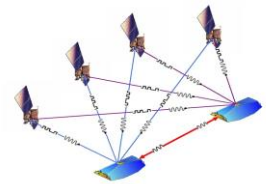

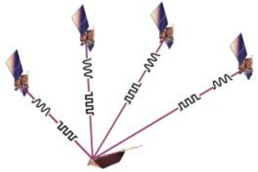

A new approach for an integrated kinematic-dynamic orbit determination of low flying satellites based on GNSS observations

Akbar Shabanloui

Geodätische Woche 2007

Session GNSS (G6)

Leipzig, Deutschland

26 Sep. 2007

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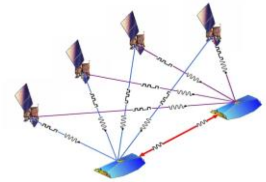
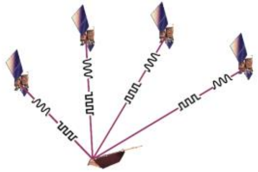
LEO Missions (Earth Explorers)



Advantages of Kinematical LEO Precise Orbit Determination (POD)



- ✓**Kinematical LEO orbits can be used to recover the gravity field of the Earth by the POD methods,**

- ✓**Analysis of altimetry observations requires kinematical orbits of altimetry satellite,**

- ✓**Atmosphere sounding requires kinematical positions of the LEO satellites**

- ✓**GNSS (GPS, GLONASS, GALILEO) methods play an important role in geometrical (subsequently kinematical) POD in addition to classical methods (e.g. SLR…)**

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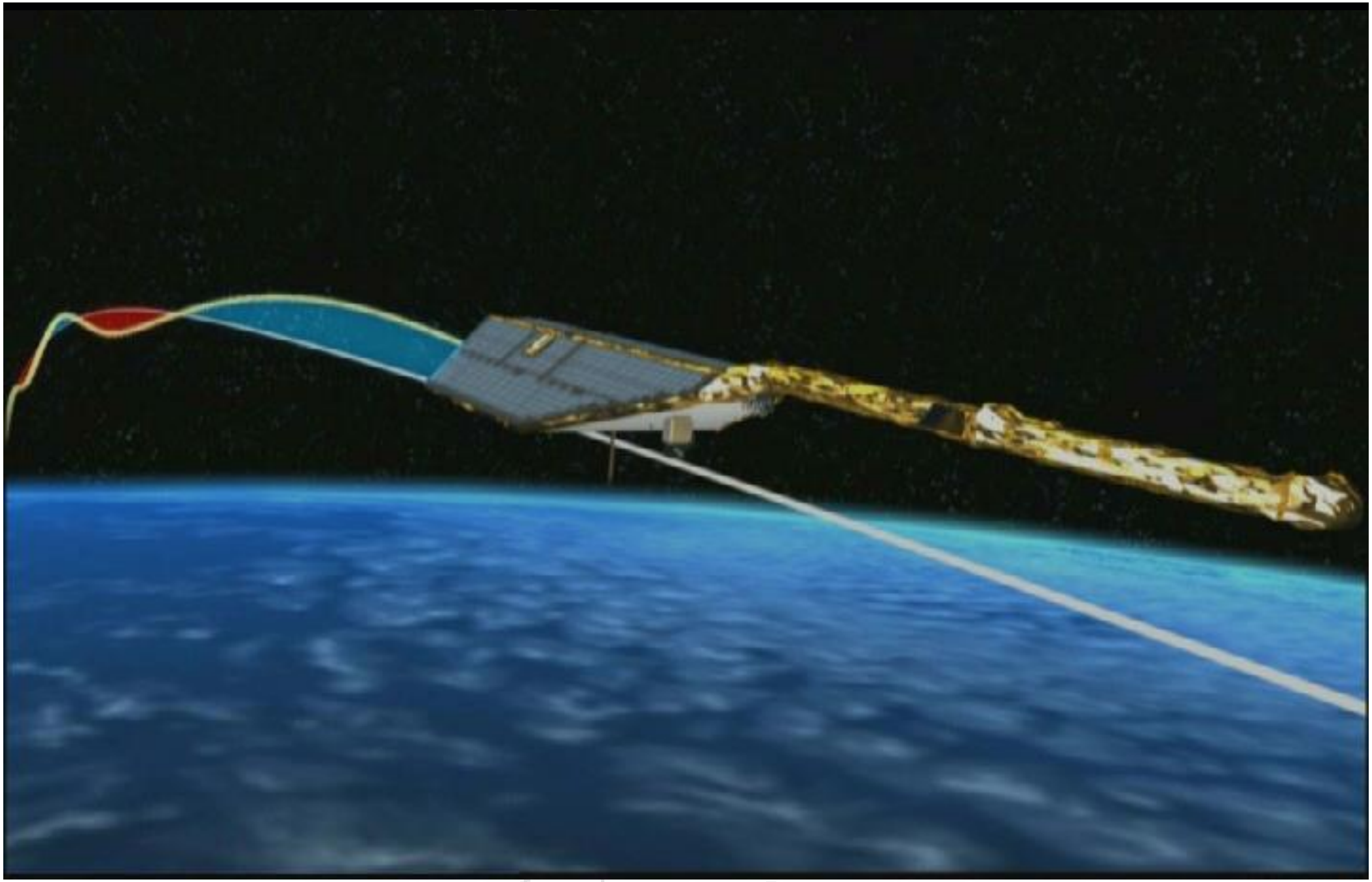
Precise Orbit Determination (POD)



Principal techniques of POD

✓Geometrical orbit determination

Only geometrical observations are used, no force models and no constraints; pointwise representation

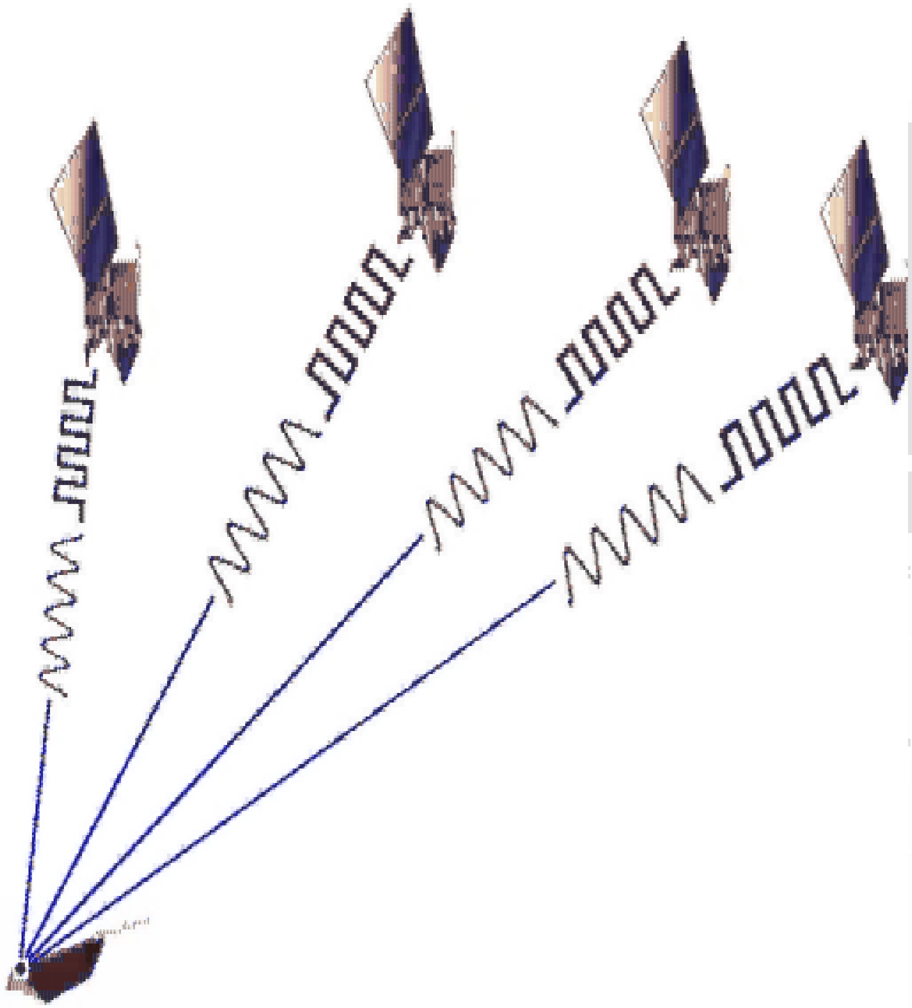
✓Kinematical orbit determination

Geometrical & kinematical observations are used, no force models used; representation by kinematical functions

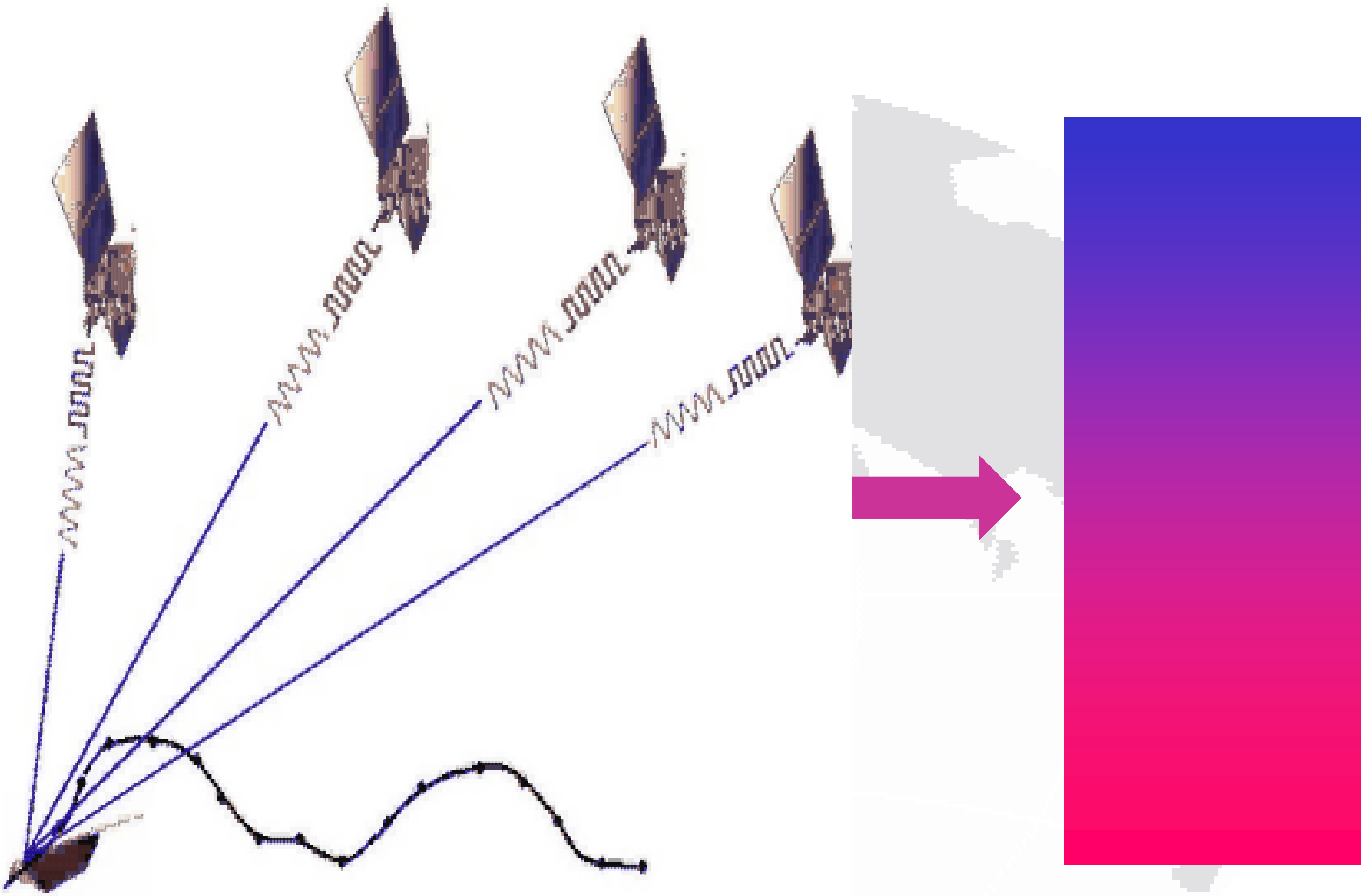
✓Dynamical orbit determination

Geometrical, kinematical & dynamical observations are used, force models necessary; pointwise representation or representation by functions

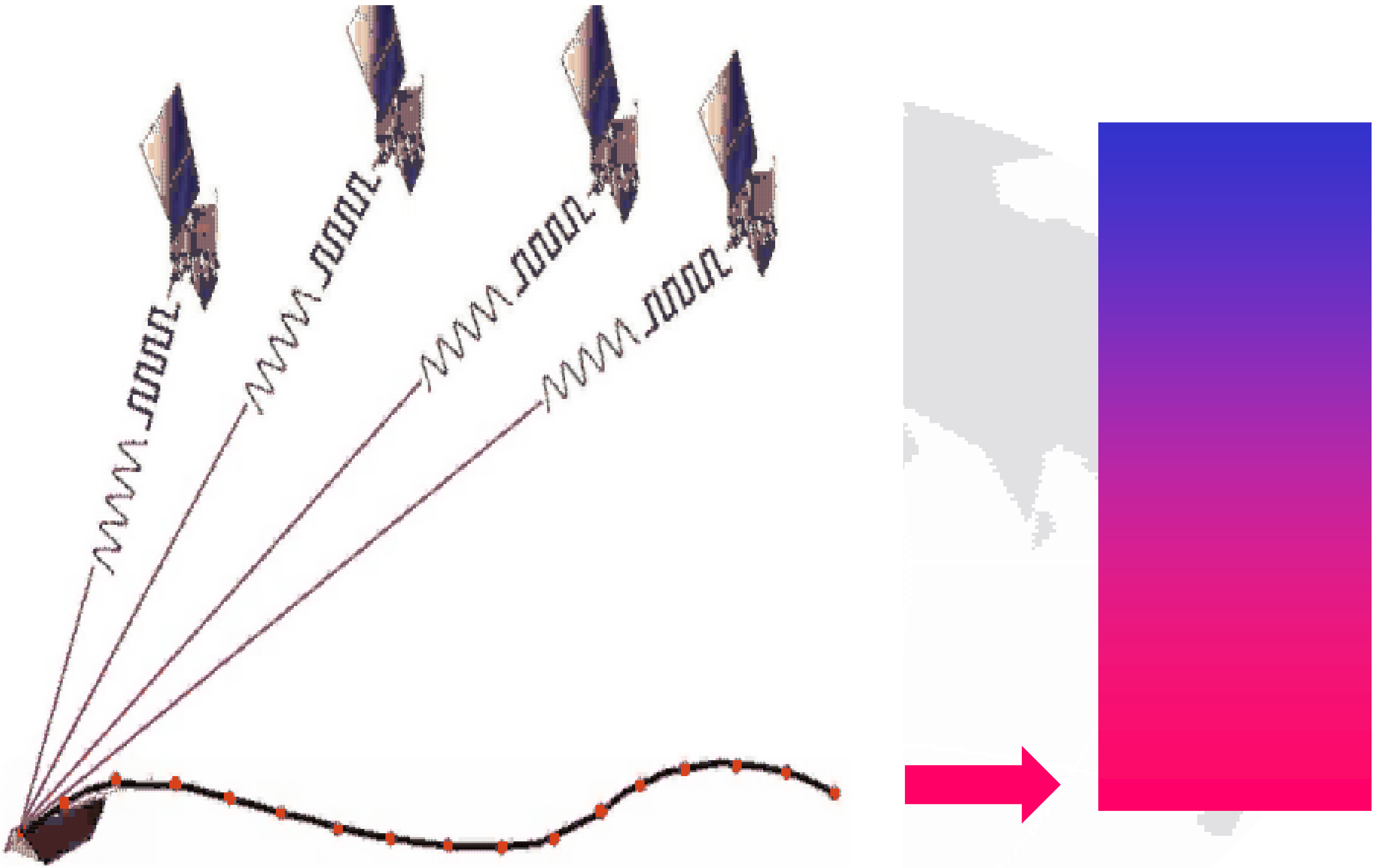
Geometrical precise orbit determination



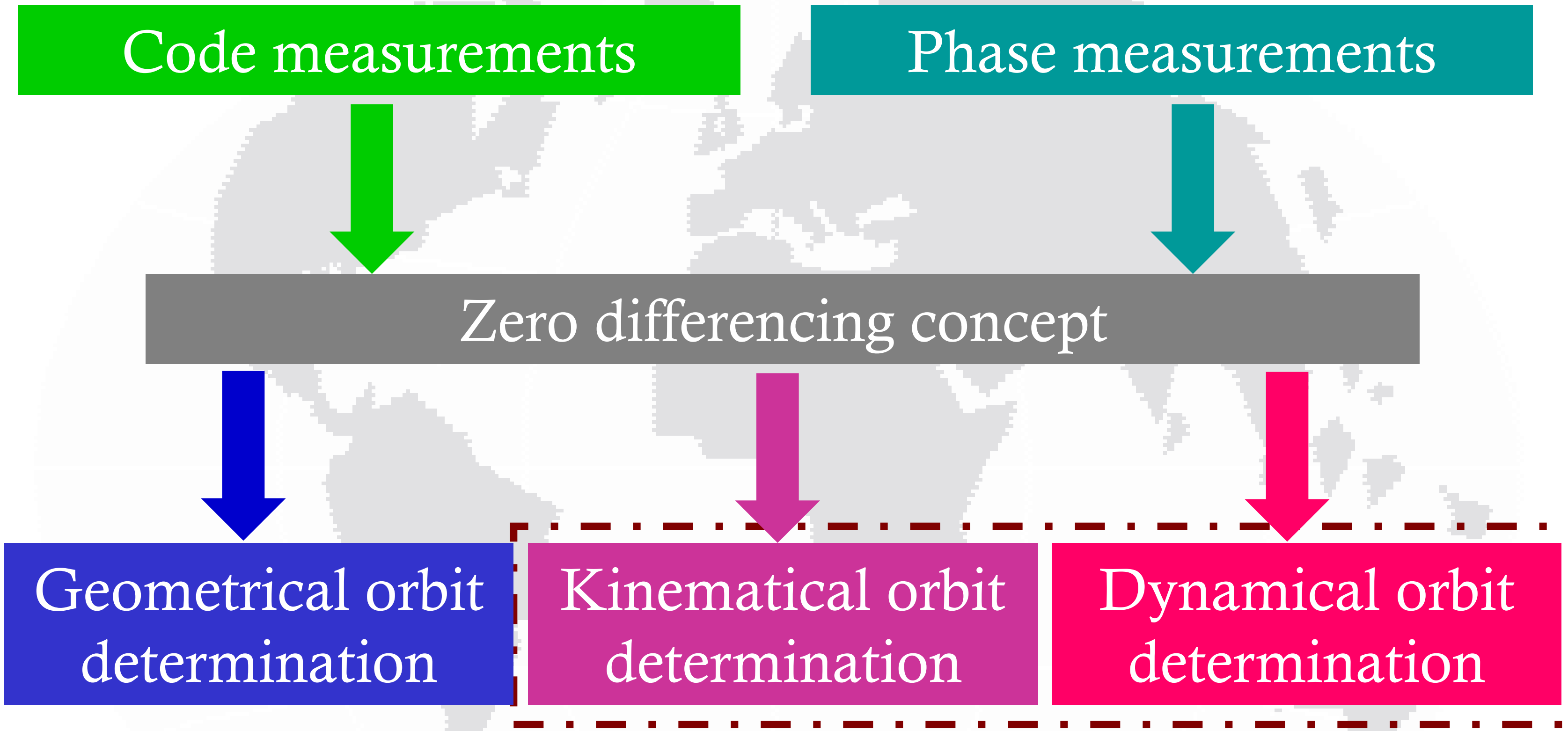
Kinematical precise orbit determination



Dynamical precise orbit determination



Processing concepts

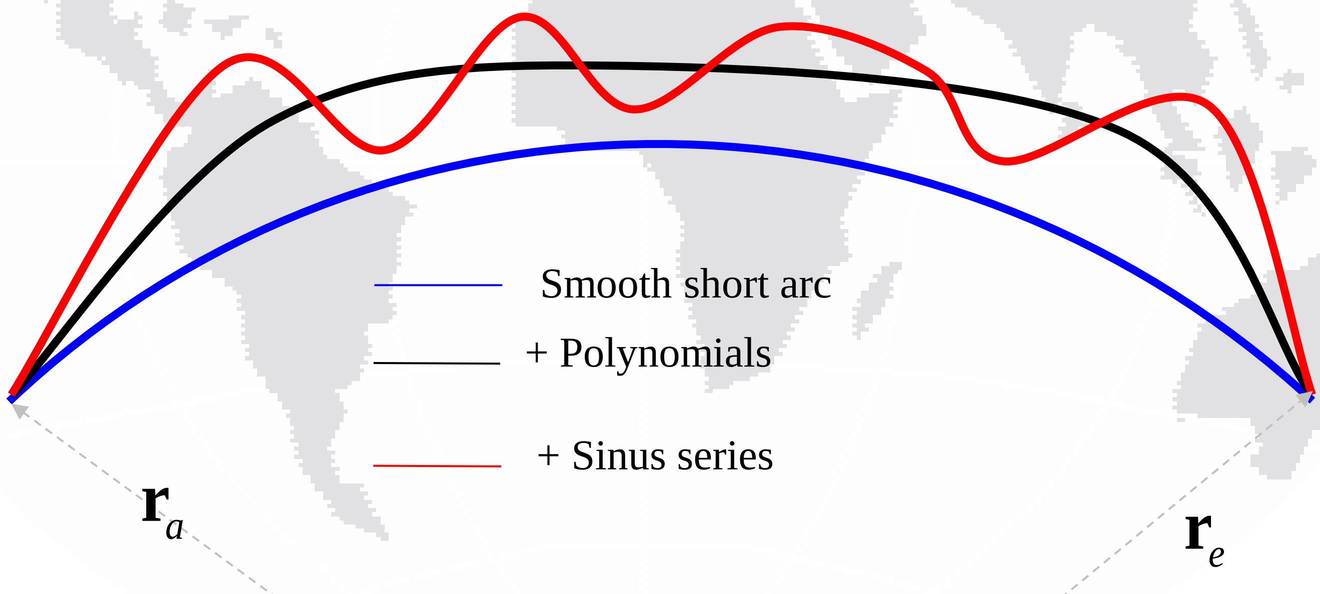


Kinematical (short arcs) POD concept

LEO short arc POD principle

Step by step presentation of LEO short arcs from the solution of Newton-Euler motion equation,

$$\mathbf{r} = \underbrace{\mathbf{r}_a \cdot \frac{\sin((1-\tau)\cdot N)}{\sin(N)} + \mathbf{r}_e \cdot \frac{\sin(\tau N)}{\sin(N)}}_{\text{Smooth short arc}} + \underbrace{\mathbf{C}^T \mathbf{P}(\tau)}_{\text{Euler-Bernoulli Polynomial}} + \underbrace{\sum_{\nu=1}^{n} \bar{\bar{\mathbf{d}}}_\nu \sin(\nu\pi\tau)}_{\text{Sinus series}}$$



representation of LEO short arc

Kinematical short arc POD-simulation

selected a-priori

$$\mathbf{r}(t) = \mathbf{r}_a \cdot \frac{\sin((1-\tau)N)}{\sin(N)} + \mathbf{r}_e \cdot \frac{\sin(\tau N)}{\sin(N)} + \mathbf{C}^T \mathbf{P}(\tau) + \sum_{\nu=1}^{n} \bar{\bar{\mathbf{d}}}_\nu \sin(\nu\pi\tau)$$

all coefficients can be estimated by a Gauss-Markov model

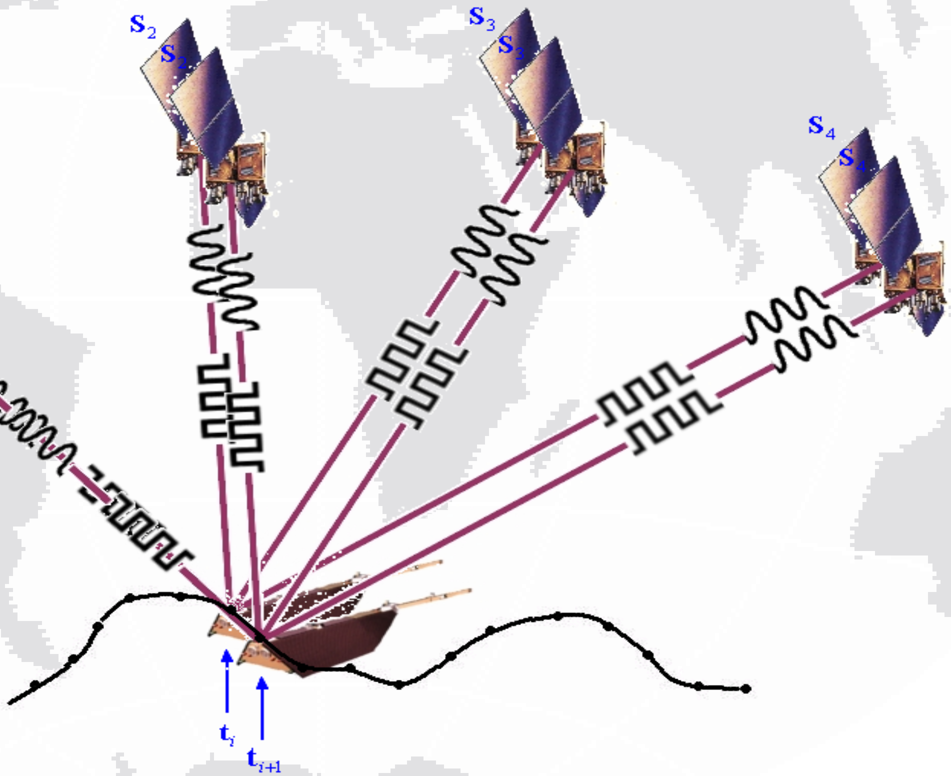
Advantage:

the kinematical orbits and other kinematical parameters can be derived directly from estimated LEO short arc parameters.

Kinematical (short arcs)
POD
simulated case

Absolute position from sequential time differenced carrier phase

- cut-off angle 15°
- simple data screening & S/N filtering
- elevation weighting



Kinematical short arc POD sequential time differenced carrier phase

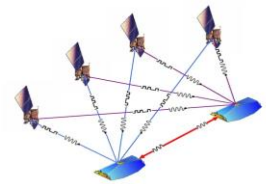
Sequential time differenced carrier phase observations between GPS satellite *s* LEO receiver *r* at epoch *t* can be written as:

$$\Delta\Phi_r^s(t_1,t_2)=\left|\mathbf{r}^s(t_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{r}^s(t_1)-\mathbf{r}_r(t_1)\right|+e_{\Delta\Phi_3}$$

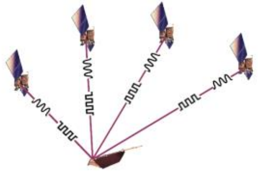
$$\Delta\Phi_{r,3}^s(t_1,t_2)=$$

$$\left|\mathbf{r}^s(t_2)-(\mathbf{r}_a\frac{\sin((1-\tau_2)N)}{\sin(N)}+\mathbf{r}_e\frac{\sin(\tau_2 N)}{\sin(N)}+\mathbf{C}_{3\times 4}\mathbf{P}(\tau_2)+\sum_{f=1}^{n}\bar{\mathbf{d}}_f\sin(\pi f\tau_2))\right|-$$

$$\left|\mathbf{r}^s(t_1)-(\mathbf{r}_a\frac{\sin((1-\tau_1)N)}{\sin(N)}+\mathbf{r}_e\frac{\sin(\tau_1 N)}{\sin(N)}+\mathbf{C}_{3\times 4}\mathbf{P}(\tau_1)+\sum_{f=1}^{n}\bar{\mathbf{d}}_f\sin(\pi f\tau_1))\right|+e_{\Delta\Phi_3}$$



Gauss-Markov model

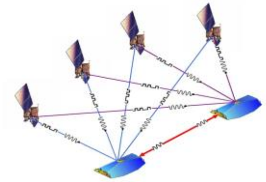


The Gauss-Markov model for two sub-sequential epochs,

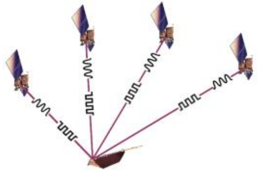
$$\underbrace{\begin{pmatrix} \Delta\Delta\Phi_r^{s_1}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_j}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_{m_1}}(t_1,t_2) \end{pmatrix}}_{\text{Residuals}} = \underbrace{\begin{pmatrix} \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \end{pmatrix}}_{\text{Design matrix}} \underbrace{[\mathbf{X}_r - \mathbf{X}_r^0]}_{\text{Unknowns}}$$

$$\mathbf{X}_r = \begin{pmatrix} x_a & y_a & z_a & \ldots & z_e & c_{11} & c_{12} & c_{13} & c_{14} & \ldots & c_{34} & \bar{\bar{d}}_{1,x} & \bar{\bar{d}}_{1,y} & \bar{\bar{d}}_{1,z} & \ldots & \bar{\bar{d}}_{n,z} \end{pmatrix}_{6+12+3n}^{\mathbf{T}}$$

$$\mathbf{\Delta\Delta\Phi} = \mathbf{A}\mathbf{\Delta X}, \qquad \Sigma_{\mathbf{\Delta\Phi}}$$



Kinematical short arc POD sequential time differenced carrier phase



- **Unknowns:** boundary values, polynomial coefficients corrections, amplitudes of Fourier series.
- **Solutions:** Gauss-Markov model
- **Convergence & accuracy:** after a few iterations, ~ cm.

✓ **LEO short arc orbit is continuous and velocities and another kinematical parameters can be derived from the estimated orbit parameters.**

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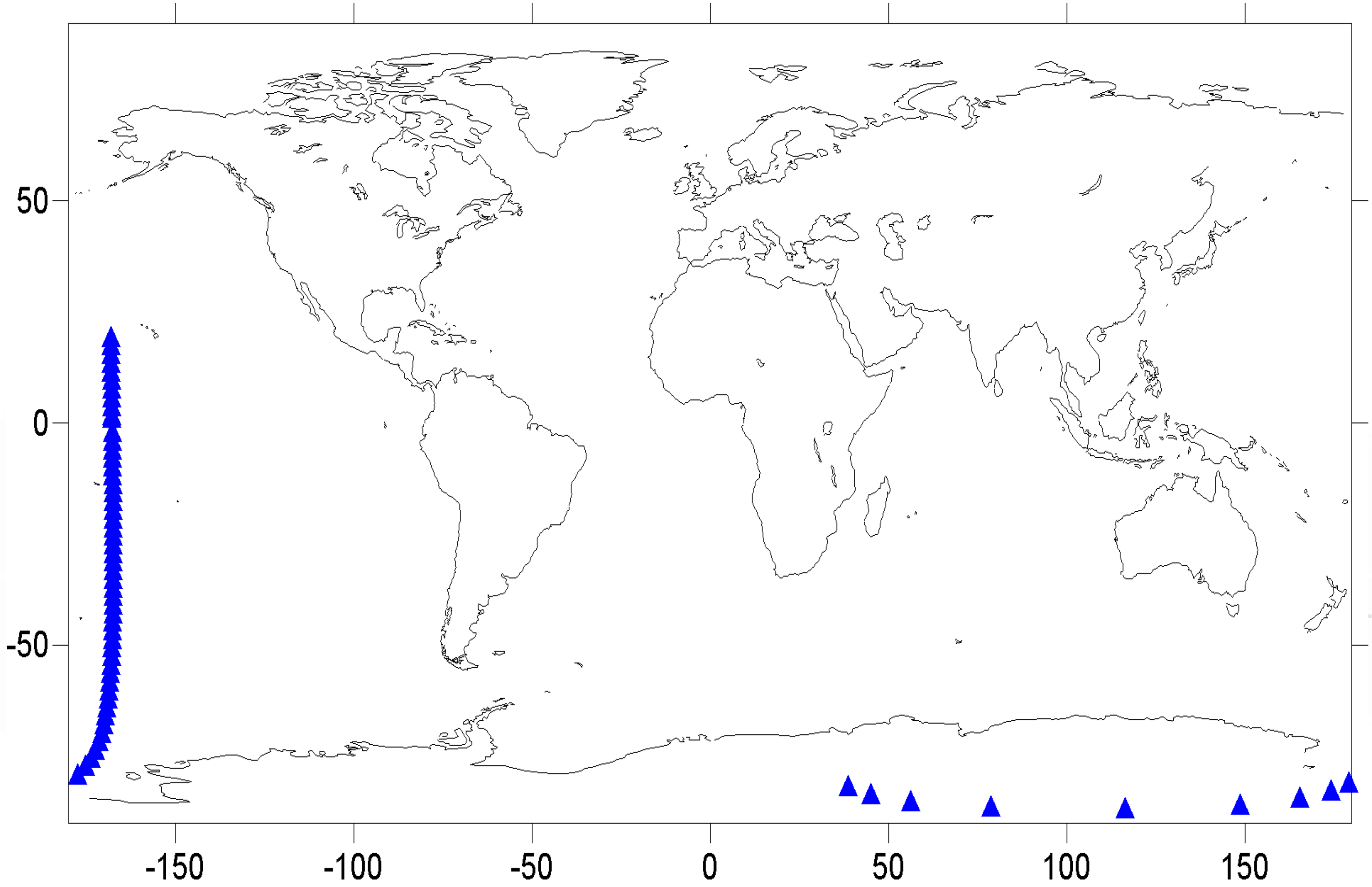
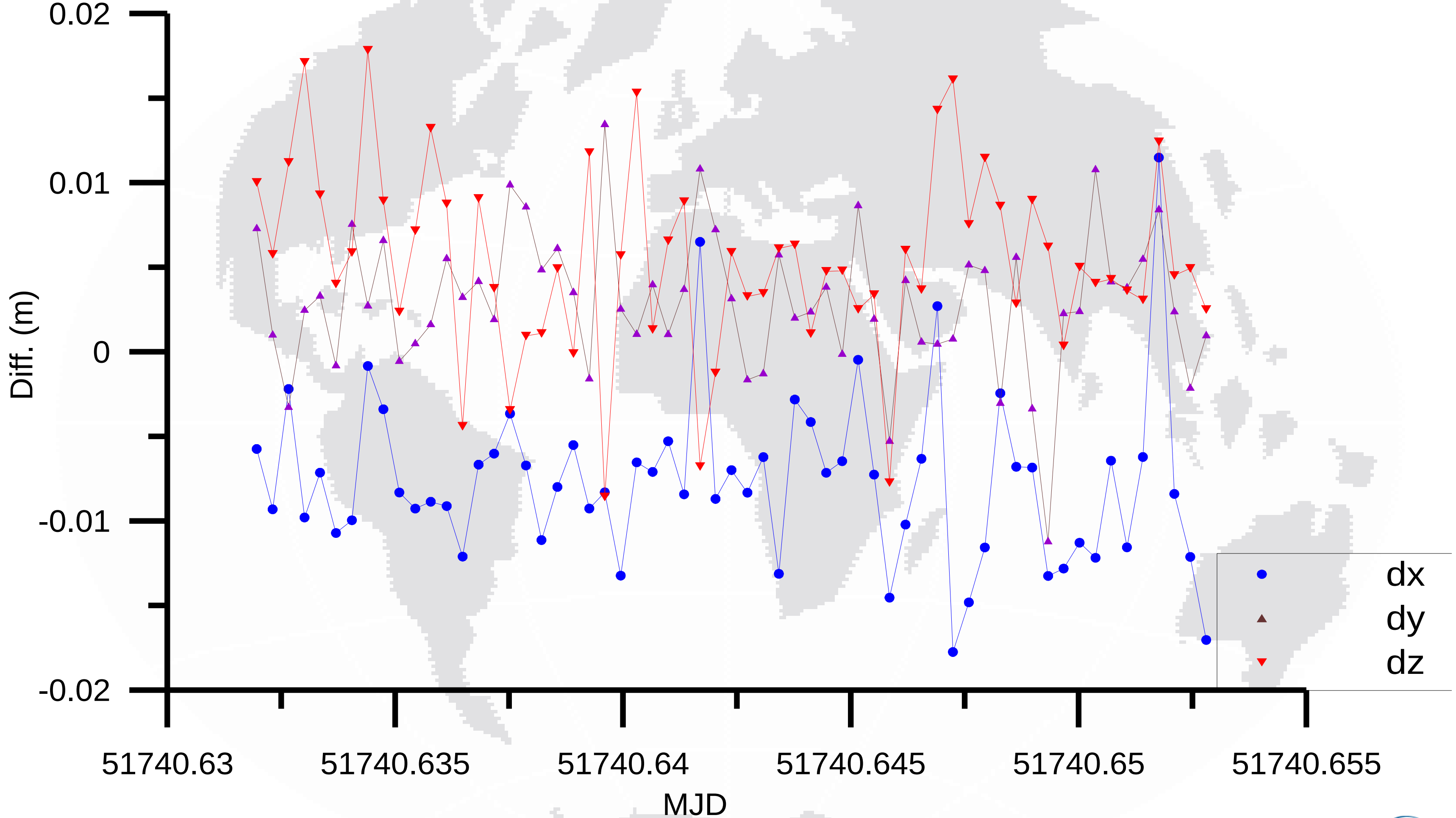
Kinematical short arc POD-simulated



30 minutes of CHAMP satellite [2000 07 15 15h 10m – 15h 40m]

Kinematical short arc POD sequential time differenced carrier phase

Difference plot between estimated short arc absolute positions with observation precision=0.01 m & given positions



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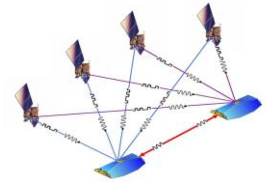
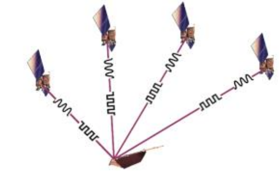
Kinematical (short arcs) POD real case



Carrier phase GPS SST observation



$$\Phi^{s}_{r,i}(t)=\left|\mathbf{R}_z(\omega_e.\varepsilon^s_r)\mathbf{r}^s(t-\varepsilon^s_r)-\mathbf{r}_r(t)\right|+c\left[dt^s(t-\varepsilon^s_r)-dt_r(t)\right]+\lambda N^s_r+$$

$$I^r_i(t)+d^s_O(t)+d^r_R(t)-d^s_R(t)+d^r_{C,i}(t)+d^r_{V,i}(t)+d_{M,P_i}(t)+e_{\Phi_i}$$

s,r	GPS, LEO indices,
ε^s_r	Travelling time between GPS & LEO,
$\mathbf{r}^s(t-\varepsilon^s_r), dt^s(t-\varepsilon^s_r)$	GPS position, clock offset at sending time,
$\mathbf{r}_r(t), dt_r(t)$	LEO position, clock offset at receiving time

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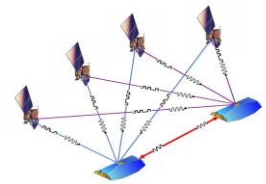
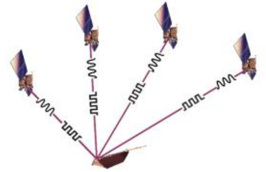


Carrier phase GPS SST observation…



$I_i^r(t)$

- for single frequency receiver, the IONEX model can be used to model the ionosphere error term,
- for dual frequency receiver, the ionosphere free combination can be used.

$d_O^s(t)$

to … used in

$d_R^s(t), d_R^r($

&

$d_{C,i}^r(t), d_{V,i}^r(t)$

th … be modeled … NTEX file.

How can the errors be eliminated or modeled in GPS LEO SST observations?

$d_{M,P_i}(t)$

multipath effect can be minimized through filtering SST observations w.r.t elevation of GPS satellites or applying the elevation weighting method or S/N filtering.

Sequential time differenced carrier phase SST

Carrier phase ionosphere-free observation at epochs (1,2)

$$\Phi^{s}_{r,3}\left(t_1\right)=\left|\mathbf{R}_z(\omega_e\varepsilon_1)\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|+\lambda_3 N^{s}_{r,3}+c\left[dt^s(t_1-\varepsilon_1)-dt_r(t_1)\right]+$$

$$d^{s}_{O}(t_1)+d^{r}_{R}(t_1)-d^{s}_{R}(t_1)+d^{r}_{C,3}(t_1)+d^{r}_{V,3}(t_1)+d_{M,\Phi_3}(t_1)+e_{\Phi_3}$$

$$\Phi^{s}_{r,3}\left(t_2\right)=\left|\mathbf{R}_z(\omega_e\varepsilon_2)\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|+\lambda_3 N^{s}_{r,3}+c\left[dt^s(t_2-\varepsilon_2)-dt_r(t_2)\right]+$$

$$d^{s}_{O}(t_2)+d^{r}_{R}(t_2)-d^{s}_{R}(t_2)+d^{r}_{C,3}(t_2)+d^{r}_{V,3}(t_2)+d_{M,\Phi_3}(t_2)+e_{\Phi_3}$$

sequential time difference ionosphere-free carrier phase observation between epochs (1,2)

$$\Delta\Phi^{s}_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2)\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

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Ionosphere free sequential time differenced carrier phase observations can be written as:

$$\Delta\Phi^{s}_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2)\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1)\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

$$\Delta\Phi^{s}_{r,3}(t_1,t_2)=$$

$$\left|\mathbf{R}_z(\omega_e\varepsilon_2)\mathbf{r}^s(t_2-\varepsilon_2)-(\mathbf{r}_a\frac{\sin((1-\tau_2)N)}{\sin(N)}+\mathbf{r}_e\frac{\sin(\tau_2 N)}{\sin(N)}+\mathbf{C}_{3\times4}\mathbf{P}(\tau_2)+\sum_{f=1}^{n}\bar{\bar{\mathbf{d}}}_f\sin(\pi f\tau_2))\right|-$$

$$\left|\mathbf{R}_z(\omega_e\varepsilon_1)\mathbf{r}^s(t_1-\varepsilon_1)-(\mathbf{r}_a\frac{\sin((1-\tau_1)N)}{\sin(N)}+\mathbf{r}_e\frac{\sin(\tau_1 N)}{\sin(N)}+\mathbf{C}_{3\times4}\mathbf{P}(\tau_1)+\sum_{f=1}^{n}\bar{\bar{\mathbf{d}}}_f\sin(\pi f\tau_1))\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

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The Gauss-Markov model for one epoch,

$$\begin{pmatrix} \Delta\Delta\Phi_r^{s_1}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_j}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_{m_1}}(t_1,t_2) \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \end{pmatrix} [\mathbf{X}_r - \mathbf{X}_r^0] + \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 0 \end{pmatrix} [\mathbf{X}_{\Delta t} - \mathbf{X}_{\Delta t}^0]$$

$$\mathbf{X}_r = \begin{pmatrix} x_a & y_a & z_a & \dots & z_e & c_{11} & c_{12} & c_{13} & c_{14} & \dots & c_{34} & \bar{\bar{d}}_{1,x} & \bar{\bar{d}}_{1,y} & \bar{\bar{d}}_{1,z} & \dots & \bar{\bar{d}}_{n,z} \end{pmatrix}^{\mathbf{T}}_{6+12+3n}$$

$$\mathbf{X}_{\Delta t} = (\Delta c\mathrm{dt}_{12} \quad \dots \quad \Delta c\mathrm{dt}_{(\mathrm{m-1})\mathrm{m}})^{\mathbf{T}}_{m-1}$$

$$\mathbf{\Delta\Delta\Phi} = \mathbf{A\Delta X}, \qquad \Sigma_{\mathbf{\Delta\Phi}}$$

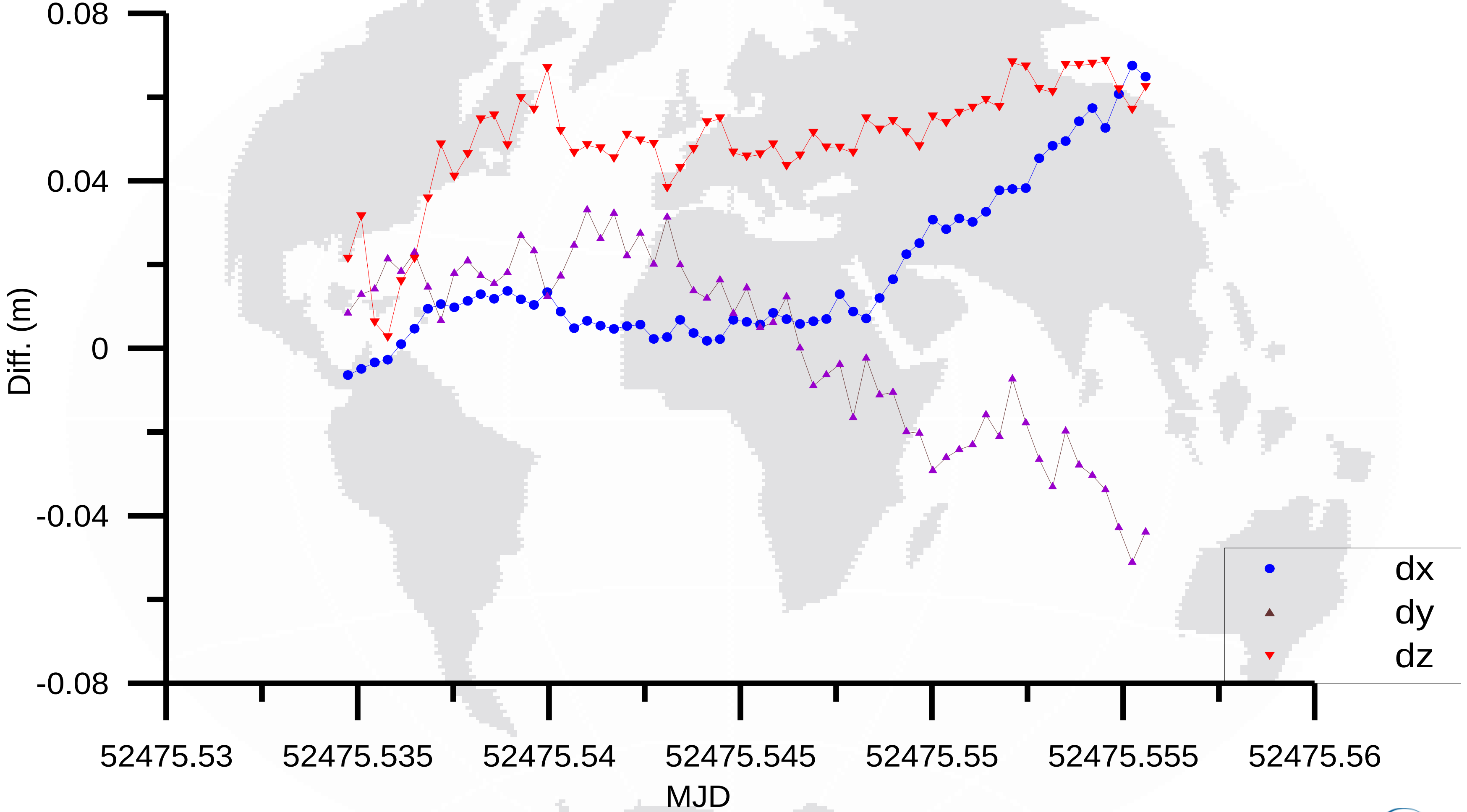
Kinematical short arc POD sequential time differenced carrier phase

- **Initial value:** unknowns initial values can be derived from code estimated positions at the first step,
- **Unknowns:** boundary values, polynomial coefficients corrections, amplitudes of Fourier series,
- **Solutions:** Gauss-Markov model,
- **Convergence & accuracy:** after ~a few iterations, ~ cm.

✓ LEO short arc orbit is continuous and velocities and another kinematical parameters can be derived from the estimated LEO short arc parameters.

Kinematical short arc POD with sequential time differenced carrier phase

Difference plot between absolute positions with carrier phase observation precision =0.01 m & given GFZ CHAMP PSO orbit



Conclusions & remarks (kinematical)

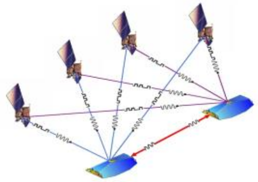
- From sequential differenced carrier phase SST observations, LEO absolute positions and clock offsets can be estimated at every epoch with a sufficient (not essential 4) number of GPS satellites and good satellite geometry,

- An accuracy of cm can be expected for the sequential time diffenced carrier phase SST data processing, but DOP! isn‘t crucial,

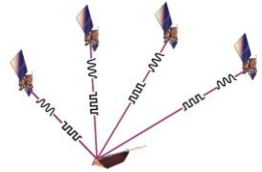
- The resulting LEO orbit is given continuous (without gaps) & smoother than geometrical POD,

- Kinematical LEO orbit can be used to recover the Earth‘s gravity field with the POD recovery concept. (kinematical parameters can be derived analytically).

Dynamical (short arcs) POD simulation case



Dynamical short arc POD



$$\mathbf{r}(t) = \mathbf{r}_a \cdot \frac{\sin((1-\tau)N)}{\sin(N)} + \mathbf{r}_e \cdot \frac{\sin(\tau N)}{\sin(N)} + \mathbf{C}^T \mathbf{P}(\tau) + \sum_{\nu=1}^{n} \bar{\bar{\mathbf{d}}}_\nu \sin(\nu\pi\tau)$$

Boundary positions have been estimated by a Gauss-Markov model

Polynomial & Fourier series coefficients have been determined from given Earth gravity field

disadvantage:

the dynamical orbits can‘t be used directly to recover the Earth gravity field, but estimated dynamical parameters can be used as initial values to kinematical POD procedure.

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Dynamical short arc POD-simulation

•Shor arcs of GRACE twin satellites above Europe and Africa have been selected,

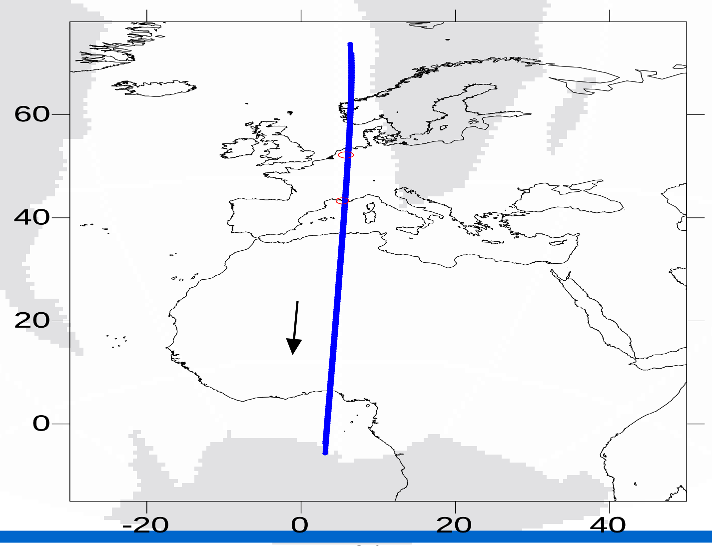
•Observations type:

-GPS high-low SST pseudo-range observations,

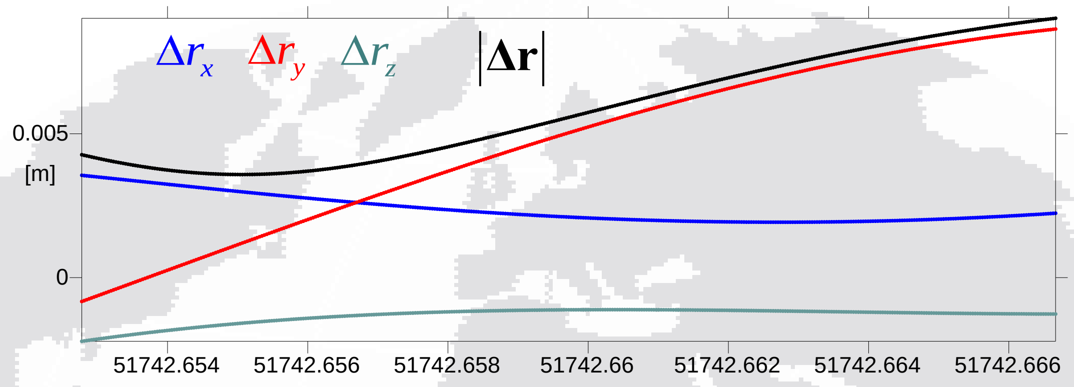
-Earth gravity field (EGM96),

•Unknowns :

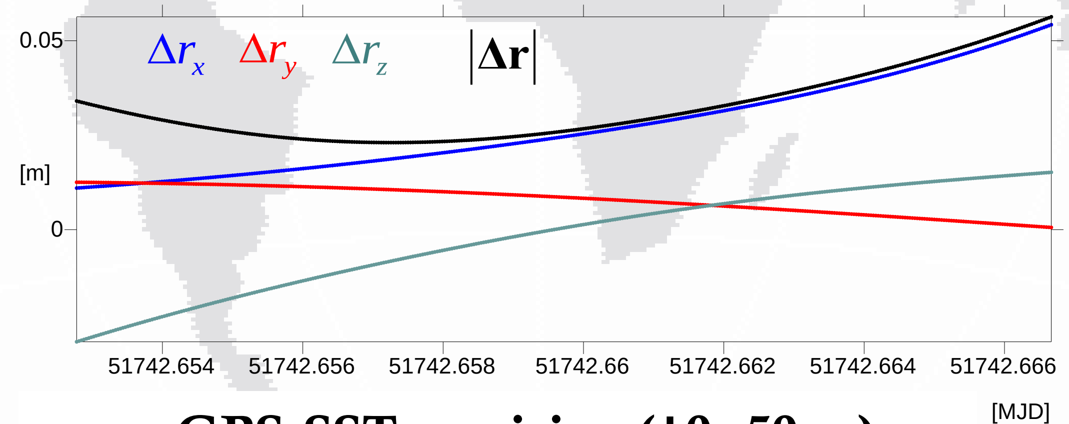
-Boundary positions at begin & end of short arc.



Dynamical short arc POD-simulation results



GPS-SST precision (±0,05 m)



GPS-SST precision (±0, 50 m)

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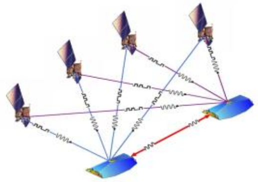
Conclusions & remarks (dynamical)

- Dynamical LEO POD can not be used for recovery of the Earth with the POD recovery concept, but estimated parameters & LEO POD can be used as initial values for further processes as kinematical POD, etc..,

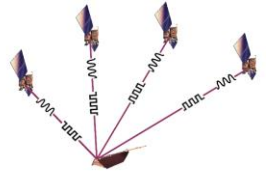
- From dynamical method, LEO positions and clock offsets can be estimated at every epoch with a sufficient enough number (not essential 4) of GPS satellites and good satellite geometry,

- The resulting LEO orbit is given continous (without gaps) and smoother than kinematical POD,

Reduced kinematical (short arcs) POD simulation case



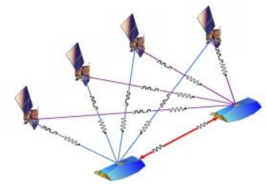
Reduced kinematical POD



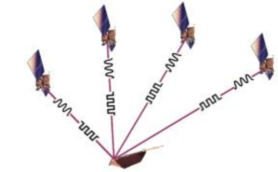
$$\mathbf{r}(t) = \mathbf{r}_a \cdot \frac{\sin((1-\tau)N)}{\sin(N)} + \mathbf{r}_e \cdot \frac{\sin(\tau N)}{\sin(N)} + \mathbf{C}^T \mathbf{P}(\tau) + \sum_{\nu=1}^{n} \bar{\bar{\mathbf{d}}}_\nu \sin(\nu \pi \tau)$$

Calculated from low degree Earth gravity field

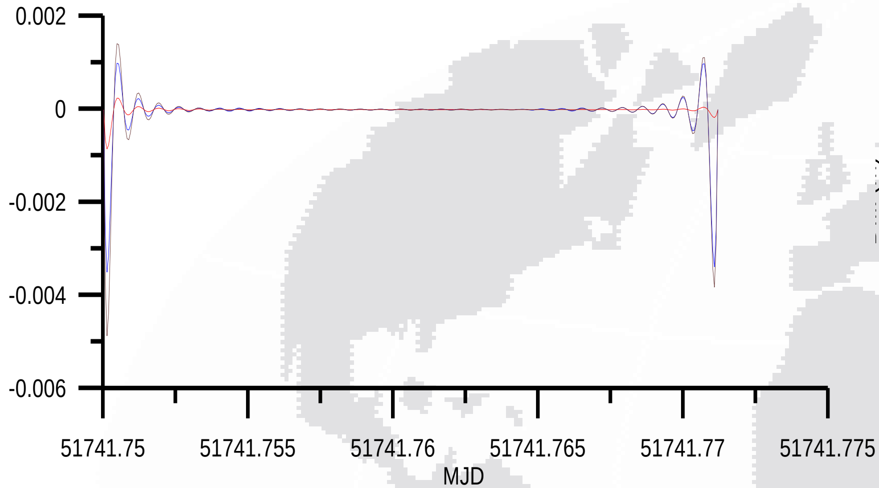
- **Sinsus analysis to difference between real orbit and dycamical low degree orbit,**
- **A little dynamical information have been used in the reduced kinematical orbit determination,**

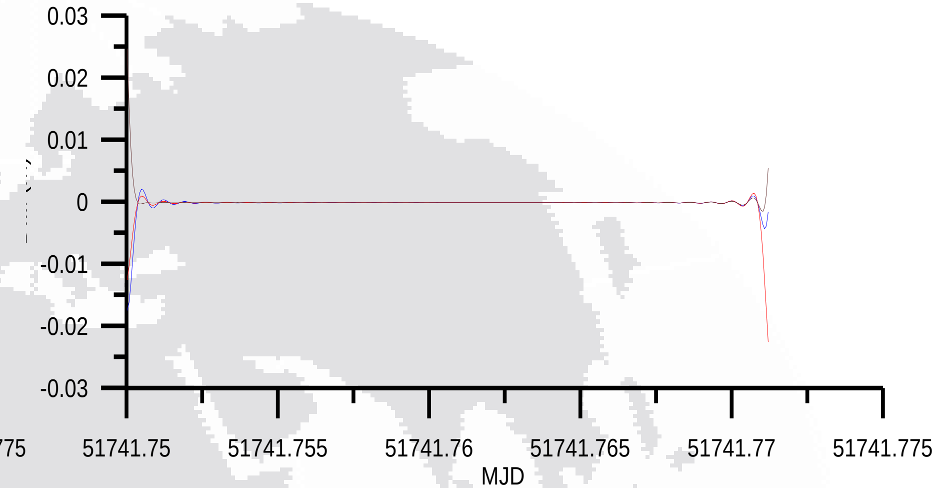


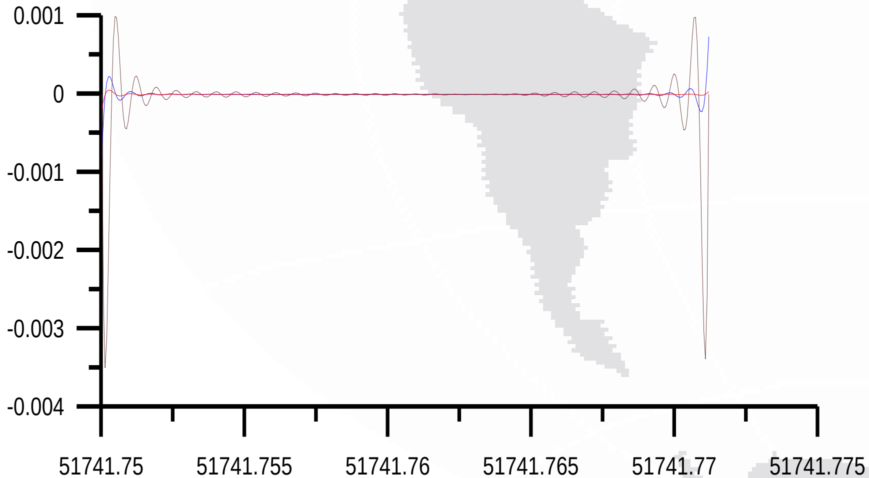
Reduced kinematical POD, results

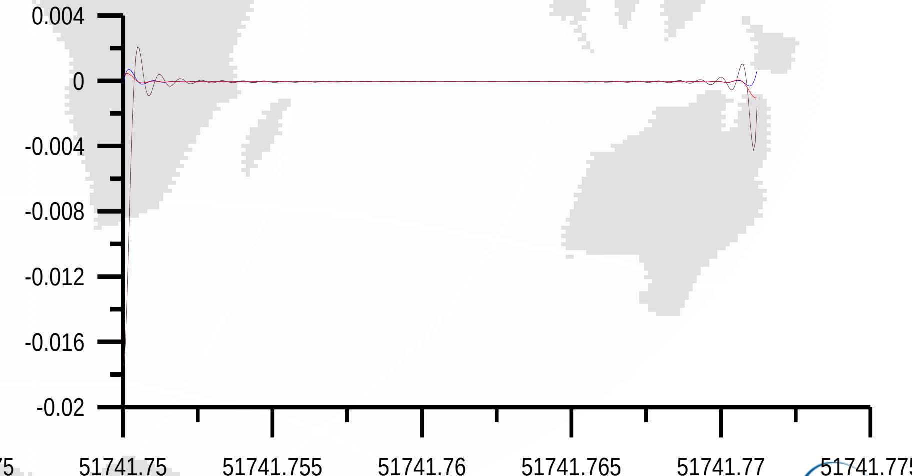


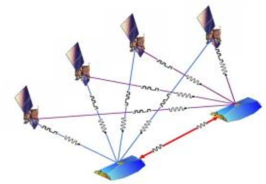
Earth gravity field deg. 02, error free	Earth gravity field deg. 02, error 05 cm



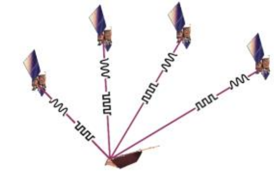




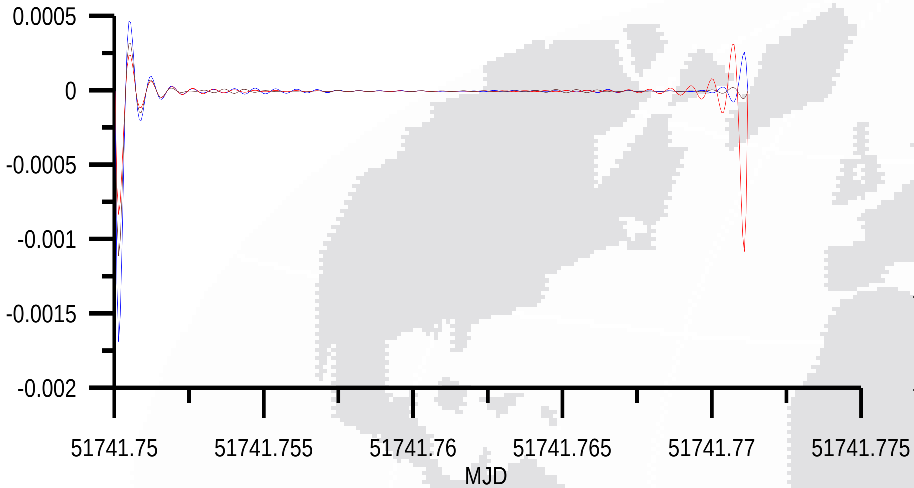


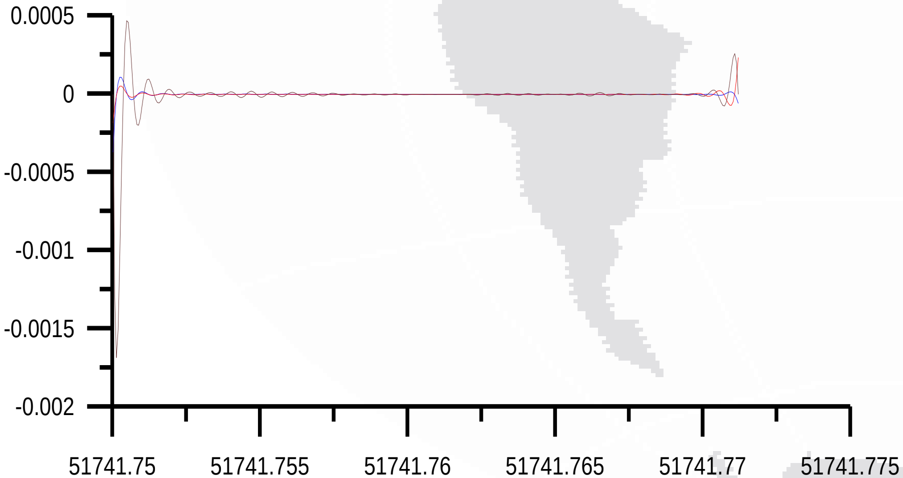


Reduced kinematical POD, results

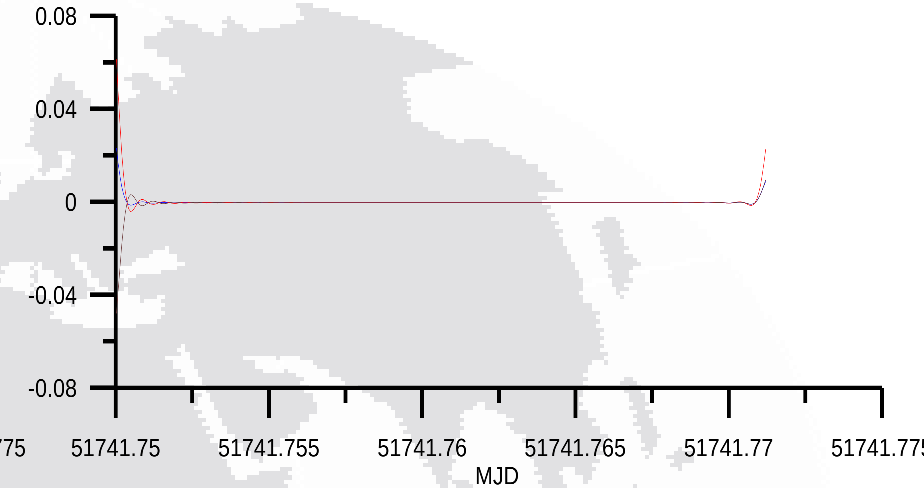


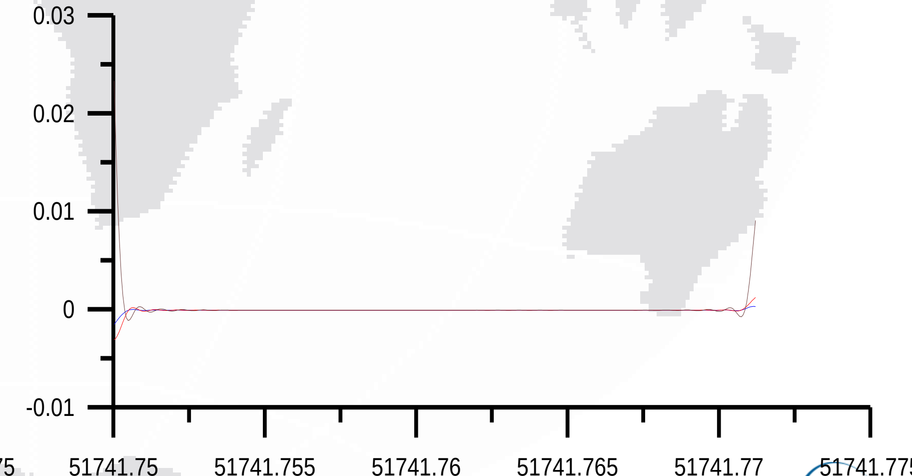
Earth gravity field deg. 06, error free





Earth gravity field deg. 06, error 05 cm





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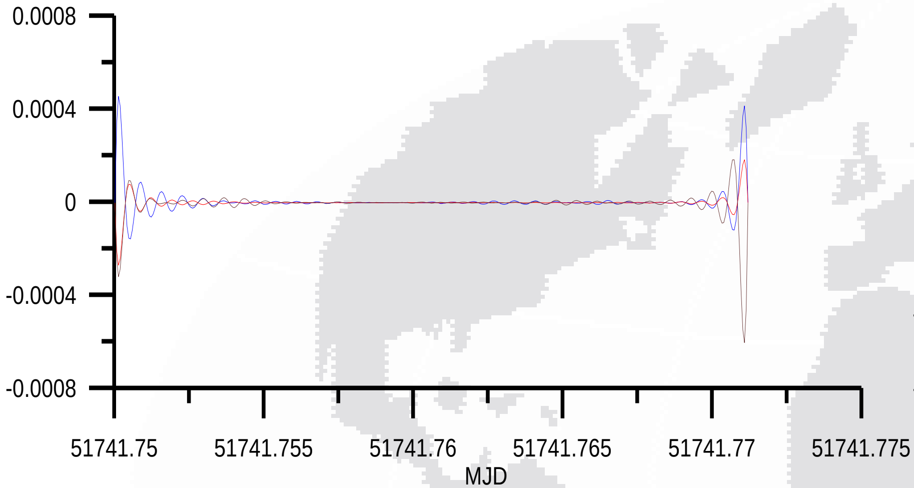
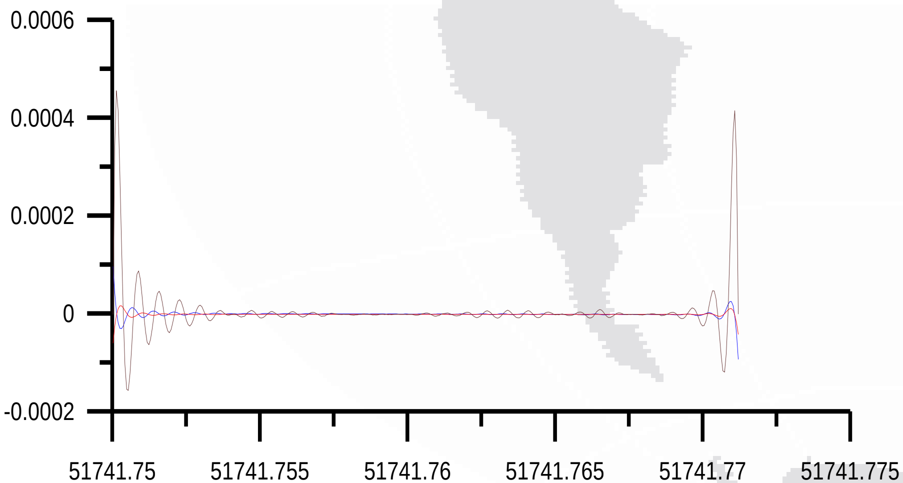
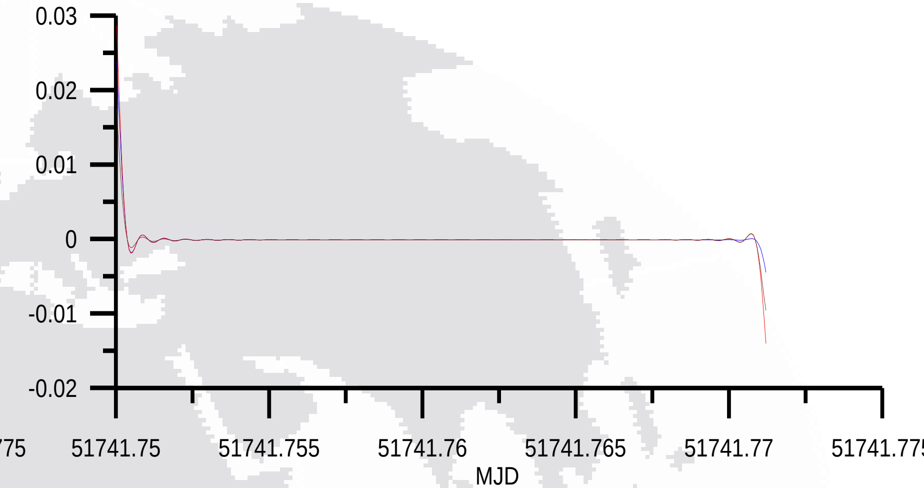
Reduced kinematical POD, results

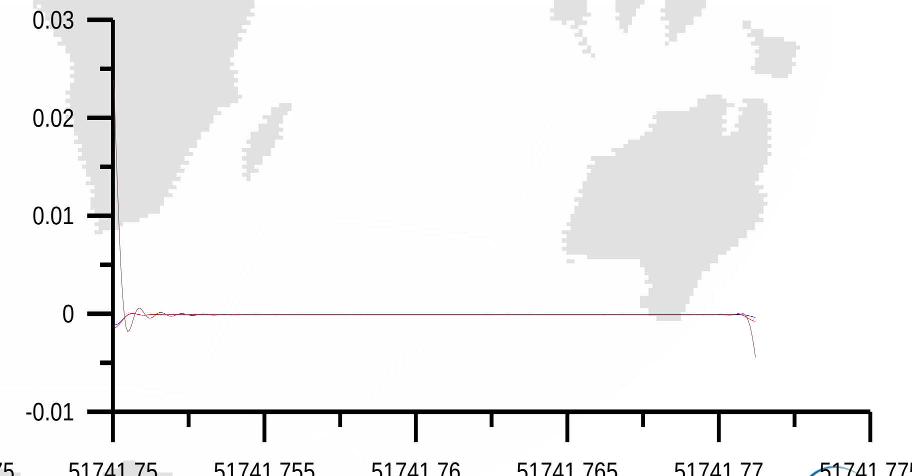
Earth gravity field deg. 12, error free





Earth gravity field deg. 12, error 05 cm





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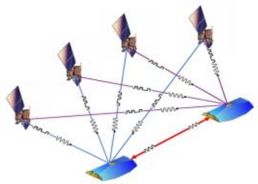
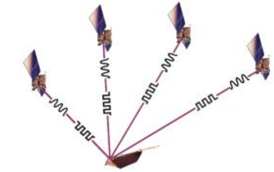
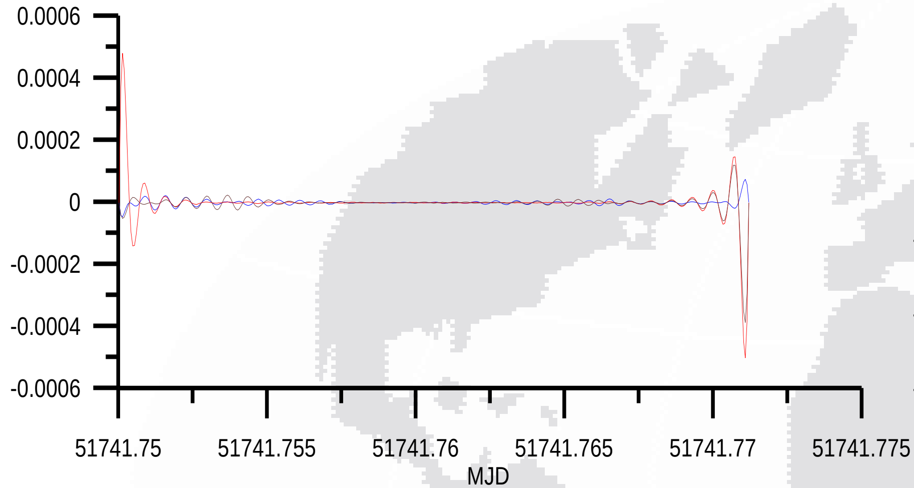
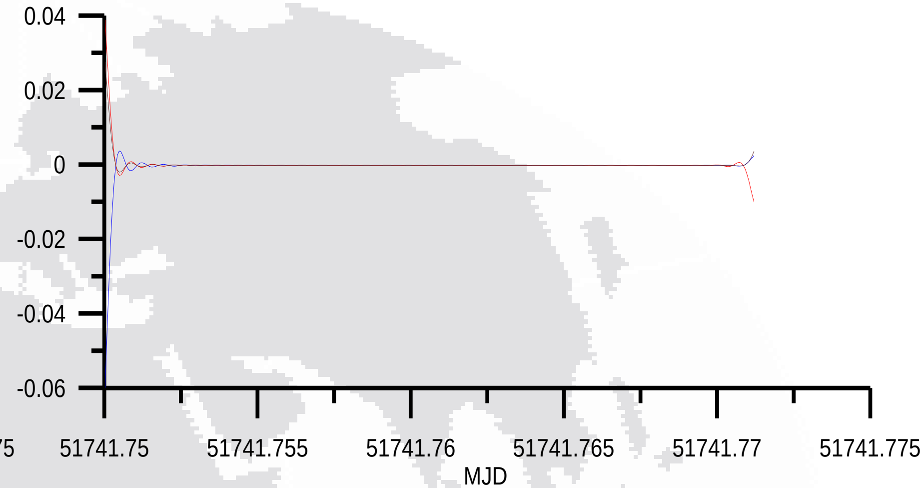


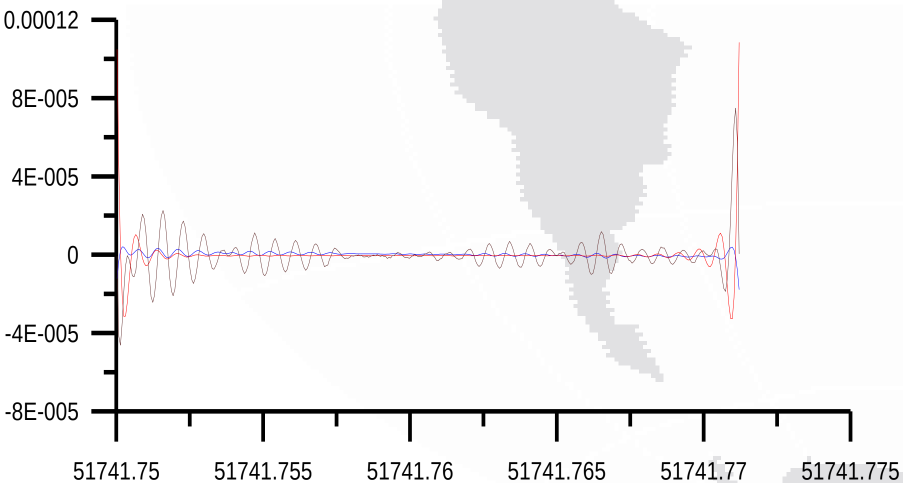
Reduced kinematical POD, results

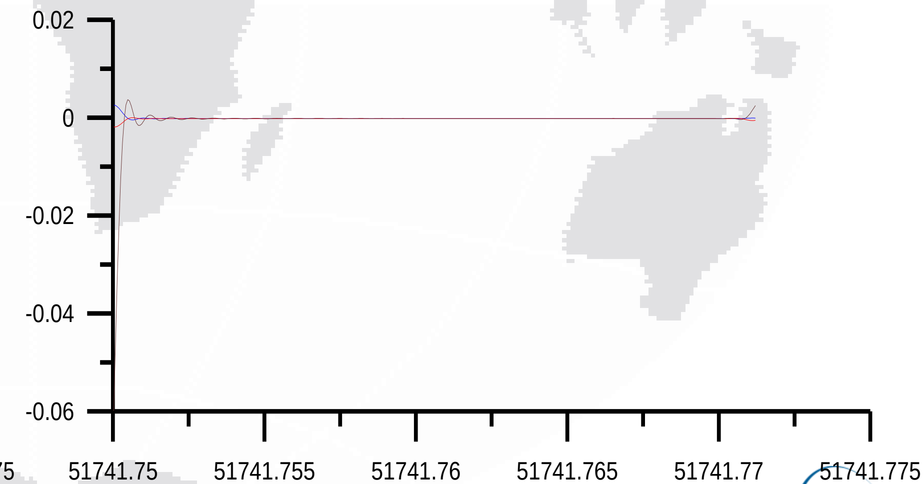


Earth gravity field deg. 16, error free	Earth gravity field deg. 16, error 05 cm









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Thank you for your attentions