A New Approach for Pure Kinematical and Reduced Kinematical Determination of a LEO Orbit based on GNSS Observations

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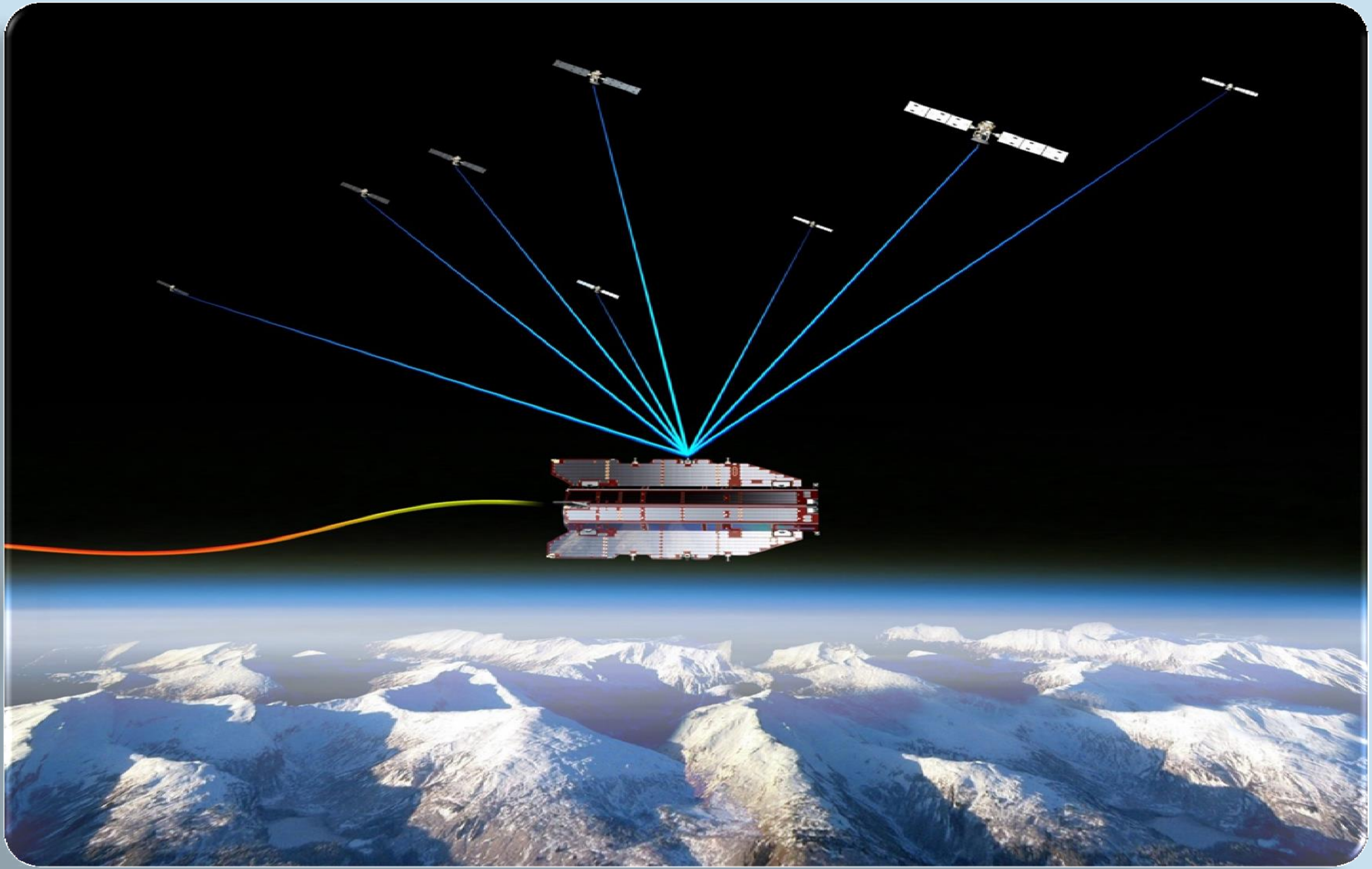
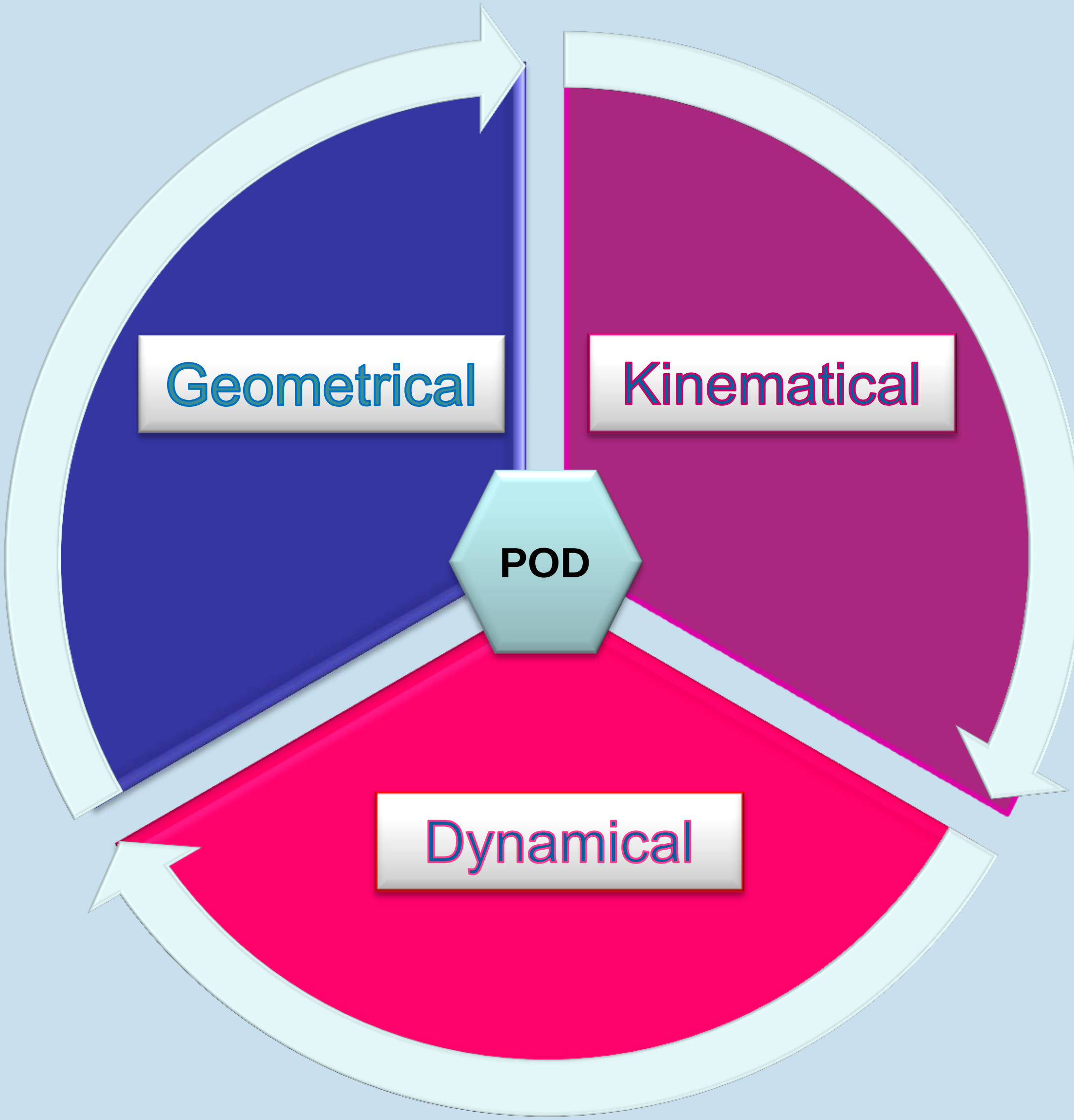
Session: Gravity 2.2
IAG 2009
31th August 2009, Buenos Aires

Precise Orbit Determination (POD)



Credit: European Space Agency

Precise Orbit Determination (POD) methods



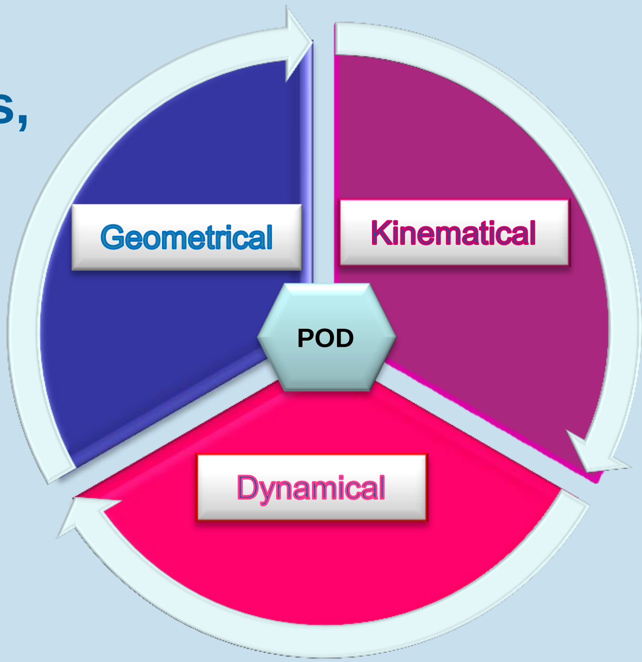
Precise Orbit Determination (POD) methods

- **Geometrical POD : point-wise, positions**

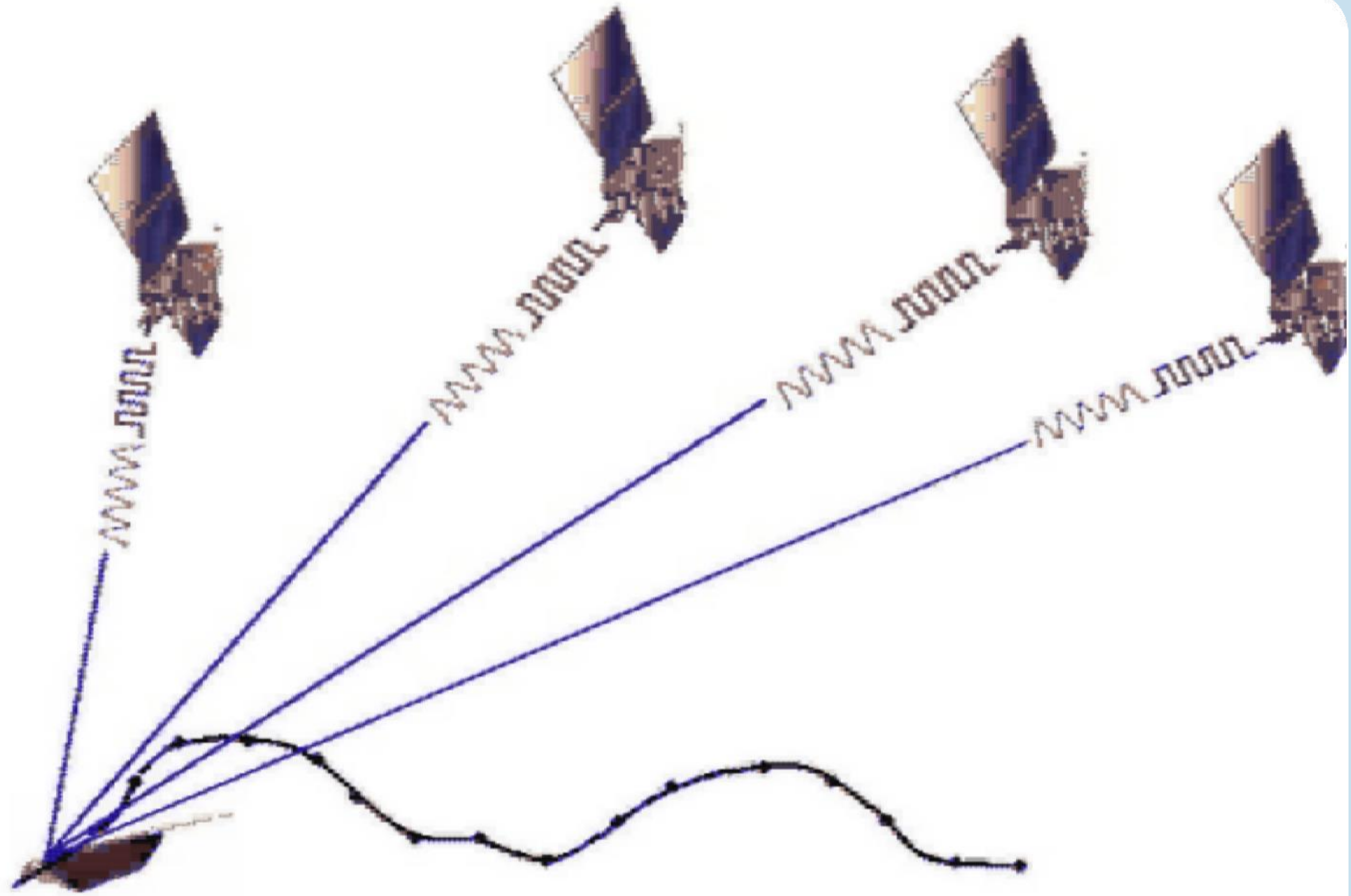
!= Kinematical POD

- **Kinematical POD : continous, positions, velocities and accelerations**

- **Dynamical POD : continous, positions, velocities and accelerations based on force function information**



Kinematical Precise Orbit Determination (KPOD)



Processing procedures

- GPOD
- KPOD
  - Method
  - Simulation
  - Real data
- RKPOD
  - Method
  - Simulation
  - Real data
- DPOD

Processing procedures

- GPOD
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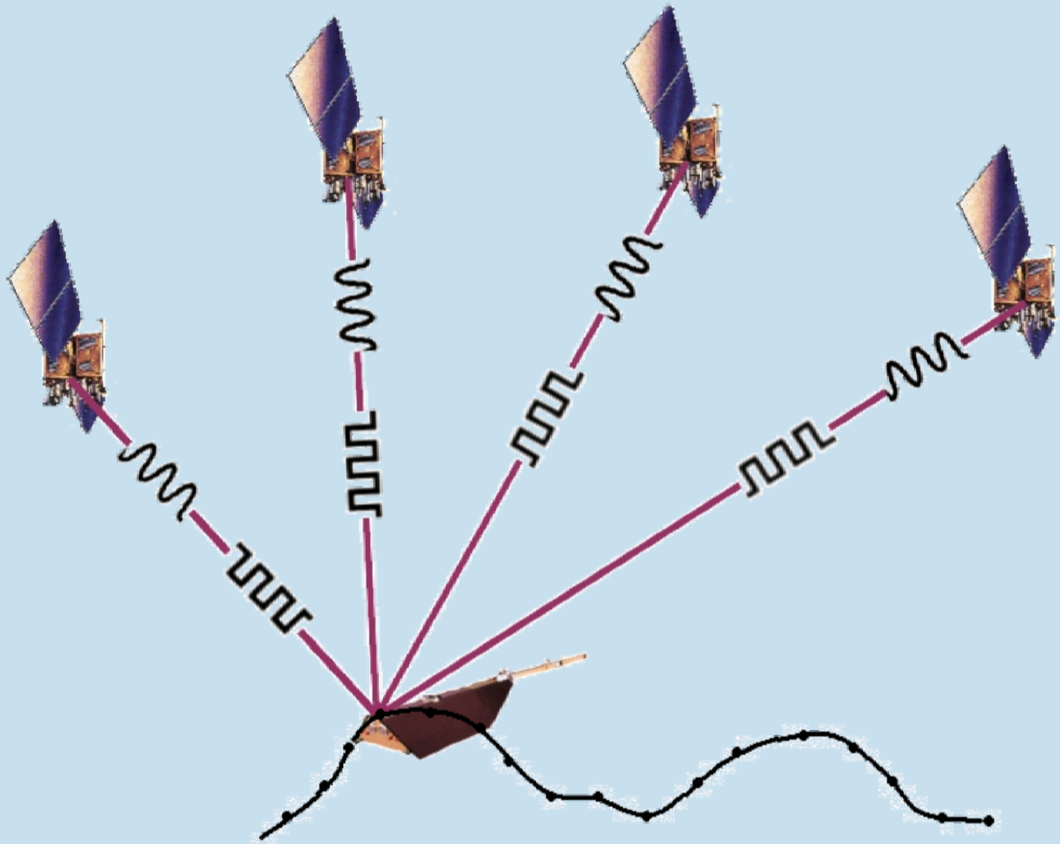
Kinematical POD – methods



Equation of motion w.r.t self-adjoint differential operator:

$$L\left(\mathbf{r}(t)\right)=\mathbf{a}(t;\mathbf{r},\dot{\mathbf{r}})$$

corresponding Fredholm's integral equation:

Reference motion

Integral kernel

$$\mathbf{r}(\tau)=\bar{\mathbf{r}}(\tau)-T^2\int_{\tau'=0}^{1}K(\tau,\tau')\mathbf{a}(\tau';\mathbf{r},\dot{\mathbf{r}})d\tau'$$

Position

$$T=t_B-t_A \quad \tau=\frac{t-t_A}{T}$$

Force function

$$L(\bar{\mathbf{r}}(\tau))=0$$

Short arc representation – ellipse mode

A satellite short arc:

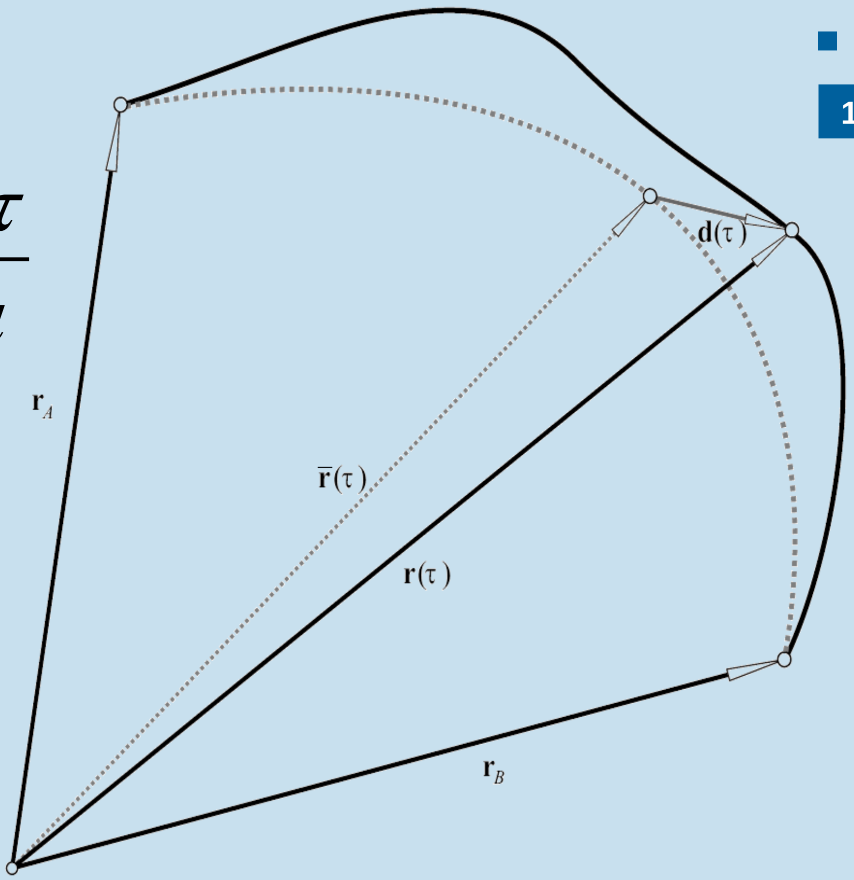
$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

elliptical reference motion:

$$\bar{\mathbf{r}}(\tau) = \mathbf{r}_A \frac{\sin \mu(1-\tau)}{\sin \mu} + \mathbf{r}_B \frac{\sin \mu\tau}{\sin \mu}$$

difference function:

$$\mathbf{d}(\tau) = \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

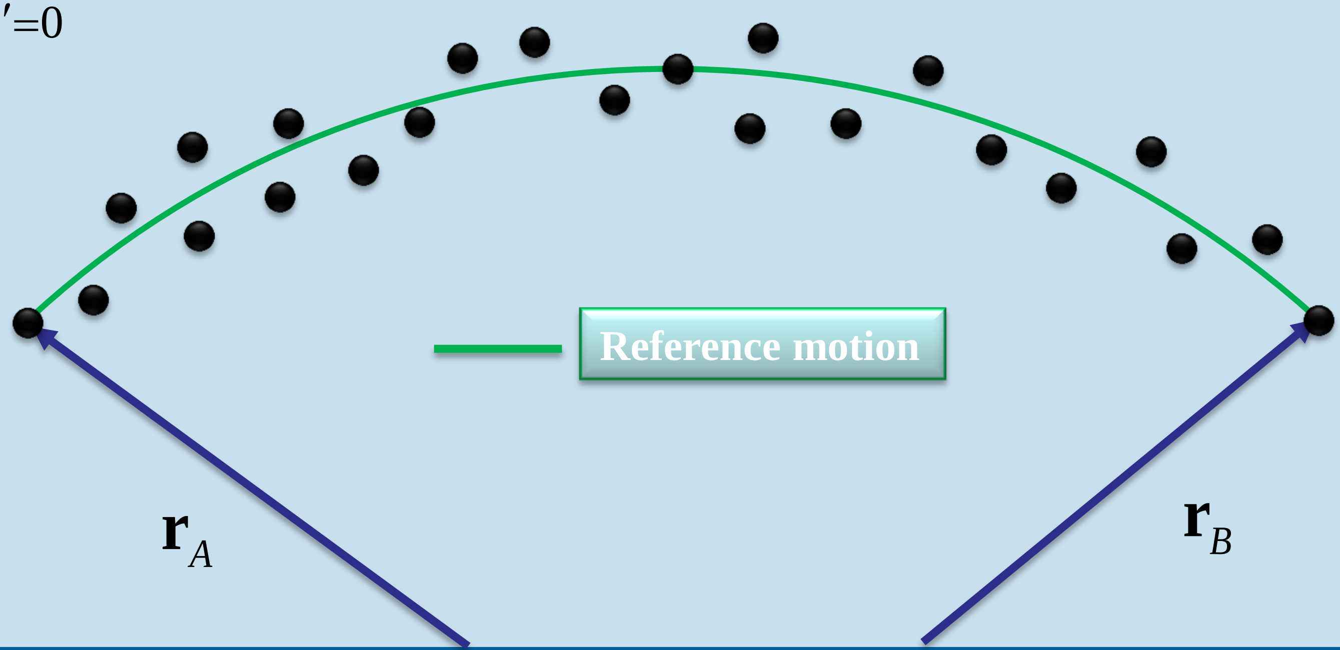


A satellite short arc can be represented:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

$$\mathbf{r}(\tau) - \bar{\mathbf{r}}(\tau) = \mathbf{d}(\tau) = \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

$$\mathbf{d}_{\upsilon} = 2 \int_{\tau'=0}^{1} \mathbf{d}(\tau) \sin(\upsilon\pi\tau') d\tau'$$



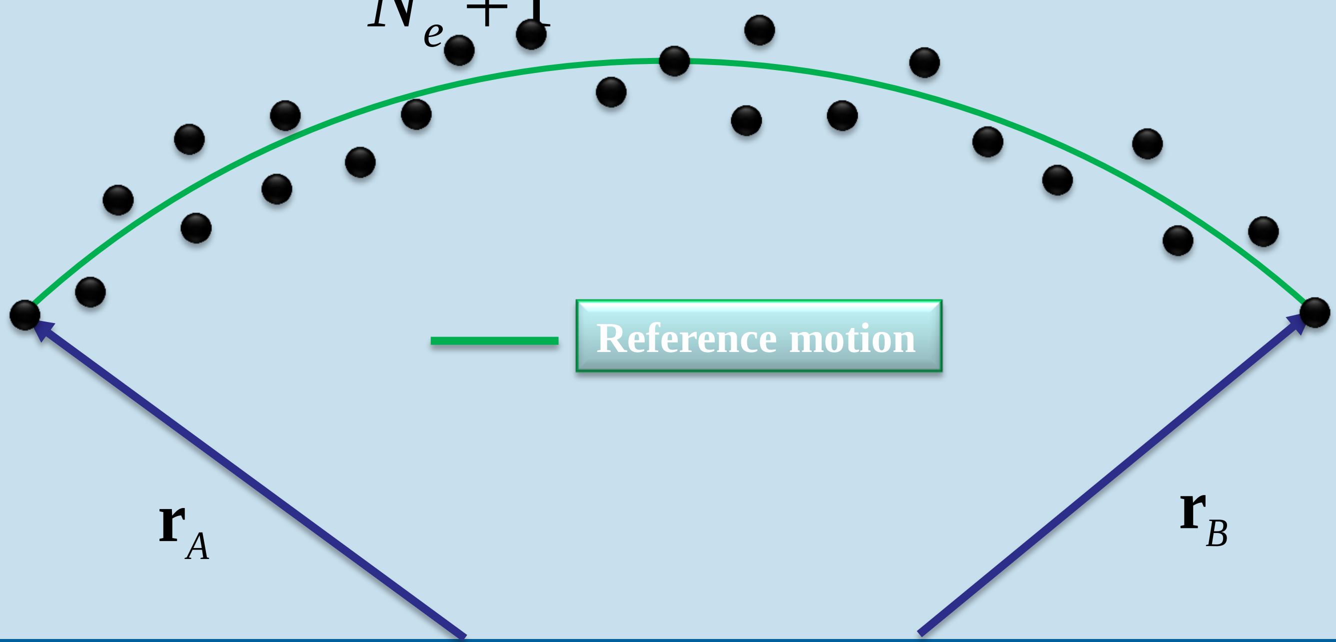
Short arc representation – Fourier series

A satellite short arc can be represented:

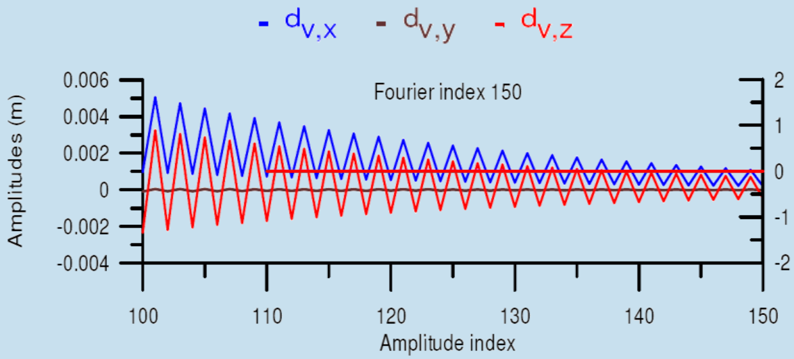
$$\mathbf{r}(\tau) - \bar{\mathbf{r}}(\tau) = \mathbf{d}(\tau) = \sum_{\upsilon=1}^{\infty} \mathbf{d}_\upsilon \sin(\upsilon\pi\tau)$$

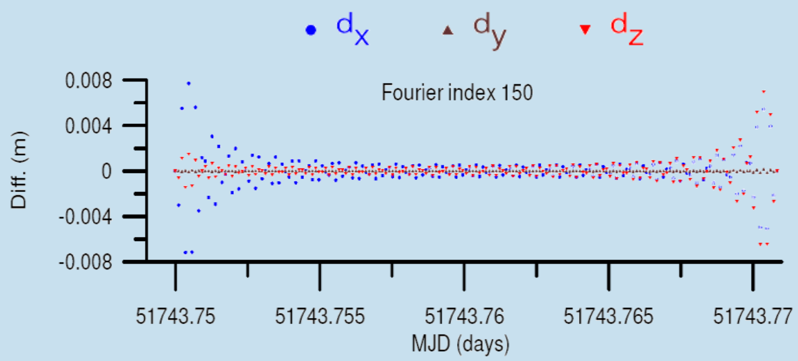
$$\mathbf{d}_\upsilon \approx \frac{2}{N_e+1} \sum_{e=1}^{N_e} \mathbf{d}(\tau_e) \sin\left(\frac{\upsilon\pi e}{N_e+1}\right)$$

$$\mathbf{d}(\tau_e) \quad \text{with} \quad \tau_e = \frac{e}{N_e+1}, \quad e = 1,\dots,N_e \quad \tau_e \in]0,1[$$



Fourier analysis – ellipse mode





ellipse mode

Amplitudes

Remainders

A satellite short arc can be represented:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

with Fourier amplitudes:

$$\mathbf{d}_{\upsilon} = 2 \int_{\tau'=0}^{1} \mathbf{d}(\tau) \sin(\upsilon\pi\tau') d\tau'$$

Fourier series amplitudes:

$$\mathbf{d}_{\upsilon} = \sum_{j=1}^{J} \frac{2(-1)^{j+1}}{(\upsilon\pi)^{2j+1}} \left[(-1)^{\upsilon} \mathbf{d}^{[2j]}(1) - \mathbf{d}^{[2j]}(0)\right] +$$

$$+\beta \frac{2}{(\upsilon\pi)^{2J+1}} \int_{\tau'=0}^{1} \mathbf{d}^{[2J+2]}(\tau') \sin(\upsilon\pi\tau') d\tau'$$

Short arc representation – Euler-Bernoulli

$$\mathbf{d}_F^{\infty} = \mathbf{d}(\tau) = \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau) =$$

$$= \sum_{j=1}^{\infty} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{\infty} \mathbf{b}_{2j+1} B_{2j+1}(\tau) = \mathbf{d}_P^{\infty}$$

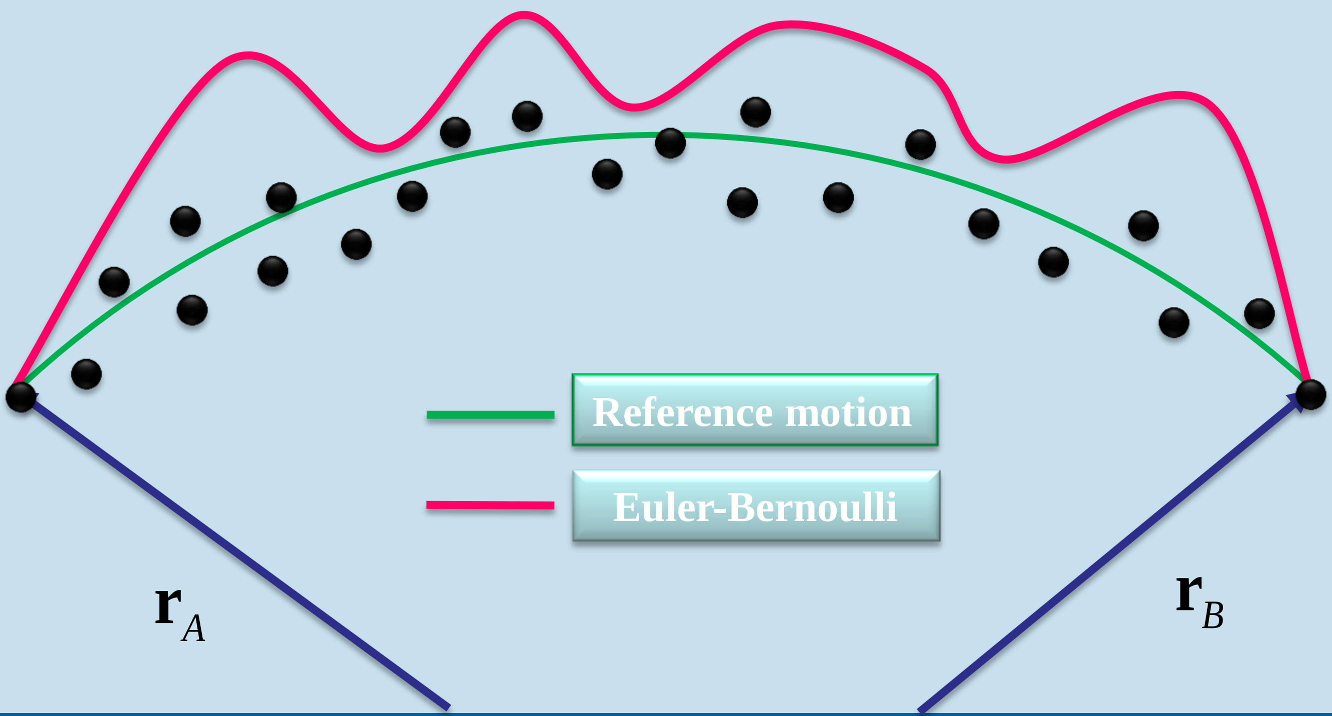
A satellite short arc can be represented:

$$\sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin\left(\upsilon\pi\tau\right) = \sum_{j=1}^{\infty} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{\infty} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$

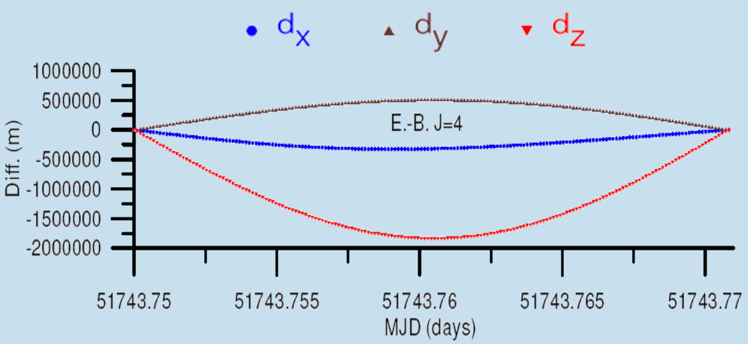
Determination of Euler-Bernoulli coefficients

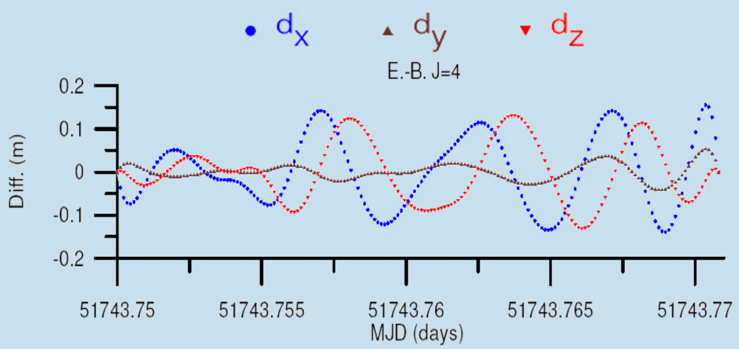
A satellite short arc can be represented with the Euler-Bernoulli term up to degree *J* as:

$$\mathbf{r}(\tau)-\overline{\mathbf{r}}(\tau)=\mathbf{d}(\tau)\approx\sum_{j=1}^{J}\mathbf{e}_{2j}E_{2j}(\tau)+\sum_{j=1}^{J}\mathbf{b}_{2j+1}B_{2j+1}(\tau)$$



Determination of Euler-Bernoulli coefficients





ellipse mode

E.-B terms

Remainders

Short arc representation

LEO orbit can be represented as:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{n} \mathbf{d}_\upsilon \sin(\upsilon\pi\tau)$$

Gibbs effect!

or

Precision!

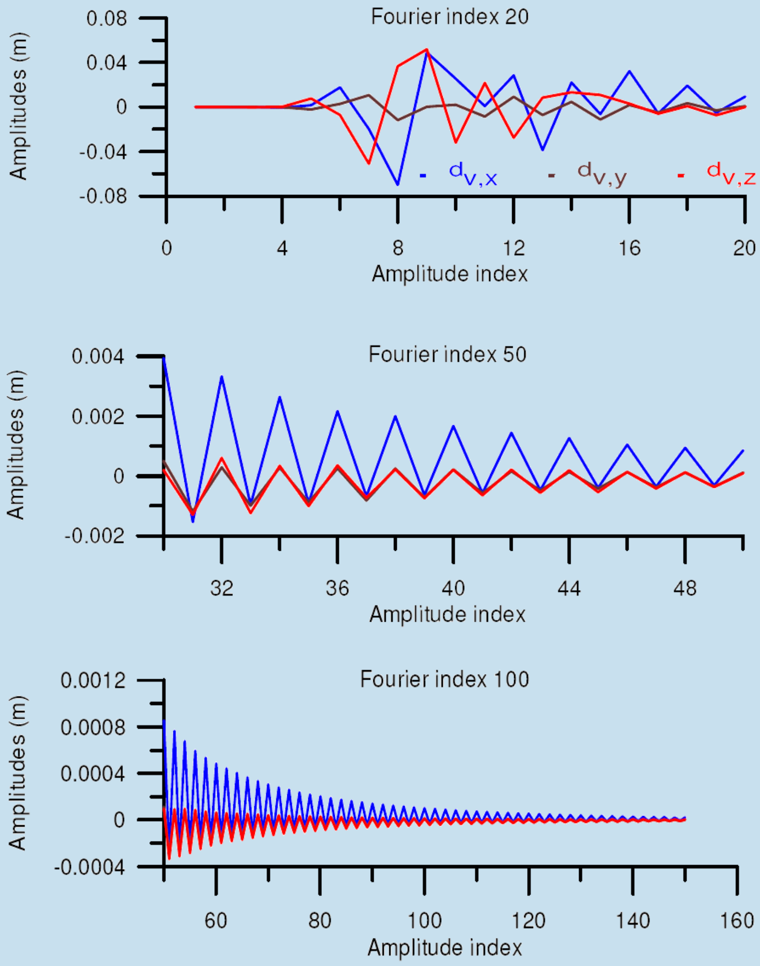
$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$

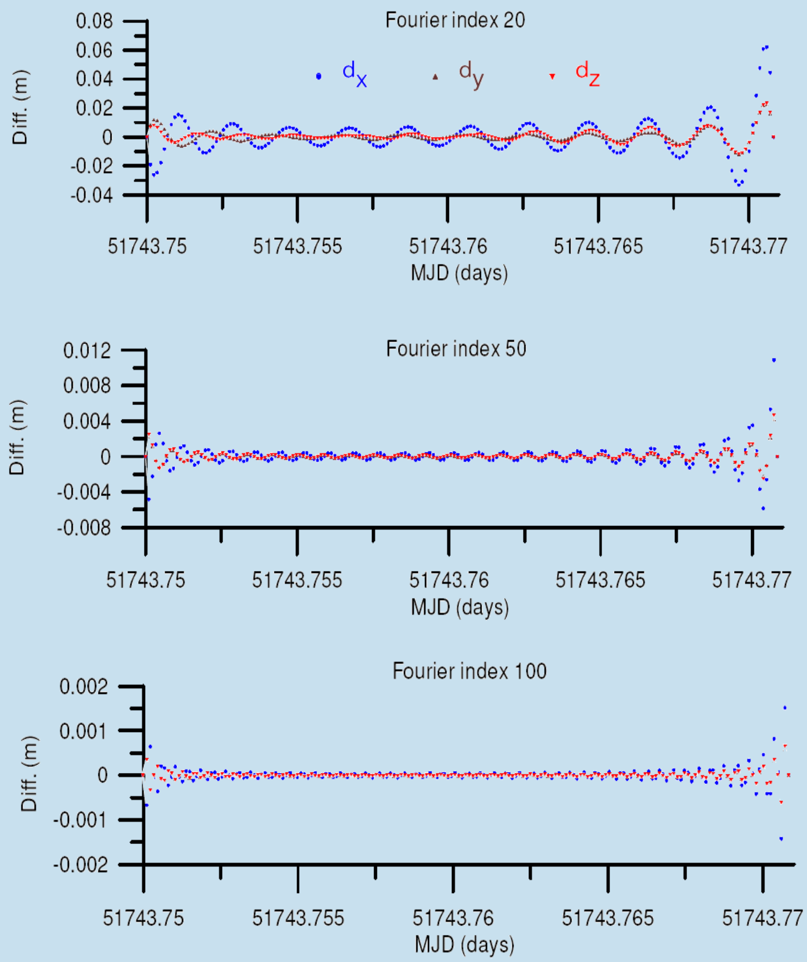
Solution?

fast convergence!

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau) + \sum_{\upsilon=1}^{\bar{n}} \bar{\mathbf{d}}_\upsilon \sin(\upsilon\pi\tau)$$

Kinematical POD – ellipse mode, J=4

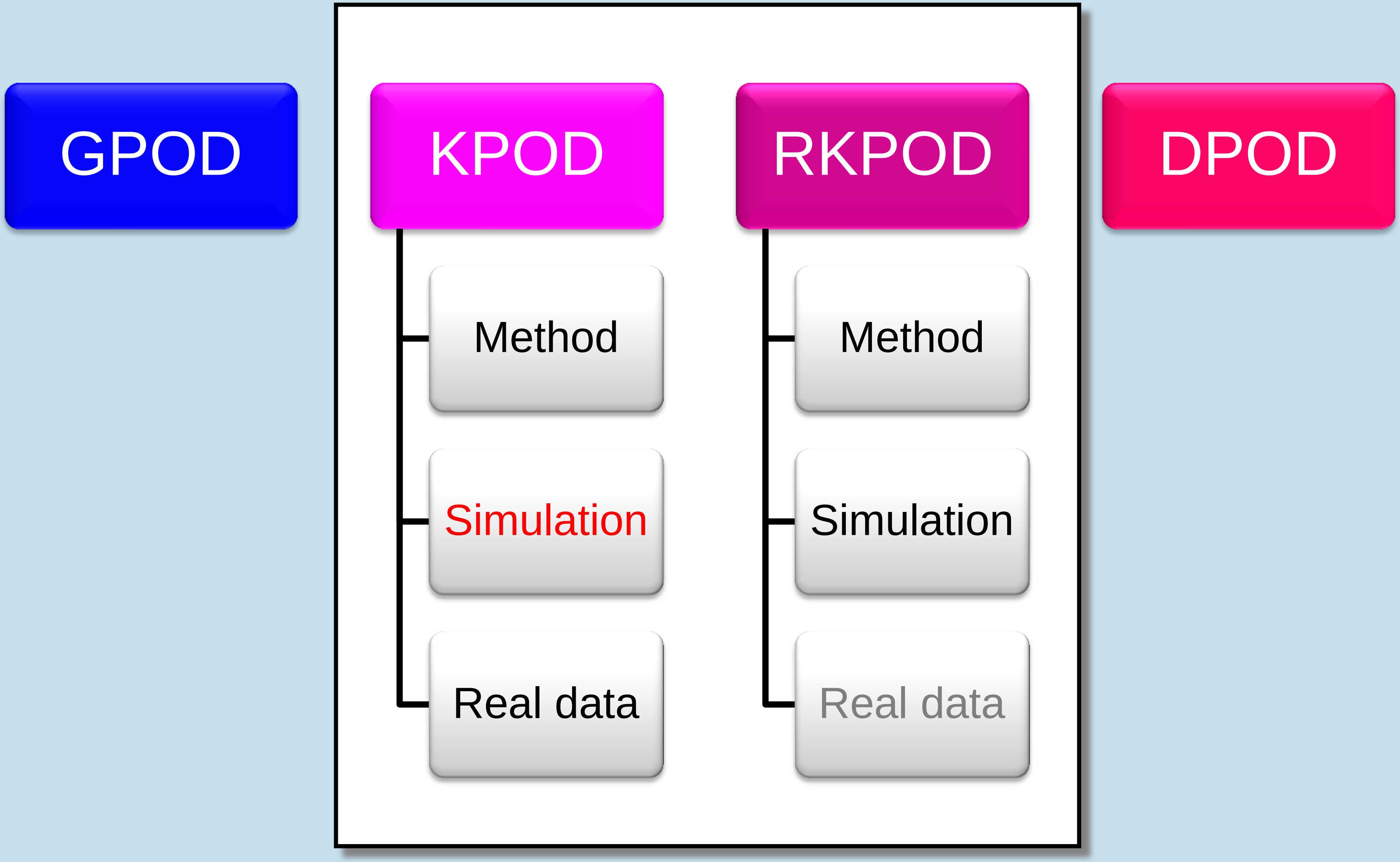




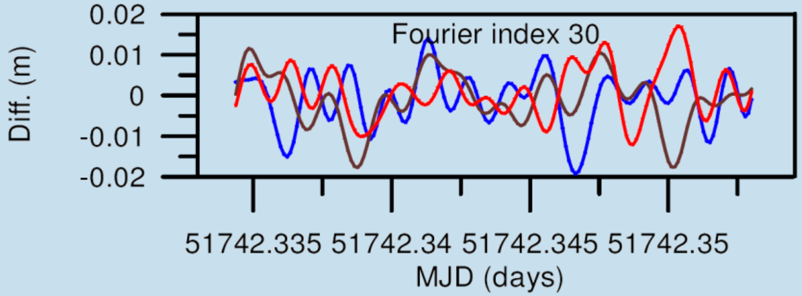
Amplitudes

Remainders

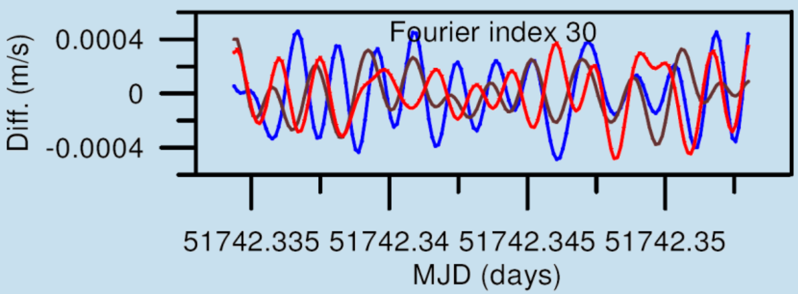
Processing procedures



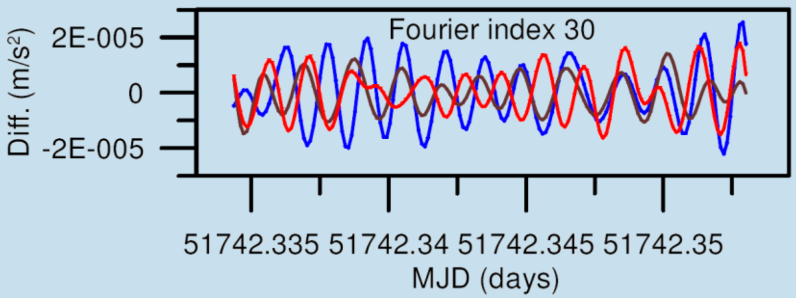
Kinematical POD – simulation (2cm noise SST)



Position differences



Velocity differences



Acceleration differences

RMS

index	Pos.(m)	Vel.(m/s)	Acc.(m/s^2)
20	0.012644	0.000353	0.000012
30	0.010717	0.000397	0.000018
40	0.011997	0.000463	0.000025
59	0.014737	0.000941	0.000077

Statistical values

Processing procedures

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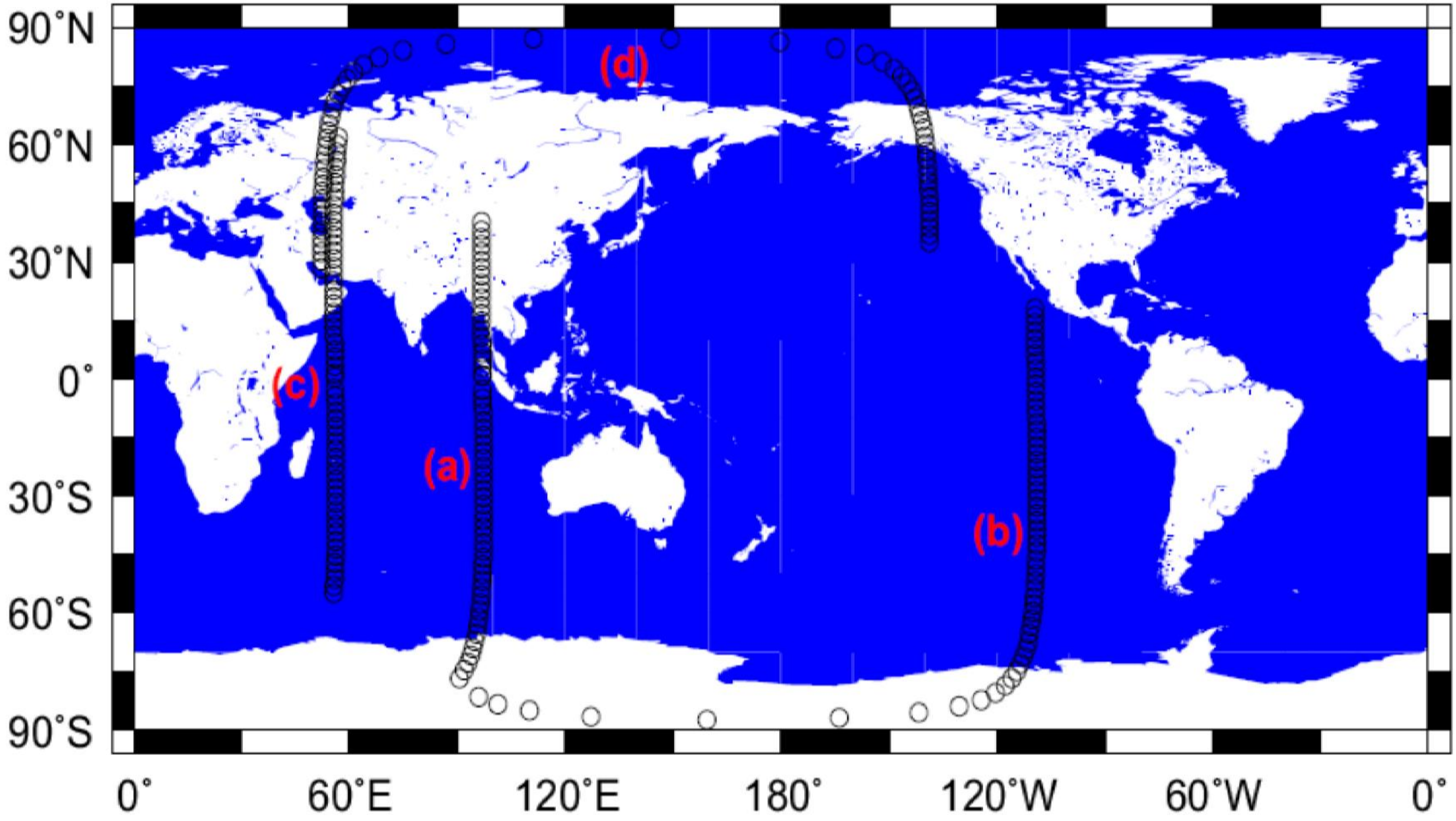
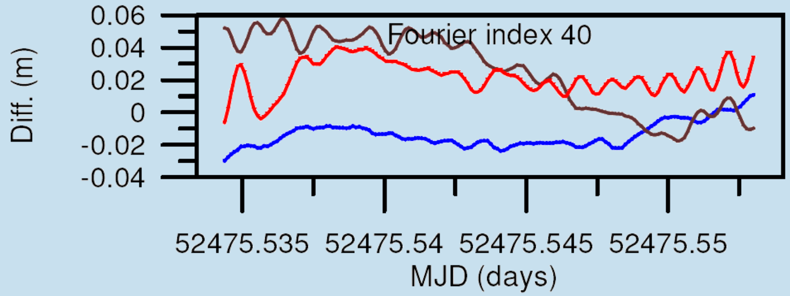
KPOD – real case

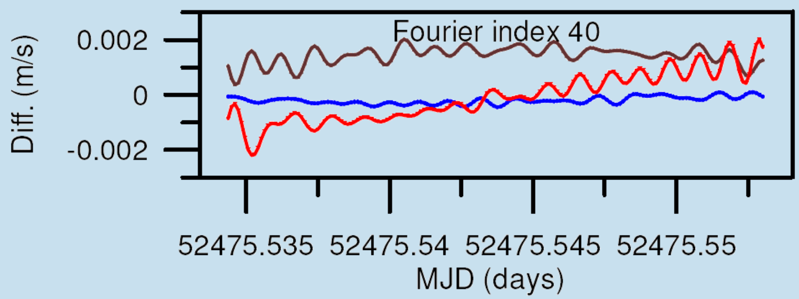


Four short arcs (30 min.) ground track of CHAMP

Kinematical POD – short arc case (b)



IGG - GFZ positions

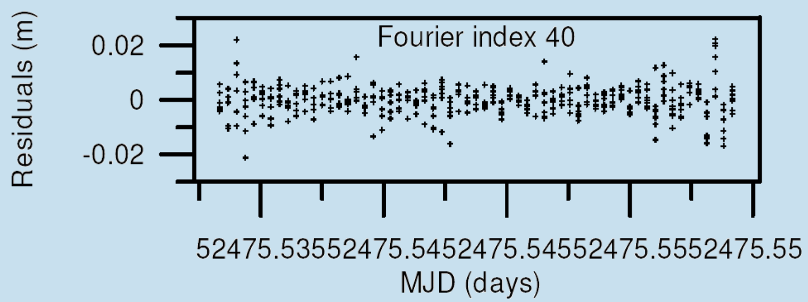


IGG - GFZ velocities

RMS

index	Pos.(m)	Vel.(m/s)
20	0.0503	0.0019
30	0.0455	0.0018
40	0.0449	0.0017
59	0.0449	0.0017

Statistical values



GPS-SST residuals

Processing procedures

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Reduced-Kinematical POD – method

Dynamical info.

$$= \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{n} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

$$\tilde{\mathbf{d}}_{\upsilon} = -\frac{2T^2}{\upsilon^2\pi^2 - \mu^2} \int_{\tau'=0}^{1} \sin(\upsilon\pi\tau)\mathbf{a}(\tau';\mathbf{r},\dot{\mathbf{r}})d\tau'$$

✓Introduction of an approximate force function $\left(\tilde{\mathbf{d}}_i \cdots \tilde{\mathbf{d}}_j\right), \mathbf{C}_{\left(\tilde{\mathbf{d}}_i \cdots \tilde{\mathbf{d}}_j\right)}$

✓Fixing only some orbit parameters $\left(\tilde{\mathbf{d}}_i \cdots \tilde{\mathbf{d}}_j\right), \mathbf{C}_{\left(\tilde{\mathbf{d}}_i \cdots \tilde{\mathbf{d}}_j\right)} \rightarrow \mathbf{0}$

✓Down- or up weighting $\mathbf{C}_{\left(\tilde{\mathbf{d}}_1 \cdots \tilde{\mathbf{d}}_n\right)}$ in relation to $\mathbf{C}_{\left(\mathbf{d}_1 \cdots \mathbf{d}_n\right)}$

Reduced Kinematical POD – method

Kinematical observation equation

$$\mathbf{l}_1 = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}, \quad \mathbf{C}_1$$

Constraints

$$\mathbf{l}_2 = \begin{pmatrix} \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}, \quad \mathbf{C}_2$$

⇩ ⇩

$$\begin{pmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}, \quad \mathbf{C}_l = \begin{pmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 \end{pmatrix}$$

⇩

$$\begin{pmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_2 \end{pmatrix} = \mathbf{N}^{-1} \begin{pmatrix} \mathbf{A}_1^T \mathbf{C}_1^{-1} \mathbf{l}_1 \\ \mathbf{A}_2^T \mathbf{C}_1^{-1} \mathbf{l}_1 + \mathbf{C}_2^{-1} \mathbf{l}_2 \end{pmatrix}, \quad \mathbf{N}^{-1} = \begin{pmatrix} \mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_1} & \mathbf{Q}_{\hat{\mathbf{x}}_1\hat{\mathbf{x}}_2} \\ \mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_1} & \mathbf{Q}_{\hat{\mathbf{x}}_2\hat{\mathbf{x}}_2} \end{pmatrix}$$

Processing procedures

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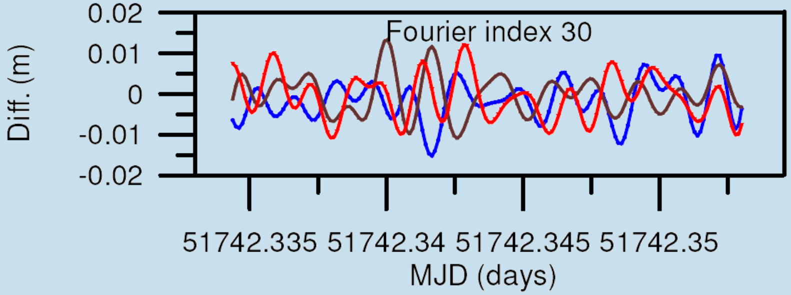
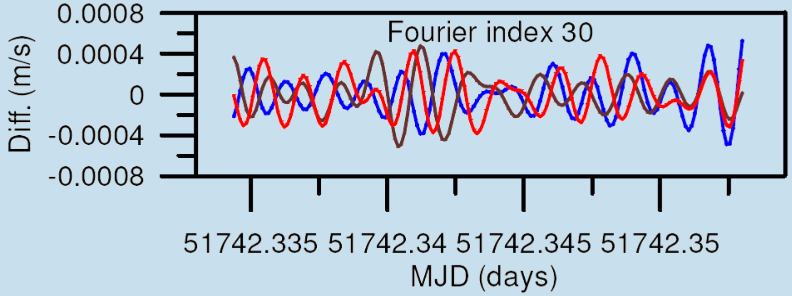
- DPOD

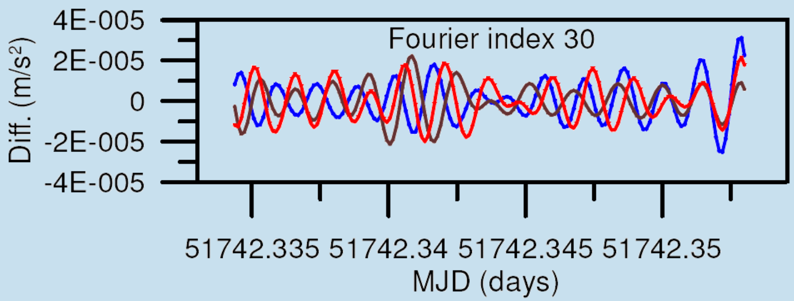
RKPOD – simulation (Amp. 1-5, 2cm, N=300)



Position differences



Velocity differences



Acceleration differences

RMS

index	Pos.(m)	Vel.(m/s)	Acc.(m/s^2)
20	0.012831	0.000316	0.000012
30	0.008873	0.000337	0.000016
40	0.014034	0.000402	0.000021
59	0.011553	0.000721	0.000056

Statistical values

Conclusions and recommendations

- GNSS-LEO satellites configuration and geometrical strength play an important role in POD,

- Kinematical POD can be used to recover the Earth's gravity field model based on the POD methods,

- No gravity field and no force models have been used in the Geometrical and Kinematical modes (advantage),

- The proposed kinematical orbit determination method is very flexible. A smooth transition from kinematical to reduced kinematical and finally dynamical or vice-versa is possible.

Thank you
for your attention