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Higher-derivative superparticle in AdS_3 space

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Employing the coset approach we construct component actions for a superparticle moving in AdS_3 with $N = (2, 0)$, $D = 3$ supersymmetry partially broken to $N = 2$, $d = 1$. These actions may contain higher time-derivative terms, which are chosen to possess the same (super)symmetries as the free superparticle. In terms of the nonlinear-realization superfields, the component actions always take a simpler form when written in terms of covariant Cartan forms. We also consider in detail the reduction to the nonrelativistic case and construct the corresponding action of a Newton-Hooke superparticle and its higher-derivative generalizations. The structure of these higher time-derivative generalizations is completely fixed by invariance under the supersymmetric Newton-Hooke algebra extended by two central charges.

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I. INTRODUCTION

The standard action of any particle moving in a flat spacetime is invariant under the target-space Poincaré group realized in a spontaneously broken manner. The spontaneously broken translations, orthogonal to the world line of particle, and the Lorentz boosts rotating these translations into world-line translations, give rise to Goldstone bosons, which appear in the particle actions. Usually, not all of these Goldstone bosons are independent of one another, and so there are additional constraints reducing the number of independent fields to a set describing the physical degrees of freedom. In the supersymmetric case the situation is more complicated, because in extended supersymmetry additional covariant constraints selecting irreducible supermultiplets have to be found. These tasks can be algorithmically solved by using the nonlinear-realization (or coset) approach [1], suitably modified for supersymmetric spacetime symmetries [2]. In this approach, the corresponding constraints are conditions on the Cartan forms (or on their θ components). Nevertheless, the coset approach fails to reproduce the superspace actions, because the superparticle Lagrangian is only *quasi-invariant* with respect to the super-Poincaré group and, therefore, it cannot be constructed in terms of Cartan forms. However, by passing to component actions and focusing on the broken supersymmetry only, one may easily construct an ansatz for the invariant action in terms of $\theta = 0$ projections of the Cartan forms. This program has been performed in our paper [3] for a superparticle moving in flat $D = 1 + 2$ spacetime with $N = 4$ supersymmetry partially broken to $N = 2$. Moreover, the possible higher time-derivative terms, possessing the same symmetry as the free superparticle, can also be constructed in terms of the Cartan forms. The role of unbroken supersymmetry is just to fix some free coefficients in the component action.

In the present paper, we investigate the application of the coset approach to a superparticle moving on AdS_3. This leads to the following complications:

(i) One has to choose a suitable parametrization of the coset space (i.e. of the set of the physical bosonic fields), which may, even in the bosonic case, drastically simplify the resulting actions.

(ii) The superspace constraints have to be properly covariantized, selecting irreducible supermultiplets which are rather distinct from the flat-spacetime case.

(iii) The fermionic components should be defined such as to render the resulting Lagrangians readable. All such choices are related by nonlinear invertible fields redefinitions [4], but an improper choice may result in highly complicated Lagrangians.

In the following we will solve all these tasks for a superparticle moving on AdS_3, with $N = (2, 0)$, $D = 3$ supersymmetry partially broken to $N = 2$, $d = 1$. We will construct the corresponding component actions in terms of the $\theta = 0$ projections of the Cartan forms and prove their invariance under the $N = (2, 0)$ AdS_3 algebra. The higher time-derivative terms share this symmetry, and the one with minimal higher derivatives (the anyonic action) can also be written in terms of Cartan forms. The analysis will then be extended by the nonrelativistic limit, which radically simplifies everything: The AdS_3 superparticle reduces to the Newton-Hooke superparticle [5,6], while the higher-derivative terms acquire quite a compact form.

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II. $N = (2,0)$ AdS$_3$ ALGEBRA AND FIXING THE BASIS

The action we are going to construct corresponds to the partial spontaneous breaking of $N = (2,0)$ AdS$_3$ supersymmetry. To start with, let us define the $N = (2,0)$ AdS$_3$ superalgebra in a standard way as (see e.g. [7])

$$\begin{aligned}
&[M_{ab}, M_{cd}] = \epsilon_{ac}M_{bd} + \epsilon_{bd}M_{ac} + \epsilon_{ad}M_{bc} + \epsilon_{bc}M_{ad}\\
&\qquad \equiv (M)_{ab,cd},\\
&[M_{ab}, P_{cd}] = (P)_{ab,cd}, \qquad [P_{ab}, P_{cd}] = -\frac{m^2}{16}(M)_{ab,cd},\\
&[M_{ab}, Q_c] = \epsilon_{ac}Q_b + \epsilon_{bc}Q_a \equiv (Q)_{ab,c},\\
&[M_{ab}, \bar{Q}_c] = (\bar{Q})_{ab,c},\\
&[P_{ab}, Q_c] = \mathrm{i}\frac{m}{4}(Q)_{ab,c}, \qquad [P_{ab}, \bar{Q}_c] = \mathrm{i}\frac{m}{4}(\bar{Q})_{ab,c},\\
&[J, Q_a] = Q_a, \qquad [J, \bar{Q}_a] = -\bar{Q}_a,\\
&\{Q_a, \bar{Q}_b\} = 2P_{ab} + \mathrm{i}\frac{m}{2}M_{ab} + \mathrm{i}m\epsilon_{ab}J.
\end{aligned} \tag{2.1}$$

Here, the generators $M_{ab} = M_{ba}$, $P_{ab} = P_{ba}$, $a, b = 1, 2$ form the bosonic AdS$_3$ algebra while the fermionic generators Q_a, $\bar{Q}_a$ together with the $U(1)$ generator J extend it to the $N = (2,0)$ AdS$_3$ one.

Note that in our basis these generators obey the following conjugation rules:

$$\begin{aligned}
&(M_{ab})^\dagger = -M_{ab}, \qquad (P_{ab})^\dagger = P_{ab},\\
&(J)^\dagger = J, \qquad (Q_a)^\dagger = \bar{Q}_a.
\end{aligned} \tag{2.2}$$

To have close relations with the previously considered case of superparticle moving in three-dimensional Poincaré spacetime [3], one has to choose the generators spanning $N = 2$, $d = 1$ super-Poincaré algebra to which the AdS$_3$ supersymmetry will be broken. One may easily check that if we define the generators $\{P, Q, \bar{Q}\}$ as

$$\begin{aligned}
&Q = Q_1 + \mathrm{i}Q_2, \qquad \bar{Q} = \bar{Q}_1 - \mathrm{i}\bar{Q}_2,\\
&P = P_{11} + P_{22} + \mathrm{i}\frac{m}{4}(M_{11} + M_{22}) + mJ,
\end{aligned} \tag{2.3}$$

then they will form the $N = 2$, $d = 1$ super-Poincaré algebra

$$\begin{aligned}
&\{Q, \bar{Q}\} = 2P,\\
&\{Q, Q\} = \{\bar{Q}, \bar{Q}\} = [P, Q] = [P, \bar{Q}] = 0.
\end{aligned} \tag{2.4}$$

The remaining bosonic generators, having the proper form in the flat limit, may be defined as follows:

$$\begin{aligned}
&P, Z = P_{11} - P_{22} - 2\mathrm{i}P_{12} + \mathrm{i}\frac{m}{4}(M_{11} - M_{22} - 2\mathrm{i}M_{12}),\\
&\bar{Z} = (Z)^\dagger,\\
&J_3 = \frac{\mathrm{i}}{4}(M_{11} + M_{22}), \qquad T = \frac{\mathrm{i}}{4}(M_{11} - M_{22} - 2\mathrm{i}M_{12}),\\
&\bar{T} = (T)^\dagger.
\end{aligned} \tag{2.5}$$

Thus, the bosonic part of the algebra (2.1), i.e. the algebra $so(2,2) \times u(1)$, acquires the form

$$\begin{aligned}
&[J_3, T] = T, \qquad [J_3, \bar{T}] = -\bar{T}, \qquad [T, \bar{T}] = -2J_3,\\
&[P, Z] = 2mZ, \qquad [P, \bar{Z}] = -2m\bar{Z},\\
&[Z, \bar{Z}] = -4mP + 4m^2J,\\
&[J_3, Z] = Z, \qquad [J_3, \bar{Z}] = -\bar{Z},\\
&[T, P] = -Z, \qquad [\bar{T}, P] = \bar{Z}, \qquad [T, \bar{Z}] = -2P + 2mJ,\\
&[\bar{T}, Z] = 2P - 2mJ.
\end{aligned} \tag{2.6}$$

Clearly, the relations (2.6) are maximally similar to the $D = 3$ Poincaré ones we used in [3] and go to them in the limit $m = 0$ (with decoupled generator J, of course).

As concerning the fermionic part of $N = (2,0)$ AdS$_3$ superalgebra (2.1), it is natural to define the generators of broken supersymmetry as

$$S = \bar{Q}_1 + \mathrm{i}\bar{Q}_2, \qquad \bar{S} = Q_1 - \mathrm{i}Q_2. \tag{2.7}$$

Then commutation relations, which include spinor generators, read

$$\begin{aligned}
&\{Q, \bar{Q}\} = 2P, \qquad \{S, \bar{S}\} = 2P - 4mJ,\\
&\{Q, S\} = 2\bar{Z}, \qquad \{\bar{Q}, \bar{S}\} = 2Z,\\
&[Z, Q] = -2m\bar{S}, \qquad [\bar{Z}, \bar{Q}] = 2mS,\\
&[Z, S] = -2m\bar{Q}, \qquad [\bar{Z}, \bar{S}] = 2mQ,\\
&[P, S] = -2mS, \qquad [P, \bar{S}] = 2m\bar{S},\\
&[T, Q] = -\bar{S}, \qquad [\bar{T}, \bar{Q}] = S,\\
&[T, S] = -\bar{Q}, \qquad [\bar{T}, \bar{S}] = Q,\\
&[J_3, Q] = -\frac{1}{2}Q, \qquad [J_3, \bar{Q}] = \frac{1}{2}\bar{Q},\\
&[J_3, S] = -\frac{1}{2}S, \qquad [J_3, \bar{S}] = \frac{1}{2}\bar{S},\\
&[J, Q] = Q, \qquad [J, \bar{Q}] = -\bar{Q}, \qquad [J, S] = -S,\\
&[J, \bar{S}] = \bar{S}.
\end{aligned} \tag{2.8}$$

III. CARTAN FORMS AND TRANSFORMATION PROPERTIES

In the coset approach [1,2], the spontaneous breakdown of S, $\bar{S}$ supersymmetry and Z, $\bar{Z}$ translations is reflected in the structure of the coset element

$$g = e^{\mathrm{i}tP} e^{\theta Q + \bar{\theta}\bar{Q}} e^{\boldsymbol{\psi} S + \bar{\boldsymbol{\psi}}\bar{S}} e^{\mathrm{i}(\mathbf{U}Z + \overline{\mathbf{U}Z})} e^{\mathrm{i}(\mathbf{\Lambda} T + \overline{\mathbf{\Lambda} T})}. \tag{3.1}$$

The $N = 2$ superfields $\mathbf{U}(t,\theta,\bar{\theta})$, $\boldsymbol{\psi}(t,\theta,\bar{\theta})$ and $\mathbf{\Lambda}(t,\theta,\bar{\theta})$ are Goldstone superfields accompanying the $N = (2,0)$ AdS_3 symmetry to $N = 2$, $d = 1$ super-Poincaré $\times U(1)^2$ breaking.[1] The transformation properties of the coordinates and the superfields are induced by the left multiplication of the coset element (3.1)

$$g_0 g = g' h, \qquad h \sim e^{\mathrm{i}\alpha J} e^{\mathrm{i}\beta J_3}.$$

The most important transformations read

(i) Unbroken supersymmetry (SUSY) $(g_0 = e^{\epsilon Q + \bar{\epsilon}\bar{Q}})$

$$\delta\theta = \epsilon, \qquad \delta\bar{\theta} = \bar{\epsilon}, \qquad \delta t = \mathrm{i}(\epsilon\bar{\theta} + \bar{\epsilon}\theta). \tag{3.2}$$

(ii) Broken SUSY $(g_0 = e^{\varepsilon S + \bar{\varepsilon}\bar{S}})$

$$\begin{aligned} &\delta_S\theta = 4m\tilde{\varepsilon}\bar{\boldsymbol{\psi}}\theta, \qquad \delta_S t = \mathrm{i}(\tilde{\varepsilon}\bar{\boldsymbol{\psi}} + \bar{\tilde{\varepsilon}}\boldsymbol{\psi})(1 - 6m\theta\bar{\theta}) - 4m(\tilde{\varepsilon}\theta\mathbf{u} - \bar{\tilde{\varepsilon}}\bar{\theta}\bar{\mathbf{u}})(1 - 2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}), \\ &\delta_S\boldsymbol{\psi} = \tilde{\varepsilon}(1 - 2m\theta\bar{\theta})(1 + 2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}) - 8\mathrm{i}m^2\boldsymbol{\psi}(\tilde{\varepsilon}\theta\mathbf{u} - \bar{\tilde{\varepsilon}}\bar{\theta}\bar{\mathbf{u}}), \qquad \delta_S\mathbf{u} = 2\mathrm{i}\bar{\tilde{\varepsilon}}\bar{\theta}(1 - 2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}})(1 - 4m^2\mathbf{u}\bar{\mathbf{u}}), \end{aligned} \tag{3.3}$$

where $\tilde{\varepsilon} = e^{2\mathrm{i}mt}\varepsilon$.

(iii) Z, $\bar{Z}$ transformations $(g_0 = e^{\mathrm{i}(bZ + \bar{b}\bar{Z})})$

$$\begin{aligned} &\delta_Z\theta = 2\mathrm{i}m\bar{\tilde{b}}\boldsymbol{\psi}(1 + 2m\theta\bar{\theta}), \qquad \delta_Z t = 4m(\tilde{b}\theta\boldsymbol{\psi} - \bar{\tilde{b}}\bar{\theta}\bar{\boldsymbol{\psi}}) - 2\mathrm{i}m(\tilde{b}\bar{\mathbf{u}} - \bar{\tilde{b}}\mathbf{u})(1 + 2m\theta\bar{\theta})(1 - 2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}), \\ &\delta_Z\mathbf{u} = \tilde{b}(1 - 4m^2\mathbf{u}\bar{\mathbf{u}})(1 + 2m\theta\bar{\theta})(1 - 2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}), \\ &\delta_Z\boldsymbol{\psi} = 2\mathrm{i}m\bar{\tilde{b}}\bar{\theta}(1 + 2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}) + 4m^2\boldsymbol{\psi}(\tilde{b}\bar{\mathbf{u}} - \bar{\tilde{b}}\mathbf{u})(1 + 2m\theta\bar{\theta}), \end{aligned} \tag{3.4}$$

where $\tilde{b} = e^{-2\mathrm{i}mt}b$.

Here, the coordinates of stereographic projections were introduced:

$$\mathbf{u} = \frac{\tanh(2m\sqrt{\mathbf{U}\bar{\mathbf{U}}})}{2m\sqrt{\mathbf{U}\bar{\mathbf{U}}}}\mathbf{U}, \qquad \lambda = \frac{\tanh(\sqrt{\mathbf{\Lambda}\bar{\mathbf{\Lambda}}})}{\sqrt{\mathbf{\Lambda}\bar{\mathbf{\Lambda}}}}\mathbf{\Lambda}. \tag{3.5}$$

The local geometric properties of the system are specified by the left-invariant Cartan forms

$$g^{-1}dg = \mathrm{i}\Omega_P P + \mathrm{i}\Omega_Z Z + \mathrm{i}\bar{\Omega}_Z\bar{Z} + \mathrm{i}\Omega_T T + \mathrm{i}\bar{\Omega}_T\bar{T} + \mathrm{i}\Omega_3 J_3 + \mathrm{i}\Omega_J J + \Omega_Q Q + \bar{\Omega}_Q\bar{Q} + \Omega_S S + \bar{\Omega}_S\bar{S}, \tag{3.6}$$

which look much more complicated than in the flat spacetime ($m \to 0$) [3]:

$$\begin{aligned} &\Omega_P = \frac{1}{1 - \lambda\bar{\lambda}}[(1 + \lambda\bar{\lambda})\hat{\Omega}_P - 2\mathrm{i}(\bar{\lambda}\hat{\Omega}_Z - \lambda\hat{\bar{\Omega}}_Z)], \qquad \Omega_Q = \frac{\hat{\Omega}_Q - \mathrm{i}\bar{\lambda}\hat{\bar{\Omega}}_S}{\sqrt{1 - \lambda\bar{\lambda}}}, \\ &\Omega_Z = \frac{1}{1 - \lambda\bar{\lambda}}[\hat{\Omega}_Z - \lambda^2\hat{\bar{\Omega}}_Z + \mathrm{i}\lambda\hat{\Omega}_P], \qquad \Omega_J = \hat{\Omega}_J - \frac{1}{1 - \lambda\bar{\lambda}}[2m\lambda\bar{\lambda}\hat{\Omega}_P - 2\mathrm{i}m(\bar{\lambda}\hat{\Omega}_Z - \lambda\hat{\bar{\Omega}}_Z)], \\ &\Omega_T = \frac{d\lambda}{1 - \lambda\bar{\lambda}}, \qquad \Omega_3 = \mathrm{i}\frac{\lambda d\bar{\lambda} - d\lambda\bar{\lambda}}{1 - \lambda\bar{\lambda}}, \qquad \Omega_S = \frac{\hat{\Omega}_S - \mathrm{i}\bar{\lambda}\hat{\bar{\Omega}}_Q}{\sqrt{1 - \lambda\bar{\lambda}}}, \end{aligned} \tag{3.7}$$

where the hatted forms read

[1] These two additional $U(1)$ groups are formed by the generators J and J_3.

$$
\begin{aligned}
\hat{\Omega}_P &= \frac{(1+4m^2\mathbf{u}\bar{\mathbf{u}})\hat{\Delta}t + 2\mathrm{i}m(\mathbf{u}d\bar{\mathbf{u}} - \bar{\mathbf{u}}d\mathbf{u}) + 8m(\mathbf{u}\boldsymbol{\psi}d\theta - \bar{\mathbf{u}}\bar{\boldsymbol{\psi}}\,d\bar{\theta})}{1-4m^2\mathbf{u}\bar{\mathbf{u}}},\\
\hat{\Omega}_Z &= \frac{d\mathbf{u} + 2\mathrm{i}m\mathbf{u}\hat{\Delta}t - 2\mathrm{i}(\bar{\boldsymbol{\psi}}d\bar{\theta} - 4m^2\mathbf{u}^2\boldsymbol{\psi}d\theta)}{1-4m^2\mathbf{u}\bar{\mathbf{u}}},\\
\hat{\Omega}_J &= -\frac{8m^3\mathbf{u}\bar{\mathbf{u}}}{1-4m^2\mathbf{u}\bar{\mathbf{u}}}\hat{\Delta}t + 2\mathrm{i}m^2\frac{\bar{\mathbf{u}}d\mathbf{u} - d\bar{\mathbf{u}}\mathbf{u}}{1-4m^2\mathbf{u}\bar{\mathbf{u}}} - 8\frac{m^2(\mathbf{u}\boldsymbol{\psi}d\theta - \bar{\mathbf{u}}\bar{\boldsymbol{\psi}}\,d\bar{\theta})}{1-4m^2\mathbf{u}\bar{\mathbf{u}}}\\
&\quad - 8m^2[dt - \mathrm{i}(\theta d\bar{\theta} + \bar{\theta}d\theta)]\boldsymbol{\psi}\bar{\boldsymbol{\psi}} + 2\mathrm{i}m(\boldsymbol{\psi}d\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{\psi}}d\boldsymbol{\psi})\\
&= 2m[dt - \mathrm{i}(\theta d\bar{\theta} + \bar{\theta}d\theta)] - m\hat{\Delta}t - m\hat{\Omega}_P,\\
\hat{\Omega}_Q &= \frac{\Delta\theta - 2\mathrm{i}m\bar{\mathbf{u}}\Delta\bar{\boldsymbol{\psi}}}{\sqrt{1-4m^2\mathbf{u}\bar{\mathbf{u}}}}, \qquad \hat{\Omega}_S = \frac{\Delta\boldsymbol{\psi} - 2\mathrm{i}m\bar{\mathbf{u}}\Delta\bar{\theta}}{\sqrt{1-4m^2\mathbf{u}\bar{\mathbf{u}}}}.
\end{aligned}
\tag{3.8}
$$

Here,

$$
\begin{aligned}
\hat{\Delta}t &= (1+4m\boldsymbol{\psi}\bar{\boldsymbol{\psi}})[dt - \mathrm{i}(\theta d\bar{\theta} + \bar{\theta}d\theta + \boldsymbol{\psi}d\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{\psi}}d\boldsymbol{\psi})] \equiv (1+4m\boldsymbol{\psi}\bar{\boldsymbol{\psi}})\Delta t,\\
\Delta\theta &= (1-2m\boldsymbol{\psi}\bar{\boldsymbol{\psi}})d\theta, \qquad \Delta\boldsymbol{\psi} = d\boldsymbol{\psi} - 2\mathrm{i}m\boldsymbol{\psi}[dt - \mathrm{i}(\theta d\bar{\theta} + \bar{\theta}d\theta)].
\end{aligned}
\tag{3.9}
$$

In what follows, we find it convenient to define the covariant derivatives similarly to the flat case, i.e. with respect to differentials Δt, $d\theta$, $d\bar{\theta}$:

$$
dt\frac{\partial}{\partial t} + d\theta\frac{\partial}{\partial\theta} + d\bar{\theta}\frac{\partial}{\partial\bar{\theta}} = \Delta t\nabla_t + d\theta\nabla + d\bar{\theta}\,\bar{\nabla}. \tag{3.10}
$$

Explicitly, they read

$$
\begin{aligned}
&\nabla_t = E^{-1}\partial_t, \qquad \nabla = D - \mathrm{i}(\bar{\boldsymbol{\psi}}\nabla\boldsymbol{\psi} + \boldsymbol{\psi}\nabla\bar{\boldsymbol{\psi}})\partial_t,\\
&\bar{\nabla} = \bar{D} - \mathrm{i}(\bar{\boldsymbol{\psi}}\,\bar{\nabla}\boldsymbol{\psi} + \boldsymbol{\psi}\bar{\nabla}\,\bar{\boldsymbol{\psi}})\partial_t,
\end{aligned}
\tag{3.11}
$$

where

$$
\begin{aligned}
&E = 1 + \mathrm{i}(\dot{\psi}\,\bar{\psi} + \dot{\bar{\psi}}\psi), \qquad D = \frac{\partial}{\partial\theta} - \mathrm{i}\bar{\theta}\partial_t,\\
&\bar{D} = \frac{\partial}{\partial\bar{\theta}} - \mathrm{i}\theta\partial_t\colon\ \{D, \bar{D}\} = -2\mathrm{i}\partial_t.
\end{aligned}
\tag{3.12}
$$

These derivatives obey the following algebra:

$$
\begin{aligned}
&\{\nabla, \bar{\nabla}\} = -2\mathrm{i}(1 + \nabla\boldsymbol{\psi}\bar{\nabla}\,\bar{\boldsymbol{\psi}} + \bar{\nabla}\boldsymbol{\psi}\nabla\bar{\boldsymbol{\psi}})\nabla_t,\\
&\{\nabla, \nabla\} = -4\mathrm{i}\nabla\bar{\boldsymbol{\psi}}\nabla\boldsymbol{\psi}\nabla_t, \qquad \{\bar{\nabla}, \bar{\nabla}\} = -4\mathrm{i}\bar{\nabla}\,\bar{\boldsymbol{\psi}}\,\bar{\nabla}\,\boldsymbol{\psi}\nabla_t,\\
&[\nabla_t, \nabla] = -2\mathrm{i}(\nabla\bar{\boldsymbol{\psi}}\nabla_t\boldsymbol{\psi} + \nabla\boldsymbol{\psi}\nabla_t\bar{\boldsymbol{\psi}})\nabla_t,\\
&[\nabla_t, \bar{\nabla}] = -2\mathrm{i}(\bar{\nabla}\,\bar{\boldsymbol{\psi}}\,\nabla_t\boldsymbol{\psi} + \bar{\nabla}\boldsymbol{\psi}\nabla_t\bar{\boldsymbol{\psi}})\nabla_t.
\end{aligned}
\tag{3.13}
$$

IV. PRELIMINARY CONSIDERATION: THE BOSONIC ACTION

Before considering the full supersymmetric AdS_3 system it makes sense to analyze its bosonic sector.

The bosonic sector of our $N=(2,0)$ supersymmetric AdS_3 superalgebra (2.1) contains the bosonic AdS_3 algebra [i.e. $so(2,2)$ algebra] commuting with the $U(1)$ algebra spanned by the generator J. In this section we are going to consider the spontaneous breakdown of this $\mathrm{AdS}_3 \times S^1$ symmetry down to $d=1$ Poincaré $\times U(1)^2$ algebra, generated by P, J and J_3 generators. Therefore, our coset element is just the $(\theta,\psi \to 0)$ limit of the full coset element (3.1):

$$
g = e^{\mathrm{i}tP}e^{\mathrm{i}(UZ+\overline{UZ})}e^{\mathrm{i}(\Lambda T+\overline{\Lambda T})}. \tag{4.1}
$$

The corresponding $(\theta, \psi \to 0)$ limit of the Cartan forms read

$$
\begin{aligned}
\omega_P &= \frac{1}{1-\lambda\bar{\lambda}}[(1+\lambda\bar{\lambda})\hat{\omega}_P - 2\mathrm{i}(\bar{\lambda}\hat{\omega}_Z - \lambda\hat{\bar{\omega}}_Z)],\\
\omega_Z &= \frac{1}{1-\lambda\bar{\lambda}}[\hat{\omega}_Z - \lambda^2\hat{\bar{\omega}}_Z + \mathrm{i}\lambda\hat{\omega}_P],\\
\omega_J &= \hat{\omega}_J - \frac{1}{1-\lambda\bar{\lambda}}[2m\lambda\bar{\lambda}\hat{\omega}_P - 2\mathrm{i}m(\bar{\lambda}\hat{\omega}_Z - \lambda\hat{\bar{\omega}}_Z)],\\
\omega_T &= \frac{d\lambda}{1-\lambda\bar{\lambda}}, \qquad \omega_3 = \mathrm{i}\frac{\lambda d\bar{\lambda} - d\lambda\bar{\lambda}}{1-\lambda\bar{\lambda}},
\end{aligned}
\tag{4.2}
$$

where

$$
\begin{aligned}
\hat{\omega}_P &= \frac{1}{1-4m^2u\bar{u}}[(1+4m^2u\bar{u})dt + 2\mathrm{i}m(ud\bar{u} - \bar{u}du)],\\
\hat{\omega}_Z &= \frac{1}{1-4m^2u\bar{u}}[du + 2\mathrm{i}mudt],\\
\hat{\bar{\omega}}_Z &= \frac{1}{1-4m^2u\bar{u}}[d\bar{u} - 2\mathrm{i}m\bar{u}dt],\\
\hat{\omega}_J &= -\frac{2m^2}{1-4m^2u\bar{u}}[4mu\bar{u}dt + \mathrm{i}(ud\bar{u} - \bar{u}du)]\\
&= m(dt - \hat{\omega}_P).
\end{aligned}
\tag{4.3}
$$

To reduce the number of independent Goldstone fields, similarly to the flat space case [3], one may impose the following conditions on the Cartan forms ω_Z and $\bar\omega_Z$ (inverse Higgs phenomenon [8]):

$$\omega_Z = 0 \Rightarrow \hat\omega_Z = -\mathrm{i}\frac{\lambda}{1+\lambda\bar\lambda}\hat\omega_P \Rightarrow \frac{\lambda}{1+\lambda\bar\lambda} = \mathrm{i}\frac{\dot u + 2\mathrm{i}mu}{1+4m^2u\bar u + 2\mathrm{i}m(u\dot{\bar u} - \bar u\dot u)},$$
$$\bar\omega_Z = 0 \Rightarrow \hat{\bar\omega}_Z = \mathrm{i}\frac{\bar\lambda}{1+\lambda\bar\lambda}\hat\omega_P \Rightarrow \frac{\bar\lambda}{1+\lambda\bar\lambda} = -\mathrm{i}\frac{\dot{\bar u} - 2\mathrm{i}m\bar u}{1+4m^2u\bar u + 2\mathrm{i}m(u\dot{\bar u} - \bar u\dot u)}, \tag{4.4}$$

and, therefore,

$$\dot u = -\mathrm{i}\frac{(2mu+\lambda)(1+2mu\bar\lambda)}{1+\lambda\bar\lambda + 2m(\bar u\lambda + u\bar\lambda)}. \tag{4.5}$$

These constraints are purely kinematic ones. Thus, to realize this spontaneous breaking of $\mathrm{AdS}_3\times U(1)$ symmetry we need one complex scalar field, $u(t)$ and $\bar u(t)$.

Using the constraints (4.4), one may further simplify the Cartan forms ω_P, ω_J (4.2) to be

$$\omega_P = \frac{1-\lambda\bar\lambda}{1+\lambda\bar\lambda}\hat\omega_P, \qquad \omega_J = mdt - m\frac{1-\lambda\bar\lambda}{1+\lambda\bar\lambda}\hat\omega_P. \tag{4.6}$$

Clearly, the simplest action, invariant under full $\mathrm{AdS}_3\times U(1)$ symmetry, is

$$S_0 = -m_0\int\omega_P = -m_0\int dt\sqrt{\left(1+\frac{2\mathrm{i}m(\dot u\bar u - u\dot{\bar u})}{1-4m^2u\bar u}\right)^2 - 4\frac{\dot u\dot{\bar u}}{(1-4m^2u\bar u)^2}} = -m_0\int dt\frac{1-\lambda\bar\lambda}{1+\lambda\bar\lambda+2m(\lambda\bar u+\bar\lambda u)}. \tag{4.7}$$

One may check that the curvature of the space with the metric

$$ds^2 = -\left(dt + \frac{2\mathrm{i}m(du\bar u - ud\bar u)}{1-4m^2u\bar u}\right)^2 + 4\frac{dud\bar u}{(1-4m^2u\bar u)^2} \tag{4.8}$$

is equal to $\mathcal{R} = -6m^2$.

Keeping in mind that the Cartan form ω_3 is shifted by the full time derivative under all transformations of the $\mathrm{AdS}_3\times U(1)$ group, the invariant anyonic term, i.e. the action which results in the at most third-order time derivative equations of motion of the fields and which possesses the invariance under full $\mathrm{AdS}_3\times U(1)$ symmetry, acquires the form

$$S_{\rm anyon} = -\int\omega_3 = \mathrm{i}\int dt\frac{\dot\lambda\bar\lambda - \dot{\bar\lambda}\lambda}{1-\lambda\bar\lambda}. \tag{4.9}$$

In terms of u, $\bar u$ and their derivatives it reads

$$S_{\rm anyon} = \int dt\left\{\frac{2\mathrm{i}(\ddot u\dot{\bar u} - \ddot{\bar u}\dot u) - 8m\dot u\dot{\bar u} + 8\mathrm{i}m^2(\dot u\bar u - \dot{\bar u}u) + 4m(\ddot u\bar u + \ddot{\bar u}u)}{\sqrt{(1-4m^2u\bar u - 2\mathrm{i}m(\dot{\bar u}u - \dot u\bar u))^2 - 4\dot u\dot{\bar u}}}\right.$$
$$\left.\times\left[1+4m^2u\bar u + 2\mathrm{i}m(u\dot{\bar u} - \bar u\dot u) + \sqrt{(1-4m^2u\bar u - 2\mathrm{i}m(\dot{\bar u}u - \dot u\bar u))^2 - 4\dot u\dot{\bar u}}\right]^{-1}\right\}. \tag{4.10}$$

The actions S_0 (4.7) and $S_{\rm anyon}$ (4.10) may be slightly simplified by passing to new variables q, $\bar q$ defined as

$$q = e^{2\mathrm{i}mt}u, \qquad \bar q = e^{-2\mathrm{i}mt}\bar u. \tag{4.11}$$

In terms of these variables the action of the AdS_3 particle reads

$$S_0 = -m_0\int dt\sqrt{\left(1-2\mathrm{i}m\frac{\dot q\bar q - \dot{\bar q}q}{1-4m^2q\bar q}\right)^2 - \frac{4\dot q\dot{\bar q}}{(1-4m^2q\bar q)^2}}, \tag{4.12}$$

while the anyonic action (4.10) is simplified to be

$$S_{\rm anyon} = \int dt\left\{\frac{2\mathrm{i}(\ddot q\dot{\bar q} - \ddot{\bar q}\dot q) + 8m\dot q\dot{\bar q}}{\sqrt{(1-4m^2q\bar q + 2\mathrm{i}m(\dot{\bar q}q - \dot q\bar q))^2 - 4\dot q\dot{\bar q}}}\right.$$
$$\left.\times\left[1-4m^2q\bar q + 2\mathrm{i}m(\dot{\bar q}q - \dot q\bar q) + \sqrt{(1-4m^2q\bar q + 2\mathrm{i}m(\dot{\bar q}q - \dot q\bar q))^2 - 4\dot q\dot{\bar q}}\right]^{-1}\right\}. \tag{4.13}$$

This action can be rewritten through the Lagrangian $\mathcal{L}_{PS2}$ of a particle on the pseudosphere and connection $\mathcal{A}_\mu$ as

$$S_{\rm anyon} = \int dt\frac{2\mathrm{i}\frac{\ddot q\dot{\bar q} - \ddot{\bar q}\dot q}{(1-4m^2q\bar q)^2} + 8m\mathcal{L}_{PS2}}{\sqrt{(1-\mathcal{A}_\mu\dot q^\mu)^2 - 4\mathcal{L}_{PS2}}(1-\mathcal{A}_\mu\dot q^\mu + \sqrt{(1-\mathcal{A}_\mu\dot q^\mu)^2 - 4\mathcal{L}_{PS2}})}. \tag{4.14}$$

Here

$$\mathcal{L}_{PS2} = \frac{\dot q\,\dot{\bar q}}{(1-4m^2 q\bar q)^2}, \qquad \mathcal{A}_\mu \dot q^\mu = 2\mathrm{i}m\frac{\dot q\,\bar q - \dot{\bar q} q}{1-4m^2 q\bar q}. \tag{4.15}$$

The AdS_3 anyon action, written in terms of the λ, $\bar\lambda$ variables (4.9), has the same form as in the flat spacetime case [3]. This analogy is slightly broken for the AdS_3 rigid particle action, which is invariant under the full $\mathrm{AdS}_3 \times U(1)$ symmetry and leads to equations of motion of at most fourth order in time derivatives,

$$\begin{aligned} S_{\text{rigid}} &= \int \frac{\omega_T \bar\omega_T}{\omega_P} \\ &= \int dt \left(1 + 2m\frac{\lambda\bar u + \bar\lambda u}{1+\lambda\bar\lambda}\right)\left(\frac{1+\lambda\bar\lambda}{(1-\lambda\bar\lambda)^3}\right)\dot\lambda\,\dot{\bar\lambda}. \end{aligned} \tag{4.16}$$

The term proportional to m is needed to provide invariance with respect to the AdS_3 symmetry, realized by left multiplications of the coset element (4.1) as follows:

$$\begin{aligned} g_0 &= e^{\mathrm{i}(aT+\bar a\bar T)} \Rightarrow \delta_T t = -\mathrm{i}a\bar u(e^{2\mathrm{i}mt}+1) \\ &\quad + \mathrm{i}\bar a u(e^{-2\mathrm{i}mt}+1), \\ \delta_T\lambda &= a - \bar a\lambda^2, \\ \delta_T u &= \frac{a}{2m}e^{-2\mathrm{i}mt}(1-4m^2u\bar u) - \frac{1}{2m}(a - 4m^2\bar a u^2), \\ g_0 &= e^{\mathrm{i}(bZ+\bar b\bar Z)} \Rightarrow \delta_Z t = -2\mathrm{i}m(be^{-2\mathrm{i}mt}\bar u - \bar b e^{2\mathrm{i}mt}u), \\ \delta_Z u &= b(1-4m^2u\bar u)e^{-2\mathrm{i}mt}, \qquad \delta_Z\lambda = 0. \end{aligned} \tag{4.17}$$

Let us finally stress that the actions (4.7), (4.10), and (4.16) we constructed are precisely the flat spacetime expressions when expressed in the terms of Cartan forms [3].

V. FULLY SUPERSYMMETRIC CASE

To construct supersymmetric component actions, invariant under both unbroken Q and broken S supersymmetries, in full analogy with the flat case [3], one has to perform four steps:

(i) impose some additional constraints to reduce the number of independent superfields and impose irreducibility constraints on the essential superfields;
(ii) find the transformation properties of the physical components under both supersymmetries;
(iii) write an ansatz for the component actions invariant under broken supersymmetries (the corresponding invariants are provided by the Cartan forms evaluated at $\theta = \bar\theta = 0$ condition);
(iv) fix the arbitrary parameters in the ansatz by demanding invariance under the unbroken supersymmetry.

Let us go through these steps.

A. Irreducibility conditions

From the beginning, in our coset (3.1) there are three independent complex superfields $\mathbf{u}$, $\boldsymbol{\psi}$, $\boldsymbol{\lambda}$ [considering the redefinitions (3.5)]. To reduce the number of independent superfields we impose the same conditions (4.4) as in the bosonic sector,

$$\left.\begin{aligned}\Omega_Z &= 0\\ \bar\Omega_Z &= 0\end{aligned}\right\} \Rightarrow \hat\Omega_Z = -\mathrm{i}\frac{\boldsymbol\lambda}{1+\boldsymbol\lambda\bar{\boldsymbol\lambda}}\hat\Omega_P, \qquad \hat{\bar\Omega}_Z = \mathrm{i}\frac{\bar{\boldsymbol\lambda}}{1+\boldsymbol\lambda\bar{\boldsymbol\lambda}}\hat\Omega_P. \tag{5.1}$$

Equating the coefficients of the differentials Δt, $d\theta$ and $d\bar\theta$ we get

$$\begin{aligned} (1-4m\boldsymbol\psi\bar{\boldsymbol\psi})\nabla_t\mathbf{u} &= -2\mathrm{i}m\mathbf{u} - \mathrm{i}\frac{\boldsymbol\lambda(1-4m^2\mathbf{u}\bar{\mathbf{u}})}{1+\boldsymbol\lambda\bar{\boldsymbol\lambda}+2m(\boldsymbol\lambda\bar{\mathbf{u}}+\bar{\boldsymbol\lambda}\mathbf{u})}, \\ (1-4m\boldsymbol\psi\bar{\boldsymbol\psi})\nabla_t\bar{\mathbf{u}} &= 2\mathrm{i}m\bar{\mathbf{u}} + \mathrm{i}\frac{\bar{\boldsymbol\lambda}(1-4m^2\mathbf{u}\bar{\mathbf{u}})}{1+\boldsymbol\lambda\bar{\boldsymbol\lambda}+2m(\boldsymbol\lambda\bar{\mathbf{u}}+\bar{\boldsymbol\lambda}\mathbf{u})} \end{aligned} \tag{5.2}$$

and

$$\nabla\mathbf{u} + 4m\mathbf{u}\boldsymbol\psi\nabla_t\mathbf{u} = 0, \qquad \bar\nabla\bar{\mathbf{u}} - 4m\bar{\mathbf{u}}\bar{\boldsymbol\psi}\nabla_t\bar{\mathbf{u}} = 0, \tag{5.3}$$

$$\begin{aligned} \bar\nabla\mathbf{u} &= -2\mathrm{i}\bar{\boldsymbol\psi}(1-4m^2\mathbf{u}\bar{\mathbf{u}} + 2\mathrm{i}m\bar{\mathbf{u}}\nabla_t\mathbf{u}), \\ \nabla\bar{\mathbf{u}} &= -2\mathrm{i}\boldsymbol\psi(1-4m^2\mathbf{u}\bar{\mathbf{u}} + 2\mathrm{i}m\mathbf{u}\nabla_t\bar{\mathbf{u}}). \end{aligned} \tag{5.4}$$

These relations simplify the form Ω_P to

$$\begin{aligned} \Omega_P &= \frac{1-\boldsymbol\lambda\bar{\boldsymbol\lambda}}{1+\boldsymbol\lambda\bar{\boldsymbol\lambda}}\hat\Omega_P, \\ \hat\Omega_P &= (1+4m\boldsymbol\psi\bar{\boldsymbol\psi})\left[\frac{\Delta t + 4m(\mathbf{u}\boldsymbol\psi d\theta - \bar{\mathbf{u}}\bar{\boldsymbol\psi}\,d\bar\theta)}{1+2m\frac{\bar{\mathbf{u}}\boldsymbol\lambda+\mathbf{u}\bar{\boldsymbol\lambda}}{1+\boldsymbol\lambda\bar{\boldsymbol\lambda}}}\right]. \end{aligned} \tag{5.5}$$

In principle, (5.2)–(5.4) solve all tasks. Indeed, using (5.2) one may express the superfields $\boldsymbol\lambda$, $\bar{\boldsymbol\lambda}$ in terms of time derivatives of $\mathbf{u}$, $\bar{\mathbf{u}}$, while (5.3) can be solved to express the fermionic superfields $\boldsymbol\psi$, $\bar{\boldsymbol\psi}$ in terms of spinor covariant derivatives of the same $\mathbf{u}$, $\bar{\mathbf{u}}$. Thus, like in the flat case [3], we remain with only one $N=2$ complex bosonic superfield $\mathbf{u}(t,\theta,\bar\theta)$, restricted by (5.3) to be covariantly chiral, with slightly modified chirality conditions. However, in what follows we are going to use as independent components the $\theta=\bar\theta=0$ projections of the superfields $\boldsymbol\psi$, $\bar{\boldsymbol\psi}$ instead of the projections of $\bar\nabla\mathbf{u}$ and $\nabla\bar{\mathbf{u}}$. Therefore, it would be useful to find the consequences of the constraints (5.2)–(5.4).

First of all, acting by ∇ on the first equation in (5.3) and by $\bar{\nabla}$ on the second one and using the algebra (3.13) of the covariant derivatives, we get the conditions

$$\nabla\boldsymbol{\psi} + 4m\mathbf{u}\boldsymbol{\psi}\nabla_t\boldsymbol{\psi} = 0, \qquad \bar{\nabla}\bar{\boldsymbol{\psi}} - 4m\bar{\mathbf{u}}\bar{\boldsymbol{\psi}}\nabla_t\bar{\boldsymbol{\psi}} = 0. \tag{5.6}$$

Note that this asserts the self-consistency of the modified chirality constraints (5.3) because

$$\begin{aligned}&\{\nabla + 4m\mathbf{u}\boldsymbol{\psi}\nabla_t, \nabla + 4m\mathbf{u}\boldsymbol{\psi}\nabla_t\} = 0,\\ &\{\bar{\nabla} - 4m\bar{\mathbf{u}}\bar{\boldsymbol{\psi}}\nabla_t, \bar{\nabla} - 4m\bar{\mathbf{u}}\bar{\boldsymbol{\psi}}\nabla_t\} = 0.\end{aligned} \tag{5.7}$$

Secondly, acting by $\bar{\nabla}$ on the first equation in (5.3) and by ∇ on the first equation in (5.4) and adding the results, after quite lengthly calculations with heavy use of (3.13), we obtain

$$\begin{aligned}\nabla\bar{\boldsymbol{\psi}} = {}& -\mathrm{i}\frac{\boldsymbol{\lambda} + 2m\mathbf{u}}{1 + 2m\bar{\mathbf{u}}\boldsymbol{\lambda}}(1 - 4m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}) - 8\mathrm{i}m^2\mathbf{u}\boldsymbol{\psi}\bar{\boldsymbol{\psi}}\\ &- 4m\mathbf{u}\boldsymbol{\psi}\nabla_t\bar{\boldsymbol{\psi}}.\end{aligned} \tag{5.8}$$

Repeating similar calculations with the second equations in (5.3), (5.4) yields the conjugated expression

$$\begin{aligned}\bar{\nabla}\boldsymbol{\psi} = {}& \mathrm{i}\frac{\bar{\boldsymbol{\lambda}} + 2m\bar{\mathbf{u}}}{1 + 2m\mathbf{u}\bar{\boldsymbol{\lambda}}}(1 - 4m\boldsymbol{\psi}\bar{\boldsymbol{\psi}}) + 8\mathrm{i}m^2\bar{\mathbf{u}}\boldsymbol{\psi}\bar{\boldsymbol{\psi}}\\ &+ 4m\bar{\mathbf{u}}\bar{\boldsymbol{\psi}}\nabla_t\boldsymbol{\psi}.\end{aligned} \tag{5.9}$$

Now we have all ingredients needed for constructing the component action.

Before closing this subsection let us visualize a more simple way to obtain (5.8) and (5.9). The idea consists in using the constraints[2]

$$\Omega_S|_{\Omega_Q,\bar{\Omega}_Q} = 0, \qquad \bar{\Omega}_S|_{\Omega_Q,\bar{\Omega}_Q} = 0. \tag{5.10}$$

Here, the notation $|_{\Omega_Q,\bar{\Omega}_Q}$ means that the Cartan forms Ω_S and $\bar{\Omega}_S$ must be expanded in the forms Ω_P, Ω_Q, $\bar{\Omega}_Q$ before nullifying their $|_{\Omega_Q,\bar{\Omega}_Q}$ projections. At first sight these conditions seem to be more complicated due to the highly nontrivial structure of the Cartan forms involved. However, this is not the case and the calculations can be simplified using the following procedure.

With the help of our definitions (3.7), the first constraint in (5.10) can be formally represented as

$$\Omega_S = \frac{\hat{\Omega}_S - \mathrm{i}\bar{\lambda}\hat{\bar{\Omega}}_Q}{\sqrt{1 - \lambda\bar{\lambda}}} = \Omega_P X, \tag{5.11}$$

[2]These conditions, being some variant of the superembedding conditions [9], were trivial in the flat case [3].

where X is some expression which is defined by this equation. The main difference between (5.10) and (5.11) is that the latter one is written as the equation on forms. Therefore, one may just substitute in (5.11) the exact expressions from (3.8) and equate on both sides the coefficients of the differentials Δt, $d\theta$ and $d\bar{\theta}$. The Δt coefficient relation yields X in terms of $\nabla_t\boldsymbol{\psi}$. Substituting this expression for X in the $d\theta$, $d\bar{\theta}$ projections of (5.11) we immediately obtain the first equation in (5.6) and also (5.9). The same procedure applied to the second constraint in (5.10) produces the second equation in (5.6) as well as (5.8).

These considerations demonstrate that the constraints (5.10) are a consequence of our basic constraints (5.1).

B. Transformation properties of the components

As we are going to construct the component actions, we need to know the transformation laws for the components. We denote the components of the superfields in the following way:

$$\begin{aligned}&\mathbf{u}|_{\theta=0} = u, \qquad \bar{\mathbf{u}}|_{\theta=0} = \bar{u}, \qquad \boldsymbol{\psi}|_{\theta=0} = \psi,\\ &\bar{\boldsymbol{\psi}}|_{\theta=0} = \bar{\psi}, \qquad \boldsymbol{\lambda}|_{\theta=0} = \lambda, \qquad \bar{\boldsymbol{\lambda}}|_{\theta=0} = \bar{\lambda}.\end{aligned} \tag{5.12}$$

Equations (5.2) evaluated at $\theta = \bar{\theta} = 0$ provide relations between λ, $\bar{\lambda}$ and the time derivatives of u, $\bar{u}$. Thus, λ, $\bar{\lambda}$ are not independent components. We introduce these variables just to simplify many expressions in what follows.

1. Broken S supersymmetry

The transformation properties of our components (5.12) under broken supersymmetry can be easily learned from (3.3). Before listing these transformations, we point out that, in contrast to the flat case [3], the superspace coordinates θ and $\bar{\theta}$ are not invariant under the broken supersymmetry (3.3):

$$\delta_S\theta = 4m\varepsilon e^{2\mathrm{i}mt}\bar{\boldsymbol{\psi}}\theta, \qquad \delta_S\bar{\theta} = -4m\bar{\varepsilon}e^{-2\mathrm{i}mt}\boldsymbol{\psi}\bar{\theta}.$$

However, the right-hand sides of these variations disappear in the limit $\theta = \bar{\theta} = 0$ and, thus, the set of the components $\{u, \bar{u}, \psi, \bar{\psi}\}$ is closed under the broken supersymmetry. The corresponding transformations read

$$\begin{aligned}&\delta_S t = \mathrm{i}(\varepsilon e^{2\mathrm{i}mt}\bar{\psi} + \bar{\varepsilon}e^{-2\mathrm{i}mt}\psi),\\ &\delta_S\psi = \varepsilon e^{2\mathrm{i}mt}(1 + 2m\psi\bar{\psi}), \qquad \delta_S\bar{\psi} = \bar{\varepsilon}e^{-2\mathrm{i}mt}(1 + 2m\psi\bar{\psi}),\\ &\delta_S u = 0, \qquad \delta_S\bar{u} = 0.\end{aligned} \tag{5.13}$$

It is rather easy to check that the expression

$$\begin{aligned}(1 + 4m\psi\bar{\psi})\Delta t|_{\theta=0} &= (1 + 4m\psi\bar{\psi})[dt - \mathrm{i}(\psi d\bar{\psi} + \bar{\psi}d\psi)]\\ &\equiv \mathcal{E}dt\end{aligned} \tag{5.14}$$

is invariant with respect to (5.13). Therefore, it is natural to define a new covariant derivative as

$$\mathcal{D}_t = \mathcal{E}^{-1}\partial_t,\qquad \mathcal{E}^{-1} = (1-4m\psi\bar{\psi})[1-\mathrm{i}(\mathcal{D}_t\psi\bar{\psi}+\mathcal{D}_t\bar{\psi}\psi)]. \tag{5.15}$$

It then immediately follows from (5.14) and (5.15) that

$$\delta_S\mathcal{D}_t u = 0,\qquad \delta_S\mathcal{D}_t\bar{u} = 0 \Rightarrow \delta_S\lambda = \delta_S\bar{\lambda} = 0. \tag{5.16}$$

2. Unbroken Q supersymmetry

The transformations under unbroken Q supersymmetry can be defined in the usual way as

$$\delta_Q f = -(\epsilon D + \bar{\epsilon}\bar{D})\mathbf{f}|_{\theta\to 0} = -(\epsilon\nabla + \bar{\epsilon}\bar{\nabla})\mathbf{f}|_{\theta\to 0} - H\partial_t,$$
$$H = \mathrm{i}\epsilon(\boldsymbol{\psi}\nabla\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{\psi}}\nabla\boldsymbol{\psi})|_{\theta\to 0} + \mathrm{i}\bar{\epsilon}(\boldsymbol{\psi}\bar{\nabla}\bar{\boldsymbol{\psi}} + \bar{\boldsymbol{\psi}}\bar{\nabla}\boldsymbol{\psi})|_{\theta\to 0}. \tag{5.17}$$

For example,

$$\delta_Q u = 2\mathrm{i}\bar{\epsilon}\bar{\psi}(1-4m^2u\bar{u}) + 4m(\epsilon u\psi - \bar{\epsilon}\bar{u}\bar{\psi})\mathcal{D}_t u - H\dot{u},$$
$$\delta_Q\psi = -(\epsilon\nabla\boldsymbol{\psi} + \bar{\epsilon}\bar{\nabla}\boldsymbol{\psi})|_{\theta\to 0} - H\dot{\psi}. \tag{5.18}$$

Another important object is the vielbein $\mathcal{E}$ (5.15) which transforms as follows:

$$\begin{aligned}\delta_Q\mathcal{E} = {} & 2\mathrm{i}(1+4m\psi\bar{\psi})\mathcal{E}[(\epsilon\nabla\boldsymbol{\psi} + \bar{\epsilon}\bar{\nabla}\boldsymbol{\psi})|_{\theta\to 0}\dot{\bar{\psi}} \\ & + (\epsilon\nabla\bar{\boldsymbol{\psi}} + \bar{\epsilon}\bar{\nabla}\bar{\boldsymbol{\psi}})|_{\theta\to 0}\dot{\psi}] \\ & - 4m\mathcal{E}(\epsilon\nabla\boldsymbol{\psi} + \bar{\epsilon}\bar{\nabla}\boldsymbol{\psi})|_{\theta\to 0}\bar{\psi} \\ & + 4m\mathcal{E}(\epsilon\nabla\bar{\boldsymbol{\psi}} + \bar{\epsilon}\bar{\nabla}\bar{\boldsymbol{\psi}})|_{\theta\to 0}\psi - \partial_t(H\mathcal{E}).\end{aligned} \tag{5.19}$$

Of course, to find the explicit form of the transformations (5.19) one has to use the relations (5.6), (5.8), (5.9) evaluated at $\theta = \bar{\theta} = 0$:

$$\begin{aligned}&(\nabla\boldsymbol{\psi})|_{\theta=0} + 4mu\psi\mathcal{D}_t\psi = 0,\\ &(\bar{\nabla}\bar{\boldsymbol{\psi}})|_{\theta=0} - 4m\bar{u}\bar{\psi}\mathcal{D}_t\bar{\psi} = 0,\\ &(\nabla\bar{\boldsymbol{\psi}})|_{\theta=0} = -\mathrm{i}\frac{\lambda+2mu}{1+2m\bar{u}\lambda}(1-4m\psi\bar{\psi}) - 8\mathrm{i}m^2u\psi\bar{\psi} - 4mu\psi\mathcal{D}_t\bar{\psi},\\ &(\bar{\nabla}\boldsymbol{\psi})|_{\theta=0} = \mathrm{i}\frac{\bar{\lambda}+2m\bar{u}}{1+2mu\bar{\lambda}}(1-4m\psi\bar{\psi}) + 8\mathrm{i}m^2\bar{u}\psi\bar{\psi} + 4m\bar{u}\bar{\psi}\mathcal{D}_t\psi.\end{aligned} \tag{5.20}$$

In particular, the transformation (5.19) acquires the form

$$\delta_Q\mathcal{E} = -\partial_t(H\mathcal{E}) + 2\mathcal{E}\frac{\lambda+2mu}{1+2m\bar{u}\lambda}\epsilon(\mathcal{D}_t\psi - 2\mathrm{i}m\psi) - 2\mathcal{E}\frac{\bar{\lambda}+2m\bar{u}}{1+2mu\bar{\lambda}}\bar{\epsilon}(\mathcal{D}_t\bar{\psi} + 2\mathrm{i}m\bar{\psi}). \tag{5.21}$$

Finally, we stress that the relations between the components u, $\bar{u}$ and λ, $\bar{\lambda}$ are given by the following expressions:

$$\mathcal{D}_t u = -2\mathrm{i}mu - \mathrm{i}\frac{\lambda(1-4m^2u\bar{u})}{1+\lambda\bar{\lambda}+2m(\lambda\bar{u}+\bar{\lambda}u)},$$
$$\mathcal{D}_t\bar{u} = 2\mathrm{i}m\bar{u} + \mathrm{i}\frac{\bar{\lambda}(1-4m^2u\bar{u})}{1+\lambda\bar{\lambda}+2m(\lambda\bar{u}+\bar{\lambda}u)}. \tag{5.22}$$

C. Actions

We are ready to construct the supersymmetric generalization of the actions (4.7) and (4.9).[3] As they have different dimension, these actions must be invariant individually.

1. Superparticle

It is easy to check that the evident ansatz

$$\int dt\mathcal{E}F(u,\bar{u},\lambda,\bar{\lambda})$$

is invariant under the broken supersymmetry for any function F because, in virtue of (5.13), (5.14), (5.16),

$$\delta_S F = 0,\qquad \delta_S(dt\mathcal{E}) = 0. \tag{5.23}$$

The desired bosonic limit (4.7) immediately fixes the function F up to constant α:

$$S_0 = -m_0\int dt\mathcal{E}\left[\alpha + \frac{1-\lambda\bar{\lambda}}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}\right]. \tag{5.24}$$

This constant α can be determined as unity either from linearized Q supersymmetry invariance or from the flat spacetime action of [3]. Let us explicitly demonstrate that the action

$$S_0 = -m_0\int dt\mathcal{E}\left[1 + \frac{1-\lambda\bar{\lambda}}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}\right] \equiv -m_0\int dt\mathcal{L} \tag{5.25}$$

is invariant under the unbroken Q supersymmetry.

Using (5.18) and (5.22), one finds that

$$\delta_Q\lambda = -2\bar{\epsilon}(\mathcal{D}_t\bar{\psi} + 2\mathrm{i}m\bar{\psi})\frac{1+2m\bar{u}\lambda}{1+2mu\bar{\lambda}}(1+\lambda\bar{\lambda}+2m(u\bar{\lambda}-\bar{u}\lambda)) + 4m(\epsilon u\psi - \bar{\epsilon}\bar{u}\bar{\psi})\mathcal{D}_t\lambda - H\partial_t\lambda,\ \delta_Q\bar{\lambda} = (\delta_Q\lambda)^\dagger. \tag{5.26}$$

[3]Higher-order corrections to the superparticle superspace action in flat spacetime were considered previously in [10].

Now, the variation of integrand in (5.25) reads

$$\delta_Q\mathcal{L} = -\partial_t(H\mathcal{L}) + 4m\mathcal{E}(\epsilon u\psi - \bar{\epsilon}\,\bar{u}\,\bar{\psi})\mathcal{D}_t\left[\frac{1-\lambda\bar{\lambda}}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}\right] + 4m\mathcal{E}(\epsilon\mathcal{D}_t\psi u - \bar{\epsilon}\mathcal{D}_t\bar{\psi}\,\bar{u})\frac{1-\lambda\bar{\lambda}}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}$$
$$- 4im\mathcal{E}\epsilon\psi\frac{(1-\lambda\bar{\lambda})(1+2mu\bar{\lambda})(\lambda+2mu)}{[1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)]^2} - 4im\mathcal{E}\bar{\epsilon}\,\bar{\psi}\frac{(1-\lambda\bar{\lambda})(1+2m\bar{u}\lambda)(\bar{\lambda}+2m\bar{u})}{[1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)]^2}. \qquad (5.27)$$

Using the relations (5.22), the last line in (5.27) may be represented as

$$4m\mathcal{E}(\epsilon\psi\mathcal{D}_t u - \bar{\epsilon}\mathcal{D}_t\bar{\psi}\mathcal{D}_t\bar{u})\frac{1-\lambda\bar{\lambda}}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)},$$

and, therefore,

$$\delta_Q\mathcal{L} = -\partial_t(H\mathcal{L}) + 4m\mathcal{E}\mathcal{D}_t\left[\frac{(\epsilon\psi u - \bar{\epsilon}\,\bar{\psi}\,\bar{u})(1-\lambda\bar{\lambda})}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}\right]$$
$$= \partial_t\left[-H\mathcal{L} + 4m\frac{(\epsilon\psi u - \bar{\epsilon}\,\bar{\psi}\,\bar{u})(1-\lambda\bar{\lambda})}{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}\right]. \qquad (5.28)$$

Thus, the action (5.25) is invariant under both the broken S and unbroken Q supersymmetries, and it is the action of the $N=(2,0)$ AdS_3 superparticle.

The AdS_3 superparticle action (5.25) may be written in terms of the Cartan forms evaluated at $\theta = d\theta = 0$ in a rather simple way as

$$S_0 = \frac{m_0}{m}\int \Omega_J|_{\theta=0}. \qquad (5.29)$$

2. Supersymmetric AdS_3 anyon

The supersymmetrization of the anyoniclike action (4.9) is more involved. To construct it, let us firstly note that, with respect to broken supersymmetry, the covariant time derivatives of the fermionic components $\mathcal{D}_t\psi$ and $\mathcal{D}_t\bar{\psi}$ transform as

$$\delta_S\mathcal{D}_t\psi = 2im\varepsilon e^{2imt}(1-2m\psi\bar{\psi}) - 4m\varepsilon e^{2imt}\bar{\psi}\mathcal{D}_t\psi,$$
$$\delta_S\mathcal{D}_t\bar{\psi} = -2im\bar{\varepsilon}e^{-2imt}(1-2m\psi\bar{\psi}) + 4m\bar{\varepsilon}e^{-2imt}\psi\mathcal{D}_t\bar{\psi}. \qquad (5.30)$$

Therefore, the S-invariant fermionic correction to the bosonic action (4.9) has the form

$$\int dt\mathcal{E}\mathcal{F}(\lambda,\bar{\lambda},u,\bar{u})(1+4m\psi\bar{\psi})(\mathcal{D}_t\psi - 2im\psi)$$
$$\times(\mathcal{D}_t\bar{\psi} + 2im\bar{\psi}).$$

Thus, our ansatz for the supersymmetric AdS_3 anyonic action reads

$$S_{\text{anyon}} = \int dt\mathcal{E}\left[i\frac{\mathcal{D}_t\lambda\bar{\lambda} - \mathcal{D}_t\bar{\lambda}\lambda}{1-\lambda\bar{\lambda}} + \mathcal{F}(\lambda,\bar{\lambda},u,\bar{u})(1+4m\psi\bar{\psi})(\mathcal{D}_t\psi - 2im\psi)(\mathcal{D}_t\bar{\psi} + 2im\bar{\psi})\right] \equiv \int dt\mathcal{L}_{\text{anyon}}, \qquad (5.31)$$

where the function $\mathcal{F}$ has to be determined by invariance under unbroken supersymmetry. The first term in (5.31) is a direct supersymmetrization of the bosonic anyon action and, by construction, is invariant under broken supersymmetry.

Due to the transformation property of all our ingredients, which roughly takes the form

$$\delta_Q\mathcal{E} \sim -\partial_t(H\mathcal{E}) + \cdots,$$
$$\delta_Q(u,\bar{u},\lambda,\bar{\lambda}) \sim -H(\partial_t u,\partial_t\bar{u},\partial_t\lambda,\partial_t\bar{\lambda}) + \cdots,$$

the H-dependent terms convert into full time derivatives. Hence, while checking invariance of the action, these terms can be ignored.

The simplest way to fix the function $\mathcal{F}$ is to consider the variation of the action (5.31) to first order in the fermions $\psi,\bar{\psi}$. At this order the variation of the integrand in (5.31) reads (we write only the ϵ part of transformations)

$$\delta_Q\mathcal{L}_{\text{anyon}} \approx 4i\epsilon(\partial_t\psi - 2im\psi)\partial_t\lambda\frac{1+2mu\bar{\lambda}}{1+2m\bar{u}\lambda}\frac{1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda)}{(1-\lambda\bar{\lambda})^2} - i\epsilon(\partial_t\psi - 2im\psi)\partial_t\lambda\cdot\frac{1-4m^2u\bar{u}}{(1+2m\bar{u}\lambda)^2}\mathcal{F}. \qquad (5.32)$$

To cancel this variation one has to choose $\mathcal{F}$ as

$$\mathcal{F} = 4\frac{(1+2m\bar{u}\lambda)(1+2mu\bar{\lambda})(1+\lambda\bar{\lambda}+2m(u\bar{\lambda}+\bar{u}\lambda))}{(1-4m^2u\bar{u})(1-\lambda\bar{\lambda})^2}. \qquad (5.33)$$

Now, it is a matter of direct but slightly complicated calculation to check that the action (5.31) is invariant under the unbroken supersymmetry to all orders in the fermionic variables. The terms which are not explicitly canceled in the variation of the integrand in (5.31) read (all terms coming from $\partial_\lambda \mathcal{F}$ and $\partial_{\bar\lambda}\mathcal{F}$ canceled trivially)

$$\begin{aligned}\delta_Q \mathcal{L}_{\text{anyon}} = & -\partial_t (H\mathcal{L}_{\text{anyon}}) + 4m\epsilon u\psi\mathcal{E}\cdot\mathcal{D}_t[\mathcal{F}(1+4m\psi\bar\psi)(\mathcal{D}_t\psi - 2\mathrm{i}m\psi)(\mathcal{D}_t\bar\psi + 2\mathrm{i}m\bar\psi)] \\ & + \epsilon\psi\mathcal{D}_t\psi\mathcal{D}_t\bar\psi\mathcal{E}\left\{2\mathrm{i}\frac{\partial\mathcal{F}}{\partial\bar u}(1-4m^2u\bar u) + 4\mathrm{i}m\mathcal{F}\left[-\lambda\frac{1-4m^2u\bar u}{1+2m\bar u\lambda} - 8\mathrm{i}\mathcal{D}_t u + 8mu\right]\right\} \\ & + \epsilon\psi\bar\psi\mathcal{D}_t\psi\mathcal{E}\cdot 4m\left\{\frac{\partial\mathcal{F}}{\partial\bar u}(1-4m^2u\bar u) - 2m\mathcal{F}\left[\frac{\lambda+2mu}{1+2m\bar u\lambda} + 2\mathrm{i}\mathcal{D}_t u - 4mu\right]\right\}. \end{aligned} \tag{5.34}$$

It is straightforward to evaluate the curved brackets, as the function $\mathcal{F}$ is already known (5.33). Substituting here the expression for $\mathcal{D}_t u$ (5.22), one finds that the last two lines in (5.34) combine to

$$\begin{aligned} & -16\mathrm{i}m\lambda\mathcal{E}\frac{(1+2m\bar u\lambda)(1+2mu\bar\lambda)}{(1-\lambda\bar\lambda)^2} \\ & \times[\epsilon\psi\mathcal{D}_t\psi\mathcal{D}_t\bar\psi - 2\mathrm{i}m\epsilon\psi\bar\psi\mathcal{D}_t\psi]. \end{aligned} \tag{5.35}$$

These terms identically cancel the last term in the first line in (5.34) (after integrating by parts and substituting expression for $\mathcal{D}_t u$). Thus, the anyonic action (5.31), with $\mathcal{F}$ given by (5.33), is invariant with respect to both supersymmetries.

Similarly to the action of the AdS_3 superparticle (5.29), the anyonic action has a rather simple shape when written in terms of Cartan forms:

$$S_{\text{anyon}} = -\int\left(\Omega_3 - 4\frac{\Omega_S\bar\Omega_S}{\Omega_P}\right)\Bigg|_{\theta=0}. \tag{5.36}$$

The supersymmetric extension of the rigid particle action (4.16) is a more complicated task. The corresponding action cannot be written in the terms of Cartan forms. The exact form of the action will be considered elsewhere.

VI. REDUCTIONS TO THE NONRELATIVISTIC CASE

The actions we constructed have quite a complicated structure. A possible way to simplify our system is considering the nonrelativistic limit. Before reducing the full supersymmetric case, let us shortly discuss possible reductions of the purely bosonic system, i.e. the nonrelativistic reductions of the AdS_3 algebra.

A. Bosonic reductions

In the bosonic case we have three possible nonrelativistic reductions of the AdS_3 algebra (2.6).

(i) The first reduction consists in the following rescaling of the generators:

$$Z\to\omega Z,\quad \bar Z\to\omega\bar Z,\quad T\to\omega T,\quad \bar T\to\omega\bar T \tag{6.1}$$

and then taking the limit $\omega\to\infty$. After performing of this step, we will finish with the following algebra:

$$\begin{aligned} &[J_3,T]=T, && [J_3,\bar T]=-\bar T,\\ &[J_3,Z]=Z, && [J_3,\bar Z]=-\bar Z,\\ &[P,Z]=2mZ, && [P,\bar Z]=-2m\bar Z,\\ &[T,P]=-Z, && [\bar T,P]=\bar Z. \end{aligned} \tag{6.2}$$

Clearly, the generator J is decoupled from the algebra while J_3 generates outer automorphisms.

The algebra (6.2) is just the Newton-Hooke algebra [11,12]. To bring the commutation relations to the conventional form [6,13,14] one has to redefine the generators as follows:

$$\begin{aligned} &H=P-mJ_3, \quad p=Z-mT,\\ &\bar p=\bar Z-m\bar T, \quad G=T, \bar G=\bar T, \quad J_3. \end{aligned} \tag{6.3}$$

In terms of these generators the nonzero commutators acquire a standard form:

$$\begin{aligned} &[H,G]=p, && [H,\bar G]=-\bar p,\\ &[H,p]=m^2G, && [H,\bar p]=-m^2\bar G,\\ &[J_3,p]=p, && [J_3,\bar p]=-\bar p,\\ &[J_3,G]=G, && [J_3,\bar G]=-\bar G. \end{aligned} \tag{6.4}$$

(ii) The second reduction includes an additional rescaling of the generator J:

$$\begin{aligned} &Z\to\omega Z, \quad \bar Z\to\omega\bar Z, \quad T\to\omega T,\\ &\bar T\to\omega\bar T, \quad J\to\omega^2 J. \end{aligned} \tag{6.5}$$

Taking now the limit $\omega\to\infty$ we finish with the same relations (6.2) and three new nonzero commutators:

$$\begin{aligned} &[Z,\bar Z]=4m^2J, \quad [T,\bar Z]=2mJ,\\ &[\bar T,Z]=-2mJ. \end{aligned} \tag{6.6}$$

The generator J now becomes the central charge generator, and the corresponding algebra is just the Bargmann-Newton-Hooke algebra [13], i.e. the central charge extension of the Newton-Hooke algebra. In terms of the generators (6.3) new nonzero commutators have the form

$$[p,\bar{G}] = 2mJ, \qquad [\bar{p},G] = -2mJ. \tag{6.7}$$

(iii) The third reduction includes the rescaling of both generators J and J_3:

$$Z \to \omega Z, \qquad \bar{Z} \to \omega \bar{Z}, \qquad T \to \omega T,$$
$$\bar{T} \to \omega \bar{T}, \qquad J \to \omega^2 J, \qquad J_3 \to \omega^2 J_3. \tag{6.8}$$

Again, after taking the limit $\omega \to \infty$, we obtain the following commutation relations:

$$[T,P] = -Z, \qquad [\bar{T},P] = \bar{Z},$$
$$[P,Z] = 2mZ, \qquad [P,\bar{Z}] = -2m\bar{Z},$$
$$[T,\bar{T}] = -2J_3, \qquad [T,\bar{Z}] = 2mJ,$$
$$[\bar{T},Z] = -2mJ, \qquad [Z,\bar{Z}] = 4m^2 J. \tag{6.9}$$

Now both generators J and J_3 become central elements in the algebra (6.9). Moreover, the $U(1)$ rotations generated previously by the generator J_3 disappeared from the algebra. Clearly, the corresponding generator V_3 with the relations

$$[V_3,T] = T, \qquad [V_3,\bar{T}] = -\bar{T},$$
$$[V_3,Z] = Z, \qquad [V_3,\bar{Z}] = -\bar{Z} \tag{6.10}$$

can be easily added to the algebra (6.9).

To pass to a conventional form of the algebra one has to redefine the generators as follows:

$$H = P - mV_3, \quad p = Z - mT, \qquad \bar{p} = \bar{Z} - m\bar{T},$$
$$G = T, \qquad \bar{G} = \bar{T}, \qquad J, \qquad J_3. \tag{6.11}$$

The full set of nonzero commutators read

$$[H,G] = p, \quad [H,\bar{G}] = -\bar{p},$$
$$[H,p] = m^2 G, \quad [H,\bar{p}] = -m^2\bar{G},$$
$$[V_3,p] = p, \quad [V_3,\bar{p}] = -\bar{p},$$
$$[V_3,G] = G, \quad [V_3,\bar{G}] = -\bar{G},$$
$$[p,\bar{p}] = -2m^2 J_3, \quad [p,\bar{G}] = 2m(J+J_3),$$
$$[\bar{p},G] = -2m(J+J_3), \quad [G,\bar{G}] = -2J_3. \tag{6.12}$$

Thus, we have two central charge extensions of the Newton-Hooke algebra [15]. The extension with the central charge J_3 exists in three-dimensional spacetime only and is called "exotic" central extension [16].

It should be clear that if we choose the coset element in the usual way as

$$g = e^{\mathrm{i}tP} e^{\mathrm{i}(uZ+\bar{u}\bar{Z})} e^{\mathrm{i}(\lambda T+\bar{\lambda}\bar{T})}, \tag{6.13}$$

then the central charges in the algebra (6.6) and in the algebra (6.9) will not have any realization on the coordinates and fields, while the Z, $\bar{Z}$ and T, $\bar{T}$ transformations will be realized in the same way for all three algebras:

$$\delta_Z u = e^{-2\mathrm{i}mt} a, \qquad \delta_Z \bar{u} = e^{2\mathrm{i}mt}\bar{a},$$
$$\delta_T u = \frac{e^{-2\mathrm{i}mt}-1}{2m} b, \qquad \delta_T \bar{u} = \frac{e^{2\mathrm{i}mt}-1}{2m}\bar{b}. \tag{6.14}$$

The advantage of the third reduction is the presence of two new Cartan forms for the central charge generators which, as we will see shortly, can be used to construct invariant actions.

With the coset element (6.13) the Cartan forms read

$$\omega_P = dt, \qquad \omega_T = d\lambda, \qquad \bar{\omega}_T = d\bar{\lambda},$$
$$\omega_3 = \mathrm{i}(\lambda d\bar{\lambda} - \bar{\lambda} d\lambda),$$
$$\omega_Z = du + 2\mathrm{i}mudt + \mathrm{i}\lambda dt, \qquad \bar{\omega}_Z = d\bar{u} - 2\mathrm{i}m\bar{u}dt - \mathrm{i}\bar{\lambda}dt,$$
$$\omega_J = -8m^3 u\bar{u}dt + 2\mathrm{i}m^2(\bar{u}du - d\bar{u}u) - 2m\lambda\bar{\lambda}dt$$
$$- 2\mathrm{i}m[\lambda(d\bar{u} - 2\mathrm{i}m\bar{u}) - \bar{\lambda}(du + 2\mathrm{i}mu)]. \tag{6.15}$$

Similarly to the previously considered bosonic case, one may impose the inverse Higgs effect conditions (4.4):

$$\omega_Z = \bar{\omega}_Z = 0,$$

which result in expressing λ, $\bar{\lambda}$ in terms of u and $\bar{u}$:

$$\lambda = \mathrm{i}\dot{u} - 2mu, \qquad \bar{\lambda} = -\mathrm{i}\dot{\bar{u}} - 2m\bar{u}. \tag{6.16}$$

Keeping in the mind that the action $\int \omega_P = \int dt$ is trivial, we have three possible invariant actions[4]:

$$S_0^{\mathrm{bos}} = \frac{1}{2m}\int \omega_J = \int dt[\dot{u}\dot{\bar{u}} - \mathrm{i}m(\dot{u}\bar{u} - u\dot{\bar{u}})], \tag{6.17}$$

$$S_{\mathrm{anyon}}^{\mathrm{bos}} = -\int \omega_3 = \int dt\{\mathrm{i}(\ddot{u}\dot{\bar{u}} - \dot{u}\ddot{\bar{u}})$$
$$+ 4\mathrm{i}m^2(\dot{u}\bar{u} - u\dot{\bar{u}}) - 8m\dot{u}\dot{\bar{u}}\}, \tag{6.18}$$

$$S_{\mathrm{rigid}}^{\mathrm{bos}} = \int \frac{\omega_T\bar{\omega}_T}{\omega_P} = \int dt[\ddot{u}\ddot{\bar{u}} - 2\mathrm{i}m(\ddot{u}\dot{\bar{u}} - \dot{u}\ddot{\bar{u}}) + 4m^2\dot{u}\dot{\bar{u}}]. \tag{6.19}$$

These actions may be slightly simplified by passing to the new variables

$$q = e^{-\mathrm{i}\gamma mt} u, \qquad \bar{q} = e^{\mathrm{i}\gamma mt}\bar{u}, \tag{6.20}$$

in which they acquire the form

[4] A nonlinear realization of the algebra (6.12) and the analysis of the system with S_0^{bos} were considered previously in [17].

$$S_0^{\rm bos} = \int dt\{\dot q\,\dot{\bar q} - \mathrm{i}m(1+\gamma)(\dot q\,\bar q - \dot{\bar q} q) + m^2\gamma(2+\gamma)q\bar q\}, \tag{6.21}$$

$$S_{\rm anyon}^{\rm bos} = \int dt\{\mathrm{i}(\ddot q\,\dot{\bar q} - \ddot{\bar q}\,\dot q) - 2m(4+3\gamma)\dot q\,\dot{\bar q} - 2m^3\gamma(2+\gamma)^2 q\bar q + \mathrm{i}m^2(2+\gamma)(2+3\gamma)(\dot q\,\bar q - \dot{\bar q} q)\}, \tag{6.22}$$

$$S_{\rm rigid}^{\rm bos} = \int dt\{\ddot q\,\ddot{\bar q} - 2\mathrm{i}m(1+\gamma)(\ddot q\,\dot{\bar q} - \ddot{\bar q}\,\dot q) + 2m^2(2+6\gamma+3\gamma^2)\dot q\,\dot{\bar q} - 2\mathrm{i}\gamma(1+\gamma)(2+\gamma)m^3(\dot q\,\bar q - \dot{\bar q} q) + m^4\gamma^2(2+\gamma)^2 q\bar q\}. \tag{6.23}$$

Choosing, for example, $\gamma = -1$ one may bring the action $S_0^{\rm bos}$ to the harmonic oscillator [6],

$$S_0^{\rm bos} = \int dt(\dot q\,\dot{\bar q} - m^2 q\bar q), \tag{6.24}$$

as it should be for a Newton-Hooke particle. Then the actions $S_{\rm anyon}^{\rm bos}$ and $S_{\rm rigid}^{\rm bos}$ with $\gamma = -1$ will give corresponding higher-derivative corrections to this action. All these actions are invariant under the algebra (6.9).

B. Supersymmetric nonrelativistic Newton-Hooke particle

It is possible to construct a nonrelativistic version of the $N = (2, 0)$ supersymmetric AdS_3 algebra and the corresponding superparticle actions. Remembering our rescaling of the bosonic subalgebra (6.5) and preserving the unbroken supersymmetry, one cannot rescale the generators $Q, \bar Q$. In addition, for keeping the relation $\{Q, S\} = 2\bar Z$ in order to realize some sort of inverse Higgs effect, it is required to rescale the S generator as $S \to \omega S$. Performing such a rescaling and taking the limit $\omega \to \infty$, we obtain the following superalgebra:

$$\begin{aligned}
&\{Q,\bar Q\} = 2P, \qquad \{Q,S\} = 2\bar Z,\\
&\{\bar Q,\bar S\} = 2Z, \qquad \{S,\bar S\} = -4mJ,\\
&[Z,Q] = -2m\bar S, \qquad [\bar Z,\bar Q] = 2mS,\\
&[P,S] = -2mS, \qquad [P,\bar S] = 2m\bar S,\\
&[T,Q] = -\bar S, \qquad [\bar T,\bar Q] = S,\\
&[T,P] = -Z, \qquad [\bar T,P] = \bar Z,\\
&[P,Z] = 2mZ, \qquad [P,\bar Z] = -2m\bar Z,\\
&[T,\bar T] = -2J_3, \qquad [T,\bar Z] = -2mJ,\\
&[\bar T,Z] = 2mJ, \qquad [Z,\bar Z] = 4m^2 J,
\end{aligned} \tag{6.25}$$

where the generators J and J_3 are still central elements of the superalgebra.

The coset element can be parameterized as before (3.1):

$$g = e^{\mathrm{i}tP} e^{\theta Q + \bar\theta \bar Q} e^{\boldsymbol\psi S + \bar{\boldsymbol\psi} \bar S} e^{\mathrm{i}(\mathbf{u} Z + \bar{\mathbf{u}} \bar Z)} e^{\mathrm{i}(\lambda T + \bar\lambda \bar T)}.$$

With such a coset element the Cartan forms are simplified to

$$\begin{aligned}
&\omega_P = \Delta t = dt - \mathrm{i}(\theta d\bar\theta + \bar\theta d\theta), \qquad \omega_Q = d\theta,\\
&\omega_T = d\lambda, \omega_3 = \mathrm{i}(\lambda d\bar\lambda - \bar\lambda d\lambda),\\
&\omega_Z = d\mathbf{u} + \mathrm{i}(2m\mathbf{u} + \lambda)\Delta t - 2\mathrm{i}\bar{\boldsymbol\psi} d\bar\theta,\\
&\omega_S = d\boldsymbol\psi - 2\mathrm{i}m\boldsymbol\psi\Delta t - \mathrm{i}(2m\bar{\mathbf{u}} + \bar\lambda)d\bar\theta,\\
&\omega_J = 2\mathrm{i}m(\bar{\boldsymbol\psi} d\boldsymbol\psi + \boldsymbol\psi d\bar{\boldsymbol\psi}) - 8m^2\boldsymbol\psi\bar{\boldsymbol\psi}\Delta t\\
&\quad - 8m^2(\mathbf{u}\boldsymbol\psi d\theta - \bar{\mathbf{u}}\,\bar{\boldsymbol\psi}\, d\bar\theta) - 8m^3\mathbf{u}\bar{\mathbf{u}}\Delta t - 2m\lambda\bar\lambda\Delta t\\
&\quad - 2\mathrm{i}m\lambda(d\bar{\mathbf{u}} - 2\mathrm{i}m\bar{\mathbf{u}}\Delta t - 2\mathrm{i}\boldsymbol\psi d\theta)\\
&\quad + 2\mathrm{i}m\bar\lambda(d\mathbf{u} + 2\mathrm{i}m\mathbf{u}\Delta t - 2\mathrm{i}\bar{\boldsymbol\psi} d\bar\theta)\\
&\quad - 2\mathrm{i}m^2(\mathbf{u} d\bar{\mathbf{u}} - \bar{\mathbf{u}} d\mathbf{u}).
\end{aligned} \tag{6.26}$$

Imposing the standard conditions of the inverse Higgs effect (4.4),

$$\omega_Z = 0, \qquad \bar\omega_Z = 0,$$

we get the equations

$$D\mathbf{u} = 0, \qquad \bar D\bar{\mathbf{u}} = 0, \tag{6.27}$$

$$\boldsymbol\psi = \frac{\mathrm{i}}{2} D\bar{\mathbf{u}}, \qquad \bar{\boldsymbol\psi} = \frac{\mathrm{i}}{2}\bar D\mathbf{u}, \qquad \lambda = \mathrm{i}(\dot{\mathbf{u}} + 2\mathrm{i}m\mathbf{u}), \qquad \bar\lambda = -\mathrm{i}(\dot{\bar{\mathbf{u}}} - 2\mathrm{i}m\bar{\mathbf{u}}), \tag{6.28}$$

where the flat spinor derivatives D, $\bar D$ were defined in (3.12). We are dealing with chiral superfields $\mathbf{u}$, $\bar{\mathbf{u}}$ and, therefore, the physical component fields may be defined in the usual way as

$$u = \mathbf{u}|_{\theta\to 0}, \qquad \bar u = \bar{\mathbf{u}}|_{\theta\to 0}, \qquad \psi = \boldsymbol\psi|_{\theta\to 0}, \qquad \bar\psi = \bar{\boldsymbol\psi}|_{\theta\to 0}. \tag{6.29}$$

The transformation properties of these components under the unbroken supersymmetry Q and the broken supersymmetry S are very simple:

$$\delta_Q u = -2\mathrm{i}\bar\epsilon\,\bar\psi, \qquad \delta_Q\bar u = -2\mathrm{i}\epsilon\psi, \qquad \delta_Q\psi = \bar\epsilon\,\dot{\bar u}, \qquad \delta_Q\bar\psi = \epsilon\dot u, \tag{6.30}$$

$$\delta_S u = 0, \qquad \delta_S \bar{u} = 0,$$
$$\delta_S \psi = \varepsilon e^{2\mathrm{i}mt}, \qquad \delta_S \bar{\psi} = \bar{\varepsilon} e^{-2\mathrm{i}mt}. \tag{6.31}$$

To construct the invariant supersymmetric actions generalizing the bosonic actions (4.7), (4.10), and (4.16), we must investigate S and Q supersymmetries. It is not hard to check that these actions are S invariant but not Q invariant. Thus, one has to find fermionic completions invariant under Q supersymmetry. It is rather easy to see that the following fermionic actions:

$$S_0^{\text{ferm}} = \int dt[\mathrm{i}(\dot{\psi}\,\bar{\psi} + \dot{\bar{\psi}}\psi) + 4m\psi\bar{\psi}], \tag{6.32}$$

$$S_{\text{anyon}}^{\text{ferm}} = \int dt[\dot{\psi}\,\dot{\bar{\psi}} - 4m^2\psi\bar{\psi}], \tag{6.33}$$

$$S_{\text{rigid}}^{\text{ferm}} = \int dt[\mathrm{i}(\ddot{\psi}\,\dot{\bar{\psi}} + \ddot{\bar{\psi}}\,\dot{\psi}) + 16m^3\psi\bar{\psi}] \tag{6.34}$$

have the proper dimension and are S invariant. Thus, our ansatz for the fully supersymmetric actions is

$$S_0 = S_0^{\text{bos}} + \gamma_0 S_0^{\text{ferm}}, \tag{6.35}$$

$$S_{\text{anyon}} = S_{\text{anyon}}^{\text{bos}} + \gamma_1 S_{\text{anyon}}^{\text{ferm}} + m\gamma_2 S_0^{\text{ferm}}, \tag{6.36}$$

$$S_{\text{rigid}} = S_{\text{rigid}}^{\text{bos}} + \gamma_3 S_{\text{rigid}}^{\text{ferm}} + m\gamma_4 S_{\text{anyon}}^{\text{ferm}} + m^2\gamma_5 S_0^{\text{ferm}}, \tag{6.37}$$

where $\gamma_0, \ldots, \gamma_5$ are constant parameters. Imposing invariance of these actions under the transformations (6.30), these constants can be uniquely fixed as

$$\gamma_0 = -1, \qquad \gamma_1 = 4, \qquad \gamma_2 = 8,$$
$$\gamma_3 = -1, \qquad \gamma_4 = -8, \qquad \gamma_5 = -4. \tag{6.38}$$

Thus, the full actions invariant under both the broken (S) and the unbroken (Q) supersymmetries read

$$\begin{aligned} S_0 &= \int dt\{\dot{u}\,\dot{\bar{u}} - \mathrm{i}m(\dot{u}\,\bar{u} - \dot{\bar{u}}u) - \mathrm{i}(\dot{\psi}\,\bar{\psi} - \psi\dot{\bar{\psi}}) - 4m\psi\bar{\psi}\}, \\ S_{\text{anyon}} &= \int dt[\mathrm{i}(\ddot{u}\,\dot{\bar{u}} - \dot{u}\,\ddot{\bar{u}}) - 8m\dot{u}\,\dot{\bar{u}} + 4\mathrm{i}m^2(\dot{u}\,\bar{u} - u\dot{\bar{u}}) \\ &\quad + 4\dot{\psi}\,\dot{\bar{\psi}} + 8\mathrm{i}m(\dot{\psi}\,\bar{\psi} - \psi\dot{\bar{\psi}}) + 16m^2\psi\bar{\psi}], \\ S_{\text{rigid}} &= \int dt[\ddot{u}\,\ddot{\bar{u}} - 2\mathrm{i}m(\ddot{u}\,\dot{\bar{u}} - \dot{u}\,\ddot{\bar{u}}) + 4m^2\dot{u}\,\dot{\bar{u}} \\ &\quad - \mathrm{i}(\ddot{\psi}\,\dot{\bar{\psi}} - \dot{\psi}\,\ddot{\bar{\psi}}) - 8m\dot{\psi}\,\dot{\bar{\psi}} - 4\mathrm{i}m^2(\dot{\psi}\,\bar{\psi} - \psi\dot{\bar{\psi}})]. \end{aligned} \tag{6.39}$$

Note that the Newton-Hooke superparticle actions S_0 and S_{anyon} may be represented, similarly to the bosonic case, as integrals of the Cartan forms (6.26) taken in the $\theta \to 0$ limit:

$$S_0 = \frac{1}{2m}\int \omega_J|_{\theta\to 0},$$
$$S_{\text{anyon}} = -\int \omega_3|_{\theta\to 0} + 4\int \left(\frac{\omega_S\bar{\omega}_S}{\omega_P}\right)_{\theta\to 0}. \tag{6.40}$$

Finally, the bosonic part of these actions may be slightly simplified, as in (6.21), by passing to the q, $\bar{q}$ variables (6.20). In addition, one may redefine the fermionic components in a similar way as

$$\psi = e^{-\mathrm{i}\rho mt}\xi, \qquad \bar{\psi} = e^{\mathrm{i}\rho mt}\bar{\xi}. \tag{6.41}$$

Then fixing the parameter ρ one may reach the desired form of the fermionic part of the actions. For example, choosing the new fermionic variables as

$$\xi = e^{-2\mathrm{i}mt}\psi, \qquad \bar{\xi} = e^{2\mathrm{i}mt}\bar{\psi}, \tag{6.42}$$

one may represent the fermionic parts of the actions (6.39) as

$$\begin{aligned} S_0 &= \int dt[\ldots - \mathrm{i}(\dot{\xi}\,\bar{\xi} - \xi\dot{\bar{\xi}})], \\ S_{\text{anyon}} &= \int dt[\ldots + 4\dot{\xi}\,\dot{\bar{\xi}}], \\ S_{\text{rigid}} - mS_{\text{anyon}} &= \int dt[\ldots - \mathrm{i}(\ddot{\xi}\,\dot{\bar{\xi}} - \dot{\xi}\,\ddot{\bar{\xi}})]. \end{aligned} \tag{6.43}$$

C. Comments

We make some comments concerning the nonrelativistic case.

(i) The structure of the bosonic actions (6.17) is completely fixed by the symmetry under the $(Z, \bar{Z})$ and $(T, \bar{T})$ transformations realized as (6.14)

$$\delta_Z u = e^{-2\mathrm{i}mt}a, \qquad \delta_Z \bar{u} = e^{2\mathrm{i}mt}\bar{a},$$
$$\delta_T u = \frac{e^{-2\mathrm{i}mt} - 1}{2m}b, \qquad \delta_T \bar{u} = \frac{e^{2\mathrm{i}mt} - 1}{2m}\bar{b}.$$

(ii) The fermionic components $(\psi, \bar{\psi})$ are invariant under the $(Z, \bar{Z})$ and $(T, \bar{T})$ transformations

$$\delta_Z\psi = \delta_Z\bar{\psi} = 0, \qquad \delta_T\psi = \delta_T\bar{\psi} = 0, \tag{6.44}$$

and, thus, the fermionic terms completing the bosonic actions to the full supersymmetric ones (6.39) are determined by their invariance under the S, $\bar{S}$ and Q, $\bar{Q}$ transformations (6.30) and (6.31).

(iii) The action S_0 (6.39) is some variant of generalized Galileon action (see e.g. [18,19]). Indeed, the transformations (6.14) are some variant of polynomial shift symmetries [20]. They go to the standard Galileon symmetries in the flat $(m \to 0)$ limit

$$\delta_Z u|_{m\to 0} = a, \qquad \delta_T u|_{m\to 0} = -\mathrm{i}bt. \tag{6.45}$$

(iv) Our supersymmetric actions (6.39) are a supersymmetrization of extended Galileon actions. Of course, these are not really Galileons, because in one dimension the higher-dimensional terms in the action always produce higher-order equations of motion. Nevertheless, our actions possess the proper extension (6.14) of the Galilean symmetries (6.44). Funnily enough, the simplest harmonic oscillator action (6.24) (as well as its supersymmetric extension) features such a symmetry, and it may be called *"extended Galileon"*.

(v) Due to the slightly nonstandard reduction we used (6.5), the actions (6.40) can be represented in terms of Cartan forms which behave as Wess-Zumino terms under (Z, T, S) transformations. Performing the first reduction (6.1), the forms ω_J, ω_3 will vanish and, therefore, the proper actions should be constructed in a standard way as Wess-Zumino terms [18].

VII. CONCLUSION

We have extended our previous analysis of a superparticle moving in flat $D = 2 + 1$ spacetime (including higher time derivatives) [3] to a superparticle moving on AdS_3, with $N = (2, 0)$, $D = 3$ supersymmetry partially broken to $N = 2$, $d = 1$. We have employed the coset approach to constructing the component actions. The higher time-derivative terms were chosen to preserve all (super)symmetries of the free superparticle in AdS_3. The actions have a nice form in terms of covariant Cartan forms. We also considered the nonrelativistic limit, in which our superalgebra turns into the Newton-Hooke superalgebra extended by two central charges, and the reduced actions describe a Newton-Hooke superparticle including higher-derivative terms.

Our consideration was purely classical. To analyze the effects of the higher-derivative terms one should quantize the system. In the present case this is much more complicated than for a superparticle moving in flat $D = 2 + 1$ spacetime. Already the quantization of the nonrelativistic systems constructed here should be quite useful. We are planning to come back to this task in future publications.

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