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Practical Model-based and Robust Control of Parallel Manipulators Using Passivity and Sliding Mode Theory

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Abstract

This chapter provides a practical strategy to realize accurate and robust control for 6 DOFs (degrees of freedom) parallel robots. The presented approach consists in two parts. The first basic part is based on the the compensation of the desired dynamics in combination with controller/observer for the single actuators. The passivity formalism offers an excellent framework to design and to tune the closed-loop dynamics, such that the desired behavior is obtained. The basic algorithm is proved to be locally robust towards uncertainties. The second part of the control strategy consists in a sliding mode controller. To keep the practical and computational efficient implementation, the proposed switching control considers explicitly only the friction model. Here we opt for the so called model-decomposition paradigm and we use additional integral action to improve robustness. The proposed approach is substantiated with experimental results demonstrating the effectiveness and success of the strategy that keeps control setup simple and intuitive.

Keywords parallel manipulators, robust control, passivity formalism, sliding mode control, desired dynamics compensation, velocity observer

1. Introduction

Due to their complexity, the practical control of parallel kinematic manipulators is challenging. The missing of appropriate control strategies plays a key role such that the promising potentials of such machines, like high dynamics and high accuracy could not be exploited satisfactorily in practice. Speaking about practical is speaking about control approaches that respect computational limitation of conventional control systems and do not require additional hardware setups, like external sensors or additional actuators.

The proposed chapter presents a complete control strategy that is suitable for parallel manipulators and that is robust to different sources of uncertainties. The issue of robust control in robotics is not quite new and has been addressed by different authors since more than two

decades. Fundamental works have been presented in (Abdallah *et al.*, 1991; Qu and Dawson, 1996). For the present study two families of robust controller are interesting: linear high gain controller, known to provide local uniform ultimate boundedness Berghuis and Nijmeijer (1994); Egeland (1987); Qu and Dawson (1996) and nonlinear structure variable or switching controller that can guarantee global stability Liu and Goldenberg (1996); Spong (1992). Even if the fundamentals of robust control have been already elaborated, their practical implementation in the industrial field has been barely considered. This is especially the case for 6 DOFs parallel robots, that are more complex and suffer especially from uncertainties due to the high coupled structure Kim *et al.* (2000). For that reason we try to close this practical gap by proposing a closed concept for the robust control of parallel manipulators.

The core of the scheme consists in the feedforward compensation of the inverse dynamics. Such type of compensation is preferable then the feedback one, since the latter requires the measurement or at least the precise knowledge of the endeffector's pose, translational and angular velocities, which is not easy to manage without additional and expensive sensors (Abdellatif and Heimann, 2007; Abdellatif *et al.*, 2005; Burdet *et al.*, 2000; Kim *et al.*, 2005; Wang and Ghorbel, 2006). The feedback controllers of the single actuators are kept linear and simple to avoid additional computational effort. The necessity of velocity error feedback for the typical stabilizing control of robotic systems is avoided by using observers of actuator's velocities Berghuis (1993); Burdet *et al.* (2000). The latter are also kept linear. The simultaneous design of controller/observer pairs is achieved by means of the passivity formalism. Both elements are tuned up, such that the closed-loop is robust against parametric uncertainties of the implemented inverse dynamics model and against the use of feedforward compensation as such, that introduces systematic errors into the control loop.

We demonstrate in this paper that the combination of desired dynamics compensation and linear robust control provides exponential ultimate boundedness. Nevertheless such approach remains conservative in the way that it demands higher feedback the more uncertainties affect the system. High feedback is limited in practice by the actuation constraints. We propose therefore to keep this basic scheme to encounter systematic or small parametric uncertainties, like those of the rigid-body model and to augment the scheme with sliding mode control. To keep the practical and computational efficient implementation, the proposed switching control considers explicitly only the friction model that is known to be more affected by time-varying uncertainties. Here we opt for the so called model-decomposition paradigm and we use additional integral action to improve robustness (Liu and Goldenberg, 1996).

The control approach is substantiated by a multitude of experimental results achieved on a directly actuated 6-DOFs parallel manipulator and by using a commercial control system. It is shown that the proposed strategy is highly appropriate to achieve high tracking accuracy at high dynamics, exploiting therefore the benefits of parallel manipulators in a practical way.

The chapter is organized as follows. Section 2 provides the reader with a preliminary discussion on the challenges that faces the control of complex parallel manipulators. Section 3 is dedicated to the passivity-based design of the control algorithm. Afterwards and in section 4, this algorithm is augmented with a sliding mode part to enhance robustness and accuracy. Section 5 provides experimental results, that substantiate the proposed control strategy.

2. Motivation and Preliminaries

2.1 Motivation for the proposed controller

Classically, the majority of model-based controller in robotics have been derived based on the famous equations of motion for Euler-Lagrangian mechanical systems:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{z})\ddot{\boldsymbol{s}} + \boldsymbol{C}(\boldsymbol{z}, \dot{\boldsymbol{s}})\dot{\boldsymbol{s}} + \boldsymbol{g}(\boldsymbol{z}), \tag{1}$$

with $\boldsymbol{\tau}$, $\boldsymbol{z}$, $\dot{\boldsymbol{s}}$ and $\ddot{\boldsymbol{s}}$ being the generalized forces, coordinates, velocities and accelerations, respectively. $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{g}$ consist in the positive definite and symmetric mass matrix, the Coriolis and centrifugal-forces matrix and the gravity vector, respectively. Notice that the generalized forces do not necessarily match with the actuating forces $\boldsymbol{Q}_a$ that correspond to the actuation variables or actuator displacements $\boldsymbol{q}_a$. From the actuation and sensing point of view, both $\boldsymbol{Q}_a$ and $\boldsymbol{q}_a$ are the only available physical interfaces to the robotic system.

The well known approaches in robotics like the computed-torque or the non-adaptive basic controller in (Slotine and Li, 1991) provide a control law that is composed of a nonlinear compensating part $\boldsymbol{u}_C$ and a stabilizing part $\boldsymbol{u}_a$, such that the actuation input is provided as

$$\boldsymbol{u} = \boldsymbol{u}_C + \boldsymbol{u}_a. \tag{2}$$

Classically, the first part $\boldsymbol{u}_C$ compensates for the nonlinear dynamics corresponding to the actual configuration $(\boldsymbol{z}, \dot{\boldsymbol{s}})$ of the robot and according to the model given by (1) or a similar variation of it. In such manner, the closed-loop dynamics is approximately linearized and could be stabilized by achieving feedback control via $\boldsymbol{u}_a$. Mostly, the latter is realized as a linear control (e.g. PD or PID) of the actuators corresponding to their respective tracking errors $\boldsymbol{e} = \boldsymbol{q}_a - \boldsymbol{q}_{a,d}$, with $\boldsymbol{q}_{a,d}$ being the desired displacements of the actuators. As it is well documented in the text book of (Qu and Dawson, 1996) and proven by a series of journal publications (Abdallah *et al.*, 1991; Berghuis and Nijmeijer, 1994; Egeland, 1987; Qu and Dorsey, 1991*a;b*), the robustness of classic model-based strategies has been demonstrated. As long as the feedback action is strong enough, the closed loop is robust against uncertainties.

The realization of any model-based control is formally straightforward for the classic open-chain robot, since the actuation or control variables coincide with the generalized ones: $\boldsymbol{z} \triangleq \boldsymbol{q}_a$ and $\dot{\boldsymbol{s}} \triangleq \dot{\boldsymbol{q}}_a = \frac{d}{dt}\boldsymbol{q}_a$. The dynamics given by (1) can be re-written in the very well known form

$$\boldsymbol{Q}_a = \boldsymbol{M}(\boldsymbol{q}_a)\ddot{\boldsymbol{q}}_a + \boldsymbol{C}(\boldsymbol{q}_a, \dot{\boldsymbol{q}}_a)\dot{\boldsymbol{q}}_a + \boldsymbol{g}(\boldsymbol{q}_a). \tag{3}$$

Since the configuration and the actuation space are the same, no mapping between both is necessary. The dynamics and therefore the control law can be calculated and derived directly from the knowledge of the actuation variables. The latter are practically always available and are provided by the actuation sensors, such incremental encoder or motor current. This advantageous case is not given for high mobility parallel manipulators. The configuration of such systems are defined with respect to the end-effector pose, velocities and accelerations $\boldsymbol{x}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ such like (1) becomes

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{x})\boldsymbol{a} + \boldsymbol{C}(\boldsymbol{x}, \boldsymbol{v})\boldsymbol{v} + \boldsymbol{g}(\boldsymbol{x}). \tag{4}$$

The computation of the nonlinear part $\boldsymbol{u}_C$ needs consequently the additional knowledge of the end-effector motion, which is not available in practice. It is therefore mandatory to have a mapping that provides the necessary but non measurable configuration variables from the measurable actuation ones. For this reason and as it is brilliantly discussed by Wang and
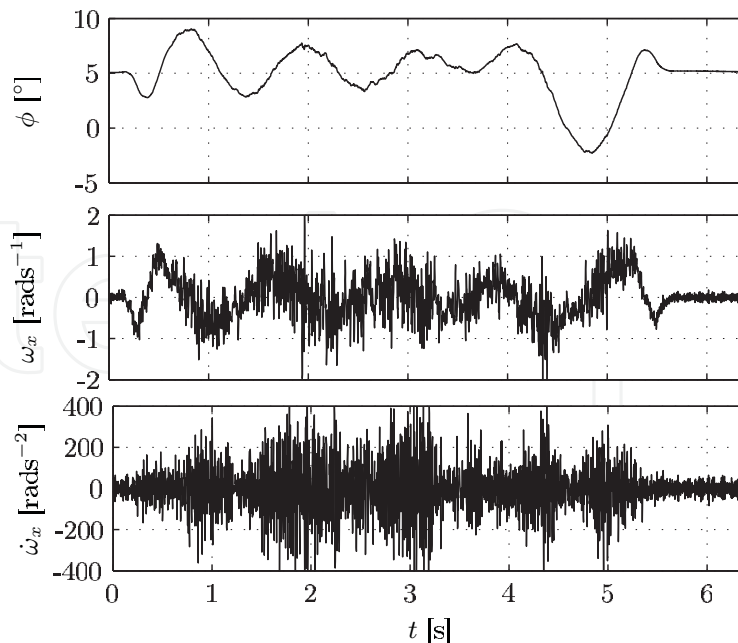
Fig. 1. Results of the direct kinematics for the first rotational DOF. Top: orientation angle, middle: angular velocity, bottom: angular acceleration.

Ghorbel (2006), a common point to model-based control schemes for parallel robots is that the direct or the forward kinematics problem (i.e. the determination of the end-effector motion given the measured joint positions) needs to be solved in real-time in order to compute the dynamics compensating term u_C (Burdet et al., 2000; Cheng et al., 2003; Kim et al., 2005; Ting et al., 2004). In general such operation do not have a closed-form solution and is achieved in an iterative numerical manner. This does not only cause a severe computational problem but yields high noisy estimation of the velocities and accelerations of the end-effector, even for very small termination conditions of the direct kinematics and especially for the rotational DOFs. The use of the Jacobian and its time derivative to calculate the velocities and accelerations may yield modest or acceptable results for lower-mobility manipulators, like reported in Cheng et al. (2003); Pietsch et al. (2005); Ren et al. (2005) and Vivas and Poignet (2005). The results are however not acceptable for accurate tracking of 6 DOFs mechanisms. Figure 1 depicts an experimental example for the first rotational DOF of a spacial parallel manipulator (see system description in section 5.1). It is obvious that both velocities and the accelerations are not suitable for providing reliable dynamics and control inputs.

A second crucial issue for the control of parallel robots is the high complexity of their dynamics, that compromises the real-time implementation of u_C. Thus, many researchers suggest the simplification of the dynamics model (Caccavale et al., 2003; Lee et al., 2003; Pietsch et al., 2005; Vivas and Poignet, 2005; Wang et al., 2007) to ensure real-time ability. This will increase the uncertainties to be counteracted by using higher feedback action. Due to the limitation of actuator energy, it is not always possible to implement high-gain control. Model approximation leads in the most of cases to a significant deterioration of the tracking quality (Abdellatif and Heimann, 2007; Denkena et al., 2006). This is especially the case for the range of high accelerations and velocities. The recommended and practical choice is to concentrate all computationally intensive terms in u_C and to keep the controller u_a linear and as simple as possible.

A third point to be considered is common to all robotic systems that are governed by (1) and stabilized by velocity feedback. The quality of actuator's velocity signal affects directly the possible range of the robust high-gain feedback (Berghuis, 1993; Slotine and Li, 1991). Since the direct measurement of actuator's velocities is not practical, the numerical calculation is highly noisy and causal filtering introduces delay, it is recommended to use observation techniques. This has been suggested in many works and in a variety of complexity (Berghuis, 1993; Celani, 2006; de Wit and Slotine, 1991). A discussion concerning this subject in relationship to fully parallel manipulators is though still missing in literature. The presented approach will contribute to close such gap.

2.2 Preliminary analysis

This section is dedicated to the analysis of different properties that are useful for the comprehension of subsequent development.

2.2.1 Passivity of parallel robots with respect to the actuation space

Rigid multi-body systems with dynamics described by (1) are known to be passive from the generalized forces τ to the generalized velocities $\dot{s}$ by satisfying following property (Ortega *et al.*, 1998)

$$\exists\, 0 < \beta < \infty, \quad \int_0^t \dot{s}^{\mathrm{T}}(t)\tau(t)dt \geq -\beta \quad \forall\, t \geq 0. \tag{5}$$

As proved in (Berghuis, 1993; Ortega *et al.*, 1998; Qu and Dawson, 1996), the passivity property results directly from the nature of the Christoffels symbols constituting the generalized Coriolis and centrifugal forces, such that the matrix $\dot{M}(z) - 2C(z, \dot{s})$ is skew symmetric $\forall\, t$. In that sense and by substituting $\dot{s}$ and τ with their corresponding values, an open-chain robot is passive from Q_{a} to $\dot{q}_{\mathrm{a}}$ (the most studied case (Berghuis, 1993; Ortega *et al.*, 1998; Slotine and Li, 1991)) and a 6 DOFs parallel robot is passive from τ to v. The latter is not directly relevant for control design, since we need the passivity of parallel robots also with respect to the actuated space from the actuation input or forces Q_{a} to the velocities of the active joints $\dot{q}_{\mathrm{a}}$. To investigate such passivity, the equations of motion (4) are transformed into the actuation space

$$Q_{\mathrm{a}} = M_{\mathrm{a}}(x)\ddot{q}_{\mathrm{a}} + C_{\mathrm{a}}(x, v)\dot{q}_{\mathrm{a}} + g_{\mathrm{a}}(x) \tag{6}$$

with

$$M_{\mathrm{a}}(x) = J^{\mathrm{T}}(x)M_{\mathrm{a}}(x)J(x)$$
$$C_{\mathrm{a}}(x, v) = J^{\mathrm{T}}(x)C(x, v)J(x) + J^{\mathrm{T}}(x)M(z)\dot{J}(x),$$
$$g_{\mathrm{a}}(x) = J^{\mathrm{T}}(x)g(x)$$

and $J(x) = \frac{\partial v}{\partial \dot{q}_{\mathrm{a}}}$ being the jacobian matrix of the robot. For non-singular jacobian[1] the mass matrix M_{a} is also positive definite. Due to the transformation, the term $C_{\mathrm{a}}(x, v)$ does not satisfy the properties of the Christoffel's symbols, such that the skew symmetry of $\dot{M}_{\mathrm{a}} - 2C_{\mathrm{a}}$ is not evident anymore. However the relevant skew-symmetric property

$$u^{\mathrm{T}}\left(\dot{M}_{\mathrm{a}} - 2C_{\mathrm{a}}\right)u = 0 \quad \forall\, u \in \mathbb{R}^6 \tag{7}$$

---

[1] The parallel manipulator is assumed to be mechanically designed, such that a singularity in the workspace is avoided

can be proven (see Appendix). The availability of the fundamental requirement (7) allows to demonstrate the passivity of 6 DOFs parallel manipulator from Q_a to $\dot{q}_a$ in a straightforward manner.

It is important to point out, that even if the dynamics equations (6) are available with respect to the actuation space, the non-measurable variables x and v are still necessary to calculate the different equation parts. Furthermore, the term $C_a(x, v)\dot{q}_a$ decreases the flexibility and variability of the control design in contrast to the case of serial robots. For the latter the Coriolis and centrifugal forces $C_a(q_a, \dot{q}_a)\dot{q}_a$ (see eq. (3)) allows a variable interchanging of desired and actual velocities in the corresponding control term to shape the energy of the closed-loop system in a more sophisticated way (Berghuis, 1993; Slotine and Li, 1991; Wen and Bayard, 1988). Due to this fact and since Coriolis and centrifugal forces are directly responsible for the global stability of Euler-Lagrange systems, the conditions on the control parameters for parallel manipulators are more conservative than those of classic open-chain systems (see section 3 for discussion).

Before proposing the control design, it is necessary to recall that the different components of the dynamics equations are bounded, that is

$$0 < \underline{m} \le \|M_a(x)\| \le \overline{m} \quad \forall x \tag{8}$$

$$\|C_a(x, v)\| \le \overline{c}\|\dot{q}_a\| \quad \forall x, v. \tag{9}$$

with $\|\cdot\|$ being the euclidean norm and where $\underline{x}$ and $\overline{x}$ denote generally the minimal and maximal eigenvalue of a Matrix X, respectively. Finally the dynamics of a robotic parallel manipulator is available in a linear form with respect to a minimal set of parameters p:

$$M_a(x)\ddot{q}_a + C_a(x, v)\dot{q}_a + g_a(x) \equiv A(x, v, a)p \tag{10}$$

which is known to be the computationally most efficient (Abdellatif *et al.*, 2005).

2.2.2 Impact of desired dynamics compensation

The desired dynamics compensation is achieved by the choice

$$u_C \triangleq M_a(x_d)\ddot{q}_{a,d} + C_a(x_d, v_d)\dot{q}_{a,d} + g_a(x_d) = A(x_d, v_d, a_d)p \tag{11}$$

where $'d'$ being the subscript that distinguishes desired variables. By considering (11), (2) and (6) and by assuming - at this stage of analysis - a perfect model knowledge the following equation

$$M_a(x)\ddot{e} + C_a(x, v)\dot{e} - u_a - \underline{\underline{\Delta}} = 0. \tag{12}$$

results for the closed-loop dynamics. The term $\underline{\underline{\Delta}}$ is equal to

$$\underline{\underline{\Delta}} = (M_a(x_d) - M_a(x))\ddot{q}_{a,d} + (C_a(x_d, v_d) - C_a(x, v))\dot{q}_{a,d} \tag{13}$$
$$+ (g_a(x_d) - g_a(x))$$

and corresponds to the systematic errors introduced by using feedforward or desired dynamics compensation instead of feedback- or actual dynamics compensation. With help of the dynamics properties (8,9) it can be demonstrated that such term is bounded (Burdet *et al.*, 2000; Qu and Dawson, 1996)

$$\|\underline{\underline{\Delta}}\| \le \overline{\alpha}\|e\| + \overline{c}v_+\|\dot{e}\| \quad \forall t, x, v \text{ and } a \tag{14}$$

with $\overline{\alpha}$ being a strict positive constant and $v_+ = \sup_t \|\dot{q}_{a,d}(t)\|$. The boundedness of the systematic error norm $\|\underline{\underline{\Delta}}\|$ is a fundamental property for the design of control schemes with desired dynamics compensation.

3. Robust Control with Desired Dynamics Compensation

The proposed control scheme is basically composed of the compensation term given by (11) and underlying independent control loops for the single actuators. These are linear control/observer combinations that are to be tuned according to the passivity formalism. This proposed basic control scheme is the adaptation of the approach proposed by (Berghuis and Nijmeijer, 1994) to the case of desired dynamics compensation. The same idea was also briefly studied by (Burdet *et al.*, 2000) but remained without successful experimental implementation.

3.1 Proposed control scheme

Based on the works in (Berghuis, 1993; Qu and Dawson, 1996) we propose for a 6 DOFs parallel manipulator the following robust and computationally high efficient controller

$$u = \underbrace{A\left(x_{\mathrm{d}}, v_{\mathrm{d}}, a_{\mathrm{d}}\right)\hat{p}}_{u_{\mathrm{C}}} \underbrace{-K_{\mathrm{D}}\left(s_1 - s_2\right)}_{u_{\mathrm{a}}}, \tag{15}$$

where $\hat{p}$ is the estimate of the real parameters. The matrix K_{D} is positive definite. The control variables are defined as follows

$$s_1 = \dot{e} + \Lambda_1 e$$
$$s_2 = \dot{\hat{e}} + \Lambda_2 \hat{e},$$

where both Λ_1 and Λ_2 are positive definite matrices, $e = q_{\mathrm{a}} - q_{\mathrm{a,d}}$ and $\hat{e} = q_{\mathrm{a}} - \hat{q}_{\mathrm{a}}$ denote the tracking and observer errors, respectively. The vectors s_1 and s_2 correspond to first order sliding tracking and observer variables, respectively (Slotine and Li, 1991). It is here important to notice that due to the assumed absence of the actuator velocity signals $\dot{q}_{\mathrm{a}}$ either s_1 nor s_2 can be calculated separately. However, their difference is obtainable for the feedback term u_{a} from the available signals. It is straightforward to prove that

$$s_1 - s_2 = \dot{\hat{q}}_{\mathrm{a}} - \Lambda_2\left(q_{\mathrm{a}} - \hat{q}_{\mathrm{a}}\right) - \dot{q}_{\mathrm{a,d}} + \Lambda_1\left(q_{\mathrm{a}} - q_{\mathrm{a,d}}\right)$$

contains only available signals. The velocity observer is proposed as suggested by Berghuis and Nijmeijer (1994)

$$\begin{aligned}
\dot{\hat{q}}_{\mathrm{a}} &= z_{\mathrm{o}} + L_{\mathrm{D}}\left(q_{\mathrm{a}} - \hat{q}_{\mathrm{a}}\right) \\
\dot{z}_{\mathrm{o}} &= \ddot{q}_{\mathrm{a,d}} + L_{\mathrm{P}}\left(q_{\mathrm{a}} - \hat{q}_{\mathrm{a}}\right)
\end{aligned} \tag{16}$$

with z_{o} being the internal observer variable, $L_{\mathrm{D}} = l_{\mathrm{D}}I + \Lambda_2$, $L_{\mathrm{P}} = l_{\mathrm{D}}\Lambda_2$ being symmetric positive definite and $l_{\mathrm{D}} > 0$ is a strict positive scalar quantity. The observer error dynamics are obtained from (16)

$$\ddot{e} = \ddot{\hat{e}} + L_{\mathrm{D}}\dot{\hat{e}} + L_{\mathrm{P}}\hat{e}$$

yielding

$$\ddot{e} = \ddot{\hat{e}} + (l_{\mathrm{D}}I + \Lambda_2)\dot{\hat{e}} + l_{\mathrm{D}}\Lambda_2\hat{e} = \dot{s}_2 + l_{\mathrm{D}}s_2. \tag{17}$$

The control error dynamics are obtained by combining (6) and (15)

$$M_{\mathrm{a}}(x)\ddot{e} + C_{\mathrm{a}}(x, v)\dot{e} + K_{\mathrm{D}}(s_1 - s_2) - \underline{\underline{\Delta}} - \underline{\Delta} = 0. \tag{18}$$

Besides the systematic errors $\underline{\Delta}$ introduced by the desired dynamics compensation (see section 2.2.2), the term $\underline{\underline{\Delta}}$ arises in the last equation. It results from model uncertainties that may be the consequence of biased parameter estimate $\hat{p}$ or unmodeled dynamics Q_{dist}. By considering the realistic assumption of bounded disturbances $\|Q_{\text{dist}}\| < \overline{Q} \ \forall\, t > 0$ we obtain an upper bound on $\underline{\underline{\Delta}}$

$$\|\underline{\underline{\Delta}}\| \le \|A\,(x_{\text{d}}, v_{\text{d}}, a_{\text{d}})\,\overline{\Delta p}\| + \overline{Q}. \tag{19}$$

where the parameter uncertainties $\overline{\Delta p}$ can be calculated by evaluating the confidence intervals of the estimate as known from the identification theory (Abdellatif *et al.*, 2008).
Considering both control and observer dynamics (17) and (18) the following closed-loop dynamics are obtained[2]

$$M_{\text{a}}\ddot{e} + C_{\text{a}}s_1 = -K_{\text{D}}s_1 + K_{\text{D}}s_2 + C_{\text{a}}\Lambda_1 e + \underline{\Delta} + \underline{\underline{\Delta}}, \tag{20}$$

$$M_{\text{a}}\dot{s}_2 + C_{\text{a}}s_2 = -K_{\text{D}}s_1 + (-l_{\text{D}}M_{\text{a}} + K_{\text{D}} + C_{\text{a}})\,s_2$$
$$- C_{\text{a}}\dot{e} + \underline{\Delta} + \underline{\underline{\Delta}}. \tag{21}$$

For the obtained nonautonomous nonlinear system the energy function

$$V = H_1 = \frac{1}{2}s_1^{\text{T}}M_{\text{a}}s_1 + \frac{1}{2}e^{\text{T}}K_1 e + \frac{1}{2}s_2^{\text{T}}M_{\text{a}}s_2 + \frac{1}{2}\hat{e}^{\text{T}}K_2\hat{e} \tag{22}$$

is a Lyapunov-function (Berghuis, 1993; Qu and Dawson, 1996), with

$$K_1 = \Lambda_1(2\Lambda_1^{-1}K_{\text{D}} - M_{\text{a}})\Lambda_1 \quad \text{and} \quad K_2 = 2\Lambda_2^{-1}K_{\text{D}}.$$

By defining the error state vector

$$z_{\text{e}} = \left[\dot{e}^{\text{T}}\ (\Lambda_1 e)^{\text{T}}\ \dot{\hat{e}}^{\text{T}}\ (\Lambda_2\hat{e})^{\text{T}}\right]^{\text{T}}$$

we obtain

$$V = \frac{1}{2}z_{\text{e}}^{\text{T}}P z_{\text{e}} \tag{23}$$

with

$$P_1 = \begin{bmatrix} \begin{bmatrix} M_{\text{a}} & M_{\text{a}} \\ M_{\text{a}} & 2\Lambda_1^{-1}K_{\text{D}} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} M_{\text{a}} & M_{\text{a}} \\ M_{\text{a}} & M_{\text{a}} + 2\Lambda_2^{-1}K_{\text{D}} \end{bmatrix} \end{bmatrix}.$$

Using (20), (21) and the skew symmetric property of $\dot{M}_{\text{a}} - 2C_{\text{a}}$ (see Appendix) the time derivative of V results in

$$\dot{V}(z_{\text{e}}, t) = -z_{\text{e}}^{\text{T}}Q z_{\text{e}} - s_2^{\text{T}}\left[l_{\text{D}}M_{\text{a}}(x) - 2K_{\text{D}} - C_{\text{a}}(x, v)\right]s_2$$
$$+ \dot{e}^{\text{T}}C_{\text{a}}(x, v)\Lambda_1 e - s_2^{\text{T}}C_{\text{a}}(x, v)\dot{e}$$
$$+ \left(s_1^{\text{T}} + s_2^{\text{T}}\right)\left(\underline{\Delta} + \underline{\underline{\Delta}}\right) \tag{24}$$

---

[2] The arguments x and v were omitted for convenience

with

$$
Q = \begin{bmatrix} \begin{bmatrix} K_D - \Lambda_1 M_a & 0 \\ 0 & K_D \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} K_D & 0 \\ 0 & K_D \end{bmatrix} \end{bmatrix} .
$$

Furthermore and according to the property (8), the time derivative of the Lyapunov-function is bounded (Qu and Dawson, 1996)

$$
\dot{V}(z_e, t) \leq \phi_0 \|z_e(t)\| - \phi_1 \|z_e(t)\|^2 + \phi_2 \|z_e(t)\|^3
$$

with

$$
\phi_0 = 2\sqrt{2} \left(\|A(x_d, v_d, a_d) \overline{\Delta p}\| + \overline{Q} \right), \tag{25}
$$

$$
\phi_1 = \underline{k_D} - \overline{\lambda_1 m} - (1 + 3\sqrt{2})\bar{c} v_+ - 2\sqrt{2}\,\overline{\lambda_1}^{-1}\overline{\alpha}, \tag{26}
$$

$$
\phi_2 = (1 + 3\sqrt{2})\bar{c} \tag{27}
$$

resulting from the error dynamics of the here studied case. For given initial error $z_e(0)$ the closed-loop system is locally uniformly and ultimately bounded when the following inequalities are fulfilled

$$
\begin{aligned}
\phi_1 &> 2\sqrt{\phi_0 \phi_2}, \\
\phi_1^2 + \phi_1 \sqrt{\phi_1^2 - 4\phi_0\phi_2} &> 2\phi_0\phi_2 \left(1 + \sqrt{\tfrac{p_M}{p_m}} \right), \\
\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2} &> 2\phi_2 \|z_e(0)\| \sqrt{\tfrac{p_M}{p_m}}.
\end{aligned} \tag{28}
$$

The variables p_m and p_M can be obtained from the eigenvalues of P_1, as given by Berghuis and Nijmeijer (1994)

$$
\begin{aligned}
p_m &= \tfrac{1}{3}\underline{p} &= \tfrac{1}{3}\min\{\underline{m}, 2\overline{\lambda_2}^{-1}\overline{\lambda_1 m}\} \\
p_M &= 3\overline{p} &= 3\max\{2\underline{\lambda_1}^{-1}\overline{k_D}, 2\underline{\lambda_2}^{-1}\overline{k_D}\}
\end{aligned} . \tag{29}
$$

It is then possible to chose the matrices K_D, Λ_1 and Λ_2 such that their eigenvalues satisfy (28,29), i.e.

$$
l_D > \underline{m}^{-1} \left[2\overline{k_D} + \bar{c} v_+ \right], \tag{30}
$$

$$
\underline{k_D} > \phi_1 + \overline{\lambda_1 m} + (1 + 3\sqrt{2})\bar{c} v_+ + 2\sqrt{2}\,\overline{\lambda_1}^{-1}\overline{\alpha} \tag{31}
$$

which finishes the control design procedure. An additional benefit is the analytical availability of the radius R of the region of final error convergence

$$
R = \frac{2\phi_0}{\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2}}. \tag{32}
$$

It is straightforward to deduce that the theoretical case of a perfect model ($\phi_0 = 0$) provides semi-global exponential stability under the regarded controller/observer combination (15,16). In contrast to the work Berghuis (1993) for serial manipulators, two major differences can be stressed out. First, the presented robust control scheme for 6 DOFs parallel manipulators uses consequently the compensation of desired dynamics. Second, the necessity of x and v for the calculation of the inverse dynamics and especially the Coriolis and centrifugal terms shrinks the theoretically possible region of attraction. Both effects yield more conservative conditions on boundedness and therefore on stability, i.e. the terms $2\sqrt{2}\,\overline{\lambda_1}^{-1}\overline{\alpha}$ and $(1 + 3\sqrt{2})\bar{c} v_+$ in (26).

3.2 Considering friction

For the sake of simplicity, friction has not been regarded in the above discussed design. This is not associated with a loss of generality, since friction preserves the passivity of the system (Berghuis, 1993; Slotine and Li, 1991). For the exemplarily case of the classic modeling approach of a superposition of coulomb friction and viscous damping, we obtain for every passive or active joint i

$$Q_{f_i} = f_{1i} \operatorname{sgn}(\dot{q}_i) + f_{2i} \dot{q}_i. \tag{33}$$

The overall friction that occurs in each actuator is obtained from (33) by means of kinematic transformation (Abdelllatif *et al.*, 2007):

$$\boldsymbol{F}_a = \left(\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{\boldsymbol{q}}_a} \right)^{\mathrm{T}} \boldsymbol{Q}_f = \boldsymbol{A}_f \left(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{a} \right) \boldsymbol{p}_f.$$

The resulting model is also linear with respect to the parameter vector $\boldsymbol{p}_f$ that groups all friction coefficients f_{1i} and f_{2i}. Consequently, the compensating term (11) can be updated by a friction part:

$$\boldsymbol{u}_C = \boldsymbol{A} \left(\boldsymbol{x}_d, \boldsymbol{v}_d, \boldsymbol{a}_d \right) \hat{\boldsymbol{p}} + \boldsymbol{A}_f \left(\boldsymbol{x}_d, \boldsymbol{v}_d, \boldsymbol{a}_d \right) \hat{\boldsymbol{p}}_f.$$

The parametric uncertainties is consequently updated by the friction parameter estimate bias and (19) may be re-written to

$$\|\boldsymbol{\Delta}\| \leq \|\boldsymbol{A} \left(\boldsymbol{x}_d, \boldsymbol{v}_d, \boldsymbol{a}_d \right) \overline{\Delta \boldsymbol{p}}\| + \|\boldsymbol{A}_f \left(\boldsymbol{x}_d, \boldsymbol{v}_d, \boldsymbol{a}_d \right) \overline{\Delta \boldsymbol{p}_f}\| + \overline{Q}.$$

As it is known for the Lyapunov-based design, the additional uncertainties yield more conservative bounds and therefore more conservative choice of the controller parameters. This is especially the case for parallel robots, since the friction forces discussed here depend on the system's configuration. As demonstrated in (Abdelllatif *et al.*, 2007), the resulting friction that is to be counteracted by an actuator j is expressed as:

$$F_{a_j} = r_{1j}(\boldsymbol{x}) \operatorname{sgn}(\dot{q}_{a_j}) + r_{2j}(\boldsymbol{x}) \dot{q}_{a_j} \tag{34}$$

and is not only dependent on the actuator velocity but also on the pose $\boldsymbol{x}$ of the manipulator. The upper bounds of $\|\underline{\boldsymbol{\Delta}}\|$ can be extended to

$$\|\underline{\boldsymbol{\Delta}}\| \leq \overline{r_2} + \overline{\alpha}\|e\| + (\overline{r_1} + \overline{c}v_+)\|\dot{e}\|$$

and integrated in the design procedure. Since r_1 and r_2 are widely varying over the workspace, their upper estimates $\overline{r_1}$ and $\overline{r_2}$ increase the conservatism of the control design, in comparison e.g. to open-chain robots, where the friction forces depend only on the actuator's velocity. The interested reader is referred to the article (Abdelllatif *et al.*, 2007) for deeper insight into the consideration of friction for parallel manipulators.

4. Augmenting the Scheme with Sliding Mode Control

We demonstrate in the previous section that the combination of desired dynamics compensation and linear feedback provides robustness in the sense of local exponential ultimate boundedness. Such approach remains conservative in the way that the robustness is achieved primarily through higher feedback. High feedback is limited in practice by the actuation constraints. Alternatively, nonlinear sliding mode (or switching) control strategy could provide

robustness in a global manner (Slotine and Li, 1991). Therefore, we propose to extend the
basic algorithm given by (2) to

$$u = u_C + u_a + u_R,$$ (35)

with u_R being the robustifying switching control term. The basic scheme is kept to encounter
systematic or parametric uncertainties, like those resulting from biased estimated rigid-body
model parameters. It is now extended with the new term u_R. Inspired by our long experience
with parallel manipulators and in order to keep the practical and computational efficient im-
plementation, the proposed switching control considers explicitly only the friction model that
is more affected by time varying uncertainties and not only by constant bias.

The proposed control approach combines and merges a multitude of schemes, that have been
proposed in early years for serial manipulators. Primarily we use the parameter-based sat-
uration principle as proposed among others by Spong (1992) and we opt for the so called
model-decomposition paradigm of Liu and Goldenberg (1996) to limit the robust action on
the friction part. Our contributions are: first, to extend such strategies to the case of desired
dynamics compensation; second, to consider the observer dynamics within the control law
and last, to implement the control for the case of complex spatial parallel manipulators.

4.1 Proposed scheme with sliding mode control

The proposed extended controller is the following:

$$u = \underbrace{A\left(x_d, v_d, a_d\right)\hat{p} + A_f\left(x_d, v_d, a_d\right)\hat{p}_f}_{u_C} \underbrace{-K_D\left(s_1 - s_2\right)}_{u_a}$$
$$+ \underbrace{A_f(x_d, v_d, a_d)\,u_f}_{u_R},$$ (36)

with u_f being dimensionally equal to the friction parameter vector p_f: $(\dim(u_f) = n_f)$ and is
a robust parametric correction vector:

$$u_{f,k} = \begin{cases} -\rho_k \dfrac{Y_{f,k}}{\|Y_{f,k}\|} & \text{if } \|Y_{f,k}\| > \epsilon_k \\ -\dfrac{\rho_k}{\epsilon_k}Y_{f,k} - K_{I,k}\int_{t_0}^t Y_{f,k}\,d\tau & \text{if } \|Y_{f,k}\| \le \epsilon_k \end{cases}, \quad \text{for } k = 1 \dots n_f.$$ (37)

and

$$Y_f = A_f^T\left(x_d, v_d, a_d\right)\left(s_1 - s_2\right).$$ (38)

Both parameters ρ_k and ϵ_k can be adjusted individually for each friction parameter $p_{f,k}$. They
correspond in the sense of saturation control to the uncertainty bounds and to the width of the
boundary layers, respectively. The parameter ρ_k depends on the modeling and the parameter
estimate precision, whereas ϵ_k is a positive control parameter, that have to be chosen with
respect to the control goals. In classical approaches the boundary layer is shaped as thin as
possible by using very small ϵ to guarantee high tracking accuracy. This implies high feedback
action with its all related disadvantages. Therefore, Liu and Goldenberg has proposed the
integral control action, given above in the second equation (37). This is motivated by the fact,
that some aspects of parametric uncertainties like estimate bias affect the system as an offset
in the parameter domain. Thus, integral action in the same domain is the adequate method to
counteract such type of uncertainties. To avoid windups due to large initial errors, the integral
action is restricted to the case when the errors are small and are within the boundary layer.

The integral term helps enlarging the boundary layer ϵ and therefore decreasing the feedback action by keeping the same tracking accuracy. It is highly recommended for use in practice. The proof of uniform ultimate boundedness can follow by combining the method shown in the previous section 2.2.1 and the procedure demonstrated in (Liu and Goldenberg, 1996). The proof - although straightforward - is too long to be put here. It shall be noticed that the Lyapunov function candidate remains the same as (22) for the region outside the boudary layer, and is extended with the term $\frac{1}{2}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{K}_{\mathrm{I}}\boldsymbol{\xi}$ in the contrary case. Here

$$\boldsymbol{\xi} = \boldsymbol{K}_{\mathrm{I}}^{-1}\Delta\boldsymbol{p}_{\mathrm{f}} - \int_{t_0}^{t}\boldsymbol{Y}_{\mathrm{f}}d\tau. \tag{39}$$

The next section provides a discussion on the provided sliding mode controller as well as its comparison to alternative approaches from literature.

4.2 Discussion

As previously mentioned, the proposed control given by (37-38) results from merging classic approaches provided for serial manipulators and their adaptation to the case of desired dynamics compensation with additional consideration of observer dynamics. In the original approaches Liu and Goldenberg (1996); Spong (1992), the vector $\boldsymbol{Y}_{\mathrm{f}}$ resulted by using only the sliding variable $\boldsymbol{s}_1$ (or similar variations of it), which is in general noisy[3] and cause the shrinking of control band width. Using the smoother variable $(\boldsymbol{s}_1 - \boldsymbol{s}_2)$ as well as the noise-free desired values $\boldsymbol{x}_{\mathrm{d}}$, $\boldsymbol{v}_{\mathrm{d}}$ and $\boldsymbol{a}_{\mathrm{d}}$ allows for more freedom when tuning the feedback gain or adjusting the boundary layer parameters ρ and ϵ. Even if the theory provides global uniform and ultimate boundedness, practically the use of sliding observer component plays a key role in the amelioration of tracking accuracy. It is believed, that the experimental studies in many publications would provide better result, if an observer has been implemented.

Even if the control laws (37-38) appear complicated, they do not cause any major losing of computational efficiency. This is due to the fact, that we consciously limited the switching control to the friction model. The related part $\boldsymbol{A}_{\mathrm{f}}$ is very simple to obtain by 36 additions and 54 multiplications (Abdellatif et al., 2005). Extending the switching control to the rigid-body part is not very efficient, since the rigid-body model requires about twenty times more computational effort. It is questionable to spend so much effort to counteract uncertainties of rigid-body parameters, that and in exception of playloads are not affected by important uncertainties. This is an additional important difference between our algorithms and other alternative robust switching controllers developed for parallel manipulators (Kim et al., 2000; 2005; 2001).

5. Experimental Study

This section is dedicated to the experimental implementation of the proposed control strategy on a 6 DOFs spatial parallel manipulator, which will be presented briefly in the first subsection.

5.1 Case study hexapod

All proposed approaches are substantiated on the hexapod PaLiDA (see Fig. 2), that has been designed and constructed by the institute of production engineering and machine tools of the university of Hannover. The machine is equipped with fast direct drives variable in length and

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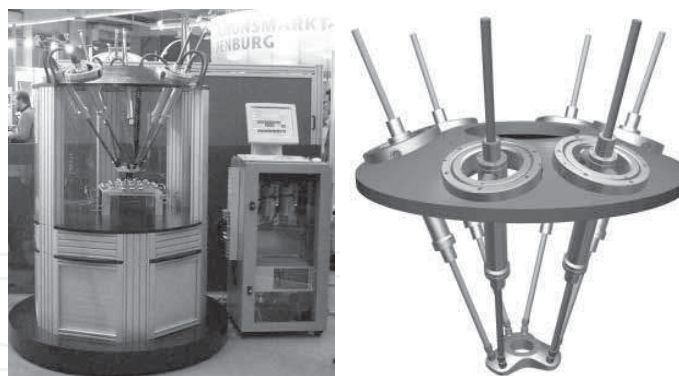[3] since the velocity errors are calculated by numerical differentiation

Fig. 2. Case study: hexapod PaLiDA. Left: presentation at the Hannover industrial fair, right: CAD model

has been designed to be a mixture of a high-speed manipulator and a tool machine (Denkena *et al.*, 2006). The application area covers fast handling and light cutting machining tasks with low process forces. Central requirement is therefore to ensure acceptable tracking errors at highest possible velocities and accelerations at the presence of disturbances. The maximal actuation force is about 230N, whereas actuator accelerations of about 2-3 g could be achieved. The internal hall sensors are affected with significant measurement noise, such that any feedback strategy of numerically differentiated variables is challenging. The control system consists in a commercial dSPACE Power-PC 604e single processor unit (333 MHz). The sample time is 0.5 ms. The proposed control approach requires (including path-planning) about 0.15 ms of computational time, which demonstrates its efficiency.

k	p	unit	$\hat{p}$
1	f_{1_α}	[Nm]	0.654
2	f_{1_β}	[Nm]	0.675
3	f_{1_1}	[N]	8.148
4	f_{1_2}	[N]	5.288
5	f_{1_3}	[N]	16.574
6	f_{1_4}	[N]	7.743
7	f_{1_5}	[N]	6.295
8	f_{1_6}	[N]	9.525
9	f_{2_1}	[Nsm^{-1}]	18.774
10	f_{2_1}	[Nsm^{-1}]	16.092
11	f_{2_1}	[Nsm^{-1}]	4.428
12	f_{2_1}	[Nsm^{-1}]	17.444
13	f_{2_1}	[Nsm^{-1}]	17.915
14	f_{2_1}	[Nsm^{-1}]	3.454

Table 1. friction parameters p_k with corresponding estimated values $\hat{p}_k$

As it has been derived and deeply discussed in former publications (see e.g. (Abdellatif and Heimann, 2007; Abdellatif *et al.*, 2005)) the dynamics model used for the feedforward controller u_C contains 24 minimal parameters. The rigid body model part corresponds to a set of
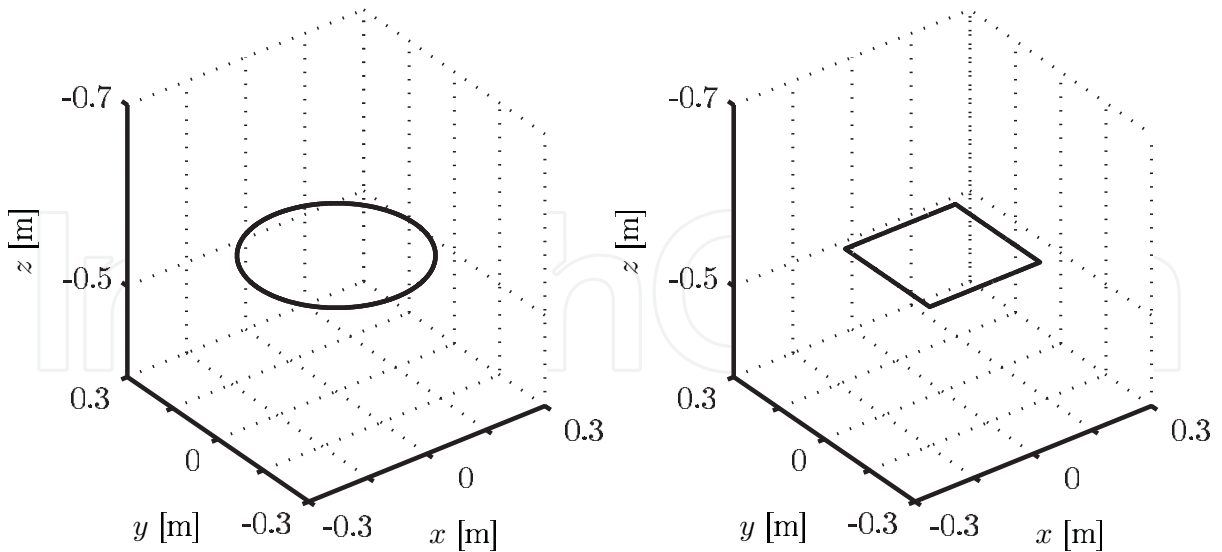
Fig. 3. Investigated motions for the experimental study, left: motion 1, right motion 2.

10 minimal parameters p. As given by the modeling approach (33), the friction of each of the 6 actuators j is modeled by a dry friction coefficient f_{1_j} and a viscous dumping coefficient f_{2_j}. Friction in the passive joints is modeled only as dry friction with a common parameter for all α_j (the first revolute joint of each strut) and another one for all β_j-joints (the second revolute joint of each strut). The friction model contains therefore 14 different parameters (see Table 1 for an overview and (Abdelllatif *et al.*, 2007) for more details).

For the experimental validation of the proposed control, two exemplarily test motions (see Fig. 3) are chosen. Both of them are demanding in point of view of achieved dynamics and velocities. The circular motion ($\varnothing = 20mm$) allows for highest actuation forces of about $230\,N$. The quadratic one (edge length$= 28mm$) allows for highest possible actuator velocities of about $1.5\,ms^{-1}$. To keep the influence of kinematic accuracy the same over the experiments, we chose both test motions in the middle of the workspace and at the same hight. The experimental comparison focuses on the two proposed schemes: the passivity based approach presented in section 3, which is denoted in the following by (P-FF) and the scheme with additional Sliding mode control (P-SM). Additionally, both proposed schemes are compared to the classic feedforward computed-force technique (CF-FF), that consists in forwarding the dynamic model by keeping the actuators controlled by standard PID (or PD) controllers (see Abdellatif *et al.* (2005)). The common control parameters for P-FF and P-SM are set equal for a meaningful comparison. For the classic CF-FF approachh, the PID parameters are tuned heuristically, but the derivative part could not has set as high as for the passivity-based schemes, due to the absence of a velocity observer. This can be stated already as an improvement of the proposed control, that allows for higher control bandwidth thanks to the integration of the observer. All different control parameters used in the following experimental study are given in Table 2.

5.2 Experimental results

The first experiment consists in comparing the three control strategies CF-FF, P-FF und P-SM. For this purpose, the identification of the model parameters has been achieved to obtain reliable estimate for $\hat{p}$ and $\hat{p}_f$ (see (Abdellatif *et al.*, 2008) and results in Table 1). To examine the robustness towards parametric uncertainties, different friction parameters $\bar{p}_f = \frac{1}{2}\hat{p}_f$ has

	K_P	k_I	K_D	Λ_1	Λ_2	l_D	ρ	ϵ	$K_I[10^3]$
CF-FF	38000	85000	1000	-	-	-	-	-	-
PF-FF	-	-	1400	42	42	180	-	-	-
PF-SM	-	-	1400	42	42	180	$\begin{bmatrix} 6.32 \\ 7.15 \\ 99.39 \\ 74.43 \\ 129.05 \\ 72.67 \\ 104.74 \\ 140.78 \\ 210.72 \\ 148.49 \\ 228.80 \\ 151.94 \\ 178.42 \\ 221.57 \end{bmatrix}$	$\begin{bmatrix} 0.06 \\ 0.06 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \\ 0.16 \end{bmatrix}$	$\begin{bmatrix} 0.31 \\ 0.35 \\ 49.69 \\ 37.21 \\ 64.52 \\ 36.33 \\ 52.37 \\ 70.39 \\ 105.36 \\ 74.24 \\ 114.40 \\ 75.97 \\ 89.21 \\ 110.78 \end{bmatrix}$

Table 2. control parameters for the experimental study

been used to compute the feedforward controller u_C. As demonstrated by our experience with the system, such choice is very realistic. Friction parameters could even vary more then here assumed.
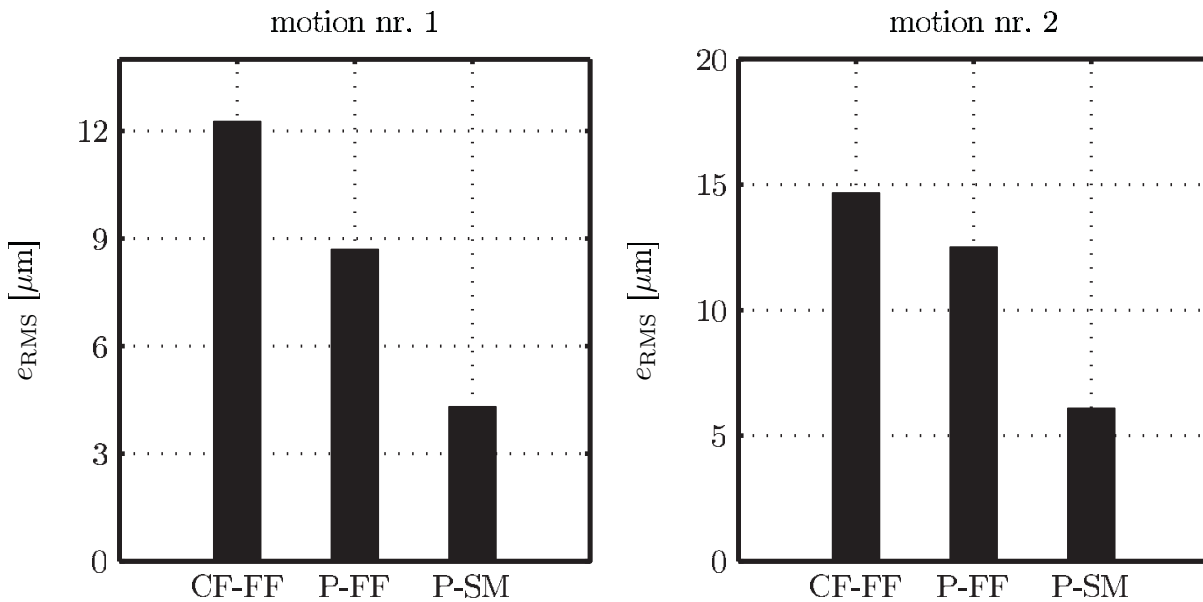
Fig. 4. Comparing the resulting root mean squares errors resulting from the two test motions and by using three different controllers.

Comparing the root mean squares of the tracking errors with respect to the actuators yields the results in Fig. 4. As expected the classic approach CF-FF shows the lowest tracking accuracy. This is due to the fact, that the highly noisy velocity error signal inhibits increasing
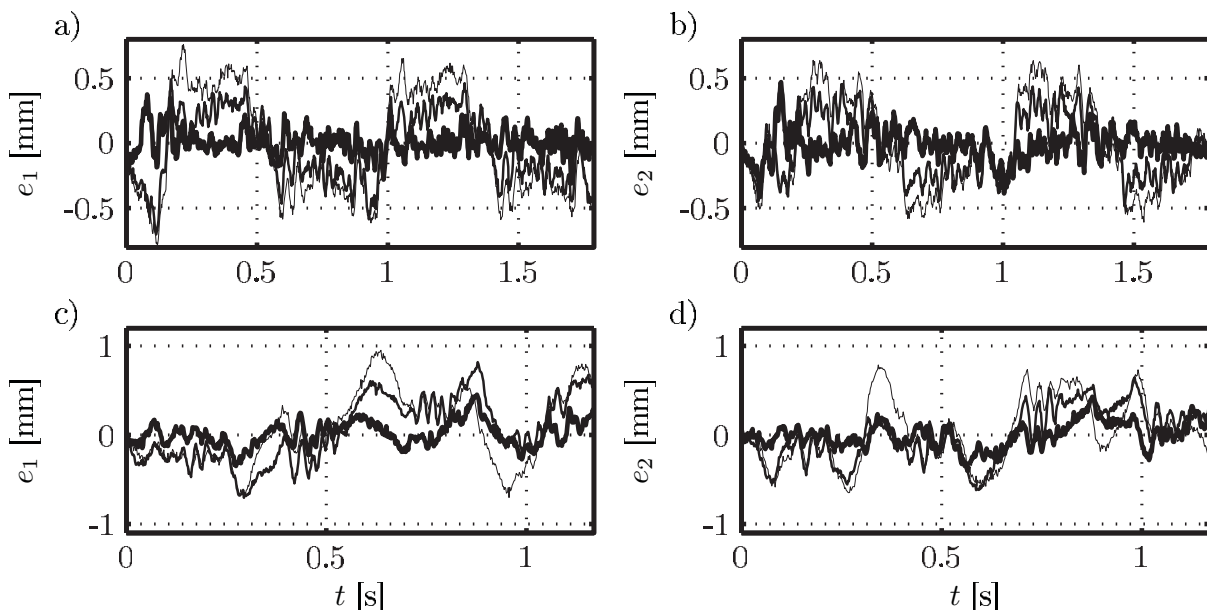
Fig. 5. Comparing the tracking errors of exemplarily chosen actuators resulting for the two test motions (top: motion nr. 1, bottom: motion nr. 2) and by using three different controllers: Thick line: P-SM, medium line: P-FF, thin line: CF-FF

the feedback action significantly without compromising the stability of the system. The use of well tuned observer is a key issue to improve the tracking performance. It is also clear that the controller augmented with the sliding mode component outperforms the other two. This can be concluded by examining the time histories of the tracking errors (see some examples depicted in 5). The approach P-FF is able to counteract the effects of the parameter uncertainty by increasing the feedback. Such operation has to be performed by the operator or by the control engineer. The feedback action should then be tuned in relationship to the estimate upper bound of the uncertainty. The switching mode robust controller is able to react autonomously on the deviation of the nominal model. Its operating mode can be illustrated for the present case by examining the robust corrections performed for the first and for the 10th friction parameters (see Fig. 6 and Fig. 7, respectively).
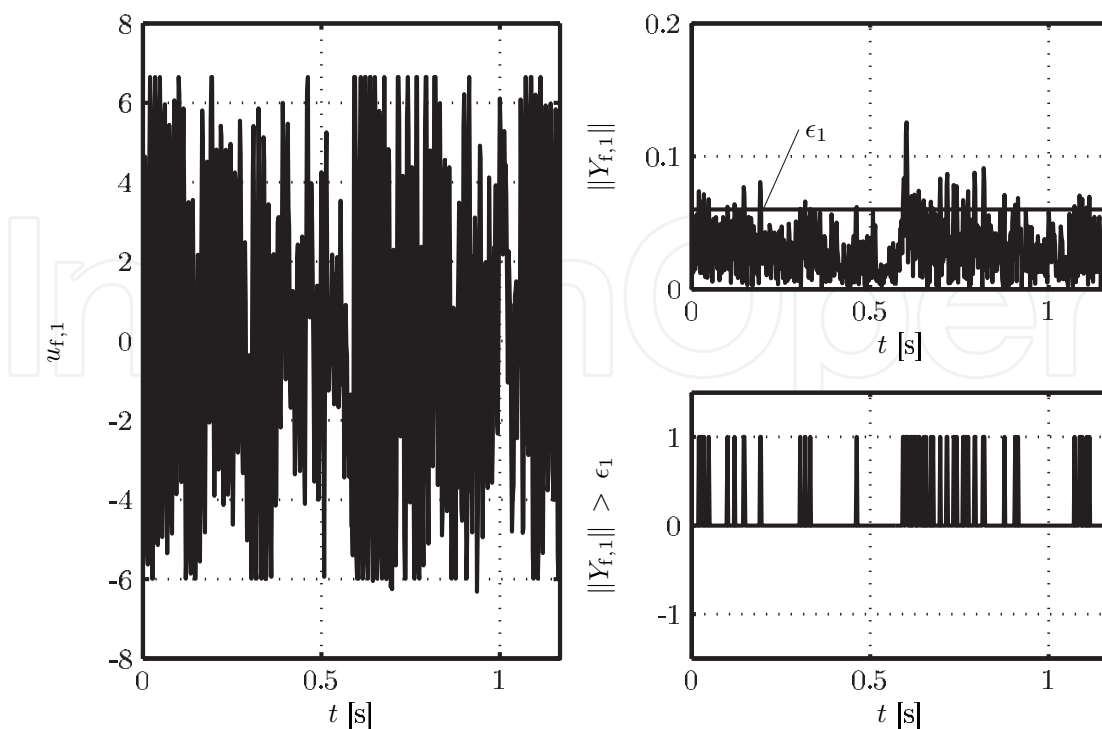
Fig. 6. Robut correction for the first entry of the input vector u_f corresponding to the first friction parameters f_{1_α}
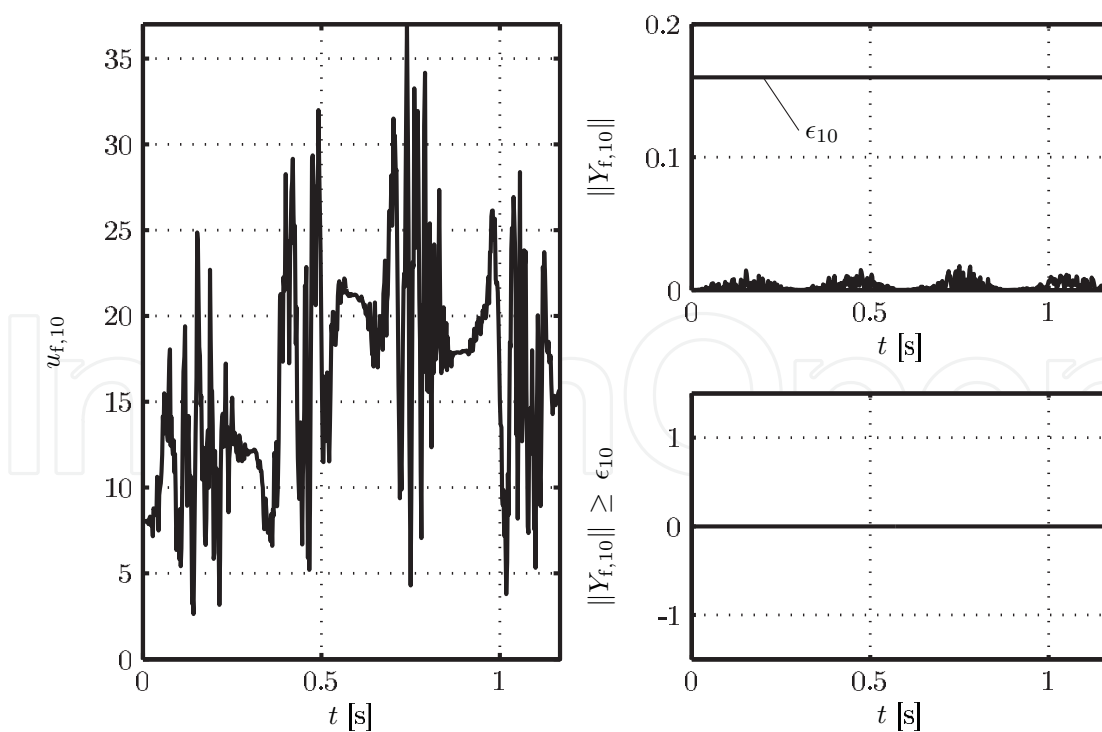
Fig. 7. Robut correction for the first entry of the input vector u_f corresponding to the 10th friction parameters f_{2_2}

As given mathematically in (36-38) the entries of the switching robust controller u_R depend on the corresponding state $\|Y_{f,k}\| > \epsilon_k$ (see right, bottom of the figures) . If the latter is fulfilled then the boundary layer is violated and the controller switches to pull back $Y_{f,k}$ within the layer. This is the case, when high uncertainty is given like it is the case for the friction parameter f_{1_α} of the passive joint. A higher switching action occurs (see left plot of Fig. 6). In the contrary case, like for $u_{f,10}$ the algorithms reacts on the error dynamics $s_1 - s_2$ by adjusting the output. As it can be better observed in Fig. 7, the output $u_{f,10}$ is highly correlated with the corresponding $Y_{f,k}$.

The case of significant initial errors showed however some drawbacks of the control approach P-SM. The most important one is that in such case the switching controller is too aggressive and leads very quickly to the violation of the actuator constraints. The design of the controller has been made without any consideration of input constraints which explains such undesirable phenomena. This issue is left for future work and for future improvement. Figure 8 shows corresponding experimental results achieved by driving the quadratic motion by significant initial errors. The exemplarily depicted tracking errors for the second actuator demonstrates that the P-SM controller yields the biggest overshoot. Additionally, it exhibits lower tracking convergence quality. Notice that we used the region of final convergence R (see eq. (32)) in Fig. 7 only for illustration purpose and in order to improve the understanding of the results. To remedy the bad behavior of the sliding mode controller in presence of high initial errors, another mild tuning of the parameters is required, e.g. increasing K_I and decreasing ϵ by 100 times and 4 times, respectively. The corresponding experimental results are denoted by P-SM'. It is clear that a tradeoff should be met between robustness and tracking accuracy, which is the classic problem in control practice.
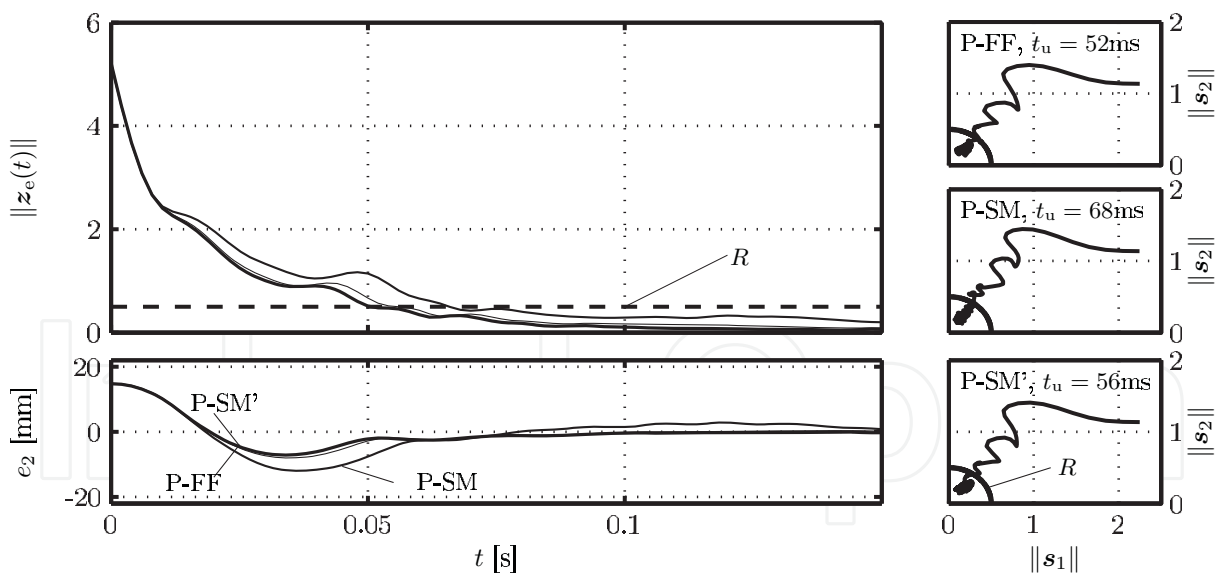

Fig. 8. Comparison of ontrol performance in case of significant initial errors. Left, top: error norm $\|z_e\|$, Left, bottom: exemplarily depicted tracking accuracy of the second actuator, on the right side: convergence of the control and observer sliding errors for the compared control approaches and with resulting convergence time t_u.

The final and concluding experiment compares the accuracy of the three control approaches in the cartesian space. Figure 9 shows the tracking performance of the circle as well as that of

a corner achieved by the algorithms CF-FF, P-FF and P-SM. In this experiment we switched back to the case of zero initial errors. The sliding mode approach outperforms the other two controllers.
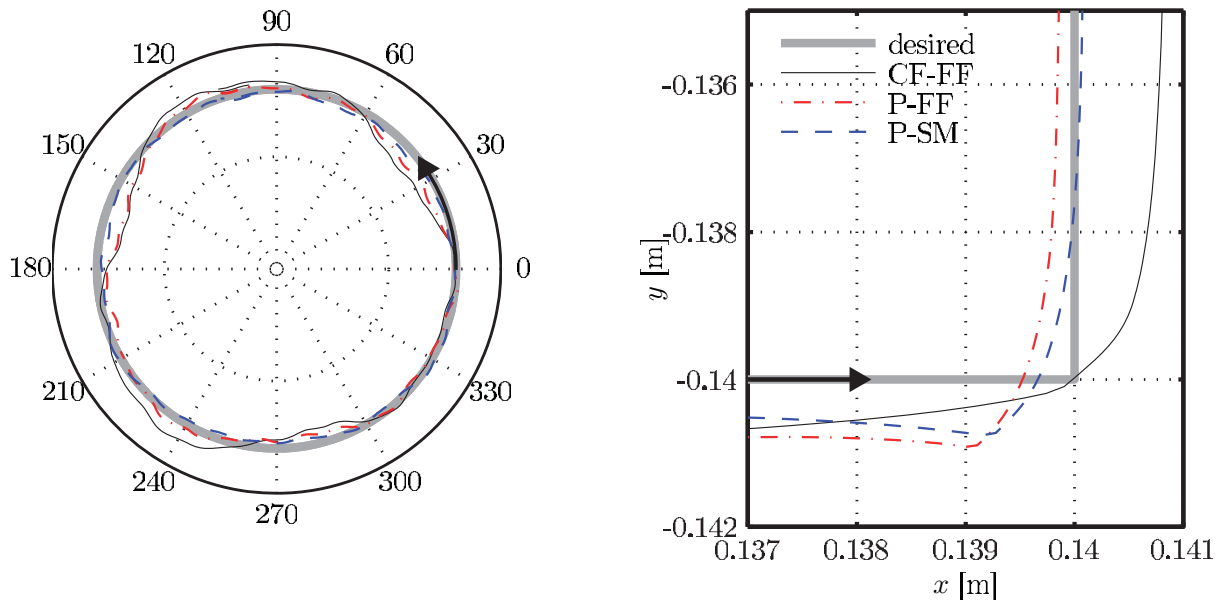

Fig. 9. Peroformance of tracking a circle (left) and a sharp corner (right) achieved by the three investigated control approaches

6. Conclusions

In this paper an experimentally approved practical methodology for the robust control of 6 DOFs parallel manipulators has been presented. A discussion on the key issues for a successful control strategy for such systems has been provided. The computational efficiency of the control has two aspects: the first one is the calculation of the complex dynamics model and the second is the determination of the end-effector motion. Both aspects can be solved by feedforward desired dynamics compensation, that is in this sense more appropriate then the feedback dynamics compensation. The use of observers for the actuator velocities allows to increase the control bandwidth. We used the passivity paradigm to develop an approach that merges the feedforward compensation technique with the observer-based feedback to provide a first basic controller, that is locally uniformly and ultimately bounded.

In a second step the basic algorithm is extended with a robust switching term. This has been designed to harmonize with the basic algorithm by consequently using desired dynamics and the consideration of the observer dynamics. The practicability of the approach is improved by restricting the robust term on the friction part of the model, which is classically affected by important and time-varying uncertainty.

The presented methodology has been investigated and substantiated by a set of experiments. It has been demonstrated that the algorithm augmented with the switching term exhibits the best performance. Nevertheless, a tradeoff between accuracy and stability should be met while tuning the controller, especially with respect to significant initial errors.

7. Appendix

Proof of the skew symmetric property of $\dot{M}_a - 2C_a$. Given the skew symmetry of $\dot{M} - 2C$ (Ortega *et al.*, 1998) following transformations hold

$$M_a = J^T M_a J,$$
$$C_a = J^T M_a \dot{J} + J^T C J,$$

substitution yields

$$\dot{M}_a - 2C_a = \frac{d}{dt}\left(J^T M_a J\right) - 2J^T M_a \dot{J} - 2J^T C J,$$
$$= J^T \left(\dot{M}_a - 2C_a\right) J + \dot{J} M J - J M \dot{J}.$$

Let $u \in \mathbb{R}^6$. It results

$$u^T \left(\dot{M}_a - 2C_a\right) u = u^T J^T \left(\dot{M}_a - 2C_a\right) J u$$
$$+ u^T \dot{J} M J u - u^T J M \dot{J} u,$$
$$= u^T \dot{J} M J u - u^T J M \dot{J} u,$$
$$= 0 \; \forall u \in \mathbb{R}^6$$

since $u^T \dot{J} M J u$ scalar, it results

$$u^T \dot{J} M J u = \left(u^T \dot{J} M J u\right)^T = u^T J M \dot{J} u.$$

yielding that

$$u^T \left(\dot{M}_a - 2C_a\right) u = 0 \quad \forall u \in \mathbb{R}^6$$

which completes the proof.

8. References

Abdallah, C., D. Dawson, P. Dorato and M. Jamshidi (1991). Survey of robust control for rigid robots. *IEEE Control Systems Magazine* **11**(2), 24–30.

Abdellatif, Houssem and Bodo Heimann (2007). *Industrial Robotics: Theory, Modeling and Control*. Chap. Model-Based Control for Industrial Robots: Uniform Approaches for Serial and Parallel Structures, pp. 523–556. Pro-Literatur Verlag.

Abdellatif, Houssem, Bodo Heimann and Jens Kotlarski (2008). *Parallel Manipulators: New Developments*. Chap. On the Robust Dynamics Identification of Parallel Manipulators: Methodology and Experiments, pp. 1–20. I-Tech Education and Publishing. Vienna, Austria.

Abdellatif, Houssem, Martin Grotjahn and Bodo Heimann (2005). High efficient dynamics calculation approach for computed-force control of robots with parallel structures. In: *Proc. of the 44th IEEE Conference on Decision and Control and the 2005 European Control Conference, CDC-ECC05*. Seville, Spain. pp. 2024–2029.

Abdelllatif, Houssem, Martin Grotjahn and Bodo Heimann (2007). Independent identification of friction characteristics for parallel manipulators. *ASME Journal of Dynamic Systems, Measurment and Control* **129**, 294–302.

Berghuis, Harry (1993). Model-based Robot Control: from Theory to Practice. PhD thesis. Universiteit Twente. Enschede, The Netherlands.

Berghuis, Harry and Henk Nijmeijer (1994). Robust control of robots via linear estimated state feedback. *IEEE Transactions on Automatic Control* **39**(10), 2159–2162.

Burdet, E., M. Honegger and A. Codourey (2000). Controllers with desired dynamic compensation and their implementation of a 6dof parallel manipulator. In: *Proc. of the 2000 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*. Takamatsu, Japan. pp. 39–45.

Caccavale, Fabrizio, Bruno Siciliano and Luigi Villani (2003). The tricept robot: Dynamics and impedance control. *IEEE/ASME Transactions on Mechatronics* **8**(2), 263–268.

Celani, Fabio (2006). A luenberger-style observer for robot manipulators with position measurements. In: *Proc. of 14th Mediterranean Conference on Control and Automation*. Ancona, Italy.

Cheng, Hui, Yiu Kuen and Zexiang Li (2003). Dynamics and control of redundantly actuated parallel manipulators. *IEEE/ASME Transactions on Mechatronics* **8**(4), 483–491.

de Wit, C. Canudas and J.-J. E. Slotine (1991). Sliding observers for robot manipulators. *Automatica* **27**(5), 859–864.

Denkena, Berend, Christian Holz and Houssem Abdellatif (2006). Model-based control of a hexapod with linear direct drives. *Int. Journal of Computer Integrated Manufacturing* **19**(5), 463–472.

Egeland, Olav (1987). On the robustness of the computed torque technique in manipulator control. *Modeling, Identification and Control* **8**(3), 149–158.

Kim, Dong Hwan, Ji-Yoon Kang and Kyo-IL Lee (2000). Robust tracking control design for a 6 dof parallel manipulator. *Journal of Robotic Systems* **17**(10), 527–547.

Kim, Hag Seong, Young Man Cho and Kyo-IL Lee (2005). Robust nonlinear task space control for a 6 dof parallel manipulator. *Automatica* **41**, 1591–1600.

Kim, Jong-Phil, Sung-Guang Kim and Jeha Ryu (2001). Robust adaptive control of a 6 hexaslide type parallel manipulator. *ICASE: The institute of Control, Automation and Sysetems Engineers, KOREA* **3**(4), 262–267.

Lee, Se-Han, Jae-Bok Song, Woo-Chun Choi and Daehie Hong (2003). Position control of a stewart platform using inverse dynamics control with approximate dynamics. *Mechatronics* **13**, 605–619.

Liu, Guangjun and Andrew A. Goldenberg (1996). Uncertainty decomposition-based robust control of robot manipulators. *IEEE Transactions on Control Systems Technology* **4**(4), 384–393.

Ortega, R., Loría, A., Nicklasson, P.J. and Sira-Ramirez, H.J., Eds. (1998). *Passivity-based Control of Euler-Lagrange Systems*. Communications and Control Engineering. Springer-Verlag.

Pietsch, Ingo T., Mathias Kreft, Oliver T. Becker, Carlos C. Bier and Jüergen Hesselbach (2005). How to reach the dynamic limits of parallel robots? an autonomous control approach. *IEEE Transactions on Automation Science and Engineering* **2**(4), 369–380.

Qu, Zhihua and Darren M. Dawson (1996). *Robust Tracking Control of Robot Manipulators*. IEEE Press. Piscataway, USA.

Qu, Zhihua and John Dorsey (1991*a*). Robust pid control of robots. *Int. Journal of Robotics and Automation* **6**(4), 228–235.

Qu, Zhihua and John Dorsey (1991*b*). Robust tracking control of robots by a linear feedback law. *IEEE Transactions on Automatic Control* **36**(9), 1081–1084.

Ren, Lu, James K. Mills and Dong Sun (2005). Controller design applied to planar parallel manipulators for trajectory tracking control. In: *Proc. of the 2005 IEEE Int. Conference on Robotics and Automation*. Barcelona, Spain. pp. 980–985.

Slotine, Jean-Jeaques E. and Weiping Li (1991). *Applied Nonlinear Control*. Patience Hall. Engelwood Cliffs, NJ.

Spong, Mark W. (1992). On the robust control of robot manipulators. *IEEE Transactions on Automatic Control* **37**(11), 1782–1786.

Ting, Yung, Yu-Shin Chen and Ho-Chin Jar (2004). Modeling and control for a gough-stewart platform cnc-machine. *Journal of Robotic Systems* **21**(11), 609–623.

Vivas, Andres and Philippe Poignet (2005). Predictive functional control of a parallel robot. *Control Engineering Practice* (13), 863–874.

Wang, Jinsong, Jun Wu, Liping Wang and Tiemin Li (2007). Simplified strategy of the dynamic model of 6-ups parallel kinematic machine for real-time control. *Mechanism and Machine Theory* **42**, 1119–1140.

Wang, Zhiyong and Fathi H. Ghorbel (2006). Control of closed kinematic chains using a singularly perturbed dynamics model. *ASME Journal of Dynamic Systems, Measurement and Control* **128**, 142–151.

Wen, John T. and David S. Bayard (1988). New class of control laws for robotic manipulators, part 1. non-adaptive case. *Int. Journal of Control* **47**(5), 1361–1385.

Robotics 2010 Current and Future Challenges

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Without a doubt, robotics has made an incredible progress over the last decades. The vision of developing, designing and creating technical systems that help humans to achieve hard and complex tasks, has intelligently led to an incredible variety of solutions. There are barely technical fields that could exhibit more interdisciplinary interconnections like robotics. This fact is generated by highly complex challenges imposed by robotic systems, especially the requirement on intelligent and autonomous operation. This book tries to give an insight into the evolutionary process that takes place in robotics. It provides articles covering a wide range of this exciting area. The progress of technical challenges and concepts may illuminate the relationship between developments that seem to be completely different at first sight. The robotics remains an exciting scientific and engineering field. The community looks optimistically ahead and also looks forward for the future challenges and new development.

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