Essays on Model Risk

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Kurzzusammenfassung

Die Bedeutung der Modellierung und Analyse von Modellrisiko ist in den letzten Jahre kontinuierlich gestiegen. Dies hängt erstens mit der häufiger werdenden Verwendung ökonometrischer Modelle in der Praxis zusammen. So erhalten diese auf Grund einer im Zuge der Solvabilitätsvorschriften für Banken und Versicherungen wie bspw. Basel II und Solvency II eingetretenen Vertiefung der Risikokultur in den Unternehmen immer häufiger Einzug in die Praxis. Zweitens setzt sich auch die Wissenschaft im Zuge der Finanzkrise wieder verstärkt mit der Risikoanalyse ökonometrischer Modelle auseinander. Diese Dissertation stellt einen Überblick hinsichtlich verschiedener Problemfelder des Begriffs Modellrisiko dar. Folglich werden in vier Aufsätzen unterschiedliche Komponenten des Modellrisikos beleuchtet und analysiert.

Nach einer Einführung in die Thematik in Kapitel 1, wird in Kapitel 2 der Effekt des Katastrophenrisikos auf den Kapitalmarkt untersucht. In einer empirischen Analyse werden die Renditen von Aktienindizes, Zinsen, Swaps sowie Wechselkursen im Hinblick auf ihre Reaktion auf Terroranschläge und Naturkatastrophen analysiert. Als Ergebnis lässt sich festhalten, dass Terroranschläge generell eine stärker Auswirkung auf die Finanzmärkte zu haben scheinen, als Naturkatastrophen. Vor allem Swap- und Wechselkursmärkte reagieren hierbei besonders sensitiv auf die Ereignisse.

Hieran anknüpfend befasst sich Kapitel 3 mit der Relevanz unterdrückter Variablen in ökonometrischen Modellen. Als Beispiel wird hier das Modell von Wilkie [1995], welches in der Versicherungswirtschaft als Standardinvestitionsmodell eine große Rolle spielt, herangezogen. Insbesondere wird die von Wilkie auferlegte Kaskadenstruktur des Modells empirisch validiert. Als Folge der Implementierung von ökonometrischen Tests auf Granger-Kausalität sowie auf unverzögerte Kausalität, kann die Kaskadenstruktur des Modells nicht aufrecht erhalten werden, wodurch das ökonometrische Problem unterdrückter Variablen induziert wird. Folglich werden falsche Schätzergebnisse erzielt, was wiederum Modellrisiko nach sich zieht.

In Kapitel 4 wird nun der Begriff des Modellrisikos genauer definiert und analysiert. Hierbei wird das gesamte Modellrisiko segmentiert in die Kategorien Schätzrisiko, Fehlspezifikationsrisiko in funktionaler Form und Fehlspezifikationsrisiko in Verteilung. Weiterhin werden die verschiedenen Arten von Modellrisiko ökonometrisch in einer operationalen Form definiert. Schließlich werden die Kategorien des Modellrisikos anhand von Inflationsdaten und einem zugehörigen Firmenmodell quantifiziert. Die empirische Analyse ergibt, dass das Fehlspezifikationsrisiko in funktionaler Form den größten Einfluss auf das gesamte Modellrisiko ausübt, während das Fehlspezifikationsrisiko in Verteilung den geringsten Anteil aufzuweisen scheint.

Die Auswirkung der Verteilungsannahme des Störterms auf das Modellrisiko wird in Kapitel 5 thematisiert. Konkret wird die Verteilungsannahme nicht länger als konstant angenommen, sondern es werden Sprünge zwischen verschiedenen parametrischen Verteilungen zugelassen. Hierbei wird die Pearsonverteilung als Brücke zwischen den Sprüngen herangezogen. Wie gezeigt werden kann, führt eine Vernachlässigung von Sprüngen in der Verteilung zu erheblichen Size- und Powereinbußen des CUSUM-Quadrat Tests.

Schlagwörter: Modellrisiko, Wilkiemodell, Ereignisanalyse, CUSUM-Quadrat Test

Short summary

The importance of dealing with model risk has risen substantially over the last few years. This is firstly due to the risen implementation of statistical models in practice. Because of new supervisory rules for banks and insurance companies such as Basel II or Solvency II the firm's risk culture is refined leading to a look into the subject of model risk. Secondly, the financial crisis has strengthened the need for science to deal with the risk analysis of econometric models in a more sophisticated way. Hence in this thesis I give an overview on the topic of model risk where in four papers various components of model risk are exemplified and analyzed.

After an introduction into the field of model risk in chapter 1, chapter 2 deals with the impact of catastrophic events on capital markets. Concretely the reaction of share indices, treasury markets, swap markets and currency markets to terror attacks as well as natural catastrophes is analyzed. For this purpose we seek the cumulative abnormal returns for various estimation and investigation periods. We find that generally terror attacks have a stronger impact on capital markets than natural disasters. As far as the differences in between the markets are concerned it may be stated that the swap and currency markets react most sensitive to the events.

Following chapter 3 deals with the relevance of omitted variables in econometric models. As an example the standard investigation model for insurance companies, the model of Wilkie [1995] is considered. Concretely the imposed cascade structure of the model is empirically validated. By running econometric tests on Granger-causality and instantaneous causality we find evidence that the cascade structure of the model cannot be maintained. Hence we detect an omitted variable bias leading to biased parameter estimates and eventually to model risk.

In chapter 4 the term model risk is defined and analyzed in more detail. Concretely total model risk is subdivided into estimation risk, misspecification risk in functional form and misspecification risk in distribution. Furthermore the various kinds of model risk are defined in an econometric sense. Finally we quantify the various categories of model risk via seeking inflation data and a respective firm model. The empirical analysis yields that misspecification risk in functional form has the strongest influence on total model risk whereas the impact of misspecification risk in distribution seems to be rather low.

The impact of the distribution assumption of the error term on total model risk is dealt with in chapter 5. Concretely the distribution is no longer being held constant but is rather allowed to jump between different families of parametric distributions. In order to model the jumps between the parametric distributions we seek the Pearson distribution. We show that neglecting jumps in the error distribution leads to severe size and power distortions of the CUSUM of squares test.

Keywords: model risk, Wilkie model, event study, CUSUM of squares test

Contents

Chapter 1

Introduction

Introduction

Over the last decades the importance of dealing with the topic of model risk in economics, finance and econometrics has risen substantially. This is basically due to the implementation of new regulatory laws for financial institutions such as Basel II and III or Solvency II. Since then solvency capital requirements need no longer be determined via the hitherto obligatory application of standard methods but the option of implementing internal models has been driven forth. The latter feature the advantage of covering the risen demands of stakeholders concerning the quality of risk management due to a refinement in the institution's risk culture. In the context of the holistic approaches of Basel II and III or Solvency II the balance sheet's forecast distribution is estimated via consultation of company models and stochastic models. With the possible utilization of internal models in order to model market risk the risk measure of the latter depends substantially on the concrete specification of the internal model. Thus there does exist a strong relationship between model risk and the resulting market risk which provides an explanation of why it it extremely important from a regulatory point of view to account for the former.

The challenging part in this topic however is to define and specify the concept of model risk in an operable way. Derman [1996] was one of the first to stress the importance of model risk for financial institutions by stating that models cannot be interpreted as an exact mapping of reality but should rather be treated as an approximation thereof. In the following Crouhy et al. [1998] were among the first trying to yield a statistical definition of model risk stating that "model risk comprises every kind of risk invoked by the application of a statistical model". This definition however features the disadvantage of being rather abstract and thus does not suffice to handle the topic of model risk in an operable way. Rebonato [2001] yields a definition for model risk merely referring to financial instrument and thus neglecting the institution's company model. Kerkhof et al. [2010] along with Sibbertsen et al. [2008] define model risk in a process-oriented way. The latter understand model risk as "every risk induced by the choice, specification and estimation of a statistical model". In this thesis we follow the latter definition of model risk.

Model risk however does not limit itself to a pure regulatory issue but can also be interpreted from an econometric point of view. In fact model risk is highly related to the theory of econometric misspecification analysis. Logically if the internal model is stochastic it has to be specified in the first place resulting in a possibly misspecified model what on the other hand may be interpreted as model risk. Therefore, putting it bluntly, the "academic" issue of specifying an appropriate statistical model for a random variable goes hand in hand with the "practical" issue of understanding model risk in terms of its implication on capital reserves.

In order to follow the idea of the structure of this thesis we may briefly look at model risk from an econometric point of view. The generic econometric process in its most general form is given by $Y = f(Z;\Theta) + u$. On the left hand side of this equation stands the vector of the variables at interest Y. These endogenous variables depend on any kind of variables Z and parameters Θ where the functional form of this relationship is given by $f(\cdot)$. This deterministic relationship is deterred by a stochastic error term u. The generic econometric process may also be interpreted as the internal model (or parts thereof) from which balance sheet positions are derived. Hence by every step in the modeling procedure such as specification of Y, $f(\cdot)$ or Z, estimation of Θ, imposing assumptions on u, collecting the data etc. model risk may be invoked.

In the first two chapters of this thesis we deal with the impact of Z and $f(\cdot)$ on Y and the implied model risk thereof. That is econometrically we deal with the question of causality, i.e. whether or not the relationship between Y and Z is monocausal in the sense that there is no feedback from Y to Z. If we can observe an interaction between the two vectors the model is misspecified with regard to an endogeneity bias which clearly invokes model risk.

Chapter 2 now examines this issue in terms of different areas of application of risk models. Concretely we analyze if the relationship between financial market risk and disaster risk is monocausal in the sense of the latter having an effect on the former via an event studyr. In terms of equation $Y = f(Z;\Theta) + u$ this means that if catastrophic events do have an impact on financial markets it should be included in Z which is not standard in practice. If however the impact of catastrophes on financial markets is neglected model risk in terms of an endogeneity bias due to omitted variables may be invoked. In fact we find that disasters do have a severe impact on financial markets. Concretely we detect that generally terrorist attacks exert a greater influence on capital markets than natural disasters where swap and currency markets as well as the German stock market react more sensitive to those events than bond and U.S. stock markets.

In chapter 3 we no longer deal with interconnections between markets but rather examine the micro structure of the financial market. For this purpose we utilize the benchmark economic scenario generator of Wilkie [1995]. Here six important financial variables are modeled via a so-called cascade model imposing causality restrictions. Hence econometrically the cascade imposes a lower triangular matrix in $f(\cdot)$ which leads to the question of whether $f(\cdot)$ marks a reasonable functional form for the benchmark model. Our findings indicate that the variables are indeed instantaneously correlated such that the assumption of a causal chain cannot be maintained. Again this result stresses the potential of induced model risk if the modeling process is not carried out appropriately.

In chapter 4 we abstract from causality issues in the sense that we look at the generic econometric equation univariately. That is given that the imposed causality structure is modeled correctly we elaborate what can be said about the contribution of different sections of model risk such as estimation risk, misspecification risk in functional form or misspecification risk in distribution to total model risk. Here we first define the different sections of model risk in an econometric sense. Following we carry out an empirical analysis for the inflation rate finding that model risk in functional form features the highest share in overall model risk. Furthermore we examine the downstream company model by using real insurance data discovering a discrepancy of 100-170 mio. € concerning the consequences of model risk of different econometric models on capital reserves. Finally we argue that according to our results the fixed Basel multiplication factor may be too rigid in order to account for model risk and should rather be motivated in connection with the concrete econometric model selected.

Eventually in chapter 5 we deal with the specification of the error term u. Here we discriminate between different kinds of unpredictability concerning u namely the situations where the distribution of the error term remains constant (intrinsic unpredictability), where it switches within a specified distribution class (extrinsic unpredictability of type I) and where it switches such that the distribution class changes (extrinsic unpredictability of type II). After formulating an econometric framework for this setting we analyze the cumulated-sum-of-squares test as an example. By deriving the limiting distribution of this test under extrinsic unpredictability of type II we analyze the properties of this test finding that it features large size and power distortions if the error distribution switches over different distribution classes. Hence we clarify that there is also a huge potential for invoking model risk when it comes to imposing assumptions concerning the error term.

Summing up it may be said that in this thesis we follow a general-to-specific analysis of the generic econometric equation $Y = f(Z;\Theta) + u$ with regard to the topic of model risk. Our results clearly signal that there is a non-negligible potential for model risk in every step of the modeling procedure with severe consequences concerning the resulting monetary effects. Hence our results indicate that an extensive and accurate econometric modeling procedure of the underlying stochastic model seems to be highly advisable.

Chapter 2

The effects of catastrophic events on capital markets

The effects of catastrophic events on capital markets

Co-authored with Gerhard Stahl.

2.1 Introduction

The question of whether catastrophic events have an effect on the performance of capital markets marks a highly important issue in many different fields of economic research such as empirical finance, macroeconometrics or risk management. Modeling capital markets properly requires the identification of important endogenous and exogenous factors that determine the behavior of market variables. One of the most practically relevant examples for an exogenous factor marks a catastrophic event. In this paper we carry out an event study in order to assess the relevance of catastrophic events concerning its impact on the capital market.

The basis of the event study literature was set by Fama et al. [1969]. According to the efficient market hypothesis introduced by Fama [1970] events cause capital market prices to change immediately after the information input in case the former are considered to impact the future development of the market variable. That is capital market prices reflect future expectations of the variable at interest perfectly. Hence if a change in prices after an event can be observed the latter serves as an explanatory variable for the market at interest.

Whether or not a dependence of catastrophic events on capital markets can be acknowledged marks also a very important topic towards a holistic approach in modern risk management. An isolated specification of internal models for catastrophic risk and capital market risk may lead to highly misleading conclusions if present dependencies between those two areas are neglected. In other words the catastrophic risk model's information and results should find consideration in the capital market model in order to improve the accuracy of the latter.

The existing literature mainly deals with the impact of catastrophic events on stock markets. Brounen and Derwall [2010] analyzed the impact of terrorist attacks on international stock markets. They found that there is a negative price reaction of international stock markets following terrorist attacks. Compared to the effects of earthquakes the consequences of terrorist attacks are more pronounced. The rebound of prices is stated to be one week. Additionally Chen and Siems [2004] investigated the return of the Dow Jones Industrial Average stock index to several terrorist attacks dating from 1915 until 2001. They conclude that although in the short-run an impact can be confirmed the intensity of terrorist attacks on stock markets has decreased with time. According to the authors this can be explained by more liquid and stable banking and financial sectors. Karolyi [2006] gives an overview of the existing literature concerning terrorist attacks and stock markets.

The consequences of natural disasters on capital markets have been studied by Koerniadi et al. [2011]. The authors differentiate between various kinds of natural disasters. Concretely they find that the impact of earthquakes, hurricanes and tornados on the capital market prevails for several weeks whilst floods, tsunamis and volcanic eruptions are playing a minor role. Worthington and Valadkhani [2004] analyze the effects of natural disasters on the Australian stock market using time series methods in order to model the abnormal returns. They identify wildfires, cyclones

and earthquakes to impact the stock market severely.
Our work extends the existing literature in three respects. First we look at a broad range of catastrophic events such as natural catastrophes and terrorist attacks and are therefore able to categorize the impact of an event more precisely. Further we do not solely concentrate on one specific market but take various financial variables into account and are thus able to draw conclusions about the sensitivity of the market to the character of the event. To our knowledge we are the first to consider reactions of swap markets as well as currency markets to catastrophic events. Finally different estimation periods are considered. Here we show that the specification of the estimation period can indeed lead to different conclusions about the significance of the event. This topic has been neglected thus far in the existing literature.
The remainder of the paper is organized as follows. After a description of the data we briefly sum up the main developments in the event study methodology in section 2.3. Following we present our results. Here we first look at aggregated events while later in this chapter the events' impacts are regarded on an individual basis. Section 2.5 concludes.

2.2 Data

Our data set includes 10 financial time series being taken from *Datastream*. Concretely we look at daily data for share prices indices (German DAX 30 and S&P 500), interest rates (U.S. and German bond yields, 5 and 10 years), U.S. and EURO swaps (1 year, to LIBOR and to EURIBOR) and currency exchange rates ($\€ and $\¥). Detailed information about the series are given in table 2.1.

Series	Start	End	Observations	Mean Return
DAX 30	01.01.1990	09.09.2011	5660	0.0123
S&P 500	01.01.1990	09.09.2011	5660	0.0007
U.S. Treasuries (5Y)	01.01.1990	09.09.2011	5660	-0.0215
U.S. Treasuries (10Y)	01.01.1990	09.09.2011	5660	-0.0133
German Treasuries (5Y)	01.01.1996	09.09.2011	4094	-0.0229
German Treasuries (10Y)	01.01.1996	09.09.2011	4094	-0.0158
U.S. Swaps	06.01.1997	09.09.2011	3829	-0.0401
EURO Swaps	04.01.1999	09.09.2011	3309	-0.0165
$\€	04.01.1999	09.09.2011	3309	0.0066
$\¥	01.01.1990	09.09.2011	5660	0.0133

Table 2.1: returns start date, end date, the number of observations and the mean daily return over the whole period in per cent for each series.

Our objective is to look at the changes in returns of these time series according to the occurrence of a specific event. Thereby we look at natural catastrophes (hurricanes, earthquakes) as well as terrorist attacks. The events we utilize are listed in table 2.2.
The procedure of the event study can then be described as follows. At first the daily returns are

No.	Event	Date
	– Natural Disasters –	
1	Typhoon Mireille	27.09.1991
2	Hurricane Andrew	23.08.1992
3	Earthquake Northridge	17.01.1994
4	Winter storm Lothar	25.12.1999
5	Hurricane Charley	11.08.2004
6	Hurricane Ivan	02.09.2004
7	Hurricane Katrina	25.08.2005
8	Hurricane Rita	20.09.2005
9	Hurricane Wilma	19.10.2005
10	Winter storm Kyrill	18.01.2007
11	Hurricane Ike	06.09.2008
12	Earthquake Japan	11.03.2011
	– Terrorist Attacks –	
13	Terror attack Oklahoma	19.04.1995
14	Terror attack WTC	11.09.2001
15	Terror attack Madrid	11.03.2004
16	Terror attack London	07.07.2005

Table 2.2: returns the date of the events being subdivided into natural catastrophes and terrorist attacks.

constructed for each series after which the abnormal returns are determined as the difference of the actual returns and the expected returns. This is done in two steps. Firstly the sample is subdivided into an estimation period (prior to the event) and an investigation period (after and including the event date). In the estimation period the estimation of the expected returns is carried out. Following the abnormal returns are constructed for the investigation period and significance tests are run.

2.3 Methodology

Let $j = (1, \ldots, J = 10)$ denote the index of the time series running from $t_{1,j}$ until $t_{T,j}$, i.e. $t_j = (t_{1,j}, \ldots, t_{T,j})$. Setting $t_{1,j} = 1\ \forall j$ the length of the jth series is given as $\tau_{1,j} = t_{T,j}$. Furthermore let $t_{e_i,j}$ be the event date for event $i = (1, \ldots, I = 16)$. Then the estimation period for the jth series concerning event i is defined as $[t_{1+h_1,j}, \ldots, t_{e_i-h_2,j}]$ with $h_1 \in \mathbb{Z}_0^+$, $h_2 \in \mathbb{Z}^+$ and $\tau_{2,j} = t_{e_i-h_2,j} - t_{1+h_1,j} \geq 1$, i.e. $h_1 \leq e_i - h_2 - 2$. The investigation period is given by $[t_{e_i,j}, \ldots, t_{e_i+h_3,j}]$ where $h_3 \in \mathbb{Z}_0^+$ and $\tau_{3,j} = t_{e_i+h_3,j} - t_{e_i,j} = h_3$. We can then determine the abnormal returns.
The percentaged daily return of variable x_j at time t is given as $\tilde{x}_{t,j} = (x_{t,j} - x_{t-1,j}) \cdot 100 / x_{t-1,j}$. From a statistical point of view the return is modeled as a random variable with constant expected value $E(\tilde{x}_{t,j}) = \mu_j\ \forall\ t = 1, \ldots, T$ plus some additive noise $\varepsilon_{t,j}$ where $\varepsilon_{t,j} \overset{iid}{\sim} N(0, \sigma_j^2)\ \forall t$. Hence the

constant-mean-return model for the return is given as

$$\tilde{x}_{t,j} = \mu_j + \varepsilon_{t,j}$$

where $E(\varepsilon_{t,j}) = 0 \ \forall \ t, j$ and $V(\varepsilon_{t,j}) = \sigma_j^2 \ \forall \ t$. The objective is now to analyze whether or not the event at interest results in an abnormal return. The latter can be defined as the actual return minus the expected return leading to

$$\hat{x}_{t,j} = \tilde{x}_{t,j} - E(\tilde{x}_{t,j}) \quad \text{for} \quad t \geq e_i.$$

Note that the abnormal return is solely defined within the investigation period. $E(\tilde{x}_{t,j}) = \hat{\mu}_j$ is estimated by the estimation period's sample average, i.e.

$$\hat{\mu}_j = \frac{1}{\tau_{2,j}} \sum_{k=h_1}^{e_i-h_2} \tilde{x}_{1+k,j}$$

resulting in $\hat{x}_{t,j} = \tilde{x}_{t,j} - \hat{\mu}_j$. For the abnormal return then (cf. Brown and Warner [1985])

$$\begin{aligned}
E(\hat{x}_{t,j}) &= 0 \quad \forall t, j, \\
Cov(\hat{x}_{s,j}, \hat{x}_{t,j}) &= \begin{cases} C_{t,j} \cdot \sigma_j^2 & , \quad s = t \\ 0 & , \quad s \neq t \end{cases} , \quad \text{with} \\
C_{t,j} &= 1 + \frac{1}{\tau_{2,j}} + \frac{\hat{x}_{t,j}^2}{\sum_{k=1}^{\tau_{2,j}} \hat{x}_{k,j}^2}.
\end{aligned}$$

Since with the assumption of the returns being normally distributed $\hat{x}_{t,j}/(\sigma_j \sqrt{C_{t,j}}) \sim N(0,1)$ and with $\hat{\sigma}_j^2 = \frac{1}{\tau_{2,j}-1} \sum_{k=h_1}^{e_i-h_2} \hat{x}_{1+k,j}^2$ it follows that

$$\frac{\hat{x}_{t,j}}{\hat{\sigma}_j \sqrt{C_{t,j}}} \sim t(\tau_{2,j} - 1). \tag{2.1}$$

Hence the significance of the abnormal return can be tested via a simple t-test with the test statistic being given by (2.1).

In practice however, it is often more interesting to see if the abnormal returns are different from zero over a specific time period, i.e. over which period the event shocks the market and at which time the shock finally decays. Thus it is rather looked at the cumulative abnormal returns $\sum_{t=e_i}^{e_i+h_3} \hat{x}_{t,j}$ with $h_3 \geq 1$. In this case $\frac{1}{\sqrt{h_3}} \sum_{t=e_i}^{e_i+h_3} \frac{\hat{x}_{t,j}}{\sigma_j \sqrt{C_{t,j}}} \sim N(0,1)$ and thus

$$\frac{1}{\sqrt{h_3}} \sum_{t=e_i}^{e_i+h_3} \frac{\hat{x}_{t,j}}{\hat{\sigma}_j \sqrt{C_{t,j}}} \sim t(\tau_{2,j} - 1). \tag{2.2}$$

Again the significance of the cumulative abnormal returns can be tested via application of a t-test based on (2.2).

Finally it may also be interesting to not only look at the effect of an individual event but to additionally regard the impact of multiple events jointly. We differentiate between the joint effect of natural disasters on capital markets and compare them to the impact of terrorist attacks. Hence we split the set of events into natural disasters with $\tilde{I}_1 = (1, \ldots, I_1 = 12)$ and terrorist attacks with $\tilde{I}_2 = (13, \ldots, I_2 = 16)$. Averaging over the events leads to

$$\breve{x}_{\tilde{t},j}^{(k)} = \frac{1}{I_k} \sum_{i \in \tilde{I}_k} \hat{x}_{e_i+\tilde{t},j}, \qquad k \in \{1,2\},\ \tilde{t} = (0, \ldots, h_3).$$

Normalizing the event dates to zero yields the vector of aggregated abnormal returns for the jth variable concerning the kth aggregate

$$\breve{x}_{\tilde{t},j}^{(k)} = (\underbrace{\breve{x}_{-\tau_{2,j},j}^{(k)}, \ldots, \breve{x}_{-h_2,j}^{(k)}}_{\text{Estimation Period}}, \ldots, \underbrace{\breve{x}_{0,j}^{(k)}, \ldots, \breve{x}_{h_3,j}^{(k)}}_{\text{Investigation Period}}). \tag{2.3}$$

The cumulative abnormal returns for aggregated events are then given by forming the sum over the investigation period such that $\sum_{\tilde{t}=0}^{h_3} \breve{x}_{\tilde{t},j}^{(k)}$. Straightforwardly the test statistic for the latter expression can then be determined according to (2.2) by replacing the time indices in respect of (2.3).

2.4 Results

We analyze the significance of abnormal returns for each financial variable listed in table 2.1. Concretely we look at the daily returns, i.e. the percentage change of the return at the event date as well as the weekly/monthly/half-yearly accumulated returns, i.e. the accumulated returns over 5/20/120 trading days. Thus we consider 4 different investigation periods. To get an overview of the results we first determine the accumulated returns at an aggregated level in section 2.4.1, i.e. we merge the abnormal returns of all natural catastrophes and of all terrorist attacks in the sample. Afterwards in section 2.4.2 we look at the effects of each individual event on each financial variable. Finally the impact of the choice of the estimation period on the results is dealt with in section 2.4.3.

2.4.1 Aggregated Events

In order to calculate the abnormal returns we impose three different kinds of estimation periods: $\tau_2 \in \{10, 100, 450\}\ \forall j$ where we choose the latter as the longest possible joint estimation period for all data. Throughout the whole analysis we set $h_2 = 1$. This seems to be a plausible assumption in this context as the events we consider are completely exogenous and hence cannot be foreseen by market participants which allows us to calculate the mean returns utilizing data as close as possible to the event date. Table 2.3 returns the significance of the cumulated abnormal returns for each market differentiated by the estimation period as well as the investigation period.

	Time Series									
h_3	DAX	$S\&P$	U.S. Gov5	U.S. Gov10	Ger Gov5	Ger Gov10	U.S. Swap	EUR Swap	U.S. EUR	U.S. JPY
	– Natural Disasters –									
	$\tau_2 = 10$									
Day	-0.2	0.3*	0.5	0.2	0.0	0.1	0.8	0.0	0.1	0.2*
Week	0.0	0.4**	-0.2	-0.1	0.8	0.8	1.6**	0.2	0.8*	0.8***
Month	2.9	0.6*	4.3**	2.7**	3.2*	2.5*	6.8***	1.3	1.7***	0.1**
6 Months	14.9***	4.2**	13.7***	6.4***	-7.4	-2.8	18.4***	-10.9**	12.2***	3.4***
	$\tau_2 = 100$									
Day	-0.4	0.2	0.3	0.1	0.0	0.1	0.6	0.0	0.0	0.2
Week	-0.8***	0.1	-1.2	-0.6	0.7	0.6	0.4	0.3	0.5	0.8***
Month	0.0	-0.6*	0.2	0.8	2.7	1.8	1.9*	1.5	0.4*	0.0
6 Months	-2.9***	-3.1***	-10.9***	-4.6*	-11.0	-7.0	-11.0	-9.5***	4.3***	3.4***
	$\tau_2 = 450$									
Day	-0.4	0.2	0.3	0.1	0.1	0.1	0.7	0.0	0.0	0.2
Week	-0.7*	0.0	-0.9	-0.4	1.0	0.8	0.8	0.7	0.4	0.7**
Month	0.3	-0.8	1.7	1.5	4.0***	2.6**	3.6***	3.4***	-0.4	-0.6
6 Months	-0.9**	-4.3***	-1.8	-0.3	-2.9	-2.3	-0.6	-2.7*	0.0	0.3*
	– Terrorist Attacks –									
	$\tau_2 = 10$									
Day	-3.3**	-0.1	0.3	0.1	-1.5*	-0.2	-2.5*	-1.2**	1.0*	0.1
Week	-0.7	1.0*	-1.0	-0.1	-0.9	-0.1	-3.9**	-2.5***	2.3**	0.8
Month	9.2**	7.3***	7.0*	4.7*	1.8	1.8	-1.0**	-5.2***	5.3***	-0.5
6 Months	46.7***	37.7***	44.6***	27.2***	18.3*	10.3*	39.7**	3.4***	19.2***	-12.1**
	$\tau_2 = 100$									
Day	-3.6***	-0.4	0.0	0.0	-1.5*	-0.1	-2.6***	-1.1*	0.9*	0.1
Week	-2.2***	-0.4	-2.1	-0.7	-0.6	0.0	-4.6***	-1.7***	1.5***	0.8
Month	3.0*	1.5	2.6	2.4	2.9	2.0	-3.9	-2.0***	2.1***	-0.6***
6 Months	9.6	3.0	17.8***	13.3***	25.3***	11.7***	22.7	22.7***	0.3***	-12.9***
	$\tau_2 = 450$									
Day	-3.6***	-0.4	0.0	0.0	-1.5**	-0.2	-2.7***	-1.1**	0.8**	0.1
Week	-2.3***	-0.4	-2.5*	-1.0	-0.9	-0.1	-4.9***	-2.0***	1.3***	0.9
Month	2.4**	1.7	0.9	1.1	1.8	1.6	-5.0***	-3.2***	1.5***	-0.2
6 Months	6.3*	4.0	8.0	5.3*	18.4**	9.1**	15.9	15.4	-3.2	-10.5***

Table 2.3: displays the cumulated abnormal returns in % for the respective series after natural catastrophes and terrorist attacks. The investigation period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is mirror by τ_2. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2) with respect to (2.3).

Concentrating first on the short-term (daily, weekly) effects of the aggregated events in table 2.3

it is obvious that terrorist attacks tend to have a higher impact on financial variables than natural catastrophes. This discrepancy becomes even more visible the longer the estimation period is chosen. We especially do not observe any impact at all of natural catastrophes on financial variables at the event day for $\tau_2 = 450$ whereas at the same time the behavior of 5 variables (DAX, German 5-year government bonds, U.S. swaps, EURO swaps, \$/€ exchange rate) seems to be clearly influenced by terrorist attacks. This can also be seen by regarding figure 2.1.

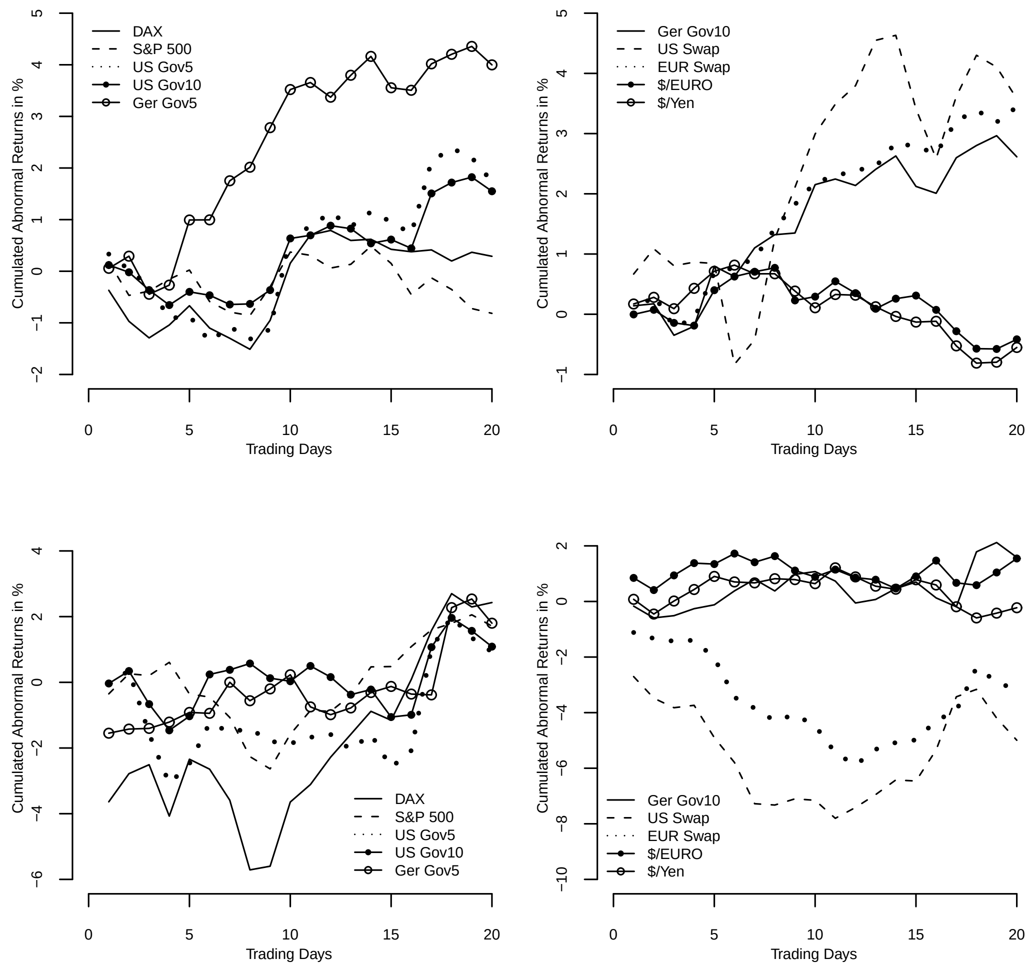

Figure 2.1: illustrates the development of the cumulated abnormal returns after and including the event days concerning natural catastrophes in the first row and concerning terrorist attacks in the second row for the respective series with $\tau_2 = 450$.

Looking at the share price indices the DAX primarily reacts to terrorist attacks whilst the S&P 500 seems relatively robust to both kinds of events. The four bond rates do not show a reaction to natural catastrophes at all which with very few exceptions also seems to be valid for terrorist attacks. Hence the long-term interest rates as well as the S&P 500 tend not to be affected by exogenous market shocks in the short run.

This statement however cannot be sustained for swaps and currencies. Here we observe highly significant negative reactions of U.S. swaps and EURO swaps to terrorist attacks which should be of no surprise as interbanking rates decrease due to negative market shocks. Again natural catastrophes do not seem to play an important role. Concerning exchange rates however the results are not as clear. Whilst there is no reaction of the $/€ ratio to natural catastrophes terrorist attacks have a positive impact on the exchange rate signifying a devaluation of the dollar. This may be explained by the huge impact of 9/11 on the dollar absorbing the effects of the remaining events (cf. also tables 2.5-2.10). The very different behavior can be observed for the $/¥ ratio. Here we do not observe a reaction to terrorist attacks whereas natural catastrophes are relevant. Actually the dollar devaluates in relation to the yen after natural disasters. An explanation may again be given by regarding the cumulated abnormal returns for the individual events. Here we detect a crucial devaluation of the dollar after the atomic catastrophe in Fukushima in March 2011 which may be explained by repatriation effects of Japanese insurers. Concerning the medium-term (monthly) effects the conclusions can roughly be maintained except for two cases. Firstly the U.S. swaps now seem to entail positive abnormal returns after natural catastrophes and secondly the impact of natural catastrophes on the $/¥ rate decays in the medium-term which seems plausible. The first finding however seems counterintuitive and may be explained by the fact that most of the analyzed natural catastrophes occurred between 2004 and 2007 where U.S. swaps rose quickly. That is the negative effects of the event may have been overcompensated by the market dynamics which can be supported by the merely weak significance of the swap's abnormal monthly cumulated return for $\tau_2 = 100$.

When it comes to the long-term (half-yearly) effects the interpretation becomes rather difficult as it seems very likely that after 6 months the effect of the event has already decayed and other (exogenous) factors or market dynamics are driving the series. Furthermore the results differ highly with τ_2. Hence it seems most plausible to look at the long-term behavior of the abnormal returns when the latter have been determined under consideration of the long-term behavior of the expected returns, i.e. with a large estimation period ($\tau_2 = 450$). In terms of natural catastrophes we solely detect an unambiguous negative long-term effect for the share price indices. Terrorist attacks on the other hand lead to a long-term increase in German government bonds and to a valorization of the dollar concerning the yen.

Summing up it may be said that we can observe a short- and medium-term effect of terrorist attacks on swap markets, the German stock market and the $/€ currency market. This effect does however decay after several months. In the long-run the German government bond market and the $/¥ currency market are affected by terrorist attacks. Concerning natural catastrophe we do not detect short-and medium-term effects except for the $/¥ value. In the long-run the stock markets seem to be negatively affected.

2.4.2 Disaggregated Events

In this section we analyze the impact of each individual event on the various markets. We change the estimation periods slightly to $\tau_2 \in \{10, 100, \tilde{T}_j\}$ where $\tilde{T}_j$ indicates the maximum number of observations for variable j meaning that we estimate the sample mean with data reaching from the beginning of the respective series up to one day before the event, i.e. setting $h_{1,j} = 0 \ \forall j$.

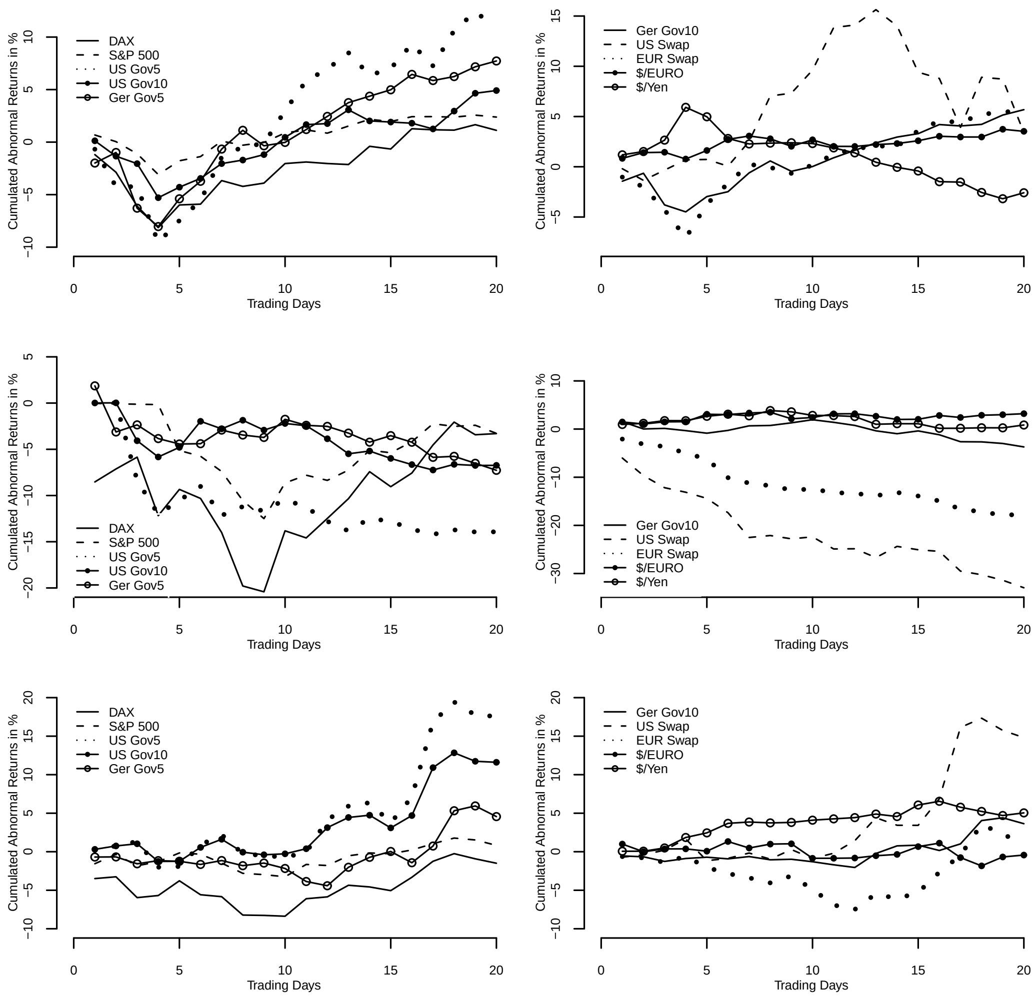

Figure 2.2: illustrates the development of the cumulated abnormal returns after and including the event day concerning the earthquake in Japan in the first row, the terrorist attacks on the World Trade Center in the second row and the terrorist attacks in Madrid in the third row for the respective series with $\tau_2 = 450$.

Tables 2.5-2.10 (cf. section 2.6) display the cumulated abnormal returns for each event. In the following we solely concentrate on unambiguous effects, i.e. we merely consider those effects that are at least significant on the 95% level for all estimation periods in order to derive clear patterns and conclusions.

By regarding the short-term (daily, weekly) effects of the individual events it becomes clear that independently of the estimation period's length the earthquake in Japan in March 2011 and the terrorist attacks on the World Trade Center and the Pentagon in September 2001 clearly have the highest impact on capital markets followed by the Madrid bombings in March 2004 (cf. also figure 2.2). After the former two events we observe negative abnormal returns in the German stock market and in the EURO swap market as well as positive abnormal returns in both currency markets. Additionally there are negative abnormal returns concerning 5-year and

10-year German government bonds following Fukushima and negative returns in the U.S. swap market after 9/11.

Another striking event for the bond market seems to have been the hurricane Ivan in September 2004. We observe significantly increasing returns in the 5-year and in the 10-year bond markets for both U.S. and German bonds. We could not detect any unambiguous impact at all for the storms Lothar, Charley, Katrina and Kyrill as well as for the earthquake in Northridge.

As far as the monthly impact of natural disasters is concerned we only observe very few unambiguous significant effects. In fact we solely observe four significant impacts: increases in the variables \$/¥ ratio, EURO swaps, U.S. swaps and \$/€ ratio concerning the events typhoon Mireille, hurricane Ivan, hurricane Ike and the earthquake in Fukushima.

In contrast we observe many series to be affected by the terrorist attacks of New York and Madrid. E.g. both currency ratios increased after the attacks whilst the S&P 500 decreased following the attack on the World Trade Center and the DAX declined after the Madrid bombings. Furthermore a decline in the swap markets after 9/11 and an increase in U.S. government bonds after the terrorist attack in Madrid can be observed. Hence we can state that there is a short-term and a medium-term effect of 9/11 on the stock markets, the currency markets and the swap markets. The effect of the earthquake in Fukushima is present in the short-run and decays in the medium term while the opposite seems to be true concerning the bombings of Madrid in 2004.

2.4.3 Sensitivity concerning the Estimation Period

In the literature the specification of the length of the estimation period τ_2 has been handled rather heterogeneously. The chosen values for τ_2 are ranging from rather low numbers of $\tau_2 = 20$ in Chen and Siems [2004] or $\tau_2 = 90$ in Brounen and Derwall [2010] to very high values of $\tau_2 = 239$ or $\tau_2 = 256$ in Brown and Warner [1985] and Cowan and Sergeant [1996].

The influence of the choice of the estimation period on the number of significant abnormal returns is displayed in table 2.4. Regarding the absolute values first we observe for each estimation period an increase of significant returns when the investigation period is enlarged. This may be explained by the fact that the longer the investigation period gets the more the variable at interest gets influenced by additional factors. Hence with an increasing investigation period the impact of the event might not be distinguished with other events or market dynamics which may lead the test to falsely signal the event to exert an influence on the variable.

Furthermore it can be seen that the overall amount of significant abnormal returns decreases with the length of the estimation period. Concretely the number declines from 212 for $\tau_2 = 10$ to 185 for $\tau_2 = 100$ and finally to 165 when the whole period of available data is utilized, i.e. for $h_1 = 0$. In other words we detect fewer significant returns when τ_2 is extended. This decline is driven by the long-term (one month or longer) investigation periods. As far as the event day is concerned we observe a remarkably low amount of abnormal returns for $\tau_2 = 10$. Hence the amount of significant cumulated abnormal returns tends to reduce when the length of the estimation period is chosen in correspondence with the length of the investigation period.

According to the percentaged changes in the number of significant abnormal returns the test

Estimation Period		Investigation Period Day	Week	Month	6 Months	Overall
$\tau_2 = 10$		7	34	65	106	212
$\tau_2 = 100$		22	36	48	79	185
$\tau_2 = \tilde{T}_j$		13	31	47	74	165
Change from...	to...					
$\tau_2 = 10$	$\tau_2 = 100$	214%	6%	-26%	-25%	-13%
$\tau_2 = 100$	$\tau_2 = \tilde{T}_j$	-41%	-14%	-2%	-6%	-12%
$\tau_2 = 10$	$\tau_2 = \tilde{T}_j$	86%	-9%	-28%	-30%	-22%

Table 2.4: the upper part mirrors the absolute number of significant abnormal returns for all events and variables at the at least 95% level differentiated by estimation and investigation periods. The lower part mirrors the percentaged changes in the number of significant abnormal returns for all events and variables at the at least 95% level when the estimation period is increased.

results seem to be relatively robust to a change in the estimation period when the investigation period is chosen to be one week. In contrast we observe a huge sensitivity for investigation periods of one day, one month and six months. The sensitivity appears to be highest concerning the enlargement of the estimation period from 10 trading days to 100 trading days. Intuitively it seems to make most sense to determine the expected returns according to the length of the investigation period in order to account for the relevant dynamics of the series, i.e. specifying the length of τ_2 correspondingly to h_3. Table 2.4 illustrates that the choice of the estimation period substantially influences the test results. This argument is especially valid for very low or high values of h_3.

2.5 Conclusion

In this paper we analyze the impact of natural disasters as well as terrorist attacks on capital markets by means of event study methodology. We find that generally terrorist attacks exert a greater influence on capital markets than natural disasters. As far as the differences in between the markets are concerned it may be stated that the swap and currency markets as well as the German stock market react rather sensitive to events whereas bond markets and the U.S. stock markets tend to be more robust.

Regarding the individual events we identify the earthquake in Japan in 2011 and the terrorist attacks on the World Trade Center and the Pentagon as most devastating. This contradicts in a way the findings of Chen and Siems [2004] stating that due to a more stable banking sector the impact of events that are dated back longer ago is found to be heavier. In fact if an unambiguous effect of an event is detected we cannot confirm the result that prices bounce back very quickly. Actually we find the impact not to decay for at least a few weeks time in the majority of cases. Furthermore the specification of the estimation period tends to play a great role for the results of the significance tests. We find that the selection of a longer estimation period reduces the number of significant returns substantially which especially holds true concerning long investigation

periods. In regard of a suitable consideration of the variable's dynamics it seems appropriate to choose not too large a distance between the length of estimation and the investigation period.
In terms of risk management approaches our findings indicate that a priori capital market risk should not be modeled independently of catastrophic risk. Our results signal that subject to the type of the market and to the length of the relevant interval the two risk categories are indeed interrelated. Hence a preceding analysis taking into account which variables might be affected over which time period seems reasonable in this case.

2.6 Appendix

	Time Series									
			U.S.	U.S.	Ger	Ger	U.S.	EUR	U.S.	U.S.
h_3	DAX	$S\&P$	Gov5	Gov10	Gov5	Gov10	Swap	Swap	EUR	JPY
	– Typhoon Mireille, 27.09.1991 –									
Day	-0.7	-0.1	-0.8*	-0.8**	-	-	-	-	-	0.6
Week	-0.4	-0.4	-0.9**	-0.5*	-	-	-	-	-	2.6**
Month	-1.2*	0.1	2.1*	4.0	-	-	-	-	-	2.2**
6 Months	15.0	7.3*	20.8	18.2***	-	-	-	-	-	0.8**
	– Hurricane Andrew, 23.08.1992 –									
Day	-0.8	-0.9	3.2**	2.3**	-	-	-	-	-	0.7*
Week	2.7	0.5	1.7**	1.4**	-	-	-	-	-	1.2
Month	16.0***	3.9*	-1.3	-3.1	-	-	-	-	-	-2.2
6 Months	76.5***	19.6***	7.2*	-7.6	-	-	-	-	-	-18.0***
	– Earthquake Northridge, 17.01.1994 –									
Day	0.4	-0.5	0.1	0.1	-	-	-	-	-	-0.1
Week	-0.3	-0.9	-0.8	-0.4	-	-	-	-	-	-0.8
Month	9.0**	-4.6**	7.1	5.4	-	-	-	-	-	1.8
6 Months	63.2***	-27.7***	47.8***	36.5***	-	-	-	-	-	4.9
	– Winter storm Lothar, 25.12.1999 –									
Day	-0.3	-0.4	-0.7	-0.7	-0.2	0.3	-0.3	0.5	-0.3	0.0
Week	-2.8	-0.7	-2.2**	-2.2**	0.1	-0.1	-1.9**	-0.9	-1.0	0.1
Month	-18.1***	-6.8**	-5.5***	-5.0***	3.4	-2.1	-4.6***	-4.5	-1.3	-2.6
6 Months	-120.8***	-33.4***	-64.1***	-69.0***	-37.0***	-51.6***	-29.8***	-8.7*	-9.8**	-5.1**
	– Hurricane Charley, 11.08.2004 –									
Day	-0.9	-0.2	0.5	0.5	1.6	1.1	3.9	1.7*	-0.2	0.5
Week	0.8	1.0	0.8	1.5	3.5*	2.6*	7.2*	3.4**	0.3	1.5*
Month	9.3*	6.7*	16.4**	12.9**	15.0***	10.7***	30.5***	18.8***	-3.8*	2.6*
6 Months	42.1***	25.2***	110.2***	80.7***	62.2***	41.5***	166.2***	78.8***	-11.0**	11.7***
	– Hurricane Ivan, 02.09.2004 –									
Day	0.2	1.0*	2.3*	1.9*	0.6	0.5	1.8	1.0	-0.1	-0.1
Week	0.5	0.4	3.0**	2.2**	5.9**	3.5**	6.6**	5.8***	0.5	0.0
Month	-2.2	-1.1	6.7**	4.0**	2.2*	0.2	6.4**	4.7***	3.4	-1.3
6 Months	-15.6*	-2.6	45.0***	29.9***	2.5*	-4.3	22.4***	1.2**	20.4***	4.0
	– Hurricane Katrina, 25.08.2005 –									
Day	-1.1	0.4	0.3	0.0	-0.5	-0.4	0.4	-0.3	0.3	0.1
Week	-1.0*	1.7	-2.3	-1.7	1.0	0.6	-2.0	0.6	1.0	-0.5
Month	2.2	3.3**	8.1**	8.3**	8.7**	6.3**	1.4**	4.4**	1.6*	-1.7
6 Months	32.5***	23.8***	64.4***	61.4***	96.5***	74.8***	25.7***	55.4***	9.0**	-11.4***
	– Hurricane Rita, 20.09.2005 –									
Day	0.7	-0.9	0.2	-0.6	-0.4	-0.2	-0.2	-0.7	0.2	-0.2
Week	1.3	-1.8**	-0.3	-1.5*	0.8	0.4	0.9	-0.4*	1.1	0.5
Month	0.4	-5.5***	-2.5	-5.0**	3.2	3.5	-1.2	1.2	5.6**	1.2
6 Months	11.5	-9.0***	-46.2***	-52.0***	-12.9	-2.4	-35.7***	-11.2**	38.5***	20.3***

Table 2.5: displays the cumulated abnormal returns in % for the respective series after natural disasters. The time period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is given by $\tau_2 = 10$. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2).

	Time Series									
τ_2	DAX	$S\&P$	U.S. Gov5	U.S. Gov10	Ger Gov5	Ger Gov10	U.S. Swap	EUR Swap	U.S. EUR	U.S. JPY
	– Hurricane Wilma, 19.10.2005 –									
Day	-1.7*	1.8*	-0.6	-0.6	-1.7*	-1.2	-0.5	-1.3*	0.2	0.4
Week	0.4	3.1**	0.1	-0.5	-1.2**	-1.4*	0.2	-0.4	1.0	1.1
Month	10.9***	10.3***	-0.4	-2.9**	2.5	-0.4	2.7**	2.9	-2.8	-0.5
6 Months	65.0***	46.6***	-20.9***	-19.4***	-21.2***	-25.0***	-1.0	-14.0**	-2.2	12.4***
	– Winter storm Kyrill, 18.01.2007 –									
Day	-0.2	-0.4	-0.8	-1.0	0.7	0.5	0.1	0.2	0.3	-0.4
Week	0.6	0.2	-0.7	-0.8	0.0	-1.0	-0.1	0.1	1.0*	0.2
Month	3.5	-0.2	-6.4**	-6.4**	-2.1	-2.8	-2.8*	-0.9	4.9***	2.0
6 Months	17.0**	-5.2	-27.2***	-25.9***	-7.0*	-13.0***	-17.6***	-0.6	25.6***	11.0***
	– Hurricane Ike, 06.09.2008 –									
Day	2.6	2.4*	2.0	0.8	3.0	2.5*	2.1**	0.8	-0.6	-0.6
Week	3.4*	2.8	4.5	5.0*	7.6**	7.2***	1.8	1.7*	1.5	-1.2
Month	1.6	-3.3	9.1	12.7**	8.6**	9.2***	19.9***	-2.7	3.9	-1.9
6 Months	4.0	6.1	49.8*	49.4***	30.7***	37.5***	-20.8	-81.2***	33.5***	-10.8*
	– Earthquake Japan, 11.03.2011 –									
Day	-1.1	0.8	-0.3	0.2	-3.0	-1.8	0.1	-1.9	0.8	1.3**
Week	-5.3***	-1.2	-5.6	-3.7	-10.4**	-4.9**	2.0	-8.2**	1.7**	5.6***
Month	3.7	4.6	18.5	7.1	-12.3**	-1.9	8.5	-12.3**	3.8***	0.1*
6 Months	-10.9	-0.4	-22.0	-25.3	-180.8***	-83.1***	56.5*	-118.0***	5.3***	21.4 ***
	– Terror attack Oklahoma, 19.04.1995 –									
Day	-0.7	-0.1	0.9*	0.4	-	-	-	-	-	-1.5
Week	2.1	1.3*	1.6**	0.2	-	-	-	-	-	-5.0**
Month	7.1**	4.4***	-0.7**	-4.3	-	-	-	-	-	-20.6***
6 Months	11.5***	14.0***	21.7***	1.1	-	-	-	-	-	-103.3***
	– Terror attack WTC, 11.09.2001 –									
Day	-7.1**	-0.8	0.1	-0.4	2.2	1.6*	-5.4*	-1.7	1.6	1.0*
Week	-2.0	0.0	-0.4	-0.9	-2.5	-0.2	-11.5***	-4.5**	3.6*	3.1***
Month	26.3**	-1.7**	-1.6	-2.4	0.3	-1.3	-21.3***	-10.8***	5.3**	2.8***
6 Months	179.4***	-1.7***	-10.6***	-11.2***	53.7***	18.9**	45.6*	28.3	13.9**	0.7**
	– Terror attack Madrid, 11.03.2004 –									
Day	-3.6**	-1.4*	1.1	1.0	-0.4	-0.2	-0.4	-0.6	1.2	0.2
Week	-4.2**	0.9	2.7	2.0	0.0	1.2	0.3	-2.0	1.2	3.4**
Month	-3.4**	5.0	36.0***	24.6***	9.6	11.4**	20.5	1.1	4.0**	8.8***
6 Months	-21.0***	19.3***	140.8***	93.8***	40.2**	53.5***	101.3***	13.3	25.2***	22.2***
	– Terror attack London, 07.07.2005 –									
Day	-1.8*	0.4	-1.4	-1.1	-6.5**	-2.0	-1.7*	-1.3*	0.3	0.5
Week	1.5	3.1**	0.7	0.5	-0.2	-1.2	-0.5	-1.1*	2.1*	1.8**
Month	6.6**	7.2***	-0.2	-1.1	-4.5	-4.8**	-2.2	-5.9***	6.5***	7.0***
6 Months	16.9***	24.3***	-26.3**	-29.3**	-38.9***	-41.6***	-27.8***	-31.4***	18.7***	32.0***

Table 2.6: displays the cumulated abnormal returns in % for the respective series after natural disasters and terrorist attacks. The time period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is given by $\tau_2 = 10$. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2).

	Time Series									
τ_2	DAX	*S&P*	U.S. Gov5	U.S. Gov10	Ger Gov5	Ger Gov10	U.S. Swap	EUR Swap	U.S. EUR	U.S. JPY
	– Typhoon Mireille, 27.09.1991 –									
Day	-0.7	-0.2	-0.8	-0.9**	-	-	-	-	-	0.5
Week	-0.9	-0.6	-1.3**	-0.9**	-	-	-	-	-	2.4***
Month	-2.9	-0.6	0.4*	2.3	-	-	-	-	-	1.4**
6 Months	4.7	3.4	10.9	8.5	-	-	-	-	-	-3.8*
	– Hurricane Andrew, 23.08.1992 –									
Day	-1.3**	-1.0*	3.4***	2.5***	-	-	-	-	-	0.8**
Week	0.4	-0.2	2.3***	2.2***	-	-	-	-	-	1.7**
Month	7.1	1.3	1.3**	0.1**	-	-	-	-	-	-0.2
6 Months	22.9***	3.9	22.3***	11.1***	-	-	-	-	-	-6.1
	– Earthquake Northridge, 17.01.1994 –									
Day	-0.3	-0.4	-0.1	0.0	-	-	-	-	-	0.1
Week	-3.7**	-0.2	-1.8	-1.1	-	-	-	-	-	-0.2
Month	-4.7**	-1.6	3.1	2.7	-	-	-	-	-	4.3**
6 Months	-19.0***	-9.8***	23.5***	19.9***	-	-	-	-	-	20.0***
	– Winter storm Lothar, 25.12.1999 –									
Day	0.5	-0.2	-0.2	-0.3	0.2	0.7	-0.1	0.6	-0.2	-0.1
Week	1.0	0.1	0.1	0.2	1.8	1.7	-0.6	-0.3	-0.6	-0.5
Month	-2.8	-3.4	3.9*	4.5*	10.5***	5.1**	0.5	-2.1	0.4	-4.9**
6 Months	-28.8***	-13.2***	-7.5	-11.9	5.0***	-8.5	1.0	5.7	0.7	-18.5***
	– Hurricane Charley, 11.08.2004 –									
Day	-1.1	-0.3	-0.7	-0.3	0.9	0.6	2.3	0.9	0.0	0.5
Week	-0.4	0.3	-5.0*	-2.7	0.0	0.0	-0.9	-0.4	1.0	1.5*
Month	4.4	4.1*	-6.8**	-3.9*	1.1	0.4	-1.8	3.4	-0.8	2.6**
6 Months	12.8*	9.6**	-28.6***	-20.0***	-21.5**	-20.1***	-27.9*	-13.8	7.0**	11.9***
	– Hurricane Ivan, 02.09.2004 –									
Day	0.5	1.1*	2.0	1.7*	0.5	0.4	1.6	1.0	-0.2	-0.1
Week	2.0*	1.0*	1.8*	1.3**	5.5***	3.3***	5.4**	5.5***	-0.2	0.1
Month	3.6**	1.2*	2.0*	0.4*	0.6*	-0.8	2.0	3.6***	0.7	-1.2
6 Months	19.1***	11.4***	16.6***	8.2**	-7.1	-10.3	-4.4	-5.5**	4.2	4.5
	– Hurricane Katrina, 25.08.2005 –									
Day	-1.3**	0.2	-0.1	-0.4	-0.9	-0.8	0.2	-0.5	0.3	0.2
Week	-2.3***	0.8	-4.5	-3.6	-1.4	-1.5	-3.0	-0.4	0.8	-0.3
Month	-3.1***	-0.4	-0.6	0.7	-1.1	-2.1	-2.4*	0.6	0.6	-0.7
6 Months	0.4**	2.0	12.3	15.2*	37.8***	24.5***	2.4	32.8***	3.2	-5.5**
	– Hurricane Rita, 20.09.2005 –									
Day	0.6	-0.8*	0.7	-0.1	0.0	0.0	0.1	-0.4	-0.1	-0.4
Week	0.7	-1.5***	2.1	1.0	2.8	1.6	2.7*	1.3	-0.3	-0.3
Month	-2.0	-4.5**	7.0**	5.3	11.4**	8.2**	5.6***	8.1***	0.2	-1.9*
6 Months	-2.8	-3.1***	10.9**	9.3*	36.2***	25.8***	5.5***	29.8***	5.8	1.7

Table 2.7: displays the cumulated abnormal returns in % for the respective series after natural disasters. The time period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is given by $\tau_2 = 100$. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2).

τ_2	DAX	$S\&P$	U.S. Gov5	U.S. Gov10	Ger Gov5	Ger Gov10	U.S. Swap	EUR Swap	U.S. EUR	U.S. JPY
					Time Series					
					– Hurricane Wilma, 19.10.2005 –					
Day	-2.2***	1.5***	-0.5	-0.5	-1.3	-0.9	-0.5	-1.1	0.3	0.3
Week	-2.0**	1.6**	0.7	0.1	0.4	0.2	-0.1	0.8	1.3	0.9
Month	1.2*	4.5***	1.9	-0.4	8.9*	6.2*	1.4	7.8***	-1.5	-1.5
6 Months	7.1	11.7***	-7.5	-4.4	17.5**	14.6**	-8.6	15.1***	5.8*	6.1*
					– Winter storm Kyrill, 18.01.2007 –					
Day	-0.3	-0.4	-0.6	-0.8	0.8	0.7	0.3	0.2	0.2	-0.4
Week	0.0	0.2	0.5	0.4	0.4	-0.2	0.7	0.0	0.1	-0.2
Month	1.1	-0.1	-1.5	-1.3	-0.5	0.4	0.3	-1.2	1.3	0.5
6 Months	2.9	-4.5**	2.1	4.5	2.5	6.0	1.0	-2.2*	4.1**	2.0
					– Hurricane Ike, 06.09.2008 –					
Day	2.3**	2.2**	1.3	0.3	2.3*	2.0**	1.8	0.6	-0.9*	-0.3
Week	2.3**	1.4	1.0	2.2	4.3**	4.7***	-0.2	0.8	0.2	0.0
Month	-3.0	-8.9	-4.9	1.4	-4.7	-0.8*	12.0**	-6.1*	-1.3	3.2*
6 Months	-23.7***	-27.4***	-34.1***	-18.4**	-49.0***	-22.5	-67.8***	-101.7***	2.4	19.8***
					– Earthquake Japan, 11.03.2011 –					
Day	-1.2*	0.6	-1.4	-0.2	-2.5	-1.7	-0.4	-1.3	0.8	1.2**
Week	-6.2***	-2.1*	-11.0*	-5.9*	-7.9**	-4.4**	-0.3	-5.2***	1.7**	5.1***
Month	0.4***	1.1	-3.1	-1.5	-2.2*	-0.2*	-0.9	-0.3*	3.8***	-2.0**
6 Months	-30.4***	-21.3***	-151.6***	-77.2***	-120.4***	-72.8***	-0.2	-45.7***	5.3***	9.1***
					– Terror attack Oklahoma, 19.04.1995 –					
Day	-0.7	-0.2	0.8	0.4	-	-	-	-	-	-1.0
Week	2.3	0.8	1.0	0.2	-	-	-	-	-	-2.6***
Month	7.9***	2.3**	-3.2	-4.2	-	-	-	-	-	-11.3***
6 Months	16.2***	1.7*	6.4	1.4	-	-	-	-	-	-47.4***
					– Terror attack WTC, 11.09.2001 –					
Day	-8.2***	-1.1	0.6	0.3	1.9**	1.5**	-5.8***	-1.9**	1.4**	1.0*
Week	-7.9***	-0.1	1.4*	1.3	-4.1**	-0.8	-13.5***	-5.3***	2.9***	2.7***
Month	2.5***	-2.5***	2.6***	2.4**	-5.9**	-3.5	-29.5***	-14.2***	2.4***	0.9**
6 Months	36.9	-3.2***	0.8	1.1	16.4	5.7	-3.1***	8.2***	-3.6	-10.5
					– Terror attack Madrid, 11.03.2004 –					
Day	-3.6***	-1.6***	0.3	0.4	-0.6	-0.5	-0.7	-0.6	1.0	0.1
Week	-4.4***	-0.4*	-1.3	-0.7	-0.6	-0.4	-1.0	-1.6	-0.1	2.6***
Month	-3.9***	-0.2**	20.1**	13.8***	7.3	5.0	15.3*	2.7	-1.1	5.6***
6 Months	-24.0***	-12.0***	45.3***	29.0***	26.5***	15.2***	70.4***	22.8***	-5.3	3.0***
					– Terror attack London, 07.07.2005 –					
Day	-1.9***	0.3	-1.0	-0.7	-5.9***	-1.5*	-1.4*	-0.7	0.2	0.2
Week	1.1	2.4**	2.3	2.3	2.7	1.1	0.7	1.8	1.7**	0.6
Month	5.4**	4.3***	6.0*	5.8**	7.4	4.5	2.5	5.5***	5.0***	2.2**
6 Months	9.2**	7.0***	11.4**	12.2***	32.9***	14.1*	0.8	37.2***	9.7***	3.1**

Table 2.8: displays the cumulated abnormal returns in % for the respective series after natural disasters and terrorist attacks. The time period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is given by $\tau_2 = 100$. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2).

τ_2	DAX	$S\&P$	U.S. Gov5	U.S. Gov10	Ger Gov5	Ger Gov10	U.S. Swap	EUR Swap	U.S. EUR	U.S. JPY
					– Typhoon Mireille, 27.09.1991 –					
Day	-0.7	-0.2	-0.8	-0.9*	-	-	-	-	-	0.5
Week	-0.7	-0.6	-1.6**	-1.2**	-	-	-	-	-	2.4**
Month	-2.3	-0.8	-0.8**	1.3	-	-	-	-	-	1.7**
6 Months	8.0	1.9	3.5**	2.1*	-	-	-	-	-	-2.2*
					– Hurricane Andrew, 23.08.1992 –					
Day	-1.4	-1.0	3.2***	2.4***	-	-	-	-	-	0.9*
Week	-0.1	-0.1	1.6***	1.7***	-	-	-	-	-	1.8**
Month	5.0	1.4	-1.7*	-2.0	-	-	-	-	-	0.5*
6 Months	10.3	4.8	4.6***	-0.8**	-	-	-	-	-	-1.5*
					– Earthquake Northridge, 17.01.1994 –					
Day	-0.2	-0.4	0.0	0.0	-	-	-	-	-	0.0
Week	-3.2*	-0.2	-1.3	-0.7	-	-	-	-	-	-0.7
Month	-2.7	-1.6	5.2	4.1	-	-	-	-	-	2.4
6 Months	-7.2	-9.7**	36.4***	28.6***	-	-	-	-	-	8.9***
					– Winter storm Lothar, 25.12.1999 –					
Day	0.8	-0.1	-0.1	-0.2	0.2	0.7	0.0	0.7	-0.2	0.0
Week	2.3*	0.5	0.5	0.5	2.0	2.1*	-0.2	0.1	-0.6	0.0
Month	2.2	-2.2	5.4**	5.9**	11.2***	6.5***	2.0	-0.5	0.5	-2.8*
6 Months	1.4	-5.6**	1.6	-3.5	9.4***	-0.1**	9.9***	15.3**	1.0	-6.0**
					– Hurricane Charley, 11.08.2004 –					
Day	-1.2	-0.3	-0.4	-0.2	1.0	0.7	2.9**	1.1	0.0	0.5
Week	-0.5	0.1	-3.5*	-1.9	0.5	0.3	2.1*	0.3	1.0	1.3
Month	3.9	3.2	-0.6	-0.7	3.0	1.7	10.1***	6.2**	-1.1	1.6*
6 Months	9.7	4.1	8.3	-0.4	-10.0	-12.2*	43.7***	2.8*	5.6*	5.9**
					– Hurricane Ivan, 02.09.2004 –					
Day	0.4	1.1	2.0*	1.7*	0.5	0.4	2.0	1.1	-0.2	-0.1
Week	1.6	0.8	1.8**	1.1**	5.4***	3.2***	7.6***	5.9***	-0.1	0.0
Month	2.1	0.1	2.0**	-0.5*	0.4*	-1.0	10.5***	5.1***	1.1	-1.5
6 Months	10.2**	5.1*	16.4***	3.1**	-7.9	-11.6	47.1***	3.5***	6.2*	2.8
					– Hurricane Katrina, 25.08.2005 –					
Day	-1.3	0.2	-0.1	-0.4	-1.0	-0.9	0.4	-0.5	0.2	0.2
Week	-1.9*	0.7	-4.4*	-3.8**	-2.0	-2.0*	-2.3	-0.5	0.5	-0.4
Month	-1.4	-0.6	-0.4	-0.1	-3.3*	-4.1**	0.0	0.1	-0.4	-1.1
6 Months	10.5	0.6	13.1	10.6	24.7**	12.5	17.4	29.4***	-2.9	-8.2**
					– Hurricane Rita, 20.09.2005 –					
Day	0.7	-0.8	0.7	0.0	-0.1	-0.1	0.2	-0.3	-0.1	-0.4
Week	1.3	-1.4*	2.4	1.2	2.6	1.3	3.3	1.5	-0.6	-0.6
Month	0.5	-4.1**	8.1**	5.8*	10.5***	6.9**	8.1**	8.7***	-1.0	-3.2**
6 Months	11.8	-0.3*	17.5***	12.6**	30.7***	17.9***	20.7***	33.6***	-1.3	-6.2***

Table 2.9: displays the cumulated abnormal returns in % for the respective series after natural disasters. The time period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is given by $\tau_2 = \tilde{T}_j$. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2).

	Time Series									
			U.S.	U.S.	Ger	Ger	U.S.	EUR	U.S.	U.S.
τ_2	DAX	*S&P*	Gov5	Gov10	Gov5	Gov10	Swap	Swap	EUR	JPY
	– Hurricane Wilma, 19.10.2005 –									
Day	-2.1*	1.5*	-0.3	-0.3	-1.3	-0.9	-0.3	-1.0	0.2	0.3
Week	-1.7	1.4	1.5	0.7	0.9	0.4	0.9	1.5	1.2	0.5
Month	2.7	3.6*	5.0	2.0	10.7***	6.9**	5.3	10.5***	-2.1	-2.9
6 Months	15.7	6.4**	11.4*	9.9	28.3***	18.9***	14.7**	31.4***	2.3	-2.4
	– Winter storm Kyrill, 18.01.2007 –									
Day	-0.2	-0.3	-0.6	-0.7	0.9	0.7	0.3	0.3	0.2	-0.5
Week	0.5	0.5	0.6	0.5	0.8	0.2	0.6	0.5	0.1	-0.4
Month	3.1	1.0	-1.2	-1.0	1.4	1.8	0.0	0.8	1.4	-0.2
6 Months	14.4*	2.3	4.1	6.3	14.1**	14.7**	-0.6	9.8*	4.4*	-2.2
	– Hurricane Ike, 06.09.2008 –									
Day	2.2*	2.0**	1.3	0.3	2.3*	2.0**	1.7	0.7	-1.0**	-0.4
Week	1.6	0.7	1.2	2.2	4.2**	4.6***	-0.2	1.0	-0.3*	-0.2
Month	-5.8	-11.6***	-4.2	1.3	-5.2	-1.2*	11.9**	-5.4	-3.5**	2.3*
6 Months	-40.3***	-43.7***	-30.2***	-18.8***	-52.3***	-24.7**	-68.9***	-97.8***	-10.8***	14.4***
	– Earthquake Japan, 11.03.2011 –									
Day	-1.2	0.7	-0.7	0.1	-2.0*	-1.4*	-0.2	-1.0	0.8	1.2**
Week	-6.0***	-1.8	-7.5***	-4.3**	-5.4***	-3.0***	0.7	-3.7***	1.6**	5.0***
Month	1.1*	2.4	10.9	4.9	7.7	5.7	3.4	5.6	3.5***	-2.6*
6 Months	-26.3***	-13.4	-67.6***	-38.7***	-60.7***	-37.6***	25.6*	-10.6	3.6***	5.5***
	– Terror attack Oklahoma, 19.04.1995 –									
Day	-0.7	-0.1	0.7	0.2	-	-	-	-	-	-0.9
Week	2.1	1.2	0.2	-0.4	-	-	-	-	-	-1.8***
Month	6.9**	3.9**	-6.0*	-6.6**	-	-	-	-	-	-8.0***
6 Months	10.3**	10.9***	-10.5***	-12.6***	-	-	-	-	-	-27.6***
	– Terror attack WTC, 11.09.2001 –									
Day	-8.5***	-0.7	0.7	0.3	1.9*	1.5*	-6.0***	-2.1***	1.5**	1.0*
Week	-9.3***	0.1	1.7**	1.4*	-4.4*	-0.9	-14.4***	-6.2***	3.1***	2.6***
Month	-3.3***	-1.3*	3.2***	2.7***	-7.3**	-3.7	-33.0***	-17.7***	3.2***	0.8**
6 Months	2.0***	-1.2	2.6***	1.6*	8.2	4.3	-24.1***	-13.0***	1.3***	-11.0
	– Terror attack Madrid, 11.03.2004 –									
Day	-3.5***	-1.6*	0.2	0.3	-0.7	-0.5	-0.7	-0.6	1.0	0.1*
Week	-3.8***	-0.2	-1.9	-1.2	-1.2	-0.7	-1.2	-2.0*	0.1	2.4**
Month	-1.5***	0.8	17.6***	11.6***	4.6	3.6	14.8**	1.1	-0.4	5.0***
6 Months	-9.7***	-5.8**	30.1***	15.9***	9.9	6.8	67.2***	12.9	-1.4	-0.1**
	– Terror attack London, 07.07.2005 –									
Day	-1.9*	0.2	-1.0	-0.7	-5.9***	-1.6**	-1.3	-0.8	0.1	0.1
Week	1.3	2.2*	2.4	2.2	2.3	0.8	1.5	1.5	1.2*	0.2
Month	5.8	3.4**	6.8**	5.7**	5.6	3.4	5.8	4.4*	3.3**	0.8
6 Months	12.1*	1.5*	15.8***	11.5***	21.8	7.3	20.7**	30.6***	-1.0	-5.4

Table 2.10: displays the cumulated abnormal returns in % for the respective series after natural disasters and terrorist attacks. The time period corresponds to the return at the event day (day), as well as the cumulated return of 5 trading days (week), 20 trading days (month) and 120 trading days (6 months) after the event. The estimation period is given by $\tau_2 = \tilde{T}_j$. $***$, $**$ and $*$ reflect significance at the 99%, 95% and 90% level with the test statistic concerning the daily effects given by (2.1) and for the other effects given by (2.2).

Kapitel 3

Modellrisiko = Spezifikation ⊕ Validierung

Modellrisiko = Spezifikation ⊕ Validierung

Verfasst mit Philipp Sibbertsen und Gerhard Stahl.

Veröffentlicht Gerhard Stahl/Philipp Sibbertsen/Philip Bertram: Modellrisiko, Spezifikation und Validierung", in: Handbuch Solvency II. Von der Standardformel zum Internen Modell, vom Governance-System zu den MaRisk VA. S. 235-256.

3.1 Einleitung

Die Umsetzung von Solvency II gibt Versicherungsunternehmen die Option, unternehmensspezifische Modelle anstatt Standardmethoden, wie sie beispielsweise in QIS 4b dargelegt sind, zur Berechnung des Solvenzkapitals zu verwenden. Interne Modelle genügen in besonderer Art den erhöhten Anforderungen von Stakeholdern an die Qualität des Risikomanagements, da nur sie es ermöglichen, auf Grund einer möglichst genauen Berechnung den unterschiedlichen Anforderungen der Stakeholder simultan Rechnung zu tragen. Interne Modelle genügen einerseits dem modernen, risikoorientierten Aufsichtsparadigma sowie andererseits auch den Bedürfnissen der Shareholder nach einer risikoadjustierten und optimierten Kapitalallokation. Neben Vorteilen in der Risikomessung, also bei den quantitativen Anforderungen, besteht gleichfalls ein Korrelat zu den qualitativen Anforderungen, da mit der Implementierung eines internen Modells eine Vertiefung der Risikokultur einhergeht. Als Beispiel hierfür mag die Vorgehensweise von Ratingagenturen dienen, ein Unternehmen bzgl. des Risikomanagements nur dann als *strong* einzustufen, falls dieses ein internes Modell zur Risikomessung verwendet. Ein solches Vorgehen wäre ohne obige Kontingenz nicht schlüssig.
Bei internen Modellen handelt es sich um große (hohe Anzahl an erklärenden Variablen), nichtlineare (eingebettete Optionen), stochastische (Modellierung zukünftiger Umweltzustände) Systeme. Im Rahmen des holistischen Ansatzes von Solvency II erfolgt damit eine Schätzung einer Prognoseverteilung des Bilanzsaldos. Hierzu finden sowohl Unternehmensmodelle (Managementregeln, freie RfB etc.), als auch stochastische Modelle Verwendung. Die Darstellung in Abb.1 zeigt schematisch die Bausteine eines internen Modells. Der Systemtheorie folgend (vgl. Luhmann [2003]) induziert die Verwendung sowohl von Standard- als auch von internen Modellen eine neue Risikokategorie: das *Modellrisiko*.
Eine operationale (bottom-up) Definition des Begriffes Modellrisiko ist zum einen auf Grund der Größe und zum anderen auf Grund der oft geringen empirischen Basis eines internen Modells kaum möglich. Dies gilt umso stärker für die aufsichtliche Definition, welche das Modellrisiko unter dem operationellen Risiko subsummiert und auch eine fehlerhafte Anwendung des Modells einschließt. In den folgenden Abschnitten wird auf das Modellrisiko am Beispiel des Wilkie-Modells (vgl. Wilkie [1995]), das stellvertretend für ökonomische Szenariogeneratoren steht, exemplifiziert. Dies ist dadurch motiviert, dass ökonomische Szenariogeneratoren den wesentlich neuen Modellbaustein in Solvency II darstellen. Doch zuvor seien noch einige grundlegende Anmerkungen geäußert.

Zum Begriff des Modellrisikos: Der Begriff ist in der Praxis insoweit umgangssprachlich interpretiert, als dass unter einer Risikosituation eine solche bezeichnet wird, unter welcher eine Verteilung über die Umweltzustände bekannt ist. Eine derartige Situation liegt beispielsweise bei bayesianischen Ansätzen, unter der Annahme, dass diese nicht fehlspezifiziert sind, vor. Solche Vorgehensweisen finden in der aktuariellen Praxis heute schon Anwendung.
Beispiel (Denuit et al. [2005]): Bezeichne $F(x)$ die Verteilung einer Verlustfunktion und $\varrho_g(X)$ ein Risikomaß im Sinne von Wang [2001], wobei $g(\cdot)$ eine konvexe Distortionsfunktion darstellt. Dann gilt:

$$\varrho_g(X) \geq E(\varrho_g(X|\Theta)),$$

wobei das letzte Integral über den Parameterraum Θ berechnet wird (vgl. Denuit et al. [2005], S.91ff). Die konvexe Detorierungsfunktion $g(\cdot)$ erlaubt eine konservative Abschätzung, die jedoch nicht mit der Nutzenfunktion aller Stakeholder vereinbar ist. Im Sinne der von Knight [1961] eingeführten Terminologie wäre indiziert von Modellunsicherheit zu sprechen.
Sicherheitsbedürfnisse erhöhen das Modellrisiko: Diese auf den ersten Blick paradoxe Aussage sei am Beispiel des Value-at-Risk für ein Signifikanzniveau von $\alpha \geq 0.99$ und einem Prognosehorizont von einem Jahr erläutert. Ersterer ist gegeben durch $VaR_\alpha = F^{-1}(\alpha)$, wobei die Verteilungsfunktion F unbekannt ist und durch die empirische Verteilungsfunktion $\hat{F}$ geschätzt werden muss. Der Schätzfehler, der hierbei gemacht werden kann, ist lediglich ein Teil des Modellrisikos und lässt sich für ein Quantil wie folgt berechnen.
Bezeichne $\hat{\xi}_n := \hat{F}_n^{-1}(\alpha)$ die Schätzung des α-Quantils ξ_α durch die empirische Verteilungsfunktion. Dann besitzt $\hat{\xi}_n$ folgende asymptotische Verteilung:

$$\hat{\xi}_n \to N\left(\xi_\alpha, \frac{\alpha(1-\alpha)}{nf(\xi_\alpha)}\right), \tag{3.1}$$

wobei $f(\cdot)$ die Dichtefunktion der zu Grunde liegenden Variable bezeichnet. Der Varianzterm in (3.1) ist unter der Annahme, dass der Träger durch die reellen Zahlen definiert ist, unter folgenden Aspekten interessant. Nach der Regel von de L'Hospital erhält man für den Limes

$$\lim_{\alpha\to 1} \frac{\alpha(1-\alpha)}{nf(\xi_\alpha)} = \lim_{\alpha\to 1} \frac{1-2\alpha}{nf'(\xi_\alpha)}, \tag{3.2}$$

welcher für festes n über den Schätzfehler entscheidet. Letzterer ist mit dem Modellfehler verknüpft, da der Limes von dem asymptotischen Verhalten der Dichte des Modells abhängt. Wie (3.2) zeigt, bestimmt die Asymptotik der Ableitung der Dichte über den Limes. Diese Annahmen werden im Regelfall nicht empirisch überprüft.
In der Wahl moderater α besteht eine Möglichkeit das obige Paradoxon aufzulösen, da für diese das Modell die Daten interpoliert und nicht wie für zu seltene Ereignisse extrapoliert. Ähnlich wie in den Ingenieurwissenschaften, müssten die Kennziffern für solche moderaten α mit einem Sicherheitsfaktor multipliziert werden.
Modellrisiko in der Praxis: Wie im unteren Teil von Abb.3.1 erläutert, handelt es sich bei der Modellierung um einen iterativen Prozess, der in Analogie zum Managen von Risiken, dasjenige Mittel der Wahl ist, um Modellrisiken zu vermeiden. Die Modellierung geht Hand in Hand mit

der Modellvalidierung sowie Sensitivitätsanalysen und Stresstests. Eine Unterlegung des Modellrisikos mit Eigenmitteln erscheint zweifelhaft, da in Folge dessen auch Modellrisiken in Standardverfahren mit Eigenmitteln zu hinterlegen wären. Die Höhe dieses Sicherheitsfaktors müsste höher als diejenige sein, welche bei internen Modellen Anwendung fände, da das Modellrisiko in internen Modellen nach Konstruktion geringer sein muss, als dasjenige für Standardverfahren. Eine Analyse des Modellrsikos kann nur für Teilmodelle bzw. für Teil-Teil-Modelle erfolgen, da diese eine Analyse im Sinne von Fehlspezifikation ermöglichen. Hierzu betrachten wir im Folgenden das Modell von Wilkie, da dieses stellvertretend für die Modellklasse der ökonomischen Szenariogeneratoren steht, die unter Solvency II die neue Modellklasse ist, welche unter allen Teilmodellen Anwendung findet.

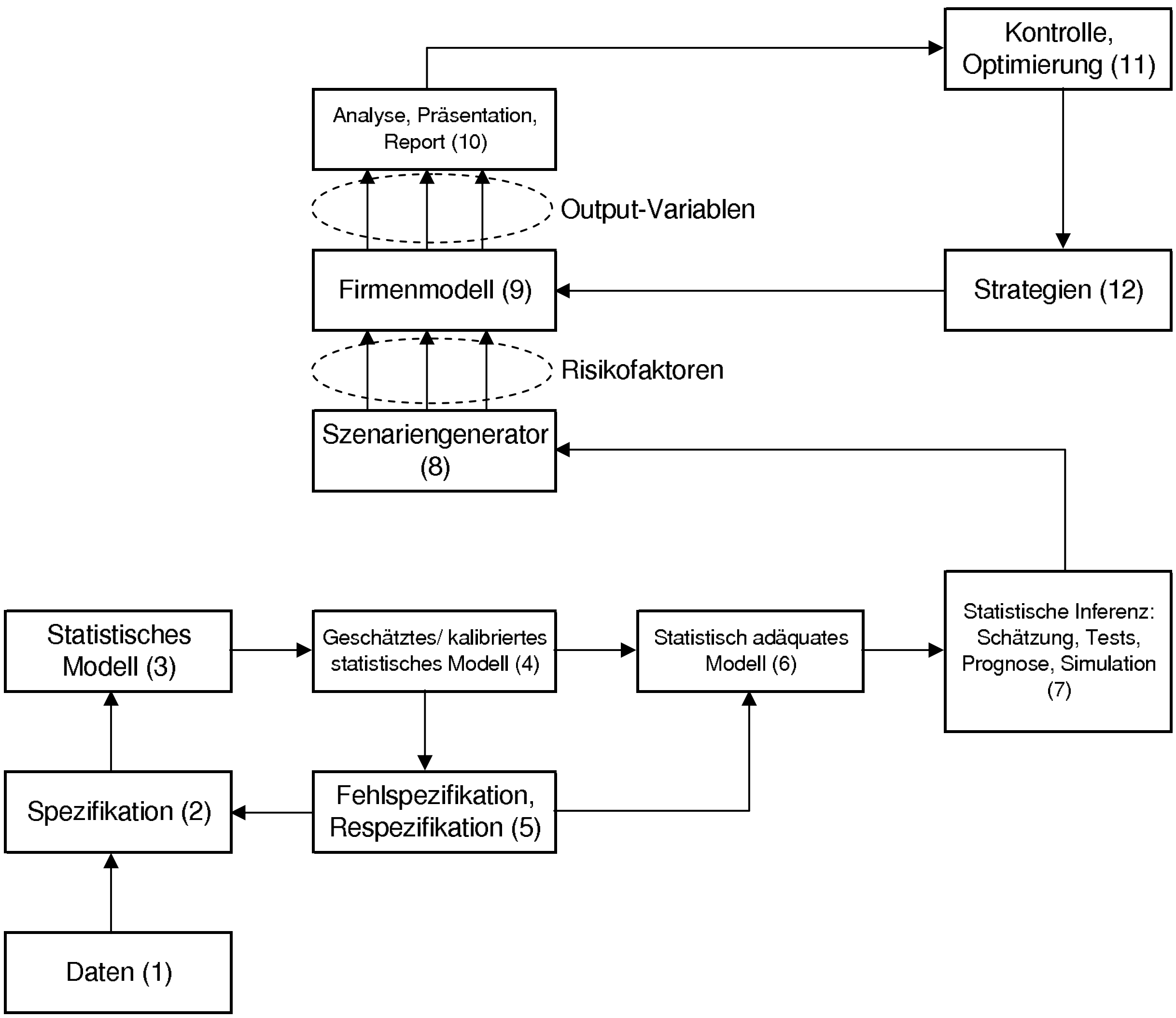

Abbildung 3.1: In dieser Abbildung werden die Bausteine des internen Modells eines Versicherungsunternehmens schematisch dargestellt. Hierbei gilt es zum einen die Feedbackstruktur des Regelkreises, welcher dem Risikomanagementprozess (bestehend aus 8–12) zu Grunde liegt, hervorzuheben. Zum anderen ist die vorgeschaltete statistische Modellierung, welcher ebenfalls ein Prozess zu Grunde liegt, nämlich der Modellierungsprozess (1–7), zu betonen. Hierbei übernimmt (5) die Aufgabe des Feedbacks.

3.2 Arten von Modellrisiko

In der Versicherungswirtschaft sind stochastische Modelle in zweierlei Hinsicht relevant: erstens bei der Tarifierung und der Bepreisung von Derivaten (Garantien in Versicherungsprodukten oder Hedges in Kapitalmarktrisiken) bzw. Portfolien (Market Consistent Embedded Value) und zweitens zur Risikoberechnung.

Während die Tarifierung zu den traditionellen Aufgaben der Versicherungswirtschaft gehört, wurde durch die neuen Solvabilitätsvorschriften für die Eigenmittelausstattung von Versicherungsunternehmen ein Aspekt verstärkt – das Modellrisiko. Die Bedeutung von letzterem hat durch die Heranziehung stochastischer Modelle im Rahmen von Solvency II stark zugenommen. Das stochastische Modell fungiert dabei als Bindeglied zwischen der Erfassung des konkreten Finanztitels und dessen Bewertung. Abb.3.2 verdeutlicht diesen Zusammenhang (vgl. im Folgenden Baumgartner et al. [2004].

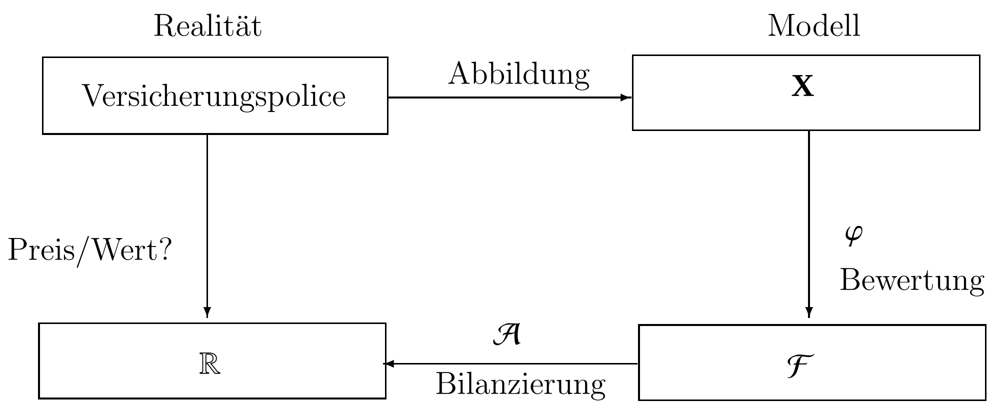

Abbildung 3.2: Zusammenhang zwischen Modell und Valuierung. Ein Finanztitel wird anhand eines Cashflows ($\mathbf{X}$) abgebildet und mit einem Bewertungsprinzip valuiert. Der bewertete Titel wird anschließend mit Hilfe von Buchungsprinzipien in die Bilanz aufgenommen.

Den Ausgangspunkt des Bewertungsprozesses stellt die Erfassung eines zu valuierenden Finanztitels durch das stochastische Modell dar. Als Beispiel sei die Modellierung einer Lebensversicherungspolice anhand eines Markov Modells angeführt. Das Modell wird dabei durch die Funktionen a_{it}, b_{ijt} und p_{ijt} beschrieben, wobei

a_{it} := Zahlung in t wenn sich der Versicherungsnehmer zum Zeitpunkt t in Zustand i befindet (z.B. Prämienzahlung),

b_{ijt} := Zahlung in $t+1$ wenn sich der Versicherungsnehmer zum Zeitpunkt t in Zustand i und zum Zeitpunkt $t+1$ in Zustand j befindet (z.B. Todesfallleistung),

p_{ijt} := Wahrscheinlichkeit, im Intervall [t,t+1) von Zustand i in Zustand j zu wechseln (Transitionswahrscheinlichkeit).

Die Versicherungspolice wird somit mit Hilfe des Markov Modells in einen Cashflow ($\mathbf{X}$) umgewandelt. Letztere sind stochastische Größen. Im zweiten Schritt werden die Policen bewertet. Die

Menge aller Lebensversicherungspolicen $\mathcal{F}$ wird hierbei durch die Abbildung φ in den Vektorraum valuierter Portfolien $\mathcal{F}$ projiziert. Hierbei wird der Erwartungswert des Cashflows $E^Q(\mathbf{X})$ unter einem Maß Q bestimmt. Die Wahl von Q kann somit z.B. einen mark-to-market oder einen mark-to-model Ansatz induzieren. Bezogen auf Abb.3.1 entspringt das Maß dem Szenariengenerator (8), während die Cashflows als Teil des Firmenmodells fungieren und als Ergebnisse im Report aufgenommen werden. Schließlich wird die valuierte Police unter Anwendung eines Buchungsprinzips (z.B. nach IFRS) in die Bilanz aufgenommen.
Berücksichtigt werden sollte hierbei, dass in jedem dieser Schritte Modelle und damit auch Modellrisiken involviert sind. Um das gesamte Modellrisiko zu erfassen, müssen somit die verschiedenen Risiken auf den einzelnen Stufen einbezogen werden.
An dieser Stelle sei nochmals betont, dass wir Modellierung als einen Prozess im Sinne von Hendry [1995], S.16ff verstehen. Dieses nimmt erneut den unteren Teil von Abb.3.1 auf und greift voraus auf Tab.3.1.
Wir beginnen mit einer allgemeinen Definition von Modellrisiko, welche an die Definition von Crouhy et al. [1998] angelehnt ist

Definition 1. *Modellrisiko bezeichnet jede Art von Risiko, das durch die Anwendung eines statistischen Modells induziert wird.*

Diesem Ansatz wird im Folgenden nicht gefolgt, da er unzureichend operational ist. Die folgende Definition ist prozessorientiert und beschreibt folglich das Risiko, dass das spezifizierte und geschätzte Modell falsch ist. Es wird also weniger auf die Eigenschaften des Modells als vielmehr auf die Diskrepanz zwischen den vorliegenden Daten und dem spezifizierten Modell abgestellt (vgl. Sibbertsen et al. [2008]).

Definition 2. *Modellrisiko im strengen Sinne bezeichnet jede Art von Risiko, das durch die Auswahl, Spezifikation und Schätzung eines statistischen Modells induziert wird.*

Damit kann Modellrisiko potentiell in jedem der drei Modellierungsschritte eintreten.

Bemerkung: Diese Definition schließt jedes Risiko, das auf menschlichem Versagen basiert aus. Dasjenige Risiko, welches aus kontaminierten Daten entstehen kann, ist hingegen in der Definition enthalten.

Die konkrete Spezifikation des Modells umfasst vier Modellierungsstufen. Die Aspekte aus Definition 2 sind hierin enthalten (s. auch Cuthbertson et al. [1992]):

1. Marginalisierung des datengenerierenden Prozesses;

2. Modellspezifikation bzgl. der Variablenauswahl;

3. Modellspezifikation bzgl. der funktionalen Form;

4. Parameterschätzung.

Eine weitere Differenzierung der Modellierungsstufen liefert Tab.3.1 (vgl. Sibbertsen et al. [2008]). Beispiele für die einzelnen Kategorien werden in Kapitel 3.3 aufgegriffen.

Komponenten des Modellrisikos	Kategorien	Prozess-stufe
Modellfehler und Fehlspezifikation	• Fehler in der analytischen Lösung	3
	• Fehlspezifikation des zugrundeliegenden stochastischen Prozesses	1
	• Unberücksichtigte Risikofaktoren	2
	• Unberücksichtigte Überlegungen	2
	• Falsche Klassifizierung/Identifikation des zugrundeliegenden Assets	2,3
	• Änderung der Marktbedingungen	1,2,3
	• Stichprobengröße	1
	• Fehler in den Variablen	2
Modellschätzung	• konkurrierende Schätzmethoden	4
	• Schätzfehler	4
	• Ausreißerproblematik	2,4
	• Schätzintervalle	4
	• Kalibrierung und Anpassung der geschätzten Parameter	4

Tabelle 3.1: Klassifikation von Modellrisiko

Tab.3.1 verdeutlicht die zwei zentralen Aufgaben bei der datenorientierten Modellierung: die Wahl der funktionalen Form und die Schätzung der Parameter. Während Fehler in ersterem implizieren, dass das Modell nicht geeignet ist, die Daten zu repräsentieren, resultieren Schätzfehler in verzerrten Parameterwerten.
Unser Ansatz zur Erfassung des Modellrisikos ist offensichtlich auch mit einigen Schwierigkeiten behaftet. So bedarf es eines alternativen Modells mit dem das zu untersuchende Modell verglichen werden kann. Konkret ist zum Vergleich also ein korrektes Modell vonnöten, welches in der Praxis schwer zu bestimmen ist. Abhilfe kann zwar dadurch geschaffen werden, dass als Alternative ein Benchmarkmodell, also dasjenige Modell, welches am nähesten an den Daten liegt, herangezogen wird. Ist dieses jedoch bekannt, so erübrigt sich der Vergleich von Modellrisiken, da das bestmögliche Modell damit bereits gefunden ist. Die Wahl eines Benchmarkmodells ist also ein schwerwiegendes Problem, welches im Kontext der Diskussion über Modellrisiko allerdings kaum übergangen werden kann.

3.3 Praxisbeipiele

3.3.1 Market Consistent Embedded Value

Der Market Consistent Embedded Value (MCEV) stellt ein Prinzip zur marktkonsistenten Bewertung von Versicherungsunternehmen aus Sicht der Eigentümer dar. Da für versicherungstechnische Verpflichtungen im Allgemeinen kein aktiver Markt existiert, müssen Modelle für eine market-to-model Bewertung der Bestände herangezogen werden. Allgemein wird von einer marktkonsistenten Bewertung gesprochen, wenn der market-to-model Preis der zu bewertenden Verpflichtung innerhalb gewisser Toleranzgrenzen repliziert werden kann. Dies bedeutet, dass in einem arbitragefreien Kapitalmarktmodell der Erwartungswert der diskontierten zukünftigen Cashflows der Kapitalanlage seinem aktuellen Marktwert entspricht. Die Cashflows der Periode

$t = 1, \dots, T$ können beschrieben werden als

$$X_t = \begin{cases} 0,0955 \cdot KE_t \cdot ZE_t, & \text{falls } KE_t^{ph} > 0 \\ (1 - r_{tax}) \cdot (KE_t \cdot ZE_t - i \cdot DR_{t-1}), & \text{falls } KE_t^{ph} \leq 0. \end{cases} \tag{3.3}$$

KE^{ph} bezeichnet den auf den Versicherungsnehmer entfallenden Anteil der erwirtschafteten Kapitalerträge (KE). Ist $KE^{ph} > 0$, wird den Versicherten eine Überschussbeteiligung gezahlt, wobei der Versicherer für $KE^{ph} \leq 0$ die gesamten Kapitalerträge einbehält. ZE beschreibt das von den Versicherungsnehmern eingesetzte Vermögen, gemessen an den gesamten Passiva, während r_{tax} den Steuersatz und i die Garantieverzinsung der Deckungsrückstellung (DR) bezeichnet. Der Wert $0,0955$ ergibt sich als der unter Berücksichtigung der Mindestzuführungsverordnung verbleibende Anteil der Kapitalerträge des Versicherers nach Steuern.
Es ist zu beachten, dass die Kapitalerträge von den stochastischen Zinserträgen abhängen. Damit stellen die Cashflows Zufallsvariablen dar und sollten somit unter Anwendung finanzmathematischer Methoden bewertet werden.
Der MCEV zum Zeitpunkt t, $\pi(t)$, ergibt sich nun als Produkt aus gegenwärtigem Diskontfaktor B_t und dem bedingten Erwartungswert der zukünftigen abdiskontierten Cashflows:

$$\pi(t) = B_t \cdot E_Q\left(\sum_{\tau=t}^{T} \frac{X_\tau}{B_\tau} \middle| \mathcal{F}_t \right). \tag{3.4}$$

Unter der Bedingung eines vollständigen, arbitragefreien Marktes, wird der Erwartungswert der Cashflows unter einem Martingalmaß Q bestimmt. Weiterhin wird $E(\cdot)$ bedingt auf die in der $\sigma-$Algebra $\mathcal{F}_t$ enthaltenen verfügbaren Informationen zum Zeitpunkt t. Da eine geschlossene Berechnungsformel für die komplexen Zahlungsströme X_τ i.A. nicht existiert, erfolgt die Bestimmung des bedingten Erwartungswertes in (3.4) häufig mit Hilfe von Simulationen.
Durch die marktkonsistente Bewertung sind folglich drei potentielle Fehlerquellen gegeben. Erstens kann die Darstellung des Portfolios an Versicherungsbeständen als Cashflows Modellfehler induzieren. So unterliegt (3.3) diversen Annahmen, bei deren Ungültigkeit X_τ fehlerhaft berechnet werden kann. In Bezug auf Tab.3.1 kann diese fehlerhafte Messung von X_τ als Fehler in den Risikofaktoren interpretiert werden.
Ein zweiter Modellfehler kann durch die Wahl des Martingalmaßes Q hervorgerufen werden. Dies wird in Tab.3.1 unter dem Stichwort Kalibrierungsfehler angeführt.
Detlefsen and Härdle [2006] zeigen, dass das Kalibrationsrisiko als wichtige Komponente des Modellrisikos nicht vernachlässigt werden sollte. So wird anhand einer Simulationsstudie demonstriert, dass bei der Bewertung von exotischen Optionen die Wahl des Fehlerfunktionals das Modellrisiko stark beeinflusst. Die Bedeutung der Wahl des Fehlerfunktionals ist dabei umso höher, je größer die Unsicherheit bzgl. der Adäquatheit des parametrischen Modells ist.
Schließlich kann ein Schätzfehler auftreten, da $\pi(t)$ nicht bekannt ist, sondern durch $\hat{\pi}(t)$ approximiert werden muss.

3.3.2 Wilkie-Modell

Das Wilkie-Modell (vgl. Wilkie [1995]) ist ein stochastisches Investitionsmodell, welches in der Versicherungwirtschaft als Benchmarkmodell für die Modellierung und Simulation ökonomischer Szenarien genutzt wird. In seiner urprünglichen Form umfasst das Gesamtmodell sechs Teilmodelle, in denen folgende Variable modelliert werden: Inflation, Dividendenrenditen, Dividendenindizes, kurz- und langfristige Zinsen sowie Wechselkurse.
Die Teilmodelle sind über eine Kaskadenstruktur miteinander verbunden (s. Abb.3.3). Hinsichtlich der Wirkungsrichtung impliziert die Kaskadenstruktur eine Kausalkette ohne Rückkopplungseffekte. Dem Inflationsmodell fällt hierbei eine besondere Rolle zu, da es als treibende Kraft des Gesamtmodells unmittelbar bzw. mittelbar in jedes nachgeordnete Teilmodell einfließt.

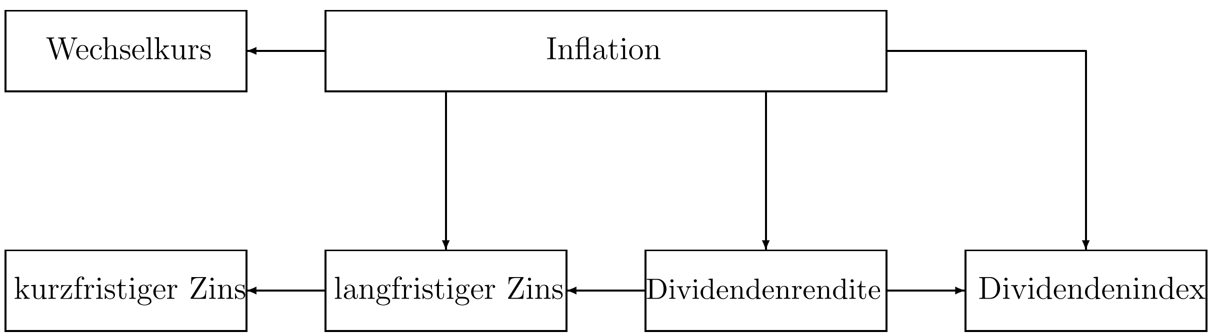

Abbildung 3.3: Schematische Darstellung des Wilkie-Modells

Die statistische Modellierung des Systems erfolgt in der Klasse der linearen State-Space Modelle. Letztere werden durch folgende Gleichungen beschrieben:

$$z_t = B_{t-1}z_{t-1} + F_{t-1}x_{t-1} + \omega_{t-1} \quad (3.5)$$

$$y_t = H_t z_t + G_t x_t + e_{t-1}. \quad (3.6)$$

z_t beschreibt dabei einen Vektor unbeobachtbarer Variablen, während die beobachtbaren endogenen und exogenen Variablen durch y_t bzw. x_t erfasst werden. ω_t und e_t sind voneinander unabhängige weiße Rauschen und B_t, F_t, H_t sowie G_t sind Koeffizientenmatrizen. (3.5) wird als Zustandsgleichung und (3.6) als Beobachtungsgleichung bezeichnet.
Der Vorteil von State-Space Modellen liegt darin, dass sie als Überbau für verschiedene Modellklassen, wie z.B. VARMAX-Modelle (vektorautoregressive moving-average Modelle mit exogenen Variablen), VARX-Modelle mit zeitinvarianten Koeffizienten oder Zeitreihenmodelle mit Trend- und Saisonkomponente fungieren. Während Wilkie also die einzelnen Gleichungen seines Modells spezifiziert und versucht diese miteinander zu verknüpfen, folgt diese Vorgehensweise einer Spezifikation des Systems in holistischer Gestalt. So wird an Stelle von Einzelgleichungen ein Gleichungssystem modelliert.
Die rekursive Schätzung der Zustandsvariable z_t erfolgt durch die Anwendung des Kalman-Filters (Kalman [1960]; Kalman and Buvy [1961]). Die Schätzung der Parametermatrizen kann mit der

Maximum-Likelihood Methode durchgeführt werden. Die exakte State-Space Darstellung des Wilkie-Modells soll im Folgenden kurz beschrieben werden.
Das Wilkie-Modell besteht gemäß Abb.3.3 aus sechs Variablen. Diese seien im Vektor $\boldsymbol{\lambda}_t = (\lambda_{1t}, \ldots, \lambda_{6t})'$ zusammengefasst. Weiterhin beschreibt $\nu = (\nu_1, \ldots, \nu_6)'$ einen Vektor von Konstanten und $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \ldots, \varepsilon_{6t})'$ ein sechsdimensionales weißes Rauschen. Die Teilmodelle sind gemäß der urprünglichen Form des Wilkie-Modells (Wilkie [1995]) wie folgt definiert.

Inflationsrate: Die Inflationsrate wird als autoregressiver Prozess erster Ordnung (AR(1)-Prozess) modelliert. Formal gilt

$$\lambda_{1t} = \nu_1 + \beta_1 \lambda_{1,t-1} + \varepsilon_{1t}.$$

Dividendenrendite: Ebenfalls anhand eines AR(1)-Prozesses erhält die logarithmierte Dividendenrendite Einzug in das Modell. Zusätzlich wird die Inflationsrate unverzögert als Erklärungsvariable in das Teilmodell aufgenommen. Es gilt

$$\ln \lambda_{2t} = \nu_2 + \alpha_1 \lambda_{1t} + \beta_2 \ln \lambda_{2,t-1} + \varepsilon_{2t}.$$

Dividendenindex: Die Wachstumsrate des Dividendenindexes wird modelliert als moving-average-Prozess erster Ordnung (MA(1)-Prozess). Als weitere Einflussfaktoren erhalten die unverzögerte Inflationsrate sowie der Störterm des Dividendenrenditemodells Einzug in das Modell.[1] Folglich ergibt sich

$$\Delta \ln \lambda_{3t} = \nu_3 + \alpha_2 \lambda_{1t} + \delta_1 \varepsilon_{2,t-1} + \delta_2 \varepsilon_{3,t-1} + \varepsilon_{3t}.$$

Langfristiger Zinssatz: Die langfristigen Zinsen werden als AR(1)-Prozess mit zusätzlichem Einfluss der unverzögerten Innovation der Dividendenrendite modelliert. Als endogene Variable fungiert in diesem Modell der logarithmierte, inflationsbereingte langfristige Zinssatz. Die Bereinigung erfolgt hierbei durch Subtraktion der zukünftig erwarteten Inflationsrate (θ). Letztere wird mit Hilfe eines exponentiell gewichteten gleitenden Durchschnitts der Inflationsrate konstruiert. Dabei kann der Parameter ρ auf Grund von Identifikationsproblemen nicht geschätzt werden. Dies bedeutet, dass sich algebraisch keine eindeutige Berechnungsformel zur Schätzung des Paramters angeben lässt. In solchen Fällen wird oft auf Erfahrungswerte zurückgegriffen, wobei Wilkie den Wert $\rho = 0.045$ wählt. Das Modell ist damit gegeben durch

$$\begin{aligned} \ln(\lambda_{4t} - \theta_t) &= \nu_4 + \beta_3 \ln(\lambda_{4,t-1} - \theta_{t-1}) + \gamma_1 \varepsilon_{2t} + \varepsilon_{4t}, \quad \text{wobei} \\ \theta_t &= (1-\rho)^t \cdot \overline{\lambda}_1 + \rho \sum_{i=1}^{t} (1-\rho)^{t-i} \lambda_{1i}. \end{aligned}$$

Kurzfristiger Zinssatz: Dieses Teilmodell modelliert die Differenz der Logarithmen von lang-

[1] Es sei darauf hingewiesen, dass der Term DM_t aus dem Wilkie-Modell in dieser Spezifikation nicht in das Modell mitaufgenommen wurde. Laut Wilkie [1995], S. 842ff ist dies durchaus sinnvoll, da der Glättungsparameter DD sich in fast allen Modellspezifikationen nicht signifikant von Null unterscheidet.

und kurzfristigem Zinssatz. Dies geschieht anhand des AR(1)-Prozesses

$$\ln\lambda_{4t} - \ln\lambda_{5t} \quad = \quad \nu_5 + \beta_4(\ln\lambda_{4,t-1} - \ln\lambda_{5,t-1}) + \varepsilon_{4t}.$$

Wechselkurs: Aufbauend auf der Kaufkraftparitätentheorie spezifiziert das Wechselkursmodell die Differenz im Logarithmus von Wechselkurs und der Differenz der Verbraucherpreisindizes der Länder j und i $(\ln Q_{jt} - \ln Q_{it})$ als endogene Variable. Letztere folgt dem AR(1)-Prozess

$$\ln\lambda_{6t} - \ln Q_{jt} + \ln Q_{it} \quad = \quad \nu_6 + \beta_6\left(\ln\lambda_{6,t-1} - \ln Q_{j,t-1} + \ln Q_{i,t-1}\right) + \varepsilon_{6t}.$$

Mit

$$\boldsymbol{y}_t = (y_{1t}, \ldots, y_{6t}) = (\lambda_{1t}, \ln\lambda_{2t}, \Delta\ln\lambda_{3t}, \ln(\lambda_{4t} - \theta_t), \ln\lambda_{4t} - \ln\lambda_{5t}, \ln\lambda_{6t} - \ln Q_{jt} + \ln Q_{it}) \tag{3.7}$$

kann das Wilkie-Modell somit durch das strukturelle VARMA(1,1)-Modell

$$A_0\boldsymbol{y}_t = \nu + A_1\boldsymbol{y}_{t-1} + M_0\boldsymbol{\varepsilon}_t + M_1\boldsymbol{\varepsilon}_{t-1} \tag{3.8}$$

dargestellt werden, wobei

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1 & 1 & 0 & 0 & 0 & 0 \\ \alpha_2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} \beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_5 \end{pmatrix}, M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \gamma_1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_1 & \delta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Mit $u_t = A_0^{-1}M_0\varepsilon_t$, $\tilde{A}_1 = A_0^{-1}A_1$, $\tilde{M}_1 = A_0^{-1}M_1M_0^{-1}A_0$ und $\tilde{\nu} = A_0^{-1}\nu$ ist die VAR(1)-Darstellung von (3.8) gegeben durch

$$Y_t = \tilde{\nu} + \boldsymbol{A}Y_{t-1} + U_t \quad \text{mit} \quad Y_t = \begin{pmatrix} y_t \\ u_t \end{pmatrix}, \quad \boldsymbol{A} = \begin{pmatrix} \tilde{A}_1 & \tilde{M}_1 \\ 0 & 0 \end{pmatrix}, \quad U_t = \begin{pmatrix} u_t \\ u_t \end{pmatrix}.$$

Die State-Space Darstellung von (3.8) ergibt sich nun über die oben angeführten Gleichungen (3.5) und (3.6), wobei $e_t = G_t = 0$, $B_t = \boldsymbol{A}$, $x_t = 1$, und $F_t = \tilde{\nu}$ jeweils für alle $t = (1, \ldots, T)$ Gültigkeit besitzt. Weiterhin ist $z_t = Y_t$ sowie $\omega_t = U_t$. Schließlich gilt $H_t = (E_6 \vdots \boldsymbol{0}) \sim (6 \times 12)$, wobei E_6 die sechsdimensionale Einheitsmatrix und $\boldsymbol{0}$ die Nullmatrix der Dimension (6×6) beschreibt.
Im Folgenden soll auf einige in Tab.3.1 beschriebene potentielle Fehlerquellen im Rahmen der Spezifikation und Schätzung des Wilkie-Modells eingegangen werden.

Strukturbrüche und Fensterbreite: Wilkie modelliert die Inflationsrate unter Heranziehung eines AR(1)-Modells. Im Folgenden verwenden wir eine leicht geänderte, jedoch kanonische Notation. Das Inflationsmodell lässt sich somit darstellen als:

$$I_t = \alpha_0 + \alpha_1 I_{t-1} + \varepsilon_t. \tag{3.9}$$

I_t bezeichnet dabei die Inflationsrate im Jahr t, α_0 die Konstante, $|\alpha_1| < 1$ den AR-Parameter und $\varepsilon_t \overset{\text{iid}}{\sim} N\left(0, \sigma^2\right)$ für alle t, beschreibt ein normalverteiltes weißes Rauschen.
Es sei nun davon ausgegangen, dass die Gesamtstichprobe $t \in \{1, \ldots, \tilde{t}, \tilde{t}+1, \ldots, T\}$ in zwei Teilstichproben $t_1 \in \{1, \ldots, \tilde{t}\}$ und $t_2 \in \{\tilde{t}+1, \ldots, T\}$ aufgeteilt werden kann, wobei t_1 eine innerhalb einer moderaten Bandbreite um ihren Mittelwert schwankende Reihe beschreibt. t_2 wiederum beinhalte außergewöhnliche Vorkommnisse wie Krisen oder Zeiten hohen Wachstums, die durch eine starke Persistenz in extremen Werten gekennzeichnet sind. Es ist nun leicht ersichtlich, dass bei Heranziehung von $t_1 + t_2$ die Paramterschätzung von α_1 in (3.9) betragsmäßig näher an Eins liegen sollte als bei einer Schätzung des Modells auf Basis von $t = t_1$. Unter der Annahme, dass (3.9) den wahren datengenerierenden Prozess (DGP) darstellt, können also bei der Modellierung durch Unterschlagung eines Teils der Stichprobe unterschiedliche Parameterwerte geschätzt werden. Somit entstünde Modellrisiko auf Basis der Nichtberücksichtigung wichtiger Prozesscharakteristika.
Dieser Aspekt wird in Tabelle 3.1 durch die Punkte *Veränderung der Marktbedingungen* sowie *Stichprobengröße* aufgegriffen. Je deutlicher die Unterschiede in den Teilstichproben hinsichtlich dieser Punkte ausfallen, desto stärker steigt die Bedeutung der Wahl der Stichprobengröße im Modellierungsprozess. Wilkie [1995] zeigt die Abhängigkeit der Parameterschätzungen im Inflationsmodell von der Wahl der Fensterbreite in Abschnitt 2.3.
Beispiel: Obige Problemstellung bei der Schätzung von (3.9) soll anhand eines empirischen Beispiels verdeutlicht werden. Als endogene Variable im Modell wird hierbei die vom *Internationalen Währungsfonds* publizierte monatliche US-amerikanische Inflationsrate im Zeitraum 1973 bis 2009 herangezogen. Eine graphische Darstellung dieser Zeitreihe liefert Abb.3.4.
Offensichtlich sind die Ausschläge der Zeitreihe während des Zeitraums der beiden Ölkrisen (1973-1982) sehr viel größer als in den Folgejahren. Bei Schätzung von (3.9) auf Basis des Zeitraumes $t_2 \in (1983, \ldots, 2009)$ ergeben sich die Parameterwerte $\hat{\alpha}_0^{t_2} = 2.99$ und $\hat{\alpha}_1^{t_2} = 0.95$, wohingegen sich bei Hinzuziehung von $t_1 \in (1973, \ldots, 1982)$ die Schätzwerte $\hat{\alpha}_0^{t_1+t_2} = 4.01$ und $\hat{\alpha}_1^{t_1+t_2} = 0.99$ ergeben. Alle Parameterschätzungen sind hochsignifikant. Die Problematik lässt sich aber in diesem Beispiel nicht nur auf die unterschiedlichen Parameterwerte beschränken. Auch die Stationaritätsuntersuchungen für die jeweiligen Fensterbreiten liefern unterschiedliche Ergebnisse. So lehnt der Augmented Dickey-Fuller-Test (vgl. Dickey and Fuller [1979]) die Nullhypothese der Instationarität auf Basis von $t = t_2$ ab, während H_0 für $t = t_1 + t_2$ nicht verworfen werden kann.
Ob sich der Anwender somit primär auf den aktuellsten Teil der Stichprobe konzentrieren sollte oder eine möglichst lange Reihe heranzuziehen ist, kann nicht pauschal beantworten werden. Die Entscheidung hängt vielmehr von den Zielen der Untersuchung (kurz- vs. langfrist Analyse/Prognose) ab.

Fehlspezifikation des DGP: Entspricht der in (3.9) unterstellte DGP dem Inflationsmodell, so gilt für OLS- (ordinary least squares) und Maximum-Likelihood-Schätzung unter klassischen

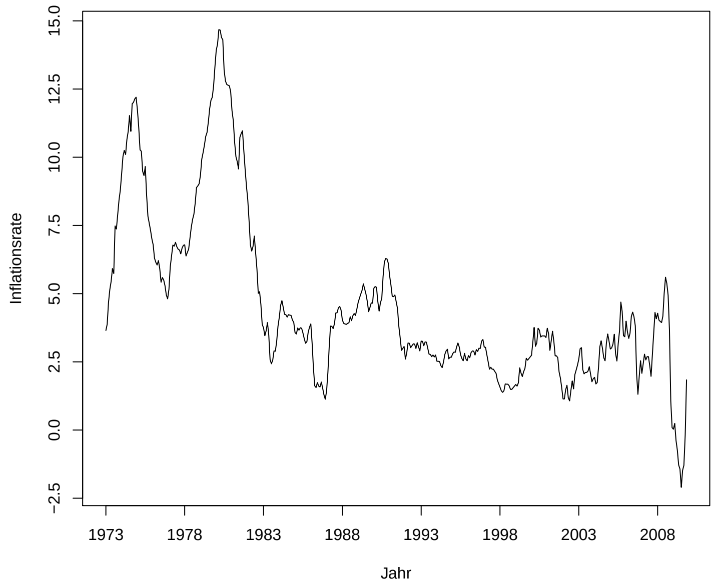

Abbildung 3.4: Monatliche Inflationsrate in den USA von 1973 bis 2009

Bedingungen (vgl. z.B. Green [2003], S.10ff) und ohne Berücksichtigung der Konstanten:

$$\widehat{\alpha}_1 = \left(X_2'X_2\right)^{-1}\left(X_2'X_1\right) = E\left(\alpha_1\right), \text{ da} \tag{3.10}$$

$$\alpha_1 = \left(X_2'X_2\right)^{-1}\left(X_2'\left(X_1-\varepsilon\right)\right), \tag{3.11}$$

wobei $X_1 = (I_T, I_{T-1}, \ldots, I_1)'$, $X_2 = (I_{T-1}, I_{T-2}, \ldots, I_0)'$ und $\varepsilon = (\varepsilon_T, \varepsilon_{T-1}, \ldots, \varepsilon_1)'$. Die Koeffizientenschätzung ist somit unverzerrt.
Ist das Modell allerdings fehlspezifiziert, so gilt (3.10) nicht. Wenn im Modell weitere Erklärungsvariablen, wie z.B. der Ölpreis oder die Zinsrate Einfluss erhalten und diese unberücksichtigt bleiben, so entstehen verzerrte Parameterschätzungen.
Die mit U bezeichneten unterdrückten Variablen werden in (3.9) von der Störgröße absorbiert, sodass gilt: $\varepsilon_t = U_t + \widetilde{\varepsilon}_t$, wobei $E(U) \neq 0$ und $\widetilde{\varepsilon}_t \overset{\text{iid}}{\sim} \left(0, \sigma^2\right)$ für alle t. Aus (3.11) ergibt sich somit $E\left(\alpha_1\right) = \widehat{\alpha}_1 - \left(X_2'X_2\right)^{-1}\left(X_2'U\right) \neq \widehat{\alpha}_1$.
Damit hat die Nichtberücksichtigung wichtiger Einflussfaktoren schwerwiegende Konsequenzen für den Modellierungsprozess. Dies stellt nicht nur auf den Einbezug zusätzlicher Variablen, sondern auch auf die Lagstruktur, also unberücksichtigte gelaggte endogene Variablen, ab. Es sollte dabei beachtet werden, dass die Unterdrückung von Einflussfaktoren bei der Modellierung lediglich eine Möglichkeit der Fehlspezifikation des DGP darstellt. Weiterhin kann letzterer in seiner funktionalen Form zwischen endogener und (schwach) exogenen Variablen oder auch hinsichtlich des Kausalzusammenhangs fehlspezifiziert sein. Abhilfe kann z.B. dadurch geschaffen werden, dass Fehlspezifikationstests herangezogen werden, die Lagstruktur mit Informationskriterien bestimmt wird oder Kausalitätstests bei der Modellierung Anwendung finden. Auf Letzteres wird in Kapitel 3.4 ausführlich eingegangen.

Konkurrierende Schätzmethoden: Auch durch die Verwendung verschiedener Schätzmethoden können Risiken im Modellierungsprozess induziert werden. Dies soll anhand eines Vergleichs der Schätzung von (3.9) nach OLS und nach GLS (generalised least squares) illustriert werden. Während sich die Parameterschätzung zwischen beiden Methoden nicht unterscheidet, da jeweils das Funktional $\sum_t \varepsilon_t^2$ minimiert wird, bestehen Unterschiede in der Schätzung der Kovarianzmatrix $E(\boldsymbol{\varepsilon\varepsilon}') = \Sigma$. Für die Varianz der Koeffizientenschätzer gilt

$$Var(\alpha_1)_{OLS} = \sigma^2 \left(X_2' X_2\right)^{-1}, \quad Var(\alpha_1)_{GLS} = \sigma^2 \left(X_2' X_2\right)^{-1} \left(X_2' \Omega X_2\right) \left(X_2' X_2\right)^{-1},$$

wobei $\Omega = \Sigma / \sigma^2$, $\sigma^2 = \sum_t \varepsilon_t^2 / (T - K)$ und K die Anzahl der zu schätzenden Parameter bezeichnet. Nach Schätzung von σ^2 und Ω lassen sich somit $\widehat{Var}(\alpha_1)_{OLS}$ und $\widehat{Var}(\alpha_1)_{GLS}$ bestimmen.
Gilt $\Omega = E$, wobei E die Einheitsmatrix beschreibt, so ist $Var(\alpha_1)_{OLS} = Var(\alpha_1)_{GLS}$. Ist die Gleichheit allerdings nicht gegeben, wird also $\varepsilon_t \overset{\text{iid}}{\sim} N\left(0, \sigma^2\right)$ verletzt, so unterscheiden sich die Varianzschätzer. Da auf letzteren die Signifikanzanalyse der Parameter basiert, hat die Auswahl des Schätzverfahrens unmittelbare Auswirkungen auf die Inferenz. So kann es beispielsweise passieren, dass $\widehat{\alpha}_1$ bei Schätzung mittels OLS signifikant und damit von Null verschieden ist, während die GLS-Methode Insignifikanz des Koeffizienten signalisiert. Ob die Variable I_{t-1} bei der Modellierung des Inflationsmodell berücksichtigt werden sollte, hinge somit von der verwendeten Schätzmethode ab. In der Praxis sollten folglich vor der Anwendung der Schätzmethode die zu Grunde liegenden Annahmen letzterer durch Tests evaluiert werden.
Schließlich sollte bemerkt werden, dass die obigen Risikofaktoren in jedem Teilmodell auftreten können. Die Risiken einzelner Teilmodelle können sich damit im gesamten Modell fortpflanzen. Dieser Effekt ist umso größer, je stärker die einzelnen Variablen untereinander korreliert sind. Ist also die Verbindung der Teilmodelle untereinander stark ausgeprägt, so ist ein insgesamt höheres Modellrisiko zu erwarten, als im Fall einer weniger starken Vernetzung.

3.4 Modellrisiko und Simultanitätsbias

Ein klassischer ökonomischer Szenariengenerator mit einer Kaskadenstruktur ist das Investitionsmodell von Wilkie [1995]. Eine Kaskadenstruktur bedeutet dabei, dass man ausgehend von einer Grundvariablen ein Gesamtmodell aufbaut, dessen Einzelkomponenten immer stärker ineinander verschachtelt werden. Die Kaskadenstruktur von Wilkies Investitionsmodell ist in Abb.3.3 gegeben.
In der Ökonometrie spricht man bei derartigen Zusammenhängen von Kausalität. Man spricht von Granger-Kausalität, wenn bei einer gegebenen Informationsmenge bis zum Zeitpunkt $t-1$ eine Variable Y zum Zeitpunkt t durch die Einbeziehung einer weiteren Variable X einen geringeren mittleren quadratischen Prognosefehler aufweist, als ohne diese Einbeziehung. Granger [1969] geht bei der Formulierung dieses Kausalitätsbegriffs davon aus, dass die Ursache der Wirkung zeitlich immer vorrausgeht. Dieser zeitliche Unterschied kann in Grangers Definition infinitesimal klein sein. Formal kann man Granger-Kausalität mit Hilfe eines VAR(p)-Modells definieren:

Definition 3. *X_1 und X_2 seien zwei Zeitreihen, die durch das folgende bivariate VAR(p)-Modell unter Vernachlässigung der Konstanten beschrieben werden:*

$$\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{pmatrix} \begin{pmatrix} X_{1,t-1} \\ X_{2,t-1} \end{pmatrix} + \ldots + \begin{pmatrix} \alpha_{11,p} & \alpha_{12,p} \\ \alpha_{21,p} & \alpha_{22,p} \end{pmatrix} \begin{pmatrix} X_{1,t-p} \\ X_{2,t-p} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

X_2 ist genau dann Granger-kausal für X_1, wenn mindestens einer der Werte $\alpha_{12,1}$ bis $\alpha_{12,p}$ nicht Null ist. Umgekehrt ist X_1 ist genau dann Granger-kausal für X_2, wenn mindestens einer der Werte $\alpha_{21,1}$ bis $\alpha_{21,p}$ nicht Null ist.

Wie aus der Definition hervorgeht, geht die Granger-Kausalität nicht notwendigerweise nur in eine Richtung, es kann auch Wechselwirkungen geben.
Besteht ein derartiger Rückkopplungszusammenhang nicht, ist also beispielsweise die Variable X_2 nicht Granger-kausal für X_1, wobei durchaus X_1 Granger-kausal für X_2 sein kann, so wird das System X_1, X_2 auch als exogen bezeichnet. Anderenfalls nennt man es endogen. Die Annahme der Exogenität ist eine der zentralen Annahmen in der Ökonometrie und eine der wichtigsten Annahmen im Bereich der linearen Regression. So kann insbesondere die Unverzerrtheit des OLS-Schätzers im linearen Regressionsmodell nur unter der Annahme exogener Regressoren gezeigt werden. Im Falle endogener Regressoren wäre der OLS-Schätzer verzerrt, man spricht vom Endogenitätsbias. Diese Aussage hat Konsequenzen weit über das einfache lineare Regressionsmodell hinaus, da zahlreiche Schätzprobleme auf das lineare Regressionsmodell zurückgeführt werden können.
Einen Spezialfall des Endogenitätsbias, den Simultanitätsbias, kann man sich am linearen Regressionsmodell gut vor Augen führen. Es sei dabei erwähnt, dass wir hier zur Vereinfachung der Darstellung keine zeitlich verzögerten Variablen betrachten. Gehen wir von dem klassischen linearen Regressionsmodell

$$y = X\beta + u$$

aus, wobei y und X nun aber gegenseitig kausal füreinander sind, das System ist also endogen. In diesem Fall ist der Regressor X nicht mehr unabhängig vom Störterm u. Den OLS-Schätzer für den Modellparameter β erhält man durch

$$\hat{\beta} = (X'X)^{-1}X'y.$$

Für einen konsistenten Schätzer muss gelten, dass $E(\hat{\beta}) = \beta$ ist. Hier erhalten wir hingegen

$$\begin{aligned} E(\hat{\beta}) &= E\left((X'X)^{-1}X'y\right) \\ &= E\left((X'X)^{-1}X'(X\beta+u)\right) \\ &= E\left((X'X)^{-1}X'X\beta + (X'X)^{-1}X'u\right) \\ &= \beta + E\left((X'X)^{-1}X'u\right). \end{aligned}$$

Der letzte Summand auf der rechten Seite ist nun nicht mehr Null und entspricht dem Endogenitätsbias. Dieser kann durchaus erheblich sein.
Wilkie [1995] betrachtet in seinem Modell einen zeitlich unverzögerten Kausalitätszusammen-

hang, sogenannte instantaneous causality, den es in Grangers Modellwelt nicht geben kann. Ein derartiger Kausalitätszusammenhang kann allerdings durch Aggregationseffekte entstehen, da in Wilkies Modell nur Quartalsdaten vorliegen und somit keine infinitesimal kleinen Zeitunterschiede betrachtet werden können. Im weiteren werden wir daher mit Bezug auf das Wilkie-Modell nur noch von Kausalität sprechen und gehen von einer instantaneous causality aus, die durch Aggregationseffekte aus der Granger-Kausalität hervorgegangen ist.
Durch die Kaskadenstruktur induziert, geht das Wilkie-Modell nicht davon aus, dass es Wechselbeziehungen zwischen den einzelnen Variablen gibt. Die Kausalitätsbeziehung geht stets nur in eine Richtung. Die Inflation ist in diesem Modell also kausal für den langfristigen Zins, umgekehrt ist der langfristige Zins aber nicht kausal für die Inflation. Das Kaskadenmodell von Wilkie ist somit ein exogenes System, indem die Modellparameter konsistent geschätzt werden können. Problematisch im Sinne eines Modellrisikos ist die Tatsache, dass Wilkie die Exogenität des Modells angenommen hat. Es ist somit eine Modellannahme des Wilkie-Modells, dass die Parametermatrizen des Modells obere Dreieckmatrizen sind. Im Sinne einer ökonometrisch sauberen Modellspezifikation hätte die Exogenität des Systems allerdings zunächst getestet werden müssen.
Eine formale Beschreibung von instantaneous causality zwischen den Größen X_1 und X_2 erhält man wie folgt:

Definition 4. *X_1 und X_2 seien zwei Zeitreihen, die durch die folgende VMA(∞)-Darstellung des bivariaten VAR(p)-Modells beschrieben werden:*

$$Y_t = \begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \Phi_{11}(B) & \Phi_{12}(B) \\ \Phi_{21}(B) & \Phi_{22}(B) \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix},$$

wobei $\Phi_{ij}(B)$ Polynome des Backshift Operators B sind und u_{it} unabhängige Fehlerterme darstellen. X_1 ist genau dann nicht unverzögert kausal für X_2, wenn $E(u_1 u_2') = 0$ ist.

Diese Bedingung kann mittels eines Wald-Tests getestet werden. Die Teststatistik folgt einer $\chi^2(N)$-Verteilung, wobei N die Anzahl der Freiheitsgrade beschreibt. Letztere sind gegeben durch die Anzahl der simultan getesteten Nullhypothesen. Nähere Informationen zu diesem Test finden sich in Lütkepohl [2005], S.104ff.
Beispiel: Um zu überprüfen, ob die Exogenitätsannahme des Wilkie-Modells gerechtfertigt ist, haben wir einen Wald-Test auf unverzögerte Kausalität auf das Modell angewendet. Die Nullhypothesen lauten somit

$$H_0 : E(u_i u_j) = 0 \qquad \text{gegen} \qquad H_1 : E(u_i u_j) \neq 0 \quad \text{für } i \neq j \in \{1, \ldots, 6\}.$$

Aufgrund der Symmetrie werden somit insgesamt 15 Hypothesen sequentiell getestet, wobei der kritische Wert zum 5% Niveau mit $\chi^2(1) = 3.84$ gegeben ist. Mit den Reihen 1 bis 6 sind die jeweiligen Transformationen der Variablen Inflation, Dividendenrendite, Dividendenindex, langfristige Zinsen, kurzfristige Zinsen und Wechselkurse gemäß (3.7) bezeichnet. Es ergaben sich folgende Teststatistiken:

i\j	1	2	3	4	5
2	3.34				
3	3.58	8.89			
4	8.19	4.13	0.02		
5	2.17	0.08	0.49	5.63	
6	0.09	1.51	6.22	0.09	0.08

Tabelle 3.2: Ergebnisse des Tests auf unverzögerte Kausalität

Daran erkennt man, dass die Nullhypothese für zahlreiche Reihenpaare zum 5%-Niveau verworfen werden muss. So ist die Dividendenrendite mit den langfristigen inflationsbereinigten Zinsen unverzögert kausal, die Exogenitätsannahme gilt hier also nicht. Im Wilkie-Modell gar nicht modelliert ist die unverzögerte Kausalität zwischen dem Dividendenindex und kaufkraftbereinigtem Wechselkurs. Lediglich verzögerte Kausalität findet man im Modell zwischen Dividendenrendite und dem Dividendenindex. Folglich erscheint es erwägenswert, ob diejenigen vom Test als signifikant ausgegebenen Korrelationen in das Modell miteinbezogen werden sollten.
Um diese Fragesetellung anhand der Prognosegüte beider Modelle zu evaluieren, haben wir den Test von Clements and Hendry [1998], S.233ff herangezogen. Dieser Encompassing-Test vergleicht die Out-of-Sample Prognosegüte zweier (oder mehrerer) Modelle. Hierfür wird folgende Hilfsregression eingeführt:

$$\mathrm{vec}(\boldsymbol{e}_{it}) = \alpha(\mathrm{vec}(\boldsymbol{e}_{it}) - \mathrm{vec}(\boldsymbol{e}_{jt})) + \mathrm{vec}(\varepsilon_{it}), \quad i, j \in \{1, 2\}.$$

Dabei stellt ε_{it} ein weißes Rauschen dar und $\boldsymbol{e}_{it}$ und $\boldsymbol{e}_{jt}$ bezeichnen die geschätzten Out-of-Sample Prognosefehler für alle Reihen der jeweiligen Modelle M_i und M_j. Getestet wird nun die Nullhypothese $H_0 : \alpha = 0$ gegen $H_1 : \alpha \neq 0$. Unter H_0 lässt sich die Prognosegüte von M_i durch Berücksichtigung von M_j nicht verbessern, während unter H_1 M_j einen Erklärungsbeitrag für die Prognose des Modells leistet. Die p-Werte der Koeffizientenschätzer sind in Tab.3.3 gegeben. Dabei bezeichnet M_1 das Modell in der von Wilkie vorgeschlagenen Form. M_2 hingegen berücksichtigt die signifikanten Korrelationen aus Tab.3.2 und kann somit als unrestringiertes Modell bezeichnet werden. π spiegelt dabei das Verhältnis aus Anzahl von Out-of-Sample- und In-Sample Beobachtungen wider.

Benchmarkmodell (M_i)	$\pi = 0.4$	$\pi = 1$
M_1	0.000	0.000
M_2	0.390	0.154

Tabelle 3.3: Ergebnisse des Tests auf Prognosegüte

Die Testergebnisse signalisieren somit, dass das unrestringierte Modell eine bessere Out-of-Sample Prognosegüte aufweist, als das Wilkie-Modell. Zusammenfassend lässt sich somit sagen, dass das Wilkie-Modell die Kausalitätsstrukturen zwischen seinen Komponenten nur unzureichend widerspiegelt. Die Exogenitätsannahme kann nicht bestätigt werden, es ist mit einem Endogenitätsbias bei der Parameterschätzung zu rechnen.

3.5 Konklusion

Der Begriff *Modellrisiko* ist eng mit den Termini *Modellspezifikation* und *Modellvalidierung* verbunden. Um Aussagen über ersteres treffen zu können, ist eine Auseinandersetzung mit der korrekten Spezifikation des Modells fundamental. Wir haben gezeigt, dass bei Nichtbeachtung dieses Aspekts das Modellrisiko erheblich zunehmen kann und daher eine konsistente Spezifikationsstrategie vonnöten ist (siehe z.B. Hendry [1995], S.16ff). Dies gilt umso mehr für diejenigen Modelle, welche ökonomischen Szenariogeneratoren zugrunde liegen, da diese nicht im Sinne von Prognosemodellen gebaut sind und daher nur in seltenen Fällen mit sogenannten Out-of-Sample-Verfahren (Backtesting-Verfahren) validiert werden können. Dieser Unterschied betont die Überprüfung des Modellbaus durch Fehlspezifikationstests, welche meist in Form von In-Sample-Tests formuliert werden. Ein hinreichend gut spezifiziertes Modell sollte schließlich anhand von Datensätzen validiert werden. Die notwendige Respezifikation bei unzureichend validierten oder fehlspezifizierten Modellen verdeutlicht den Prozesscharakter dieses Vorganges. Somit sollte auch das Modellrisiko prozessorientiert verstanden werden:

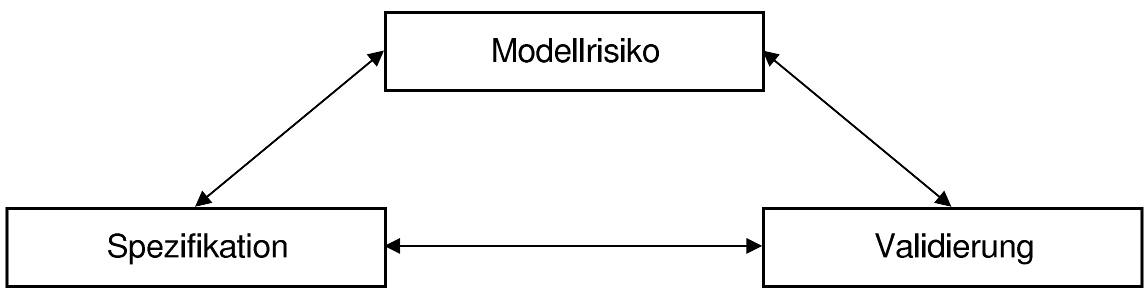

Abbildung 3.5: Modellrisiko, Spezifikation und Validierung.

In unserer Darstellung wurden die topologischen Argumente von Davies [2008] nicht explizit berücksichtigt. Letzterer übt Kritik an den konventionellen klassischen sowie bayesianischen statistischen Verfahren und favorisiert die von der Konvergenz in Verteilung erzeugte Topologie für eine konsistente Spezifikation, Validierung und Inferenz in statistischen Modellen. Diese topologischen Aspekte sollten neben der Elaboration von Spezifikationstests Gegenstand weiterführender Forschung sein.

Chapter 4

About the impact of model risk on capital reserves: A quantitative analysis

About the impact of model risk on capital reserves: A quantitative analysis

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4.1 Introduction

From a financial institution's point of view the importance of dealing with model risk has risen substantially since the implementation of new regulatory laws such as Basel II or Solvency II. Since then the option of implementing internal models instead of the hitherto obligatory application of standard methods as e.g. being documented in QIS 4b for the calculation of the solvency capital requirement has been driven forth. Internal models are particulary suitable for covering the risen demands of stakeholders concerning the quality of risk management as the incorporation of sophisticated and flexible mathematical methods can be fulfilled. Another advantage of internal models apart from the improved risk measurement marks the refinement of the risk culture. This might be exemplified by the procedure of rating agencies demanding the existence of an internal model in order for the company to be rated *strong* concerning its risk management. Internal models can be defined as large (high amount of explanatory variables), nonlinear (embedded options), stochastic (modeling future states of nature) systems. In the context of the holistic approach of Basel II and Solvency II an estimation of the balance sheet's forecast distribution is carried out by consulting company models (management rules, provision for premium refunds etc.) as well as stochastic models.

Nevertheless the implementation of internal models implies one thus far not satisfactorily handled issue: the topic of model risk. Without consideration of the latter the capital reserves are determined by the standard approach of risk management. That is portfolio risk is subsumed as the aggregate of the marginal distributions of the risk factors market risk, credit risk and operational risk applying a suitable aggregation method (for a discussion of this topic cf. Rosenberg and Schuermann [2006]) and reporting a risk measure thereof. With the possible utilization of internal models in order to model market risk the risk measure of the latter depends substantially on the concrete specification of the internal model. Thus there does exist a strong relationship between model risk and the resulting market risk which should be accounted for when it comes to the determination of capital reserves. In this context we understand model risk as every risk induced by the choice, specification and estimation of a statistical model.[1]

In order for model risk to be considered as a separate risk factor an operational quantification of the former should be provided. Although some authors like Crouhy et al. [1998] or Cont [2006] made several proposals for an abstract coverage of the topic there does not exist an unambiguous method for the quantification of model risk thus far. In the literature there are basically two approaches dealing with the question of measuring model risk: the bayesian model averaging approach (cf. e.g. Brock et al. [2003]) and the worst-case approach (cf. e.g. Kerkhof et al.

[1]Note that human failure is captured under operational risk.

[2010]). Although from a practical point of view there is no such thing as obligatory capital charges for induced model risk the Basel Committee (cf. BCBS [1996]) suggests a so-called multiplication factor of three with regard to market risk in order to account for model risk. Stahl [1997] showed that the multiplication factor may be interpreted as the relation of the risk measure of the underlying under different (parametric or non-parametric) distributions. This interpretation corresponds closely to the worst-case approach of measuring model risk. Hence in this paper we follow the idea of Kerkhof et al. [2010] fragmenting model risk into estimation risk, misspecification risk and identification risk and analyze its impact on capital reserves. Our approach features the following new aspects concerning the topic of model risk and capital reserves.

By using real insurance data we do not only analyze the model risk of the underlying but also take the company model into account what, to our knowledge, has not been done before. Whilst the existing literature does not differentiate between the statistical model and the company model resulting in the assumption that the underlying marks a concrete balance sheet position we take the whole structure of the internal model into account. Concretely we utilize a specific company model, the model for pension liabilities of a large German insurance company, and demonstrate its interaction with the econometric model for the underlying under the aspect of model risk. By taking a broad range of time series models into account which differ in their functional forms we are able to refine the definition of model risk further by discriminating between misspecification risk in functional form and misspecification risk in distribution and are thus able to quantify its contribution to overall model risk.

The paper proceeds as follows. A formal definition of the various types of model risk is carried out in section 4.2. Afterwards the results of the empirical study are presented in section 4.3. Here we briefly describe the specification and estimation results for the underlying. Then we look at the implications concerning model risk with respect to the underlying and the company model. Section 4.4 concludes.

4.2 Measuring Model Risk

4.2.1 Econometric Setting

One of the main tasks of risk management is to determine risk measures, denoted by π, of economic or financial variables of interest, X. Statistically X is considered to be a random variable originating from the stochastic process $\{X_t\}_{-\infty}^{\infty}$ with time index t being defined on the probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ consisting of event space, σ-algebra and a probability measure. Assume furthermore that X is distributed according to some density function $f(X)$, short $X \sim f(X)$. Obviously π is then some function $q(\cdot)$ of $f(X)$, i.e. $\pi = q(f(X))$. Throughout the paper we specify π as the value-at-risk being defined as $VaR_j(p) = \inf\{x \in \mathbb{R} | \mathbb{P}(X \leq x) \geq p\}$[2] where p denotes the confidence level.

[2] We additionally utilized the expected shortfall being proposed by Artzner et al. [1999]: $ES_j(p) = E_{\mathbb{P}}(x \in \mathbb{R} | x \geq \inf\{x \in \mathbb{R} | \mathbb{P}(X \leq x) \geq p\})$ as a risk measure. Since the results do not differ qualitatively we solely report the results for the value-at-risk. Quantitative results for the expected shortfall are available upon request.

In case $f(X)$ is known π marks the market risk of the underlying X. In practice however $f(X)$ is unknown in two respects. First only a finite number of values for the random variable X can be observed. In other words the data $\{x_t\}_0^T$, $T < \infty$ are assumed to be realizations originating from the stochastic process $\{X_t\}_{-\infty}^{\infty}$, i.e. $\{x_t\}_0^T$ is an approximation for the unknown process $\{X_t\}_{-\infty}^{\infty}$. The second part that has to be approximated in $q(f(X))$ concerns the character or shape of the density function $f(\cdot)$ which is also unknown in practice. Denoting the estimation or assumption of $f(\cdot)$ under model i by $f_i(\cdot)$ leads to the potential risk that $f(\cdot) \neq f_i(\cdot)$ and thus $q(f(X)) \neq q(f_i(X))$. These two concepts define the two components of total model risk: estimation risk and misspecification risk.

In order to overcome the gap between $\{x_t\}$ and $\{X_t\}$ the frequentist's approach assumes the data to be generated by a so-called data generating process (DGP). The DGP connects the theoretical distribution of X with its empirical counterpart by introducing the k-dimensional parameter space $\Theta \subseteq \mathbb{R}^k$ which determines the character of the empirical distribution function. The population parameter $\theta \in \Theta$ marks the point in Θ that generates the data. The generic econometric DGP is given by the model

$$x_t = H(z_t|\theta) + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim G. \tag{4.1}$$

$z_t = (x_{t-1}, \ldots, x_0, y_t, \ldots, y_0) = (\tilde{x}_{t-1}, \tilde{y}_t)$ contains all kinds of lagged endogenous $(\tilde{x}_{t-1})$ and/or exogenous $(\tilde{y}_t)$ explanatory variables, $H(\cdot)$ describes the functional form of the relationship and ε_t denotes an error term being distributed according to distribution function G. Put differently $H(z_t|\theta) = E_t(x_t|\daleth_{t-1}) := \mu_t$ may be interpreted as the expected mean of x_t at time t conditioned on the information set $\daleth_{t-1}$, i.e. under consideration of all information available up to and including time $t-1$. Hence the density function of x is given by $f(x) = f(\mu + \varepsilon)$. With $\mu \sim H$ and the assumption of no endogeneity in 4.1 leads to the convolution of $f(x)$ being given as

$$f(x) = \int_{-\infty}^{\infty} H(x - \varepsilon) G(\varepsilon) d\varepsilon. \tag{4.2}$$

4.2.2 Definitions of Model Risk

Let us now introduce the empirical counterpart of (4.2) as being given as

$$f_i(x) = f_{jk}(x) = \int_{-\infty}^{\infty} H_j(x - \varepsilon) G_k(\varepsilon) d\varepsilon. \tag{4.3}$$

The empirical or assumed model is defined as M_i implying that the data is distributed according to $f_i(x)$. Then $\Psi : j \oplus k \to M_i$ returns the empirical or assumed model specification consisting of a functional form specification H_j and a error distribution assumption G_k where $j \in \mathcal{J}$ and $k \in \mathcal{K}$ denote sets of feasible functional form and distribution specifications resulting in model set $\mathcal{I} \ni i$. In other words every combination of a specification of functional form j and distribution k leads to the concrete model specification i. (4.2) and (4.3) are assumed to be related via $f_{ab}(x) = f(x)$. Hence choosing $k = a$ and $j = b$ yields the true density specification of x which is obviously never possible in practice as a and b cannot be observed. This enables us to analyze the topic of model risk according to specification i with respect to three different assumptions concerning the choice of j and k in (4.3):

$$\begin{aligned} A_1 &: \quad a = j \quad \text{and} \quad b = k \\ A_2 &: \quad a \neq j \quad \text{and} \quad b = k \\ A_3 &: \quad a = j \quad \text{and} \quad b \neq k. \end{aligned}$$

These three assumptions are now referred to for defining model risk in an operational way. Let the three resulting density functions for x according to model specification i with respect to assumptions $1-3$ be defined as $f_{i|A1} = f_{jk}(x|a = j, b = k)$, $f_{i|A2} = f_{jk}(x|a \neq j, b = k)$, $f_{i|A3} = f_{jk}(x|a = j, b \neq k)$ whereas $f_{jk}(x) = f_i(x)$ describes the unconditional empirical density of x.
The connection of the density functions under the three assumptions and the resulting risk measure concerning model specification i is described by $\pi_i = q(f_i(x))$. Since π_i marks a point estimate of a quantile of $f_i(x)$ a confidence interval $[\pi_i \pm \eta_i(\alpha)]$ for the estimate at level $1-\alpha$ can be defined. Via the functional delta method eta can be derived as $\eta_{iT}(\alpha) = z_{1-\alpha/2}\hat{\sigma}_i{}^3$, where $\hat{\sigma}_i = \sqrt{p(1-p)/T \cdot f_i(\pi_i(p))^2}$ with p denoting the risk measure's confidence level. The corresponding conditional values are denoted by $\pi_{i|A_\gamma}$ and $\eta_{i|A_\gamma}$ with $\gamma \in \{1,2,3\}$ where $i|A_\gamma$ indicates that it is referred to $f_{i|A_\gamma}$ in the respective formula. π and σ describe the point estimate and standard deviation with respect to the true density function $f(\cdot)$. Note that for large T the confidence interval diminishes as $\lim_{T\to\infty} \eta = 0$.
Under A_1 $f_i(\cdot) = f_{i|A1}(\cdot) = f(\cdot)$ is not misspecified, i.e. $f(\cdot)$ is modeled correctly which is why there is no misspecification risk under A_1. There is however estimation risk as $f_{i|A_1}(x) \neq f(X)$ as $T < \infty$ in $\{x_t\}_0^T$. Consequently the quotient of the upper bound of the confidence interval and the point estimate under A_1 yields an operational definition for estimation risk according to model specification i.

Definition 5 (Estimation Risk)**.** *Let $\Pi_{i|A_1} = \pi_{i|A_1} + \eta_{i|A_1}$ be the upper bound of the $1-\alpha$ confidence interval of the risk measure's point estimate according to model specification i under the assumption that $a = j$ and $b = k$ in (4.3). Estimation risk of model i is then given by $R_{1i} = \Pi_{i|A_1} \cdot \pi_i^{-1}$.*

The second component of model risk arises from the fact that $f(\cdot)$ is unknown in practice. Hence practically A_1 can never be fulfilled. Therefore the empirical density function under model i, $f_i(\cdot)$, is seen as an approximation for $f(\cdot)$ resulting in the risk of misspecifying the population density function when j and/or k are chosen incorrectly. On top there is again estimation uncertainty even when $f(\cdot)$ is found as described earlier. Therefore total model risk concerning model i consists of estimation risk and misspecification risk and can be defined as the difference of π regarding the true model and the imposed model specification i.
Obviously $f(\cdot)$ is unknown in practice and has to be approximated adequately. A natural estimate of $f(\cdot)$ marks the empirical density function $\tilde{f}(\cdot)$ seeking the sample analogue of X, $x \equiv \{x_t\}_{t=0}^T$

[3] For notational convenience we drop the index T in the following.

with $T < \infty$. Recalling that by the Glivenko–Cantelli theorem

$$P\left(\lim_{T\to\infty} \sup_x |\tilde{f}(x) - f(X)| = 0\right) = 1$$

the definition of total model risk can be formulated as follows.

Definition 6 (Total Model Risk)**.** *Let $\Pi = \pi + \eta$ be the upper bound of the $1-\alpha$ confidence interval of the risk measure's point estimate according to the empirical density function $\tilde{f}(\cdot)$ and let π_i be the point estimate of π under model specification i in (4.3). Total model risk of model i is then given by $R_{2i} = \Pi \cdot \pi_i^{-1}$.*

The inequality $f_i(\cdot) \neq f(\cdot)$ holds when either the functional form assumption is wrong, i.e. $j \neq a$ and/or the error distribution is modeled incorrectly, i.e. $k \neq b$. Consequently misspecification risk may be further differentiated into misspecification risk in functional form and misspecification risk in distribution depending on whether A_2 or A_3 is met. Note however that as $T < \infty$ in empirical modeling there is still estimation risk. Thus the term misspecification risk may be misleading as rather a combination of estimation risk and misspecification risk is defined. Straightforwardly the two types of misspecification risk may be formulated.

Definition 7 (Misspecification Risk in Functional Form)**.** *Let $\Pi_{i|A_2} = \pi_{i|A_2} + \eta_{i|A_2}$ be the upper bound of the $1-\alpha$ confidence interval of the risk measure's point estimate according to model specification i under the condition that $a \neq j$ and $b = k$ in (4.3). Misspecification risk in functional form of model i is then given by $R_{3i} = \Pi_{i|A_2} \cdot \pi_i^{-1}$.*

Thus R_{3i} returns a measure of what happens when A_1 is imposed whilst A_2 is true. In other words the effect of falsely imposing $j = a$ whereas actually $j \neq a$ holds true is quantified under the condition that the error distribution is specified correctly. Straightforwardly comparing π under A_1 and A_3 yields a definition for misspecification risk in distribution.

Definition 8 (Misspecification Risk in Distribution)**.** *Let $\Pi_{i|A_3} = \pi_{i|A_3} + \eta_{i|A_3}$ be the upper bound of the $1-\alpha$ confidence interval of the risk measure's point estimate according to model specification i under the condition that $a = j$ and $b \neq k$ in (4.3). Misspecification risk in functional form of model i is then given by $R_{4i} = \Pi_{i|A_3} \cdot \pi_i^{-1}$.*

Since so far solely the underlying has been regarded in the following we should look at model risk concerning the company model. The latter models the variable at interest (denoted by L) which depends on the underlying via the relationship $L = g(x)$ where $g(\cdot)$ denotes a continuously differentiable function.[4] Then the density of L with respect to $f_i(x)$ is given by $\hat{f}_i(L) = f_i(x) \cdot |dh(L)/dL|$ where $h(L) = g^{-1}(L)$. Hence the various definitions of market risk and model risk (total, in distribution and in functional form) can easily be transferred to the company model by substituting $f_i(x)$ for $\hat{f}_i(L)$. Note however that we cannot define an estimation error in this setting since the company model cannot be handled like an econometric model.

4.3 Empirical Study

In this section we show that the definition of model risk is not only an academic issue but also marks a highly relevant topic in practice due to its monetary implications. Concretely

[4] A concrete specification of $g(\cdot)$ is given in section 4.3.

we exemplify the monetary changes concerning the capital reserves from an internal company model that occur when using different definitions of model risk. For this purpose we use a real company model stemming from a large German insurance company. Concretely we deal with the company's pension model and its implied capital reserves. The pension liabilities depend on two economic variables: inflation and interest rates. Whereas the former functions as an adjustment for the obligation in terms of salary rates the latter is used as a discount rate. The pension liabilities are then given by

$$L = \beta_0 \cdot (1 + \beta_1 \cdot I)^{\beta_2} \cdot (1 + \beta_3 \cdot Y)^{-\beta_2}, \tag{4.4}$$

where I denotes inflation, Y denotes the interest rate and $\beta = (\beta_0, \ldots, \beta_3)$ returns a vector of coefficients. Due to reasons of concealment β cannot be reported in this paper. Fig.4.1 however gives an idea of the shape of the function. In order to derive the distribution of the pension

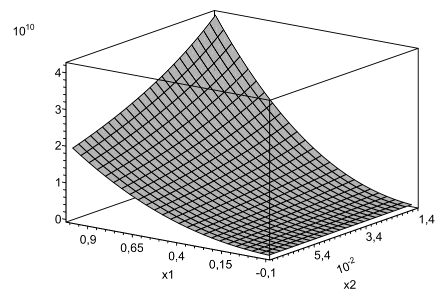

Figure 4.1: illustrates the shape of the pension function. x_1 describes the inflation rate, the interest rate is given by x_2 and L is displayed on the z-axis. Note that the combination of high inflation rates and low interest rates leads the function to rise quickly.

liabilities from which the capital reserves are determined different inflation and interest rate scenarios have to be considered. Concretely we simulate $J = 10,000$ inflation scenarios for $I = 15$ different specifications of various econometric time series models. Following we calculate $L_{j,i}$ according to (4.4) with $j = (1, \ldots, J = 10,000)$ and $(i = 1, \ldots, I = 15)$ resulting in the array of pension liabilities $L \sim (J \times K)$. The concrete specification of the inflation models is described in the next section.

4.3.1 Inflation Models

In order not to further complicate the procedure we solely deal with modeling the inflation rate for a start. As far as the interest rate is concerned scenarios having been developed internally by the insurance company are utilized (for a brief overview of the interest rate scenario's distribution cf. panel (a) in Fig.4.2). This is legitimized by considering the position of the inflation rate at the top of the cascade in the benchmark Wilkie model (Wilkie [1995]) mirroring its particular importance. In case a misspecified inflation model is utilized the misspecification error transmits throughout the whole system. This means that dealing with the inflation model should be of the highest priority when it comes to specifying scenarios that are to outperform the Wilkie model.

In order to carry out a consistent procedure of model specification a hierarchy of the univariate time series models being used in practice is very helpful. In the first level we discriminate between linear and nonlinear models. Recall that by the Wold decomposition any zero-mean purely non deterministic stationary process $\{y_t\}_{t=1}^T$ can be written in the form

$$y_t = \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i} \tag{4.5}$$

where $\sum_i \|\Psi_i\|^2 < \infty$ and $\{\varepsilon_t\}_{t=1}^T$ is a stationary sequence of centered and uncorrelated random variables with common variance Σ. A process $\{y_t\}_{t=1}^T$ is said to be linear when $\{\varepsilon_t\} \overset{iid}{\sim} (0,\Sigma)$ in (4.5). Otherwise the process can be declared nonlinear. Note that the nonlinearity can occur in the mean as well as in the volatility.

This thought leads to the discrimination of three classes of time series models in our model selection procedure: linear models, nonlinear models in the mean and nonlinear models in the volatility.

The linear models are described by the $ARFIMA(p,d,q)$ model class being given as

$$\Theta(L)^{-1}\Phi(L)(1-L)^d y_t = \varepsilon_t, \tag{4.6}$$

where $\{y_t\}_{t=1}^T$ describes the time series of interest and $\{\varepsilon_t\}_{t=1}^T$ forms a white noise process. A possibility to model nonlinearity in the mean in this setting was proposed by Hsu [2005]. By rewriting the time series $y_t = \mu_t + \varepsilon_t$ as the sum of a deterministic part μ_t and a stochastic part ε_t the former can be modelled as $\mu_t = \mu_1 + \sum_{i=1}^n \lambda_i \cdot I(l_i < t \le l_{i+1})$ where n denotes the number of breaks, l_i are the break points, $I(\cdot)$ denotes the indicator function and $\lambda_i = \mu_{i+1} - \mu_i$. Note that structural changes in the mean are a typical example of the occurance of spurious long memory (cf. Diebold and Inoue [2001] or Engle and Smith [1999]). By neglecting the mean shifts the estimation of the fractional differencing parameter d might be biased quite heavily. That is why Hsu [2005] proposed to first determine the number of break points in the model and thereafter estimate the $ARFIMA$ parameters and the time of the breaks simultaneously. Whereas the former is done via application of the LIC information criterion described in Lavielle and Moulines [2000] the estimation is carried out by a modified local Whittle method.

A further possibility to model nonlinearity in the mean is given by the $STAR$ model introduced by Chan and Tong [1986] and popularized by Granger and Teräsvirta [1993] and Teräsvirta [1994]. It is given by

$$\begin{aligned} y_t \quad = \quad & (\phi_{0,1} + \phi_{1,1} y_{t-1} + \ldots + \phi_{p_1} y_{t-p_1})(1 - G(y_{t-1};\gamma,c)) \\ & +(\phi_{0,2} + \phi_{1,2} y_{t-1} + \ldots + \phi_{p_2} y_{t-p_2}) G(y_{t-1};\gamma,c) + \varepsilon_t \end{aligned} \tag{4.7}$$

with $G(\cdot)$, γ and c denoting transition function, smoothness parameter and threshold value.

Finally nonlinearities in the volatility may be handled by the $APARCH$ model class which was introduced by Ding et al. [1993] and is defined as

$$y_t = \mu + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{j=1}^{q} \varepsilon_{t-j} + \varepsilon_t, \tag{4.8}$$

$$\varepsilon_t = h_t^{1/\delta} \cdot \nu_t,$$

$$h_t = \omega + \sum_{k=1}^{K} \alpha_k \left(|\varepsilon_{t-k}| - \psi_k \cdot \varepsilon_{t-k}\right)^{\delta} + \sum_{l=1}^{L} \beta_l \cdot h_{t-l} \tag{4.9}$$

where μ and ω are constants, a, α and β are vectors of coefficients and $\{\nu_t\} \overset{iid}{\sim} (0,\Sigma)$. The specification and estimation of the models is described in the next section.

4.3.2 Estimation Results

The modeling of the inflation rate has been carried out by using monthly US inflation data for the period 01/1954 until 02/2010 taken from *Datastream*. The time series and its empirical density estimate are plotted in panel (b) and (c) of Fig.4.2. The specified models and its parameters

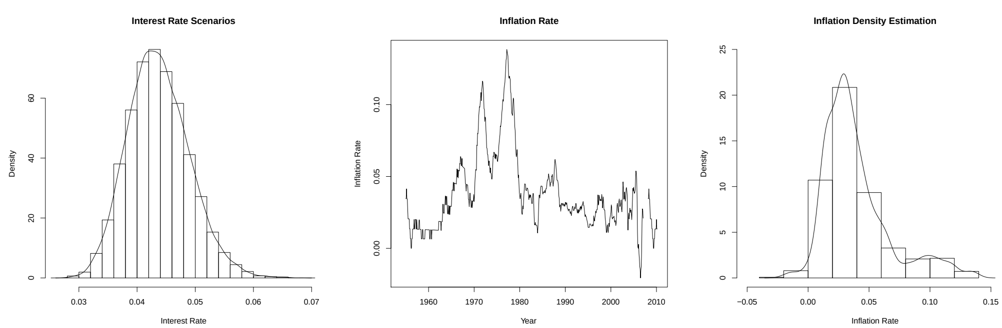

Figure 4.2: describes the histogram and density estimation of the interest rate scenarios generated by the insurers internal economic scenario generator, the monthly US inflation rate being calculated as the difference of the log consumer price indexes in regard to the respective value from the previous year and the corresponding empirical density estimate.

are given in Tab.4.1. The *ADF* test (cf. Dickey and Fuller [1979]) as well as the *KPSS* test (cf. Kwiatkowski et al. [1992]) indicate the series to be $I(1)$ which is why henceforward its first difference is utilized. The procedure concerning the simulation of the inflation rates is given as follows. For each of the $I = 15$ models M_i, $i = 1,\ldots,I$, the parameters are estimated. Following forecast values $\hat{y}_{t+h}$ with $h = 1,\ldots,H$ are derived, where the forecast period is chosen to equal $H = 118$. This value accounts for the fact that (4.4) necessitates the 10-year ahead inflation rate while having monthly data up to 02/2010. The forecast values are then given by

$$\hat{y}_{t+h} = E(y_{t+h}|\Omega_{t+h-1}) + \varepsilon_{t+h}, \quad h = 1,\ldots,H \tag{4.10}$$

where Ω_{t+h-1} is the information set consisting of all relevant information up to and including time $t+h-1$ and ε being a gaussian error term. The annual inflation rate is given by the annual mean value. This procedure is replicated $N = 10{,}000$ times for each of the $I = 15$ models yielding the empirical distributions which are summarized in Tab.4.4 (cf. section 4.5).

Equation	i	Model	Parameter										Notation
			p	q	d	p_1	p_2	c	γ	K	L	ψ	
(4.6)	1	M_1	1	0	0	-	-	-	-	-	-	-	Wilkie, AR(1)
	2	M_2	2^*	2^*	0	-	-	-	-	-	-	-	ARMA(2,2)
	3	M_3	0	0	0.178^{**}	-	-	-	-	-	-	-	ARFIMA(0,d,0)
	4	M_4	2^*	2^*	0.118^{**}	-	-	-	-	-	-	-	ARFIMA(p,d,q)
	5	M_5	0	0	0.083^{**}	-	-	-	-	-	-	-	HSU
(4.7)	6	M_6	-	-	-	1	1	0.005^{**}	40^{**}	-	-	-	STAR(1,c,γ)
	7	M_7	-	-	-	13^*	13^*	-0.012^{**}	40^{**}	-	-	-	STAR(p,c,γ)
(4.8)	8	M_8	0	0	0	-	-	-	-	1	0	0	ARCH(1)
	9	M_9	0	0	0	-	-	-	-	4^*	0	0	ARCH(K)
	10	M_{10}	0	0	0	-	-	-	-	1	1	0	GARCH(1,1)
	11	M_{11}	1	0	0	-	-	-	-	1	0	0	ARMA(1,0)-ARCH(1)
	12	M_{12}	1	0	0	-	-	-	-	2^*	0	0	ARMA(1,0)-ARCH(K)
	13	M_{13}	1	1	0	-	-	-	-	1	1	0	ARMA(1,1)-GARCH(1,1)
	14	M_{14}	0	0	0	-	-	-	-	1	0	0.083^{**}	APARCH(1)
	15	M_{15}	0	0	0	-	-	-	-	1	1	0.102^{**}	APARCH(1,1)

Table 4.1: offers an overview of the specified models. * signifies that the respective lag order has been chosen via information criteria. ** marks estimated values. Note that in M_6 and M_7 γ was respectively estimated to equal 40 signifying that the regime-switch is not carried out smoothly. In fact a threshold autoregressive (TAR) model is specified. The model specifications reported here are the most striking ones regarding its impact on the pension function. We examined a broad range of further specifications which can be reported upon request.

4.3.3 Simulation Results

Inflation Models

The results for the resulting inflation distributions are summarized in Tab. 4.4 (cf. section 4.5). The first striking result marks the fact that the differences of the inflation distributions mainly focuses on its tails. Whereas the central part of the distributions is rather homogenous the more extreme quantiles and the range differ considerably. This is especially driven by those models belonging to the class of *GARCH* processes (i.e. M_{10}, M_{13} and M_{15}). Although these models forecast quite plausible 10-year ahead inflation rates of approximately 2% in the mean its worst case scenarios of e.g. 170% deflation do not seem to be very realistic.
An explanation for these features can be given by more thoroughly looking at the autocorrelation function of the $GARCH(1,1)$ process. Bollerslev [1986] and Bollerslev [1988] showed that the kth autocorrelation of the squared errors in the $GARCH(1,1)$ process is given by

$$\rho_1 = \alpha_1 + \frac{\alpha_1^2\beta_1}{1-2\alpha_1\beta_1-\beta_1^2} \tag{4.11}$$

$$\rho_k = (\alpha_1+\beta_1)^{k-1}\rho_1 \quad \text{for } k = 2,3,\ldots \tag{4.12}$$

Note that the decay factor of (4.12) is $\alpha_1+\beta_1$. If the sum is close to 1 the autocorrelations will decline only very gradually (although an exponential decline is still given). In our case the sum of the estimated coefficients from the respective *GARCH* models are in each of the three cases very close to 1 i.e. the *GARCH* models feature slowly decaying autocorrelation functions. This leads to the result that draws of exceptionally high error terms during the simulation process (4.10) hardly decline in this model class explaining the extreme scenarios. Note that the fact of the sum of the estimated parameters in $GARCH(1,1)$ models being close to 1 is commonly found

in empirical research. E.g. Taylor [1986] estimate $GARCH(1,1)$ models for 40 different financial time series finding in all but six cases that $0.97 \leq \alpha_1 + \beta_1 < 1$.
It should furthermore be mentioned that Bollerslev [1986] and Bollerslev [1988] conditioned (4.11)-(4.12) on the validity of $(\alpha_1 + \beta_1)^2 + 2\alpha_1^2 < 1$ signifying that the kurtosis of ε_t is finite. If however this cannot be maintained, which is the case in our analysis, Ding and Granger [1996] show that for $\alpha_1 + \beta_1 < 1$ and $(\alpha_1 + \beta_1)^2 + 2\alpha_1^2 \geq 1$ the $GARCH(1,1)$ model is still covariance stationary with infinite fourth moment. In this case (4.12) is approximately valid with $\rho_1 \approx \alpha_1 + \beta_1/3$.
Note further that the HSU model (M_5) features a lower mean than the other models. This is due to the fact that we determined $n = 1$ break point via the LIC criterion at $t^* = 302$ which corresponds to 02/1979. By looking at Fig.4.2 it becomes clear that after t^* the trend in the inflation rate is declining what explaining the lower mean of M_5 even over 10,000 replications.
The risk measures being defined in section 4.2 are summarized in Tab.4.2. By first concentrating

i	Model	π_i	R_{1i}	R_{2i}	R_{3i}	R_{4i}
1	M_1	0.15	1.088	0.816	0.845	0.988
2	M_2	0.15	1.085	0.786	0.807	0.997
3	M_3	0.24	1.096	0.493	0.499	1.016
4	M_4	0.09	1.068	1.408	1.362	1.057
5	M_5	0.15	1.286	0.808	0.814	1.018
6	M_6	0.14	1.098	0.854	0.874	1.000
7	M_7	0.14	1.079	0.813	0.819	1.015
8	M_8	0.12	1.093	0.994	1.029	0.994
9	M_9	0.14	1.116	0.868	1.029	0.868
10	M_{10}	0.21	1.189	0.589	1.029	0.589
11	M_{11}	0.14	1.085	0.849	0.893	0.974
12	M_{12}	0.15	1.088	0.833	0.877	0.975
13	M_{13}	0.26	1.278	0.421	0.134	3.231
14	M_{14}	0.12	1.077	1.055	1.029	1.055
15	M_{15}	0.36	1.568	0.375	1.029	0.375

Table 4.2: returns measures of market risk (π), estimation risk (R_1), total model risk (R_2), model risk in functional form (R_3) and model risk in distribution (R_4) being defined in section 4.2 for each of the models in Tab. 4.1 with $p = 0.99$ and $\alpha = 0.05$.

on the estimation risk with regard to $f_i(X)$ it becomes clear that for the majority of the models the estimation risk lies somewhere between 5 and 10 percent. The $GARCH$ model class again forms an exception with estimation risks up to almost 57% which again can be attributed to its near-integratedness and high volatilities.
R_{2i} mirrors the total model risk of the underlying, that is specification, estimation and forecasting risk of the respective inflation model. Hence R_{2i} accounts for the risk that the chosen model differs from the true model. Remember that in practice R_{2i} is interpreted as the Basel multiplication factor and is set equal to three apart from some slight possible modifications. Hence utilizing an internal model necessitates to multiply π_i by the factor three when it comes to reporting market risk. Obviously this means that under this regulation there is an incentive to select the model

that implies the lowest value of π.

Note that apart from M_4 and M_{14} total model risk is smaller than 1 which is a rather unusual result. A possible explanation is given by the fact that in many popular examples the underlying marks a financial market variable exhibiting the stylized fact of fat-tailedness. This results in the empirical density function having a higher kurtosis than most of the standard parametric models which is why the risk measure of the empirical distribution exceeds the risk measure of the parametric distribution with the resulting multiplication factor exceeding 1. Inflation however is not a monetary but a real variable usually not featuring these stylized facts. Hence in many cases the parametric distribution possesses a much higher kurtosis than its empirical counterpart. Thus our results mirror the following trade-off. Those models implying a low market risk are penalized by a multiplication factor greater than 1 concerning the capital reserves. If a model reports a high market risk it is compensated by a multiplication factor smaller than 1.

Focusing on R_{3i} and R_{4i} we can state that model risk in functional form differs much more in between the models than model risk in distribution. This result seems to be rather intuitive as long as the empirical error distribution is close to the normal distribution. Further total model risk is in many cases very close to model risk in functional form indicating that the latter explains a big part of the former. On the other hand model risk in distribution is mostly very close to 1 except for the $GARCH$ model class. Note further that there are five models exhibiting the same model risk in functional form of 2.9%. For those models $H(z_t|\theta) = 0$ in (4.1) which means that there are no short-term dependencies in the DGP. Hence model risk in functional forms reduces to the estimation error when ε is drawn from the empirical distribution. In other words model risk in distribution exclusively determines total model risk when there are no short-term dependencies and thus no functional form in the process.

Hence we can conclude that market risk as well as model risk differs substantially between the various econometric models. Whereas theoretically there is a trade-off between market risk and model risk as both depend on the functional form and the distribution of the underlying practically the Basel multiplication factor is fixed which leads to the fact that the model inducing the lowest market risk implies the lowest capital reserves. In this context we do not intent to bother about the (political) question of whether setting $R_2 = 3$ marks a reasonable approach or not. We show however that if R_2 is interpreted and motivated as a measure of model risk the treatment of holding it constant over different models does not seem to be a plausible approach. The implication of these result in terms of monetary values is outlined next.

Company Model

Once the inflation scenarios have been determined the corresponding pension liabilities are calculated by (4.4) via the calibrated parameters β. As was argued in section 4.2 the market risk of L if the value-at-risk is chosen as a risk measure is given by $\hat{\pi}_i(p) = \inf\{L \in \mathbb{R} | \int_{-\infty}^{L} \hat{f}_i(L|\hat{\theta};\beta)dL \geq p\}$ with $\hat{f}_i(L) = f_i(x) \cdot |dh(L)/dL|$. (4.4) automatically yields $h(L) = \beta_1^{-1}((L/\beta_0(1+\beta_3 Y)^{-\beta_3})^{1/\beta_1} - 1)$ which is why $|dh(L)/dL| = (\beta_1\beta_2 L)^{-1}(L/\beta_0(1+\beta_3 Y)^{-\beta_3})^{1/\beta_1}$. We may then analyze the impact of the different model risk definitions in terms of monetary values. Since the exact values of $\hat{\pi}$ cannot be reported we again normalize with regard to the benchmark model M_1. Under the Basel accord capital reserves of model M_i in relation to M_1 are given by $CR_{Basel;i} = \hat{\pi}_i \cdot 3/(\hat{\pi}_1 \cdot 3)$ whereas $CR_{New;i} = \hat{\pi}_i \cdot R_{2i}/(\hat{\pi}_1 \cdot R_{21})$ describes the capital reserves under our proposed measure normalized with regard to the first model. As a stylized example capital reserves with respect to market risk are calculated as 8%[5] of the risk weighted assets which are given as a measure for market risk times δ, i.e. $CR = 0.08 \cdot \hat{\pi} \cdot \delta$. Setting $\hat{\pi} = 800$[6] leads to the difference of the two model risk definitions being given as

$$\Delta CR_i = 0.08 \cdot 800(3 - R_{2i}). \tag{4.13}$$

In other words ΔCR_i returns the excess capital reserves in Mio. € under model i when the Basel approach is applied instead of R_{2i}. Tab.4.3 mirrors the monetary implications concerning the capital reserves.

i	Model	$CR_{Basel;i}$	$CR_{new;i}$	ΔCR_i in Mio.€
1	M_1	1.00	1.00	139.75
2	M_2	1.04	0.96	141.67
3	M_3	1.73	0.60	160.48
4	M_4	0.68	1.72	101.89
5	M_5	1.01	0.99	140.28
6	M_6	0.98	1.05	137.35
7	M_7	1.02	1.00	139.94
8	M_8	0.86	1.22	128.39
9	M_9	0.98	1.06	136.45
10	M_{10}	1.39	0.72	154.25
11	M_{11}	0.99	1.04	137.63
12	M_{12}	1.00	1.02	138.66
13	M_{13}	2.07	0.52	165.08
14	M_{14}	0.82	1.29	124.47
15	M_{15}	2.47	0.46	167.98

Table 4.3: returns the capital reserves of model i under the Basel accord ($CR_{Basel;i}$) and under R_{2i} in relation to the benchmark model M_1. $CR_{new;i}$ mirrors the difference in million € of total capital reserves between the Basel approach and our definition of model risk according to (4.13).

It is of no surprise that the results displayed in Tab.4.2 are rediscovered in the results for CR_{Basel} in Tab.4.3. As M_4 features the lowest value of the market risk measure π this model exhibits the lowest model risk compensation with respect to a constant multiplication factor. On the other hand M_{15} clearly induces the highest capital reserves due to its high market risk measure. Again the models of the *GARCH* class feature distinct differences in the tails in comparison with the other models. By recapitulating the shape of the pension function (cf. Fig.4.1) this result should not be surprising. Remember that the *GARCH* inflation scenarios exhibit values in its right tail that are well above 0.4 which is exactly the area where (4.4) increases rapidly. By

[5]This value is proposed in the Basel accord (cf. BCBS [1996]).

[6]The exact market risk of the insurer's pension liabilities cannot be reported. The amount of 800 Mio.€ however yields a reasonable approximation.

looking at the pension liabilities' distribution[7] in more detail we can state that as the pension function is leveraged by the inflation scenarios the center of the distributions differ slightly more than the scenarios itself. Nevertheless the most striking deviations are once more found in the distributions' tails.

It can further be stated that the (unreported) values for the value-at-risk differ enormously in terms of monetary values. Concretely the discrepancy of the model with the lowest value-at-risk (M_4) and the model with the highest value-at-risk (M_{15}) lies around $5,000$ million €. Of course, one might argue that common sense allows the exclusion of the $GARCH$ model class but then still the difference adds up to approximately $2,000$ million € (M_4 vs. M_3). Regarding the expected shortfall the differences in the pension liabilities are even more striking going from $18,000$ million € without exclusion of the $GARCH$ model class to $3,000$ million € without consideration of M_{10}, M_{13} and M_{15}. Generally it becomes clear that both a high range and excess kurtosis in the econometric models produces the kurtosis in the pension liabilities' distribution to rise resulting in large values for the value-at-risk and the expected shortfall.

Looking at CR_{new} leads to the opposite implication. Here we observe that the models already entailing high values of π are "compensated" by a low multiplication factor. Intuitively that seems to be a reasonable procedure: If the econometric model induces a rather conservative (i.e. high) value-at-risk the model should be "rewarded" by a smaller multiplication factor if the latter is indeed interpreted as a measure of model risk. Another finding when comparing CR_{Basel} with CR_{New} is that for the former the variation over the models is much stronger. This should also be of no surprise as via consideration of $\tilde{f}(x)$ CR_{New} is a measure relative to the historical data at hand whereas the Basel definition rather returns a somewhat absolute parameter. Finally the differences of the two approaches concerning its monetary implications are given by ΔCR. We can see that the difference approximately lies between 100 and 170 million € which makes up about $13-21\%$ of the primary market risk measure. As $R_{2i} < 3 \forall i$ ΔCR is positive for all models meaning that according to our definition of model risk the capital reserves compensation for market risk are smaller than in the Basel approach. This, of course is not a general result but rather caused by the above mentioned situation that the inflations' distributions implied by $M_1 - M_{15}$ are mostly heavier tailed than its empirical counterpart (except for M_4 and M_{14}). If the empirical distribution features heavier tails than the induced model's distribution, ΔCR could very well be negative implying that more capital reserves should be reported in our definition compared to the Basel approach. Hence again we would like to mention that our results should not be interpreted in an absolute way such that our definition of model risk leads to higher or fewer capital reserves. In our opinion however R_2 yields a much better founded multiplication factor if the latter is interpreted as a measure of model risk. As the next section shows that marks a very interesting result since M_4 is the model which is indeed chosen by application of an empirical model specification strategy.

[7] Note again that we cannot report concrete values of the pension liabilities' distributions due to reasons of concealment.

4.3.4 Empirical Model Specification Strategy

The process of finding an appropriate model for the inflation scenarios marks a widely debated task among practitioners. In empirical work a specific model class is often chosen rather ad hoc and a suitable specification procedure is only very rarely carried out. Even if previous work attests a specific model to work very well for the economic variable at interest consulting a different data set might lead to a completely converse implication. That is why we carry out a data driven approach concerning the process of model specification. Our strategy consists of at most three steps and is given as follows. At first we try to find the best model in the class of linear time series. Once this model has been found, it is tested for remaining unspecified nonlinearity. If the latter cannot be rejected the best linear model is tested against each of the nonlinear model classes given in Tab.4.1 being represented by their most general form. In other words we discriminate between the cases of short memory, long memory and spurious long memory. Whilst the decision between short and long memory is the decision between $ARFIMA$ and $ARMA$, spurious long memory can be invoked by a nonlinear behavior of the process.

The selection of the most suited linear model might at first sight be thought of as an easy task since almost every conventional time series model is nested in (4.6). I.e. by selecting the lag orders in (4.6) first and estimating the corresponding parameters thereafter one might be able to impose zero restrictions on some of the parameters leading to sub models of the $ARFIMA$ class. Hence a general-to-specific modeling procedure equivalent to the Box-Jenkins approach for $ARMA$ models might be applied. There are however certain caveats in this argumentation.

Note that there are several ways to estimate the fractional differencing parameter in (4.6) which are based on the periodogram of the process. These include e.g. the GPH estimator of Geweke and Porter-Huwak [1983] or the Whittle estimator (cf. Robinson [1995]). The spectrum of a covariance stationary process $\{y_t\}_{t=1}^{T}$ is given as

$$f(\lambda) = |1 - \exp(-i\lambda)|^{-2d} f^*(\lambda), \quad -\pi \leq \lambda \leq \pi, \quad |d| < 0.5 \tag{4.14}$$

with $f^*(\lambda)$ representing the short-term correlation structure of the model and $i = \sqrt{-1}$. In practice (4.14) is approximated by the estimation function

$$\log I(\lambda_k) = c + dX_k + \varepsilon_k, \quad k = 1, \ldots, m \tag{4.15}$$

for $m \leq T/2$, $X_k = -2\log(\sin\lambda_k)$, $\lambda_k = 2\pi k/T$ and $I(\lambda) = (2\pi T)^{-1}|\sum_{t=1}^{T} y_t \exp(it\lambda)|^2$ for the sample y_t, $t = 1, \ldots, T$. (4.15) defines the periodogram where c and d can be estimated via linear regression. However, as Hurvich et al. [1998] point out, the procedure of estimating d by (4.15) leads to a bias if there are short-term correlations in the model, i.e. if $f^*(\lambda)$ is not a constant. This induces that if (4.6) contains $ARMA$ components no statements about the parameters' significance should be made for estimators based on (4.15).

Hence there are two possibilities for avoiding this shortcoming. Either one applies a different estimator not being based on the periodogram such as the nonparametric estimator proposed by Hurst [1951] or the maximum likelihood estimator of Beran [1995] determining all parameters simultaneously. Or the application of tests discriminating between short and long memory should

be carried out. We decide for the second procedure as it is, to our knowledge, not assured that alternative estimators are robust against *ARMA* processes. Concretely we apply two tests in order to discriminate between short and long memory. Firstly we employ the test of Lo [1991] and secondly we apply the test of Davidson and Sibbertsen [2009].
Lo [1991] specifies a modified rescaled range estimator given by

$$\hat{Q}_T = \frac{\max_{0<i\leq T}\{\sum_{t=1}^{i}(y_t-\overline{y})\} - \min_{0<i\leq T}\{\sum_{t=1}^{i}(y_t-\overline{y})\}}{\hat{\sigma}_T} \quad \text{and} \tag{4.16}$$

$$\hat{\sigma}_T = T^{-1}\sum_{t=1}^{T}(y_t-\overline{y})^2 + 2T^{-1}\omega_j(q)\left(\sum_{t=j+1}^{q}(y_t-\overline{y})(y_{t-j}-\overline{y})\right) \tag{4.17}$$

where $\{y\}_{t=1}^{T}$ denotes the process of interest with mean $\overline{y} = T^{-1}\sum_{t=1}^{T} y_t$ and $\omega_j(q) = 1-(j/(q+1))$ for $q < T$. Hence (4.16) can be interpreted as the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Note that (4.17) is the heteroscedasticity and autocorrelation consistent variance estimator with the weights $\omega_j(q)$ being those suggested by Newey and West [1987]. Hence in case the process is short-range dependent $\hat{\sigma}_T$ controls for the autocovariances making (4.16) able to discriminate between short-range and long-range dependence. As $T^{-1/2}\hat{Q}_T$ is asymptotically distributed as the range of a standard brownian bridge under the Null of short-range dependence the latter can be tested against the alternative of long-range dependence.
The bias test of Davidson and Sibbertsen [2009] is based on (4.14) and tests $H_0 : f^*(\lambda) = cons$ vs. $H_1 : f^*(\lambda) \neq cons$. The test statistic is given by

$$TS = \frac{\hat{d}_1 - \hat{d}_2}{SE(\hat{d}_1 - \hat{d}_2)} \tag{4.18}$$

where $\hat{d}_1$ and $\hat{d}_2$ denote alternative estimators of the fractional differencing parameter with $SE(\cdot)$ being a suitable estimation for the difference's standard deviation being derived in Davidson and Sibbertsen [2009]. The authors further proved in their paper that $TS \xrightarrow{d} N(0,1)$ under certain conditions. Choosing $\hat{d}_1$ to be the estimator regressing $I(\lambda_k)$ onto $(X_k, 1)$ in (4.15) whilst $\hat{d}_2$ is derived if $I(\lambda_k)$ is regressed onto $(X_k, 1, h_1(\lambda_k), \ldots, h_{pT}(\lambda_k))$, where $h_j(\lambda_k) = \cos(j\lambda_k)/\sqrt{\pi}$ is the jth order Fourier frequency, one can test the Null of an $ARFIMA(0,d,0)$ process against the alternative of an $ARFIMA(p,d,q)$ process with either $p > 0$ and/or $q > 0$. Note that (4.18) is a simple type of the Hausman [1978] test as under the Null $\hat{d}_1$ is consistent and asymptotically efficient, but biased and inconsistent under the alternative, whereas $\hat{d}_2$ is consistent under both hypotheses.
With a p-value being very close to zero the Lo test clearly rejects the Null of short-range dependence. The p-value of the bias test equals 0.045 indicating that there are *ARMA* components in the process at the 5% level. Having in mind the result of d being actually different from zero we can now be relatively sure that the presence of the fractional differencing parameter is due to long-range dependence and not spuriously caused by *ARMA* components although the latter are indeed present. Thus in the next step we estimate the parameters of the $ARFIMA(p,d,q)$ model simultaneously by the method of Beran [1995] after selecting the lag orders via the Schwarz

information criterion leading to the values reported in Tab.4.4 for M_4.
Once a linear model has been specified and estimated it is tested against remaining nonlinearity. Note that there are several linearity tests in the literature (for an overview cf. Granger and Teräsvirta [1993]). We focused on testing against unspecified remaining nonlinearity as from a practical point of view it is not feasible to carry out different types of tests for every kind of nonlinear model. Thus we applied the popular test of Tsay [1986] performing considerably well in small samples as has been shown in simulation studies (cf. e.g. Tsay [1986] or Pena and Rodriguez [2005]). The test can be described as follows.
At first a linear model (in our case M_4) is fitted to the time series and the residuals of the linear fit $\hat{\varepsilon}_t$ are computed. Secondly $h = M(M+1)/2$ proxy variables, where M stands for the autoregressive order of the process, are defined. The proxy variables are represented by $z_t = vech(Y_t^{'} Y_t)$ where $Y_t = (y_{t-1}, \ldots, y_{t-M})$ and $vech(\cdot)$ denotes the column stacking operator using only those elements on or below the main diagonal of each column. Hence z_t consists of several squares and cross products of the series typifying the nonlinearity. Thirdly each of the proxy variables is regressed against Y_t and the h corresponding residuals are denoted by $\hat{u}_t$. Finally the model

$$\hat{\varepsilon}_t = \xi \cdot \hat{u}_t + \iota_t \tag{4.19}$$

where ι is white noise and $\xi = (\xi_1, \ldots, \xi_h)$ denotes a vector of coefficients is specified. (4.19) is estimated by OLS and $H_0 : \xi_1 = \cdots = \xi_h = 0$ vs. $H_1 : \xi_i \neq 0$, for at least one $i = 1, \ldots, h$ is tested consulting a conventional F-test. Under the Null no remaining nonlinearity covered by the proxy variables can be detected. Having utilized the test we do not find remaining nonlinearity in M_4 as the Tsay test reported a p-value of 0.75 for $M = 3$.[8] Hence we can state that M_4 is chosen by the empirical specification procedure as the model that fits the data best.

4.4 Conclusion

In this paper we elaborate a definition of model risk as being interpreted as every risk induced by the choice, specification and estimation of a statistical model. We further differentiate model risk into estimation risk, model risk functional form and model risk in distribution. Afterwards we compare our definition of model risk with the standard definition under the Basel accord. As a toy model we looked at the interaction of an econometric model and the corresponding pension liabilities of a large German insurance company as an example of a company model under the focus of induced model risk. For this purpose we model the inflation rate with 15 different time series models representing most of the conventional time series models in the literature. We then look at the impact of model risk on capital reserves with the former functioning as a multiplication factor concerning market risk under the two different definitions of model risk.
The first striking result marks the fact that the different econometric models feature rather decent (between 5 and 10 %) measures of estimation risk. With model risk in distribution also being rather small total model risk is mainly caused by model risk in functional form. By using real insurance data we then determine the corresponding pension liabilities finding that

[8] M was determined by information criteria. Choosing different orders did not alter the test's outcome.

induced model risk differs remarkably depending on the econometric model that is applied. In general the model risk rises if the range and/or the kurtosis of the inflation scenarios increases. Concretely the discrepancy between the models might add up to several million € concerning capital reserves. We find that under the Basel accord the model featuring the lowest market risk measure is “rewarded” with the lowest resulting capital reserves as the multiplication factor remains constant. In the context of our definition of model risk the opposite is true. The model featuring the highest market risk is “compensated” for its conservative estimation by a low multiplication factor. In our view holding the multiplication factor constant counteracts the motivation of model risk which is to link capital reserves to the concrete econometric model specification. Our proposed definition overcomes this caveat as the measure directly depends on the input data at hand. Comparing these two approaches of model risk as to the consequences in terms of capital reserves we find the difference to add up to 100-170 mio. € in our example. Finally we apply a data driven specification strategy in order to specify the inflation model resulting in the model with the lowest market risk and thus the model with the lowest induced capital reserves under the Basel approach being chosen.

Our analysis might be refined in two respects. First, we solely focus on cascade models as economic scenario generators. Here our work might easily be extended via utilization of (structural) multivariate models covering topics such as cointegration or causality. Furthermore consulting an alternative company model being dependent of more than two economic variables offers the analysis of the cascade structure of the Wilkie model in general and possible improvements thereof.

Secondly we did not deal with errors which might occur by leveraging the pension function. I.e. the selection of model points as well as the calibration of the company model was neglected. Especially the first aspect is worth considering as it is still unclear which scenarios should be selected such that an appropriate fit of the company model function is achieved. Considering that an unrepresentative selection might lead to a bad fit misleading statements concerning the pension liabilities and the induced model risk might be concluded.

4.5 Appendix

	Model							
Statistic	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
Minimum	-0.18	-0.21	-0.33	-0.09	-0.21	-0.16	-0.14	-0.16
1%-Quantile	-0.10	-0.11	-0.20	-0.04	-0.13	-0.10	-0.07	-0.08
5%-Quantile	-0.07	-0.08	-0.13	-0.03	-0.09	-0.07	-0.04	-0.05
1st Quartile	-0.01	-0.02	-0.04	0.00	-0.03	-0.02	0.00	-0.01
Median	0.02	0.02	0.02	0.02	0.01	0.02	0.04	0.02
Mean	0.02	0.02	0.02	0.02	0.01	0.02	0.04	0.02
3rd Quartile	0.06	0.06	0.09	0.04	0.05	0.05	0.07	0.05
95%-Quantile	0.11	0.12	0.18	0.07	0.11	0.10	0.11	0.09
99%-Quantile	0.15	0.15	0.24	0.09	0.15	0.14	0.14	0.12
Maximum	0.22	0.25	0.52	0.12	0.23	0.21	0.22	0.18
1st Moment	0.02	0.02	0.02	0.02	0.01	0.02	0.04	0.02
2nd Moment	0.05	0.06	0.10	0.03	0.06	0.05	0.05	0.04
3rd Moment	-0.02	-0.03	0.02	0.01	-0.03	-0.05	0.00	-0.03
4th Moment	-0.04	-0.01	0.00	-0.01	-0.02	-0.05	-0.05	0.02
	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}	
Minimum	-0.25	-1.09	-0.19	-0.20	-1.70	-0.15	-1.24	
1%-Quantile	-0.10	-0.18	-0.10	-0.11	-0.22	-0.08	-0.25	
5%-Quantile	-0.06	-0.08	-0.06	-0.07	-0.11	-0.05	-0.12	
1st Quartile	-0.01	-0.01	-0.01	-0.01	-0.02	-0.01	-0.03	
Median	0.02	0.02	0.02	0.02	0.02	0.02	0.02	
Mean	0.02	0.02	0.02	0.02	0.02	0.02	0.02	
3rd Quartile	0.05	0.05	0.06	0.06	0.06	0.05	0.06	
95%-Quantile	0.10	0.12	0.11	0.11	0.15	0.09	0.17	
99%-Quantile	0.14	0.21	0.14	0.15	0.26	0.12	0.36	
Maximum	0.35	0.84	0.20	0.23	0.74	0.20	2.95	
1st Moment	0.02	0.02	0.02	0.02	0.02	0.02	0.02	
2nd Moment	0.05	0.07	0.05	0.05	0.09	0.04	0.11	
3rd Moment	0.07	-0.33	-0.03	-0.01	-1.4	-0.05	3.42	
4th Moment	1.14	18.5	0.04	0.15	27.2	0.06	78.53	

Table 4.4: returns some descriptive statistics of the $J = 10{,}000$ simulated inflation paths according to the respective model concerning Tab.4.1. The 4th moment corresponds to excess kurtosis compared to the normal distribution.

Chapter 5

Discriminating between different kinds of unpredictability

Discriminating between different kinds of unpredictability

5.1 Introduction

The properties and assumptions of the error term in an econometric model play a major role in the econometric modeling procedure. Not only are both specification and estimation issues involved but as a consequence the interpretation of the model's outcome and thus the conclusions drawn from an empirical analysis highly depend on the assumptions with regard to the noise term. As the stochastic part of the model the error term naturally cannot be observed and only be predicted in a very limited sense (i.e. if e.g. autocorrelated errors are assumed) resulting in the need for the modeler to meet several assumptions concerning the error.

Usually assumptions concerning a) the type of distribution, b) the distribution's constancy for different draws and c) the dependence for various draws of the error term are made. Specifying a) a gaussian distribution which is b) constant over each draw with c) independent draws yields the most popular assumption of *iid* normal errors. Most of the conventional estimation and testing procedures are applied to *iid* errors under normality. Over the past decades there has been a vast part of literature dealing with deviations from the normality assumption which can be subsumed under the concept of robust statistics dealing with fat tails and outliers. Dropping the assumption of independent draws can also be handled rather easily via the concept of autocorrelated errors which is standard in the econometric literature.

In this paper we analyze deviations from assumption b), that is the consequences of errors not being identically distributed. Hence we try to formalize the issue of jumps within the error distribution. Looking at isolated switches in the first moment of the distribution refers to the mean-shift literature whereas isolated switches in the second moment may be modeled by means of heteroscedastic models. When it comes to switches in more than one moment at the same time the theory of mixture distributions may be referred to which especially in the case of normality may be handled in a feasible way.

Our contribution in this paper is to examine and try to formalize the situation where the error distribution switches as a whole (a concept that we call extrinsic unpredictability of type II). Hence we give up the assumption that merely one of the moments switch or that the distribution indeed switches but nevertheless stays in the assumed distribution class. For this purpose we utilize the Pearson distribution functioning as the "transfer" from one distribution class to another. In section 5.2.1 we first formulate different kinds of unpredictability econometrically. Following we provide an operable concept of how to handle this issue in empirical practice in section 5.2.2. Finally we yield an example in terms of the cumulated-sum-of-squares test in section 5.3. Here we examine the size and power properties of the test in sections 5.3.3 and 5.3.4 if extrinsic unpredictability of type II is present. Section 5.4 concludes.

5.2 Types of Unpredictability

5.2.1 Theoretical Framework

In the following we derive definitions for three different kinds of unpredictability: intrinsic unpredictability, extrinsic unpredictability of type I and extrinsic unpredictability of type II. Concerning the difference between intrinsic and extrinsic unpredictability we follow the framework of Hendry [2011].

Definition 9 (Intrinsic Unpredictability). *Let $\{X_t\}_{t=-\infty}^{\infty}$ be a stochastic process with density function $f(\cdot)$. X_{t+h}, $h \in \mathbb{Z}^+$ is intrinsically unpredictable at time t over a time period $\mathcal{T} = (t+1, \ldots, \infty)$ with respect to the information set $\Im_t$ if its conditional distribution equals its unconditional distribution such that $f(X_{t+1}|\Im_t) = f(X_{t+1})$.*

Hence intrinsic unpredictability is been formalized as intrinsic stochastic variation within a known distribution. This results into the fact that the best way to model or forecast a random variable is by its unconditional distribution. In other words there is no contribution of the variable's history in terms of model or forecast issues. Extrinsic unpredictability on the other hand may be defined as follows.

Definition 10 (Extrinsic Unpredictability). *Let $\{X_t\}_{t=-\infty}^{\infty}$ be a stochastic process with its density function at time t being given as $f_t(\cdot)$. X_{t+h}, $h \in \mathbb{Z}^+$ is extrinsically unpredictable at time t over a time period $\mathcal{T} = (t+1, \ldots, \infty)$ if its distribution shifts in unanticipated ways such that $f_t(\cdot) \neq f_{t+1}(\cdot)$.*

Thus the density function of X_t is no longer be assumed to be constant over time resulting in the need of time dating each moment of the distribution. As an economic example one may consider the case where a financial variable switches from a thin-tailed to a heavy-tailed distribution once extreme market events like crises or bubbles occur which would correspond to a switch of the second and/or third and fourth moment of the distribution. Likewise one could think of a change in monetary policy with a modified inflation target causing the first moment of the inflation's distribution to switch. Hendry and Mizon [2010] show that under extrinsic unpredictability the results of conditional expectation theory do no longer hold and thus many of the conventional econometric methods are no longer applicable. Hence the point of interest concerns the question of how extrinsic unpredictability can be formalized and incorporated into econometric methods. In terms of formalization we further elaborate two different types of extrinsic unpredictability. Concretely we differentiate between two types of jumps:

(*i*) The distribution jumps in its moments but stays within the assumed distribution class.

(*ii*) The distribution jumps in its moments such that it jumps out of the assumed distribution class.

Obviously (*i*) and (*ii*) (henceforth ExtI and ExtII) are special cases of the above defined extrinsic unpredictability. Naturally ExtII marks the challenging part of the analysis.

Definition 11 (Extrinsic Unpredictability of Type I). *Let $\{X_t\}_{t=-\infty}^{\infty}$ be a stochastic process with density function $f(\cdot)$. Let further $\mathfrak{F} \leftarrow f(x;\Theta)$ denote a class of a parametric distribution being defined through density function $f(x;\Theta)$ with parameter vector Θ and $f(x;\theta_k) \to \mathfrak{F}_k \subset \mathfrak{F}$ with*

$\theta_k \subset \Theta$, $k \in \{i, j\}$, $i \neq j$. X_{t+h}, $h \in \mathbb{Z}^+$ is extrinsically unpredictable of type I at time t over a time period $\mathcal{T} = (t+1, \ldots, \infty)$ if its distribution shifts in unanticipated ways such that $f_t(\cdot) \neq f_{t+1}(\cdot)$ where $f_t(\cdot) \in \mathfrak{F}_i$ and $f_{t+1}(\cdot) \in \mathfrak{F}_j$.

In other words under ExtI the density shifts from parametrization θ_i to θ_j, i.e. from $\mathfrak{F}_i$ to $\mathfrak{F}_j$ while the distribution class itself does not change. As a very simple example consider the case where $f_t = N(0,1)$ and $f_{t+1} = N(1,1)$, i.e. the parameter vector of the normal distribution jumps from $\theta_1 = (\mu_1, \sigma_1) = (0,1)$ to $\theta_2 = (\mu_2, \sigma_2) = (1,1)$. Let us now turn to the more advanced case of ExtII.

Definition 12 (Extrinsic Unpredictability of Type II)**.** *Let $\{X_t\}_{t=-\infty}^{\infty}$ be a stochastic process with density function $h(\cdot)$. Let further $\mathcal{H} \leftarrow h(\cdot)$ denote a class of distributions being defined through $h(\cdot)$. Let further $\mathfrak{F} \leftarrow f(x;\Theta)$ and $\mathcal{G} \leftarrow g(x;\Delta)$ be defined as classes of a parametric distribution with parameter vectors Θ and Δ where $\mathfrak{F}, \mathcal{G} \subset \mathcal{H}$ and $\mathfrak{F} \cap \mathcal{G} = \emptyset$. X_{t+h}, $h \in \mathbb{Z}^+$ is extrinsically unpredictable of type II at time t over a time period $\mathcal{T} = (t+1, \ldots, \infty)$ if its distribution shifts in unanticipated ways such that $h_t(\cdot) \neq h_{t+1}(\cdot)$ where $h_t(\cdot) \in \mathfrak{F}$ and $h_{t+1}(\cdot) \in \mathcal{G}$.*

Hence under ExtII, X jumps to different distribution classes through different parameterizations in $h(\cdot)$. Put differently if the parameter vector in $h(\cdot)$ changes over time, X shifts from distribution class $\mathfrak{F}$ to distribution class $\mathfrak{G}$ which are nonnested with respective density functions $f(\cdot)$ and $g(\cdot)$. Note that under ExtI merely Θ or Δ change yet $\mathfrak{F}$ or $\mathfrak{G}$ are never left. So obviously ExtI may be interpreted as a special case ($\mathfrak{F} = \mathcal{G}$) of the more general framework of ExtII.

5.2.2 Econometric Implementation

In order to make Def.12 operable $\mathcal{H}$ has to be specified. The most natural choice for a distribution class in which the bigger part of the conventional parametric distributions are nested marks the Pearson distribution (cf. Pearson [1894-1896]). Its density function $h(x)$ is defined through the solutions of the differential equation

$$\mathcal{H} \leftarrow \frac{\partial h(x)}{\partial x} = h(x)\frac{x-a}{b_0 + b_1 x + b_2 x^2} \tag{5.1}$$

where (a, b_0, b_1, b_2) denotes the parameter vector of the distribution which is related to the moments of $h(x)$ via

$$k b_0 \mu'_{k-1} + ((k+1)b_1 - a)\mu'_k + ((k+2)b_2 + 1)\mu'_{k+1} = 0, \quad k = 1, 2, \ldots \tag{5.2}$$

where the $\mu'_k = E(x^k)$ describe the raw moments of $h(x)$. Note that through the value of

$$\begin{aligned}
\kappa &= \frac{\beta_1(\beta_2+3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \quad \text{where} \quad \beta_1 = \left(\frac{\mu_3^2}{\mu_2^3}\right), \quad \beta_2 = \left(\frac{\mu_4}{\mu_2^2}\right), \\
\mu_2 &= \mu'_2 - (\mu'_1)^2, \\
\mu_3 &= \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3 \quad \text{and} \\
\mu_4 &= \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^4
\end{aligned}$$

with μ_i denoting the ith central moment of $h(\cdot)$, the type of the distribution can be defined. Overall there are seven particular types of Pearson distributions. Under certain conditions (cf. e.g. Johnson et al. [1994], chapter 12) the parametric distribution classes are nested into Pearson type distributions. Hence note that for the choice of the Pearson distribution type the value of κ is sufficient whereas κ is merely necessary for the selection of particular parametric distribution class. A detailed overview of this relationship yields table 5.1. The parameters of $h(x)$ may then

Value	Type of Pearson distribution	Possible parametric distribution
$\kappa < 0$	Type I	beta type I
$\kappa = 0$	Type VII, Type II	cauchy, student, beta type I, gaussian
$0 < \kappa < 1$	Type IV	complex gamma, beta
$\kappa = 1$	Type V	inverse gamma
$\kappa > 1$	Type VI	beta type II, F-distribution
$\kappa \pm \infty$	Type III	gamma, chi-squared

Table 5.1: displays the relation between the value of κ, the type of the Pearson distribution and the corresponding parametric distribution class on the basis of Nagahara [2004].

be determined via (5.2), i.e. by solving the system

$$\begin{aligned}(b_1 - a) + (2b_2 + 1)\mu'_1 &= 0\\ b_0 + (2b_1 - a)\mu'_1 + (3b_2 + 1)\mu'_2 &= 0\\ 2b_0\mu'_1 + (3b_1 - a)\mu'_2 + (4b_2 + 1)\mu'_3 &= 0\\ 3b_0\mu'_2 + (4b_1 - a)\mu'_3 + (5b_2 + 1)\mu'_4 &= 0\end{aligned} \tag{5.3}$$

for the parameters (a, b_0, b_1, b_2) where the μ'_i are replaced by their empirical counterparts $\hat{\mu}'_i$.

5.3 Example: CUSUM of Squares Test

5.3.1 Standard Results

As an example for the presence of extrinsic unpredictability we consider the cumulated-sum-of-squares test (henceforth CUSQ). Consider the linear regression model

$$y_t = x'_t\beta_t + \nu_t, \quad t \in \mathcal{T} \tag{5.4}$$

with $\mathcal{T} = \{1, \ldots, T\}$. y_t describes the dependent variable, the regressors are given by $x'_t = (x_{1t}, \ldots, x_{kt})$ and the k-dimensional vector of coefficients is given by β_t. Further ν_t denotes a stationary and ergodic error term. The CUSQ introduced by Brown et al. [1975] tests the Null of $H_0 : \beta_t = \beta^*$

$\forall t \in \mathcal{T}$ where

$$CUSQ = \max_{k+1 \le r \le T} \sqrt{T} \left| S_T^{(r)} - \frac{r-k}{T-k} \right| \quad \text{and}$$
$$S_T^{(r)} = \left(\sum_{t=k+1}^{r} \nu_t^2 \right) / \left(\sum_{k+1}^{T} \nu_t^2 \right). \tag{5.5}$$

Ploberger and Krämer [1986] show that the limiting distribution of (5.5) depends on the distribution of the ν_t. Concretely they derive that

$$\sqrt{T} \cdot CUSQ \Rightarrow \sqrt{\Omega} \sup_{r \in [0,1]} |BB(r)| \tag{5.6}$$

where $\sqrt{\Omega} = \sqrt{\mu_4 - \sigma^4}/\sigma^2$, $\sigma^2 = E(\nu_t^2)$, $\mu_4 = E(\nu_t^4)$, $\Rightarrow$ denotes weak convergence under the Skorohod topology and $BB(r) = W(r) - rW(1)$ with $W(r)$ being defined as a unit Wiener process on $[0,1]$. If ν is gaussian, $\Omega = 2$ and the familiar result of $\sqrt{T} \cdot CUSQ \Rightarrow \sqrt{2} \sup_{r \in [0,1]} |BB(r)|$ is derived. Note that the limiting distribution of the CUSQ as given in (5.6) is thus derived under intrinsic unpredictability, i.e. under Def.9.

5.3.2 Limiting Distribution under ExtII

In order to derive the limiting distribution of the CUSQ under ExtII we may first formulate an error process that is of type ExtII. Let

$$\{\nu_t\}_{t \in \mathcal{T}} = \begin{cases} \nu_{1t} & , 1 \le t \le [\lambda T] \\ \nu_{2t} & , [\lambda T] + 1 \le t \le T \end{cases} \tag{5.7}$$

be an stochastic process where $\mathcal{T} = (1, \ldots, T)$, $0 \le \lambda \le 1$ and $[\lambda T]$ denotes the largest integer less or equal to $\lambda \cdot T$. Note that if $[\lambda T] < 1$ or $[\lambda T] > T - 1$ there is no switch in ν_t and the process maintains in the second or first regime respectively. Consider further assumption 1:

Assumption 1. *(a) $\{\nu_{it}\}$, $i \in \{1,2\}$ are iid processes with $E(\nu_{1t}) = E(\nu_{2t}) = 0 \ \forall \ t \in \mathcal{T}$ and $V(\nu_{1t}) = V(\nu_{2t}) = \sigma^2 \ \forall \ t \in \mathcal{T}$. (b) $\nu_{it} \sim h_i = h(a_i, b_{0i}, b_{1i}, b_{2i})$ where $i \in \{1,2\}$ and $h_1 \in \mathfrak{F}$, $h_2 \in \mathcal{G}$ with $\mathfrak{F}, \mathcal{G} \subset \mathcal{H}$ and $\mathcal{H} \leftarrow h(\nu_t)$ where $h(\nu_t)$ is given by (5.1).*

That is under A1 the distribution of ν switches at time $[\lambda T]$ from $h_1(\cdot)$ to $h_2(\cdot)$ where

$$\frac{\partial h_i(\nu_i)}{\partial \nu_i} = h_i(\nu_i) \frac{\nu_i - a_i}{b_{0i} + b_{1i}\nu_i + b_{2i}\nu_i^2}, \quad i \in \{1,2\}. \tag{5.8}$$

Then (5.7) with unknown λ together with A1 marks an extrinsically unpredictable process of type II in accordance with Def.12. In other words assuming ν_t in (5.4) to be of type (5.7) enables us to derive the properties of the CUSQ if the error term switches in its distribution.
Theorem 1 (proven in section 5.5.2) now returns one of the main results of this paper. Here we show that under extrinsic unpredictability of type II the CUSQ statistic converges to a different limiting distribution than in the case of intrinsically unpredictable error processes.

Theorem 1. *Let assumption 1 be fulfilled and let the error terms in (5.4) be given by (5.7),* *then*

$$CUSQ \Rightarrow \sqrt{2+(1-\lambda)\frac{6a_2^2}{b_0}+\lambda\frac{6a_1^2}{b_0}}\sup_{r\in[0,1]}|BB(r)|$$

where $\Rightarrow$ *denotes weak convergence under the Skorohod topology,* $BB(r) = W(r) - rW(1)$ *with* $W(r)$ *being defined as a unit Wiener process on* $[0,1]$ *and* a_1*,* a_2 *and* b_0 *denote the parameters of the Pearson distribution according to (5.8).*

Remark 1. *Under normality and given that* $h_1(v) = h_2(v)$*, i.e. under intrinsic unpredictability* $\lambda = 1$ *and* $a_1 = 0$ *leading to the familiar convergence rate of* $1/\sqrt{2}$ *of the CUSQ.*

Remark 2. *Theorem 1 is supported for parameter values* (i) $b_{10} = b_{20} = b_0 < 0$*,* (ii) $b_{21} = b_{22} = b_2 = 0$ *and* (iii) $|a_2| < \sqrt{\frac{|b_0|}{3(1-\lambda)} - \frac{\lambda}{1-\lambda}a_1^2}$ *in (5.8).*

Remark 3. *Assuming normality in the first regime theorem 1 is supported for parameter values* $|a_2| < \sqrt{\frac{|b_0|}{3(1-\lambda)}}$ *in (5.8).*

Whereas remark 1 seems obvious, the first two parts of remark 2 assure that A1 is met (cf. section 5.5.2 for a proof hereof). Part (iii) assures a positive convergence rate of the CUSQ under ExtII, i.e. $2+(1-\lambda)6a_2^2/b_0 + \lambda 6a_1^2/b_0 > 0$ is solved for a_2. Remark 3 then results directly from remark 2 by setting $a_1 = 0$ as is the case under normality in the first regime. Note finally that naturally remarks 2 and 3 cannot be supported for the limiting case of intrinsic unpredictability, i.e. if $\lambda = 1$.

Summing up we can state that under ExtII, that is if a possible jump in the distribution of the error term is accounted for we elaborate a different rate of convergence of the CUSQ than in the standard case of intrinsic unpredictability. Hence variations in the statistic's critical values and thus size and power distortions are highly expected in this context. These issues are dealt with in next subsections.

5.3.3 Size Results

Recall that under intrinsic unpredictability the rate of convergence of the CUSQ is equal to $\Omega^{-1/2}$ with $\Omega = \mu_4 - \sigma^4/\sigma^4$. and $CUSQ/\sqrt{\Omega} \Rightarrow \sup_{r\in[0,1]}|BB(r)|$ with the distribution function equalling

$$F_X := P\left(\frac{CUSQ}{\sqrt{\Omega}} \leq x\right) = 1 + 2\sum_{k=1}^{\infty}(-1)^k \exp(-2k^2x^2) \tag{5.9}$$

(cf. Billingsley [1968], p.85). Under ExtII however the rate of convergence is given by $\Omega^0 = 2+(1-\lambda)\frac{6a_2^2}{b_0}+\lambda\frac{6a_1^2}{b_0}$. Therefore defining $\frac{CUSQ}{\sqrt{\Omega}} =: X$, $\frac{CUSQ}{\sqrt{\Omega^0}} =: Y$ and $Y = g(X) := A \cdot X$ with $A := \sqrt{\frac{\Omega}{\Omega^0}}$ yields

$$P\left(\frac{CUSQ}{\sqrt{\Omega^0}} \leq x\right) = P(g(X) \leq x) = \int_{-\infty}^{x} f_X\left(g^{-1}(Y)\right)|g^{-1'}(Y)|dY.$$

As $f_X = 1-8\sum_{k=1}^{\infty}(-1)^k k^2 x\exp(-2k^2x^2)$, $g^{-1} = Y/A$ and $|g^{-1'}| = 1/|A|$ the distribution function of the CUSQ under ExtII is given as

$$P\left(\frac{CUSQ}{\sqrt{\Omega^0}} \leq x\right) = 1 + \int_{-\infty}^{x}\left(2\sum_{k=1}^{\infty} -4k^2(-1)^k x\exp(-2k^2(x/A)^2)/A^2\right)dx. \tag{5.10}$$

Recalling that under intrinsic unpredictability $\Omega = 2+6a_1^2/b_0$ leads to

$$A = \sqrt{\frac{b_0+3a_1^2}{b_0+3(\lambda a_1^2+(1-\lambda)a_2^2)}}. \tag{5.11}$$

Inserting (5.11) into (5.10) enables us to derive the exact distribution of the CUSQ under ExtII in dependence of the parameters of the Pearson distribution (a_1, a_2, b_0).
Tables 5.2-5.4 display the size of the one-sided CUSQ, i.e. testing $H_0 : \beta_t = \beta^*$ for $t \leq [\lambda T]$ versus $H_1 : \beta_t = \beta^{**} \neq \beta^*$ for $t > [\lambda T]$. Hence the test is considered under the null with respect to a_2 and $-b_0 = \sigma^2$. Throughout the analysis we set $a_1 = 0$, that is we look at cases where the error distributions jumps from normality to some other distribution defined by $a_2 =: a$ (henceforward we abandon the index for notational convenience). Panel 1 of Fig.5.1 illustrates the results for the one-sided CUSQ.
As can clearly be detected the test is undersized if jumps in higher moments of the error distribution are neglected. Actually the size declines monotonically with increasing $|a|$. This result is due to the fact that A decreases with increasing $|a|$ leading to a faster rate of convergence concerning the right hand side in (5.10). Hence we observe decreasing critical values with increasing $|a|$ which is also supported by regarding Tab.5.8. Trivially the size of the one-sided CUSQ is always maintained if $|a| = 0$ corresponding to the limiting case of intrinsic unpredictability. It can further be observed that the size also declines with declining λ. This result seems very intuitive as lower values of λ go along with earlier breaks in the error distribution. Hence if the latter are neglected the effect of an early break in the sample is expected to lead to much heavier size distortions than a rather late timing of the break. What becomes especially clear by looking at Fig.5.1 is the fact that the impact of the break time gets higher for larger breaks. Put differently, the higher the break in the error distribution (i.e. $|a|$) the higher the impact of an early break in the sample gets leading to severe size distortions in the CUSQ.
Concerning the two-sided CUSQ, i.e. testing $H_0 : \beta_t = \beta^*\ \forall t \in \mathcal{T}$ against the alternative that at least one β_t differs from β^*, the opposite behavior can be detected. Tables 5.5-5.7 and Fig.5.1 reflect that when neglecting the possibility of jumps between different error distributions the CUSQ is oversized while the size gets higher with increasing $|a|$. Again an explanation of this rather counterintuitive result can be found by regarding the critical values which are given in Tab.5.9. The finding of decreasing upper critical values with increasing $|a|$ is again supported. Additionally we find the lower critical values to be also decreasing with increasing $|a|$ which again is supported by the argument that A decreases with increasing $|a|$. The point however is that the lower value decreases at a lower rate than the upper value resulting in an increasing overall size of the two-sided test. Again we find the familiar results that the size distortion becomes higher

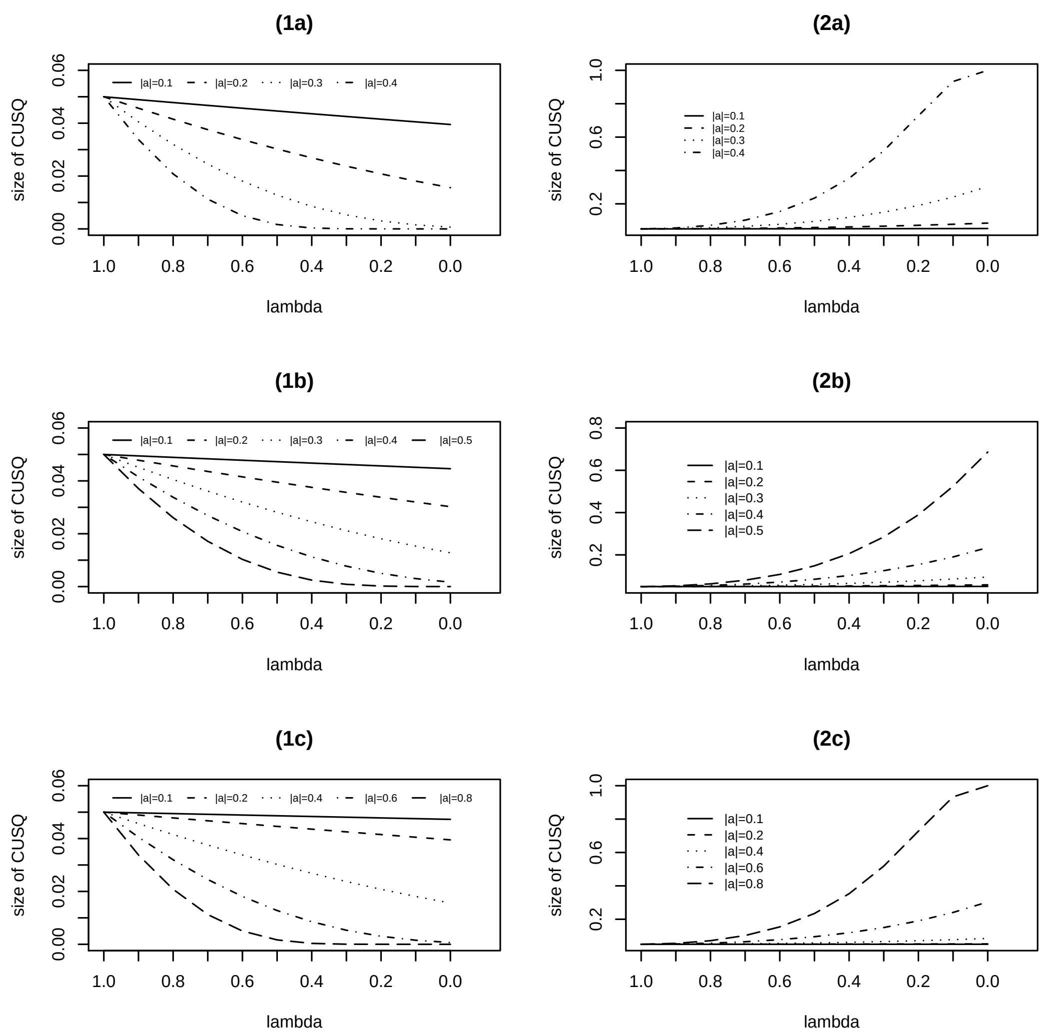

Figure 5.1: displays the size results of the CUSQ under ExtII. Panels 1 and 2 refer to the one-sided and two-sided version of CUSQ respectively whereas panels a-c refer to the values for $-b_0$ of 0.5, 1 and 2 respectively.

with increasing $|a|$ and decreasing λ whilst a combination of both makes the CUSQ virtually useless if ExtII is neglected.

5.3.4 Power Results

Concerning the power of the test we follow the setting of Deng and Perron [2008]. Referring to (5.4) we assume the data to be generated by

$$\begin{aligned} y_t &= x_t'\beta_t + \nu_t \\ \beta_t &= \beta + \delta 1(t > [\theta T]), \end{aligned} \tag{5.12}$$

where $1(\cdot)$ denotes the indicator function taking value 1 if the event in brackets holds, δ describes the impact of the switch in β and $0 < \theta < 1$ mirrors the timing of the break. Considering the OLS residuals of (5.12) Deng and Perron [2008] derived the power of the CUSQ under intrinsic unpredictability for arbitrary error distributions. Theorem 2 below (proven in section 5.5.2) now derives the power of the CUSQ with OLS residuals under ExtII.

Theorem 2. *Let assumption 1 be fulfilled and let the error terms in (5.12) be given by (5.7), then*

$$\begin{aligned}
T^{-1/2} CUSQ^{OLS} &\xrightarrow{p} \frac{|(2\theta-1)\theta(1-\theta)(\delta' R\delta)|}{\sqrt{d(\lambda,\delta,\theta)}}, \quad \textit{where} \\
d(\lambda,\delta,\theta) &= \lambda\mu_4^{(1)} + (1-\lambda)\mu_4^{(2)} - \sigma_\nu^4 - 8\sigma_\nu^2\theta(1-\theta)(\delta' R\delta) - \theta^2(1-\theta)^2(\delta' R\delta)^2 + \theta(1-\theta) \\
&\quad \times (3\theta^2 - 3\theta + 1) g(\delta), \quad \textit{and} \\
g(\delta) &= p\lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} (\delta' x_t x_t' \delta)^2,
\end{aligned}$$

with $\xrightarrow{p}$ *denoting convergence in probability,* $\mu_4^{(i)} = E(\nu_i^4)$ *and* $R := p\lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} x_t x_t'$.

Note that the enumerator of the limit does not change under ExtII. Hence $CUSQ^{OLS}$ gets very low if $\theta \approx 0.5$, i.e. when the break in coefficients appears near the mid-point of the sample. This is a familiar result in the context of OLS residuals as the least squares procedure allocates the sum of squared residuals equally to both regimes if the break takes place at mid-sample. Thus also under ExtII the $CUSQ^{OLS}$ is not able to detect a break in this situation.

In order to make our power results under ExtII comparable to those of Deng and Perron [2008] under intrinsic unpredictability we followed their framework of conducting a monte carlo analysis for the finite sample properties of the two-sided $CUSQ^{OLS}$. For this purpose we define the vector of regressors as $x_t = \{x_{1t}, x_{2t}\} = \{1, (-1)^t\}'$ where the break in coefficient is defined more accurately as $\delta = b(\cos(\psi), \sin\psi)'$ with ψ being defined as the angle between the constant and the slope coefficient. We specify ψ to take values of $(0°, 45°, 90°)$. That is if $\psi = 0°$ or $\psi = 90°$ the break occurs solely in the constant or the slope coefficient respectively. For $\psi = 45°$ both coefficients break in equal parts. Further we consider break dates for the coefficients of $\theta = (0.3, 0.5, 0.7)$ and for the error distribution of $\lambda = (0.3, 0.5, 0.7)$. The error distribution breaks in sense of the setting of the proceeding analysis, i.e. from a standard normal distribution to a different distribution class defined by a. For the latter parameter we consider values of $a = (-0.5, -0.4, \dots, 0.5)$ whereas b_0 is held constant to -1 throughout the analysis. Finally the sample size is set to $T = 120$, $\alpha = 0.05$ and the asymptotic critical values given in Tab.5.9 are utilized in order to evaluate the power properties. Tables 5.10-5.12 describe the results.

As expected we detect that the $CUSQ^{OLS}$ has very small power for $\theta = 0.5$ regardless of the size of the break in ν. Further the angle of the break does not seem to play a great role for the power of the test. Although there are some discrepancies especially between $\psi = 0°$ and $\psi = 45°, 90°$ the main findings do not seem to depend on the value of ψ. Further we observe that in finite samples the test is severely oversized for large $|a|$ under the Null of no break in β, it is undersized for small values of $|a|$.

Concerning the influence of a we can elaborate three results. First the power of the test is higher when additional to a break in coefficients the error distribution breaks. Second we observe a higher power of the test (especially for high a) when the break in $f(\nu)$ takes place early in the sample, i.e. when λ is small. So obviously an early break in the distribution of ν "helps" the

$CUSQ^{OLS}$ in the detection of breaks in coefficients. Of course this result may as well be caused spuriously as the test might not be able to discriminate between breaks in β and breaks in ν.
Thirdly we detect power asymmetries concerning θ. That is especially for large $|a|$ the power differs profoundly as to when the break in β occurs. This effect differs with the sign of a. For large negative a we observe that for $\psi = 45°, 90°$ the power of the test is higher if β breaks later in the sample which marks a rather counterintuitive results. On the other hand for large positive a we observe a higher power for earlier breaks in β for all values of ψ.
Summing up it may be said that under extrinsic unpredictability of type II the $CUSQ^{OLS}$ features a substantially higher power than under intrinsic unpredictability. This effect gets stronger the higher the break in $f(\nu)$ and (apart from some few special cases) the earlier the break in $f(\nu)$ occurs in the sample.

5.4 Conclusion

In this paper we deal with different kinds of unpredictability and its implications concerning econometric modeling. Given a statistical distribution defined through a vector of parameters we define and discriminate between the cases where the parameters remain constant (intrinsic unpredictability), change within a distribution class (extrinsic unpredictability of type I) and change such that the distribution switches (extrinsic unpredictability of type II). For the latter type we conduct the Pearson distribution class and show that a switch of the Pearson parameters may lead to type II unpredictability. We further show that when the error process is of type II conventional tests may result in severe size and power distortions. As an example we consider the cumulated-sum-of-squares test of Brown et al. [1975]. For the latter we derive the limiting distribution and the power function in the presence of type II unpredictability. Here we find severe size and power distortions if extrinsic unpredictability of type II is present. Since the limiting distribution of the test under ExtII contains the nuisance parameter a of the Pearson distribution a natural future step in the analysis would be to formulate a (nearly) optimal test under the Null concerning the constancy of β in the sense of Elliot et al. [2012]. Besides alternative examples regarding the consequences of extrinsic unpredictability may be formulated and evaluated in future work.

5.5 Appendix

5.5.1 Tables

Size Results

	λ										
100α	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
						$\lvert a\rvert = 0.1$					
1	1.00	0.97	0.94	0.91	0.88	0.85	0.82	0.79	0.77	0.74	0.71
5	5.00	4.89	4.78	4.67	4.57	4.46	4.36	4.25	4.15	4.05	3.95
10	10.00	9.82	9.64	9.47	9.29	9.12	8.94	8.77	8.60	8.43	8.26
						$\lvert a\rvert = 0.2$					
1	1.00	0.88	0.77	0.66	0.57	0.49	0.41	0.34	0.28	0.23	0.19
5	5.00	4.57	4.15	3.76	3.38	3.02	2.69	2.37	2.08	1.81	1.56
10	10.00	9.29	8.60	7.93	7.28	6.65	6.04	5.46	4.91	4.38	3.88
						$\lvert a\rvert = 0.3$					
1	1.00	0.74	0.53	0.36	0.23	0.14	0.08	0.04	0.02	0.01	0.00
5	5.00	4.05	3.20	2.45	1.81	1.28	0.85	0.53	0.30	0.15	0.07
10	10.00	8.43	6.96	5.60	4.38	3.30	2.38	1.62	1.02	0.59	0.30
						$\lvert a\rvert = 0.4$					
1	1.00	0.57	0.28	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00
5	5.00	3.38	2.08	1.12	0.50	0.17	0.03	0.00	0.00	0.00	0.00
10	10.00	7.28	4.91	2.98	1.55	0.63	0.17	0.02	0.00	0.00	0.00

Table 5.2: mirrors the size of the one-sided CUSQ under unpredictability of type II for $b_0 = -0.5$.

	λ										
100α	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
						$\lvert a\rvert = 0.1$					
1	1.00	0.98	0.97	0.95	0.94	0.92	0.91	0.89	0.88	0.86	0.85
5	5.00	4.94	4.89	4.84	4.78	4.73	4.67	4.62	4.57	4.51	4.46
10	10.00	9.91	9.82	9.73	9.64	9.55	9.47	9.38	9.29	9.2	9.12
						$\lvert a\rvert = 0.2$					
1	1.00	0.94	0.88	0.82	0.77	0.71	0.66	0.62	0.57	0.53	0.49
5	5.00	4.78	4.57	4.36	4.15	3.95	3.76	3.57	3.38	3.20	3.02
10	10.00	9.64	9.29	8.94	8.60	8.26	7.93	7.60	7.28	6.96	6.65
						$\lvert a\rvert = 0.3$					
1	1.00	0.86	0.74	0.63	0.53	0.44	0.36	0.29	0.23	0.18	0.14
5	5.00	4.51	4.05	3.61	3.20	2.81	2.45	2.12	1.81	1.53	1.28
10	10.00	9.20	8.43	7.68	6.96	6.27	5.60	4.98	4.38	3.82	3.3
						$\lvert a\rvert = 0.4$					
1	1.00	0.77	0.57	0.41	0.28	0.19	0.12	0.07	0.04	0.02	0.01
5	5.00	4.15	3.38	2.69	2.08	1.56	1.12	0.77	0.50	0.30	0.17
10	10.00	8.60	7.28	6.04	4.91	3.88	2.98	2.20	1.55	1.02	0.63
						$\lvert a\rvert = 0.5$					
1	1.00	0.65	0.39	0.21	0.10	0.04	0.01	0.00	0.00	0.00	0.00
5	5.00	3.71	2.61	1.71	1.03	0.55	0.24	0.08	0.02	0.00	0.00
10	10.00	7.84	5.89	4.19	2.77	1.66	0.86	0.36	0.11	0.02	0.00

Table 5.3: mirrors the size of the one-sided CUSQ under unpredictability of type II for $b_0 = -1$.

100α	λ 1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
						$\lvert a\rvert = 0.1$					
1	1.00	0.99	0.98	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.92
5	5.00	4.97	4.94	4.92	4.89	4.86	4.84	4.81	4.78	4.75	4.73
10	10.00	9.96	9.91	9.87	9.82	9.78	9.73	9.69	9.64	9.6	9.55
						$\lvert a\rvert = 0.2$					
1	1.00	0.97	0.94	0.91	0.88	0.85	0.82	0.79	0.77	0.74	0.71
5	5.00	4.89	4.78	4.67	4.57	4.46	4.36	4.25	4.15	4.05	3.95
10	10.00	9.82	9.64	9.47	9.29	9.12	8.94	8.77	8.6	8.43	8.26
						$\lvert a\rvert = 0.3$					
1	1.00	0.93	0.86	0.80	0.74	0.68	0.63	0.58	0.53	0.48	0.44
5	5.00	4.75	4.51	4.28	4.05	3.83	3.61	3.40	3.20	3.00	2.81
10	10.00	9.60	9.20	8.81	8.43	8.05	7.68	7.32	6.96	6.61	6.27
						$\lvert a\rvert = 0.4$					
1	1.00	0.88	0.77	0.66	0.57	0.49	0.41	0.34	0.28	0.23	0.19
5	5.00	4.57	4.15	3.76	3.38	3.02	2.69	2.37	2.08	1.81	1.56
10	10.00	9.29	8.60	7.93	7.28	6.65	6.04	5.46	4.91	4.38	3.88
						$\lvert a\rvert = 0.5$					
1	1.00	0.81	0.65	0.51	0.39	0.29	0.21	0.15	0.10	0.07	0.04
5	5.00	4.33	3.71	3.13	2.61	2.13	1.71	1.35	1.03	0.76	0.55
10	10.00	8.90	7.84	6.84	5.89	5.01	4.19	3.44	2.77	2.17	1.66
						$\lvert a\rvert = 0.6$					
1	1.00	0.74	0.53	0.36	0.23	0.14	0.08	0.04	0.02	0.01	0.00
5	5.00	4.05	3.20	2.45	1.81	1.28	0.85	0.53	0.30	0.15	0.07
10	10.00	8.43	6.96	5.60	4.38	3.30	2.38	1.62	1.02	0.59	0.30
						$\lvert a\rvert = 0.7$					
1	1.00	0.66	0.4	0.22	0.11	0.05	0.02	0.00	0.00	0.00	0.00
5	5.00	3.73	2.65	1.76	1.08	0.59	0.27	0.10	0.03	0.00	0.00
10	10.00	7.89	5.97	4.29	2.87	1.75	0.94	0.42	0.14	0.03	0.00
						$\lvert a\rvert = 0.8$					
1	1.00	0.57	0.28	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00
5	5.00	3.38	2.08	1.12	0.50	0.17	0.03	0.00	0.00	0.00	0.00
10	10.00	7.28	4.91	2.98	1.55	0.63	0.17	0.02	0.00	0.00	0.00

Table 5.4: mirrors the size of the one-sided CUSQ under unpredictability of type II for $b_0 = -2$.

100α	λ 1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
						$\lvert a\rvert = 0.1$					
1	1.00	1.00	1.01	1.01	1.02	1.02	1.03	1.04	1.06	1.07	1.08
5	5.00	5.01	5.02	5.03	5.05	5.07	5.10	5.13	5.16	5.19	5.23
10	10.00	10.01	10.03	10.05	10.08	10.11	10.14	10.18	10.23	10.27	10.33
						$\lvert a\rvert = 0.2$					
1	1.00	1.02	1.06	1.12	1.20	1.32	1.47	1.65	1.87	2.14	2.47
5	5.00	5.05	5.16	5.32	5.54	5.83	6.19	6.63	7.15	7.76	8.48
10	10.00	10.08	10.23	10.45	10.75	11.14	11.62	12.2	12.89	13.69	14.61
						$\lvert a\rvert = 0.3$					
1	1.00	1.07	1.26	1.60	2.14	2.95	4.12	5.77	8.07	11.25	15.61
5	5.00	5.19	5.67	6.51	7.76	9.53	11.9	15.02	19.02	24.06	30.34
10	10.00	10.27	10.93	12.05	13.69	15.95	18.92	22.71	27.43	33.18	40.06
						$\lvert a\rvert = 0.4$					
1	1.00	1.20	1.87	3.30	5.99	10.85	19.35	33.62	55.86	84.52	99.97
5	5.00	5.54	7.15	10.25	15.42	23.44	35.29	51.82	72.77	93.31	100.00
10	10.00	10.75	12.89	16.85	23.19	32.49	45.29	61.73	80.51	96.23	100.00

Table 5.5: mirrors the size of the two-sided CUSQ under unpredictability of type II for $b_0 = -0.5$.

100α	1.0	0.9	0.8	0.7	0.6	λ 0.5	0.4	0.3	0.2	0.1	0.0
						$\lvert a\rvert = 0.1$					
1	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.02	1.02	1.02
5	5.00	5.00	5.01	5.01	5.02	5.03	5.03	5.04	5.05	5.06	5.07
10	10.00	10.01	10.01	10.02	10.03	10.04	10.05	10.07	10.08	10.09	10.11
						$\lvert a\rvert = 0.2$					
1	1.00	1.01	1.02	1.03	1.06	1.08	1.12	1.16	1.2	1.26	1.32
5	5.00	5.02	5.05	5.10	5.16	5.23	5.32	5.42	5.54	5.67	5.83
10	10.00	10.03	10.08	10.14	10.23	10.33	10.45	10.59	10.75	10.93	11.14
						$\lvert a\rvert = 0.3$					
1	1.00	1.02	1.07	1.15	1.26	1.41	1.6	1.84	2.14	2.51	2.95
5	5.00	5.06	5.19	5.39	5.67	6.04	6.51	7.08	7.76	8.58	9.53
10	10.00	10.09	10.27	10.55	10.93	11.43	12.05	12.8	13.69	14.74	15.95
						$\lvert a\rvert = 0.4$					
1	1.00	1.06	1.20	1.47	1.87	2.47	3.30	4.44	5.99	8.07	10.85
5	5.00	5.16	5.54	6.19	7.15	8.48	10.25	12.53	15.42	19.02	23.44
10	10.00	10.23	10.75	11.62	12.89	14.61	16.85	19.69	23.19	27.43	32.49
						$\lvert a\rvert = 0.5$					
1	1.00	1.13	1.51	2.26	3.55	5.66	9.02	14.27	22.28	34.19	51.03
5	5.00	5.34	6.29	8.02	10.76	14.82	20.57	28.46	38.96	52.41	68.62
10	10.00	10.48	11.76	14.02	17.50	22.48	29.23	38.03	49.07	62.29	76.97

Table 5.6: mirrors the size of the two-sided CUSQ under unpredictability of type II for $b_0 = -1$.

100α	1.0	0.9	0.8	0.7	0.6	λ 0.5	0.4	0.3	0.2	0.1	0.0
						$\lvert a\rvert = 0.1$					
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01
5	5.00	5.00	5.00	5.01	5.01	5.01	5.01	5.02	5.02	5.02	5.03
10	10.00	10.00	10.01	10.01	10.01	10.02	10.02	10.03	10.03	10.04	10.04
						$\lvert a\rvert = 0.2$					
1	1.00	1.00	1.01	1.01	1.02	1.02	1.03	1.04	1.06	1.07	1.08
5	5.00	5.01	5.02	5.03	5.05	5.07	5.10	5.13	5.16	5.19	5.23
10	10.00	10.01	10.03	10.05	10.08	10.11	10.14	10.18	10.23	10.27	10.33
						$\lvert a\rvert = 0.3$					
1	1.00	1.01	1.02	1.04	1.07	1.10	1.15	1.20	1.26	1.33	1.41
5	5.00	5.02	5.06	5.12	5.19	5.28	5.39	5.52	5.67	5.85	6.04
10	10.00	10.04	10.09	10.17	10.27	10.4	10.55	10.73	10.93	11.17	11.43
						$\lvert a\rvert = 0.4$					
1	1.00	1.02	1.06	1.12	1.20	1.32	1.47	1.65	1.87	2.14	2.47
5	5.00	5.05	5.16	5.32	5.54	5.83	6.19	6.63	7.15	7.76	8.48
10	10.00	10.08	10.23	10.45	10.75	11.14	11.62	12.2	12.89	13.69	14.61
						$\lvert a\rvert = 0.5$					
1	1.00	1.04	1.13	1.28	1.51	1.83	2.26	2.82	3.55	4.48	5.66
5	5.00	5.10	5.34	5.73	6.29	7.04	8.02	9.25	10.76	12.61	14.82
10	10.00	10.15	10.48	11.01	11.76	12.75	14.02	15.59	17.5	19.79	22.48
						$\lvert a\rvert = 0.6$					
1	1.00	1.07	1.26	1.60	2.14	2.95	4.12	5.77	8.07	11.25	15.61
5	5.00	5.19	5.67	6.51	7.76	9.53	11.90	15.02	19.02	24.06	30.34
10	10.00	10.27	10.93	12.05	13.69	15.95	18.92	22.71	27.43	33.18	40.06
						$\lvert a\rvert = 0.7$					
1	1.00	1.12	1.49	2.20	3.42	5.40	8.53	13.39	20.77	31.7	47.24
5	5.00	5.33	6.24	7.89	10.5	14.35	19.78	27.21	37.09	49.77	65.20
10	10.00	10.47	11.69	13.85	17.18	21.91	28.31	36.66	47.15	59.77	73.99
						$\lvert a\rvert = 0.8$					
1	1.00	1.20	1.87	3.30	5.99	10.85	19.35	33.62	55.86	84.52	99.97
5	5.00	5.54	7.15	10.25	15.42	23.44	35.29	51.82	72.77	93.31	100.00
10	10.00	10.75	12.89	16.85	23.19	32.49	45.29	61.73	80.51	96.23	100.00

Table 5.7: mirrors the size of the two-sided CUSQ under unpredictability of type II for $b_0 = -2$.

Critical Values

100α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
					$\sigma^2 = 0.5$				
1	2.30	2.23	2.01	1.56	0.46	–	–	–	–
5	1.92	1.86	1.67	1.30	0.38	–	–	–	–
10	1.73	1.68	1.51	1.17	0.35	–	–	–	–
					$\sigma^2 = 1$				
1	2.30	2.27	2.16	1.97	1.66	1.15	–	–	–
5	1.92	1.89	1.80	1.64	1.38	0.96	–	–	–
10	1.73	1.70	1.62	1.48	1.25	0.87	–	–	–
					$\sigma^2 = 2$				
1	2.30	2.28	2.23	2.14	2.01	1.82	1.56	1.18	0.46
5	1.92	1.91	1.86	1.79	1.67	1.52	1.30	0.99	0.38
10	1.73	1.72	1.68	1.61	1.51	1.37	1.17	0.89	0.35

Table 5.8: mirrors the critical values of the one-sided CUSQ under unpredictability of type II for $\lambda = 0$ and $-b_0 = \sigma^2$.

100α		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
						$\sigma^2 = 0.5$				
1	U	2.45	2.37	2.13	1.66	0.49	–	–	–	–
	L	0.59	0.57	0.51	0.40	0.12	–	–	–	–
5	U	2.09	2.03	1.82	1.42	0.42	–	–	–	–
	L	0.68	0.66	0.59	0.46	0.14	–	–	–	–
10	U	1.92	1.86	1.67	1.30	0.38	–	–	–	–
	L	0.73	0.71	0.64	0.50	0.15	–	–	–	–
						$\sigma^2 = 1$				
1	U	2.45	2.41	2.30	2.09	1.77	1.22	–	–	–
	L	0.59	0.58	0.55	0.50	0.43	0.29	–	–	–
5	U	2.09	2.06	1.96	1.79	1.51	1.05	–	–	–
	L	0.68	0.67	0.64	0.58	0.49	0.34	–	–	–
10	U	1.92	1.89	1.80	1.64	1.38	0.96	–	–	–
	L	0.73	0.72	0.69	0.63	0.53	0.37	–	–	–
						$\sigma^2 = 2$				
1	U	2.45	2.43	2.37	2.28	2.13	1.94	1.66	1.26	0.49
	L	0.59	0.59	0.57	0.55	0.51	0.47	0.40	0.30	0.12
5	U	2.09	2.08	2.03	1.95	1.82	1.65	1.42	1.08	0.42
	L	0.68	0.67	0.66	0.63	0.59	0.54	0.46	0.35	0.14
10	U	1.92	1.91	1.86	1.79	1.67	1.52	1.30	0.99	0.38
	L	0.73	0.73	0.71	0.68	0.64	0.58	0.50	0.38	0.15

Table 5.9: mirrors the critical values of the two-sided CUSQ under unpredictability of type II for $\lambda = 0$ and $-b_0 = \sigma^2$.

Power Results

θ	$\lambda\backslash b$	0	0.5	1	2	5	0	0.5	1	2	5
		$a=-0.5$					$a=-0.4$				
0.3	0.3	19.86	21.44	28.26	79.44	100	11.00	11.82	20.18	78.76	100
	0.5	15.90	17.54	24.38	75.22	100	8.48	9.60	14.84	74.26	100
	0.7	10.34	11.20	17.18	75.68	100	6.42	7.00	13.64	74.36	100
0.5	0.3	19.86	17.80	11.48	1.96	0	11.00	9.72	5.80	0.58	0
	0.5	15.90	15.36	10.52	1.90	0	8.48	7.92	4.90	0.86	0
	0.7	10.34	9.72	6.58	1.16	0	6.42	5.90	3.54	0.42	0
0.7	0.3	19.86	18.34	21.84	69.90	100	11.00	11.50	16.26	69.42	100
	0.5	15.90	16.12	19.94	69.32	100	8.48	8.88	14.82	69.76	100
	0.7	10.34	10.98	16.66	68.22	100	6.42	7.24	13.54	68.88	100
		$a=-0.3$					$a=-0.2$				
0.3	0.3	6.44	7.18	14.14	77.12	100	4.74	5.52	11.64	76.82	100
	0.5	5.50	6.20	12.90	75.20	100	4.20	4.72	10.72	75.54	100
	0.7	4.86	5.24	11.24	75.40	100	3.64	4.02	9.70	75.32	100
0.5	0.3	6.44	5.58	3.20	0.38	0	4.74	4.14	2.16	0.14	0
	0.5	5.50	4.92	2.92	0.26	0	4.20	3.68	1.94	0.18	0
	0.7	4.86	4.18	2.36	0.30	0	3.64	3.06	1.70	0.18	0
0.7	0.3	6.44	6.40	12.84	70.00	100	4.74	5.12	10.66	71.40	100
	0.5	5.50	6.18	12.24	70.56	100	4.20	4.82	10.28	71.52	100
	0.7	4.86	5.36	11.22	70.70	100	3.64	4.52	10.74	70.68	100
		$a=-0.1$					$a=0.1$				
0.3	0.3	3.90	4.22	10.00	75.74	100	3.54	3.60	8.80	74.80	100
	0.5	3.34	3.74	10.34	75.86	100	3.58	3.82	8.92	74.94	100
	0.7	3.62	3.98	9.16	75.60	100	4.10	4.22	8.76	73.40	100
0.5	0.3	3.90	2.80	1.36	0.10	0	3.54	2.88	1.40	0.10	0
	0.5	3.34	2.64	1.28	0.12	0	3.58	2.66	1.26	0.06	0
	0.7	3.62	2.90	1.58	0.14	0	4.10	3.02	1.50	0.10	0
0.7	0.3	3.90	3.86	8.66	72.04	100	3.54	3.76	9.78	73.64	100
	0.5	3.34	3.74	9.34	72.74	100	3.58	3.98	9.68	73.14	100
	0.7	3.62	3.84	9.78	72.72	100	4.10	4.10	9.24	73.04	100
		$a=0.2$					$a=0.3$				
0.3	0.3	4.58	4.48	9.74	73.18	100	7.30	6.82	12.26	72.40	100
	0.5	3.64	3.56	10.08	75.96	100	5.68	5.60	11.18	73.64	100
	0.7	4.08	4.06	10.24	75.00	100	5.30	5.06	10.54	73.90	100
0.5	0.3	4.58	3.64	1.80	0.16	0	7.30	5.54	2.50	0.14	0
	0.5	3.64	2.66	1.26	0.08	0	5.68	3.90	1.80	0.08	0
	0.7	4.08	2.92	1.58	0.08	0	5.30	3.90	1.84	0.16	0
0.7	0.3	4.58	4.98	10.98	74.86	100	7.30	7.02	12.36	74.44	100
	0.5	3.64	3.60	8.84	72.82	100	5.68	5.72	11.60	74.66	100
	0.7	4.08	3.98	8.48	72.14	100	5.30	4.84	9.84	73.08	100
		$a=0.4$					$a=0.5$				
0.3	0.3	10.56	9.62	16.18	73.16	100	19.16	23.66	32.66	81.66	100
	0.5	8.50	8.06	14.12	72.74	100	15.22	18.08	25.10	75.90	100
	0.7	6.50	6.76	12.92	74.58	100	11.10	12.70	18.40	73.70	100
0.5	0.3	10.56	8.36	4.30	0.32	0	19.16	19.04	14.82	3.36	0
	0.5	8.50	6.20	3.20	0.22	0	15.22	17.74	13.38	2.98	0
	0.7	6.50	5.18	2.56	0.32	0	11.10	11.72	8.90	2.22	0
0.7	0.3	10.56	10.24	15.90	74.84	100	19.16	19.00	23.38	68.56	100
	0.5	8.50	7.74	13.56	75.18	100	15.22	16.64	21.04	68.68	100
	0.7	6.50	5.70	10.34	71.56	100	11.10	14.46	20.30	70.04	100

Table 5.10: mirrors the power of the two-sided CUSQ under unpredictability of type II for $b_0 = -1$ and $\psi = 0°$ concerning different values of the break time in coefficients (θ), in the error distribution (λ), the impact of the coefficient's break (b) and the impact of the distribution's break (a).

θ	$\lambda\backslash b$	0	0.5	1	2	5	0	0.5	1	2	5
		$a=-0.5$					$a=-0.4$				
0.3	0.3	19.86	15.70	20.06	68.38	100	11.00	8.62	14.04	69.44	100
	0.5	15.90	13.82	18.70	69.14	100	8.48	7.60	12.88	70.42	100
	0.7	10.34	9.08	14.84	70.60	100	6.42	6.48	12.70	72.10	100
0.5	0.3	19.86	14.30	7.88	0.68	0	11.00	7.96	3.80	0.14	0
	0.5	15.90	10.18	4.68	0.40	0	8.48	5.12	1.94	0.16	0
	0.7	10.34	6.90	3.40	0.24	0	6.42	4.60	2.30	0.14	0
0.7	0.3	19.86	17.34	22.52	82.66	100	11.00	10.00	15.70	81.70	100
	0.5	15.90	13.16	17.58	81.80	100	8.48	7.42	13.56	79.90	100
	0.7	10.34	6.64	9.96	73.76	100	6.42	5.08	8.44	73.52	100
		$a=-0.3$					$a=-0.2$				
0.3	0.3	6.44	5.48	10.76	69.24	100	4.74	4.34	9.48	71.76	100
	0.5	5.50	5.46	11.32	72.62	100	4.20	4.26	9.86	72.52	100
	0.7	4.86	4.52	10.38	71.94	100	3.64	3.58	9.68	72.24	100
0.5	0.3	6.44	4.78	2.42	0.14	0	4.74	3.60	1.38	0.04	0
	0.5	5.50	3.24	1.34	0.04	0	4.20	3.02	1.32	0.02	0
	0.7	4.86	3.34	1.58	0.06	0	3.64	2.60	1.34	0.02	0
0.7	0.3	6.44	6.52	12.38	79.50	100	4.74	4.40	9.04	76.84	100
	0.5	5.50	4.96	10.32	77.36	100	4.20	4.18	9.26	77.60	100
	0.7	4.86	4.14	8.08	75.20	100	3.64	3.44	8.58	75.22	100
		$a=-0.1$					$a=0.1$				
0.3	0.3	3.90	3.80	8.90	72.18	100	3.54	4.32	9.90	73.94	100
	0.5	3.34	3.50	9.26	72.84	100	3.58	3.72	8.92	73.60	100
	0.7	3.62	3.52	9.08	73.04	100	4.10	3.92	9.60	73.02	100
0.5	0.3	3.90	2.72	1.20	0.02	0	3.54	2.66	1.34	0.18	0
	0.5	3.34	2.22	1.20	0.06	0	3.58	2.92	1.66	0.16	0
	0.7	3.62	2.74	1.30	0.06	0	4.10	3.36	1.98	0.10	0
0.7	0.3	3.90	3.82	8.90	75.12	100	3.54	4.06	9.76	72.62	100
	0.5	3.34	3.72	8.96	75.70	100	3.58	4.58	11.22	72.70	100
	0.7	3.62	3.66	8.94	75.24	100	4.10	4.70	10.62	74.52	100
		$a=0.2$					$a=0.3$				
0.3	0.3	4.58	5.48	12.14	76.10	100	7.30	8.88	15.38	75.82	100
	0.5	3.64	4.26	9.74	74.08	100	5.68	6.52	12.56	74.54	100
	0.7	4.08	4.56	10.86	74.44	100	5.30	5.82	10.96	72.38	100
0.5	0.3	4.58	4.08	2.56	0.44	0	7.30	6.76	4.42	0.66	0
	0.5	3.64	3.34	1.92	0.18	0	5.68	5.96	3.82	0.46	0
	0.7	4.08	3.28	2.10	0.26	0	5.30	4.82	3.24	0.40	0
0.7	0.3	4.58	5.04	12.04	72.02	100	7.30	7.92	14.60	70.64	100
	0.5	3.64	4.36	10.88	71.56	100	5.68	7.22	14.28	72.00	100
	0.7	4.08	5.10	11.12	73.32	100	5.30	6.76	14.30	72.90	100
		$a=0.4$					$a=0.5$				
0.3	0.3	10.56	13.64	22.00	79.06	100	19.16	18.54	24.04	73.62	100
	0.5	8.50	10.16	16.90	73.80	100	15.22	14.14	19.80	73.12	100
	0.7	6.50	7.48	13.58	73.14	100	11.10	10.48	15.48	72.52	100
0.5	0.3	10.56	10.06	7.42	1.30	0	19.16	15.52	8.54	0.92	0
	0.5	8.50	9.60	7.32	1.34	0	15.22	10.72	5.82	0.72	0
	0.7	6.50	6.68	4.76	1.06	0	11.10	8.42	4.50	0.52	0
0.7	0.3	10.56	11.40	17.94	68.82	100	19.16	17.16	21.04	76.52	100
	0.5	8.50	10.12	16.60	69.04	100	15.22	12.78	16.74	76.18	100
	0.7	6.50	8.88	16.08	70.50	100	11.10	8.96	12.08	69.50	100

Table 5.11: mirrors the power of the two-sided CUSQ under unpredictability of type II for $b_0 = -1$ and $\psi = 45°$ concerning different values of the break time in coefficients (θ), in the error distribution (λ), the impact of the coefficient's break (b) and the impact of the distribution's break (a).

		$a=-0.5$					$a=-0.4$				
θ	$\lambda \backslash b$	0	0.5	1	2	5	0	0.5	1	2	5
	0.3	19.86	15.96	20.22	68.98	100	11.00	8.84	14.04	69.86	100
0.3	0.5	15.90	14.00	19.18	69.24	100	8.48	7.62	12.64	69.58	100
	0.7	10.34	9.12	14.70	70.62	100	6.42	6.50	12.40	71.94	100
	0.3	19.86	14.68	8.12	0.70	0	11.00	8.16	3.92	0.14	0
0.5	0.5	15.90	10.54	5.14	0.42	0	8.48	5.26	2.16	0.14	0
	0.7	10.34	6.88	3.52	0.38	0	6.42	4.68	2.32	0.16	0
	0.3	19.86	17.46	22.82	82.54	100	11.00	10.06	15.82	81.56	100
0.7	0.5	15.90	13.48	18.06	81.44	100	8.48	7.48	13.58	80.16	100
	0.7	10.34	6.88	10.48	74.76	100	6.42	5.30	8.88	74.62	100
		$a=-0.3$					$a=-0.2$				
	0.3	6.44	5.56	10.40	69.78	100	4.74	4.32	9.78	71.36	100
0.3	0.5	5.50	5.40	11.02	71.98	100	4.20	4.38	9.98	72.24	100
	0.7	4.86	4.50	10.32	71.62	100	3.64	3.58	9.30	72.06	100
	0.3	6.44	4.80	2.48	0.16	0	4.74	3.52	1.32	0.04	0
0.5	0.5	5.50	3.24	1.36	0.10	0	4.20	3.00	1.40	0.04	0
	0.7	4.86	3.26	1.58	0.04	0	3.64	2.62	1.34	0.04	0
	0.3	6.44	6.24	12.40	80.08	100	4.74	4.72	9.22	77.50	100
0.7	0.5	5.50	4.96	10.42	78.00	100	4.20	4.22	9.76	78.42	100
	0.7	4.86	4.12	8.28	76.56	100	3.64	3.72	8.98	75.18	100
		$a=-0.1$					$a=0.1$				
	0.3	3.90	3.78	8.82	71.92	100	3.54	4.22	9.76	73.56	100
0.3	0.5	3.34	3.32	9.22	72.68	100	3.58	3.86	8.94	72.88	100
	0.7	3.62	3.80	8.96	72.56	100	4.10	3.76	9.52	72.58	100
	0.3	3.90	2.70	1.30	0.06	0	3.54	2.74	1.36	0.14	0
0.5	0.5	3.34	2.32	1.10	0.06	0	3.58	2.84	1.48	0.16	0
	0.7	3.62	2.82	1.26	0.08	0	4.10	3.40	2.06	0.10	0
	0.3	3.90	3.72	8.78	76.04	100	3.54	3.86	9.94	73.72	100
0.7	0.5	3.34	3.84	8.78	76.72	100	3.58	4.48	11.30	74.12	100
	0.7	3.62	3.68	9.08	75.96	100	4.10	4.70	10.90	75.70	100
		$a=0.2$					$a=0.3$				
	0.3	4.58	5.46	11.86	75.10	100	7.30	8.82	15.36	75.04	100
0.3	0.5	3.64	4.00	9.56	73.72	100	5.68	6.32	12.12	74.12	100
	0.7	4.08	4.54	10.96	73.86	100	5.30	5.74	10.84	72.00	100
	0.3	4.58	4.14	2.60	0.44	0	7.30	6.72	4.34	0.62	0
0.5	0.5	3.64	3.34	1.74	0.20	0	5.68	5.86	3.60	0.44	0
	0.7	4.08	3.26	2.06	0.24	0	5.30	4.94	3.26	0.40	0
	0.3	4.58	5.14	11.96	72.8	100	7.30	7.66	14.66	71.60	100
0.7	0.5	3.64	4.12	10.82	72.64	100	5.68	6.94	14.02	72.52	100
	0.7	4.08	4.90	11.08	73.78	100	5.30	6.72	14.60	73.52	100
		$a=0.4$					$a=0.5$				
	0.3	10.56	13.26	21.66	78.30	100	19.16	23.26	31.90	80.84	100
0.3	0.5	8.50	10.14	16.54	73.58	100	15.22	17.76	24.80	75.60	100
	0.7	6.50	7.46	13.74	72.96	100	11.10	12.42	17.90	72.74	100
	0.3	10.56	10.00	7.00	1.16	0	19.16	18.84	14.04	3.06	0
0.5	0.5	8.50	9.36	6.80	1.20	0	15.22	17.14	12.84	2.90	0
	0.7	6.50	6.44	4.46	1.14	0	11.10	11.36	8.44	2.14	0
	0.3	10.56	11.16	17.76	69.44	100	19.16	18.74	23.00	69.86	100
0.7	0.5	8.50	10.08	16.46	69.84	100	15.22	15.96	20.68	69.80	100
	0.7	6.50	8.58	15.58	71.60	100	11.10	14.04	19.94	70.26	100

Table 5.12: mirrors the power of the two-sided CUSQ under unpredictability of type II for $b_0 = -1$ and $\psi = 90^\circ$ concerning different values of the break time in coefficients (θ), in the error distribution (λ), the impact of the coefficient's break (b) and the impact of the distribution's break (a).

5.5.2 Proofs

Proof of Theorem 1

Assumption 2. *Let* $\{u_t\}_{t\geq 1}$ *be defined as a mean-zero stochastic process ,* $\|\cdot\|_p$ *as the* L_p*-norm of a random variable,* $W(s)$ *as a unit Wiener process being defined on* $[0,1]$*,* $\Rightarrow$ *as weak convergence under the Skorohod topology,*$[Tr]$ *as the largest integer less than or equal to* $r \cdot T$ *with* $0 < r \leq 1$*,* $\xi_t = u_t^2/\sigma^2 - 1$ *where* $\sigma^2 := \lim\limits_{T\to\infty} T^{-1}\sum_{t=1}^{T} E(u_t^2)$ *and* $S_T = \sum_{t=1}^{T}\xi_t$*.*

Assumption 3. *Let furthermore* $x_t u_t$ *and* u_t^2 *be (a) short memory processes with (b) bounded fourth moments while* $p\lim\limits_{T\to\infty}\sum_{t=1}^{[Tr]} x_t x_t' = \lim\limits_{T\to\infty}\sum_{t=1}^{[Tr]} E(x_t x_t') = \tilde{X}(r)$ *where the latter describes a non-random, absolutely integrable, positive definite matrix.*

Lemma 1. *(cf. Ploberger and Krämer [1986] and Deng and Perron [2008])*
Under assumptions 2 and 3

(a) $\sum\limits_{1\leq n\leq T} T^{-1/2}\|\sum_{t=1}^{n} x_t u_t\| = O_p(1)$

(b) $\lim\limits_{T\to\infty}\Omega_T = \Omega$ *where* $\Omega_T = Var(T^{-1/2}S_T)$

(c) $T^{-1/2}\sum_{t=1}^{[Tr]}\xi_t \Rightarrow \Omega^{1/2}W(r)$

(d) $\sum\limits_{n\geq 1}(n^{1/2}\log^{1/2}(n))^{-1}\|\sum_{t=1}^{n} x_t u_t\| = O_p(1)$

leading to the result that

$$CUSQ \Rightarrow \sqrt{\Omega}\sup_{r\in[0,1]}|BB(r)|. \tag{5.13}$$

A proof of (a) is given in Deng and Perron [2008]), while (b)-(d) are proven in Qu and Perron [2007] and Corradi [1999].
Let us now turn to the case where u_t is of type ExtII. Hence under Def.12 together with A1 ν_t is given by (5.7).

Corollary 1. *Under A1, A3 is fulfilled for* $b_{2i} \in \{\mathbb{R}\backslash -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, -\frac{1}{8}, -\frac{1}{9}\}$*.*

Proof. Under A1, A3(a) is trivially fulfilled due to its iid character.
Define $\omega_t := \nu_t^2$. Then $E(\omega_t^4) = \begin{cases} E\left(\omega_{1t}^4\right) & ,1\leq t\leq[\lambda T] \\ E\left(\omega_{2t}^4\right) & ,[\lambda T]+1\leq t\leq T. \end{cases}$
Recursive insertion into (5.2) yields

$$\begin{aligned} E(\omega_{it}^4) \quad = \quad & ((3b_{2i}+1)(4b_{2i}+1)(5b_{2i}+1)(6b_{2i}+1)(7b_{2i}+1)(8b_{2i}+1)(9b_{2i}+1))^{-1}\cdot \\ & (100800\, b_{1i}{}^4 b_{0i}{}^2 b_{2i} + 4145\, b_{1i}{}^2 a_i{}^2 b_{0i}{}^2 - 14164\, b_{1i}{}^3 a_i b_{0i}{}^2 - 5176\, a_i{}^2 b_{0i}{}^3 b_{2i}{}^2 \\ & +105\, b_{0i}{}^4 - 20160\, b_{1i}{}^6 b_{0i} - a_i{}^6 b_{0i} + 50424\, b_{1i}\, a_i b_{0i}{}^3 b_{2i}{}^2 - 3300\, b_{1i}\, a_i{}^3 b_{0i}{}^2 \\ & b_{2i} - 82896\, b_{1i}{}^3 a_i b_{0i}{}^2 b_{2i} + 25052\, b_{1i}{}^2 a_i{}^2 b_{0i}{}^2 b_{2i} - 445\, b_{1i}{}^2 a_i{}^4 b_{0i} + 3135\, b_{1i}{}^3 \\ & a_i{}^3 b_{0i} + 24552\, b_{1i}{}^5 a_i b_{0i} - 12154\, b_{1i}{}^4 a_i{}^2 b_{0i} + 33\, b_{1i}\, a_i{}^5 b_{0i} + 20160\, b_{0i}{}^4 b_{2i}{}^3 \\ & -141\, a_i{}^2 b_{0i}{}^3 - 3690\, b_{1i}{}^2 b_{0i}{}^3 + 10920\, b_{0i}{}^4 b_{2i}{}^2 + 1890\, b_{0i}{}^4 b_{2i} + 25\, a_i{}^4 b_{0i}{}^2 + \\ & 17832\, b_{1i}{}^4 b_{0i}{}^2 - 120960\, b_{1i}{}^2 b_{0i}{}^3 b_{2i}{}^2 + 160\, a_i{}^4 b_{0i}{}^2 b_{2i} - 530\, b_{1i}\, a_i{}^3 b_{0i}{}^2 \\ & +1451\, b_{1i}\, a_i b_{0i}{}^3 - 43224\, b_{1i}{}^2 b_{0i}{}^3 b_{2i} - 1742\, a_i{}^2 b_{0i}{}^3 b_{2i} + 17470\, b_{1i}\, a_i b_{0i}{}^3 b_{2i}) \end{aligned}$$

Hence $E(\omega_{it}^4) \in \mathbb{R}$ for $b_{2i} \in \{\mathbb{R}\backslash -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, -\frac{1}{8}, -\frac{1}{9}\}$. □

Corollary 2. *Under A1-A3 together with (5.7),* $\Omega = 2 + (1-\lambda)\frac{6a_2^2}{b_0} + \lambda\frac{6a_1^2}{b0}$.

Proof.
With $\mu_4^{(i)} := E\left(v_{it}^4\right)$ for $i \in \{1,2\}$

$$\begin{aligned}
\Omega_T &:= Var(T^{-1/2}S_T) = T^{-1}E(S_T S_{T'}) = T^{-1}E\left(\sum_{t=1}^{T}\xi_t^2\right) = T^{-1}E\left(\sum_{t=1}^{T}\left(\frac{v_t^2}{\sigma^2}-1\right)^2\right) \\
&= T^{-1}\left(\frac{1}{\sigma^4}\sum_{t=1}^{T}E(v_t^4) - \frac{2}{\sigma^2}\sum_{t=1}^{T}E(v_t^2) + T\right) = \frac{\sum_{t=1}^{T}E(v_t^4)}{T\sigma^4} - 1 \\
&= \frac{\sum_{t=1}^{[\lambda T]}E(v_t^4) + \sum_{t=[\lambda T+1]}^{T}E(v_t^4)}{T\sigma^4} - 1 = \frac{[\lambda T]\mu_4^{(1)} + [T-\lambda T]\mu_4^{(2)}}{T\sigma^4} - 1 \\
\Rightarrow \Omega &= \lim_{T\to\infty}\Omega_T = \sigma^{-4}\left(\mu_4^{(2)} - \sigma^4 + (\mu_4^{(1)} - \mu_4^{(2)})\lim_{T\to\infty}\left(\frac{[\lambda T]}{T}\right)\right) \\
&= \frac{\mu_4^{(2)} + \lambda(\mu_4^{(1)} - \mu_4^{(2)})}{\sigma^4} - 1
\end{aligned}$$

Via (5.2), A1(a) together with (5.8) implies that

$$\begin{aligned}
E(v_i) &= \frac{a_i - b_{1i}}{2b_{2i}+1} \overset{!}{=} 0 \\
E(v_i^2) - E^2(v_i) &= -\frac{b_{0i} + (2b_{1i} - a)E(v_i)}{3b_{2i}+1} - E^2(v_i) \overset{!}{=} -b_{0i} \\
E(v_i) &\overset{!}{=} E(v), \quad i \in \{1,2\} \\
E(v_i^2) - E^2(v_i) &\overset{!}{=} E(v^2) - E^2(v) = -b_0, \quad i \in \{1,2\}
\end{aligned}$$

leading to $a_i = b_{1i}$, $b_{0i} = b_0$ and $b_{2i} = b_2 = 0$. Note that due to the latter condition Corollary 1 is always supported. As further

$$\begin{aligned}
E\left(v_i^3\right) &= -\frac{2b_0E(v_i) + (3b_{1i} - a_i)E(v_i^2)}{4b_{2i}+1} \\
E\left(v_i^4\right) &= -\frac{3b_0E(v_i^2) + (4b_{1i} - a_i)E(v_i^3)}{5b_{2i}+1}
\end{aligned}$$

it follows that $E\left(v_i^4 | a_i = b_{1i}, b_{2i} = 0\right) = 3b_0^2 + 6a_i^2b_0$ leading to

$$\begin{aligned}
\Omega &= \frac{3b_0^2 + 6a_2^2b_0 + \lambda(3b_0^2 + 6a_1^2b_0 - (3b_0^2 + 6a_2^2b_0))}{\sigma^4} - 1 \\
&= 2 + (1-\lambda)\frac{6a_2^2}{b_0} + \lambda\frac{6a_1^2}{b_0}
\end{aligned}$$

□

Via Lemma1 , Corollary1 and Corollary2 together with A1-A3 yield Theorem 1.

Proof of Theorem 2

Deng and Perron [2008] derive that for arbitrary error distributions the CUSQ with OLS residuals converges in the following way:

$$\begin{aligned}
T^{-1/2}CUSQ^{OLS} &= \frac{\sup_{r\in[0,1]}|T^{-1/2}(\sum_{s=1}^{[Tr]}\hat{v}_s^2 - \frac{[Tr]}{T}\sum_{s=1}^{T}\hat{v}_s^2)|}{\sqrt{T^{-1}\sum_{t=1}^{T}\hat{v}_t^4-(T^{-1}\sum_{t=1}^{T}\hat{v}_t^2)^2}} + o_p(1) \xrightarrow{p} \frac{|(2\theta-1)\theta(1-\theta)(\delta' R\delta)|}{\sqrt{d(\delta,\theta)}}, \quad \text{where}\\
d(\delta,\theta) &= p\lim_{T\to\infty}\left(T^{-1}\sum_{t=1}^{T}\hat{v}_t^4 - \left(T^{-1}\sum_{t=1}^{T}\hat{v}_t^2\right)^2\right)\\
&= \theta(1-\theta)(3\theta^2-3\theta+1)g(\delta) - \theta^2(1-\theta)^2(\delta' R\delta)^2 + o_p(\|\delta\|^4), \quad \text{and}\\
g(\delta) &= p\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}(\delta' x_t x_t'\delta)^2.
\end{aligned}$$

Noting that the OLS residuals of (5.12) are given by

$$\hat{v}_s = v_s + x_s'\delta 1_{(s>[T\lambda])} - x_s'\left(\sum_{t=1}^{T}x_t x_t'\right)^{-1}\sum_{t=1}^{T}x_t x_t'\delta 1_{(t>[T\lambda])} - x_s'\left(\sum_{t=1}^{T}x_t x_t'\right)^{-1}\sum_{t=1}^{T}x_t v_t \tag{5.14}$$

and seeking assumption 1(a) we need merely to derive the limit of $T^{-1}\sum_{t=1}^{T}\hat{v}_t^4 = \mu_4 - 6\sigma_v^2\theta(1-\theta)(\delta' R\delta) + \theta(1-\theta)(3\theta^2-3\theta+1)g(\delta)$ in order to derive the convergence of $T^{-1/2}CUSQ^{OLS}$ under ExtII in theorem 2.

Proof. Defining

$$\begin{aligned}
A_s &:= v_s\\
B_s &:= x_s'\delta 1_{(s>[T\lambda])}\\
C_s &:= x_s'\left(\sum_{t=1}^{T}x_t x_t'\right)^{-1}\sum_{t=1}^{T}x_t x_t'\delta 1_{(t>[T\lambda])}\\
D_s &:= x_s'\left(\sum_{t=1}^{T}x_t x_t'\right)^{-1}\sum_{t=1}^{T}x_t v_t
\end{aligned}$$

yields

$$\begin{aligned}
\sum_{s=1}^{T}\hat{v}_s^4 = \sum_{s=1}^{T} & 12A_sB_s{D_s}^2 - 12B_sC_s{D_s}^2 - 12A_s{B_s}^2C_s - 12A_s{C_s}^2D_s + {B_s}^4 + {C_s}^4 + {D_s}^4 + {A_s}^4 + 24A_sB_sC_sD_s\\
& -12{A_s}^2B_sD_s - 12B_s{C_s}^2D_s + 12{A_s}^2C_sD_s - 12A_sC_s{D_s}^2 - 12A_s{B_s}^2D_s - 12{A_s}^2B_sC_s + 4{A_s}^3B_s\\
& -4{A_s}^3C_s - 4{A_s}^3D_s + 6{A_s}^2{B_s}^2 - 4A_s{D_s}^3 - 4{B_s}^3C_s - 4{B_s}^3D_s + 6{B_s}^2{C_s}^2 + 6{B_s}^2{D_s}^2 - 4B_s{D_s}^3\\
& -4A_s{C_s}^3 + 4A_s{B_s}^3 + 6{A_s}^2{C_s}^2 + 6{A_s}^2{D_s}^2 + 4{C_s}^3D_s + 6{C_s}^2{D_s}^2 + 4C_s{D_s}^3 - 4B_s{C_s}^3 + 12A_sB_s{C_s}^2\\
& +12{B_s}^2C_sD_s.
\end{aligned}$$

Note that since $p\lim_{T\to\infty}T^{-1}\sum_{t=1}^{T}x_tv_t = 0$ and $p\lim_{T\to\infty}T^{-1}\sum_{s=1}^{T}v_s = 0$ terms including powers of D_s as well as terms including A_s converge to zero. Further $p\lim_{T\to\infty}T^{-1}\sum_{s=1}^{T}A_s^3 = 0$ which is a direct consequence from (5.3) and remark 2(*ii*). Hence

$$p\lim_{T\to\infty}\sum_{s=1}^{T}\hat{v}_s^4 = p\lim_{T\to\infty}\sum_{s=1}^{T}{A_s}^4+{B_s}^4+{C_s}^4+6{A_s}^2{B_s}^2+6{A_s}^2{C_s}^2-12{A_s}^2B_sC_s+6{B_s}^2{C_s}^2-4{B_s}^3C_s-4B_s{C_s}^3.$$

With

$$\begin{aligned}
p\lim_{T\to\infty}\sum_{s=1}^{T}A_s^4 &= \mu_4\\
p\lim_{T\to\infty}\sum_{s=1}^{T}B_s^4 &= (1-\theta)g(\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}C_s^4 &= (1-\theta)^4g(\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}A_s^2B_s^2 &= \sigma_\nu^2(1-\theta)(\delta'R\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}A_s^2C_s^2 &= \sigma_\nu^2(1-\theta)^2(\delta'R\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}A_s^2B_sC_s &= \sigma_\nu^2(1-\theta)^2(\delta'R\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}B_s^2C_s^2 &= (1-\theta)^3g(\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}B_s^3C_s &= (1-\theta)^2g(\delta)\\
p\lim_{T\to\infty}\sum_{s=1}^{T}B_sC_s^3 &= (1-\theta)^4g(\delta)
\end{aligned}$$

the limit of $\sum_{s=1}^{T}\hat{v}_s^4$ is given as

$$\begin{aligned}
p\lim_{T\to\infty}\sum_{s=1}^{T}\hat{v}_s^4 &= \mu_4+(1-\theta)g(\delta)+(1-\theta)^4g(\delta)+6\sigma_\nu^2(1-\theta)(\delta'R\delta)+6\sigma_\nu^2(1-\theta)^2(\delta'R\delta)-12\sigma_\nu^2(1-\theta)^2\\
&\quad\times(\delta'R\delta)+6(1-\theta)^3g(\delta)-4(1-\theta)^2g(\delta)-4(1-\theta)^4g(\delta)\\
&= \mu_4-6\sigma_\nu^2\theta(1-\theta)(\delta'R\delta)+\theta(1-\theta)(3\theta^2-3\theta+1)g(\delta).
\end{aligned}$$

As $p\lim_{T\to\infty}\sum_{s=1}^{T}\hat{v}_s^2=\sigma_\nu^2+\theta(1-\theta)(\delta'R\delta)$ we get the result of

$$\begin{aligned}
p\lim_{T\to\infty}\left(\sum_{s=1}^{T}\hat{v}_s^4-\left(\sum_{s=1}^{T}\hat{v}_s^2\right)^2\right) &= \mu_4-6\sigma_\nu^2\theta(1-\theta)(\delta'R\delta)+\theta(1-\theta)(3\theta^2-3\theta+1)g(\delta)-(\sigma_\nu^4+2\sigma_\nu^2\theta(1-\theta)\\
&\quad\times(\delta'R\delta)+\theta^2(1-\theta)^2(\delta'R\delta)^2)\\
&= \mu_4-\sigma_\nu^4-8\sigma_\nu^2\theta(1-\theta)(\delta'R\delta)-\theta^2(1-\theta)^2(\delta'R\delta)^2+\theta(1-\theta)(3\theta^2-3\theta+1)\\
&\quad\times g(\delta).
\end{aligned}$$

Inserting $\mu_4 = \lambda\mu_4^{(1)} + (1-\lambda)\mu_4^{(2)}$ yields theorem 2. $\square$

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