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Møller scattering and Lorentz-violating Z bosons

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ABSTRACT

Lorentz-symmetry breakdown in weak-interaction physics is studied. In particular, the CPT-even Lorentz-violating contributions to the Z boson in the minimal Standard-Model Extension are considered, and in this context polarized electron–electron scattering is investigated. Corrections to the usual parity-violating asymmetry are determined at tree level. Together with available data, this result can be used to improve existing estimates for the Lorentz-violating k_W coefficient by two orders of magnitude. Some implications for past and future experiments are mentioned.

1. Introduction

A key concept in our present understanding of classical spacetime is Lorentz symmetry. For over a century, this symmetry has been scrutinized experimentally with ever increasing sensitivity, and no compelling evidence for deviations from Lorentz invariance has been found to date. However, the fate of Lorentz symmetry becomes less clear when the effects of quantum physics on spacetime are considered. In fact, various theoretical approaches to this problem can accommodate departures from Lorentz invariance [1–8] with a size that is expected to be governed by the Planck scale.

For the description of the ensuing Lorentz-breaking effects at presently attainable energies, the Standard-Model Extension (SME) framework has been developed [9,10]. The SME is based on effective field theory, incorporates the usual Standard Model and General Relativity as limiting cases, and contains general Lorentz- and CPT-violating operators of arbitrary mass dimension. This framework is therefore expected to provide an adequate characterization of low-energy departures from Lorentz symmetry regardless of their Planck-scale origin. For about two decades, the SME has served as the basis for both experimental [11–17] and theoretical [18–23] analyses of Lorentz violation. The SME has also been employed to study certain phenomenological effects of spacetime torsion [24].

For practical reasons, the overwhelming majority of past Lorentz-symmetry tests has involved stable or quasistable particles. Only comparatively few analyses have involved, e.g., weak-interaction physics [25–27], and only some of these have placed actual constraints on the SME coefficients for the heavy vector bosons [26,27]. The present work is aimed at analyzing a different set of phenomenological effects in this context that could potentially be used for alternative measurements of SME coefficients associated with the massive gauge bosons. More specifically, we consider the tree-level corrections to electron–electron scattering arising from Lorentz-symmetry violations that involve internal Z-boson lines. The leading effects in this context depend on the Lorentz-breaking $k_{\phi\phi}$ and k_W coefficients, which are both CPT even. Throughout, we work within the minimal SME (mSME), which is a subset of the full SME that restricts attention to power-counting renormalizable Lorentz-violating operators.

The structure of our analysis is as follows. Section 2 provides a brief review of the relevant aspects of electroweak symmetry breaking within the mSME. In Sec. 3, we present the calculation of the cross section for electron–electron scattering. A discussion of experimental signatures and possible measurements is given in Sec. 4. A summary and outlook is contained in Sec. 5. Throughout, we work in natural units $\hbar = c = 1$, and our convention for the Minkowski metric is $\eta^{\mu\nu} = \mathrm{diag}\,(+,-,-,-)$.

2. Basics

The flat-spacetime mSME is constructed to contain all Lorentz-violating power-counting renormalizable operators that are compatible with various key principles of physics. For example, spacetime-translation symmetry ensuring 4-momentum conserva-

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tion, unitarity, and the usual SU(3) × SU(2) × U(1) gauge structure are typically taken as properties of the mSME. In the present weak-interaction context, the latter requirement of gauge symmetry is best displayed before electroweak symmetry breaking. For this reason, we begin by reviewing the Lorentz-violating pieces of the unbroken Higgs and gauge-boson sectors in the mSME.

In the Higgs sector, the above conditions permit the following Lorentz-violating lagrangian contributions [9]:

$$\mathcal{L}^{\text{CPT-even}}_{\text{Higgs}} = \tfrac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi + \text{h.c.} - \tfrac{1}{2}(k_{\phi B})^{\mu\nu}\phi^\dagger\phi B_{\mu\nu} - \tfrac{1}{2}(k_{\phi W})^{\mu\nu}\phi^\dagger W_{\mu\nu}\phi\,, \quad (1)$$

$$\mathcal{L}^{\text{CPT-odd}}_{\text{Higgs}} = i(k_\phi)^\mu\phi^\dagger D_\mu\phi + \text{h.c.} \quad (2)$$

Here, D^μ is the ordinary gauge-covariant derivative. As usual, ϕ denotes the Higgs doublet, and $B^{\mu\nu}$ and $W^{\mu\nu}$ are the respective $\text{U}_\text{Y}(1)$ and SU(2) gauge-field strengths. The dimensionless Lorentz-breaking coefficient $k_{\phi\phi}$ is CPT even and can have symmetric real and antisymmetric imaginary parts. This coefficient is usually chosen to be traceless

$$\text{Re}\,(k_{\phi\phi})^\mu{}_\mu = 0\,, \quad (3)$$

so as to avoid introducing unwanted Lorentz-invariant contributions. The coefficients $k_{\phi B}$ and $k_{\phi W}$ are also CPT conserving and dimensionless, and they must be real and antisymmetric. The coefficient k_ϕ is CPT odd, has dimensions of mass, and must be real.

The gauge-field sector also possesses both CPT-even and CPT-odd Lorentz-violating operators. The CPT-even ones are given by [9]

$$\mathcal{L}^{\text{CPT-even}}_{\text{gauge}} = -\tfrac{1}{4}(k_B)_{\kappa\lambda\mu\nu}B^{\kappa\lambda}B^{\mu\nu} - \tfrac{1}{2}(k_W)_{\kappa\lambda\mu\nu}\text{Tr}(W^{\kappa\lambda}W^{\mu\nu})\,. \quad (4)$$

The coefficients k_B and k_W are real and dimensionless. Moreover, they possess the symmetries of the Riemann tensor and a vanishing double trace, so each has 19 independent components.

The CPT-odd Lorentz-breaking contributions take the following form [9]:

$$\mathcal{L}^{\text{CPT-odd}}_{\text{gauge}} = (k_1)_\kappa\epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu} + (k_0)_\kappa B^\kappa + (k_2)_\kappa\epsilon^{\kappa\lambda\mu\nu}\text{Tr}\left(W_\lambda W_{\mu\nu} + \tfrac{2}{3}igW_\lambda W_\mu W_\nu\right)\,. \quad (5)$$

Here, the couplings k_1 and k_2 are real and have dimensions of mass, while k_0 is also real and has dimensions of mass cubed. In what follows, we will disregard these terms:[1] they are undesirable from a theoretical perspective because they are all associated with negative contributions to the energy [9].

In principle, the matter sector of mSME also contains a number of Lorentz-violating operators. In the present context of Møller scattering, the mSME coefficients for the electron field might be expected to be of relevance. However, we can safely disregard them on phenomenological grounds because they are constrained many orders of magnitude beyond the reach of present-day and near-future Møller-scattering experiments [11].

To determine the low-energy phenomenology associated with the above lagrangian contributions (1), (2), and (4), we need to consider the issue of spontaneous electroweak SU(2) × U(1) symmetry breakdown. To this end, we employ the usual approach of implementing unitary gauge and representing the Higgs doublet as

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ r_\phi \end{pmatrix}\,. \quad (6)$$

Note that this is justified, as the corrections (1), (2), and (4) have been constructed such that the gauge structure of the theory is left unaffected. Paralleling the conventional case, we also define the fields

$$W^\pm_\mu = \tfrac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)\,, \quad (7)$$

$$Z^0_\mu = W^3_\mu\cos\theta_W - B_\mu\sin\theta_W\,, \quad (8)$$

$$A_\mu = B_\mu\cos\theta_W + W^3_\mu\sin\theta_W\,, \quad (9)$$

where θ_W denotes the ordinary weak angle.

The next step is the extraction of the static potential $V(r_\phi, A_\mu, Z^0_\mu, W^\pm_\mu)$ for the gauge and Higgs fields. It has the general form

$$V(r_\phi, A_\mu, Z^0_\mu, W^\pm_\mu) = V_0(r_\phi) + \delta V(r_\phi, A_\mu, Z^0_\mu, W^\pm_\mu)\,. \quad (10)$$

Here, $V_0(r_\phi)$ denotes the ordinary Lorentz-invariant contribution given by

$$V_0(r_\phi) = -\frac{1}{2}\mu^2 r_\phi^2 + \frac{\lambda}{4!}r_\phi^4\,, \quad (11)$$

where μ and λ are the conventional Higgs-potential parameters. The covariant derivatives contained in Eqs. (1) and (2) lead to the following Lorentz-violating corrections $\delta V(r_\phi, A_\mu, Z^0_\mu, W^\pm_\mu)$ to the static potential:

$$\begin{aligned}\delta V(r_\phi,\ A_\mu, Z^0_\mu, W^\pm_\mu) = &\ \tfrac{1}{2}\sqrt{g^2+g'^2}\,k^\mu_\phi\,Z^0_\mu\,r_\phi^2 \\ &- \tfrac{1}{8}(g^2+g'^2)\left(\eta^{\mu\nu} + \text{Re}\,k^{\{\mu\nu\}}_{\phi\phi}\right)Z^0_\mu\,Z^0_\nu\,r_\phi^2 \\ &- \tfrac{1}{4}g^2\left(\eta^{\mu\nu} + \text{Re}\,k^{\{\mu\nu\}}_{\phi\phi} + i\,\text{Im}\,k^{[\mu\nu]}_{\phi\phi}\right)W^-_\mu\,W^+_\nu\,r_\phi^2\,, \end{aligned} \quad (12)$$

where g and g' are the usual SU(2) and $\text{U}_\text{Y}(1)$ couplings, respectively. Curly (square) brackets denote (anti)symmetrization, e.g., $k^{[\mu\nu]}_{\phi\phi} \equiv \tfrac{1}{2}(k^{\mu\nu}_{\phi\phi} - k^{\nu\mu}_{\phi\phi})$.

Extremizing the static potential requires

$$\frac{\partial V}{\partial r_\phi} = 0\,, \qquad \frac{\partial V}{\partial A_\mu} = 0\,, \quad (13)$$

$$\frac{\partial V}{\partial Z^0_\mu} = 0\,, \qquad \frac{\partial V}{\partial W^\pm_\mu} = 0\,. \quad (14)$$

The second one of Eqs. (13) is trivial since V is independent of A_μ, which can therefore be chosen freely. This feature is compatible with the surviving $\text{U}_\gamma(1)$ gauge symmetry. The remaining set of simultaneous equations is solved by [9]

$$\langle r_\phi\rangle = a\left(1 - \frac{1}{\mu^2}(\text{Re}\,\hat{k}_{\phi\phi})^{-1}_{\mu\nu}\,k^\mu_\phi\,k^\nu_\phi\right)^{1/2}\,, \quad (15)$$

$$\langle Z^0_\mu\rangle = \frac{1}{q}\sin 2\theta_W\,(\text{Re}\,\hat{k}_{\phi\phi})^{-1}_{\mu\nu}\,k^\nu_\phi\,, \quad (16)$$

$$\langle W^\pm_\mu\rangle = 0\,, \quad (17)$$

where we have defined $\hat{k}^{\mu\nu}_{\phi\phi} \equiv \eta^{\mu\nu} + k^{\mu\nu}_{\phi\phi}$ and $a \equiv \sqrt{6\mu^2/\lambda}$. We note that the quantity $\text{Re}\,(\hat{k}_{\phi\phi})^{-1}_{\mu\nu}$ always exists for perturbative mSME coefficients $|(k_{\phi\phi})^{\mu\nu}| \ll 1$.

An interesting questions concerns the identification of experimental signatures for such Lorentz-violating effects. One avenue to pursue in this context is discussed in the next section.

[1] This may not require setting to zero coefficients with the same symmetries in other SME sectors [21].

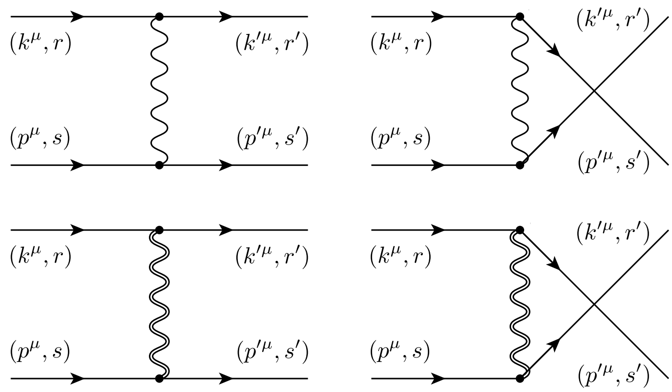


Fig. 1. Leading Lorentz-invariant tree-level contributions to electron–electron scattering. A solid line represents an electron. It is labeled by one of the four external 4-momenta $k^\mu, k'^\mu, p^\mu, p'^\mu$ as well as by one of the helicity observables r, r', s, s'. A single wavy line denotes the ordinary Lorentz-invariant photon propagator, and a double wavy line the usual Z-boson propagator. The vertices are conventional.

3. Electron–electron scattering

Electron–electron scattering, also known as Møller scattering, is dominated by the electromagnetic interaction. However, Z-boson exchange can also contribute to the amplitude for this process via the bottom two Feynman diagrams in Fig. 1. The corresponding effects are small but measurable, they have in the past been used to investigate the weak charge

$$Q_W^e = 4\sin^2\theta_W - 1 \tag{18}$$

of the electron [28], and future measurements with improved sensitivity are planned [29]. Electron–electron scattering might therefore provide a window for an independent study of Lorentz breakdown in the Z boson.

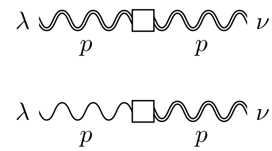
To determine the Lorentz-violating Z-boson effects in electron–electron scattering within the mSME, we assume that mSME coefficients are small and hence can be treated perturbatively. This assumption is a natural one in light of the expected Planck suppression of Lorentz-breaking effects. The first step is therefore to represent the mSME coefficients diagrammatically, so they can be added to the usual set of electroweak Feynman rules.

We begin by expanding the corrections (1), (2), and (4) about the electroweak vacuum expectation values (15) and (16):

$$r_\phi = \langle r_\phi \rangle + h\,, \tag{19}$$

$$Z^0_\mu = \langle Z^0_\mu \rangle + Z_\mu\,. \tag{20}$$

The resulting lagrangian expression contains various constant terms that can be dropped, as they fail to contribute to the equations of motion. Terms linear in the fields are absent, which is a direct consequence of expanding around an extremum of the static potential. The leading-order Lorentz-violating corrections are thus quadratic in the physical fields. Third- or higher-order terms also appear, but we may disregard them: their contribution to electron–electron scattering would be associated with further suppression factors arising from additional powers of the electromagnetic or weak coupling constants. Among the set of the quadratic terms itself, we can further narrow down relevant contributions. In particular, we will ignore effects resulting from internal Higgs lines due their expected further suppression relative to Z-boson lines. For these reasons, the dominant Lorentz-violating effects are controlled by the following quadratic lagrangian terms involving A_μ and Z_μ:



$$= -2i(1-\tan^2\theta_W)\,(k_W)_{\kappa\lambda\mu\nu}\,p^\kappa p^\mu + iM_Z^2\,\mathrm{Re}(k_{\phi\phi})_{\lambda\nu}$$

$$= -2i\tan\theta_W\,(k_W)_{\kappa\lambda\mu\nu}\,p^\kappa p^\mu$$

Fig. 2. Feynman rules for Lorentz-violating corrections to the Z boson. The corrections relevant in the present context take the form of propagator insertions. The single and double wavy lines represent the conventional Lorentz-invariant photon and Z-boson propagators, respectively.

$$\begin{aligned}\delta\mathcal{L}^{(2)}_{A,Z} = &-\tfrac{1}{4}(k_B\cos^2\theta_W + k_W\sin^2\theta_W)_{\kappa\lambda\mu\nu}\,F^{\kappa\lambda}F^{\mu\nu}\\ &-\tfrac{1}{4}(k_W\cos^2\theta_W + k_B\sin^2\theta_W)_{\kappa\lambda\mu\nu}\,Z^{\kappa\lambda}Z^{\mu\nu}\\ &-\tfrac{1}{4}\sin 2\theta_W\,(k_W-k_B)_{\kappa\lambda\mu\nu}\,F^{\kappa\lambda}Z^{\mu\nu}\\ &+\tfrac{1}{2}M_Z^2\,\mathrm{Re}\,(k_{\phi\phi})_{\mu\nu}\,Z^\mu Z^\nu\,,\end{aligned} \tag{21}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$. We also remind the reader that we have set to zero the mSME coefficients of the external-leg electrons because of existing tight experimental constraints.

Before inferring the diagrammatic representation of the mSME coefficients relevant for the present context of electron–electron scattering from Eq. (21) above, a further simplification can be made. Numerous previous measurements and observations in the mSME's photon sector have constrained the quantity

$$(k_F)_{\kappa\lambda\mu\nu} \equiv (k_B\cos^2\theta_W + k_W\sin^2\theta_W)_{\kappa\lambda\mu\nu} \tag{22}$$

appearing in Eq. (21) to precisions well beyond those that can be achieved in electron–electron scattering [11]. For our present purposes, we can therefore effectively set $(k_F)_{\kappa\lambda\mu\nu} = 0$, so that k_B can be expressed in terms of k_W:

$$(k_B)_{\kappa\lambda\mu\nu} = -\tan^2\theta_W\,(k_W)_{\kappa\lambda\mu\nu}\,. \tag{23}$$

This phenomenological simplification together with Eq. (21) yield the Feynman rules displayed in Fig. 2. These rules govern the dominant Lorentz-violating effects in Møller scattering. They may also give rise to certain effects in the photon sector, but a thorough study of these lies outside our present scope.

With these Lorentz-violating Feynman rules, the leading mSME corrections to the Lorentz-invariant amplitude can be determined and are shown as Feynman graphs in Fig. 3. The corresponding measurements typically involve incoming longitudinally polarized, relativistic electrons incident on a fixed unpolarized target. Subsequently, the outgoing Møller electrons are counted for a limited range of scattering angles [28,29]. We therefore need to consider incoming states of definite helicity, average of the spin states of the fixed-target electrons, and sum over outgoing electron spins. This yields two squared matrix elements $\overline{|\mathcal{M}_R|^2}$ and $\overline{|\mathcal{M}_L|^2}$ for incoming right-handed and left-handed beam electrons, respectively. The general structure of these is given by

$$\begin{aligned}\overline{|\mathcal{M}_R|^2} &= \overline{|\mathcal{M}_R^0|^2} + \delta\overline{|\mathcal{M}_R|^2}\,,\\ \overline{|\mathcal{M}_L|^2} &= \overline{|\mathcal{M}_L^0|^2} + \delta\overline{|\mathcal{M}_L|^2}\,,\end{aligned} \tag{24}$$

where $\overline{|\mathcal{M}_R^0|^2}$ and $\overline{|\mathcal{M}_L^0|^2}$ are the conventional Lorentz-invariant contributions, which can be inferred from the literature [30]. The Lorentz-violating effects of the physical system under consideration are encoded in the corrections $\delta\overline{|\mathcal{M}_R|^2}$ and $\delta\overline{|\mathcal{M}_L|^2}$. The next step is therefore the determination of explicit expressions for these in terms of the relevant kinematical quantities and the k_W and $k_{\phi\phi}$ coefficients.

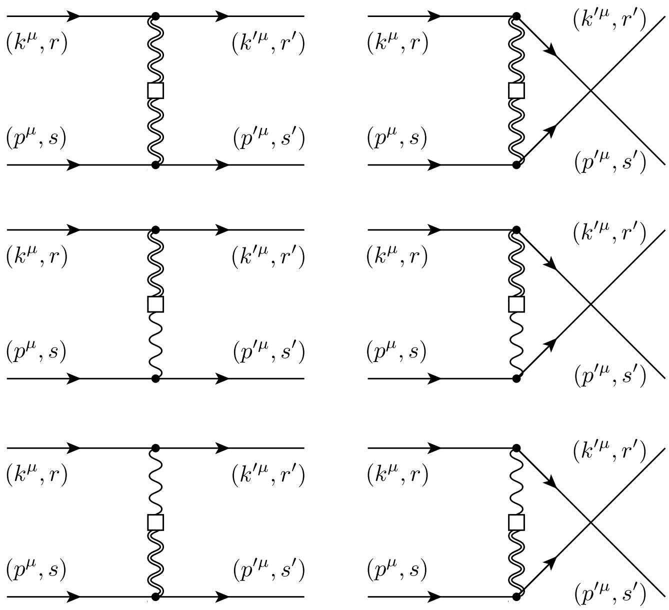


Fig. 3. Dominant Lorentz-violating corrections to the processes shown in Fig. 1. The notation is as before. In particular, the 3-point vertices are the conventional Lorentz-symmetric ones, and the insertions are those from Fig. 2.

As an aside before embarking on the calculation of $\delta\overline{|\mathcal{M}_R|^2}$ and $\delta\overline{|\mathcal{M}_L|^2}$, we consider the limit

$$\mathrm{Re}\,(k_{\phi\phi})^{\mu\nu} \to \eta^{\mu\nu} \tag{25}$$

for the first insertion in Fig. 2 This limit is incompatible with our earlier assumptions, such as the trace-free condition (3) and is therefore not useful for phenomenological purposes. However, it may be of mathematical interest in the evaluation of Møller matrix elements for the following reason. In this limit, the $k_{\phi\phi}$ term of the insertion becomes Lorentz invariant, and the corresponding pieces of the top two mSME diagrams in Fig. 3 cancel the bottom two conventional Lorentz-invariant diagrams in Fig. 1. The usual Lorentz-symmetric contributions arising from Z-boson exchange should then be absent from the expressions for our Møller matrix elements. Solely for the purpose of making explicit these cancellations in our final result, we will carry the trace piece of $k_{\phi\phi}$ along in what follows. But again, for actual phenomenological purposes in the present context, $k_{\phi\phi}$ must be small and traceless.

The calculation of $\delta\overline{|\mathcal{M}_R|^2}$ and $\delta\overline{|\mathcal{M}_L|^2}$ is somewhat tedious, but essentially proceeds in the usual way and includes the diagrams in Fig. 3 with the Lorentz-breaking insertions. The four external momenta k^μ, k'^μ, p^μ, and p'^μ are constrained by energy–momentum conservation, leaving three independent momenta. To cast the results in a relatively compact form, we choose the following three combinations of 4-momenta as our kinematical variables

$$S_\mu = k_\mu + p_\mu\,, \tag{26}$$

$$T_\mu = k'_\mu - k_\mu\,, \tag{27}$$

$$U_\mu = p_\mu - k'_\mu\,, \tag{28}$$

with a notation inspired by the ordinary Mandelstam variables. We then obtain

$$\begin{aligned}
\delta\overline{|\mathcal{M}_R|^2} = {}& \frac{2e^4 (k_W)^{\kappa\mu\lambda\nu}}{s\,M_Z^2\,y^2(1-y)^2\cos^2\theta_W} \times \\
& \Big\{(y-1)\Big[(2-4y+y^2)Q_W^e + 1 - 2y\Big]\,S_\kappa T_\mu S_\lambda T_\nu \\
& + y\Big[(1-2y-y^2)Q_W^e + 1 - 2y\Big]\,S_\kappa U_\mu S_\lambda U_\nu \\
& + \Big[(2-y+y^2)Q_W^e + 1\Big]\,T_\kappa U_\mu T_\lambda U_\nu \\
& - s\,y\,(1-y)\,\big[y\,(2-y)\,Q_W^e + 1\big]\,\eta_{\mu\nu}\,T_\kappa T_\lambda \\
& - s\,y\,(1-y)\Big[(1-y^2)Q_W^e + 1\Big]\,\eta_{\mu\nu}\,U_\kappa U_\lambda\Big\} \\
& - \frac{e^4 (Q_W^e + 1)\,\mathrm{Re}\,(k_{\phi\phi})^{\mu\nu}}{2M_Z^2\,y\,(1-y)\sin^2 2\theta_W} \times \\
& \Big\{(Q_W^e - 1)\Big[(1-2y+2y^2)\,S_\mu S_\nu - T_\mu T_\nu - U_\mu U_\nu\Big] \\
& + s\Big[(1-y+y^2)Q_W^e + 1 + y - y^2\Big]\,\eta_{\mu\nu}\Big\}\,,
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
\delta\overline{|\mathcal{M}_L|^2} = {}& \frac{2e^4 (k_W)^{\kappa\mu\lambda\nu}}{s\,M_Z^2\,y^2(1-y)^2\cos^2\theta_W} \times \\
& \Big\{(y-1)\Big[(2-4y+y^2)\,Q_W^e - 1 + 2y\Big]\,S_\kappa T_\mu S_\lambda T_\nu \\
& + y\,\Big[(1-2y-y^2)Q_W^e - 1 + 2y\Big]\,S_\kappa U_\mu S_\lambda U_\nu \\
& + \Big[(2-y+y^2)Q_W^e - 1\Big]\,T_\kappa U_\mu T_\lambda U_\nu \\
& - s\,y\,(1-y)\,\big[y\,(2-y)\,Q_W^e - 1\big]\,\eta_{\mu\nu}\,T_\kappa T_\lambda \\
& - s\,y\,(1-y)\Big[(1-y^2)Q_W^e - 1\Big]\,\eta_{\mu\nu}\,U_\kappa U_\lambda\Big\} \\
& - \frac{e^4 (Q_W^e - 1)\,\mathrm{Re}\,(k_{\phi\phi})^{\mu\nu}}{2M_Z^2\,y\,(1-y)\sin^2 2\theta_W} \times \\
& \Big\{(Q_W^e + 1)\Big[(1-2y+2y^2)\,S_\mu S_\nu - T_\mu T_\nu - U_\mu U_\nu\Big] \\
& + s\Big[(1-y+y^2)Q_W^e - 1 - y + y^2\Big]\,\eta_{\mu\nu}\Big\}\,.
\end{aligned} \tag{30}$$

Here, $s = S^2 = (k+p)^2$ is the usual Mandelstam variable that provides a measure for the center-of-mass energy of the system. Following Ref. [30], we have set $y = -s^{-1}T^2 = -s^{-1}(k'-k)^2$, which governs the scattering angle. The above expressions are correct to order M_Z^{-2}, and the ultrarelativistic approximation of dropping all explicitly appearing m has been implemented. Equations (29) and (30) represent the main theoretical result of this work. As per our above discussion, the last lines of both Eqs. (29) and (30) contain the Lorentz-invariant trace part of $k_{\phi\phi}$ and should be omitted in phenomenological applications.

With the results (29) and (30) at hand, essentially all Lorentz-violating observables for typical electron–electron scattering measurements can now be determined in a straightforward way. For example, the conventional relation between the squared matrix element and the cross section [31] remains valid because the external electron legs have been taken as the usual Lorentz-invariant ones:

$$d\sigma_h = \frac{1}{(4\pi)^2}\,\frac{\overline{|\mathcal{M}_h|^2}}{|k^0\vec{p} - p^0\vec{k}|}\,\frac{d^3k'}{2k'^0}\,\frac{d^3p'}{2p'^0}\,\delta^{(4)}(k+p-k'-p')\,. \tag{31}$$

Here, we have abbreviated the helicity $h \in \{R, L\}$, and we have assumed the incoming 3-momenta $\vec{k}$ and $\vec{p}$ to be directed along the same line.

An observable that is particularly interesting for both theoretical and experimental purposes is the asymmetry

$$A \equiv \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = \frac{\overline{|\mathcal{M}_R|^2} - \overline{|\mathcal{M}_L|^2}}{\overline{|\mathcal{M}_R|^2} + \overline{|\mathcal{M}_L|^2}} . \tag{32}$$

In the present context, this asymmetry consists of two contributions $A = A_0 + \delta A$, where A_0 is the usual Lorentz-invariant piece given by [30]

$$A_0 = \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{y\,(1-y)}{(y^2-y+1)^2}\, s\, Q_W^e . \tag{33}$$

Our results (29) and (30) yield the first-order Lorentz-violating piece δA:

$$\begin{aligned}\delta A = {} & \frac{G_F}{\sqrt{2}\pi\alpha} \frac{(k_W)^{\kappa\mu\lambda\nu}}{(y^2-y+1)^2} \frac{\sin^2\theta_W}{s}\Big[(1-2y)\,y\,S_\kappa U_\mu S_\lambda U_\nu \\ & -(2y^2-3y+1)\,S_\kappa T_\mu S_\lambda T_\nu + T_\kappa U_\mu T_\lambda U_\nu \\ & - s\,(1-y)\,y\,\eta_{\mu\nu}\,(T_\kappa T_\lambda + U_\kappa U_\lambda)\Big] \\ & - \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{y\,(1-y)}{(y^2-y+1)^2}\, s\, Q_W^e \,\frac{\mathrm{Re}\,(k_{\phi\phi})^\mu{}_\mu}{4} .\end{aligned} \tag{34}$$

As before, we have worked to order M_Z^{-2} and in the ultrarelativistic limit by neglecting explicit occurrences of m. In the above expression, we have also eliminated M_Z in favor of the fine-structure constant α and Fermi constant G_F.

We mention that we again kept the trace part of $k_{\phi\phi}$ for the purely theoretical purpose of studying the Lorentz-symmetric limit (25). In this limit, $\mathrm{Re}\,(k_{\phi\phi})^\mu{}_\mu \to 4$, so that the last term in Eq. (34) precisely offsets the Lorentz-symmetric contribution A_0, as expected from the aforementioned cancellation of Z-exchange diagrams. As discussed above, $\mathrm{Re}\,(k_{\phi\phi})^\mu{}_\mu$ is largely irrelevant for Lorentz-violation searches because it is a scalar. In any case, this term is eliminated by the trace-free condition (3). Since there are no other $k_{\phi\phi}$ terms in Eq. (34), the asymmetry (32) is insensitive to this mSME coefficient at leading order. However, other potential observables, such as

$$B_h \equiv \frac{d\sigma_h}{d\sigma_R + d\sigma_L} = \frac{\overline{|\mathcal{M}_h|^2}}{\overline{|\mathcal{M}_R|^2} + \overline{|\mathcal{M}_L|^2}} , \tag{35}$$

where again $h \in \{R, L\}$, do contain nontrivial $k_{\phi\phi}$ contributions and are perhaps better suited for measurements of this particular mSME coefficient.

The above expressions for the asymmetry A hold at tree level. In general, these results are modified by radiative corrections. The conventional Lorentz-invariant asymmetry A_0 is constructed such that parity-conserving effects cancel, which makes A_0 much smaller than the corresponding generic tree-level effects of this type, such as B_h. Numerically, A_0 turns out to be of the same order as radiative corrections predominantly arising from γ–Z mixing diagrams and the anapole-moment diagram [32]. This comparatively large radiative effect is therefore of paramount importance for the interpretation of any measurement of the conventional Lorentz-invariant asymmetry A_0. An analogous tree-level cancellation is absent from the Lorentz-violating contribution δA, as can be inferred from Eqs. (29) and (30). It is therefore expected that Lorentz-breaking loop effects represent only small corrections to our tree-level δA and can be disregarded.

4. Experimental tests

With the theoretical prediction (34) for the effects of a Lorentz-violating Z boson on electron–electron scattering, a comparison to experimental data, and thus measurements of k_W become possible. For a comprehensive experimental investigation of k_W, all of its 19 independent components should be considered. However, the purpose of this section is merely to outline a particular experimental signature of k_W in polarized Møller scattering and to estimate the potential sensitivity of such measurements to k_W. It is therefore sufficient to employ a simplified form of k_W. Our choice

$$\begin{aligned}(k_W)^{\mu\nu\rho\sigma} = {} & \tfrac{1}{2}\Big[\eta^{\mu\rho}\zeta^{\{\nu}\xi^{\sigma\}} - \eta^{\mu\sigma}\zeta^{\{\nu}\xi^{\rho\}} \\ & + \eta^{\nu\sigma}\zeta^{\{\mu}\xi^{\rho\}} - \eta^{\nu\rho}\zeta^{\{\mu}\xi^{\sigma\}}\Big]\end{aligned} \tag{36}$$

is parametrized by

$$\zeta^{\{\mu}\xi^{\nu\}} \equiv \tfrac{1}{2}(\zeta^\mu\xi^\nu + \zeta^\nu\xi^\mu) , \quad \zeta^\mu = (1, \vec{0}) , \quad \xi^\mu = (0, \vec{\xi}) , \tag{37}$$

which essentially reduces the number of Lorentz-violating coefficients to the three components of $\vec{\xi}$. We remark in passing that the analogous three coefficients in the photon sector obey the weakest experimental bounds and have therefore also been studied separately as a special case [33,16]. With this simplification, the asymmetry correction (34) takes the form

$$\begin{aligned}\delta A = {} & \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{\zeta^\mu\xi^\nu + \zeta^\nu\xi^\mu}{(y^2-y+1)^2} \sin^2\theta_W \Big[k'_\mu k'_\nu + y\, p_\mu p_\nu \\ & + (1-y)\, k_\mu k_\nu - 2(1-y)\, k_\mu k'_\nu - 2y\, p_\mu k'_\nu\Big] ,\end{aligned} \tag{38}$$

where we have reinstated the original momentum variables with p'_μ eliminated by energy–momentum conservation, and where we have taken the ultrarelativistic limit $m \to 0$, as before.

In the flat-spacetime SME, the Lorentz-violating k_W, and thus ζ and ξ, are taken as x^μ independent, so that their components in cartesian inertial coordinates are constant as well. However, being components of a tensor, the individual ζ^μ and ξ^μ do depend on the coordinate system, in which ζ and ξ are expressed. This shows that a meaningful comparison of ξ^μ measurements between different experiments requires a common standardized coordinate system. Sun-centered celestial equatorial coordinates are usually selected as this standard frame [11], and these coordinates are also employed in what follows. In particular, the components of ζ and ξ specified in Eq. (37) are understood to refer to this Sun-centered frame.

The electron momenta k_μ, p_μ, and k'_μ are usually measured in an Earth-based laboratory frame. As result of the rotation and orbital motion of the Earth, the transformation between the Sun-centered coordinates and the laboratory frame is in general time dependent. This transformation is dominated by the change of orientation (i.e., the rotation transformation) arising from the Earth spinning about its axis. This spinning motion also leads to a boost transformation due to the rotational velocity of laboratories located off the Earth's axis. A second, larger boost transformation results from the Earth's orbital motion around the Sun. In some circumstances, these two boost effects need to be taken into account. Nonetheless, both boosts involve nonrelativistic speeds, so that the corresponding effects are subdominant and are therefore neglected here. We also mention that strictly speaking an Earth-based laboratory frame fails to be inertial. However, for our present purposes the rotation is adiabatically slow, so that we can disregard non-inertial effects.

To set up the explicit transformation between laboratory and Sun-centered coordinates, the laboratory frame needs to be specified. A typical choice for a location at colatitude χ has the xy-plane parallel to the local surface of the Earth, such that the x-axis is pointing South and the y-axis East. A right-handed system then has the z-axis pointing vertically upward. The corresponding rotation matrix is [13]

$$R^{Jj}(t) = \begin{pmatrix} \cos\chi\cos\Omega_\oplus t & -\sin\Omega_\oplus t & \sin\chi\cos\Omega_\oplus t \\ \cos\chi\sin\Omega_\oplus t & \cos\Omega_\oplus t & \sin\chi\sin\Omega_\oplus t \\ -\sin\chi & 0 & \cos\chi \end{pmatrix}. \quad (39)$$

Here, $\Omega_\oplus = 2\pi/(23\text{ h }56\text{ min})$ denotes the Earth's sidereal angular frequency, $J = X, Y, Z$ are the spatial Sun-frame components, and $j = x, y, z$ the spatial laboratory-frame components. With this matrix the components of the laboratory momenta for the beam electrons, the target electrons, and the collected Møller electrons can be transformed into the Sun-centered frame, which yields explicit expressions for $k^\mu = (E_k, \vec{k})$, $p^\mu = (m, \vec{0})$, and $k'^\mu = (E_{k'}, \vec{k}')$, respectively. Note in particular the time dependence of $R^{Jj}(t)$: 3-momenta perceived as constant in the laboratory will generally change direction when viewed in the Sun-centered frame, i.e., $\vec{k} = \vec{k}(t)$ and $\vec{k}' = \vec{k}'(t)$.

With these considerations, the Lorentz-violating asymmetry correction (38) becomes

$$\delta A(t) = \frac{G_F}{\sqrt{2}\pi\alpha} \frac{\sin^2\theta_W}{(y^2 - y + 1)^2} \left[c_1 \vec{k}(t) + c_2 \vec{k}'(t)\right] \cdot \vec{\xi}\,, \quad (40)$$

where the coefficients c_1 and c_2 are given by

$$c_1 = (1 - y)(E_k - E_{k'})\,, \quad (41)$$

$$c_2 = E_{k'} - (1 - y)E_k - y\,m\,. \quad (42)$$

The result (40) illustrates two general features of Lorentz violation characterized by the anisotropic SME coefficient $\vec{\xi}$. First, the rotation of the laboratory with respect to $\vec{\xi}$ will typically generate a time-dependent asymmetry correction $\delta A = \delta A(t)$. Second, the usual azimuthal symmetry of the experimental set-up may not necessarily translate into azimuthal symmetry in the scattering process: for $c_2 \neq 0$, the azimuth dependence of the scattered-electron momentum $\vec{k}'$ can contribute to δA due to the presence of $\vec{\xi}$.

Our next step is to express c_1 and c_2 in terms of the beam energy E_k and the previously introduced variable y, which essentially provides a measure for the scattering angle. These quantities are unaffected by the transformation (39), so we may proceed in the laboratory frame. With the help of

$$E_{k'} = \left(1 - \frac{E_k + m}{E_k} y\right) E_k \simeq (1 - y)E_k\,, \quad (43)$$

where the second step applies in the ultrarelativistic limit, we find

$$c_1 = y\,(1 - y)\,E_k\,, \quad (44)$$

$$c_2 = -y\,m\,. \quad (45)$$

It is apparent that $c_2 \simeq \mathcal{O}(m)$ can be neglected in comparison to $c_1 \simeq \mathcal{O}(E_k)$. We remark that for this reason the aforementioned Lorentz-violating azimuthal dependence, although present in general, fails to generate leading-order effects in the specific situation at hand. For an incoming-beam direction that is parallel to the local surface of the Earth and points in a direction α East of South, these considerations lead to

$$\begin{aligned} \delta A(t) &= \frac{G_F}{\sqrt{2}\pi\alpha} \frac{E_k\,y\,(1 - y)\sin^2\theta_W}{(y^2 - y + 1)^2} \vec{k}(t) \cdot \vec{\xi} \\ &= \frac{G_F}{\sqrt{2}\pi\alpha} \frac{E_k^2\,y\,(1 - y)\sin^2\theta_W}{(y^2 - y + 1)^2} \times \\ &\quad \left[\sqrt{\xi_X^2 + \xi_Y^2}\sqrt{1 - \cos^2\alpha\sin^2\chi}\,\cos\Omega_\oplus t + c_0\right]. \end{aligned} \quad (46)$$

We have again implemented the ultrarelativistic approximation. In the second step, we have absorbed an irrelevant phase into the definition of the time t. The constant $c_0 \equiv \xi_Z \cos\alpha \sin\chi$ is of less interest in the present context; it represents a constant shift in the asymmetry, which is difficult to disentangle from Lorentz-symmetric effects. The key result is the square-root term, which predicts sidereal variations of the asymmetry δA with an amplitude depending on the Lorentz-violating coefficients ξ_X and ξ_Y.

The E158 experiment at the Stanford Linear Accelerator Center (SLAC), located at a colatitude of $\chi = 53°$, has performed a measurement of this type [28]. The orientation of SLAC's End Station A, which determined the incoming-beam direction, points $\alpha = 123°$ East of South. Beam electrons with either $E_k = 45.0$ GeV or $E_k = 48.3$ GeV were incident on atomic electrons of a stationary liquid-hydrogen target, where the two beam energies correspond to opposite longitudinal polarizations of the incoming electrons. The resulting scattered Møller electrons were counted in the range $\frac{1}{2} < y < \frac{3}{4}$. For this experimental input, the asymmetry A was measured with a statistical uncertainty of 1.4×10^{-8}.

This measurement permits an estimate of the sensitivity of the E158 experiment to the Lorentz-violating coefficients ξ_X and ξ_Y. An integration of the scattering data on time scales of a day or longer would have washed out the predicted sidereal effects. However, if large enough, these variations would have likely been interpreted as part of the statistical uncertainty. We therefore presume that the amplitude

$$a_\oplus \equiv \frac{\sqrt{1 - \cos^2\alpha\sin^2\chi}\,E_k^2\,y\,(1 - y)}{2\,M_Z^2\,(y^2 - y + 1)^2\cos^2\theta_W}\sqrt{\xi_X^2 + \xi_Y^2} \quad (47)$$

of the sidereal variations must be in the order of or less than the statistical uncertainty of the measurement $a_\oplus \lesssim 1.4 \times 10^{-8}$. This gives

$$\sqrt{\xi_X^2 + \xi_Y^2} \lesssim 3.4 \times 10^{-7}\,. \quad (48)$$

To arrive at this result, we have made two assumptions. First, we have conservatively taken the smaller of the above two beam energies E_k. Second, the scattered Møller electrons were collected over the aforementioned range $\frac{1}{2} < y < \frac{3}{4}$. As this corresponds only to a $\pm 22\%$ variation of $a_\oplus$ about its average value, we have used our differential cross-section result (47) with the fixed, conservative value of $y = \frac{3}{4}$, at which $a_\oplus$ is smallest [28].

We are only aware of a single previous study that has attempted to estimate the size of the components $(k_W)_\alpha{}^{\mu\alpha\nu}$, which are related to the present $\vec{\xi}$ coefficient by $2(k_W)_\alpha{}^{J\alpha 0} = \xi^J$. That study was based on astrophysical observations and involved the W boson. More specifically, certain values of the components $(k_W)_\alpha{}^{\mu\alpha\nu}$ may lead to photon decay $\gamma \to W^+ + W^-$ at high energies. The observed survival of ultrahigh-energy photons from cosmological sources then leads to a one-sided estimate at the 10^{-5} level [27]. The present result, based on laboratory physics involving the Z boson, represents an improvement by two orders of magnitude with different methods in a different system. For this reason, dedicated analyses in the context of previous and future polarized-electron scattering experiments taking the full data sample and all 19 components of k_W into consideration would be competitive and of definite interest.

5. Summary and outlook

This work has considered power-counting renormalizable Lorentz-violating contributions to Z-boson physics. We have determined the dominant Feynman rules that govern these effects and employed them to calculate the tree-level Lorentz-violating corrections to electron–electron scattering. These general results, contained in Eqs. (29) and (30), form the theoretical basis for experimental investigations of the leading-order Lorentz-breaking effects arising from the Z boson in polarized Møller scattering of electrons.

We have applied our results in the case of a simplified k_W, given by Eqs. (36) and (37), which has the number of independent coefficients reduced from 19 to the three components of $\vec{\xi}$. This sample case has shown that the differential cross-section asymmetry can in principle depend on the azimuth, even if the system is set up with azimuthal symmetry, as expected when Lorentz invariance is broken. Moreover, in a terrestrial laboratory, the rotation of the Earth will typically lead to sidereal oscillations in the asymmetry.

For the specific experimental conditions of the E158 experiment at SLAC, the published measurement yields an estimated constraint at the level of 10^{-7} on the components $(k_W)_\alpha{}^{J\alpha 0}$ for $J = X, Y$. This result improves existing astrophysical estimates by two orders of magnitude and encourages a dedicated analysis of the full E158 data that includes all 19 components of k_W. The proposed MOLLER experiment at Jefferson Lab offers the potential for an increased sensitivity of 0.7×10^{-9} to the asymmetry, albeit at the lower electron energy of 12 GeV. With this input, the MOLLER experiment may be able to provide a further overall improvement by a factor of about 1.4. Moreover, the different beam directions and colatitudes for E158 and MOLLER are likely to provide experimental access to different component combinations of the full $(k_W)^{\mu\nu\rho\sigma}$.

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