Geometrical LEO Precise Orbit Determination (POD) with only sequential time differenced GPS SST carrier phase observations

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Abstract

High-low GPS LEO (Low Earth Orbiter) SST (Satellite to Satellite Tracking) observations play an important role to determine geometrical, precise, 3D orbits of LEO satellites. The ambiguity parameters in the Zero Difference (ZD) technique are not integer any more, and the carrier phase observations have to be solved in the float mode. If the difference between two sequential epochs has been built, and the observation rate is small then on the one hand, the ambiguity parameters cancel out (if there are no cycle slips in the carrier phase observations), on the other hand many errors in the ZD observations can be eliminated or mitigated (e.g. antenna phase centre offsets and its variations, multi-path, etc.). Therefore, the sequential time differenced carrier phase observation has been proven to be very efficient for the LEO precise orbit determination. In this paper, as a first step to determine geometrical LEO precise orbits, initial LEO absolute positions have been estimated based on the Bancroft method with an accuracy of a few meters. These absolute positions can be used subsequently as initial values for LEO positions based on pseudo-range ionosphere free observations. To avoid cycle slips in the carrier phase observations, at first, a 15° elevation cut-off angle has been applied to the observations, secondly, with the estimated positions in the code pseudo-range process and with the help of the receiver clock offset between two sequential epochs, the cycle slips have been eliminated in the iterative process. In this method, the estimated LEO orbit is determined point-wise (geometrical, not kinematical) and the geometrical configuration (DOP) of GPS satellites plays an important role in the data processing.

This method has been tested with simulated data (SC7 - Special Commission 7), which are available from the homepage of the Institute for Geodesy and Geo-information of the university Bonn, (www.geod.uni-bonn.de), and with real LEO data (CHAMP & GRACE) provided by GFZ, Potsdam, Germany.

Formulation

The carrier phase SST observation between the GPS satellite s and the LEO satellite r at frequency i with respect to the ambiguity parameter and all the error terms can be written as:

$$\begin{aligned}\Phi^s_{r,i}(t) &= \rho^s_r(t)+\lambda_i N^s_{r,i}+\lambda_i\left[\phi^0_{i,r}-\phi^0_{i,s}\right]+c\left[dt^s(t-\tau^s_r)-dt_r(t)\right]-I^r_i(t)+\\ &+d^s_O(t)+d^r_R(t)-d^s_R(t)+d^r_{C,i}(t)+d^r_{V,i}(t)+d_{M,\Phi_i}(t)+\varepsilon_{\Phi_i}(t).\end{aligned} \tag{1}$$

If we assume that all systematic error effects on the LEO and the GPS satellites have been represented by the specific models, then all the error effects can be summarized as:

$$\begin{aligned}e^s_{r,\Phi_i}(t) &= \lambda_i\left[\phi^0_{i,r}-\phi^0_{i,s}\right]+cdt^s(t-\tau^s_r)-I^r_i(t)+d^s_O(t)+\\ &+d^r_R(t)-d^s_R(t)+d^r_{C,i}(t)+d^r_{V,i}(t)+d_{M,\Phi_i}(t),\end{aligned} \tag{2}$$

then the carrier phase observation equation at frequency i at epoch t can be rewritten as:

$$\Phi^s_{r,i}(t)=\rho^s_r(t)+\lambda_i N^s_{r,i}-cdt_r(t)+e^s_{r,\Phi_i}(t)+\varepsilon_{\Phi_i}(t). \tag{3}$$

Here at epoch t, $\Phi^s_{r,i}$ is the observed carrier phase between the GPS satellite and the LEO GPS receiver at frequency of i in unit of length, ρ^s_r is the geometrical distance between the GPS and LEO satellites, $N^s_{r,i}$ is the ambiguity parameter, λ_i is the wavelength of the given GPS signal at frequency of i and $e^s_{r,i}$ is the summation of all error effects on the LEO and the GPS satellites, finally ε_{Φ_i} is the remaining errors that can not be modelled in the carrier phase observations. To eliminate the ionosphere effect on the carrier phase observations, the linear combination (L_3) of the carrier phase observation at frequencies $L_1$and L_2has been used. The ionosphere free carrier phase observations at two subsequent epochs, i.e. t_1,t_2, for the same GPS satellite can be written as:

$$\Phi^s_{r,3}(t_1)=\rho^s_r(t_1)+\lambda_3 N^s_{r,3}-cdt_r(t_1)+e^s_{r,\Phi_3}(t_1)+\varepsilon_{\Phi_3}(t_1), \tag{4}$$

$$\Phi^s_{r,3}(t_2)=\rho^s_r(t_2)+\lambda_3 N^s_{r,3}-cdt_r(t_2)+e^s_{r,\Phi_3}(t_2)+\varepsilon_{\Phi_3}(t_2). \tag{5}$$

The difference between the carrier phase observations at two subsequent epochs can be rewritten as:

$$\Delta\Phi^s_{r,3}(t_1,t_2)=\rho^s_r(t_2)-\rho^s_r(t_1)-cdt_r(t_2)+cdt_r(t_1)+e^s_{r,\Phi_3}(t_2)-e^s_{r,\Phi_3}(t_1)+\varepsilon_{\Phi_3}(t_1,t_2). \tag{6}$$

Obviously, in the differenced carrier phase observation, the ambiguity term has been cancelled out. To use the carrier phase observations in the precise positioning procedure, the pseudo-range between the LEO and the GPS satellites at two subsequent epochs with respect to GPS signal travelling time and Sagnac effect can be written as:

$$\rho^s_r(t_1)=\left|\mathbf{R}_Z(\omega_e\tau^s_{r,1})\mathbf{r}^s(t_1-\tau^s_{r,1})-\mathbf{r}_r(t_1)\right|,\quad \rho^s_r(t_2)=\left|\mathbf{R}_Z(\omega_e\tau^s_{r,2})\mathbf{r}^s(t_2-\tau^s_{r,2})-\mathbf{r}_r(t_2)\right|, \tag{7}$$

$$\mathbf{r}_r(t)=\begin{bmatrix}x_r(t) & y_r(t) & z_r(t) & 0\end{bmatrix}^{\mathrm T},\ \mathbf{R}_Z(\omega_e\tau^s_r)\mathbf{r}^s(t-\tau^s_r)=\mathbf{r}^s_m(t)=\begin{bmatrix}x^s(t) & y^s(t) & z^s(t) & 0\end{bmatrix}^{\mathrm T},$$

$$\rho^s_r(t)=\left|\mathbf{r}^s_m(t)-\mathbf{r}_r(t)\right|,\ \rho^s_r(t)=\left|\left(x^s(t)-x_r(t)\right)^2+\left(y^s(t)-y_r(t)\right)^2+\left(z^s(t)-z_r(t)\right)^2\right|^{1/2}.$$

By inserting Eq. (7) in Eq. (6), ionosphere free observation equation can be formulated as:

$$\begin{aligned}\Delta\Phi^s_{r,3}(t_1,t_2) &= \left|\mathbf{r}^s_m(t_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{r}^s_m(t_1)-\mathbf{r}_r(t_1)\right|-\left[cdt_r(t_2)-cdt_r(t_1)\right]+\\ &+e^s_{r,\Phi_3}(t_2)-e^s_{r,\Phi_3}(t_1)+\varepsilon_{\Phi_3}(t_1,t_2).\end{aligned} \tag{8}$$

If we assume that n GPS satellites $s_i, i=1,n$ and the absolute positions and clock offsets are available at two sequential epochs. The sequential time differenced carrier at two given sequential epochs. Then the Gauss-Markov model for the GPS satellite s can be built as:

$$\Delta\Phi^s_{r,3}(t_1,t_2)=\Delta\Phi^s_{r,3,0}(t_1,t_2)+\left.\frac{\partial\Delta\Phi^s_{r,3}(t_1,t_2)}{\partial\mathbf{X}_r}\right|_{\mathbf{X}=\mathbf{X}_0}(\mathbf{X}-\mathbf{X}_0), \tag{9}$$

$$\Delta\Delta\Phi^s_{r,3}(t_1,t_2)=\left.\frac{\partial\Delta\Phi^s_{r,3}(t_1,t_2)}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X}^0}(\Delta\mathbf{X}). \tag{10}$$

with the misclosure vector, $\Delta\Delta\Phi^s_{r,3}(t_1,t_2)$, and the unknown vector correction, $\Delta\mathbf{X}$, (corrections to absolute position, clock offset at two sequential epochs) as:

$$\Delta\Delta\Phi^s_{r,3}(t_1,t_2)=\mathbf{A}^s_r(t_1,t_2)\Delta\mathbf{X}, \tag{11}$$

$$\Delta\Delta\Phi^s_{r,3}(t_1,t_2)=\Delta\Phi^s_{r,3}(t_1,t_2)-\Delta\Phi^s_{r,c}(t_1,t_2),\ \Delta\mathbf{X}=\mathbf{X}-\mathbf{X}_0. \tag{12}$$

The partial derivatives of the sequential time differenced carrier phase observation with respect to the receiver absolute positions and clock offsets can be formulated as:

$$\begin{aligned}\frac{\partial\Delta\Phi^s_{r,3}(t_1,t_2)}{\partial\mathbf{X}} &= \frac{\partial[\rho^s_r(t_2)-cdt_r(t_2)]}{\partial\mathbf{X}}-\frac{\partial[\rho^s_r(t_1)-cdt_r(t_1)]}{\partial\mathbf{X}_r},\\ &=\mathbf{A}^s_r(t_1,t_2)=\begin{bmatrix}-\mathbf{A}^s_r(t_1) & \mathbf{A}^s_r(t_2)\end{bmatrix}\end{aligned} \tag{13}$$

with the design matrix at time t:

$$\begin{aligned}\mathbf{A}^s_r(t) &= \begin{pmatrix}\dfrac{x_r(t)-x^s(t)}{\rho^s_r(t)} & \dfrac{y_r(t)-y^s(t)}{\rho^s_r(t)} & \dfrac{z_r(t)-z^s(t)}{\rho^s_r(t)} & -1\end{pmatrix}\\ &=\begin{pmatrix}e^s_{r,x}(t) & e^s_{r,y}(t) & e^s_{r,z}(t) & -1\end{pmatrix}.\end{aligned} \tag{14}$$

The Gauss-Markov Model for all sequential time differenced carrier phase observation equations reads:

$$\mathbf{\Delta\Delta\Phi}=\mathbf{A\Delta X},\qquad \textstyle\sum_{\mathbf{\Delta\Phi}} \tag{15}$$

$$\begin{pmatrix}\Delta\Delta\Phi^{s_1}_{r,3}(t_1,t_2)\\ \vdots\\ \Delta\Delta\Phi^{s_{m_1}}_{r,3}(t_1,t_2)\\ \Delta\Delta\Phi^{s_1}_{r,3}(t_2,t_3)\\ \vdots\\ \Delta\Delta\Phi^{s_{m_2}}_{r,3}(t_2,t_3)\\ \vdots\\ \Delta\Delta\Phi^{s_1}_{r,3}(t_{n-1},t_n)\\ \vdots\\ \Delta\Delta\Phi^{s_{m_n}}_{r,3}(t_{n-1},t_n)\end{pmatrix}=\begin{pmatrix}-\mathbf{A}^{s_1}_r(t_1) & \mathbf{A}^{s_1}_r(t_2) & \mathbf{0} & \vdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ -\mathbf{A}^{s_{m_1}}_r(t_1) & \mathbf{A}^{s_{m_1}}_r(t_2) & \mathbf{0} & \vdots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{A}^{s_1}_r(t_2) & \mathbf{A}^{s_1}_r(t_3) & \vdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{0} & -\mathbf{A}^{s_{m_2}}_r(t_2) & \mathbf{A}^{s_{m_2}}_r(t_3) & \vdots & \mathbf{0} & \mathbf{0}\\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & -\mathbf{A}^{s_1}_r(t_{n-1}) & \mathbf{A}^{s_1}_r(t_n)\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & -\mathbf{A}^{s_{m_n}}_r(t_{n-1}) & \mathbf{A}^{s_{m_n}}_r(t_n)\end{pmatrix}\begin{pmatrix}dx_1\\ dy_1\\ dz_1\\ dcdt_1\\ dx_2\\ dy_2\\ dz_2\\ dcdt_2\\ \vdots\\ dx_n\\ dy_n\\ dz_n\\ dcdt_n\end{pmatrix}$$

If the number of all GPS SST observations is larger than $4n$ (n is the number of epochs) and if the design matrix $\mathbf{A}$ is regular, then it holds for the vector of unknowns:

$$\mathbf{\Delta\hat X}=\left(\mathbf{A}^{\mathrm T}\mathbf{W}_{\mathbf{\Delta\Phi}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm T}\mathbf{W}_{\mathbf{\Delta\Phi}}\mathbf{\Delta\Delta\Phi},\quad \mathbf{W}_{\mathbf{\Delta\Phi}}=\sigma_0^2\textstyle\sum_{\mathbf{\Delta\Phi}}^{-1},\quad \mathbf{C}_{\mathbf{\Delta\hat X}}=\left(\mathbf{A}^{\mathrm T}\mathbf{W}_{\mathbf{\Delta\Phi}}\mathbf{A}\right)^{-1}. \tag{16}$$

After an convergent iterative estimation procedure the corrections to the unknowns, the receiver (LEO) absolute positions and the clock offsets at all epochs can be estimated as follows:

$$\mathbf{\hat X}=\mathbf{X}_0+\mathbf{\Delta\hat X},\quad \mathbf{C}_{\mathbf{\hat X}}=\mathbf{C}_{\mathbf{\Delta\hat X}}. \tag{17}$$

Design matrix rank defect

The design matrix $\mathbf{A}_{p\times q}$ with $\mathbf{x}_q, |\mathbf{x}|\neq 0$,

$$\mathbf{Ax}=0, \tag{18}$$

has a rank defect, which is caused by the clock offsets. This can be seen by setting the values zero for the position unknowns and one for the clock offsets, as follows,

$$\mathbf{x}_{4n}=\begin{pmatrix}0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 1\end{pmatrix}^{\mathrm T}. \tag{19}$$

Then the linear dependency of the clock offsets follows

$$\mathbf{Ax}=0. \tag{20}$$

The rank defect is of the order one, because if only one clock offset value of the vector $\mathbf{x}$ is changed to zero, then the design matrix becomes regular.

To remove the rank defect of the design matrix in equation (15), the following alternative is possible:

1) elimination of the clock offset parameter at the first epoch in the estimation procedure (this corresponds to the choice of the value zero for the "clock offset datum"),
2) inserting code pseudo-range observations at the first epoch in the estimation procedure.

If the clock offset parameter at the first epoch has been eliminated then the equation (15) can be rewritten as:

$$\mathbf{\Delta L}=\mathbf{A\Delta X},\qquad \textstyle\sum_{\mathbf{\Delta L}} \tag{21}$$

$$\begin{pmatrix}\Delta\Delta\Phi^{s_1}_{r,3}(t_1,t_2)\\ \vdots\\ \Delta\Delta\Phi^{s_{m_1}}_{r,3}(t_1,t_2)\\ \Delta\Delta\Phi^{s_1}_{r,3}(t_2,t_3)\\ \vdots\\ \Delta\Delta\Phi^{s_{m_2}}_{r,3}(t_2,t_3)\\ \vdots\\ \Delta\Delta\Phi^{s_1}_{r,3}(t_{n-1},t_n)\\ \vdots\\ \Delta\Delta\Phi^{s_{m_n}}_{r,3}(t_{n-1},t_n)\end{pmatrix}=\begin{pmatrix}-\mathbf{\bar A}^{s_1}_r(t_1) & \mathbf{A}^{s_1}_r(t_2) & \mathbf{0} & \vdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ -\mathbf{\bar A}^{s_{m_1}}_r(t_1) & \mathbf{A}^{s_{m_1}}_r(t_2) & \mathbf{0} & \vdots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{A}^{s_1}_r(t_2) & \mathbf{A}^{s_1}_r(t_3) & \vdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{0} & -\mathbf{A}^{s_{m_2}}_r(t_2) & \mathbf{A}^{s_{m_2}}_r(t_3) & \vdots & \mathbf{0} & \mathbf{0}\\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & -\mathbf{A}^{s_1}_r(t_{n-1}) & \mathbf{A}^{s_1}_r(t_n)\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & -\mathbf{A}^{s_{m_n}}_r(t_{n-1}) & \mathbf{A}^{s_{m_n}}_r(t_n)\end{pmatrix}\begin{pmatrix}dx_1\\ dy_1\\ dz_1\\ dx_2\\ dy_2\\ dz_2\\ dcdt_2\\ \vdots\\ dx_n\\ dy_n\\ dz_n\\ dcdt_n\end{pmatrix}$$

with the modified design matrix $\mathbf{\bar A}^s_r(t_1)$ at the first epoch,

$$\begin{aligned}\mathbf{\bar A}^s_r(t_1) &= \begin{pmatrix}\dfrac{x_r(t_1)-x^s(t_1)}{\rho^s_r(t_1)} & \dfrac{y_r(t_1)-y^s(t_1)}{\rho^s_r(t_1)} & \dfrac{z_r(t_1)-z^s(t_1)}{\rho^s_r(t_1)}\end{pmatrix}\\ &=\begin{pmatrix}e^s_{r,x}(t_1) & e^s_{r,y}(t_1) & e^s_{r,z}(t_1)\end{pmatrix}.\end{aligned} \tag{22}$$

In other words, the column corresponding to the clock offset at the first epoch in Eq. (15), has been removed to get regularity for the design matrix. Now, the design matrix is regular and the unknown parameters (positions at all tracked epochs and relative clock offsets at all epochs w.r.t the given clock offset at the first epoch) can be estimated. Obviously, the estimated clock offsets are relative to the first epoch. This principle disadvantage is not relevant in our application!

Data pre-processing

Our data pre-processing strategy is based on an epoch wise processing of GPS SST observations. In this method, the code observations are processed for each epoch, the receiver absolute position is estimated and the receiver clock is synchronized to GPS time in a first step. In a second step, the carrier phase differences between subsequent epochs are processed to detect large cycle slips and outliers in the observations. If we assume that at the sequential epoch i (epoch i, $i+1$), m identical GPS satellites are tracked by the LEO GPS receiver, then the receiver clock offset difference between two sequential epochs for every tracked GPS satellite can be calculated from sequential time differenced carrier phase observation and from the absolute positions at two sequential epochs:

$$\begin{aligned}\Delta cdt^s_r(t_{12}) &= \left|\mathbf{r}^s_m(t_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{r}^s_m(t_1)-\mathbf{r}_r(t_1)\right|-\Delta\Phi^s_{r,3}(t_1,t_2)+\\ &+e^s_{r,\Phi_3}(t_2)-e^s_{r,\Phi_3}(t_1)+\varepsilon_{\Phi_3}(t_1,t_2).\end{aligned} \tag{23}$$

If the receiver clock offset differences at every sequential epoch have been calculated from this equation then the clock offset difference vector can be arranged as:

$$\mathbf{\Delta cdt}_r(t_{12})=\begin{pmatrix}cdt^{s_1}_r & \cdots & cdt^{s_i}_r & \cdots & cdt^{s_m}_r\end{pmatrix}^{\mathrm T}. \tag{24}$$

The mean value and the standard deviation of all calculated clock offset differences can be written as:

$$\overline{\Delta cdt}_r=\mathrm{E}(\mathbf{\Delta cdt}_r),\quad \sigma_{\Delta cdt_r}. \tag{25}$$

If it holds

$$\varepsilon^s_r=\left|\Delta cdt^s_r\text{-}\overline{\Delta cdt}_r\right|\leq 3\sigma_{\Delta cdt_r}, \tag{26}$$

then the GPS satellite s is accepted to the data processing. If the condition in Eq. (26) is not fulfilled then the GPS satellite s is marked as outlier and the GPS data processing will be performed without GPS satellite s. The key factor that limits the performance of the pre-screening approach is the quality of the a-priori knowledge of the positions. For the point positioning based on code observation this is not a serious problem. But the quality of the a-priori information is critical for the screening of the sequential time differenced carrier phase observations. To avoid too many rejections it is advisable to set the threshold not too small. This implies that the screening procedure has to be iterated to find carrier phase observations without outliers.

Numerical results

To test the proposed strategy to estimate LEO satellites absolute positions from sequential time differenced carrier phase observations a simulation example as well as a real orbit of CHAMP will be considered. Based on the SC7 dataset one revolution has been selected and carrier phase observations have been simulated. The observations are contaminated with white noise of a standard deviation of 1 cm. In Fig. (1), the simulated and real orbits of CHAMP are shown and in Fig. (2) and Fig. (3) results of both examples are shown. A cut-off angle of 15° for the LEO GPS SST link has been applied.

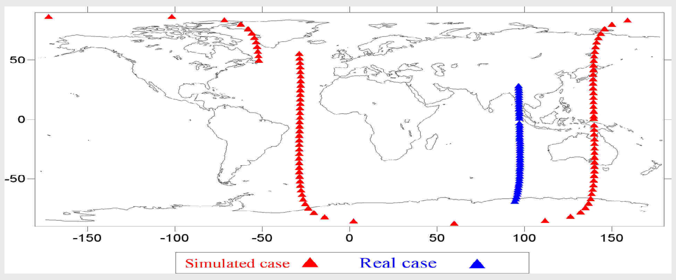

Fig. (1): One revolution of the CHAMP simulation at 2000 07 17 18:00-19:35 and a 25 minutes real short arc of CHAMP at 2002 07 20 12:50-13:15.

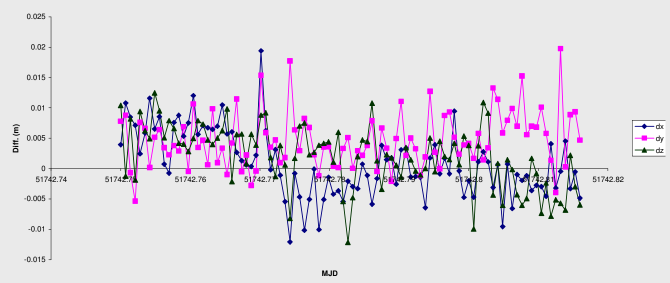

Fig. (2): Difference graph between the estimated absolute positions from sequential time differenced carrier phase observations and the error free absolute positions from SC7.

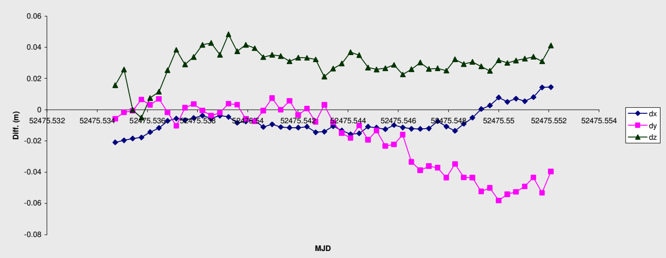

Fig. (3): Difference graph between estimated absolute positions of CHAMP from time differenced carrier phase observations and the CHAMP PSO absolute positions from GFZ data base.

Conclusions

The proposed POD strategy can be characterized by the following properties:

- the LEO satellite orbits are of pure geometrical type, point-wise, 3D,
- carrier phase observations are used in a sequential way by zero differencing,
- time differenced carrier phase observations are free from ambiguity parameters,
- the proposed technique can be easily modified to represent a reduced geometrical precise orbit determination technique with less noise,
- simulation and real case results demonstrate an cm-accuracy is possible.

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