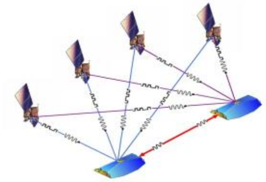

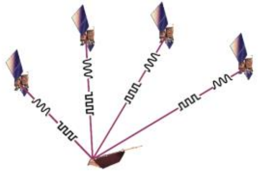

Zero difference geometrical precise orbit determination of low flying satellites with GPS-SST observations

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University of Bonn
Germany

Geodetic Week
Munich, 11th October 2006

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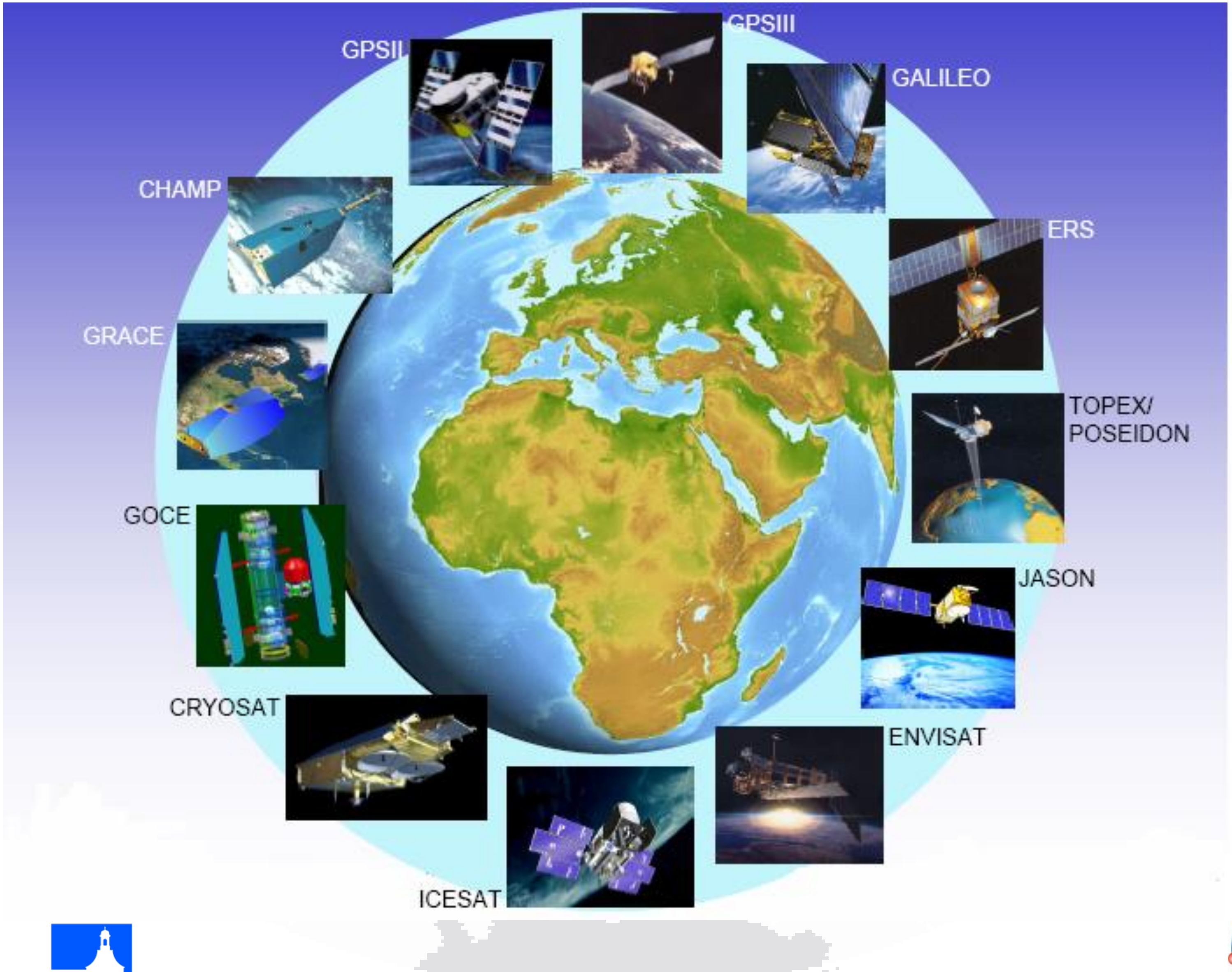
LEO Missions (Earth Explorers)



Advantages of LEO POD

- Precise LEO orbits can be used to recover the gravity field of the Earth by the POD method,
- Analysis of altimetry observations require precise orbits,
- Atmosphere sounding requires precise positions of the LEO satellites,
- GNSS (GPS, GLONASS, GALILEO) methods play an important role in POD in addition to classical methods (e.g. SLR, DORIS).

Principal techniques of orbit determination

✓Geometrical orbit determination

Only geometrical observations have to be used, no force models and no constraints; pointwise representation

✓Kinematical orbit determination

Geometrical, kinematical observations have to be used, no force models are used; representation by kinematical functions

✓Dynamical orbit determination

Geometrical, kinematical & dynamical observations have to be used, but complete force models; pointwise representation or representation by functions

Processing concepts

Code measurements

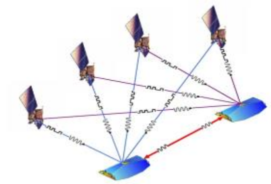
Phase measurements

Zero differencing procedure

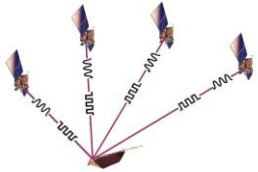
Geometrical orbit determination

Kinematical orbit determination

Dynamical orbit determination



Concept of Geometrical & Zero Difference PPP (P^3)



✓Zero Difference:

Only SST observations between LEO satellite and GPS satellites have to be used (pseudo-range & carrier phase)

✓Geometrical:

Only geometrical relations between LEO and GPS satellites have to be used, no force models and no constraints

✓Precise:

All effects on GPS observations, precise and final GPS satellites positions & clock offsets have to be used to estimate precise LEO position at every epoch.

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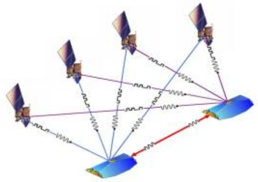
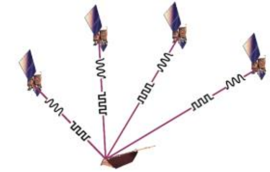
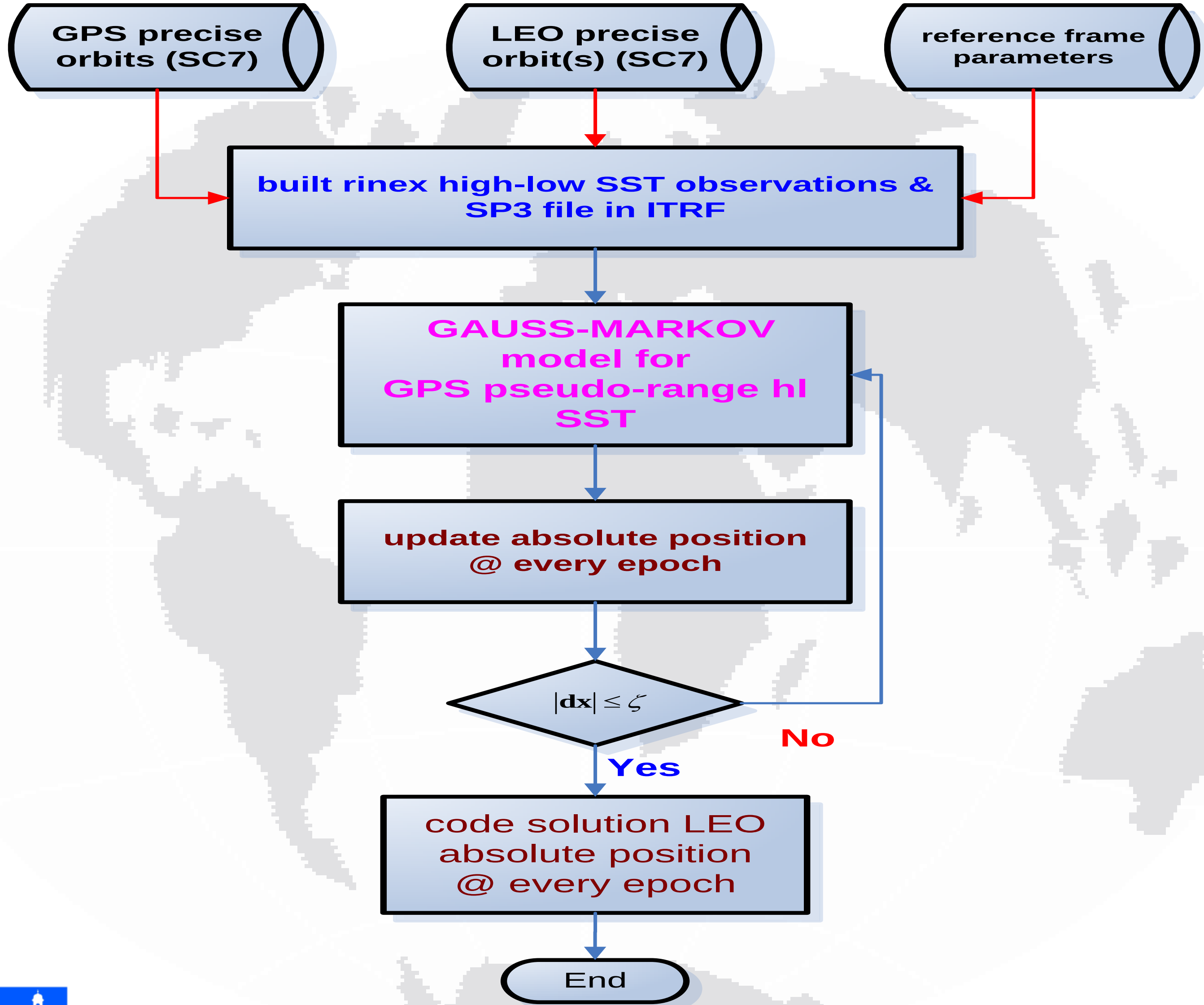
Zero difference geometrical PPP

simulated case

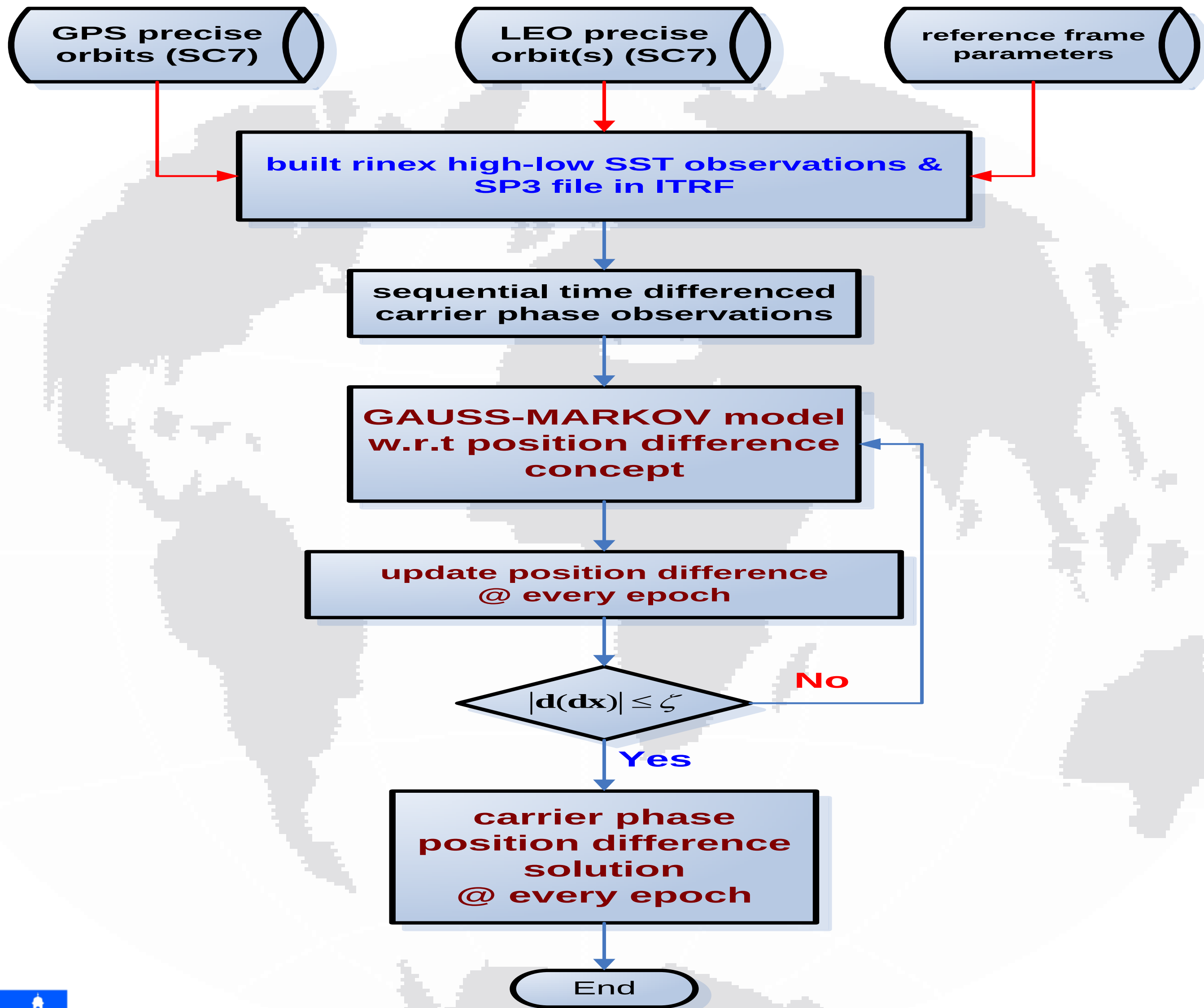


Absolute point positioning with high-low pseudo-range SST





Position difference with high-low time differenced carrier phase SST



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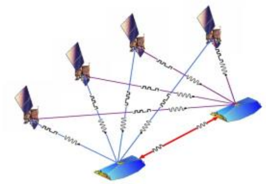
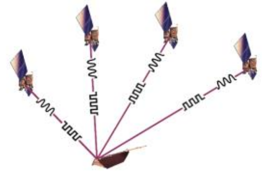
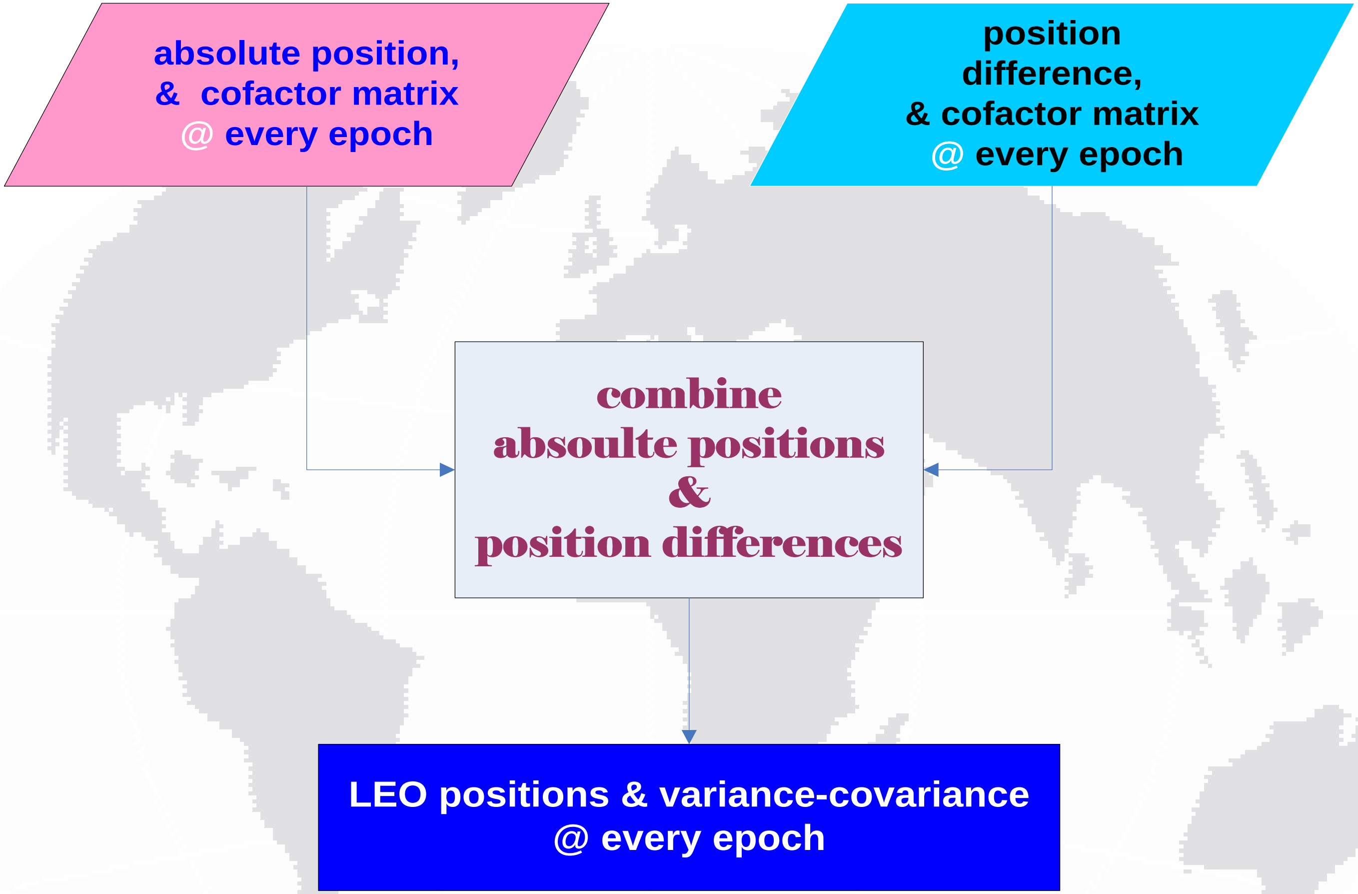


Combination strategy





combination method (simulation)

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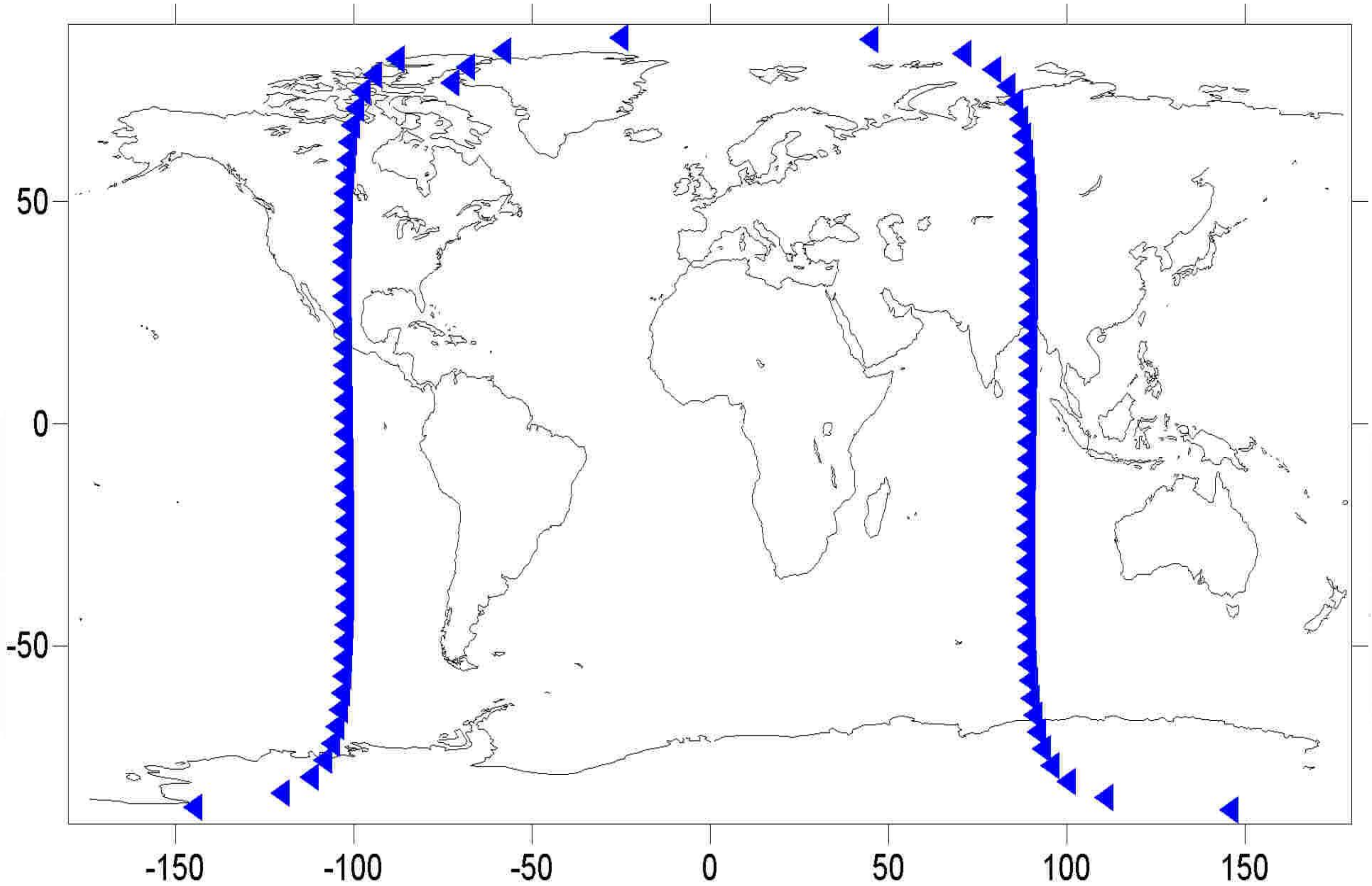
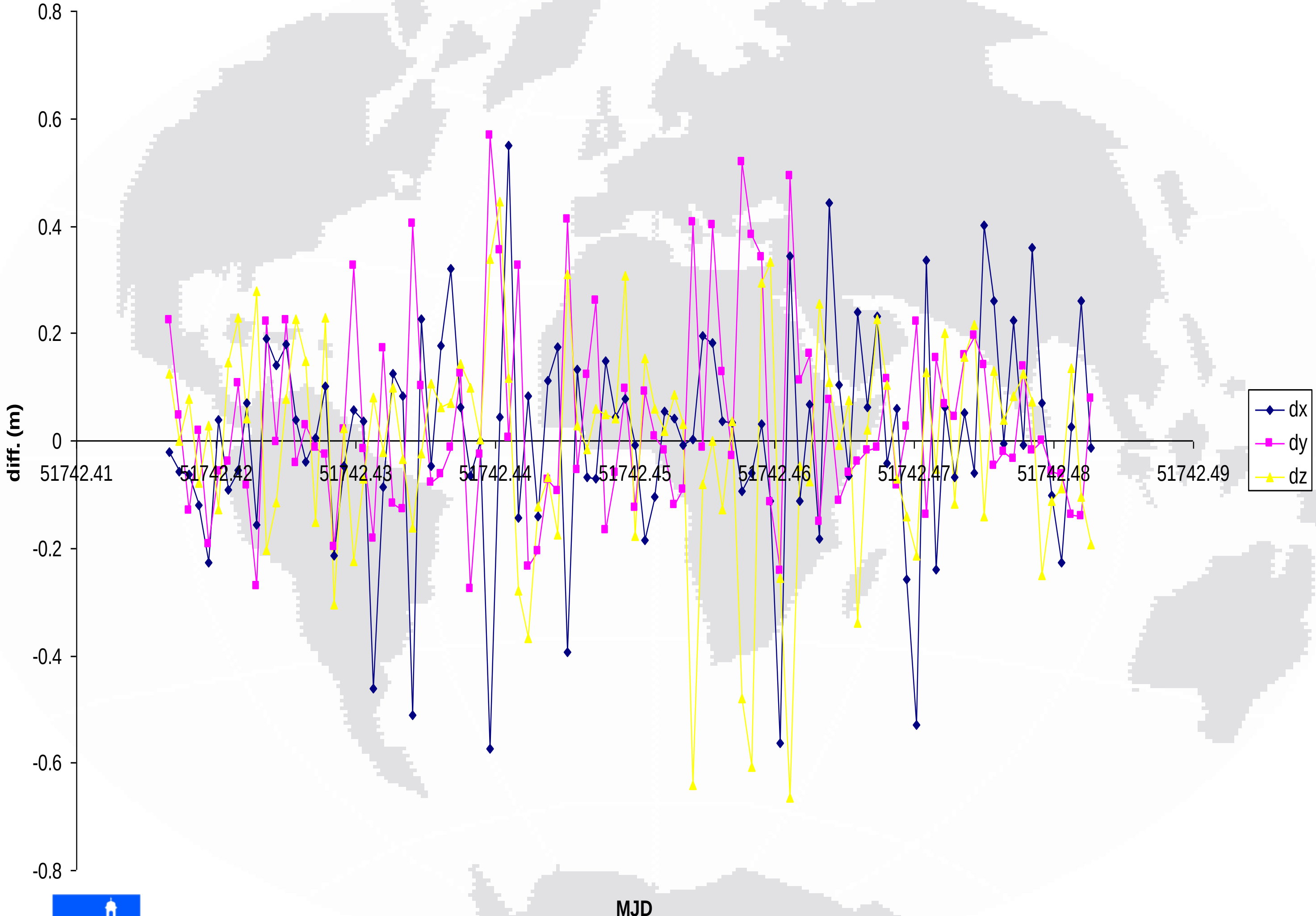
Zero Difference Geometrical PPP



one revolution of CHAMP satellite [2000 07 17 10h 00m – 11h 35m]

Absolute position residuals

Difference between CHAMP estimated absolute positions and the original given absolute positions SigC=0.50 m



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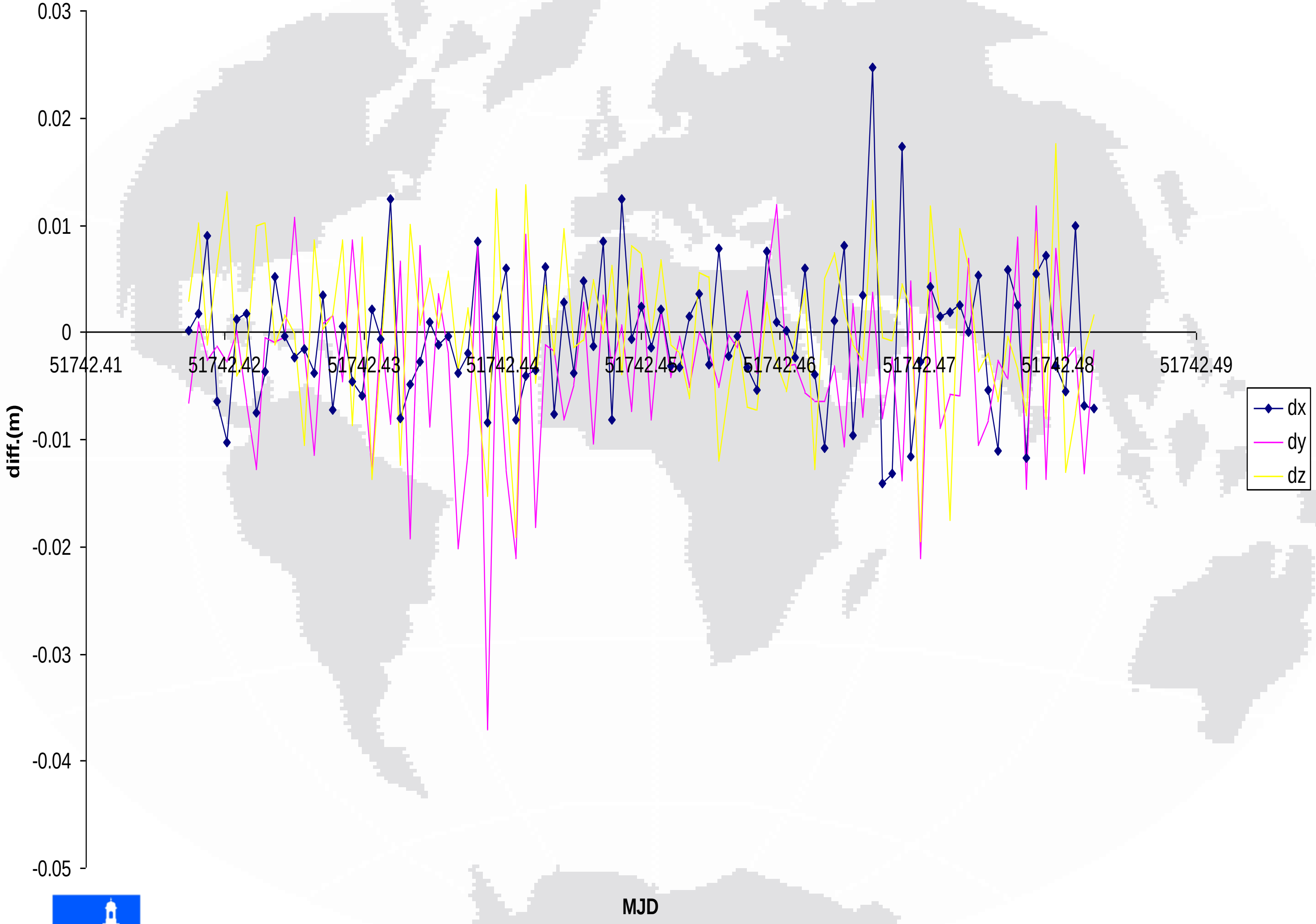
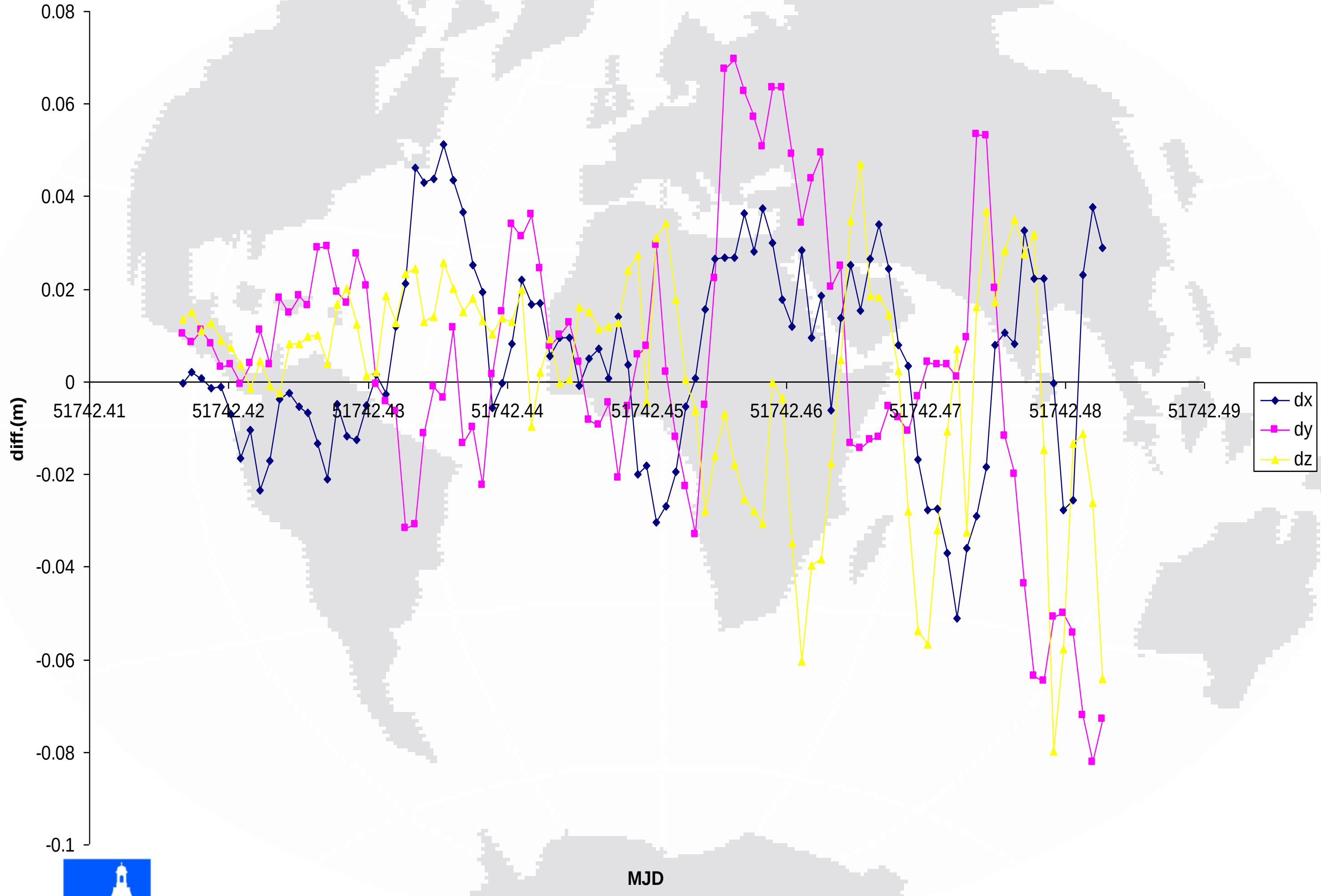
Position difference residuals

Difference between CHAMP estimated position differences & given position differences, SigP=0.01 m



CHAMP combined residuals

Difference between CHAMP combined positions & given positions SigC=0.50m & SigP=0.01cm



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Zero difference geometrical PPP

real case

Geometrical ZD LEO POD-procedure

- at first, initial positions & clock offsets have been estimaed with Bancroft model.

- LEO approximation absolute positions & clock offsets have been improved with the code pseudorange observations in accuracy of code observations. (~ meter)

- LEO absolute position & clock offset differences have been estimated in accuracy of carrier phase sequential time difference observations.(~ cm)

- LEO absolute positions & clock offsets from the code, position & clock offset differences from the carrier phase observations have been combined to estimate final LEO positions & clock offsets at every epoch.

Geometrical zero difference PPP

- Geometrical LEO orbit has been estimated in zero difference processing mode of GPS observations.

- IGS GPS final orbits with accuracy of ~cm, Earth rotation parameters (ERP) from IERS centre have been used in the procedure.

- because of ZD procedure, many corrections must be applied to GPS satellite positions , code and sequential time difference of carrier phase observations (e.g. GPS antenna mass centre offset, relativistic effect,…)

- no Earth gravity filed and no force models have been used in the geometrical mode (advantage of the geometrical method)

Absolute position from code pseudo-range

code pseudo-range GPS SST observations:

$$P_{r,i}^{s}(t) = \left|\mathbf{R}_z(\omega_e.\varepsilon_r^s)\mathbf{r}^s(t-\varepsilon_r^s) - \mathbf{r}_r(t)\right| + c\left[dt^s(t-\varepsilon_r^s) - dt_r(t)\right] + I_i^r(t) +$$

$$d_O^s(t) + d_R^r(t) - d_R^s(t) + d_{C,i}^r(t) + d_{V,i}^r(t) + d_{M,P_i}(t) + e_{P_i}$$

s, r	GPS, LEO indices,
ε_r^s	Travelling time between GPS & LEO,
$\mathbf{r}^s(t-\varepsilon_r^s), dt^s(t-\varepsilon_r^s)$	GPS position, clock offset at sending time,
$\mathbf{r}_r(t), dt_r(t)$	LEO position, clock offset at receiving time

Absolute position from code pseudo-range…

$I_i^r(t)$

•for single frequency receiver, the IONEX model can be used to model the ionosphere error term,
•for dual frequency receiver, the ionosphere free combination can be used.

$d_O^s(t)$

to … used in
r…

$d_R^s(t), d_R^r($…

… &

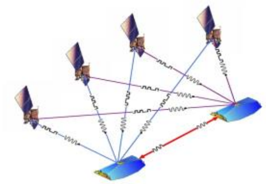
$d_{C,i}^r(t), d_{V,i}^r(t)$

… be modeled
t… …NTEX file.

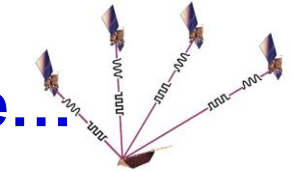
$d_{M,P_i}(t)$

multipath effect can be minimized through filtering SST observations w.r.t elevation of GPS satellites or applying the elvation weighting method or S/N filtering.

How the errors can be eliminated
or
modeled in GPS LEO SST observations?



Absolute position from code pseudo-range…



ionosphere free code pseudo-range GPS SST observations:

$$P_{r,3}^{s}\left(t\right)-c\left[dt^{s}(t-\varepsilon_{r}^{s})\right]-d_{R}^{r}(t)-d_{R}^{s}(t)+d_{C,3}^{r}(t)+d_{V,3}^{r}(t)+d_{M,P_3}(t)=\Delta L_{r,3}^{s}\left(t\right)=$$

$$\left|\mathbf{R}_{z}(\omega_{e}.\varepsilon_{r}^{s})\mathbf{r}^{s}(t-\varepsilon_{r}^{s})-\mathbf{r}_{r}(t)\right|-c\left[dt_{r}(t)\right]+e_{P_3}$$

$\Downarrow$ with linearaz

$\Delta L_{r,}^{s}$

$$\mathbf{A}_{r}^{s}(t)=\left(\frac{x_{r}(t)-x^{s}(t-\tau_{r}^{s})}{\rho_{r}^{s}(t)},\quad \frac{y_{r}\ldots}{\ldots}\right.$$

$\Downarrow$

$$\delta\hat{\mathbf{r}}(t)=(\mathbf{A}^{T}\mathbf{P}_{l}\mathbf{A})^{-1}\mathbf{A}^{T}\mathbf{P}_{l}\Delta\mathbf{L}(t)$$

the observation equations can be adjusted through least square sence in
- *single*
- *batch*

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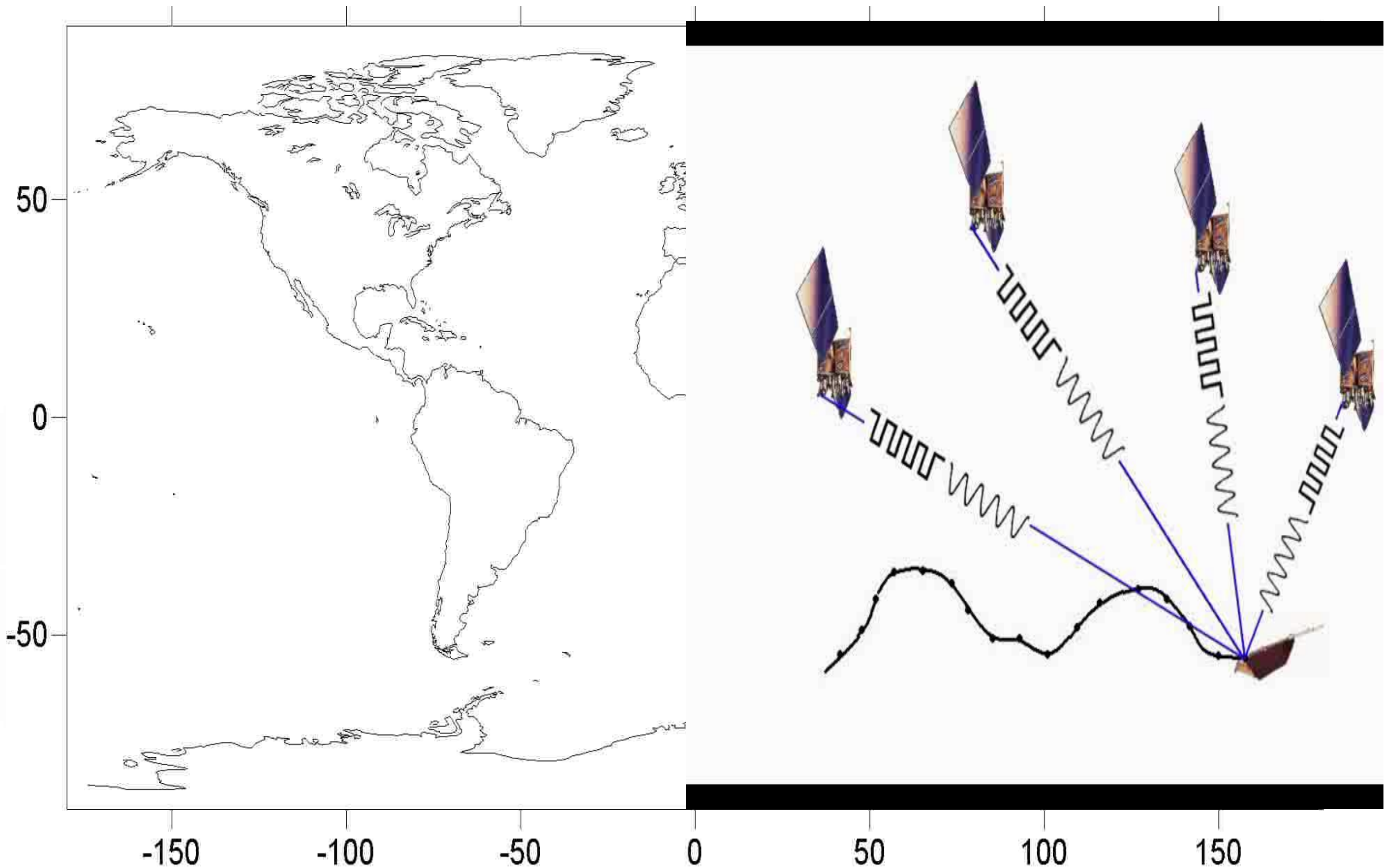
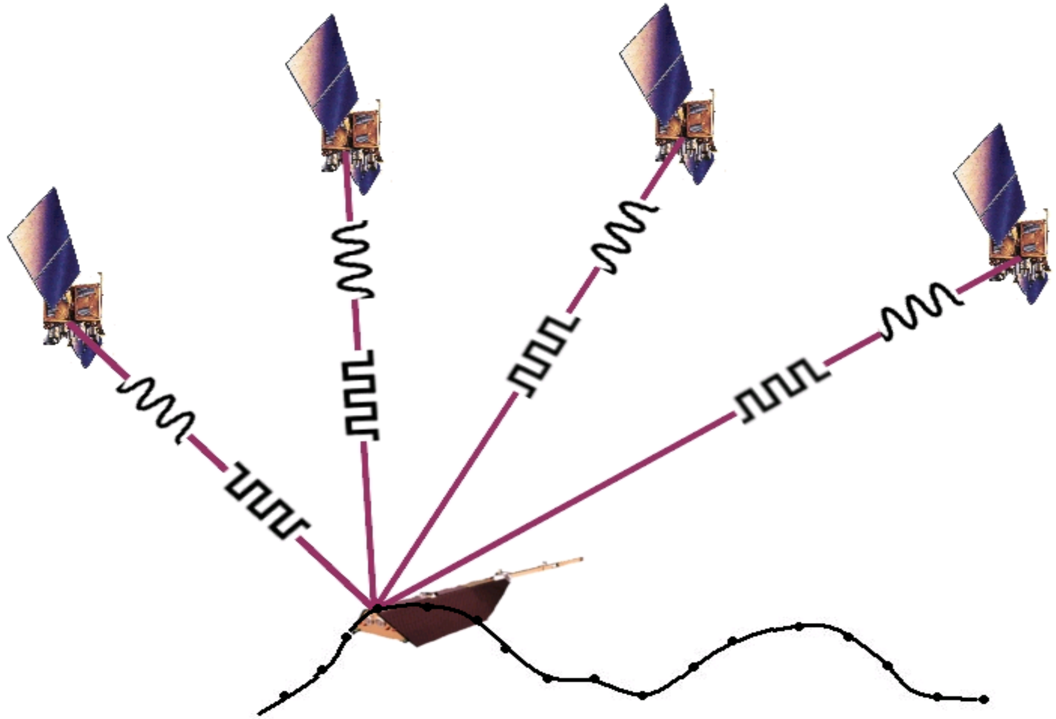
Absoulte position from code pseudo-range…



25 minutes of CHAMP satellite [2002 07 20 12h 50m – 13h 15m]

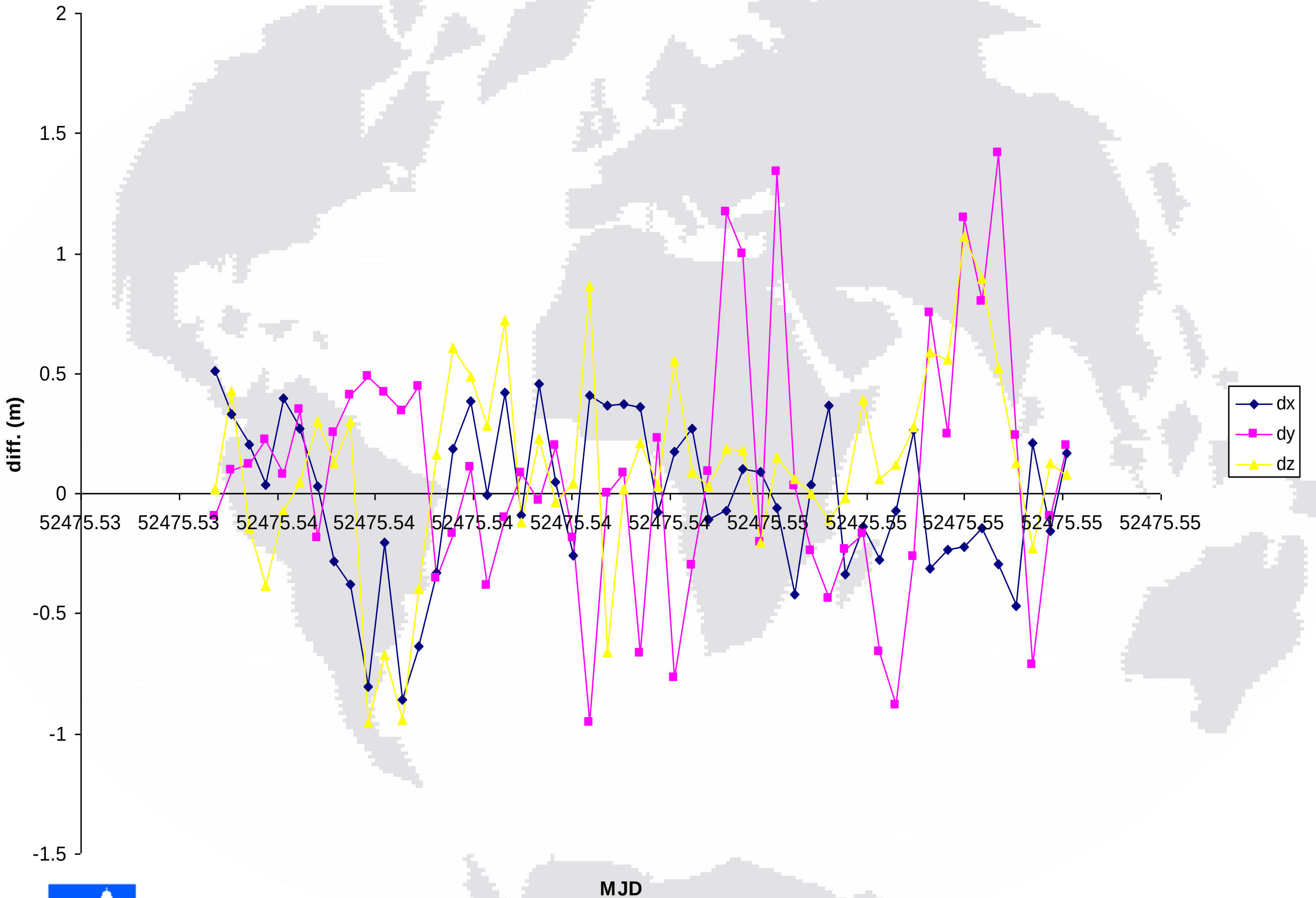
Absolute position from code pseudo-range…

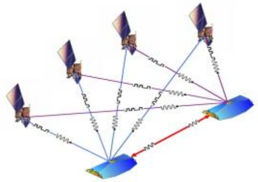
- cut-off angle 15 deg.
- majority voting procedure & S/N filtering,
- elevation weighting has been applied.



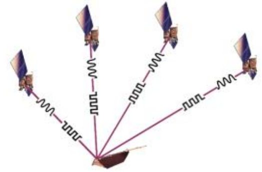
Absolute position from code pseudo-range…

difference between code solution & GFZ PSO CHAMP orbit
SigC=1.0, Cut-off=15, Beta=10





Position difference from sequential time differenced carrier phase…



carrier phase ionosphere-free observation at epochs (1,2)

$$\Phi^{s}_{r,3}\left(t_1\right)=\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|+\lambda_3 N^{s}_{r,3}+c\left[dt^s(t_1-\varepsilon_1)-dt_r(t_1)\right]+$$

$$d^s_O(t_1)+d^r_R(t_1)-d^s_R(t_1)+d^r_{C,3}(t_1)+d^r_{V,3}(t_1)+d_{M,\Phi_3}(t_1)+e_{\Phi_3}$$

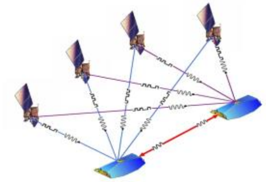
$$\Phi^{s}_{r,3}\left(t_2\right)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|+\lambda_3 N^{s}_{r,3}+c\left[dt^s(t_2-\varepsilon_2)-dt_r(t_2)\right]+$$

$$d^s_O(t_2)+d^r_R(t_2)-d^s_R(t_2)+d^r_{C,3}(t_2)+d^r_{V,3}(t_2)+d_{M,\Phi_3}(t_2)+e_{\Phi_3}$$

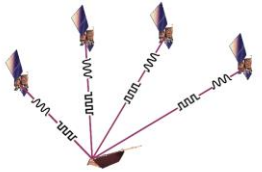
sequential time difference ionosphere-free carrier phase observation between epochs (1,2)

$$\Delta\tilde{\Phi}^{s}_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$



Position difference from sequential time difference carrier phase…



$$\Delta\tilde{\Phi}^{s}_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

$\Downarrow$ with linearazation

$$\Delta\tilde{\Phi}^{\mathrm{GPS}i}(t_1,t_2)=\Delta\tilde{\Phi}_0^{\mathrm{GPS}i}(t_1,t_2)+\frac{\partial\Delta\tilde{\Phi}^{\mathrm{GPS}i}(\ldots)}{\ldots}\Big|$$

$$\Delta\mathrm{P}=\left[\Delta x_{12}-\Delta x_{12}^0\quad\Delta y_{12}-\ldots\right.$$

$$\frac{\partial\Delta\tilde{\Phi}^{\mathrm{GPS}i}(t_1,t_2)}{\partial\mathrm{P}}=\left(\frac{x_{\mathrm{LEO}}(t_1)+\Delta x_{12}-x^{\mathrm{GPS}i}(t_2)}{\rho_{\mathrm{LEO}}^{\mathrm{GPS}i}(t_2)}\ldots\right.$$

$$=\mathbf{A}_{\mathrm{LEO}}^{\mathrm{GPS}i}(t_1,t_2)$$

the observation equations can be adjusted through least square sence in

single batch

$\Downarrow$

$$\delta\hat{\mathbf{r}}(t)=(\mathbf{A}^T\mathbf{P}_l\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}_l\Delta\mathbf{L}(t)$$

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Position difference from sequential time differenced carrier phase…

advantages:

- ✓**No ambiguity parameter,**

- ✓**Estimation of precise position differences between two sequential epochs ,**

- ✓**Code solution result with accuracy of meter at first epoch is enough to estimate position difference with accuracy of cm.**

- ✓**Negligiblity of the correlations between two sequential epochs carrier phase observations in single solution (or as batch solution ?)**

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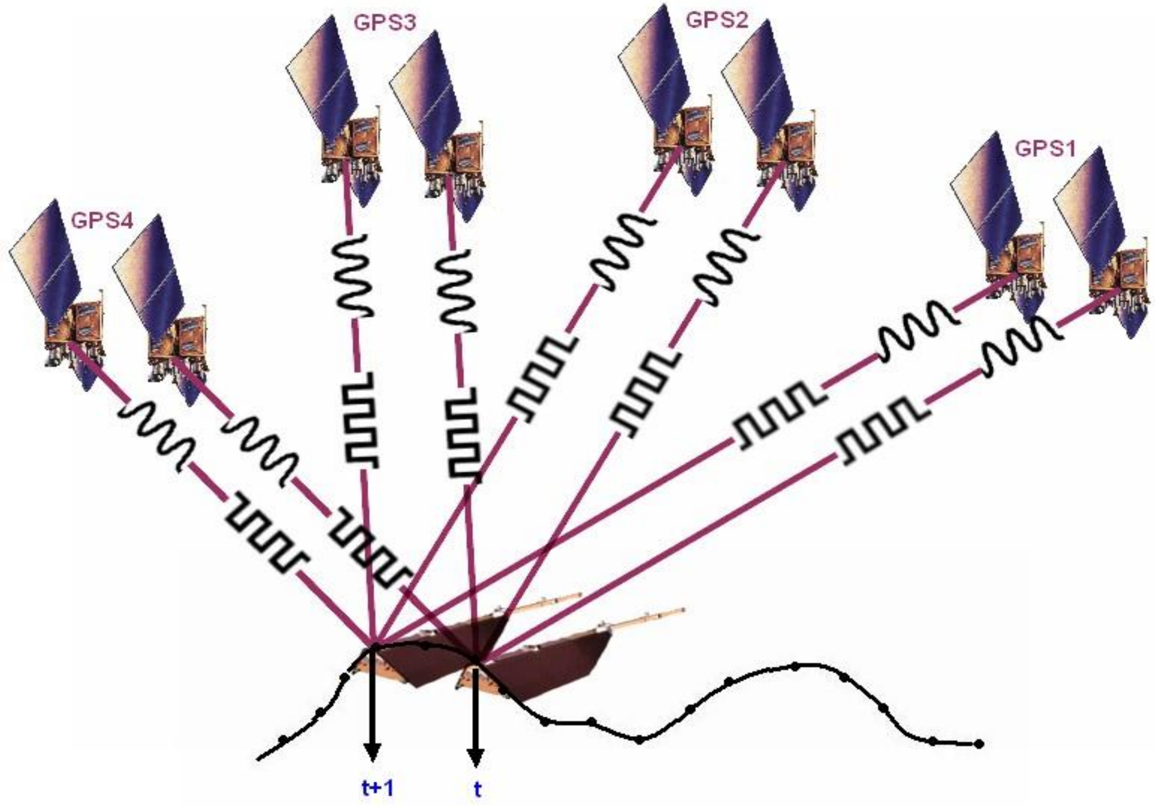
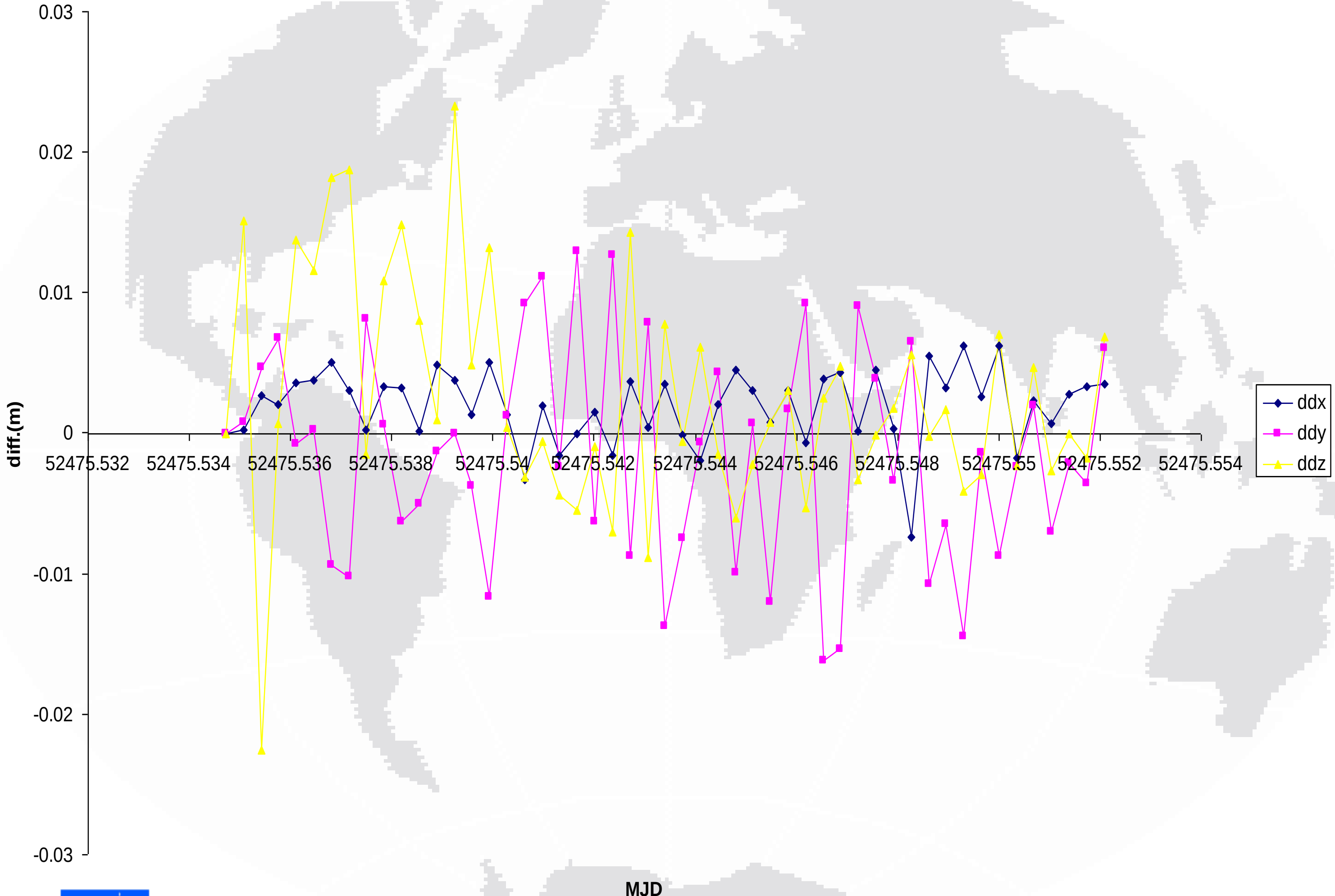
Position difference from sequential time differenced carrier phase

- cut-off angle 15 deg.
- majority voting procedure & S/N filtering,
- elevation weighting has been applied.



Position difference from sequential time differenced carrier phase…

Difference between estimated position difference & position difference from GFZ CHAMP PSO, SigP=0.01m



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Combination absolute position & position difference in the result phase

$$\mathbf{E}.\hat{\mathbf{x}}_i - \mathbf{x}_{c,i} = \mathbf{v}_{c,i}$$

$$\mathbf{E}.[\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_{i-1}] - [\mathbf{x}_{\phi,i} - \mathbf{x}_{\phi,i-1}] = \mathbf{v}_{\Delta\phi,i}$$

$$\Rightarrow \mathbf{Ax} = \mathbf{L} + \mathbf{v}, \quad \mathbf{P}$$

$$\mathbf{L} = [\mathbf{x}_{c,1}, \mathbf{x}_{c,2}, \Delta\mathbf{x}_{\phi,2}, \mathbf{x}_{c,3}, \Delta\mathbf{x}_{\phi,3}, \ldots, \mathbf{x}_{c,n}, \Delta\mathbf{x}_{\phi,n}]^{\mathrm{T}} \qquad \hat{\mathbf{x}} = [\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3, \ldots, \hat{\mathbf{x}}_{n-1}, \hat{\mathbf{x}}_n]^{\mathrm{T}}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ -\mathbf{E} & \mathbf{E} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E} & \mathbf{E} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{E} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{E} & \mathbf{E} \end{pmatrix},$$

$$\Downarrow$$

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{PA})^{-1}\mathbf{A}^T\mathbf{PL}, \quad \mathbf{C}_{\hat{\mathbf{x}}} = (\mathbf{A}^T\mathbf{PA})^{-1}$$

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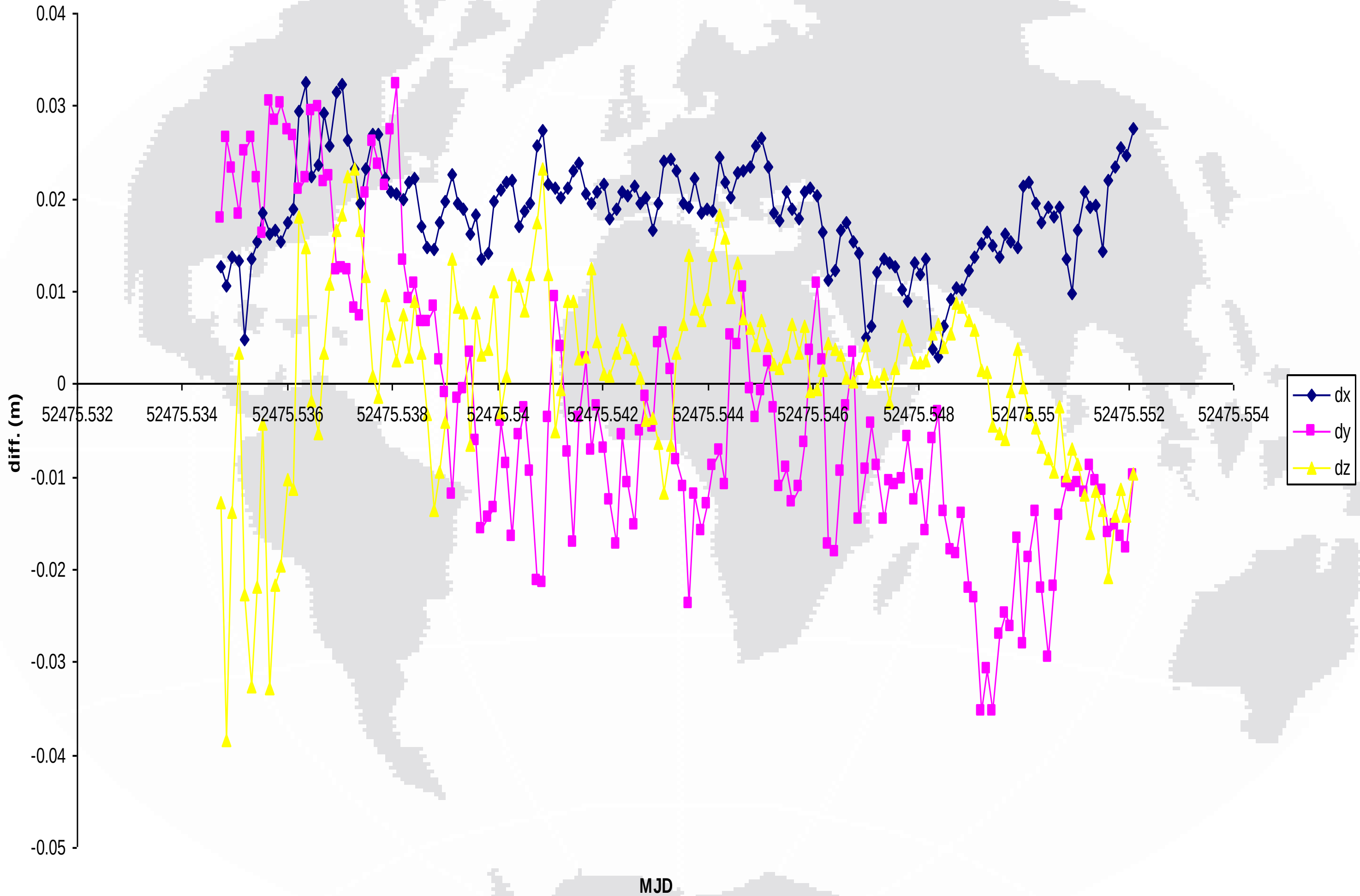
Combination absolute position & position difference in the result phase…

Difference between combined positions & and GFZ CHAMP PSO orbit, SigC=1.0m, SigP=0.01m



Conclusions & remarks

- **From the code & carrier phase SST observations, LEO positions and clock offsets can be estimated at every epoch with enough number of GPS satellites (>4) and good satellite geometry (sufficient DOP).**

- **An accuracy of cm can be expected for the combined GPS SST data processing, but depends on the GPS satellites geometry (DOP)!**

- **The resulting LEO orbit is given pointwise with noise!**

- **Geometrical LEO orbit can be used to recover the Earth‘s gravity field with the POD recovery concept!**

Thank you for your

attentions