Comment on "Model of Fermions with Correlated Hopping (Integrable Cases)"

In a recent Letter [1], Karnaukhov studied the properties of electrons on a one-dimensional lattice allowing for general two-site hopping integrals ($c_{i\sigma}^\dagger$ and $c_{i\sigma}$ are canonical electron creation and annihilation operators, $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$)

$$\mathcal{T} = -\sum_i \sum_{\sigma=\uparrow\downarrow} (c_{i\sigma}^\dagger c_{i+1\sigma} + \text{H.c.}) \times [t_0 - X(n_{i,-\sigma} + n_{i+1,-\sigma}) + \overline{X} n_{i,-\sigma} n_{i+1,-\sigma}] - t_3 \sum_i (c_{i+1,\uparrow}^\dagger c_{i+1,\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow} + \text{H.c.}) \tag{1}$$

(where $X = t_0 - t_1$, $\overline{X} = t_0 - 2t_1 + t_2$ in [1]) in the presence of a Hubbard type on-site interaction with coupling constant U. To identify cases integrable by the *Bethe ansatz* method, he computes the two-particle scattering matrix and identifies two special cases where this S matrix is of the form

$$S_{12} = \frac{\vartheta(k_1) - \vartheta(k_2) \pm iP_{12}}{\vartheta(k_1) - \vartheta(k_2) \pm i}, \tag{2}$$

(P_{12} is a spin permutation operator) and hence satisfies a Yang-Baxter equation. While this is a necessary condition for the *Bethe Ansatz* to hold, it is not sufficient. Direct computation shows, however, that the only additional constraint on the hopping amplitudes t_i appears in the three-particle sector and relates the three-particle coupling $\overline{X}$ to the two-particle ones. In [1] two cases are derived where the S matrix is of the form (2) and solved by the *Bethe Ansatz*. The only difference between the corresponding models is the sign of the local Coulomb interaction U.

Here we want to point that (i) these models are only two of a one parametric family (although the generic behavior is reproduced by these special solutions) and (ii) the analysis of the ground-state properties of the system in [1] is incorrect.

Repeating the investigation of the S matrix, it is straightforward to show that the solutions are the special points $t_2 = \frac{1}{2}t_0$ and $t_0 = \frac{1}{2}t_2$ of the general solution (2) of the factorization equation with

$$\vartheta(k) = \frac{1}{2}\frac{t_2}{t_0 - t_2}\tan(k/2) \tag{3}$$

for $\frac{1}{2}U = -t_3 = t_0 - t_2 \neq 0$. The condition $t_0 t_2 = t_1^2$ for the three-particle coupling remains unchanged. Note that the sign of t_1 is not fixed by these relations: In fact, mapping $c_{i\sigma} \rightarrow c_{i\sigma}(1 - 2n_{i,-\sigma})$ has the only effect of changing $t_1 \rightarrow -t_1$. Hence the resulting model depends on a single parameter $\alpha = t_0/t_2$ (and additional chemical potential and magnetic field terms) with different solutions for $\alpha > (<)1$.

The observation complements a recent proof of integrability for this model within the framework of the quantum inverse scattering method (QISM) based on a gl(2|1)-invariant rational R matrix [2]. Hence it is a "higher spin version" of the supersymmetric t-J model. As in the latter one expects several possible *Bethe Ansatz* for the diagonalization of the corresponding transfer matrix (see, e.g., [3]). These have not yet been obtained within the algebraic framework of the QISM. However, one possible set of *Bethe Ansatz* equations is a straightforward generalization of Eqs. (13) and (14) of Ref. [1], obtained within a coordinate *Bethe Ansatz*.

The analysis of these equations allows one to study the thermodynamics of the models, e.g., ground state energy and asymptotic behavior of correlation functions at temperature $T = 0$. The resulting properties resemble those of the Hubbard model below half filling, although the explicit dependence on the system parameters is different, of course. Without external magnetic field H, the ground state is found to be a spin singlet for *all* densities. The energy difference to the ferromagnetic state claimed to be the ground state for $\alpha = 2$ below half filling in Ref. [1] is $\sim (\pi^2 \ln 2) n_e^4/3$ for small densities.

In the repulsive case $\alpha > 1$ [(a) in [1]] the system has massless spin and charge excitations, while in the attractive regime $\alpha < 1$ only charge density waves are gapless. The identification of the correct ground state and low-lying excitations is crucial for the computation of the critical exponents for the system (see, e.g., [4]). In fact, the results obtained in the sequel [5] by the same author for the attractive regime are wrong as they rely on the existence of massless spin excitations. Details of our calculations will be given elsewhere [6].

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[1] I. N. Karnaukhov, Phys. Rev. Lett. **73**, 1130 (1994).
[2] A. J. Bracken, M. D. Gould, J. R. Links, and Y.-Z. Zhang, Phys. Rev. Lett. **74**, 2768 (1995).
[3] F. H. L. Eßler and V. E. Korepin, Phys. Rev. B **46**, 9147 (1992).
[4] H. Frahm and V. E. Korepin, Phys. Rev. B **42**, 10553 (1990); **43**, 5653 (1991).
[5] I. N. Karnaukhov, Phys. Rev. B **51**, 7858 (1995).
[6] G. Bedürftig and H. Frahm, University of Hannover Report No. ITP-UH-12/95.

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