus, we test our analytical findings. This case is denoted as *Simulation Study* m_0.

Table 5.8 summarizes the different case settings.

Table 5.8: Case Settings within the Multi-sector Approach

	Analytical Study (s)	Simulation Study (m_0)	Case 1 (m_1)	Case 2 (m_2)
Amount of Sectors	1	1	4	4
Bonds per Sector	LHP	500	500	500
Recovery Rate of Bonds	0.5	0.5	0.5	0.5
ρ^1	0.25	0.25	0.25	0.5
ρ^2	-	-	0.25	0.4
ρ^3	-	-	0.25	0.3
ρ^4	-	-	0.25	0.2
δ^1	0.25	0.25	0.25	0.4
δ^2	-	-	0.25	0.5
δ^3	-	-	0.25	0.6
δ^4	-	-	0.25	0.7

Notes: The table provides an overview of case settings within the multi-sector (m) approach in contrast to the setting in the analytical single-sector approach (s). The borrowers' asset correlation ρ^j with $j \in \{1, 2, 3, 4\}$ is homogeneous across sectors j in *Case 1* and inhomogeneous in *Case 2*. The sectoral dependency on systematic risk influences is expressed by δ^j with $j \in \{1, 2, 3, 4\}$ which is also homogeneous in *Case 1* and inhomogeneous in *Case 2*. The simulation study (m_0) exhibits the same parameter settings as our analytical study except the amount of bonds in the underlying pool.

Both settings - those in *Case 1* and in *Case 2* - may overcome some shortcomings of a single-sector approach, e.g., the underestimation of sectoral diversification effects. However, *Case 2* overcomes additionally various simplifications of *Case 1* given by constant correlation assumptions for both ρ^j and δ^j. Thus, our asset-securitization model gets closer to observable real-world CDOs. With inhomogeneous sectors and homogeneous borrowers in each sector more valuable insights into risk characteristics of structured products may be provided.[135] While results labeled with s refer to our single-sector approach

[135] Our findings hold even if we implement inhomogeneous borrowers located in inhomogeneous sectors. For tractability we focus solely on the introduced case settings.

(analytical), results of the multi-sector simulations are generally indicated by m_c with $c \in \{0,1,2\}$. In addition to our simulation study m_0 which tests our analytical findings, we introduce m_1 and m_2 to distinguish between *Case 1* and *Case 2*, respectively.

Next, we calibrate the risk profile of tranches $T_{[A^*_{m_c}, D^*_{m_c})}$ on the specific risk characteristics of reference bond B^* in order to ensure an appropriate comparison of both products with regard to their product-specific behavior related to variations of Y^*.

Based on case-wise simulated portfolio loss distributions, we assign the risk-adjusted attachment points $A^*_{m_c}$ and detachment points $D^*_{m_c}$ of tranches m_c (compare Section 5.3.4). Eventually, tranche $T_{[A^*_{m_c}, D^*_{m_c})}$ and bond B^* again exhibit the same risk profile in terms of PDs and ELs.

In comparison to the single-sector approach, our multi-sector simulations clearly show the inter-sectoral diversification effects related to the main characteristics of structured products: inter-sectoral pooling and tranching lead to a default-risk reduction of structured claims. For this reason, the attachment point decreases from $A^*_s = 7.44\%$ to $A^*_{m_1} = 4.07\%$, while the detachment point $D^*_s = 11.10\%$ decreases to $D^*_{m_1} = 5.28\%$ in order to match the risk profile of the reference bond B^*, which is determined by a PD of 0.324% and an EL of 0.162%.

Regarding Equation (5.26), we may observe sectoral diversification effects for $\delta^j < 1$. Apart from idiosyncratic risk, the borrowers' asset returns are solely influenced by sectoral changes if $\delta^j = 0$. The borrowers' idiosyncratic risk contributions can be diversified in case of $\rho^j < 1$ and are perfectly diversified in case of the LHP (compare Gordy 2000, 2003). Finally, we evaluate the following pool-specific dependency measures from Equation (5.26):

- The intra-sectoral asset correlation (pairwise asset correlation among borrowers within a single sector)

$$corr(Z_m^{B_{i,j}}, Z_m^{B_{k,j}}) = \rho^j \quad \text{for} \quad i \neq k. \tag{5.27}$$

- The inter-sectoral asset correlation (pairwise asset correlation of borrowers located in different sectors)

$$corr(Z_m^{B_{i,j}}, Z_m^{B_{k,l}}) = \sqrt{\rho_i \cdot \delta_j \cdot \rho_k \cdot \delta_l} \quad \text{for} \quad j \neq l. \tag{5.28}$$

- The cross-sector correlation (pairwise sector correlation)

$$corr(U^j, U^l) = \sqrt{\delta^j \cdot \delta^l} \quad \text{for} \quad j \neq l. \tag{5.29}$$

Intra-sectoral correlations referring to Equation (5.27) are presented in Table 5.8. According to Equation (5.28), the average inter-sectoral asset correlation is 10.93% (22.15%) in *Case 1* (*Case 2*). Under consideration of Equation (5.29), we calculate an average cross-sector correlation of 25% (54.23%) in *Case 1* (*Case 2*). Thus, each setting leads to both sectoral and idiosyncratic risk diversification.

Referring to *Case 1*, the two upper charts in Figure 5.7 show the product-specific PD sensitivities (y-axes) to realizations of Y^* (x-axes) with $Y^* \in [-3, 3]$. Analogously, both lower charts in Figure 5.7 show the product-specific sensitivities of expected losses (y-axes) to Y^* (x-axes).

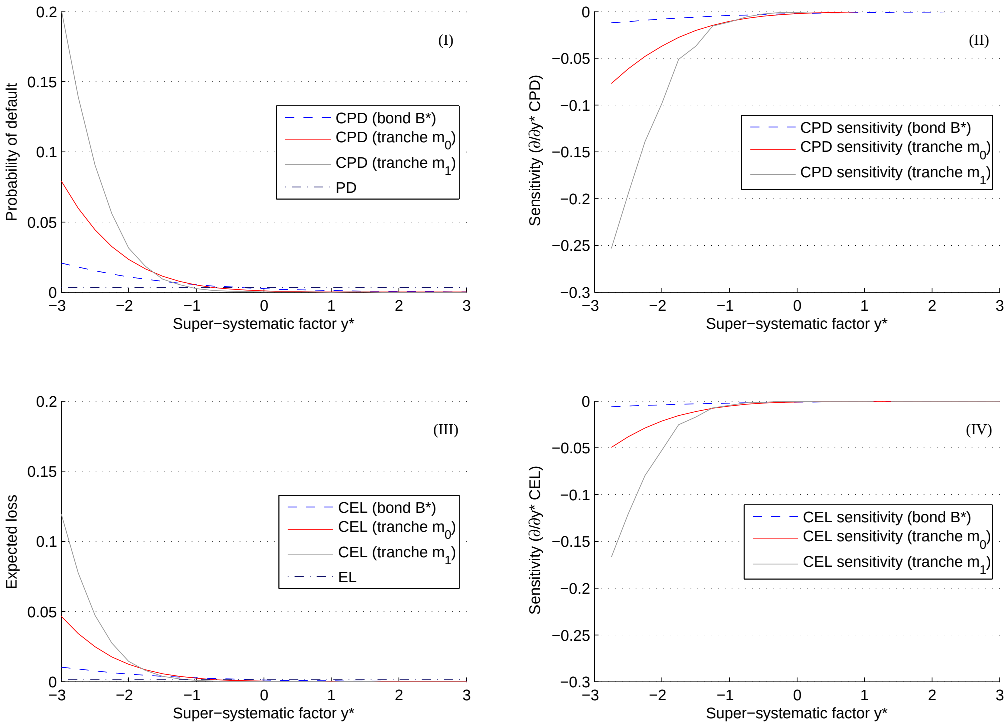
Comparing Figure 5.7 with our analytical findings – plotted in Figures 5.1 and 5.3 – we conclude that the already stated effects due to systematic influences are not only confirmed but also intensified by simulating multi-sector securitizations.[136] Despite corresponding risk profiles, tranche m_1 with $T_{[4.07\%, 5.28\%)}$ seems to be much more affected by systematic risk – in terms of PDs and ELs – than both the reference bond B^* and tranche m_0 with $T_{[7.86\%, 10.95\%)}$. In contrast to our single-sector approach (tranche m_0), the function of tranche m_1 gets steeper with decreasing realizations of Y^*. Thus, we derive throughout higher sensitivities to Y^* for tranche m_1 than for tranche m_0 in economic downturns, as shown in the upper and lower chart on the right of Figure 5.7. Finally, we suggest that the sensitivity of structured products to systematic risk increases with the degree of sectoral diversification within CDO structures.

Table 5.9 shows the impacts of a massive economic downturn on both product-specific PDs and ELs for *Case 1*. With regard to various rated tranches $T_{[A^*_{m_1}, D^*_{m_1})}$, respective risk measures are provided, e.g., the ratio between CPD and PD, for $Y^* = -5$ in order to underline the effect size in such a downturn scenario.

A direct comparison with findings shown in Tables 5.2 and 5.4 indicates that impacts of such economy downturns are consistently higher on multi-sectoral pooled debt claims than on single-sectoral pooled ones. In line with Figure

[136]Fore more details see Sections 5.4.1 and 5.4.2.

Figure 5.7: Systematic Risk Sensitivity in a Multi-sector Approach (Case 1)



Notes: The figure for *Case 1* compares effects on both conditional probabilities of default (CPDs) (I+II) and conditional expected losses (CELs) (III+IV) related to several realizations of the super-systematic risk factor Y^* in a multi-sector approach. The tranche seniority is determined with $A^*_{m_1} = 4.07\%$ and $D^*_{m_1} = 5.28\%$. It refers to a simulated pool containing debt claims from four homogenous sectors (compare Table 5.8). The results refer to identical default probabilities of bond B^*, tranche m_0 and tranche m_1 ($\pi^{B^*} = 0.324\% = \pi^{T_{[4.07\%,5.28\%)}}_{m_1} = \pi^{T_{[7.86\%,10.95\%)}}_{m_0}$), a default rate $\pi = 1.180\%$ of the simulated pool (SP), a recovery rate $R^{B^*} = 50\%$ of bond B^* and a recovery rate $R = 50\%$ of the SP. The correlation parameters δ^j and ρ^j are set to $\delta^j = \rho^j = 0.25$.

5.7, we conclude from Table 5.9 that the sensitivity of multi-sectoral pooled products to systematic risk is much higher in terms of both PDs and ELs. Furthermore, the systematic risk exposure increases in line with the seniority of tranche m_1. For example, under very bad economic conditions, which are again indicated by $y^* = -5$, the CPD of tranche $T_{[4.07\%,5.28\%)}$ ('Aa3-rated') is at least 2.4 times higher than the respective tranche CPD referring to single-sector debt claims (m_0) and nearly 14 times higher than the respective CPD of bond B^*.[137] In case of an 'AAA'-rated tranche m_1, its CPD is 29 times higher than the respective bond CPD and at least 3.2 times higher than the respective CPD of tranche m_0. The results of *Simulation Study* m_0 confirm our analytical findings (compare column *Tranche* s with column *Tranche* m_0).

[137]Regarding already presented CRC migrations, we achieve similar results.

Table 5.9: Case 1: Economic Impacts on PDs and ELs in a Multi-sector Approach

$Y^* = -5$							
Rating	PD (%)	CPD (%) Bond	Tranche s	Tranche m_0	Tranche m_1	$\frac{\text{CPD Tranche } m_1}{\text{PD}}$	$\frac{\text{CPD Tranche } m_1}{\text{CPD Bond}}$
Aaa	0.086	2.579	23.183	23.266	74.729	868.940	28.976
Aa1	0.141	3.643	28.703	27.778	81.642	579.024	22.411
Aa2	0.195	4.553	32.789	31.956	85.891	440.468	18.865
Aa3	0.324	6.416	39.864	36.946	90.062	277.968	14.037
A2	0.746	11.064	53.026	52.292	95.312	127.764	8.615
Baa1	1.180	14.759	60.752	60.124	97.247	82.413	6.589
Baa2	2.024	20.468	69.880	68.953	98.730	48.779	4.824
Baa3	3.081	26.131	76.691	75.936	99.422	32.269	3.805
Rating	EL (%)	CEL (%) Bond	Tranche s	Tranche m_0	Tranche m_1	$\frac{\text{CEL Tranche } m_1}{\text{EL}}$	$\frac{\text{CEL Tranche } m_1}{\text{CEL Bond}}$
Aaa	0.043	1.290	16.321	14.869	64.799	1506.949	50.232
Aa1	0.071	1.822	20.573	20.735	72.210	1024.258	39.632
Aa2	0.098	2.277	23.787	22.140	77.182	791.607	33.896
Aa3	0.162	3.208	29.528	28.652	83.532	515.631	26.039
A2	0.373	5.532	40.785	38.928	90.040	241.394	16.276
Baa1	0.590	7.380	47.819	46.666	93.765	158.924	12.705
Baa2	1.012	10.234	56.622	55.154	95.940	94.802	9.375
Baa3	1.541	13.065	63.661	62.779	97.585	63.346	7.469

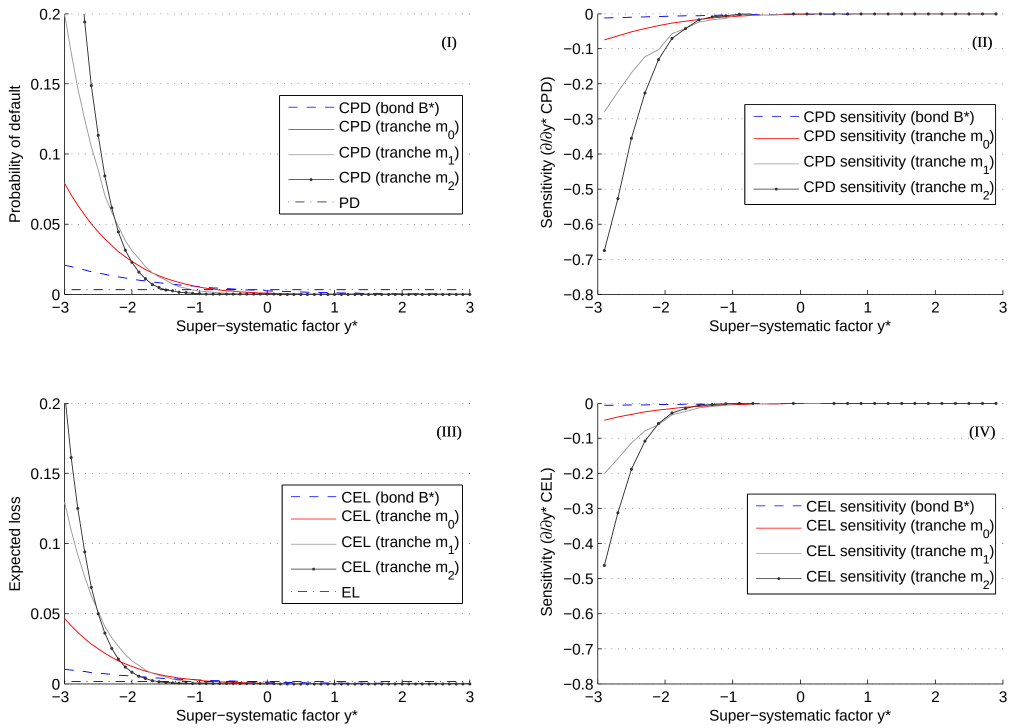
Notes: The table provides a comparison of several risk measures referring to tranches $T_{[A^*_{m_1}, D^*_{m_1})}$ with various ratings in a multi-sector (m) approach. The realization of $Y^* = -5$ reflects a strong economic downturn. The analytical results in column *Tranche s* correspond approximately to those of our multi-sector approach considering just one homogeneous sector (compare column *Tranche m_0*). Values in column *Tranche m_1* describe the results of our m-approach based on four homogeneous sectors.

Even in *Case 2*, we observe the risk characteristics of multi-sectoral CDOs. Figure 5.8 contrasts risk characteristics of tranche m_2 with those of tranche m_1, tranche m_0, and reference bond B^*.[138] The upper chart on the left-hand side of Figure 5.8 refers to product-specific PDs (y-axis), while the lower-left chart denotes the products' ELs (y-axis) related to variations of $Y^* \in [-3, 3]$ (x-axes). The two charts on the right-hand side show the respective sensitivities (y-axes) to Y^* (x-axes).

Similarly to the plotted CPD functions in Figure 5.8, the CEL functions of tranche m_1 and tranche m_2, respectively, underline that they are much more affected by variations of Y^* than corresponding functions of tranche m_0. But in relation to tranche m_1, the sensitivity to systematic risk of tranche m_2 increases more rapidly, the worse the economic downturn is (compare the two charts on the right). However, in bad states of the global economy the sensitivity to systematic risk is clearly higher for all securitizations than the sensitivity of the reference bond B^*.

[138]The behavior of reference bond B^* related to changes of Y^* is independent from presented case settings since the reference bond B^* is not affected by pooling and tranching activities (securitizations).

Figure 5.8: Systematic Risk Sensitivity in a Multi-sector Approach (Case 2)



Notes: The figure for *Case 2* compares effects on both conditional probabilities of default (CPDs) (I+II) and conditional expected losses (CELs) (III+IV) related to several realizations of the super-systematic risk factor Y^* in a multi-sector approach. The tranche seniority is determined with $A^*_{m_2} = 7.36\%$ and $D^*_{m_2} = 10.95\%$. It refers to a pool containing debt claims from four inhomogeneous sectors (compare Table 5.8). The results refer to identical default probabilities of i) bond B^*, ii) tranche m_0, iii) tranche m_1 and iiii) tranche m_2. Thus, $\pi^{B^*} = 0.324\% = \pi_{m_2}^{T[7.36\%,10.95\%)} = \pi_{m_1}^{T[4.07\%,5.28\%)} = \pi_{m_0}^{T[7.86\%,10.95\%)}$. The default rate of the pool is $\pi = 1.180\%$; the recovery rate of the pool is $R = 50\%$; the recovery rate of bond B^* is $R^{B^*} = 50\%$.

For $Y^* \geq 0$, the CPD functions of all products are below the PD function, which is jointly shared. This indicates that the default probabilities conditional on Y^* are lower than the unconditional PD of 0.324% for all good states of the global economy. Since the unconditional PD equals conditional default expectations over all possible realizations of Y^*, all CPD functions must cross the PD function at negative realizations of Y^* ($Y^* < 0$). We conclude that the intersections – where the CPD functions cross the PD threshold from below – are determined by the products' overall dependency on i) systematic risk, ii) sector risk and iii) idiosyncratic risk. Since ii) and iii) can be diversified in securitizations, tranche m_2 and tranche m_1 may reveal the highest degree of risk diversification. In comparison to both multi-sector CDO tranches, the tranche m_0 exhibits a lower degree of sectoral diversification. Bond B^* is simultaneously exposed to systematic, sectoral and idiosyncratic risk without any diversification effects. Diversifying sectoral risk as well as idiosyncratic risk (diversification effect) requires an adjustment of tranche seniority in order to maintain the unconditional default risk of 0.324%. Both diversification effects lead additionally to an increase of systematic risk exposure in securitizations (concentration effect). This concentration effect is the higher, the more senior a CDO tranche is. Along decreasing values of Y^*, we expect therefore that the products' CPDs cross the PD threshold in the following order: first, the CPD function of bond B^*, second the one of tranche m_0 and last the ones of the multi-sector CDO tranches m_1 and m_2, respectively. We confirm these theoretical findings in the two charts on the left-hand side of Figure 5.8. Due to different settings (see Table 5.8), tranche m_1 and tranche m_2 are not directly comparable in terms of risk diversification and concentration as well. However, what we observe is a higher systematic risk exposure in tranche m_2 than in tranche m_2 since the CPD function of tranche m_2 is much steeper after crossing the PD function than that of tranche m_1 (compare the two charts on the right-hand side).[139] As already indicated by Figure 5.7, the sensitivity of securitized tranches to systematic risk is heavily increasing through multi-sectoral pooling and tranching, even if inhomogeneous sectors are considered.

Table 5.10 addresses the impacts of an economic downturn ($Y^* = -3$) on the products' PDs and ELs in a multi-sector approach with inhomogeneous sectors. Of special interest are several risk measures related to various rated tranches m_2.

For example, the 'Aa3'-rated tranche m_2 with attachment point $A^*_{m_2} = 7.36\%$ and detachment point $D^*_{m_2} = 10.95\%$ is at least 5.2 times stronger affected by the economic downturn than tranche m_0 (see also Figure 5.8). The CPD of an 'AAA'-rated tranche m_2 is at least 1.8 times higher in *Case 2* than in *Case1*.[140] In such an economic downturn, the CPD of an 'Aa1'-rated tranche m_2 is already 151 times higher than the products' PD and at least 20 times higher than the CPD of an 'Aa1'-rated bond. We obtain similar results with regard to CELs, which are also denoted in Table 5.10. Overall, the impacts on CPDs and CELs are even worse in very bad states of the global economy.

We conclude that the differences between case-specific CPDs and CELs are due to multi-sectoral pooling and tranching across sectors with inhomogeneous sensitivities to systematic risk. Expanding our analytical results, we find that

[139] All these results correspondingly hold for conditional expected losses.

[140] In *Case 1*, the CPD of an 'AAA'-rated tranche m_1 is 7.010% for $Y^* = -3$.

Table 5.10: Case 2: Economic Impacts on PDs and ELs in a Multi-sector Approach

$Y^* = -3$							
Rating	PD (%)	CPD (%)				$\frac{\text{CPD Tranche } m_2}{\text{PD}}$	$\frac{\text{CPD Tranche } m_2}{\text{CPD Bond}}$
		Bond	Tranche s	Tranche m_0	Tranche m_2		
Aaa	0.086	0.689	2.954	3.006	13.148	152.884	19.083
Aa1	0.141	1.044	4.301	4.269	21.342	151.365	20.443
Aa2	0.195	1.369	5.475	5.293	26.672	136.779	19.483
Aa3	0.324	2.082	7.904	7.079	37.179	114.751	17.857
A2	0.746	4.097	14.034	13.847	63.506	85.128	15.501
Baa1	1.18	5.9	18.893	18.663	75.198	63.727	12.745
Baa2	2.024	8.989	26.313	26.043	88.860	43.903	9.885
Baa3	3.081	12.39	33.506	33.326	94.740	30.750	7.646
Rating	EL (%)	CEL (%)				$\frac{\text{CEL Tranche } m_2}{\text{EL}}$	$\frac{\text{CEL Tranche } m_2}{\text{CEL Bond}}$
		Bond	Tranche s	Tranche m_0	Tranche m_2		
Aaa	0.043	0.344	1.683	1.487	5.821	135.374	16.922
Aa1	0.0705	0.522	2.481	2.634	11.091	157.324	21.248
Aa2	0.0975	0.684	3.184	2.872	13.829	141.832	20.217
Aa3	0.162	1.041	4.665	4.474	20.638	127.394	19.825
A2	0.373	2.048	8.517	7.904	40.856	109.533	19.949
Baa1	0.59	2.95	11.678	11.285	51.056	86.536	17.307
Baa2	1.012	4.495	16.661	16.198	66.456	65.668	14.784
Baa3	1.5405	6.195	21.681	21.530	80.123	52.011	12.933

Notes: The table provides a comparison of several risk measures referring to tranches $T_{[A^*_{m_2}, D^*_{m_2})}$ related to various ratings in a multi-sector (m) approach. The realization of $Y^* = -3$ reflects an economic downturn. The analytical results in column *Tranche s* correspond approximately to those of our multi-sector approach considering just one homogenous sector (compare column *Tranche* m_0). Values in column *Tranche* m_2 describe the results of our m-approach based on four inhomogeneous sectors.

the effect size is positively correlated with the degree of sectoral diversification across structured debt claims. Thus, the higher the degree of sectoral diversification within structured financial products, the higher is the sensitivity of the products' CPD and CEL to changes in the global economy.

In accordance to recent theoretical findings, we conclude further that sectoral diversification leads to decreasing default risk of structured finance products (diversification effect), but we also see an increase of their systematic risk exposure (concentration effect). Thereby, the concentration effect within a given CDO structure is positively correlated with tranche seniority.

Our findings hint at shortcomings of credit ratings which focus solely on product-specific PDs and ELs. We conclude that particularly structured finance instruments, e.g., CDOs or STCDOs, are much more affected by systematic risk than corporate bonds. Additionally, we show that systematic risk characteristics of securitized tranches differ in seniority. Especially, tranches of high seniority are tremendously affected by systematic influences which is pointed out by our sensitivity analysis. This higher degree of systematic risk within securitized tranches, particularly in the most senior tranches, is not reflected appropriately by standardized ratings provided by CRAs (Rösch &

Scheule 2009). Especially, ratings of securitized tranches should account for their specific systematic risk characteristics in order to quantify their inherent risk in a proper manner. Concerning systematic risk exposures, our results underline that market participants should avoid applying historically developed interpretations of corporate bond ratings directly to structured financial instruments, even if these structured instruments exhibit a high degree of intersectoral diversification.

5.5 Summary

The Global Financial Crisis has shown that both default probabilities (PDs) and expected losses (ELs) of structured financial instruments are much more affected by systematic influences than PDs and ELs of comparably rated bonds. While rating agencies primarily consider *unconditional* risk measures - PDs and ELs - they omit product-specific sensitivities to systematic risk. Since these *unconditional* risk measures are averaging out extreme scenarios of the global economy, we explicitly model this kind of scenario within our analytical study to examine impacts on (1) impairment risks and (2) expected losses related to tranches and bonds with corresponding unconditional risk profiles. Monte Carlo simulations considering multi-sectoral securitizations improve the robustness of our analytical findings.

The simple analytical model expands theoretical findings of the recent literature and provides a general framework for quantifying *conditional* probabilities of default (CPDs) and *conditional* expected losses (CELs). We demonstrate that in economic downturns the impairment risk of securitization is many times higher than the respective default risk of straight bonds, even though their *unconditional* PDs are identical. Analogously, product-specific expected losses related to systematic influences differ widely as well. Overall, tranche CPDs and CELs react much more sensitively to systematic influences than bond CPDs and CELs, which is indicated by several risk ratios. In fact, the revealed systematic risk sensitivities vary not only between financial instruments, but also within securitizations depending on tranche seniorities. Thus, high-seniority tranches may exhibit the highest degree of systematic exposures. Furthermore, the systematic risk exposures seem to be even higher in sectorally well-diversified CDO structures than in less diversified securitizations. Our Monte Carlo simulation study shows that the systematic risk exposure of securitized tranches is depending on inter-sectoral diversification effects caused by multi-sectoral pooling and tranching. Once again, the systematic risk exposures increase in line with tranche seniorities.

By introducing conditional risk clusters (CRCs), we account for cluster migrations related to systematic risk in the sense of a 'point-in-time' perspective. We show that the downside risk of tranches conditional on systematic influences is many times higher in contrast to straight bonds. This crucial effect also seems to be positive correlated with subordination in securitization.

In general, rating agencies provide ratings which do not sufficiently reflect the degree of systematic risk exposures inherent in both financial instruments. Particularly, ratings of tranches should account for revealed systematic risk characteristics in order to quantify their overall risk in a proper way. We suggest that our CRC approach provides an reasonable starting point for further research in risk measures anticipating product-based systematic risk.

Finally, our approach provides market participants with essential information for risk-adjusted investment decisions since the downside risk of securitization due to changes in the global economy is obviously striking. Further, risk managers may benefit in terms of hedging credit portfolio risks, in particular with respect to risk contributions of structured instruments. Risk traders may gain reasonable insights into product-specific risk profiles for pricing and hedging single-tranche CDO swaps or bespoke CDOs. On the other hand, our findings may be useful for regulatory authorities to provide risk-adjusted capital rules in order to ensure stability of financial markets since many banks are highly involved in dealing with structured finance instruments.

For tractability our analytical analyses are based on a Gaussian copula approach. Even within this simple model framework, effects of systematic risks are striking. Comprehensive Monte Carlo simulations show that reported results are boosted under consideration of a student-t distribution for Y^*, even for lower degrees of freedom.

However, especially for structured products research is suggested in examining dependency structures within credit risky portfolios, e.g., default correlations or counterparty risk (compare Brigo & Chourdakis 2009). Not only with respect to borrower-specific dependencies, but also with regard to unobservable systematic risk factors influencing simultaneously all parties. Thus far, we have focussed our study only on impacts of controlled systematic risk changes. In this context, it also might be valuable to address related issues like parameter uncertainty or model risk (see Coval et al. 2009*b*, Heitfield 2009).

Addressing the importance of systematic influences for modeling credit risk is one step to improve the understanding of structured finance instruments. Finally, market participants, e.g., banks, investors and regulatory authorities, should be provided with all relevant information reflecting product-specific risk characteristics to return more confidence and transparency in structured instruments, credit markets, and credit ratings as well.

Chapter 6

Valuation of Systematic Risk in the Cross-section of Credit Default Swap Spreads

The content of this chapter refers to the working paper 'Valuation of Systematic Risk in the Cross-section of Credit Default Swap Spreads' by Löhr, S., Claußen, A., Rösch, D., and Scheule, H., 2012.

6.1 Introduction

During the Global Financial Crisis (GFC) the spreads of Credit Default Swaps (CDS) heavily increased across most CDS dealings on corporate debt claims, which was triggered by high numbers of corporate defaults on bonds and loans.[141] While 31 Moody's-rated corporate issuers defaulted in 2006 on a total of 10,388 USD billion of loans and bonds, the number of defaulted issuers increased to 261 in 2009 on a total of 328,864 USD billion (Moody's 2010*b*). In fact, the CDS spreads on high-rated debt claims, e.g., 'AAA'-rated bonds, increased much more rapidly than those on lower-rated credit assets, which may indicate a mismatch between credit ratings and the related default

[141]Similar to insurance contracts, CDS – as credit derivatives – are linked to credit-risky assets such as corporate bonds, loans etc. In their role as protection seller, CDS investors periodically receive premium payments for covering losses in the underlying credit assets. These losses may be due to default events such as interest shortfalls or principal impairments, see Gandhi et al. (2012). Thus, in the absence of arbitrage, the fair CDS spread (risk premium) theoretically compensates for the default risk of the underlying credit asset.

risk.

On the corporate debt market this phenomenon takes part in the so-called credit spread puzzle which is already addressed by several authors (Amato & Remolona 2003, Hui 2010). Apart from addressing corporate default risk (Giesecke, Longstaff, Schaefer & Strebulaev 2011), several empirical studies recently looked beyond theoretical contingent claims and accounted for other pricing factors such as liquidity (De Jong & Driessen 2006, Bongaerts et al. 2011, Friewald et al. 2012). As suspected by Collin-Dufresne et al. (2001), Hui (2010) and Iannotta & Pennacchi (2011) for corporate debt, other authors also identified systematic risk factors driving CDS spreads (e.g., Amato 2005, Blanco et al. 2005, Gala et al. 2010, Gandhi et al. 2012). However, numerous theoretical as well as empirical studies find that common risk factors are especially affecting the default risk of credit derivatives, e.g., single-name CDS or baskets of CDS, and thus may also be important in pricing related dealings such as synthetic Collateralized Debt Obligations (see, e.g, Coval et al. 2009*a*).

Most of the recent studies analyze time-series properties of credit spreads or credit spread changes by focusing on time-series regressions. An exception are Friewald et al. (2012) who use Fama-Macbeth cross-sectional regressions to show that liquidity is priced in bond markets after controlling for other factors such as credit ratings. Summarizing, the current literature on both bond and CDS markets focuses on the identification of credit spread drivers and aims to answer the question how these determinants are priced.

Our paper contributes to credit spread determinants in several aspects. Firstly, we explicitly address systematic risk exposures of CDS contracts and identify at least three systematic risk factors beyond Merton's (1974) structural theory as important drivers for CDS spread changes. Thus, we suggest the *Credit Market Climate*, the *Market Volatility* and the *Cross-market Correlation* as common determinants of CDS spread changes. The latter common spread determinant is provided to indicate the effects of a global contagion across credit and stock markets.

Secondly, based on our CDS database from 2004 to 2010 containing weekly spread data of 339 U.S. firms we show that credit ratings are not sufficiently covering the overall credit risk priced in CDS spreads. We find that systematic risk is generally priced beyond the ratings of U.S. firms located in numerous economic sectors, e.g., financials, industrials and consumer goods.

Thirdly, we extend the current literature by applying a two-pass regression approach to CDS markets (compare Fama & MacBeth 1973) and thus we show that systematic risk exposures are cross-sectionally priced in swap markets.[142] In the first pass, we identify common determinants of credit spread changes and provide contract-specific sensitivities (betas) to common risk factors by time-series regressions. In the second pass, we examine by cross-section regressions how these betas are cross-sectionally priced in CDS spreads after controlling for i) several individual factors such as credit ratings, contract liquidity and firm leverage and ii) sectoral influences. Thus, we calculate premiums for these systematic risk betas, similar to the CAPM's beta premium.[143] We find that these determinants of CDS spread changes are priced across several economic sectors, particularly in times of financial distress. Especially, common risks related to the *Credit Market Climate*, the *Market Volatility* and the *Cross-market Correlation* are rewarded in the cross-section of CDS spreads, even after controlling for other important pricing elements such as credit ratings and liquidity. The results of the cross-section regressions show that our set of variables – composed of systematic and idiosyncratic risk measures – allows us to explain about 80% of the observed CDS spreads in normal market environments. Even in times of financial turmoil, our model setup achieves an explanatory power of about 90%. Furthermore, the OLS regression results are robust with respect to the inclusion of the Fama-French factors and other firm-specific factors such as the firm's leverage ratio and market capitalization. Our findings suggest that systematic risk is a decisive pricing factor, even if we control for individual risk factors and sectoral influences.

Our empirical findings are important for at least three fields. Firstly, the contributions are relevant for asset pricing as they identify variables which determine spreads of swap contracts referring to credit risky assets. While previous literature analyzes price impacts of credit ratings (e.g., Ederington & Goh 1993, 1998), we explicitly address price impacts of systematic risk in CDS spreads beyond ratings. Extending the current literature related to CDS and corporate debt, our findings are not only relevant for the valuation of CDS, but may also provide further insight into the pricing of corporate bonds, as bonds are also exposed to systematic risk, see Collin-Dufresne et al. (2001)

[142] The two-pass regression approach was first proposed by Fama & MacBeth (1973) to evaluate the cross-section of stock returns.

[143] According to the Capital Asset Pricing Model (CAPM), market participants can fully diversify firm-specific (idiosyncratic) risks, but not market (systematic) risk which is therefore compensated by a risk (beta) premium (compare Sharpe 1964).

and Hui (2010).

Secondly, the results are important for the regulation of financial markets. As pointed out by Iannotta & Pennacchi (2011), there is a mismatch between regulatory capital for banks derived from credit ratings and credit spreads, as the latter might account for systematic risk, while credit ratings do not appropriately reflect systematic risk. Current regulatory capital requirements for banks primarily focus on credit ratings, and therefore banks – or financial investors in general – are subject to misaligned incentives if systematic risk is priced: within a specific rating grade, banks may choose those investments with highest systematic risk exposures due to the higher risk premiums linked to these products. This might be a threat for financial institutions, or even the whole financial system. By providing empirical evidence for the pricing of systematic risk on CDS markets beyond ratings, our paper also contributes to this discussion.

Thirdly, our findings might be important for pricing structured finance securities such as Collateralized Debt Obligations (CDOs). Since, for example, synthetic CDOs take on credit exposures through including CDS contracts, this work may also provide first insight into the valuation of such structured products, which are particularly exposed to systematic risk (see Coval et al. 2009*a*, Rösch & Scheule 2010).[144]

The remainder of the paper is organized as follows. In Section 6.2, we provide the theoretical framework for our empirical analysis by introducing systematic and idiosyncratic spread determinants. Further, we describe the database and briefly discuss the proxies used. In Section 6.3, we firstly introduce the regression models within the two-pass approach and secondly provide the methodology to test whether corporate ratings are appropriately reflecting systematic risk. Thirdly, we provide our results and check the robustness of our findings by expanding our model framework to i) the Fama-French factors, ii) further firm-specific factors and iii) a principal component analysis. Section 6.4 concludes.

[144]Popular synthetic CDOs are credit indices such as the North American CDX and iTraxx Europe credit index families.

6.2 Determinants of Credit Default Swap Spreads

6.2.1 Theoretical Framework

Black & Scholes (1973) and Merton (1974) introduced an intuitive option-pricing framework for valuing corporate equity and debt. This structural framework by Merton (1974) provides an attractive approach to credit risk. In structural models, the default event is usually triggered when the firm's assets fall below a critical threshold.[145] The value of firm assets follows a simple random walk (firm value process) and the default threshold is a function of the amount of debt outstanding.

The values of debt claims are determined under the risk-neutral measure by computing the present value of their expected future cash flows discounted at the risk-free rate. Since a credit default swap extracts and transfers the default risk of corporate debt, CDS investors – in their role as protection seller – periodically receive a premium payment (premium leg) for covering losses in underlying debt claims (protection leg). In the absence of arbitrage and in the presence of risk-neutral valuation, the present value (PV) of the premium leg equals the PV of the protection leg. Hence, depending on the underlying debt claim future expected cash flows – namely the protection and premium payments – of the related CDS are analogously discounted to determine the fair CDS spread.[146]

Motivated by the structural framework, we uniquely define the CDS spread $S_{\vartheta,t}$ of contract ϑ at time t through 1) the price of underlying debt claims, 2) its related contractual cash flows, 3) the time-specific risk-free rate r_t, 4) common state variables $\mathcal{Y}_t$, which are affecting cross-sectionally all credit spreads simultaneously and 5) idiosyncratic state variables $\mathcal{V}_{\vartheta,t}$, which are firm-specific. Thus, we define credit spreads similarly to Collin-Dufresne et al. (2001) extended by the common state variables $\mathcal{Y}_t$. This leads to

$$S_{\vartheta,t} := S_{\vartheta,t}\left(C_{\vartheta,t}(F_{\vartheta,t}), r_t, \mathcal{Y}_t, \mathcal{V}_{\vartheta,t}\right) \tag{6.1}$$

with contractual payments $C_{\vartheta,t}$ depending on the firm value $F_{\vartheta,t}$.[147] Based

[145] Structural models were further investigated - among others - by Black & Cox (1976), Leland (1994), Longstaff & Schwartz (1995), Briys & de Varenne (1997), Gordy (2000), Collin-Dufresne & Goldstein (2001) and Gordy (2003).

[146] For more detailed information compare Amato (2005).

[147] See Collin-Dufresne et al. (2001) for more detailed information.

on this theoretical framework, credit spread changes are determined given the current values of the time-specific variables $\mathcal{Y}_t$ and $\mathcal{V}_{\vartheta,t}$, respectively. Referring to the structural framework, we may predict i) determinants of CDS spread changes, and ii) whether changes in these variables should be positively or negatively correlated with changes in the CDS spreads.

Similar to other authors, we propose some common state variables reflecting systematic risk:[148]

1. *Changes in the Spot Rate.* In theory, the static effect of a higher spot rate is to increase the risk-neutral drift of the firm value process (Longstaff & Schwartz 1995, Duffee 1998). The higher drift reduces the firm's probability of default and thus the price of related derivatives offering protection against default losses. We therefore expect that CDS spreads are negatively correlated with the risk-less interest rate.

2. *Changes in the Slope of the Yield Curve.* Independent from the structural framework, some authors argue that the interest term-structure is upon other factors mainly driven by i) the interest level and ii) the slope characteristics (Blanco et al. 2005).

   Often, the slope of the yield curve is seen as an indicator of economic wealth: while a positive slope indicates a prosperous economy, a negative one reflects expectations of an economic downturn. Hence, the CDS spread may decrease if an increasing slope of the interest curve indicates higher expected short rates, as also argued by Collin-Dufresne et al. (2001) for credit spreads.[149] By contrast, a decreasing term-structure may indicate an economic downturn leading to higher losses given default since recoveries are assumed to be positively correlated to the macroeconomy (Frye 2000, Altman 2008, Bade et al. 2011). In this way, the liquidation risk for corporate debt may be higher leading to widening CDS spreads.

3. *Changes in the Market Volatility.* Since debt claims exhibit characteristics similar to a short position in a put, it follows from the option-pricing framework that option prices increase with increasing volatility.

[148]Since systematic risk affects all market participants simultaneously, we aim to approximate this kind of risk by common risk variables. Note that state variables are generally not necessary in Merton's structural approach.

[149]Note that rising future short-term rates may lead to lower default probabilities and thus to lower CDS spreads.

Intuitively, with an increase of volatility the firm's default probability increases and thus the related CDS spread increases due to the higher default risk.

4. *Changes in the Credit Market Climate.* The *Credit Market Climate* may reflect the market view of the overall credit risk. If the global economy is turning down in line with decreasing recoveries, the weakening market conditions should increase the firms' default risk as well as related losses. Thus, the increased credit risk on credit markets may lead to an increase of the overall credit spread level. The *Credit Market Climate* can be seen as a common market factor similar to the market index in the CAPM. It should strongly be affected by economic conditions. Therefore, we expect a cross-sectional increase of default risk due to weakening economic conditions leading to increased CDS spread levels. Hence, the CDS spreads should be positively correlated with the *Credit Market Climate.*

5. *Changes in the Cross-market Correlation.* Foresi & Wu (2005) argue for equity markets that downside movements in any index are likely to be highly correlated with those in other markets as a result of global contagion. Expanding this argument to credit markets, we expect higher CDS spreads if cross-market correlations increase, because the prospects for risk diversification on global markets decrease. In turn, we expect lower CDS spreads if the dependencies across various markets – such as credit, equity, and exchange markets – decrease.

Lastly, firm-specific or idiosyncratic spread determinants are proposed and discussed individually.

1. *Physical Default Probability.* Within the structural framework, the difference between the physical probability of default (PD) and the risk-neutral PD indicates the risk aversion of market participants. Under the risk-neutral measure, the drift parameter μ of the asset value process is changed to the risk-less rate r from which it follows that the risk-neutral PD is composed of the physical PD plus a correction term accounting for the risk aversion. By controlling for the physical PD, we quantify the premium for pure default risk apart from other major determinants. In line with intuition and ceteris paribus, the higher (lower) the firm's physical PD, the higher (lower) the CDS spread should be.

2. *Swap Liquidity.* Analogous to other authors who show that liquidity is priced in credit spreads of corporate bonds, we assume that CDS investors also claim a premium compensating for liquidity risk. Transferring these empirical findings to CDS markets, the contract's liquidity is expected to determine the CDS spread. Intuitively, CDS spreads should rise if the contracts' liquidity, for example, measured by its trading volume, decreases and vice versa. Eventually, we expect a negative relationship between *Swap Liquidity* and swap spread.

6.2.2 Empirical Data

Our empirical study refers to a comprehensive data set of single-name CDS spreads provided by Markit. Overall, we analyze dollar-denoted CDS spreads of 339 U.S. American entities from January 6^{th}, 2004 to December 27^{th}, 2010.[150] By splitting the entire period into two different subsamples, we account for different market conditions before the GFC and in times of market turbulences during the GFC. Firstly, we define the period from January 6^{th}, 2004 to June 18^{th}, 2007 as time prior to the GFC (Pre-GFC). Secondly, we define the period from June 19^{th}, 2007 to December 27^{th}, 2010 as times of financial distress during the GFC.[151]

Table 6.1 summarizes the sample periods for the time-series regressions (TSR) and for the cross-sectional regressions (CSR).[152] The amount of related CDS spread observations and the number of considered entities are also denoted.

Overall, we investigate 124,413 weekly CDS spreads from 339 different issuers in the entire period, in which the number of CDS spreads per entity is 367. The Pre-GFC sample contains 180 weekly spreads per entity, which leads to 61,020 weekly observations in total. In the GFC sample, we examine 63,393 weekly CDS spreads with 187 observation per entity.

[150] The contracts' document clause is MR. The seniority is SNRFOR (senior unsecured debt). For more information compare Markit (2008). Thereby, we select only contracts which have at least 47 weekly spread notations in each year.

[151] On June 18^{th}, 2007 it is reported for the first time that Merrill Lynch seizes collateral from a Bear Stearns hedge fund invested heavily in subprime loans, which may have caused strong spread increases on credit markets over the following days.

[152] The corresponding regression models are introduced in the next section.

Table 6.1: Sample Periods of Multiple Time-series and Cross-section Regressions

Multiple Time-series and Cross-section Regressions				
Sample		Entire period	Pre-GFC	GFC
Maturity	From:	6^{th} of Jan 04	6^{th} of Jan 04	19^{th} of Jun 07
	Until:	27^{th} of Dec 10	18^{th} of Jun 07	27^{th} of Dec 10
	Amount:	339	339	339
Entities	Obs. per entity:	367	180	187
	Sum of obs.:	124,413	61,020	63,393

Notes: The table summarizes the sample maturities as well as the amount of CDS spread observations (obs.) covered by each sample. The period of the Pre-GFC reflects the time interval prior to the financial crisis and the GFC describes the time period during the crisis. Based on each sample, multiple time-series regressions as well as cross-sectional regressions are conducted.

The U.S. companies are divided over ten economic sectors, e.g., financials (16.81%), industrials (14.16%) and consumer goods (13.57%). Table 6.2 summarizes the amount of firms located in each sector and provides the sector-specific average spreads by sample.

Table 6.2: Investigated Economic Sectors

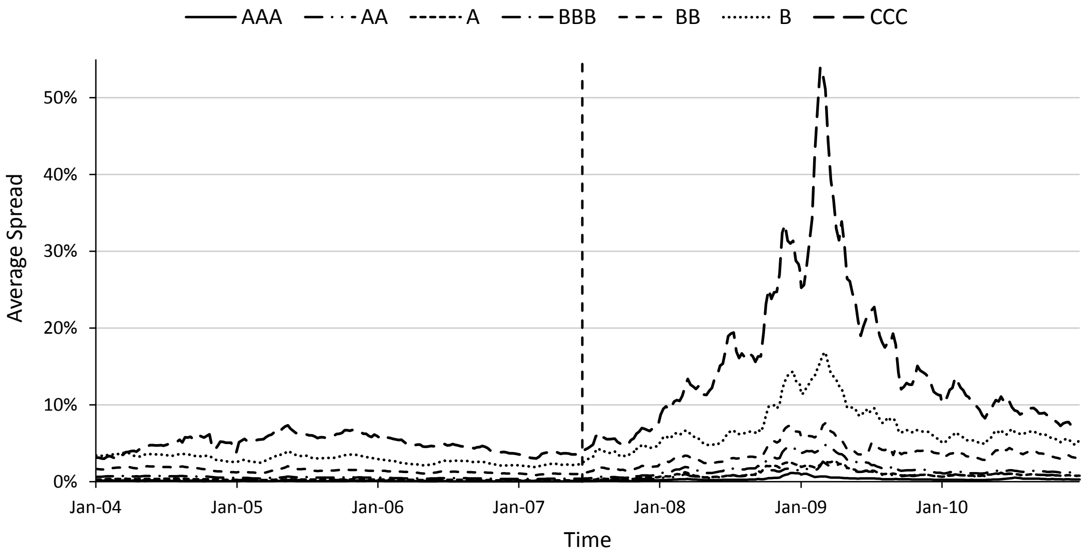
			Mean Spread		
U.S. Sector	Count	Count in %	Entire	Pre-GFC	GFC
Basic Materials	22	6.49	0.0184	0.0113	0.0253
Consumer Goods	46	13.57	0.0216	0.0113	0.0316
Consumer Services	58	17.11	0.0320	0.0162	0.0471
Financials	57	16.81	0.0220	0.0042	0.0389
Health Care	16	4.72	0.0137	0.0074	0.0198
Industrials	48	14.16	0.0123	0.0077	0.0168
Oil & Gas	29	8.55	0.0128	0.0082	0.0174
Technology	14	4.13	0.0156	0.0109	0.0202
Telecommunications	12	3.54	0.0291	0.0230	0.0349
Utilities	37	10.91	0.0119	0.0073	0.0163
Overall	339	100	0.0189	0.0107	0.0268

Notes: The table reports the amount of U.S. entities located in ten economic sectors and denotes the sector-specific mean CDS spreads by sample (Entire, Pre-GFC and GFC).

Since we investigate a wide range of U.S. American firms, we may obtain a broad insight into the cross-sectional determinants of CDS spreads. The sector-specific average spreads clearly vary by sample and even across sectors. In order to account for sector-specific influences, we implement sector dummies in our CSR model.

Furthermore, all underlying contracts of the CDS are rated on a rating scale from 'AAA' to 'CCC'.[153] In Figure 6.1, we plot the time series of average CDS spreads per rating grade from January 6^{th}, 2004 to December 27^{th}, 2010 (x-axis). The y-axis denotes the average CDS spreads.

Figure 6.1: Average Spreads by Rating



Notes: This figure shows time series of average CDS spreads for various rating grades, e.g., 'AAA', 'AA', 'A', from January 6^{th}, 2004 to December 27^{th} 2010. The spread function of 'AAA'-rated contracts (black line) is below all other spread functions since highest creditworthiness is linked to the lowest risk premium. In turn, the 'CCC'-based CDS spread function (dashed line) is located above all others. The entire sample is divided into the period prior to the financial crisis (Pre-GFC) and the GFC by the dashed vertical line.

The average spread level generally varies depending on the rating grades: the average CDS spread of 'AAA'-rated underlyings (black line) is throughout below all other grade-specific average spreads, as theoretically assumed above. By contrast, 'CCC'-rated contracts (dashed line) exhibit the highest average CDS spreads since they reflect the highest default risk. All grade-specific functions show that average spreads are rapidly increasing across all rating grades during the turmoil of the GFC.

Next, we choose the following proxies for the identified systematic state variables.

1. *Spot Rate*. The spot rate (SR) is approximated by changes in government bonds, as also suggested by other authors in the recent literature

[153]The rating scale contains average ratings referring to Moody's and S&P ratings. For more details compare www.markit.com.

(compare Blanco et al. 2005, Avramov et al. 2007).[154] We use 5-year Treasury bill rates provided by the U.S. Department of the Treasury.[155]

2. *Slope of the Yield Curve.* Analogous to Collin-Dufresne et al. (2001), among others, we define the slope of the term structure (STS) as the difference between the long-term and the short-term Treasury bill rate. To capture slope effects, we use changes in spread differences on U.S. Treasury bills with 2-year and 10-year maturity. The slope may be interpreted as an indicator of the economic health and expectations of future short rates. Respective Treasury bill rates are also provided by the U.S. Department of the Treasury.

3. *Market Volatility.* As benchmark for the *Market Volatility*, we assume the VIX index provided by the Chicago Board Options Exchange. The VIX measures market expectation of near-term volatility conveyed by stock index option prices.[156] By using a wider range of strike prices rather than just at-the-money series, the VIX index is additionally incorporating information from the volatility 'skew'. Thus, the VIX may not only reflect investors' consensus view of future expected stock market volatility: since out-of-the money put options as well as in-the-money call options are considered for short maturities, the index may also be seen as an indicator for negative jumps in the S&P 500 index causing investors' fear. According to Collin-Dufresne et al. (2001), an increasing probability and magnitude of large negative jumps in the firm value should rise credit spreads, and thus CDS spreads (Blanco et al. 2005).

4. *Climate of Credit Markets.* As S&P 500 index returns are suggested to approximate the overall state of the economy (see Collin-Dufresne et al. 2001, Blanco et al. 2005), we analogously assume the index spread changes of the 5-year (5Y) CDX NA IG credit index (CDX) as proxy

[154] However, due to several reasons, e.g., taxation treatment, scarcity premiums and benchmark status issues, it is often criticized that government bonds are no ideal proxy for the unobservable risk-free rate. In this concern, 5-year swap rates for dollars and euros are often proposed as a better proxy. For an insightful discussion see Blanco et al. (2005). We also incorporate corresponding swap rates for robustness.

[155] Other maturities such as 1 year, 2 years and 10 years are also investigated, but not reported since they lead to similar results.

[156] The VIX uses a weighted average of options with a constant maturity of 30 days to expiration. The options refer to the S&P 500 index.

for the credit market conditions. The CDX is one of the most popular CDS indices covering a cross-sectoral basket of the 125 most liquid North American (NA) investment grade (IG) single-name CDS.[157] Index spreads of the CDX are provided by Markit.

5. *Cross-market Correlation.* We consider the average of quarterly cross-correlations referring to returns on numerous i) exchange, ii) equity and iii) credit markets. In this context, we suggest some indices to calculate the applied *Cross-market Correlation* (CMC), e.g., S&P 500, DAX 30, 5Y CDX NA IG, Dow Jones Industrial Average, Nikkei 225.

Figure 6.2 shows the times series of the systematic state variables from January 6^{th}, 2004 to December 27^{th}, 2010 (x-axes). The y-axes denote the states of the respective proxies. The dashed vertical lines divide the entire sample period into the samples Pre-GFC and GFC.

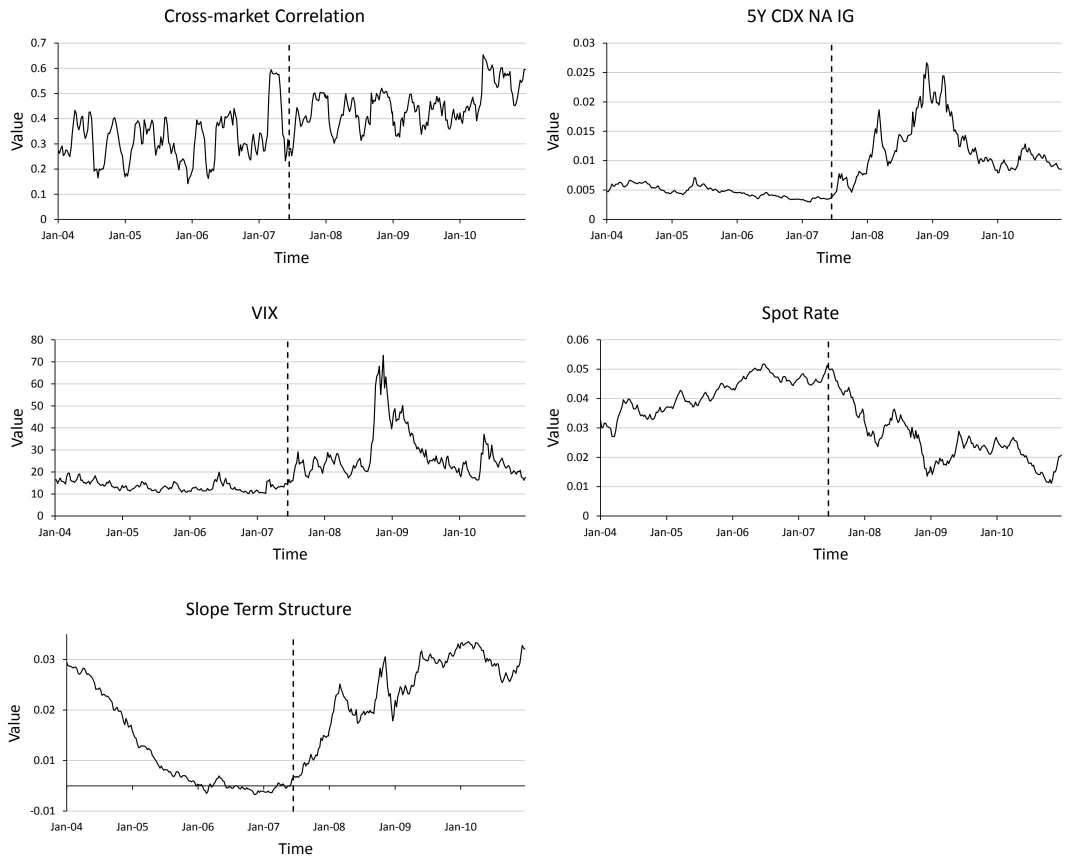
Time series of the *Cross-market Correlation* (upper-left chart) fluctuated within the entire period in a moderate range between 0.13 (min) and 0.63 (max) with mean 0.36 and standard deviation (STD) 0.09.

As intuitively expected, the index spread of the CDX (upper-right chart) was moving sideways with relatively low volatility before the GFC. Indeed, during the GFC the volatility of the CDX strongly increased as well as its spread level. While its mean was denoted at almost 47 basis points (bp), its STD was at 9.8 bp prior to the crisis. In contrast to the Pre-GFC, the mean of the CDX was three times higher (136 bp) during the GFC, while its STD was six times higher (59 bp). The maximum spread was observed at the end of 2008 denoting at 280 bp, the minimum spread of 29 bp in January 2007, a few months before the GFC began.

The VIX index (mid-left chart) moved sideways from January 2004 until June 2007 with moderate volatility (index mean 13.6 and STD 2.2), increased clearly in the beginning of the GFC and reached its historical peak at around 80.9 in December 2008. Similarly to the other systematic risk factors, the mean of the VIX was clearly higher in times of the crises (2.3 times higher) than in moderate economic conditions and also its related STD (6.1 times higher). In the beginning of 2009, the VIX index turned clearly back on the index level reached in January 2008.

[157] The composition of the basket is fixed until maturity and included CDS contracts are equally weighted. For a detailed description of the numerous CDX indices refer to www.markit.com.

Figure 6.2: Time Series of Systematic State Variables



Notes: This figure shows time series of systematic state variables from January 6^{th}, 2004 to December 27^{th}, 2010. The following proxies for the state variables are plotted: the *Cross-Market Correlation* refers to the average cross-correlation across several market indices (upper-left). The time series of the 5Y CDX IG index spread represents the *Credit Market Climate* (upper-right). The *Market Volatility* is indicated by the time series of the VIX index (mid-left). The *Spot Rate* is approximated by the 5Y T-bill rate (mid-right). For the *Slope of The Term Structure*, we present time series of the difference between the 10Y and the 2Y T-bill rate (lower-left). The dashed vertical lines divide the entire sample into the two sub-samples (Pre-GFC and GFC).

The *Spot Rate* in terms of the 5-year Treasury bill rate (mid-right chart) was about 3% in January 2004, moved around 5% in June 2007 and then decreased rapidly to 1.5% in 2009 due to the market turbulences on the credit markets.

A decreasing *Slope of the Term-structure* (lower-left chart), which we observed before the GFC began in June 2007, indicates expectations of an economic downward movement (compare BIS 2009*a*). Increasing slope values as observed during the turmoil on financial markets, in turn, may have predicted an economic up-turn in the aftermath.

Eventually, the time series show that each systematic risk factor moves clearly different before the GFC than during the financial turmoil, as it is indicated by the factors' period-specific means and standard deviations. Basically motivated by the chronology of the GFC, the determination of our subsamples is also confirmed by both CDS spread descriptives and time-series analysis of the systematic state variables.

The correlation matrix in Table 6.3 refers to changes (Δ) in the systematic state variables identified above and reflects the linear dependency structure across these changes. The upper triangle of the matrix refers to correlations in the Pre-GFC and the lower triangle shows cross-correlations in the crisis.

Table 6.3: Sample-specific Correlation Matrix of Common Risk Factors

		Pre-GFC				
		ΔCMC	ΔCDX	ΔVIX	ΔSR	ΔSTS
	ΔCMC		0.0571	0.264	-0.0254	0.0726
	ΔCDX	0.0441		0.4116	-0.1067	-0.0734
GFC	ΔVIX	0.0673	0.6382		-0.1656	0.0219
	ΔSR	-0.0534	-0.4530	-0.3158		0.1949
	ΔSTS	-0.1589	0.0522	0.1152	0.1021	

Notes: The table shows the cross-correlations related to changes (Δ) in five systematic risk variables, namely the *Cross-market Correlation* (CMC), the CDX index, the VIX index, the *Spot Rate* (SR), and the *Slope of the Term Structure* (STS). While the upper triangle of the matrix refers to the cross-correlations of the Pre-GFC, the lower triangle shows the correlations of the GFC.

According to Table 6.3, the proxy for the *Cross-market Correlation* and the proxies for the interest risk – *Slope of the Term Structure* and *Spot Rate* – exhibit the lowest overall Δ-dependencies on the other systematic risk factors in both samples. Table 6.3 also shows that the dependencies generally increase during the GFC. Nevertheless, most cross-correlations denote at low levels (about 0.10). We observe the highest correlation between the VIX and the CDX with 0.41 before the GFC and 0.64 during the financial crisis.[158]

In the following, proxies for idiosyncratic or individual risk are provided.

1. *Physical Probability of Default.* Since a credit rating generally reflects an opinion of the obligor's creditworthiness, the highest-rated obligors ('AAA'-rated) are assumed to exhibit the lowest probability of default (PD), while lowest rated ones ('C'-rated) exhibit the highest PD. Credit-rating agencies (CRA), for example, link their classical rating grades (ordinal scaled) to historical default rates of corporate bonds (Moody's 2010*b*).[159] Hence, we use average ratings provided by Markit as proxy

[158]Even if the correlations are solely moderate, the presence of multi-collinearity in systematic state variables may generally distort the interpretations of regression results.

[159]Referring to the three major rating agencies – Moody's, S&P and Fitch – the rating grades are monotonically increasing with the obligor's creditworthiness (compare Moody's 2012).

for the firm's physical default risk, similarly to Friewald et al. (2012).[160]

As already examined by Abid & Naifar (2006), we assume that the absolute CDS spread level is determined by the related obligor rating. The worse the rating of the obligor the higher is the CDS spread level and vice versa, all else equal. For simplicity, we apply a shortened rating scale which summarizes the available rating metrics. In our cross-sectional regressions, we account for five different rating classes RC_1 to RC_5, where the latter indicates lowest creditworthiness and RC_1 highest.[161]

CRAs such as Moody's and S&P's provide ratings that are rather stable through business cycles (through-the-cycle ratings), see Moody's (1999) and S&P's (2008). Thus, macroeconomic point-in-time information is rather neglected in such a through-the-cycle approach (Moody's 1999).[162] Since CRAs mainly address firm-specific risks in their rating metrics rather than states of the global economy (common risk) (S&P's 2008), we consider credit ratings primarily as proxy for idiosyncratic risk.

2. *Swap Liquidity.* As proposed by Gala et al. (2010) and Gandhi et al. (2012), we incorporate the contract's amount of trades (trading depth) to proxy its liquidity. The data were provided by Markit and denoted as *Swap Liquidity* (LIQ).

Overall, each single-risk proxy i) reveals for itself significant explanatory power in respective univariate regressions, ii) significantly contributes to the explanation of the endogenous variable in our CSR and iii) has the power to innovate. Especially the latter condition is important in terms of multicollinearity: all of our systematic risk factors provide for themselves additional explanatory power, even if considered lastly in a normalized regression

[160] Recall that Markit's average ratings are based on available Moody's and S&P ratings, see www.markit.com

[161] Due to the number of available ratings, RC_1 includes rating grades 'AAA', 'AA' and 'A' , RC_2 reflects 'BBB' ratings, RC_3 accounts for rating grade 'BB', RC_4 refers to 'B' ratings and RC_5 to 'CCC' ratings. The data set contains only one 'AAA'-rated and six 'AA'-rated entities. Even if we exclude these entities from our regression approach and thus solely group 'A'-rated firms in RC_1, we obtain similar regression results.

[162] More detailed information to CRAs and their rating systems can be found in Krahnen & Weber (2001) or in Löffler (2004, 2012).

framework.[163] In other words, the explanatory power of each proxy is i) not completely covered by the ensemble of other regressors, independent of the introduction order, and ii) its explanatory power is not the product of the entire ensemble.

6.3 Empirical Evidence for Pricing Systematic Risk in CDS Spreads

6.3.1 Models in the Two-pass Regression Approach

In the first step of our two-pass regression procedure (compare Fama & MacBeth 1973), we estimate the CDS spread sensitivities (betas) to the proposed systematic state variables by multiple time-series regressions (TSR). For each CDS referring to entity $\vartheta \in \{1, ..., 339\}$ with CDS spread $S_{\vartheta,t}$ at time t we estimate the following time-series regression model, which was methodologically proposed by Collin-Dufresne et al. (2001) for credit spreads and also applied by Ericsson et al. (2009) and by Friewald et al. (2012).

$$\begin{aligned}\Delta S_{\vartheta,t} = \alpha_\vartheta + \beta_\vartheta^{CMC} \cdot \Delta CMC_t + \beta_\vartheta^{CDX} \cdot \Delta CDX_t + \beta_\vartheta^{VIX} \cdot \Delta VIX_t \\ + \beta_\vartheta^{SR} \cdot \Delta SR_t + \beta_\vartheta^{STS} \cdot \Delta STS_t + \varepsilon_{\vartheta,t}.\end{aligned} \tag{6.2}$$

$\Delta S_{\vartheta,t}$ denotes the spread change of the contract related to firm ϑ at time t. α_ϑ describes the intercept, $\beta_\vartheta^{(\cdot)}$ denotes the coefficients of included regressors, Δ refers generally to changes in the state variables and $\varepsilon_{\vartheta,t}$ is the residual.[164]

In the second step, we examine the cross-section of CDS spreads by cross-section regressions, similarly to Friewald et al. (2012) who apply this type of regression to corporate bond spreads. Thus, our TSR beta estimates are used as regressors in the cross-sectional regression (CSR), along with additional

[163] The order of regressors does not matter in basic OLS regressions, but within OLS regressions based on normalized regressors which were additionally conducted for robustness. Within a normalized framework, the regressors are corrected for observable co-variances. In the end, the regressors' covariance matrix is a diagonal matrix with variances equal to one. The regressors' means are also standardized and equal null. The results accounting for multi-collinearity are not separately reported since they solely confirm the presented findings.

[164] Linkage of shortened declarations according to Section 6.2.2: *Cross-market Correlation* (CMC), *Credit Market Climate* (CDX), *Market Volatility* (VIX), *Spot Rate* (SR) and *Slope of the Term Structure* (STS).

variables such as the proposed individual risk factors. In the basic model setup, we consider the firm's ratings and the contract's liquidity. Additionally, we account for firm-specific sectoral influences by sector dummies. In Section 6.3.4, we add further idiosyncratic risk factors, e.g., the firm's *Leverage Ratio* and *Market Capitalization*, as well as further systematic risk betas related to the Fama-French factors in order to check the robustness of our findings.

After calculating the entities' average CDS spreads $\overline{S}_\vartheta$ by sample, we estimate the following cross-section regression model for each sample

$$\begin{aligned}\overline{S}_\vartheta = \alpha + \gamma^{CMC} \cdot \hat{\beta}_\vartheta^{CMC} + \gamma^{CDX} \cdot \hat{\beta}_\vartheta^{CDX} + \gamma^{VIX} \cdot \hat{\beta}_\vartheta^{VIX} + \gamma^{SR} \cdot \hat{\beta}_\vartheta^{SR} \\ + \gamma^{STS} \cdot \hat{\beta}_\vartheta^{STS} + \gamma^{LIQ} \cdot LIQ_\vartheta + \boldsymbol{\gamma}^{RC} \cdot \boldsymbol{RC}_\vartheta + \boldsymbol{\gamma}^{SI} \cdot \boldsymbol{SI}_\vartheta + \varepsilon_\vartheta^{CS}, \quad (6.3)\end{aligned}$$

where $\varepsilon_\vartheta^{CS}$ denotes the cross-sectional residual. $\hat{\beta}_\vartheta^{(\cdot)}$ denotes the parameter estimates of TSR regressors. LIQ denotes the swap's average liquidity, **RC** and **SI** represent the firm-specific *Rating Classes* and *Sector Indicators*, respectively, which are included as dummy variables.[165] α denotes the intercept and $\gamma^{(\cdot)}$ are the cross-sectional slope parameters. $\boldsymbol{\gamma}^{RC}$ and $\boldsymbol{\gamma}^{SI}$ represent vectors of estimators referring to the sector-specific and rating-specific dummy variables.

Table 6.4 gives a brief regressor overview and shows the predicted signs of coefficients related to the TSR and the CSR in line with the theoretical expectations presented in Section 6.2.1.

Table 6.4: Overview of Common Risk Factors and Predicted Signs

Variable	Description	Predicted Sign β (TSR)	γ (CSR)
	Systematic Risk Factors in Time-series Regressions		
ΔCMC	Change in the Cross-market Correlation	+	+
ΔCDX	Change in CDX index spread	+	+
ΔVIX	Change in implied volatility of S&P 500	+	+
ΔSR	Change in yield on 5-year Treasury yield	-	-
ΔSTS	Change in 10-year minus 2-year Treasury yield	-	-
	Idiosyncratic Risk Factors in the Cross-section Regression		
LIQ_ϑ	Liquidity of CDS Contract		-
RC_ϑ	Rating Dummy for Class 1 to 5		+
SI_ϑ	Sector Indicator for Sector 1 to 10		

Notes: The table shows included regressors of both the multiple time-series regressions (TSR) and the cross-section regressions (CSR). The predicted signs for the respective regression coefficients of the TSR (β) and CSR (γ) are also denoted.

[165] RC_1 (SI_1) represents the reference rating class (sector) included in the intercept.

For example, the estimates of the TSR which refer to changes in the CDX credit index should be positive since in theory an increase of the CDX index spread should commonly widen the CDS spreads. Alternatively, for the LIQ we expect a negative relationship to the CDS spread. Hence, an increase of the swaps's liquidity should lead to a decrease of the CDS spread and vice versa.

6.3.2 Systematic Risk Beyond Ratings

Firstly, we examine whether the firms' ratings have cross-sectional explanatory power with respect to the CDS spreads of 339 entities. Table 6.5 reports the regression results of rating-based CSRs.

Table 6.5: Cross-section Regressions by Rating Dummies

	Entire Period	Pre-GFC	GFC
Intercept	0.0065***	0.0025*	0.0103***
	(0.0017)	(0.0013)	(0.0028)
BBB-rated	0.0039*	0.0027*	0.005⁻
	(0.0021)	(0.0016)	(0.0034)
BB-rated	0.0217***	0.0116***	0.03***
	(0.0027)	(0.0021)	(0.0043)
B-rated	0.0461***	0.0252***	0.0676***
	(0.0031)	(0.0023)	(0.0049)
CCC-rated	0.0741***	0.0493***	0.1111***
	(0.0048)	(0.004)	(0.0077)
R^2	59.29%	45.29%	55.39%
No. Entities	339	339	339

Notes: The table summarizes the rating-based results of cross-section regressions. The parameters are statistical significant at the 1%-level (***), the 5%-level (**) and the 10%-level (*). R^2 denotes the coefficient of determination. The number of entities (No. Entities) reflects the amount of entities considered in the cross-section regressions.

The intercept includes the reference rating class RC_1 referring to 'AAA', 'AA' and 'A' ratings. Thus, the intercept represents the basic spread level of swap contracts related to high-rated obligors. Furthermore, Table 6.5 shows that the worse the firm's rating the higher is the general risk premium for that CDS contract, which is in line with our expectations. The risk premiums seem to be higher in the financial crisis than prior to the GFC, which holds across all rating classes. The results show that firm ratings represent relevant information for pricing swap contracts cross-sectionally. A comparison of R^2 based on a comparable number of observations (see Table 6.3) indicates that ratings may explain more of the spread variation in times of financial distress than in moderate economic conditions (45.29% vs. 55.39%). These results may also indicate that market participants, who were involved in pricing swap contracts, relied more intensively on ratings during the GFC than prior to the crisis. Since we observe sample-specific differences in the rating-based spread level, which are 'averaged' out over the entire period, we focus our empirical study in the following solely on the two samples Pre-GFC and GFC.

A simple preliminary test to examine whether CDS spreads are reflecting systematic risk beyond the risks reflected by CRA ratings is to compare the rating-based mean CDS spreads of contracts having different sensitivities to systematic risk.[166] Similarly to Iannotta & Pennacchi (2011), we conduct univariate TSRs for each systematic regressor by rating class ($RC_1 - RC_5$). For each sample period, the sensitivity to systematic risk is measured by the regressor's beta. CDS contracts exhibiting systematic risk sensitivities above or equal to the sample median are defined as contracts with high systematic risk exposures and thus attributed to Portfolio 1. Contracts with sensitivities below the sample median are attributed to Portfolio 2 (low systematic risk exposures). Afterwards, the portfolios' mean spreads are calculated by rating class and tested for equality (t-test).

Table 6.6 reports the median betas (β), the portfolio-specific mean spreads and the t-test results for each systematic risk factor, rating class and sample.

The median betas are monotonically increasing with rating classes. Hence, contracts related to the worse credit rating exhibit the highest betas to systematic risk. This may be due to the increase in the rating-specific spread level. Thus, swap contracts of bad-rated firms may not necessarily exhibit the highest sensitivities to systematic risk. Although the sensitivities to systematic risk are not comparable across rating classes, the contracts' systematic risk sensitivities vary widely around the median beta within each rating class. This indicates that systematic risk exposures are underestimated in parts by CRAs. This finding holds for all regressors and both samples.

Eventually, most of the portfolio-specific mean spreads are significantly differing from each other in each sample and across all regressors. In most cases, we observe higher average spreads for portfolios composed of high risk contracts.[167] From these empirical findings we conclude that CDS with higher

[166] Iannotta & Pennacchi (2011) provide a similar test for credit spreads on corporate bonds.

[167] The order of sensitivities is descending for each rating class. Thus, we observe the highest systematic risk concentration in *Portfolio 2*, if the regressor's sensitivity to systematic risk is expected to be negative, as it is the case in terms of the *Spot Rate*. Again, the higher interest risk sensitivity leads in average to higher CDS spreads.

Table 6.6: Systematic Risk Indication by Rating Class

		Pre-GFC			GFC		
			Mean CDS Spread			Mean CDS Spread	
	Rating Class	Median Beta	Portfolio 1 (above median)	Portfolio 2 (below median)	Beta Median	Portfolio 1 (above median)	Portfolio 2 (below median)
CMC	1	$\overset{+}{<}0.0001$	0.0024***	0.0025	0.0009	0.0128***	0.0079
	2	0.0001	0.0059***	0.0044	0.0014	0.0182***	0.0125
	3	0.0002	0.014	0.0141	0.0058	0.0518***	0.0293
	4	0.0014	0.0383***	0.0170	0.0187	0.1002***	0.0557
	5	0.0037	0.078***	0.0255	0.0228	0.1315***	0.1112
CDX	1	0.2074	0.0029***	0.0020	0.3926	0.0154***	0.0055
	2	0.4223	0.0063***	0.0041	0.5809	0.0206***	0.0102
	3	11,635	0.0194***	0.0090	11,506	0.0606***	0.0208
	4	16,865	0.0385***	0.0168	23,389	0.0984***	0.0574
	5	23,129	0.0781***	0.0254	46,139	0.1571***	0.0856
VIX	1	$\overset{+}{<}0.0001$	0.0029***	0.0020	0.0001	0.0154***	0.0055
	2	$\overset{+}{<}0.0001$	0.0062***	0.0042	0.0001	0.0201***	0.0107
	3	0.0001	0.0189***	0.0094	0.0002	0.0561***	0.0251
	4	0.0001	0.0384***	0.0168	0.0005	0.0977***	0.0582
	5	0.0002	0.0609***	0.0426	0.0011	0.1416***	0.1011
SR	1	-0.0144	0.0024***	0.0025	-0.1502	0.0054***	0.0150
	2	-0.0235	0.0043***	0.0060	-0.2388	0.0096***	0.0206
	3	-0.0632	0.0089***	0.0191	-0.5638	0.0186***	0.0614
	4	-0.0817	0.0219***	0.0333	-0.9909	0.0514***	0.1045
	5	-0.1624	0.0472***	0.0563	-15,973	0.1209	0.1219
STS	1	-0.0076	0.0026***	0.0023	0.0025	0.0117***	0.0090
	2	-0.0075	0.0056***	0.0047	-0.0210	0.0141***	0.0164
	3	-0.0046	0.0156***	0.0126	0.0177	0.0413*	0.0394
	4	0.0501	0.0322***	0.0231	-0.0470	0.0648***	0.0911
	5	-0.0400	0.0472***	0.0563	-0.2102	0.1416***	0.1011

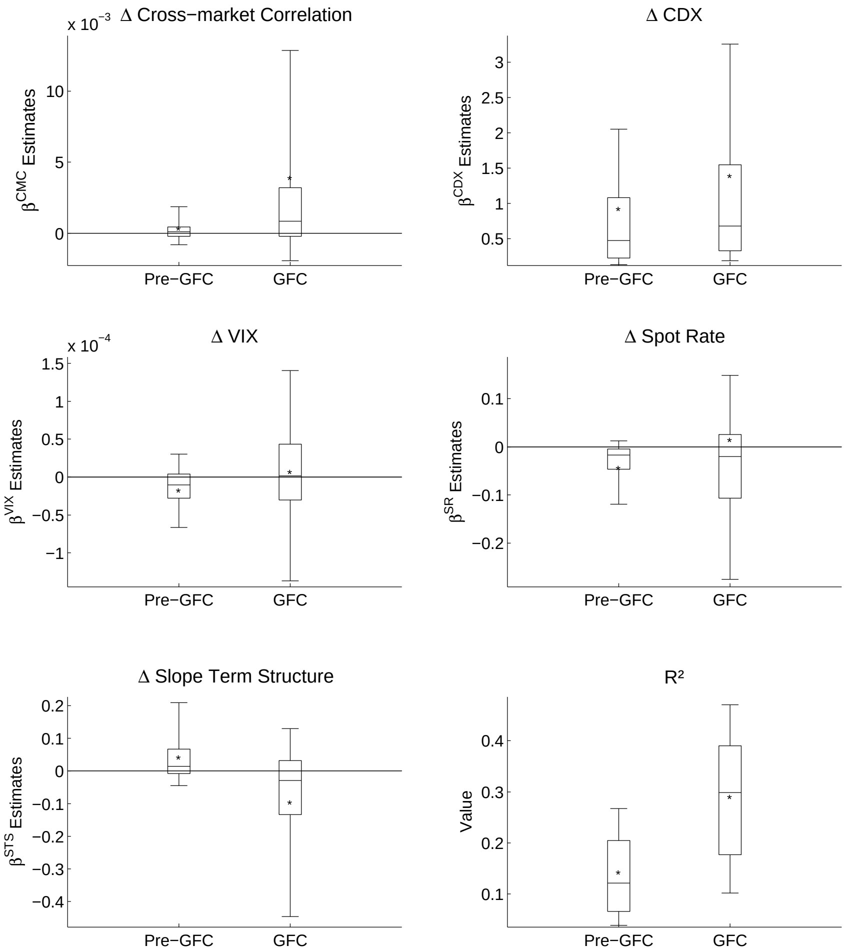
Notes: This table reports mean CDS spreads per rating class depending on the sensitivity of CDS spread changes to five systematic risk factors: *Cross-market Correlation* (CMC), CDX index , VIX index, *Spot Rate* (SR) and *Slope of the Term Structure* (STS). Univariate regressions are conducted on CDS contracts in order to evaluate the median sensitivity (*Median Beta*) to the systematic risk proxies in each rating class. Afterwards portfolios are established in dependence on estimated betas. Portfolio 1 contains all CDS with betas above the median, while Portfolio 2 includes those with betas below the median. The results are reported for both samples, the Pre-GFC and the GFC. ***, **, and * indicate the statistical significance (1%-, 5%-, and 10%-level) of the t-test for equality of mean CDS spreads for contracts with beta estimates below and above the median.

systematic risk exposures are in general higher priced and that this systematic risk is not sufficiently reflected by the related credit rating.

6.3.3 Results of Time-series and Cross-section Regressions

Figure 6.3 shows boxplots summarizing the estimation results of multiple time-series regressions across 339 entities by regressor and by sample. All of the state variables in regression (6.2) have some ability to explain changes in the CDS spreads. Further, the signs of the estimated coefficients mostly correspond with our rationale.

Figure 6.3: Estimation Results of Time-series Regressions



Notes: This figure provides boxplots referring to estimates of time-series regressions for the Pre-GFC and the GFC. Additionally, the lower-right chart shows boxplots related to the coefficients of determination (R^2). In each boxplot, the upper whisker$^+$ refers to the 90 percentile, while the lower whisker$^-$ refers to the 10 percentile. Asterisks denote the means.

With respect to the GFC (right boxplot in each chart), the regression results show that signs of estimates agree on average with our expectations, except the betas of the *Spot Rate* proxy. These beta estimates are expected to be negative, which is on average only fulfilled prior to the GFC.[168] Hence, in the pre-crisis the SR corresponds to expectations and thus an increase in the SP tends on average to a decrease of CDS spreads across all firms. In times of financial distress, the beta estimates of the *Cross-market Correlation* are on average positive and thus CDS spreads tend to increase with increasing market

[168]While the median beta is negative, the mean beta is positive due to a few outliers.

correlation. Coefficients of the slope proxy (SMT) are mainly negative during the *GFC*. As suggested in theory, positive expectations of the economic health leads to a decrease in CDS spreads across most of the firms. We find further that the betas of the CDX index spread changes are positive throughout all samples. As theoretically expected, there is a positive relationship between CDX spread changes and CDS spread changes.

Regarding the Pre-GFC (left boxplot in each chart), the signs of betas correspond on average to theory except in case of the VIX and the STS. In terms of the VIX (STS), the respective beta estimates are on average negative (positive) before the crisis and thus contrary to our rationale.[169] Analogous to the empirical findings of Longstaff & Schwartz (1995), Duffee (1998) and Blanco et al. (2005) for credit spread changes, we find that an increase in the risk-free rate (SR) lowers the CDS spread for at least 75% of the firms prior to the crisis.

Similarly to other empirical studies (Collin-Dufresne et al. 2001, Ericsson et al. 2009, Friewald et al. 2012), the coefficient of determination R^2 ranges in average between 14.37% and 29.08%, as shown in the lower-right chart of Figure 6.3. We find that the explanatory power of our applied systematic risk factors depends on the sample period. Our systematic state variables explain CDS spread changes much better in times of market turbulences than in moderate market conditions. This finding may justify the selection of our proxies for systematic state variables.

By contrast, most recent studies (e.g., Collin-Dufresne et al. 2001, Ericsson et al. 2009) primarily consider idiosyncratic risk factors in their TSR, but do not provide a cross-sectional spread examination, except Friewald et al. (2012). In our two-pass approach such individual risk factors are methodically omitted in the TSR (pass one), but explicitly considered in the CSR (pass two).[170] Nevertheless, we achieve comparable explanatory power in our first pass by focusing on systematic risk factors.

In the second step, we run the cross-sectional regressions – see Equation (6.3) – according to our two-pass regression methodology to identify significant cross-sectional pricing factors and their specified weights or spread premiums

[169]Note that there are still entities whose beta estimates meet our rationale, but not on average.

[170]We are explicitly targeting at the product's sensitivity to systematic risk based on weekly data points. To avoid distortions due to rather time-constant firm-specific risk factors such as the firm ratings or corporate debt, we omit these factors in the first pass.

(γ) in pricing CDS.

Table 6.7 shows the gamma estimates of the CSR for the two samples (Pre-GFC and GFC). While the left column in each sample reports results without

Table 6.7: Table of Cross-section Estimates

	Pre-GFC		GFC	
Intercept	0.0098$^{***/***}$	0.0095$^{***/***}$	0.0165$^{***/***}$	0.0164$^{***/***}$
	(0.0014/0.0016)	(0.0017/0.0016)	(0.0033/0.0021)	(0.0036/0.002)
CMC	2.4531$^{***/***}$	2.5994$^{***/***}$	0.3227$^{***/**}$	0.3335$^{***/**}$
	(0.2492/0.8104)	(0.2455/0.8938)	(0.0673/0.1546)	(0.0678/0.159)
CDX	0.0074$^{***/***}$	0.0075$^{***/***}$	0.0127$^{***/***}$	0.0126$^{***/***}$
	(0.0004/0.0015)	(0.0004/0.0015)	(0.0006/0.0014)	(0.0006/0.0015)
VIX	41.1653$^{***/***}$	41.1929$^{***/**}$	33.0365$^{***/***}$	32.8229$^{***/***}$
	(5.7257/15.8017)	(5.6241/16.4716)	(3.6158/8.0606)	(3.674/8.5242)
SP	0.0072$^{*/-}$	0.0077$^{**/-}$	-0.0144$^{***/***}$	-0.0144$^{***/***}$
	(0.0039/0.0104)	(0.0039/0.0108)	(0.0018/0.003)	(0.0019/0.003)
STS	-0.0002$^{-/-}$	0.0002$^{-/-}$	-0.009$^{***/***}$	-0.0093$^{***/***}$
	(0.0013/0.0061)	(0.0013/0.0062)	(0.0012/0.0022)	(0.0012/0.0023)
LIQ	-0.0008$^{***/***}$	-0.0008$^{***/***}$	-0.0021$^{***/***}$	-0.0019$^{***/***}$
	(0.0001/0.0001)	(0.0001/0.0001)	(0.0004/0.0003)	(0.0004/0.0003)
BBB-rated	-0.0001$^{-/-}$	-0.0002$^{-/-}$	0.0027$^{-/***}$	0.0029$^{*/***}$
	(0.001/0.0003)	(0.001/0.0004)	(0.0017/0.0005)	(0.0017/0.0005)
BB-rated	0.0017$^{-/*}$	0.0012$^{-/-}$	0.0122$^{***/***}$	0.0129$^{***/***}$
	(0.0013/0.001)	(0.0013/0.0011)	(0.0022/0.001)	(0.0023/0.0012)
B-rated	0.0077$^{***/***}$	0.0066$^{***/***}$	0.0258$^{***/***}$	0.0258$^{***/***}$
	(0.0016/0.0014)	(0.0016/0.0015)	(0.0028/0.0023)	(0.003/0.0024)
CCC-rated	0.0125$^{***/***}$	0.0108$^{***/***}$	0.0445$^{***/***}$	0.0457$^{***/***}$
	(0.0028/0.0032)	(0.0028/0.0034)	(0.0043/0.0051)	(0.0045/0.0056)
Sector Dummies	No	Yes	No	Yes
R^2	81.70%	83.18%	89.89%	90.18%
No. Entities	339		339	

Notes: This table shows the estimation results referring to the cross-section regressions (CSR) of Equation (6.3) under consideration of both systematic and idiosyncratic risk factors. Systematic risk factors are the *Cross-market Correlation* (CMC), the CDX index, the VIX index, the *Spot Rate* (SR) and the *Slope of the Term Structure* (STS). Idiosyncratic risk factors are represented by the *Swap liquidity* (*LIQ*) and the firm's rating. Sector dummies account for the sector in which the firm is operating. The results are provided for each subsample based on weekly CDS spread data. The parameters are statistical significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). Values in parenthesis describe the parameters' standard deviation (STD). Shanken-corrected STDs and significances are separated by a slash. R^2 denotes the coefficient of determination. The number of entities (No. Entities) refers to the amount of entities considered in the CSR.

sector dummies, the right column shows results under consideration of sector dummies which account for sectoral influences. Standard deviations are reported in parentheses and significance levels are marked with asterisks. In contrast to the additional individual variables in the CSR, which are deterministic, the betas of our systematic state variables are statistically estimated. Thus, they are generally stochastic and hence possibly misspecified. To account for related parameter estimation errors, we also report corrected standard deviations and corrected significances for the gamma estimates, as suggested by

Shanken (1992).[171]

While the TSR estimates indicate the firm-specific sensitivity to the systematic risk factors, the CSR estimates may be interpreted as average pricing weights for the systematic risk factors across all CDS spreads. We find that mostly the CSR estimates are significantly differing from null. Thus, TSR estimates are either positively ($\gamma > 0$) or negatively ($\gamma < 0$) priced. For example: given a positive beta (β), we observe with respect to a positive gamma ($\gamma > 0$) that the CDS spread increases if the firms sensitivity to that common risk factor increases.

In times of financial distress (GFC), all systematic risk sensitivities (TSR-betas) exhibit significant explanatory power to the cross-section of CDS spreads, even if the standard deviations of gamma estimates are Shanken corrected (separated by slash). This means that the contracts' sensitivities to the systematic risk proxies are significantly priced in CDS contracts across all economic sectors. Thereby, the signs of all gamma coefficients correspond to our economic expectations.

In the pre-crisis, we observe a slight mismatch between theory and empirical findings with respect to the interest risk proxies. Prior to the GFC, the gamma estimates indicate that a higher sensitivity to the *Spot Rate* leads to a spread increase, but the corrected t-statistic shows that these estimates are not significantly priced. The *Slope of the Term Structure* also lacks statistical significance in the pre-crisis, but becomes statistically significant in the GFC. Therefore, we conjecture that market participants view the STS as an indicator of economic wealth, which is particularly priced in economic downturns, but less relevant in moderate economic conditions.

Particularly gammas of the CDX, the CMC and the VIX reach high statistical significance in both samples, even if we control for idiosyncratic risks, sector dummies and Shanken-corrected t-statistics. Thus, market participants seem to demand a positive risk premium depending on the *Cross Market Correlation*, the *Credit Market Climate* and the *Market Volatility*, independent from the sample period.

As found in previous literature, liquidity is also an economically and statistically significant pricing determinant, which is contract-specific. The estimates of the *Swap Liquidity* are statistically significant across all samples. According

[171]The Shanken corrections are separated by slash. For a thorough description of the applied correction procedures compare Shanken (1992), Shanken & Zhou (2007).

to our rationale, market participants claim a risk premium for the market liquidity of the CDS. Thus, in the cross-section an increase of the *Swap Liquidity* leads to a decrease of swap spreads and vice versa.

Regarding the rating classes, our empirical results confirm our expectations and show that the CDS spreads monotonically increase with decreasing firm rating. Again, we observe a strong increase of basic spread levels across all rating classes during the GFC. This general increase in CDS spreads may be particularly due to extremely high default rates of investment grade bonds in this period, which may have caused many rating downgrades of these financial instruments as well. Hence, we suspect that the firms' rating information significantly determines the CDS spread levels across both samples, correction methods and swap contracts. As expected, we conclude that a high-rated firm may benefit from its higher creditworthiness by receiving a reduction in its CDS spread (lower spread level).

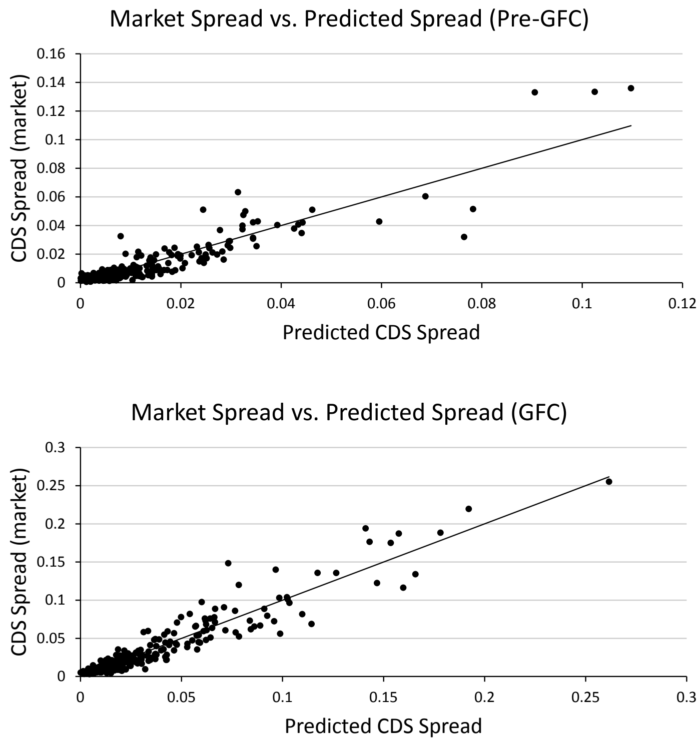
Moreover, we find that our empirical results hold across all economic sectors examined, since the inclusion of sector dummies affects our estimation results only slightly. Thus, we conclude that the introduced risk factors have economy-wide impacts on the pricing of swap contracts, beyond sectoral influences.

While the entire ensemble of risk factors account for almost 90.18% of the spread variation during the GFC, the models R^2 is clearly lower prior to the crisis (83.18%). Thus, we find that the explanatory power of the regressor ensemble is depending on the sample period and that the regressors best fit CDS spreads in the crisis. This finding also indicates that systematic risk betas of CDS contracts are particularly priced in economic downturns coming along with increasing statistical significances of our systematic risk proxies in the GFC.

In Figure 6.4, we compare predicted CDS spreads with observed market ones by sample in order to indicate the accuracy of our CSR model. The x-axes of the two charts denote the predicted CDS spreads and the y-axes denote the market CDS spreads.

Referring to regression (6.3), both scatter plots visualize the quality of our proposed CSR model. Since the spread predictions in the lower chart ($R^2 =$ 90.18%) are less scattering than in the upper one ($R^2 =$ 83.18%), we suggest that our CSR model – which is explicitly addressing systematic risk – reaches the highest model accuracy in times of global financial distress.

Figure 6.4: CDS Spread Comparison (market spread vs. model spread)



Notes: This figure shows the comparison of market CDS spreads (y-axes) and model spreads (x-axes). The spread predictions are based on the estimation results related to the basic CSR model in Equation (6.3). While the upper chart refers to the period prior to the GFC (Pre-GFC), the lower chart shows the results for the GFC.

6.3.4 Robustness

In the following, we extend our basic regression approach in several ways to show the robustness of our empirical findings. Firstly, we test whether our results hold, even if the three Fama-French (FF) factors are included in our basic models. Thereby, we also examine if the FF factors provide additional explanatory power in the cross-section of swap spreads beyond the basic risk components (compare Fama & French 1993). Secondly, we examine euro/dollar swap rates as alternate proxies for the *Spot Rate* and the *Slope of the Term Structure*. Thirdly, we conduct a principal component analysis (PCA) referring to the residuals of the multiple time-series regressions. The PCA may help to identify further potential candidates for systematic risk. Related to the PCA, we conduct new cross-sectional regressions in which we include the eigenvector of the first major component. By this, we test if this unknown systematic risk factor is cross-sectionally priced in the CDS spreads. Fourthly, we add the firm's *Leverage Ratio* and *Market Capitalization* as two more idiosyncratic risk factors to the basic regression model. Lastly, we provide the regression results of the entire model in which all model extensions are simultaneously considered.[172]

Table 6.8 shows CSR results referring to the first three model extensions. Estimation results are presented for both sample periods (Pre-GFC and GFC) under consideration of sector and rating dummies. Respective standard deviations are reported in parentheses. Additionally, corrections for the standard deviations and significances are provided as suggested by Shanken (1992), which are separated by a slash.

In the first case (left column), the three Fama-French benchmark returns are included in the TSR. Afterwards, the estimated betas are added to the basic CSR model in Equation (6.3). The *Fama-French excess Return* (FFR) describes the excess[173] return on the market, *Small Minus Big* (SMB) represents the performance of small stocks relative to big stocks, and *High Minus Low* (HML) denotes the performance of value stocks relative to growth stocks (compare Fama & French 1993). Commonly, the Fama-French factors are used by investors seeking for portfolio benchmark returns and by academics to explain the cross-section of stock returns.

We find that there is a negative relationship between the FFR and CDS spreads which is also statistically significant. This empirical result also follows economic intuition since positive excess returns may indicate a prosperous global economy with lower default risk in general. Thus, the CDS spreads should increase if the excess returns decrease and vice versa. In contrast to the FFR, we observe shifts in the signs of estimators with respect to SMB and HML across samples. These sign changes makes further interpretations somewhat difficult, even if the estimates reach statistical significance in both samples.

On the one hand, the R^2 increases from 83.19% to 89.04% through the consideration of the Fama-French factors in the pre-crisis. On the other hand, the R^2 remains on the same level with respect to the GFC (90.18% vs. 90.57%).

[172]Not reported are robustness checks related to i) various window sizes of the cross-sectional regressions, e.g., rolling or fixed, and ii) other alternate proxies for, e.g., the *Slope of the Term Structure*, *Spot Rate* and *Cross-market Correlation*. These analyses lead to similar regression results as the already reported ones.

[173]The excess return is defined as the difference between the return of the market portfolio (R_m) and the risk-less rate (r) (compare Fama & French 1993).

Table 6.8: Table of Case-specific Cross-section Estimates

Case	Fama-French Factors		Alternate Interest Rates		Principal Component	
Sample	Pre-GFC	GFC	Pre-GFC	GFC	Pre-GFC	GFC
Intercept	0.0085***/***	0.0155***/***	0.0098***/***	0.0155***/***	0.0098***/***	0.0221***/***
	(0.0014/0.001)	(0.0036/0.0021)	(0.0017/0.0015)	(0.0037/0.002)	(0.0019/0.0017)	(0.0042/0.0029)
CMC	1.3524***/***	0.3823***/***	2.3556***/***	0.3015***/**	2.6067***/***	0.3237***/**
	(0.222/0.2979)	(0.0768/0.132)	(0.256/0.7025)	(0.0687/0.1521)	(0.2465/0.9009)	(0.0673/0.1578)
CDX	0.0062***/***	0.0114***/***	0.0073***/***	0.0121***/***	0.0075***/***	0.0126***/***
	(0.0004/0.0008)	(0.0007/0.0012)	(0.0004/0.0013)	(0.0006/0.0015)	(0.0004/0.0016)	(0.0006/0.0015)
VIX	32.3192***/***	27.8042***/***	49.522***/***	33.5933***/***	41.4919***/**	32.8809***/***
	(4.8401/9.7092)	(3.9545/6.1847)	(5.445/17.3408)	(3.6465/8.393)	(5.6819/16.6877)	(3.6428/8.5889)
SR (T-bills)	-0.0059*/−	-0.0152***/***			0.0076*/−	-0.0148***/***
	(0.0033/0.0051)	(0.0019/0.0033)			(0.0039/0.0108)	(0.0019/0.003)
STS (T-bills)	0.0047***/−	-0.0081***/***			0.0002−/−	-0.0096***/***
	(0.0013/0.0035)	(0.0012/0.0025)			(0.0013/0.0062)	(0.0012/0.0024)
SR (Swap Rate)			0.0025−/−	-0.0165***/***		
			(0.0042/0.0084)	(0.0018/0.0029)		
STS (Swap Rate)			-0.001−/−	-0.0093***/***		
			(0.0013/0.0058)	(0.0012/0.002)		
FFR	-0.065***/**	-0.075***/***				
	(0.0106/0.0265)	(0.0083/0.0257)				
SMB	0.0213**/*	-0.0128**/−				
	(0.0085/0.0124)	(0.0056/0.0105)				
HML	0.0151**/−	-0.0262***/***				
	(0.0066/0.0131)	(0.0033/0.0072)				
LIQ	-0.0007***/***	-0.0017***/***	-0.0008***/***	-0.0019***/***	-0.0008***/***	-0.002***/***
	(0.0001/0.0001)	(0.0004/0.0003)	(0.0001/0.0001)	(0.0004/0.0003)	(0.0001/0.0002)	(0.0004/0.0003)
PC 1					-0.0075−/−	-0.0863**/**
					(0.0189/0.0164)	(0.0338/0.04)
Dummies (Rating & Sector)	Yes	Yes	Yes	Yes	Yes	Yes
R^2	89.04%	90.57%	83.22%	90.08%	83.19%	90.38%
No. Entities	339	339	339	339	339	339

Notes: This table shows the estimation results referring to the cross-section regressions under consideration of three different cases: in the first case, three Fama-French Factors are added as regressor to the basic model of Equation (6.2). Thus, the explanatory power of betas related to the Fama-French excess return (FFR), *Small Minus Big* (SMB) and *High Minus Low* are tested in the presence of the basic risk factors (*Cross-market Correlation* (CMC), CDX index, VIX index, *Spot Rate* (SR), *Slope of the Term Structure* (STS) and *Swap Liquidity* (LIQ)). In the second case, runtime equivalent swap rates are considered in the basic CSR model instead of Treasury bills. In the third case, the basic model is expanded to the first component (PC 1) of the principal component analysis. In all three cases, dummy variables account for both the firm's rating and economic sector. The results are provided for each sample (Pre-GFC and GFC). The parameters are statistical significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). Values in parenthesis describe the parameters' standard deviation (STD). Shanken-corrected STDs and significances are separated by a slash. R^2 denotes the coefficient of determination.

Thus, we conclude that the Fama-French factors may increase the explanatory power of the basic model in times of moderate economic movements, but that the additional pricing information is strongly limited in times of an economic downturn.

In the second case (middle column), we examine the 5-year euro/dollar swap rate as alternate proxy for the *Spot Rate*, since some authors in the recent literature suggest swap rates as interest rate proxies rather than Treasury bills.[174] Furthermore, the *Slope of the Term Structure* is now approximated by the difference of the 10-year swap rate and the 2-year swap rate. The new ensemble of systematic risk factors achieves similar high R^2, whereat the gamma coefficients are roughly similar to those of the basic model. Since the coefficients of determination vary not more than 0.1% in each sample, we suggest that alternate interest rate proxies provide similar pricing information.

In the third case (right column), we conduct a principal component analysis (PCA) on the residuals of the multiple TSR to identify potential candidates for systematic risk omitted in this empirical study so far. By this, we examine if the TSR residuals are jointly driven by unknown systematic risk factors and we specify these principal components, similar to Collin-Dufresne et al. (2001). To test whether the specified principal components are priced by market participants in our CDS spreads cross-sectionally, we run subsequent second-pass regressions (CSR) in which we additionally include the eigenvector of the first major component.

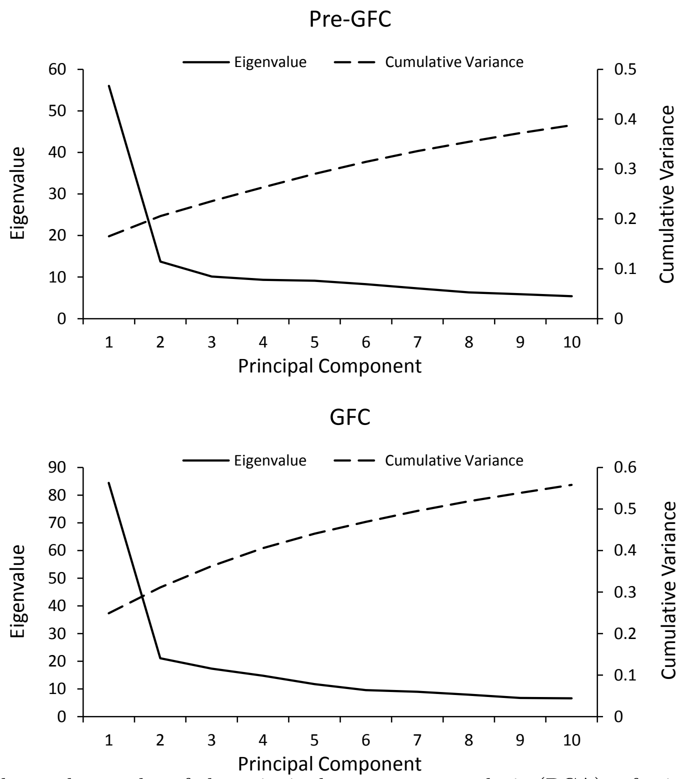
Results of the PCA are plotted in Figure 6.5. While the upper chart shows the results of the PCA related to the Pre-GFC, the lower chart contains the PCA results for the GFC. The primary y-axes show the eigenvalues, the secondary y-axes denote the cumulative variance of identified components that are denoted on the x-axes.

Both charts demonstrate that the PCA leads to similar results in each sample. According to the scree-test, the residuals of the time-series regressions are mainly driven by one major risk component that accounts for almost 17% of the cumulated variance prior to the GFC and for almost 25% during the GFC.

The right column of Table 6.8 summarizes the estimation results of the CSR after adding the first principal component. The results show that the influence of the first component ($PC1$) is negatively estimated in both samples. Moreover, the principal component is significantly priced during the GFC, but not

[174]For literature remarks see Footnote 13.

Figure 6.5: Principal Component Analysis of Time-series Residuals



Notes: This figure shows the results of the principal component analysis (PCA) referring to the residuals of the time-series regressions. The PCA is provided for both subsamples. The upper chart refers to the Pre-GFC, the lower chart to the GFC. In each chart, the x-axis denotes the principal components and the primary y-axis reports the corresponding eigenvalues. The secondary y-axes show the cumulative variance of the principal components.

in the pre-crisis. Since the component is unknown, economic interpretations are somewhat difficult. But the coefficient indicates that there may be a source for systematic risk that is negatively correlated with CDS spreads. Thus, the swap spreads increase when the component's value decreases and vice versa.

Overall, the PCA indicates that there are some systematic drivers responsible for the shared variance of TSR residuals, but these drivers are not priced without restrictions cross-sectionally. Therefore, the use of the PCA is strongly limited. From the small pricing impact of the PCA component in combination with the relatively high explanatory power of our basic model framework, one may conclude that our valuation framework already considers the most important systematic as well as idiosyncratic spread drivers and thus provides valuable insight into the pricing of swap contracts.

Table 6.9 reports the empirical results related to the CSR based on the last two model extensions. The estimation results refer to the Pre-GFC and the GFC. Standard deviations are reported in parentheses. The shanken-corrected standard deviations and significances are separated by slash, see Shanken (1992).

Table 6.9: Table of Cross-section Estimates Including Additional Risk Factors

Case	Idiosyncratic Risk Factors		Entire Model	
Sample	Pre-Crisis	GFC	Pre-Crisis	GFC
Intercept	$0.0029^{-/-}$	$0.0125^{-/*}$	$0.0026^{-/-}$	$0.0184^{**/**}$
	(0.0037/0.0024)	(0.0079/0.0071)	(0.0032/0.0016)	(0.0071/0.0089)
CMC	$2.8977^{***/***}$	$0.5783^{***/*}$	$1.8085^{***/***}$	$0.7574^{***/**}$
	(0.2552/1.0955)	(0.0775/0.3461)	(0.2538/0.3952)	(0.0817/0.3358)
CDX	$0.0056^{***/***}$	$0.0138^{***/***}$	$0.0052^{***/***}$	$0.0092^{***/***}$
	(0.0004/0.0017)	(0.0007/0.0032)	(0.0004/0.0012)	(0.0008/0.0024)
VIX	$48.5248^{***/**}$	$23.6418^{***/-}$	$50.5089^{***/***}$	$12.4364^{***/-}$
	(6.4512/19.9699)	(4.6039/14.6251)	(5.6951/15.5018)	(4.6586/9.3532)
SR (T-bills)	$0.0172^{***/-}$	$-0.0052^{-/-}$	$-0.0072^{*/-}$	$-0.0126^{***/*}$
	(0.0035/0.0207)	(0.0032/0.0057)	(0.0042/0.0083)	(0.0029/0.0069)
STS (T-bills)	$-0.0006^{-/-}$	$-0.0105^{***/*}$	$0.0058^{***/-}$	$-0.0045^{***/-}$
	(0.0016/0.0093)	(0.0015/0.0055)	(0.0016/0.0058)	(0.0014/0.0075)
FFR			$-0.1065^{***/**}$	$-0.0518^{***/-}$
			(0.0142/0.0522)	(0.0084/0.041)
SMB			$0.0035^{-/-}$	$-0.0299^{***/-}$
			(0.0141/0.022)	(0.0069/0.0285)
HML			$0.0206^{**/-}$	$-0.0402^{***/***}$
			(0.0083/0.0223)	(0.0037/0.0134)
LIQ	$-0.0008^{***/***}$	$-0.0017^{***/***}$	$-0.0008^{***/***}$	$-0.0007^{*/-}$
	(0.0001/0.0002)	(0.0005/0.0007)	(0.0001/0.0002)	(0.0004/0.0006)
MC	$0.0003^{-/-}$	$0^{-/-}$	$0.0002^{-/*}$	$-0.0002^{-/-}$
	(0.0003/0.0002)	(0.0006/0.0007)	(0.0002/0.0001)	(0.0005/0.0007)
LR	$0.0055^{**/***}$	$0.0034^{-/-}$	$0.0064^{***/***}$	$-0.0013^{-/-}$
	(0.0027/0.002)	(0.0054/0.0048)	(0.0023/0.0015)	(0.0045/0.0063)
PC 1			$0.0223^{*/-}$	$-0.0768^{***/*}$
			(0.013/0.0151)	(0.0274/0.0419)
Dummies (Rating & Sector)	Yes	Yes	Yes	Yes
R^2	91.29%	91.63%	93.87%	94.43%
No. Entities	225	225	225	225

Notes: This table shows the estimation results referring to the cross-section regressions under consideration of two different cases: in the first case, two more idiosyncratic risk factors – the firm's *Market Capitalization* and *Leverage Ratio* – are added to the basic CSR model of Equation (6.3). The *Entire Model* (case two) contains the basic risk factors (*Cross-market Correlation* (CMC), CDX index, VIX index, *Spot Rate* (SR), *Slope of the Term Structure* (STS) and *Swap Liquidity* (LIQ)), the three Fama-French factors (Fama-French excess Return (FFR), *Small Minus Big* (SMB) and *High Minus Low* (HML), additional idiosyncratic risk factors (*Market Capitalization* and *Leverage Ratio*) and the first component (PC 1) of the principal component analysis as further systematic risk factor. Dummy variables are included to account for both the firm's rating and economic sector. The results are provided for both samples (Pre-GFC and GFC) based on weekly CDS spread data of 225 entities. The parameters are statistical significant at the 1%-level (***), the 5%-level (**), and the 10%-level (*). The values in parenthesis describe the parameters' standard deviations (STD). Shanken-corrected STDs and significances are separated by a slash. R^2 denotes the coefficient of determination.

Results reported in the left column refer to the basic model under consideration of two more idiosyncratic factors: the firm's *Market Capitalization* (MC) and *Leverage Ratio* (LR). Independent from Merton's structural framework, we suppose that the firm's size indicates somehow the robustness of the firm against, e.g., economic downturns (compare Blume et al. 1998, Tang & Yan 2007). We suggest that firms characterized by a large and well-diversified asset portfolio exhibit both a higher resistance to external shocks and a greater power to innovate, even in market turbulences (compare Porter 1987, Hitt et al. 1996). Thus, we expect a positive risk premium for firms that are less market capitalized. Finally, we measure the firm size by the natural logarithm of the market value of the firm's equity (market capitalization) (compare Blume et al. 1998) and additionally calculate the book-to-market equity ratio based on a COMPUSTAT database.[175]

According to the structural theory, the default threshold is a function of outstanding debt claims. The higher the leverage, the higher is the probability that the asset value process undergoes the critical threshold. Hence, the default probability is increasing with increasing leverage. Therefore, we may expect a positive relationship between the leverage ratio and the observed CDS spread. Among others, Welch (2004) found that stock returns capture changes in leverage appropriately. Approximated by stock returns, Avramov et al. (2007) identified leverage as main driver for credit spread changes. We approximate leverage by the following leverage ratio

$$\frac{\text{Book Value of Debt}}{\text{Market Value of Equity} + \text{Book Value of Debt}}$$

to proxy the firm's health according to Collin-Dufresne et al. (2001). Respective data is provided by COMPUSTAT. As this analysis requires additional data from COMPUSTAT the number of entities is reduced to 225.

According to the left column of Table 6.9, our main results also hold with respect to the inclusion of these two firm-specific variables.[176] We find that the MC does not provide significant explanatory power – neither prior to the GFC nor during the GFC. By contrast, the firm's LR constitutes a significant pricing determinant in moderate economic conditions which also corresponds

[175] Since the book-to-market equity ratio reaches no significance in our model framework, we solely focus on the firm's market capitalization.

[176] Slight differences may be due to the lower amount of entities in this model setup.

to economic expectations: across all economic sectors an increase in the firm's leverage leads to an increase in the swap's risk premium. Overall, the inclusion of these idiosyncratic risk factors lead to an increase of the R^2 from 83.18% to 91.29% in the Pre-GFC, but causes solely small benefits in times of the GFC, where the R^2 increases only from 90.18% in the basic model to 91.63% in the extended model.

To check whether the effect sizes related to each model extension are complementary or not, we estimate the last model case in which the basic two-pass approach is simultaneously extended to the Fama-French factors, the *Leverage Ratio*, the *Market Capitalitzation* and the first principal component.[177] The respective regression results are reported in the right column of Table 6.9.

With respect to the models R^2, the entire ensemble of risk factors accounts for almost 94% of the CDS spread variation in both samples, which is highest compared to the R^2 of all other regression models.[178] We find that the main results are confirmed in the *Entire Model*: again, the OLS regression results show that all estimates of the systematic risk variables reach statistical significance, independent from the sample period.[179] Thus, all systematic risk proxies are significantly priced in the cross-section of CDS spreads. Apart from the *Slope of the Term Structure*, all of these variables additionally meet economic expectations. But even though, e.g., the time-series characteristics of systematic risk variables vary by sample in terms of both their means and standard deviations (compare Figure 6.2), the quality of the *Entire Model* is almost identical in both samples. Since the *Entire Model* exhibits high explanatory power independent from the sample, this regression model seems to be robust against subsampling in some extent.

Each case-specific model extension confirms for itself the results of the basic approach. Thus, we identify the *Credit Market Climate* (CDX), the *Cross Market Correlations* (CMC) and the *Market Volatility* (VIX) as most important systematic risk factors in the cross-sectional pricing process of swap contracts. The corresponding risk sensitivities (betas) are positively priced across all samples and model cases. This result indicates a positive correlation between these

[177]Here, Treasury bills constitute the reference interest rates.

[178]Note that the models' R^2 are not directly comparable with each other due to different numbers of entities in the data sets.

[179]The Shanken-corrected t-value of the VIX is not statistically significant in this model setup. Such distortions may generally be due to i) the lower amount of entities, ii) the higher number of regressors or iii) effects of multi-collinearity.

risk proxies and the cross-section of credit spreads. We find that CDS spreads significantly rise if one of these risk factors increases and vice versa which is in line with economic expectations. The applied model extensions may help to increase the model's explanatory power particularly with respect to moderate economic conditions. Additionally, we confirm liquidity as a further decisive determinant in pricing swap contracts. Corresponding to expectations, the contract's liquidity reveals a negative relationship to the CDS spread in both samples and we observe significant negative gamma estimates across all models. Hence, the results show that the contract's sensitivity to liquidity risk is compensated through a respective premium widening the spread if the liquidity of the contract decreases. Referring to the rating classes, the estimates are statistically significant in most cases. The empirical results show that market participants claim a higher risk premium for investing in low-rated swap contracts reflecting a lower creditworthiness of the rated obligor. This risk premium is monotonically increasing with rating classes and paid in the cross-section of CDS spreads. All these findings hold, even if we account for the economic sector in which the firm is operating.

Eventually, we conclude that systematic risk is generally affecting spreads of swap contracts relying on debt assets. Even if it is hard to measure the pricing impact of the systematic risk factors exactly, we demonstrate that specific systematic risk variables such as the *Credit Market Climate*, the *Cross-market Correlation* and the *Market Volatility* may play a major role in pricing credit default swaps. We find that the systematic risk exposures of CDS contracts vary by rating class and even within each rating class. We further show that these systematic risk exposures are priced beyond ratings. Although the explanatory power of our systematic risk determinants may generally vary by regressor and by sample, we find that the influence of most systematic risk factors particularly increases in economic downturns. Overall, we argue in this empirical study from both an economic and a statistical perspective in order to demonstrate the relevance of the provided systematic risk factors for pricing CDS contracts, even in the presence of major idiosyncratic risk factors, other systematic risk proxies and sectoral influences.

6.4 Summary

The recent Global Financial Crisis (GFC) has shown that macroeconomic shocks, e.g., caused by the U.S. housing crisis, may have strong impacts on global financial markets, particularly on the credit markets. Indeed, many credit market participants suffered from unexpected high default rates on corporate bonds or related financial instruments such as credit default swaps or collateralized debt obligations (compare Moody's 2009*a*, 2011*b*).

Motivated by i) these market-specific impacts of systematic risk and ii) other authors who show that corporate credit spreads are driven by both economic risk factors as well as firm-specific factors (Collin-Dufresne et al. 2001, Eom 2004, Longstaff et al. 2005, Iannotta & Pennacchi 2011), we explicitly address the sensitivity of credit default swap spreads to systematic risk and evaluate related pricing impacts.

Firstly, we introduce a set of systematic risk factors that reflects in theory several systematic state variables such as the *Credit Market Climate*, the *Market Volatility*, the *Spot Rate* and the *Slope of the Term Structure*. Additionally, we provide the *Cross-market Correlation* as proxy for global contagion.

Secondly, we show that swap contracts, which are highly sensitive to systematic risk, are higher priced than contracts having a lower sensitivity. We also demonstrate that credit ratings are not reflecting these systematic risks appropriately and that systematic risk sensitivities may vary by rating grade and economic environment. Especially, these findings may provide first insight for regulatory authorities who develop risk-adjusted capital requirements for banks based on ratings.

Thirdly, we demonstrate in a two-pass regression approach according to Fama & MacBeth (1973) that CDS spread changes sensitively react to changes in the proposed systematic risk variables and that these sensitivities (betas) to systematic risk are cross-sectionally priced. In the first pass, we calculate the factor betas by multiple time-series regressions. In the second pass, we test whether these systematic risk sensitivities are cross-sectionally priced in CDS spreads, even in the presence of firm-specific risk factors suggested from numerous authors in the recent literature on both corporate debt and CDS. While the firm's rating as well as the swap's liquidity is considered in the basic approach, we account for the firm's logarithmized *Market Capitalization* and *Leverage Ratio* in further model extensions.

We find that most betas of our systematic state variables are significantly priced in each sample, even if idiosyncratic risk variables and sector dummies are included. Our basic ensemble of risk factors explains about 83% of the cross-section of CDS spreads before the crisis and about 90% during the crisis. Thus, systematic risk seems to be priced particularly in economic downturns. Moreover, we identify the firm's rating, *Leverage Ratio* and the contract-specific *Swap Liquidity* as most important individual risk factors in pricing swap contracts. Thus, our results also correspond to findings of other authors in the recent literature. While the firm's rating is mostly significantly priced and its gamma estimates correspond to economic expectations, those of other risk factors, such as the *Market Capitalization* do not. Results related to the firm's *Leverage Ratio* are plausible from an economic point of view in both samples and this proxy is also significantly priced prior to the GFC.

Related to our systematic risk factors, we find that particularly the sensitivity to the *Credit Market Climate* – approximated by the 5-year CDX NA IG credit index spread – is significantly influencing the cross-section of CDS spreads. From an economic perspective, we observe a positive sensitivity of CDS spread changes to changes in the CDX which leads to a positive risk premium in the contracts' cross-section. If the credit climate gets worse, the CDS spreads significantly increase and vice versa. Hence, our empirical findings show that investors on CDS markets are monetarily compensated for this kind of common risk.

Furthermore, we find that the suggested *Cross-market Correlation* also significantly explains CDS spreads. To approximate the prospects of risk diversification across, e.g., stock, credit and exchange markets, we calculate the average cross-correlation related to specified markets. Both beta and gamma estimates also satisfy economic expectations: the higher (lower) the cross-market correlation the higher (lower) is the related systematic risk since market participants are more (less) constrained in their diversification efforts. Thus, we observe increasing CDS spreads in line with an increasing *Cross-market Correlation* (positive pricing effect) due to a positive sensitivity of CDS spreads to cross-correlation movements.

With the VIX index – indicating the *Market Volatility* – we identify another important determinant for the valuation of systematic risk in CDS spreads. Throughout positive beta as well as gamma estimates, which are also statistically significant, confirm our theoretical expectations and suggest that market participants are positively rewarded for the market risk expressed through the volatility on stock markets. We find that if the volatility on stock markets is high (low) swap investors may receive a high (low) risk premium included in the CDS spread.

In order to check the robustness of our empirical findings, we provide further model extensions: to account for parameter estimation risk related to our two-pass regression approach, particularly to our cross-section regressions, we firstly provide corrected t-statistics for the gamma estimates, as proposed by Shanken (1992). Even if the related significances slightly differ, the primary tendency of our main results hold. Moreover, we extend our analysis to the Fama-French Factors (Fama & French 1993). In both samples, the model accuracy increases in terms of the coefficient of determination (R^2), but this effect is particularly observable in moderate economic conditions. The inclusion of swap rates instead of Treasury bills in order to approximate the interest rate risk in terms of the *Spot Rate* and the *Slope of the Term Structure* leads to R^2, which are similarly as high as in the basic model. Eventually, the R^2 do not differ more than 0.1% in total. Therefore, we conclude that swap rates provide comparable pricing information to Treasury bills. Through a principal component analysis – similarly to Collin-Dufresne et al. (2001) – we identify at least one major component responsible for the shared variance of TSR residuals. We find that this principal component is significantly priced in CDS spreads across all entities during the GFC, but not prior to the crisis. Eventually, the results related to each model extension show that our main empirical findings hold, even in the presence of these additional risk factors or proxy alternatives.

Apart from our findings, further research is suggested in other systematic risk variables such as market recovery risk, or counter-party risk since both factors may represent other relevant determinants of CDS spreads omitted in this study (compare Brigo & Chourdakis 2009, Gandhi et al. 2012). Thereby, both risk variables may be evaluated either referring to credit markets in general (systematic) or explicitly as swap-specific (idiosyncratic) risk factors.

On the one hand, we are aware that there may exist other proxies that more appropriately measure the identified systematic risk variables. On the other hand, results may in parts generally be due to 'failure' in the proposed risk proxies since they depend strongly on the measurement technique and the quality of the data source. Even in terms of our *Cross-market correlation*, several other index compilations seem to be economically plausible as well. Thus, modifications of the measurement technique may either confirm or contradict our findings, even in a large set of risk factors, where also multi-collinearity may cause further distortions. Proper interpretations of our empirical findings are even more complicated if single state variables are conversely discussed in the recent literature. This might be the case, for example, in terms of the economic meaning and effect size related to the *Market Capitalization*.

In general, our empirical study provides a valuable insight into the valuation of systematic risk in CDS spreads. We suggest that at least three of our systematic risk factors reflect decisive determinants in pricing credit default swaps in line with economic expectations. These systematic determinants may also play a decisive role in the valuation of synthetic CDOs since this type of asset securitizations consists of CDS contracts.[180] Thus, our empirical study indicates not only the impacts of systematic risk on the valuation of swap contracts, but also offers scope for further research in the valuation of structured securities.

[180]Compare Chapter 2.

Chapter 7

Conclusion and Suggested Research Outlook

In this cumulative thesis, the main contributions are provided in the Chapters 2, 4, 5 and 6. At the end of each chapter, the main findings are summarized in detail and in context of the addressed audience. Hence, this section primarily suggests some chapter-specific research outlooks and afterwards concludes briefly.

In Chapter 2, Asset-backed Securities are introduced and the relevance of asset securitizations in global financial markets is highlighted. Recent developments in markets for structured products, especially for CDOs, show that the investors' confidence has slightly returned since 2010 leading to an increased demand for such products during the last two years. But nowadays, the collateral of securitizations is focused on more secured asset types such as investment grade bonds. In fact, the European market for securitization grows faster than its U.S. counterpart. With respect to current and future valuation models, market developments will be of special interest since the types of collateral may play an important role for model specifications and the selection of included risk or pricing factors. Future market developments will show whether asset securitizations have the potential to reach their former popularity in terms of issuance and trading volumes.

In Chapter 4, basic correlation concepts – represented by latent asset correlations and default correlations – are extended to implied correlations as crucial parameters for the valuation of structured securities. Using base correlations, our proposed dynamic regression correlation models – FERM and

MERM – account for both fixed tranche-specific effects and stochastic time effects in order to increase the forecast accuracy related to tranche spreads of the iTraxx Europe credit index. In both correlation skew models, we consider distributional properties of numerically inverted base correlations (means) instead of point estimates to lower the related parameter estimation risk. The results suggest that particularly our MERM offers a prosperous concept for modeling and forecasting tranche spreads by considering tranche-specific fixed effects and stochastic time effects simultaneously. Another beneficial aspect of our dynamic correlation regression approach is the independence to parameter assumptions in some extent due to the model framework.[181] Thus, parameter risk is rather less important in this model setup which may increase the models' applicability in practice. Our empirical findings may be checked in subsequent empirical studies on an expanded database of iTraxx tranche spreads, or even based on a corresponding CDX database. Furthermore, it might be valuable to extend our proposed model framework to further systematic risk factors, for example, to those identified in Chapter 6. Then, it could be evaluated whether common risk factors such as the *Credit Market Climate*, the *Cross-market Correlation* and the *Market Volatility* significantly explain base correlations and thus might increase the forecast accuracy of our proposed dynamic correlation models. Other correlation approaches in the recent literature such as random factor loadings, proposed by Andersen & Sidenius (2005*b*), aim – similarly to our models – to reduce the limitations of a single-correlation approach. By extending the standard Gaussian copula model, this approach targets at better fits of tranche spreads than in the market standard model. Through the inclusion of two or more correlation regimes depending on states of the global economy, Andersen & Sidenius (2005*b*) address highly significant pricing effects such as fat tails and correlation skews in synthetic CDO tranches. In this context, Longstaff & Rajan (2008) provide another top-down model approach which accounts for a systematic, a sectoral and an idiosyncratic risk factor as decisive determinants of CDX tranche spreads. Eventually, each of the three model approaches follows the same idea from a different perspective. Thus, further research may focus, for example, on a broad model comparison of these three approaches to evaluate the models' applicability in practice for

[181]Recall that distributional assumptions must be made with respect to the stochastic time effect, if considered. But parameter assumptions of the basic valuation model, e.g., related to recovery rates and the risk-less rate, are compensated by the numerical inverted level of base correlations (compare Appendix A).

best spread fits and forecasts. The results of such a model comparison may offer valuable insights particularly for market participants dealing and hedging single-tranche CDO swaps.

In Chapter 5, we demonstrate the systematic risk sensitivity of structured securities by decomposing the single-risk factor of the basic Gaussian copula model into a sectoral and a super-systematic risk component. We show that in securitizations pooling and tranching generally lead i) to effects of idiosyncratic risk diversification and ii) to effects of systematic risk concentration. The results show that particularly securitized tranches exhibit high systematic risk exposures monotonically increasing with the tranche's seniority, while idiosyncratic risks are fully diversified. These effects may be even higher in multiple structured securitizations such as CDO squared (CDO^2). By applying a Monte Carlo approach to several CDO^2 structures and accounting for heterogenous borrowers and heterogeneous sectors, subsequent research studies may provide additional insights into the risk characteristics of multiple structured products related to systematic risk under consideration of rather realistic scenarios, beyond the simplifying assumptions of the LHP. Furthermore, our results hint at shortcomings of current credit rating metrics provided by CRAs. Our findings suggest to develop ratings which reflect more appropriately the overall risk characteristics of financial instruments. Compared to corporate bonds, CDO tranches are especially exposed to systematic risk and thus a new rating metric is required which additionally indicates the products' systematic risk exposures and resulting sensitivities to systematic risk. Thus, further research is suggested in risk measures which do not only focus on the first moment of the loss distribution, but also anticipate impacts of cyclical effects such as systematic shocks in economic downturns. Moreover, our results may improve the current understanding of the natural behavior of asset securitizations related to systematic risk influences in order to develop pricing models appropriately anticipating product-specific risks. Apart from the assessment of systematic risk, aspects of model risk might be addressed within our extended model framework by applying different copula approaches, e.g., archimedean copulae, to the joint distribution of the borrowers' asset returns or the distributions of risk factors in the model setup.[182] Similar to Tarashev (2010), aspects of parameter risk may also be examined for several specifications of our extended

[182]Beyond elliptical copulae such as the Gaussian or t-copula which are already examined in the presented Monte Carlo approach.

model in order to quantify impacts of systematic risk on the portfolio's (or tranche's) value at risk (VaR), as monetary measure of credit risk.[183]

In Chapter 6, we identify the *Credit Market Climate*, *Cross-market Correlation* and the *Market Volatility* as relevant systematic risk factors driving spread changes of CDS contracts referring to numerous U.S. firms which are operating in a broad variety of economic sectors. Cross-sectional regression results show that the sensitivities to these common risk factors are significantly priced in the cross-section of CDS spreads, even after controlling for individual risk factors such as firm ratings, swap liquidity etc. and sectoral influences. Particularly, the examination of price impacts related to other systematic risk factors such as market recovery risk or individual counter-party risk offers scope for further research (compare Brigo & Chourdakis 2009, Gandhi et al. 2012). As Longstaff & Rajan (2008) find that sectoral risk is a major determinant in pricing CDX tranche spreads, the examination of sector-specific risk factors in subsequent studies may also provide valuable contributions to the pricing of CDS spreads. Moreover, it might be worthwhile combining the panel regression approach proposed in Chapter 4 with our findings related to systematic CDS spread determinants in order to develop a new valuation approach for structured securities such as synthetic CDOs. The application of a dynamic panel regression approach may help to identify common determinants of tranche spreads and thus offers scope for further research. Analogous to findings in the recent literature on CDS and corporate debt, the empirical results of our applied two-pass regression approach in Chapter 6 confirm that particularly the physical default risk represents an important individual risk factor in pricing swap contracts. With respect to the suggested panel regressions, the approximation of the tranches' physical default risk might be problematic, if tranche ratings of CRAs are not available. Thus, it is suggested to develop a model approach for the estimation of the tranche-specific physical default risk before applying dynamic panel regressions to the cross-section of tranche spreads. Controlling for other relevant spread determinants – proposed, for example, in Chapter 6 – may generally help to achieve more valid regression results and to avoid misleading interpretations.

In conclusion, this thesis explicitly addresses several aspects of systematic risk related to structured securities such as CDOs and CDS. The findings contribute to the discussion about the risk properties of structured finance

[183]For a good introduction to the VaR concept see Hull (2007).

products related to macroeconomic (systematic) influences and improves the understanding of the 'natural' behavior of securitizations caused by movements of the global economy. By providing the dynamic correlation regression models – FERM and MERM – we offer a prosperous approach for pricing and forecasting STCDO spreads. Thus, we contribute to the discussion of correlation modeling in the context of structured finance securities. The identified three systematic risk factors – *Credit Market Climate*, *Cross-market Correlation* and the *Market Volatility* – which are significantly priced in the cross-section of CDS spreads, may also be highly relevant for the valuation of CDOs, since structured securities are especially exposed to systematic risk, as found in the recent literature in a rather general manner (see, e.g., Coval et al. 2009*b*). Hence, these results may be relevant for asset pricing in general and particularly for the pricing of tranches referring to the two most popular credit index families, namely the iTraxx Europe and CDX index families. Overall, the findings presented in this thesis may help to anticipate systematic risk sensitivities of credit-linked instruments in order to return more transparency and confidence in structured products, or even in credit-related financial markets. The results also offer valuable insights for both market participants involved in dealing or hedging examined financial instruments and regulatory authorities developing risk-adjusted capital requirements for banks. Additionally, this thesis suggests further research in i) measurement tools for systematic risk, ii) the development of rating metrics appropriately reflecting the products' exposures to systematic risk and iii) the application of appropriate valuation models for CDS and CDOs explicitly accounting for systematic risk, as one of the most important sources of credit risk in current financial markets.

Appendix A

Sensitivity Analysis

In order to address the effects of parameter variations, we provide some sensitivity analysis related to changes of the risk-less interest rate r. Based on historical Treasury bills, we calculate impacts of varying interest rates on the forecast performance of our three correlation models MERM, FERM, and ACM.

With regard to *Sample 3*, the average treasury rate - in the meaning of averaging the 1 to 5 years treasury rates - varies between 1.6% (minimum) and 5.27% (maximum) with a mean of 4%.

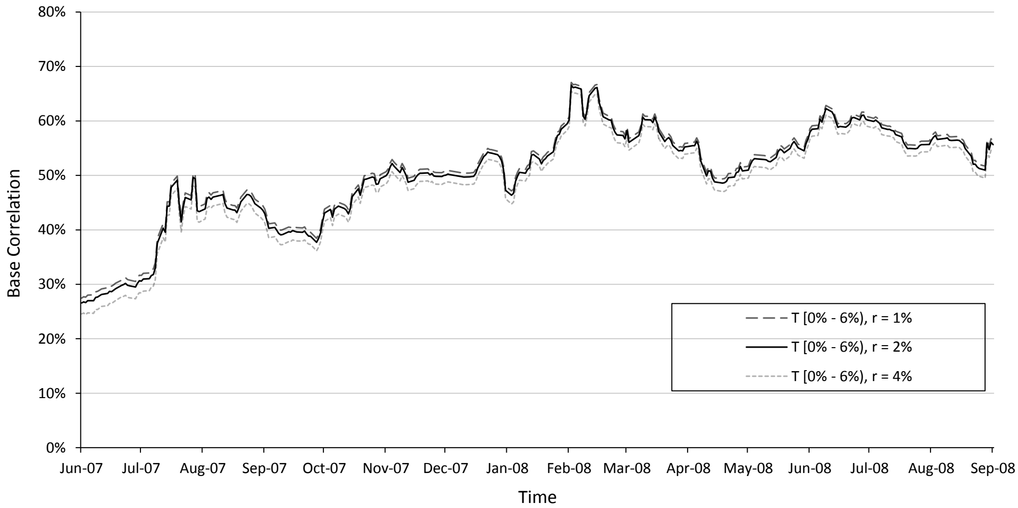
In Figure A.1, the time-series of *base correlations* (y-axis) are plotted for the mezzanine tranche $T_{[0\%,6\%)}$ from June 19^{th}, 2007 to September 19^{th}, 2008 (x-axis) depending on three different flat term structures of the risk-less interest rate r with $r \in \{1\%, 2\%, 4\%\}$.[184]

As we can see, an increase of the interest rate leads to a slight decrease of *base correlations* and vice versa (ceteris paribus). The dashed line ($r = 1\%$) is above the black line ($r = 2\%$) throughout *Sample 2*. In terms of $r = 4\%$, the *base correlations* (grey-dashed line) are throughout below the black line. Referring to historical T-bills, a real term-structure can easily be implemented, but eventually time-series of resulting *base correlations* will be located somewhere between the plotted functions denoting the empirical border of Treasury bill rates.

Concerning our analysis, implied correlations that are numerically inverted are completely covering shifts in r due to the applied pricing methodology,

[184]Note that the deviations due to changes of r are quite small, even in times of financial distress. Thus, we focus on *Sample 2* only for illustration purposes.

Figure A.1: Sensitivity of Base Correlations



Notes: This figure shows time-series of base correlations related to various flat term structures of the risk-less interest rate r with $r \in \{1\%, 2\%, 4\%\}$ for *Sample 2* with respect to the mezzanine tranche $T_{[0\%,6\%)}$.

compare Equation (4.5).[185] Eventually, our dynamic correlation approach is somewhat independent from parameter assumptions and thus the risk-less interest rate r as well as other parameter settings, e.g., recovery rates, are of secondary importance.

For example: related to Equation (4.18), Table A.1 provides RMSFEs referring to our three correlation models, each regression window (40-, 50- and 60-days), each sample and each tranche under consideration of two different flat term structures of the risk-less interest rate r with $r \in \{1\%, 4\%\}$.

The effects of varying interest rates are marginal in terms of our dynamic correlation models MERM and FERM. Apparent differences are due to our forecast procedure in which we simulate *base correlations* in order to calculate our spread forecasts. As already indicated, we find that a 'real' term structure solely influences the ACM forecasts. Thus, the RMSFEs of the ACM are dependent on the applied term structure. Implementing a T-bill term structure may help to reduce the RMSFEs of our ACM forecasts. However, the decrease of the RMSFEs is on average not more than 2.43% in *Sample 3*. Compared to our dynamic correlation models, the ACM still provides RMSFEs which are on average 15 times higher. Therefore, we suggest that the resulting benefits of a 'real' term structure are somewhat negligible.[186]

[185]Note that the risk-less interest rate r determines the time-specific discount factors Q_{t_j}.
[186]We obtain similar results by applying an interest term structure based on swap rates.

Table A.1: Interest Rate Sensitivity in Terms of RMSFEs by Sample

	Window	Sample 1 (r = 1%) Tranche(in %)					Sample 2 (r = 1%) Tranche (in %)					Sample 3 (r = 1%) Tranche(in %)				
		0-3	3-6	6-9	9-12	12-22	0-3	3-6%	6-9	9-12	12-22	0-3	3-6	6-9	9-12	12-22
MERM	40	0.044	0.039	0.054	0.070	0.066	0.051	0.064	0.070	0.080	0.081	0.053	0.064	0.078	0.092	0.093
	50	0.044	0.039	0.054	0.071	0.067	0.051	0.064	0.071	0.080	0.081	0.053	0.064	0.078	0.093	0.093
	60	0.044	0.039	0.053	0.070	0.067	0.051	0.065	0.071	0.080	0.081	0.053	0.064	0.078	0.092	0.093
FERM	40	0.044	0.039	0.055	0.072	0.068	0.051	0.065	0.072	0.083	0.082	0.053	0.064	0.079	0.095	0.094
	50	0.044	0.039	0.055	0.072	0.070	0.051	0.065	0.073	0.082	0.082	0.053	0.064	0.079	0.095	0.095
	60	0.044	0.039	0.055	0.073	0.068	0.051	0.066	0.072	0.085	0.081	0.053	0.064	0.080	0.096	0.094
ACM	40	0.340	1.435	1.990	1.780	0.733	0.230	1.043	0.948	0.717	0.436	0.292	1.285	1.614	1.402	0.605

	Window	Sample 1 (r = 4%) Tranche (in %)					Sample 2 (r = 4%) Tranche (in %)					Sample 3 (r = 4%) Tranche (in %)				
		0-3	3-6	6-9	9-12	12-22	0-3	3-6	6-9	9-12	12-22	0-3	3-6	6-9	9-12	12-22
MERM	40	0.044	0.040	0.055	0.057	0.070	0.051	0.064	0.071	0.080	0.081	0.053	0.064	0.078	0.087	0.094
	50	0.044	0.040	0.055	0.058	0.074	0.051	0.064	0.071	0.080	0.081	0.053	0.064	0.079	0.088	0.096
	60	0.044	0.039	0.054	0.059	0.073	0.051	0.064	0.071	0.081	0.081	0.053	0.064	0.078	0.089	0.097
FERM	40	0.044	0.040	0.057	0.061	0.070	0.051	0.065	0.072	0.083	0.081	0.053	0.064	0.080	0.090	0.095
	50	0.044	0.040	0.056	0.060	0.075	0.051	0.065	0.074	0.081	0.082	0.053	0.064	0.081	0.089	0.097
	60	0.044	0.039	0.057	0.064	0.077	0.051	0.065	0.073	0.083	0.081	0.053	0.064	0.080	0.091	0.097
ACM	40	0.368	1.398	1.935	1.724	0.704	0.197	1.025	0.924	0.693	0.419	0.301	1.254	1.570	1.358	0.582

Notes: The table shows root mean square forecast errors (RMSFE) that are i) model-specific (MERM, FERM and ACM), ii) window-specific (40-, 50- and 60-days) and iii) tranche-specific for different interest rats r with $r \in \{1\%, 4\%\}$. The results refer to each sample-specific forecast period. With respect to our dynamic correlation models slight RMSFE differences are solely due to the applied forecast methodology. Tranche boundaries are denoted in percent.

Appendix B

Forecast Test Statistics

In this section, we examine the forecast accuracy of our three correlation models by comparing market spreads with our model-specific spread forecasts in each sample $p \in \{1, 2, 3\}$. We estimate the intercept $\theta_{0,p}$ and the coefficient $\theta_{1,p}$ of each tranche spread forecast based on

$$s_{t,p} = \theta_{0,p} + \theta_{1,p} \cdot \hat{s}_{t,p} + \epsilon_{t,p} \tag{B.1}$$

with $\epsilon_{t,p} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ i.i.d.

Applying regression model B.1, the hypothesis $H_0 : \theta_{q,p} = 0$ is tested with $q \in \{0, 1\}$. Additionally, a F-test is provided for $H_0 : \theta_{1,p} = 1$.

Table B.1 summarizes the regression results referring to Equation B.1 as well as corresponding p-values of the second test statistic. The results also refer to the forecast period of *Sample 3* and each of our three correlation models, each regression window (40-, 50- and 60-days), and each tranche.

Referring to our dynamic regression correlation models – FERM and MERM – we find i) that the intercepts θ_0 are not significantly differing from zero and ii) the coefficients $\theta_{1,p}$ of spread forecasts $\hat{s}_{t,p}$ are significantly different to zero. These findings hold for all tranches and all regression windows. Since the estimates $\theta_{1,p}$ are close to 1 across all tranches and regression windows as well, we apply a F-test for $\theta_{1,p} = 1$. The p-values of the secondary test statistic (F-test) indicate that the estimated coefficients $\theta_{1,p}$ are not significantly differing from 1 (compare the last column of Table B.1). On the other hand, the estimation results related to the ACM show that the intercepts θ_0 as well as the spread forecast coefficients $\theta_{1,p}$ are differing from zero with a statistical significance at the 10%-quantile across all tranches. Further, the secondary test statistic indicates that the coefficients $\theta_{1,p}$ are different to 1 at the statistical significance level of 0.01%.

Table B.1: Forecast Test Statistics

Window	Tranche	Intercept θ_0 Estimate	Intercept θ_0 STD	Forecast Coefficient θ_1 Estimate	Forecast Coefficient θ_1 STD	R^2	Test Statistic (p-value)
MERM							
40-days	0% − 3%	0.001	0.001	0.996^{***}	0.005	0.981	0.423
	3% − 6%	0.000	0.000	0.997^{***}	0.004	0.988	0.547
	6% − 9%	0.000	0.000	0.998^{***}	0.004	0.990	0.604
	9% − 12%	0.000	0.000	0.995^{***}	0.004	0.986	0.258
	12% − 22%	0.000	0.000	0.996^{***}	0.004	0.986	0.339
50-days	0% − 3%	0.001	0.001	0.997^{***}	0.005	0.981	0.517
	3% − 6%	0.000	0.000	0.997^{***}	0.004	0.988	0.477
	6% − 9%	0.000	0.000	0.996^{***}	0.004	0.990	0.355
	9% − 12%	0.000	0.000	0.993^{***}	0.004	0.987	0.112
	12% − 22%	0.000	0.000	0.994^{***}	0.004	0.986	0.187
60-days	0% − 3%	0.001	0.001	0.996^{***}	0.005	0.981	0.497
	3% − 6%	0.000	0.000	0.997^{***}	0.004	0.988	0.449
	6% − 9%	0.000	0.000	0.996^{***}	0.004	0.990	0.296
	9% − 12%	0.000	0.000	0.992^{***}	0.004	0.987	0.068
	12% − 22%	0.000	0.000	0.993^{***}	0.004	0.986	0.136
FERM							
40-days	0% − 3%	0.001	0.001	0.997^{***}	0.005	0.981	0.571
	3% − 6%	0.000	0.000	0.997^{***}	0.004	0.988	0.495
	6% − 9%	0.000	0.000	0.996^{***}	0.004	0.990	0.269
	9% − 12%	0.000	0.000	0.991^{***}	0.005	0.986	0.055
	12% − 22%	0.000	0.000	0.992^{***}	0.005	0.986	0.078
50-days	0% − 3%	0.001	0.001	0.997^{***}	0.005	0.981	0.516
	3% − 6%	0.000	0.000	0.997^{***}	0.004	0.988	0.423
	6% − 9%	0.000	0.000	0.996^{***}	0.004	0.989	0.352
	9% − 12%	0.000	0.000	0.991^{***}	0.004	0.986	0.040
	12% − 22%	0.000	0.000	0.992^{***}	0.005	0.986	0.072
60-days	0% − 3%	0.001	0.001	0.996^{***}	0.005	0.981	0.479
	3% − 6%	0.000	0.000	0.997^{***}	0.004	0.988	0.486
	6% − 9%	0.000	0.000	0.996^{***}	0.004	0.989	0.266
	9% − 12%	0.000	0.000	0.991^{***}	0.005	0.986	0.056
	12% − 22%	0.000	0.000	0.992^{***}	0.005	0.986	0.081
ACM							
-	0% − 3%	0.085^{***}	0.002	0.638^{***}	0.008	0.913	0.000
	3% − 6%	-0.002^{***}	0.000	0.551^{***}	0.004	0.971	0.000
	6% − 9%	-0.001^{***}	0.000	0.597^{***}	0.004	0.973	0.000
	9% − 12%	0.000^{**}	0.000	0.600^{***}	0.005	0.954	0.000
	12% − 22%	0.000^{***}	0.000	0.639^{***}	0.006	0.941	0.000

Notes: The table provides test statistics related to tranche-specific spread forecasts for each correlation model (MERM, FERM, ACM) and each window size (40-, 50-, 60-days) as well. The results refer to the forecast period of *Sample 3*. While the parameter estimates are tested for $H_0 : \theta_q = 0$ with $q \in \{0, 1\}$, the test ,statistic is testing the coefficient θ_1 of spread forecasts for $H_0 : \theta_1 = 1$. STD denotes the standard deviation of parameter estimates. ***,** and * are indicating statistical significance at the 0.1%-, 5%- and 10%-quantile. R^2 denotes the coefficient of determination.

Eventually, our test statistics confirm results of Figure 4.9 and underline that all three correlation models provide valuable forecast performance. As revealed in Figure 4.9, the ACM spread forecasts are more scattering in contrast to both dynamic regression correlation models which leads to lower R^2 across all tranches (compare column 7 of Table B.1). Thus, our MERM and FERM seem to provide more reliable spread forecasts leading to their superiority in terms of the R^2 metric.

Appendix C

Proof of Equation (5.15)

Conditional default probability $\pi^{T_{[A,D)}}(y^*)$ *of tranche* $T_{[A,D)}$

We derive the conditional default probability $\pi^{T_{[A,D)}}(y^*)$ of tranche $T_{[A,D)}$ in dependence of the super-systematic factor Y^* in three steps:

Firstly, we determine the default rate $L(U, Y^* \mid U = u, Y^* = y^*)$ of the LHP with recovery rate $R = 0$ in dependence on realizations of both $Y^* = y^*$ and the sectoral component $U = u$:

$$
\begin{aligned}
L(U, Y^* \mid U = u, Y^* = y^*) &= \mathbb{P}\left(D^B = 1 | U = u, Y^* = y^*\right) \\
&= \mathbb{P}\left(Z^B < \Phi^{-1}(\pi^B) | U = u, Y^* = y^*\right) \\
&= \mathbb{P}\left(\varepsilon^B < \frac{\Phi^{-1}(\pi^B) - \sqrt{\rho \cdot \delta} \cdot Y^*}{\sqrt{1-\rho}} \right. \\
&\qquad \left. - \frac{\sqrt{\rho - \rho \cdot \delta} \cdot U}{\sqrt{1-\rho}} | U = u, Y^* = y^*\right) \\
&= \Phi\left(\frac{\Phi^{-1}(\pi^B) - \sqrt{\rho \cdot \delta} \cdot y^* - \sqrt{\rho - \rho \cdot \delta} \cdot u}{\sqrt{1-\rho}}\right)
\end{aligned}
\tag{C.1}
$$

Secondly, we provide the cumulative loss distribution function of the LHP conditional on $Y^* = y^*$:

$$\begin{aligned}
\mathbb{P}\left(L(U, Y^* \mid Y^* = y^*) \leq x\right) &= \mathbb{P}\left(\Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{\rho\cdot\delta}\cdot y^* - \sqrt{\rho - \rho\cdot\delta}\cdot U}{\sqrt{1-\rho}}\right) \leq x\right) \\
&= \mathbb{P}\left(U \leq \frac{\Phi^{-1}(x)\cdot\sqrt{1-\rho} - \Phi^{-1}(\pi) + \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{\rho-\rho\cdot\delta}}\right) \\
&= \Phi\left(\frac{\Phi^{-1}(x)\cdot\sqrt{1-\rho} - \Phi^{-1}(\pi) + \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{\rho-\rho\cdot\delta}}\right)
\end{aligned} \tag{C.2}$$

with $x \in [0, 1]$. By inserting the tranche-specific attachment point A into Equation (C.2) and considering the LHP recovery rate $R \in [0, 1)$, we obtain the conditional default probability $\pi^{T_{[A,D)}}(y^*)$ of tranche $T_{[A,D)}$

$$\begin{aligned}
\pi^{T_{[A,D)}}(y^*) &= \mathbb{P}\left(D^{T_{[A,D)}} = 1 | Y^* = y^*\right) \\
&= \mathbb{P}\left(L(U, Y^* \mid Y^* = y^*) > \frac{A}{1-R}\right) \\
&= 1 - \mathbb{P}\left(L(U, Y^* \mid Y^* = y^*) \leq \frac{A}{1-R}\right) \\
&= \Phi\left(\frac{\Phi^{-1}(\pi) - \sqrt{1-\rho}\cdot\Phi^{-1}\left(\frac{A}{1-R}\right) - \sqrt{\rho\cdot\delta}\cdot y^*}{\sqrt{\rho-\rho\cdot\delta}}\right)
\end{aligned} \tag{C.3}$$

for $A < 1-R$. Since LHP losses are limited to $1-R$, it follows that $\pi^{T_{[A,D)}}(y^*) = 0$ for $A \geq 1-R$.

Appendix D

Proof of Equations (5.9) and (5.16)

Expected losses of tranche $T_{[A,D)}$

To calculate the percentage loss $L^{T_{[A,D)}}$ of tranche $T_{[A,D)}$, we consider Equation (5.4) or as an alternative

$$L^{T_{[A,D)}} = \frac{1}{D-A}\left[(L-A)^{+} - (L-D)^{+}\right], \tag{D.1}$$

where the random variable L denotes the portfolio loss. $(\cdot)^{+} = \max(\cdot, 0)$.

Due to linearity of $\mathbb{E}(\cdot)$, we solely calculate expectations of $\mathbb{E}\left((L-A)^{+}\right)$. Generally, the LHP's recovery rate R may vary but we can reduce our proof on the special case of $R = 0$ since

$$\mathbb{E}\left((L-A)^{+}\right) = (1-R) \cdot \mathbb{E}^{0}\left(\left(L - \frac{A}{1-R}\right)^{+}\right) \tag{D.2}$$

is valid with $\mathbb{E}^{0}(\cdot)$ describing expectations of portfolio losses exhibiting a recovery rate $R = 0$. The latter case can be calculated by

$$\begin{aligned} \mathbb{E}^{0}\left((L-A)^{+}\right) &= \int_{A}^{1} (x-A) \cdot g(x)dx \\ &= 1 - A - \int_{A}^{1} \mathbb{P}(L \leq x)dx, \end{aligned} \tag{D.3}$$

where $g(x)$ with $x \in [0,1]$ is the density function of losses in the LHP. $\mathbb{P}(L \leq x)$ denotes the respective cumulative loss distribution.

Equation (D.3) holds for all cumulative loss distributions $\mathbb{P}(L \leq x)$. Therefore, we insert results of both Equations (5.6) and (C.2) for $\mathbb{P}(L \leq x)$ in order to calculate expected tranche losses in the regarded special case ($R = 0$). Integrating and using the following identity (compare Andersen & Sidenius 2005*b*))

$$\int_{-\infty}^{\gamma} \Phi(\alpha \cdot x + \beta) \cdot \varphi(x) dx = \Phi_2 \left(\frac{\beta}{\sqrt{1+\alpha^2}}, \gamma; \frac{-\alpha}{\sqrt{1+\alpha^2}} \right) \tag{D.4}$$

leads to corresponding expected losses

1. in the basic single-factor model:

$$\mathbb{E}^0 \left((L-A)^+ \right) = \Phi_2 \left(-\Phi^{-1}(A), \Phi^{-1}(\pi); \sqrt{1-\rho} \right). \tag{D.5}$$

2. in the expanded model conditional on Y^*:

$$\mathbb{E}^0 \left((L-A)^+ | Y^* = y^* \right) = \Phi_2 \left(-\Phi^{-1}(A), \tilde{c}; \tilde{\varrho} \right) \tag{D.6}$$

with

$$\tilde{c} = \frac{\Phi^{-1}(\pi) - \sqrt{\rho \cdot \delta} \cdot y^*}{\sqrt{1 - \rho \cdot \delta}} \qquad \text{as well as} \qquad \tilde{\varrho} = \frac{-\sqrt{1-\rho}}{\sqrt{1-\rho \cdot \delta}}.$$

By combining these results, we obtain Equation (5.9) and Equation (5.16).

Appendix E

Proof of Equation (5.25)

Calculating the partial derivative of the bivariate Gaussian cumulative distribution function.

$\varphi(x, \mu, \sigma)$ denotes the Gaussian density function; $\varphi_2(x, y; \rho)$ the bivariate Gaussian density function; $\Phi(x, \mu, \sigma)$ the Gaussian cumulative distribution function, and $\Phi_2(x, y; \rho)$ the bivariate Gaussian cumulative distribution function. μ represents the mean, σ the standard deviation and ρ describes the correlation between X and Y in the bivariate case.

Referring to the bivariate normal distribution, it is commonly known that the conditional probability distribution function is normally distributed. This leads in the bivariate case to:

$$\varphi_2(\overline{x}, y; \rho | \overline{x} = x) = \varphi\left(y, \rho \cdot x, \sqrt{1-\rho^2}\right). \tag{E.1}$$

According to the law of total probability

$$\varphi_2(x, y; \rho) = \varphi(x) \cdot \varphi_2\left(\overline{x}, y; \rho | \overline{x} = x\right), \tag{E.2}$$

we rewrite the bivariate Gaussian density function

$$\varphi_2(x, y; \rho) = \varphi(x) \cdot \varphi\left(y, \rho \cdot x, \sqrt{1-\rho^2}\right). \tag{E.3}$$

Based on these results, the partial derivative of the bivariate Gaussian cumulative distribution function can be calculated as follows:

$$\begin{aligned}
\frac{\partial}{\partial x}\left(\Phi_2(a\cdot x+b,y;\rho)\right) &= \frac{\partial}{\partial x}\left(\int\limits_{-\infty}^{a\cdot x+b}\int\limits_{-\infty}^{y}\varphi_2(\overline{x},\overline{y};\rho)\ d\overline{y}\ d\overline{x}\right)\\
&= \frac{\partial}{\partial x}\left(\int\limits_{-\infty}^{x}\int\limits_{-\infty}^{y}a\cdot\varphi_2(a\cdot\overline{x}+b,\overline{y};\rho)\ d\overline{y}\ d\overline{x}\right)\\
&= \int\limits_{-\infty}^{y}a\cdot\varphi_2(a\cdot x+b,\overline{y};\rho)\ d\overline{y}\\
&= \int\limits_{-\infty}^{y}a\cdot\varphi(a\cdot x+b)\cdot\varphi\left(\overline{y},\rho\cdot(a\cdot x+b),\sqrt{1-\rho^2}\right)\ d\overline{y}\\
&= a\cdot\varphi(a\cdot x+b)\cdot\Phi\left(y,\rho\cdot(a\cdot x+b),\sqrt{1-\rho^2}\right) \quad \text{(E.4)}
\end{aligned}$$

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