Comment on “Model of Fermions with Correlated Hopping (Integrable Cases)”

In a recent Letter [1], Karnaukhov studied the properties of electrons on a one-dimensional lattice allowing for general two-site hopping integrals ($c_i^\dagger \sigma$ and $c_i\sigma$ are canonical electron creation and annihilation operators, $n_i\sigma = c_i^\dagger \sigma c_i\sigma$)

$$T = -\sum_i \sum_{\sigma=\uparrow,\downarrow} (c_i^\dagger \sigma c_{i+1}\sigma + H.c.)$$

$$\times [t_0 - X(n_{i,-\sigma} + n_{i+1,-\sigma}) + \bar{X} n_{i,-\sigma} n_{i+1,-\sigma}]$$

$$- t_3 \sum_i (c_i^\dagger_{i+1}\sigma c_i c_{i+1}\sigma + H.c.)$$

(1)

(Where $X = t_0 - t_2$, $\bar{X} = t_0 - 2t_1 + t_2$ in [1]) in the presence of a Hubbard type on-site interaction with coupling constant $U$. To identify cases integrable by the Bethe ansatz method, he computes the two-particle scattering matrix and identifies two special cases where this $S$ matrix is of the form

$$S_{12} = \frac{\theta(k_1) - \theta(k_2) \pm iP_{12}}{\theta(k_1) - \theta(k_2) \pm i}$$

(2)

($P_{12}$ is a spin permutation operator) and hence satisfies a Yang-Baxter equation. While this is a necessary condition for the Bethe ansatz to hold, it is not sufficient. Direct computation shows, however, that the only additional constraint on the hopping amplitudes $t_1$ appears in the three-particle sector and relates the three-particle coupling $\bar{X}$ to the two-particle ones. In [1] two cases are derived where the $S$ matrix is of the form (2) and solved by the Bethe ansatz. The only difference between the corresponding models is the sign of the local Coulomb interaction $U$.

Here we want to point that (i) these models are only two of a one-parametric family (although the generic behavior is reproduced by these special solutions) and (ii) the analysis of the ground-state properties of the system in [1] is incorrect.

Repeating the investigation of the $S$ matrix, it is straightforward to show that the solutions are the special points $t_2 = \frac{1}{2}t_0$ and $t_0 = \frac{1}{2}t_2$ of the general solution (2) of the factorization equation with

$$\theta(k) = \frac{1}{2} \frac{t_2}{t_0 - t_2} \tan(k/2)$$

(3)

for $\frac{1}{2}U = -t_3 = t_0 - t_2 \neq 0$. The condition $t_0 t_2 = t_1^2$ for the three-particle coupling remains unchanged. Note that the sign of $t_1$ is not fixed by these relations: In fact, mapping $c_i\sigma \rightarrow c_i\sigma (1 - 2n_{i,-\sigma})$ has the only effect of changing $t_1 \rightarrow -t_1$. Hence the resulting model depends on a single parameter $\alpha = t_0/t_2$ (and additional chemical potential and magnetic field terms) with different solutions for $\alpha > (<) 1$.

The observation complements a recent proof of integrability for this model within the framework of the quantum inverse scattering method (QISM) based on a $gl(2|1)$-invariant rational $R$ matrix [2]. Hence it is a “higher spin version” of the supersymmetric $t$-$J$ model. As in the latter one expects several possible Bethe ansatz for the diagonalization of the corresponding transfer matrix (see, e.g., [3]). These have not yet been obtained within the algebraic framework of the QISM. However, one possible set of Bethe ansatz equations is a straightforward generalization of Eqs. (13) and (14) of Ref. [1], obtained within a coordinate Bethe ansatz.

The analysis of these equations allows one to study the thermodynamics of the models, e.g., ground state energy and asymptotic behavior of correlation functions at temperature $T = 0$. The resulting properties resemble those of the Hubbard model below half filling, although the explicit dependence on the system parameters is different, of course. Without external magnetic field $H$, the ground state is found to be a spin singlet for all densities. The energy difference to the ferromagnetic state claimed to be the ground state for $\alpha = 2$ below half filling in Ref. [1] is $\sim (\pi^2/2)n^2/3$ for small densities.

In the repulsive case $\alpha > 1$ [(a) in [1]] the system has massless spin and charge excitations, while in the attractive regime $\alpha < 1$ only charge density waves are gapless. The identification of the correct ground state and low-lying excitations is crucial for the computation of the critical exponents for the system (see, e.g., [4]). In fact, the results obtained in the sequel [5] by the same author for the attractive regime are wrong as they rely on the existence of massless spin excitations. Details of our calculations will be given elsewhere [6].

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