Geometrical and Kinematical Precise Orbit Determination of GOCE

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7th October 2010
Cologne, Germany

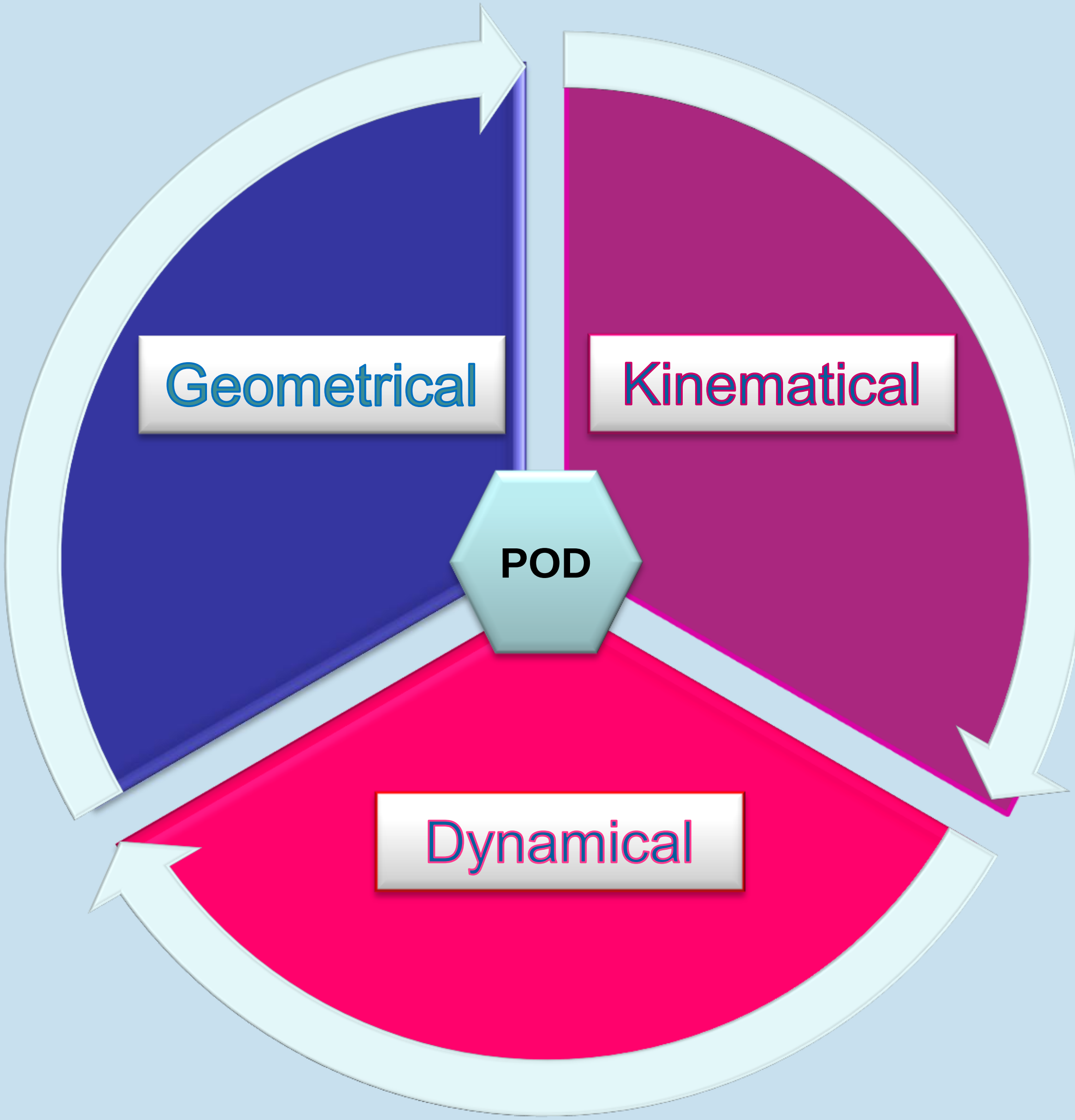
Outline

- Precise Orbit Determination (POD) principle
- Geometrical Precise Orbit Determination (GPOD)
- Kinematical Precise Orbit Determination (KPOD)
- GOCE Lagrange receiver (clock)
- Zero difference estimation procedure
- Results
- Conclusions

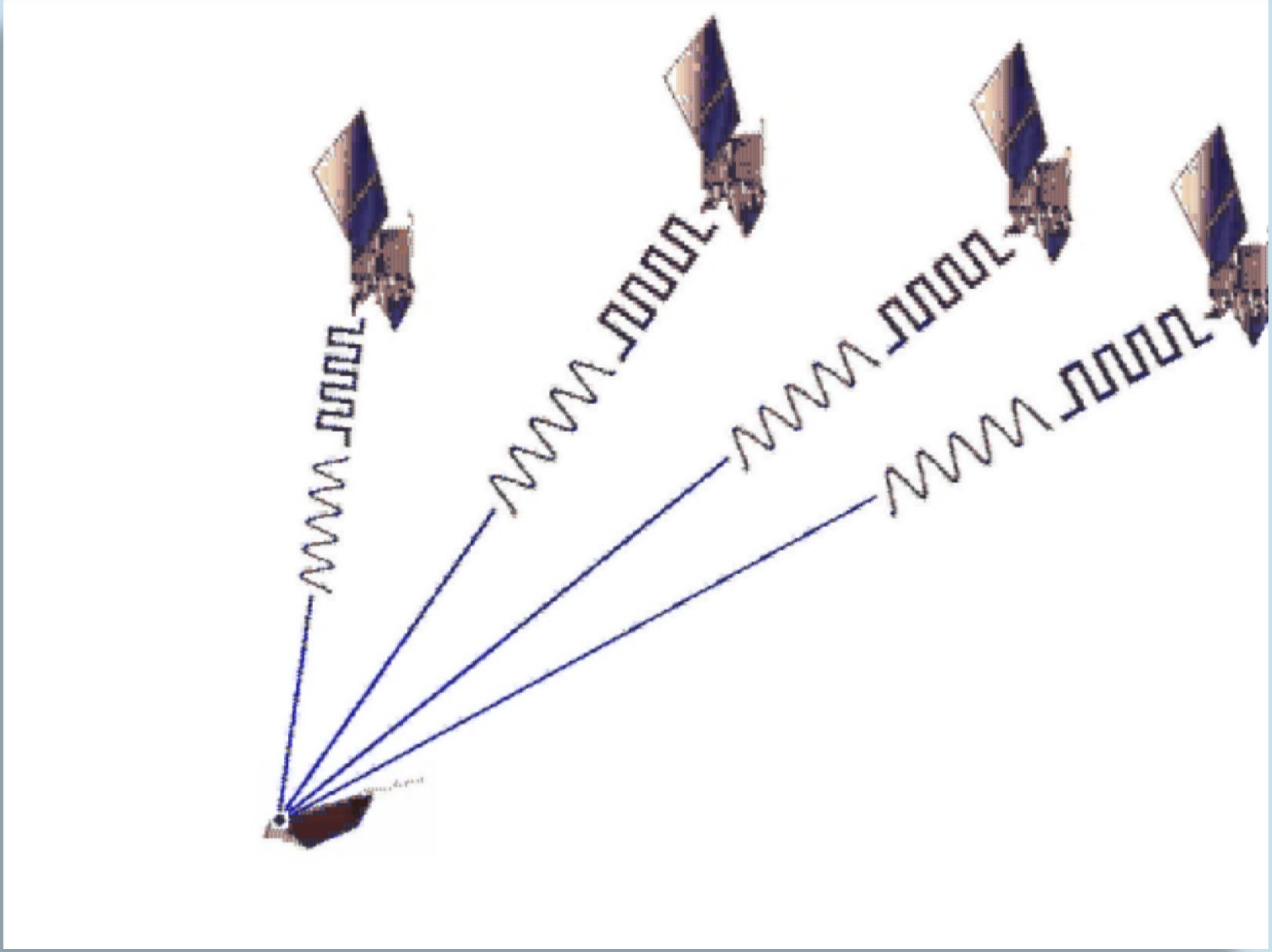
Precise Orbit Determination (POD)

Credit by ESA

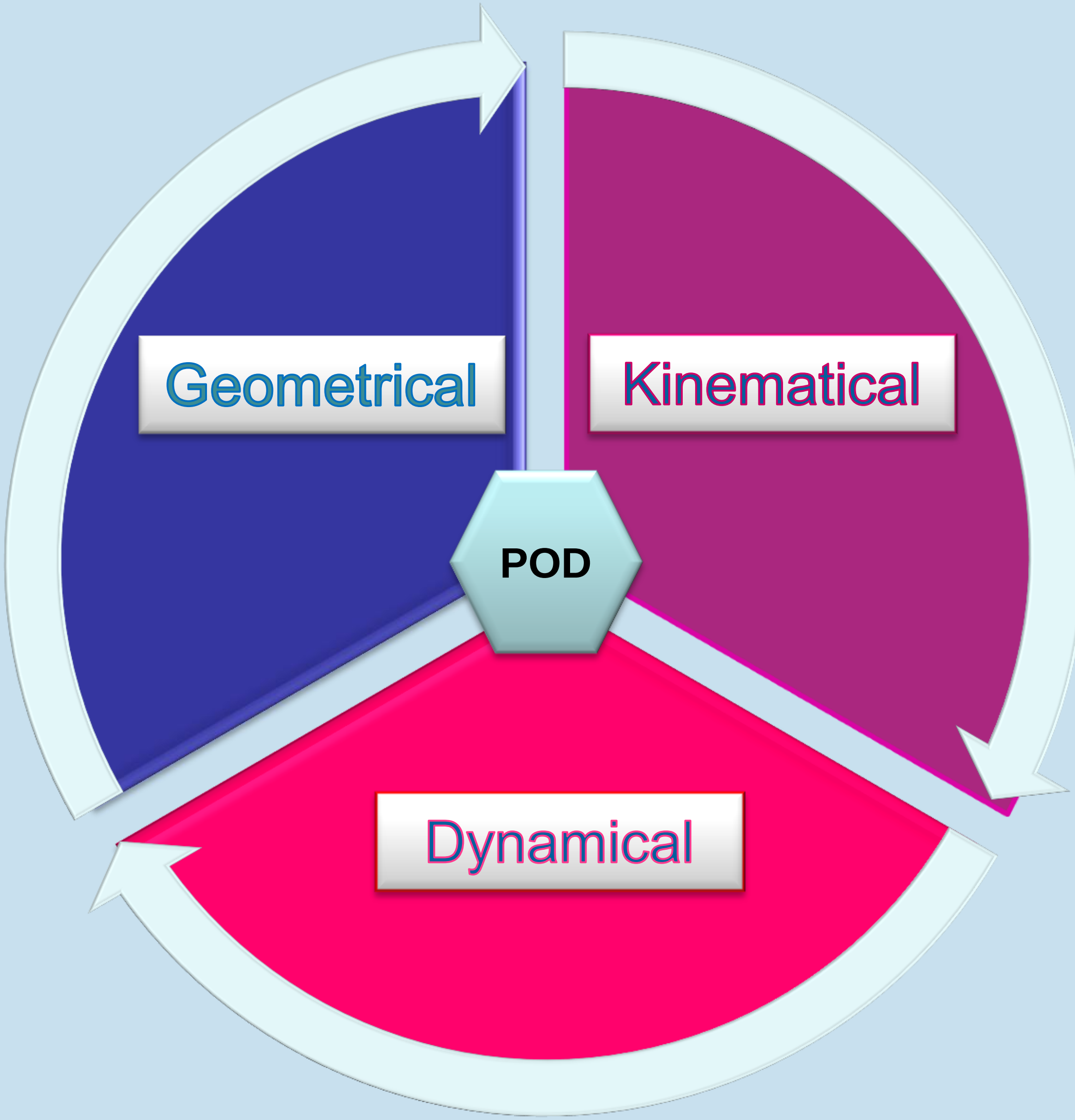
Precise Orbit Determination (POD) methods

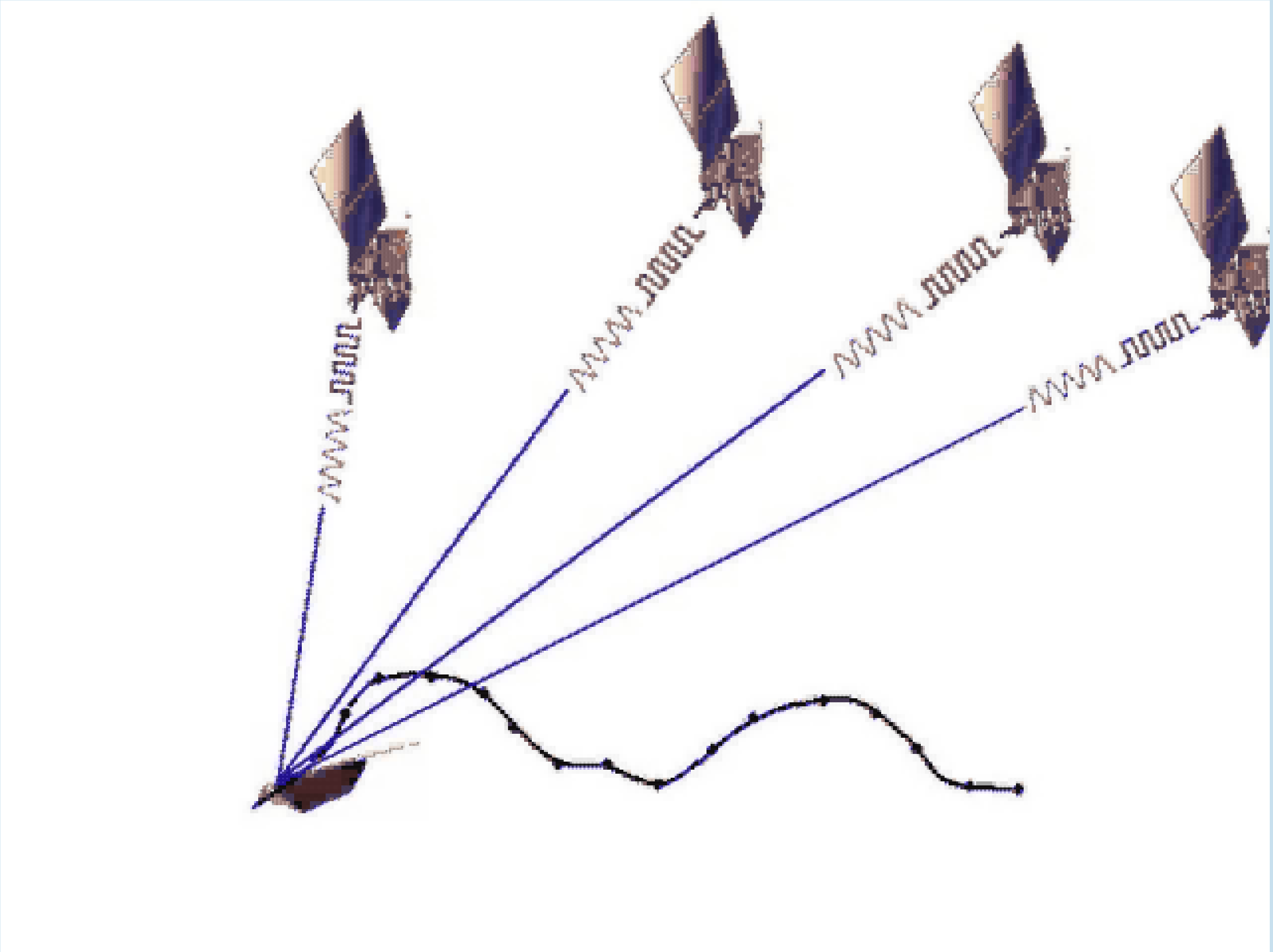


Geometrical Precise Orbit Determination (GPOD)



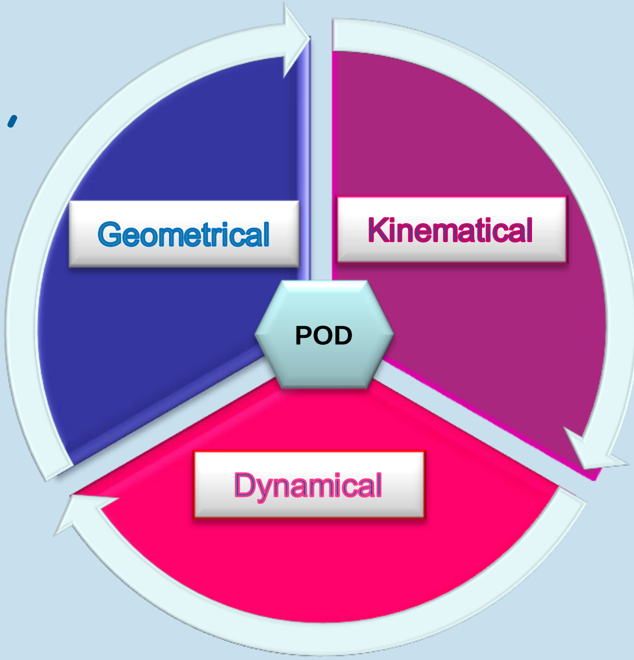
Precise Orbit Determination (POD) methods



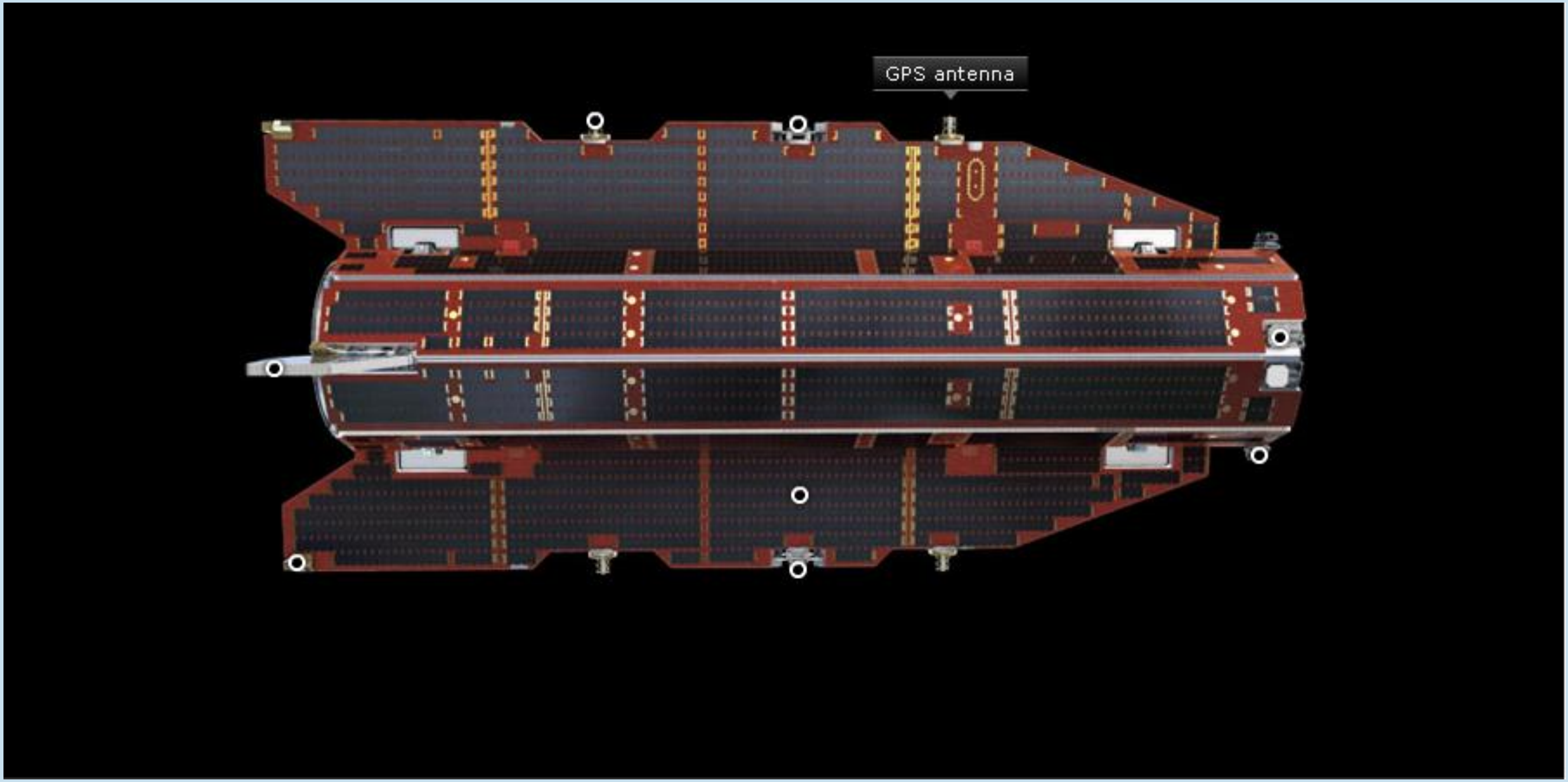


Precise Orbit Determination (POD) methods

- **Geometrical POD** : point-wise, positions != KPOD, e.g. Bern
- **Kinematical POD** : continous, positions, velocities and accelerations
- **Dynamical POD** : continous, positions, velocities and accelerations based on force function information

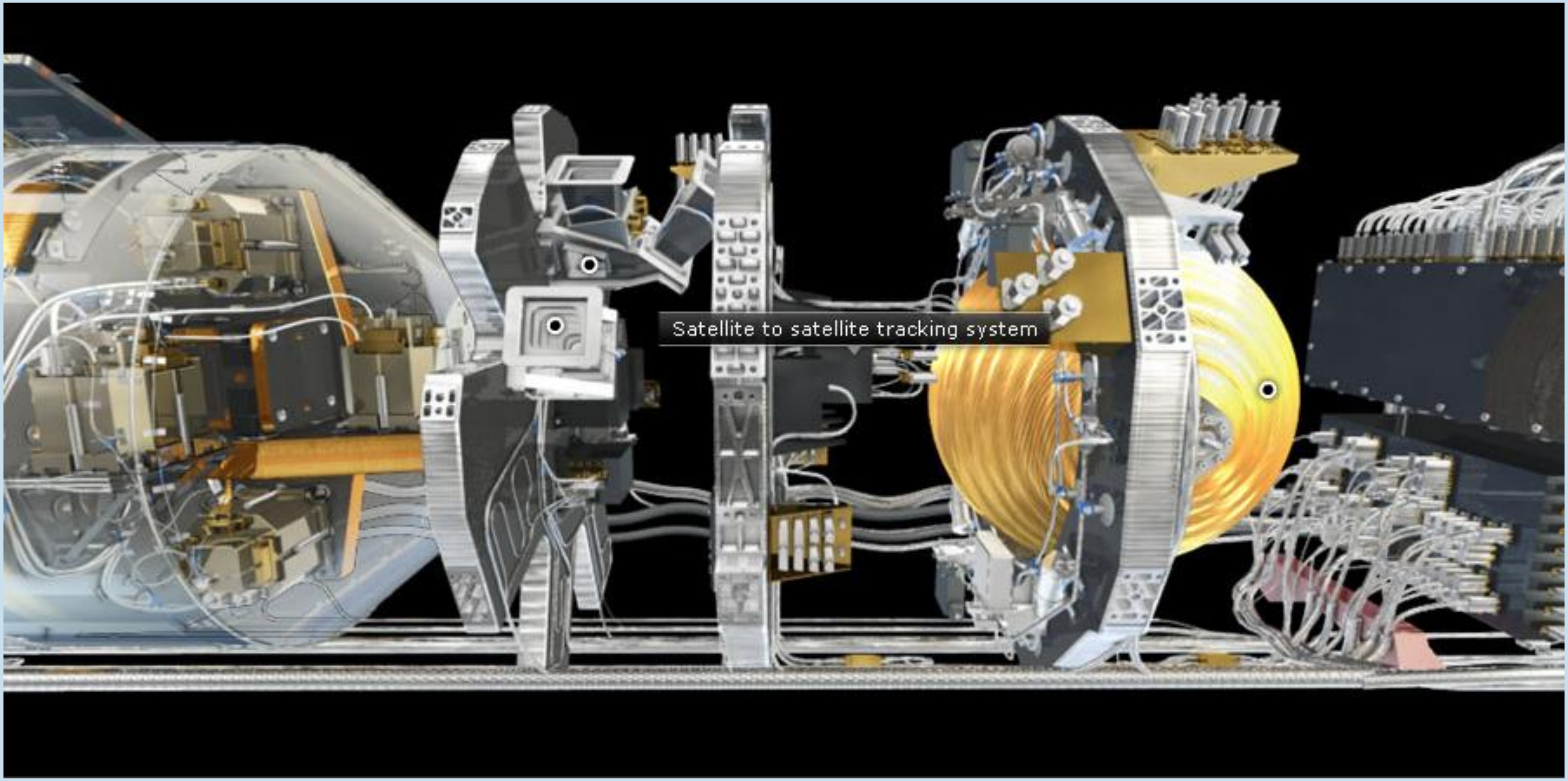


GPS LAGRANGE receiver onboard GOCE



Credit by ESA

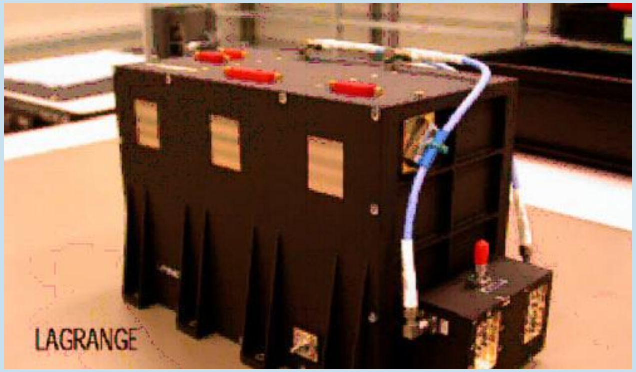
GNSS receiver on-board GOCE



Credit by ESA

GNSS receiver on-board GOCE

- LAGRANGE (Laben GNSS Receiver for Advanced Navigation, Geodesy and Experiments)
  - 12 chanels, dual frequency (L1 and L2) GPS/GLONASS
  - The clock of the GOCE LAGRANAGE receiver is not steered to integer seconds (free clock system)
    - Interpolation of SST observations (Data Screening with triple differenced method)
    - Interpolation of GPS orbits (Zero differenced)

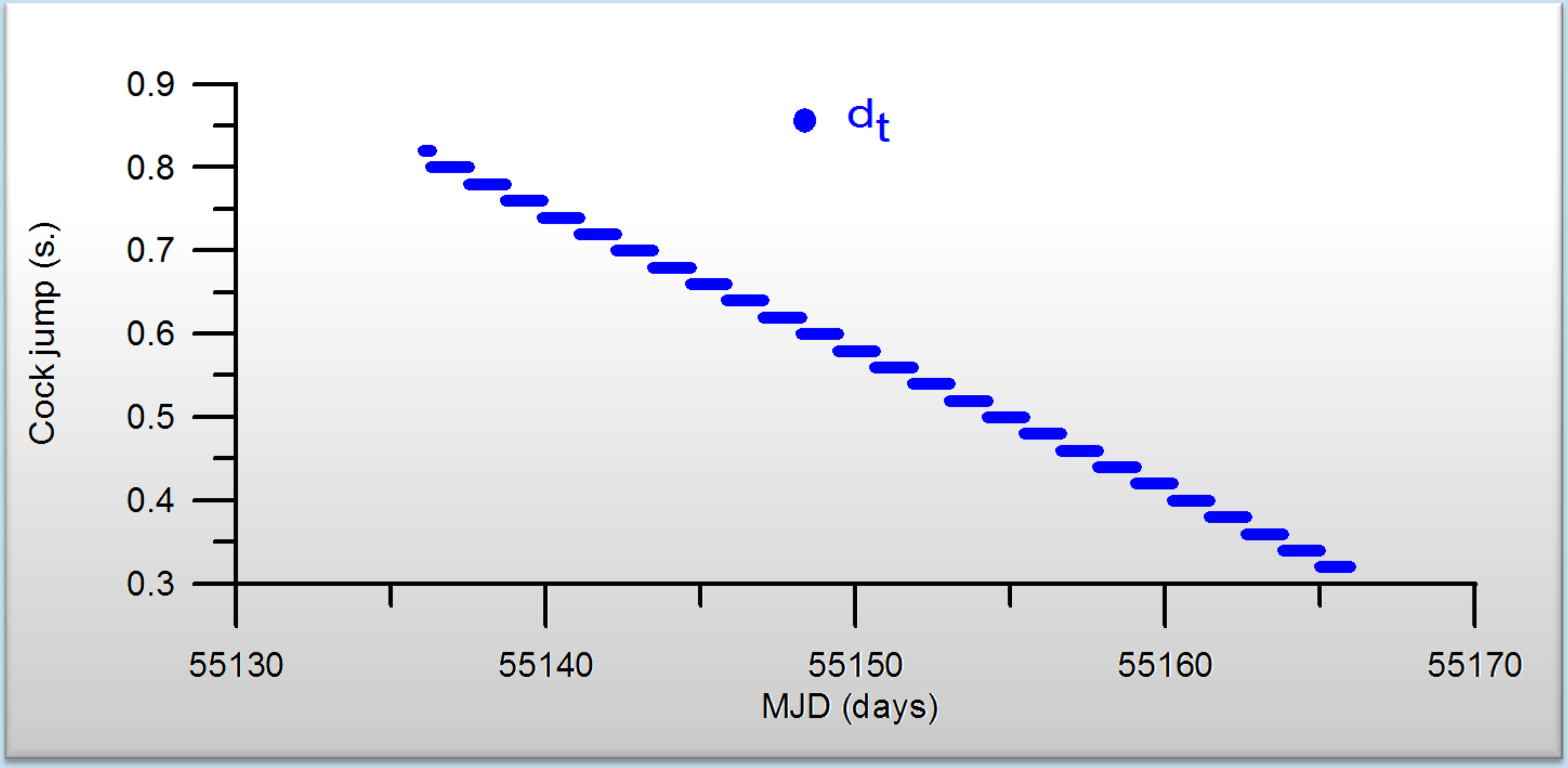


GNSS receiver on-board GOCE (Rinex SST)

Clock is not steerd to be integer

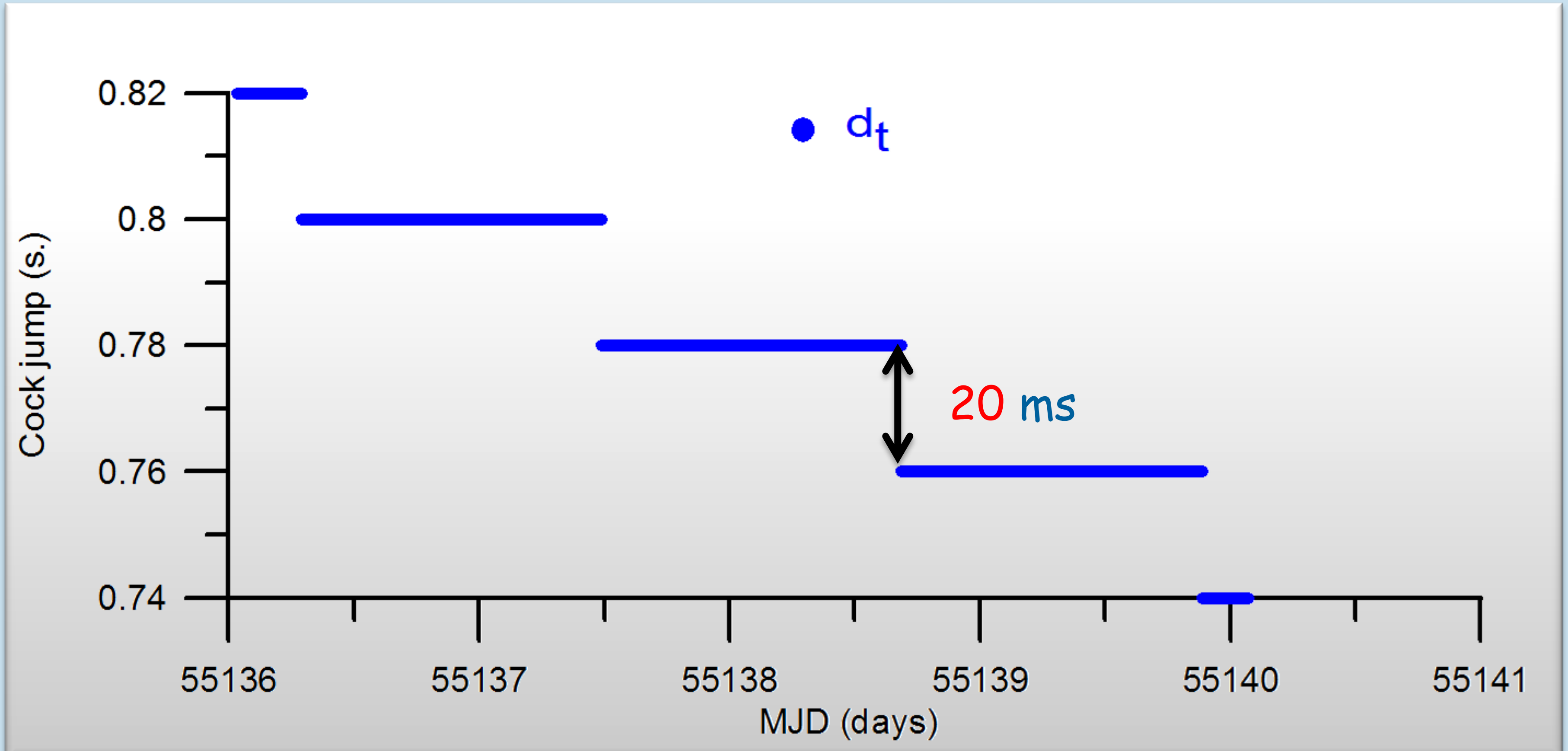
```
     2.20           OBSERVATION DATA    GPS                 RINEX VERSION / TYPE
SST_L1NOM_P 02.16   ESA                 29-Jan-10 16:53:16  PGM / RUN BY / DATE
GOCE                                                        MARKER NAME
SPACEBORNE                                                  MARKER TYPE
Kiruna              ESA                                     OBSERVER / AGENCY
Main                LABEN               3.2                 REC # / TYPE / VERS
1                   RYMSA                                   ANT # / TYPE
        0.6899       -0.0046       -1.1755                  ANTENNA: DELTA X/Y/Z
        0.0000        0.0000       -1.0000                  ANTENNA: B.SIGHT XYZ
     1.000                                                  INTERVAL
     1     1                                                WAVELENGTH FACT L1/2
     8    L1    L2    C1    P1    P2    S1    S2    CH      # / TYPES OF OBSERV
  2009    11     2    12     0    0.7799998                 TIME OF FIRST OBS
  2009    11     2    12    59   29.7799936                 TIME OF LAST OBS
     0                                                      RCV CLOCK OFFS APPL
                                                            END OF HEADER
 09 11  2 12 15 30.7800000  0 11 17 32 11 14 28  9 19 27 26 20 22
 -18682885.84200 -13467931.56500  19787021.70300  19787020.76600  19787018.19500
        47.25000        38.12500         0.00000
 -21235717.23700 -15022391.51300  19311520.43000  19311519.06200  19311518.27300
        46.62500        35.18800         1.00000
 -25948184.88400 -18683843.74600  18557424.93000  18557423.85200  18557421.78100
        44.06200        35.81200         2.00000
 -17114597.83100 -12088234.96000  20190959.61700  20190957.53900  20190955.82000
        43.00000        30.62500         3.00000
 -14313629.49800 -10192727.64200  20547552.26600  20547550.61700  20547548.82800
        44.12500        31.06200         4.00000
 -10848177.10100  -5025368.26200  21912937.10200  21912935.60200  21912933.95300
        42.37500        28.43800         5.00000
  -2430062.53600   -759916.91900  22715136.58600  22715134.43800  22715133.43800
        36.81200        24.37500         6.00000
  -8781453.66200  -4550814.80200  21167404.22700  21167402.78900  21167401.50800
        41.93800        27.56200         7.00000
 -14602364.77300  -8316038.34400  21258921.50000  21258919.21100  21258917.78100
        44.18800        32.18800         8.00000
 -14690624.07000 -10659461.43000  20720050.23400  20720047.79700  20720046.36700
        42.68800        28.68800        10.00000
  -3228686.36900  -1849801.82500  22928182.01600  22928181.07800  22928177.00800
        39.06200        21.43800        11.00000
```

Clock jumps of receiver (Nov. 2009)



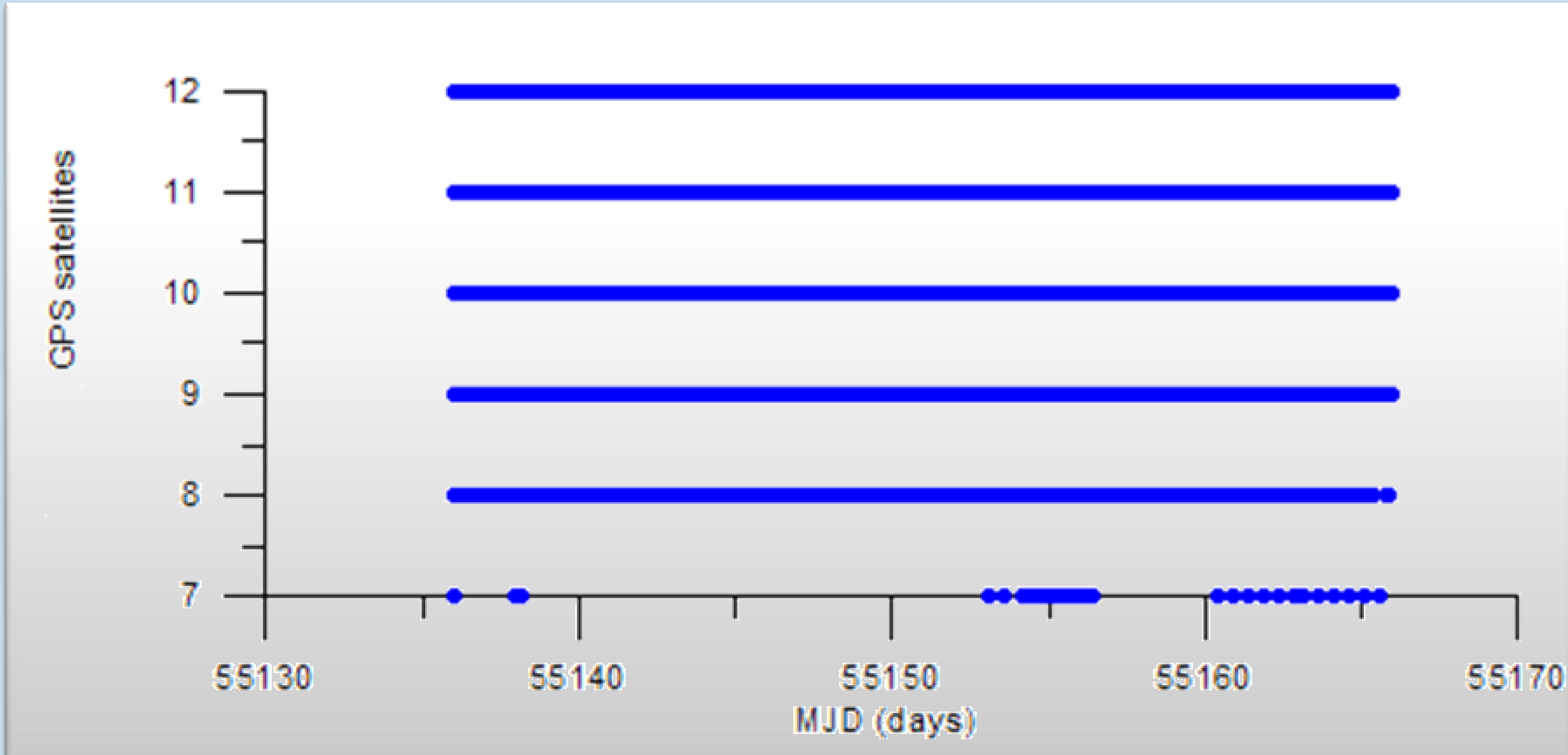
GPS LAGRANGE receiver clock behavior!

Clock behavior of receiver (Nov. 2009)



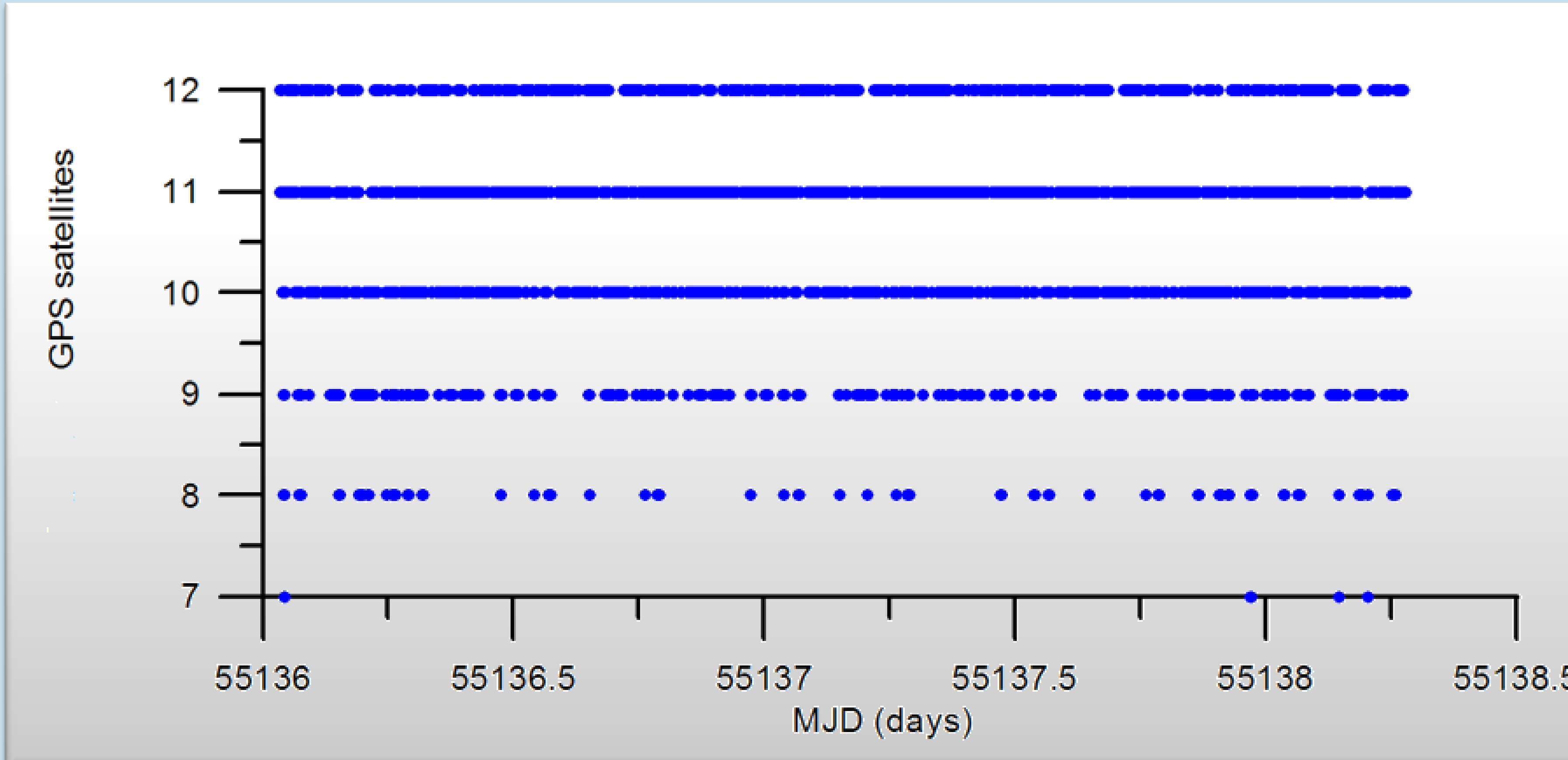
Clock jumps of 20 ms at ~27 hours can be seen

GPS visibility onboard GOCE (Nov. 2009)



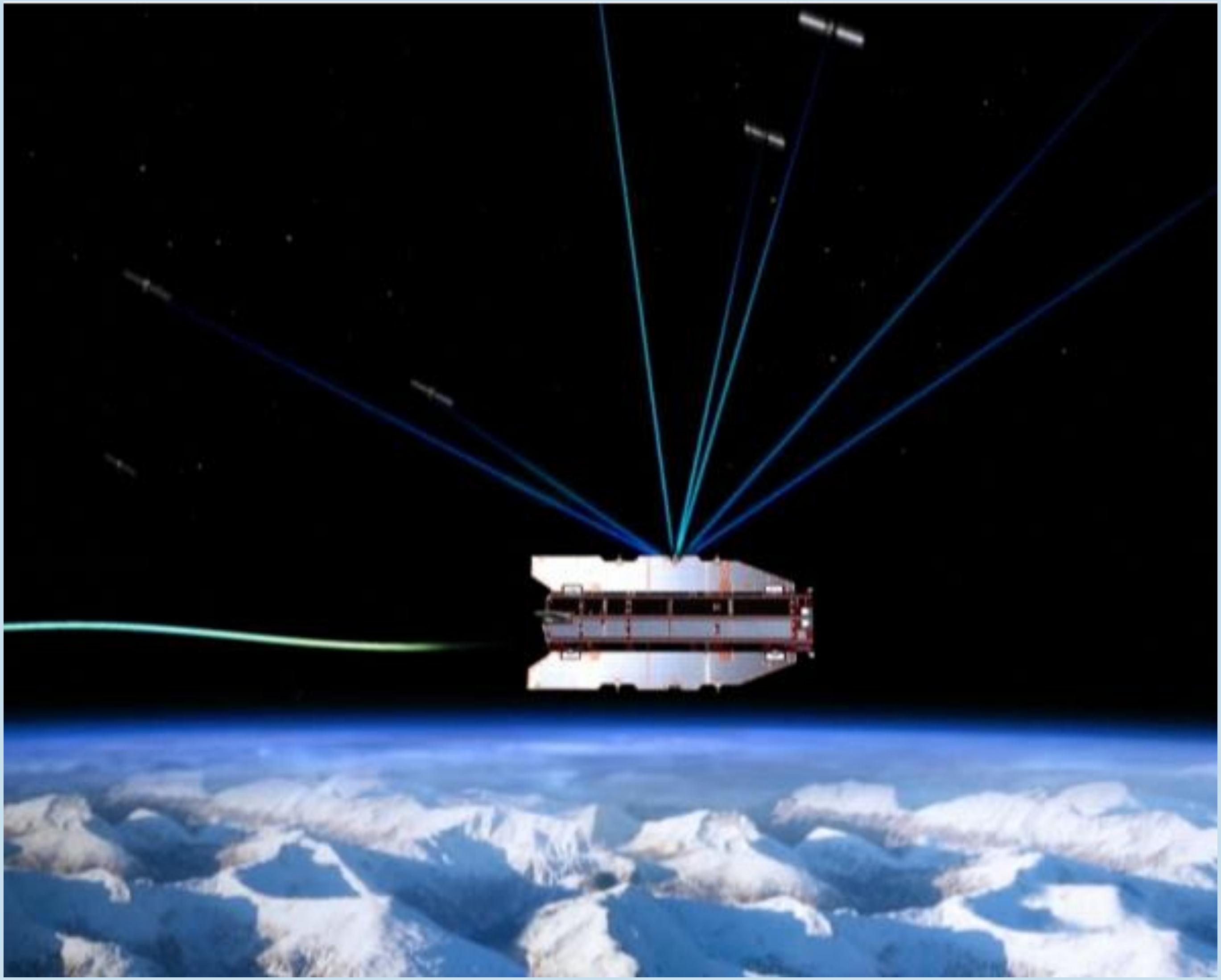
7 ‹ Number of GPS satellites (PRNs) ‹ 12

GPS visibility onboard GOCE (Nov. 2009)



7 < Number of GPS satellites (PRNs) < 12

Zero differenced GPOD



Credite by ESA

Zero differenced GPOD

Zero Difference

Only connection between LEO satellite and GPS satellites,

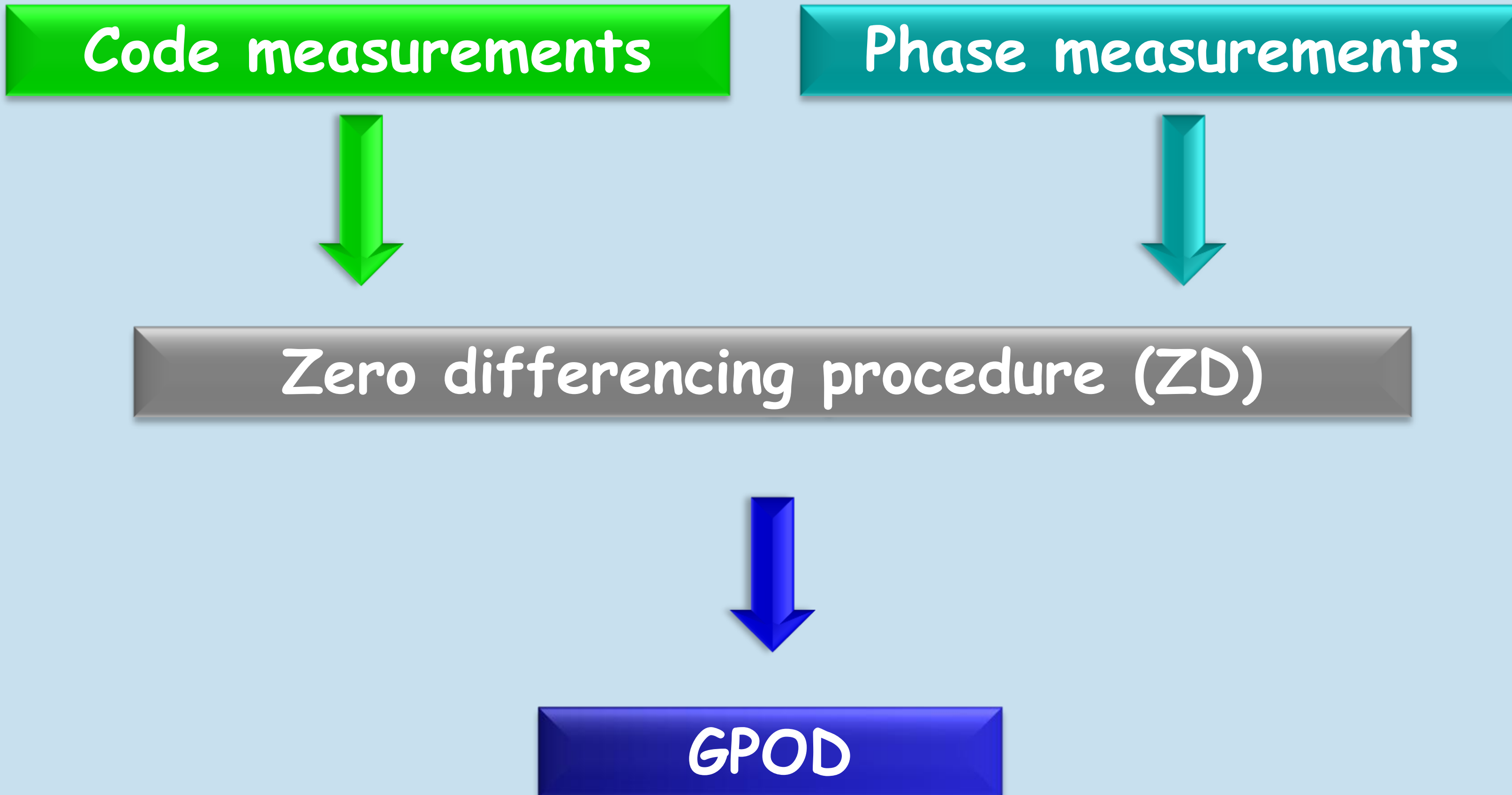
Geometrical

Only pure geometrical relations between LEO and the GPS satellites have to be used, no force models and no constraints,

Precise

Consideration all effects on GPS-SST observations and using precise GNSS satellites ephemerides.

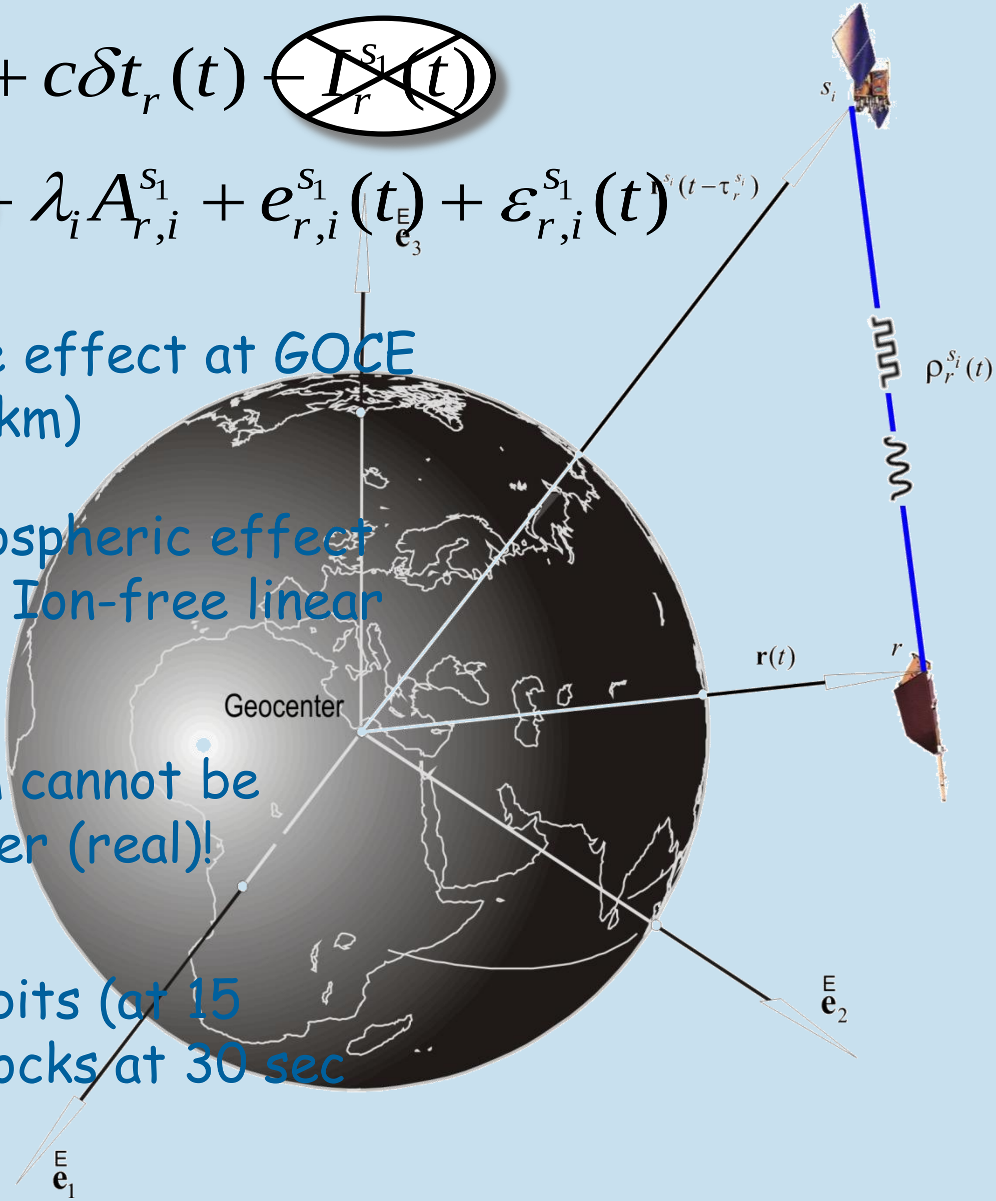
Processing concept



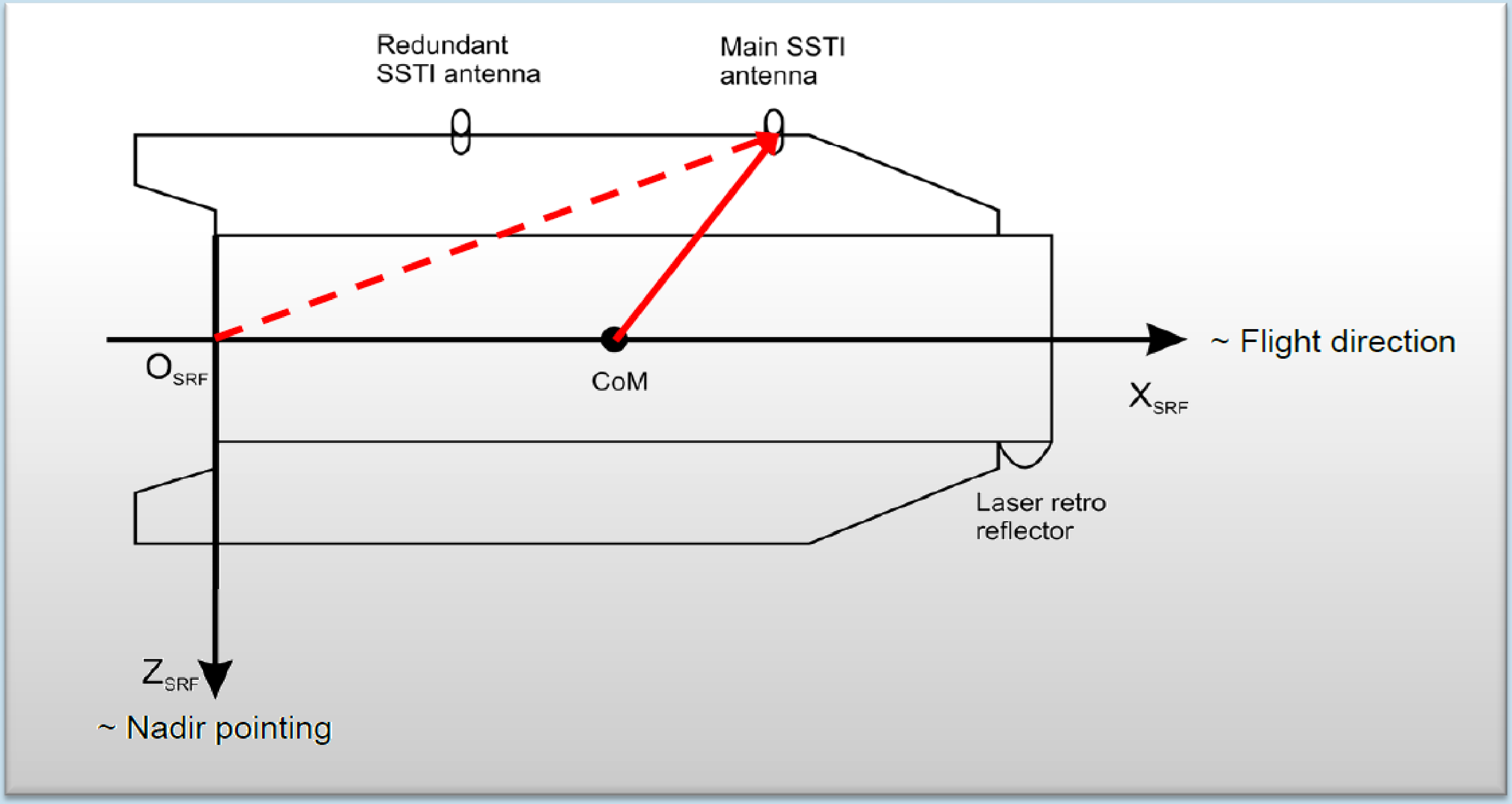
Precise Orbit Determination (POD)

$$\Phi^{S_1}_{r,i}(t) = \rho^{S_1}_r(t) + c\delta t_r(t) \cancel{- I^{S_1}_r(t)}$$
$$\cancel{+ T^{S_1}_r(t)} + \lambda_i A^{S_1}_{r,i} + e^{S_1}_{r,i}(t) + \varepsilon^{S_1}_{r,i}(t)$$

- No troposphere effect at GOCE altitude (~250 km)
- First order ionospheric effect eliminated with Ion-free linear combination
- Ambiguity term cannot be solved as Integer (real)!
- GPS precise orbits (at 15 minutes) and clocks at 30 sec

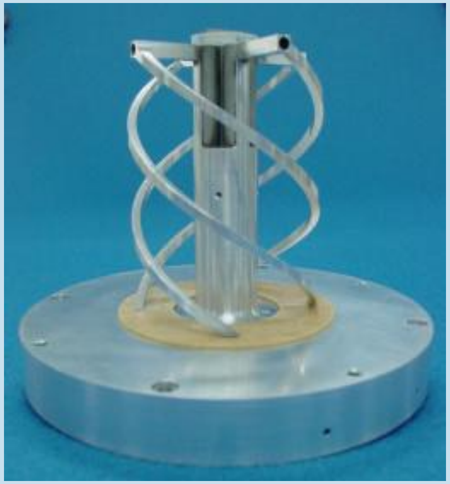


GPS Antenna offsets



GPS antenna offsets …

- GPS receiver offset with respect to GOCE reference frame is constant
- L1 and L2 (L3) Phase Center Offsets (PCO) are derived from IGS ANTEX (ANTenna Exchange format)
- Phase Center Variation (PCV) can be empirical estimated based on carrier phase residuals! (or ANTEX?)
- Offset with respect to center of mass (COM) is slowly varing!



Geometrical POD – hl-SST

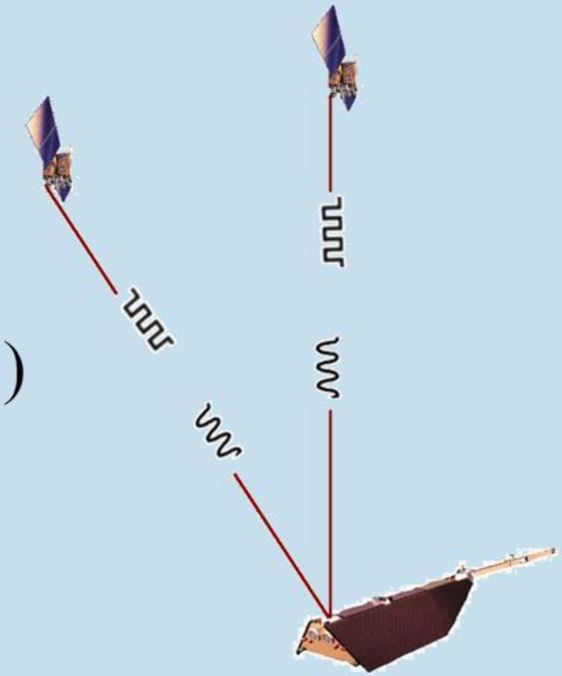
$$\Phi^{s_1}_{r,i}(t) = \rho^{s_1}_{r}(t) + c\delta t_r(t) +$$
$$+ \lambda_i A^{s_1}_{r,i} + e^{s_1}_{r,i}(t) + \varepsilon^{s_1}_{r,i}(t)$$

$$\Delta\Phi_{r,i}^{s_1}(t) = \mathbf{a}_{\mathbf{x}}^{s_1}(t)\Delta\mathbf{x}$$

Geometrical POD – hl-SST

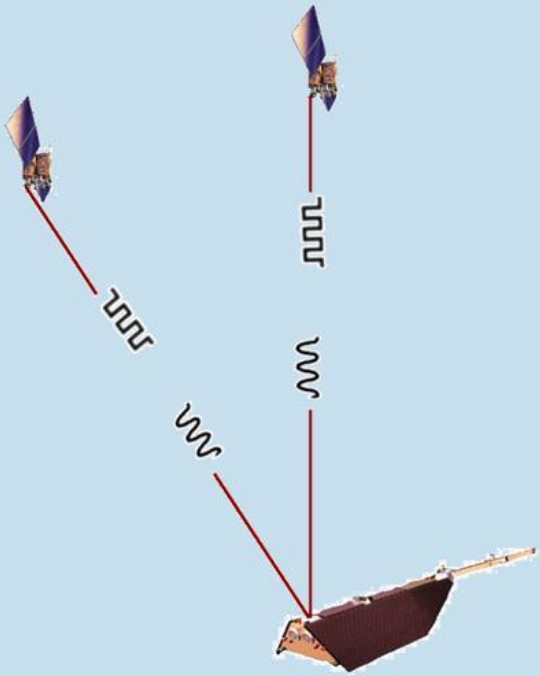
$$\Delta\Phi_{r,i}^{s_1}(t) = \mathbf{a}_{\mathbf{x}}^{s_1}(t)\Delta\mathbf{x}$$

$$\Phi_{r,i}^{s_2}(t) = \rho_r^{s_2}(t) + c\delta t_r(t) + \\ + \lambda_i A_{r,i}^{s_2} + e_{r,i}^{s_2}(t) + \varepsilon_{r,i}^{s_2}(t)$$



$$\Delta\Phi_{r,i}^{s_1}(t) = \mathbf{a}_{\mathbf{x}}^{s_1}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_2}(t) = \mathbf{a}_{\mathbf{x}}^{s_2}(t)\Delta\mathbf{x}$$



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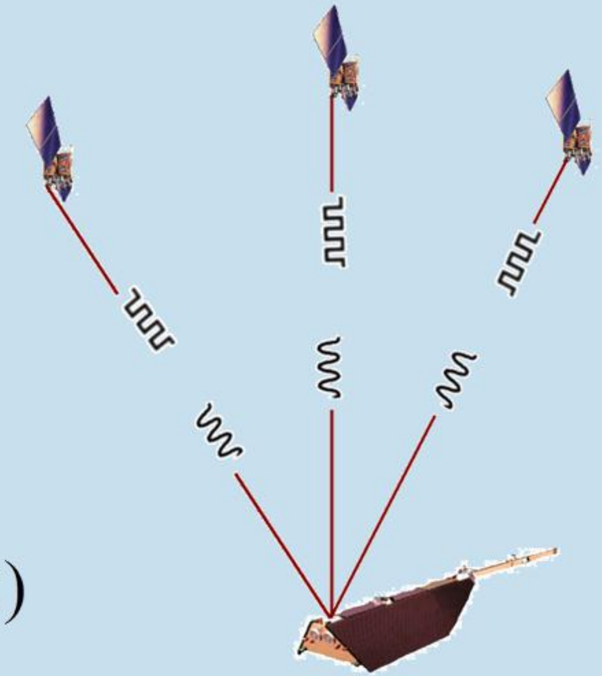
$$\Delta\Phi_{r,i}^{s_1}(t) = \mathbf{a}_{\mathbf{x}}^{s_1}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_2}(t) = \mathbf{a}_{\mathbf{x}}^{s_2}(t)\Delta\mathbf{x}$$

$$\Phi_{r,i}^{s_3}(t) = \rho_r^{s_3}(t) + c\delta t_r(t) + \\ + \lambda_i A_{r,i}^{s_3} + e_{r,i}^{s_3}(t) + \varepsilon_{r,i}^{s_3}(t)$$



Geometrical POD – hl-SST

$$\Delta\Phi_{r,i}^{s_1}(t) = \mathbf{a}_{\mathbf{x}}^{s_1}(t)\Delta\mathbf{x}$$

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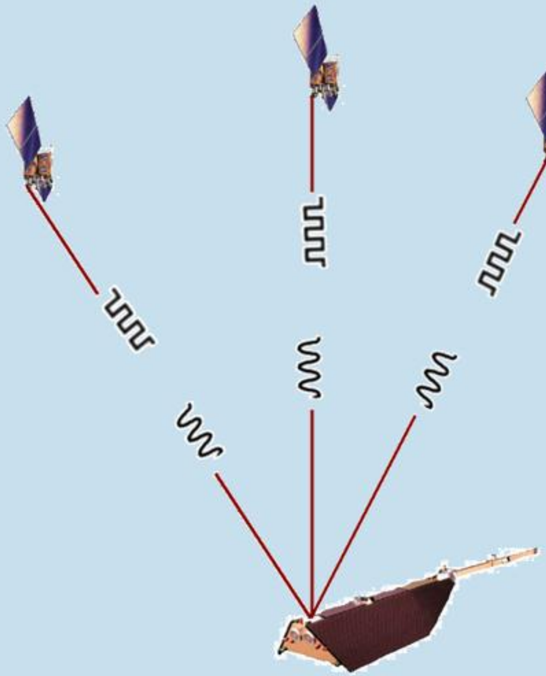

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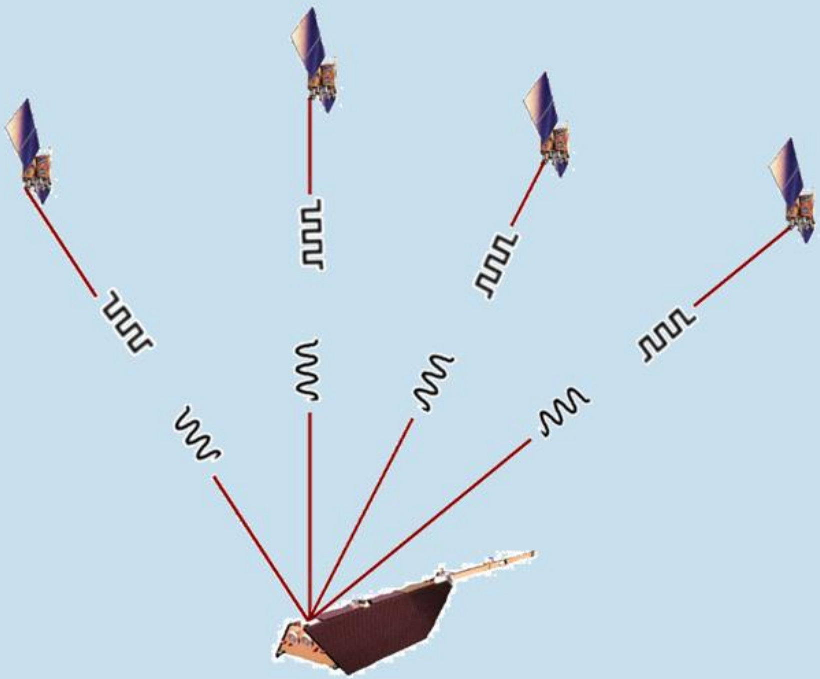
$$\Delta\Phi_{r,i}^{s_3}(t) = \mathbf{a}_{\mathbf{x}}^{s_3}(t)\Delta\mathbf{x}$$

$$\Phi_{r,i}^{s_4}(t) = \rho_r^{s_4}(t) + c\delta t_r(t) + + \lambda_i A_{r,i}^{s_4} + e_{r,i}^{s_4}(t) + \varepsilon_{r,i}^{s_4}(t)$$



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Geometrical POD – hl-SST

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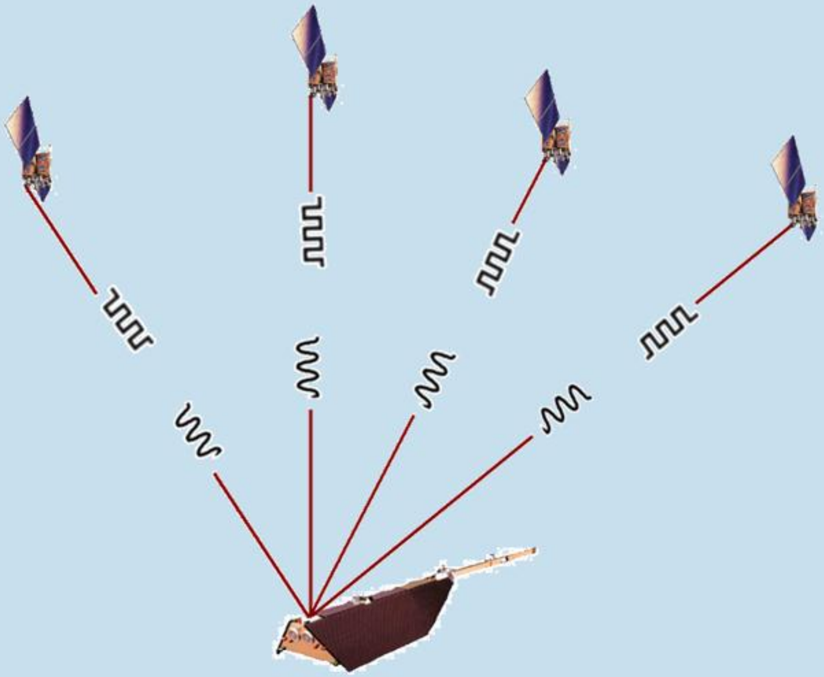
$$\Delta\Phi_{r,i}^{s_3}(t) = \mathbf{a}_{\mathbf{x}}^{s_3}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_4}(t) = \mathbf{a}_{\mathbf{x}}^{s_4}(t)\Delta\mathbf{x}$$



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Geometrical POD – hl-SST

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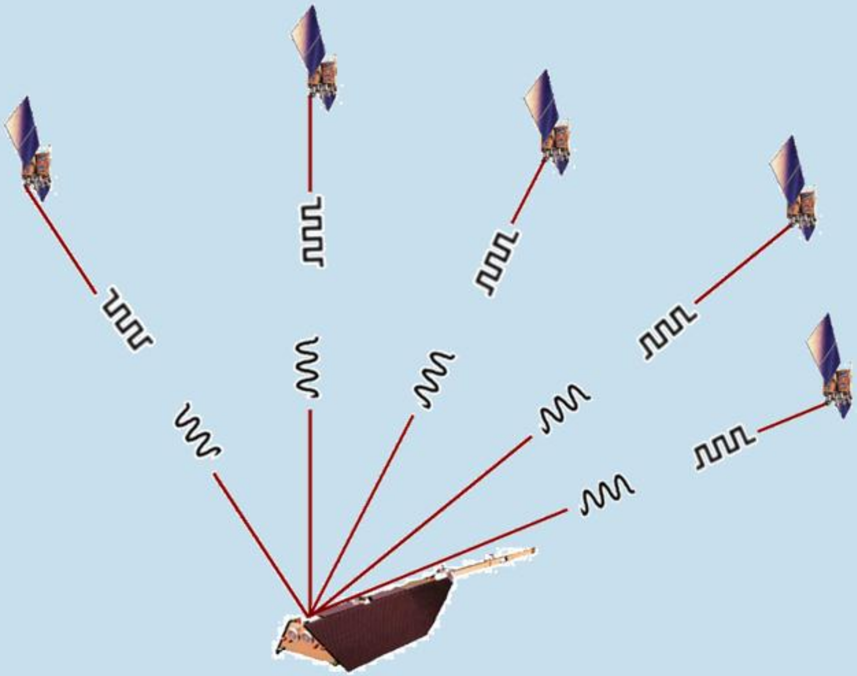
$$\Delta\Phi_{r,i}^{s_2}(t) = \mathbf{a}_{\mathbf{x}}^{s_2}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_3}(t) = \mathbf{a}_{\mathbf{x}}^{s_3}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_4}(t) = \mathbf{a}_{\mathbf{x}}^{s_4}(t)\Delta\mathbf{x}$$

$$\Phi_{r,i}^{s_5}(t) = \rho_r^{s_5}(t) + c\delta t_r(t) + $$
$$+ \lambda_i A_{r,i}^{s_5} + e_{r,i}^{s_5}(t) + \varepsilon_{r,i}^{s_5}(t)$$



Geometrical POD – hl-SST

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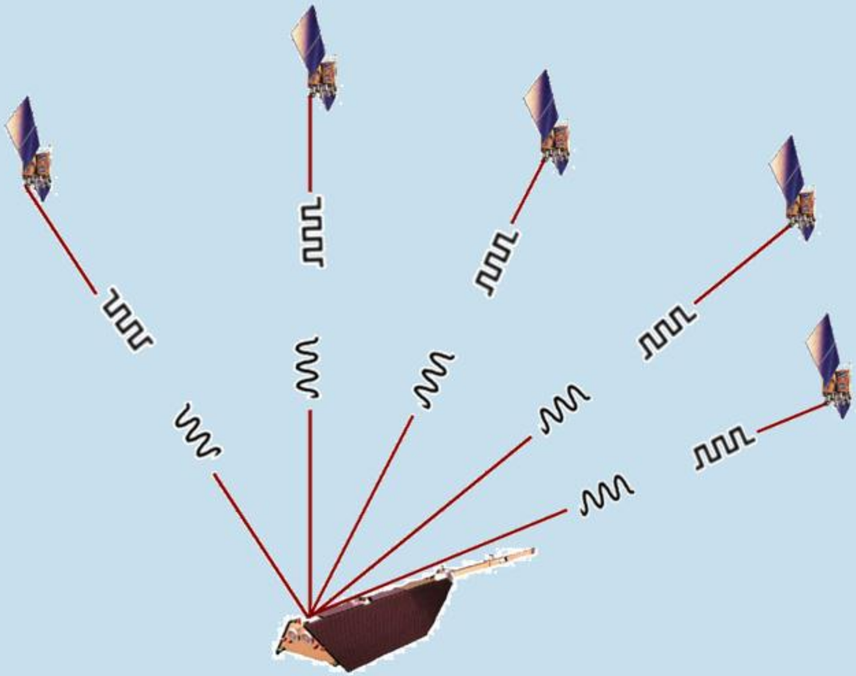
$$\Delta\Phi_{r,i}^{s_2}(t) = \mathbf{a}_{\mathbf{x}}^{s_2}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_3}(t) = \mathbf{a}_{\mathbf{x}}^{s_3}(t)\Delta\mathbf{x}$$

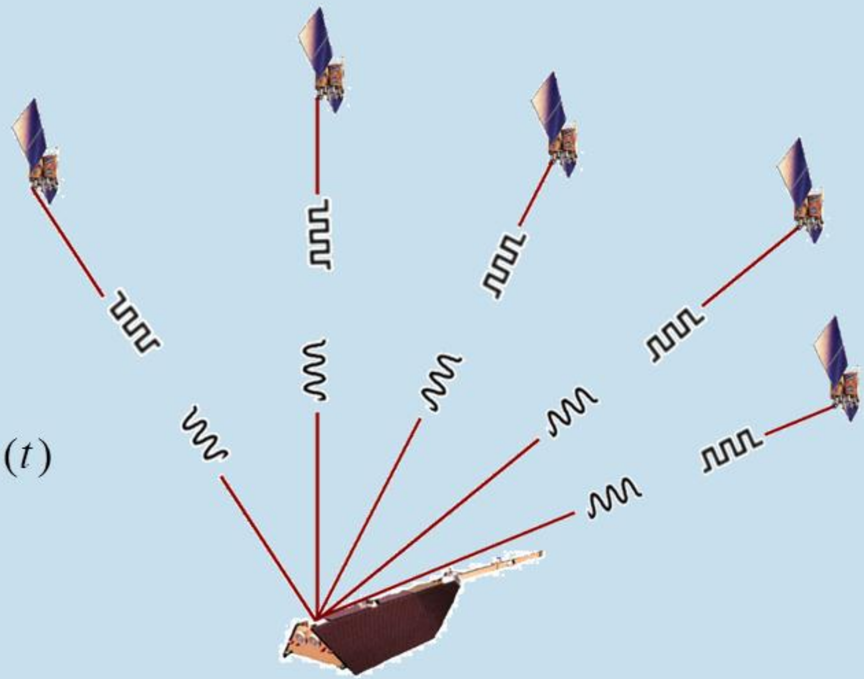
$$\Delta\Phi_{r,i}^{s_4}(t) = \mathbf{a}_{\mathbf{x}}^{s_4}(t)\Delta\mathbf{x}$$

$$\Delta\Phi_{r,i}^{s_5}(t) = \mathbf{a}_{\mathbf{x}}^{s_5}(t)\Delta\mathbf{x}$$



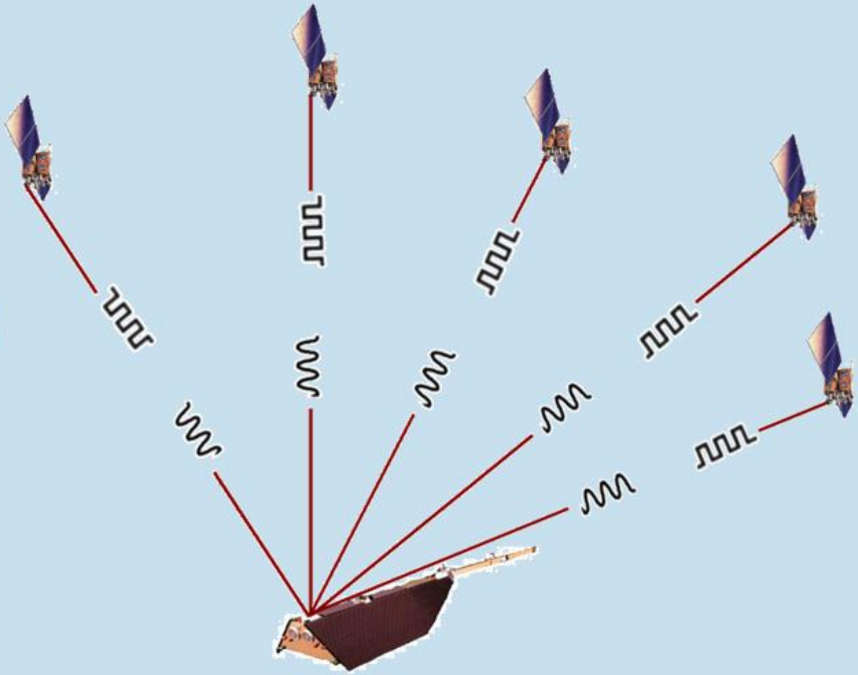
Geometrical POD – hl-SST

$$\begin{pmatrix} \Delta\Phi_{r,i}^{s_1}(t) \\ \Delta\Phi_{r,i}^{s_2}(t) \\ \Delta\Phi_{r,i}^{s_3}(t) \\ \Delta\Phi_{r,i}^{s_4}(t) \\ \Delta\Phi_{r,i}^{s_5}(t) \end{pmatrix} = \begin{pmatrix} \mathbf{a}_{\mathbf{x}}^{s_1}(t) \\ \mathbf{a}_{\mathbf{x}}^{s_2}(t) \\ \mathbf{a}_{\mathbf{x}}^{s_3}(t) \\ \mathbf{a}_{\mathbf{x}}^{s_4}(t) \\ \mathbf{a}_{\mathbf{x}}^{s_5}(t) \end{pmatrix} \Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t)}$$



Geometrical POD – hl-SST

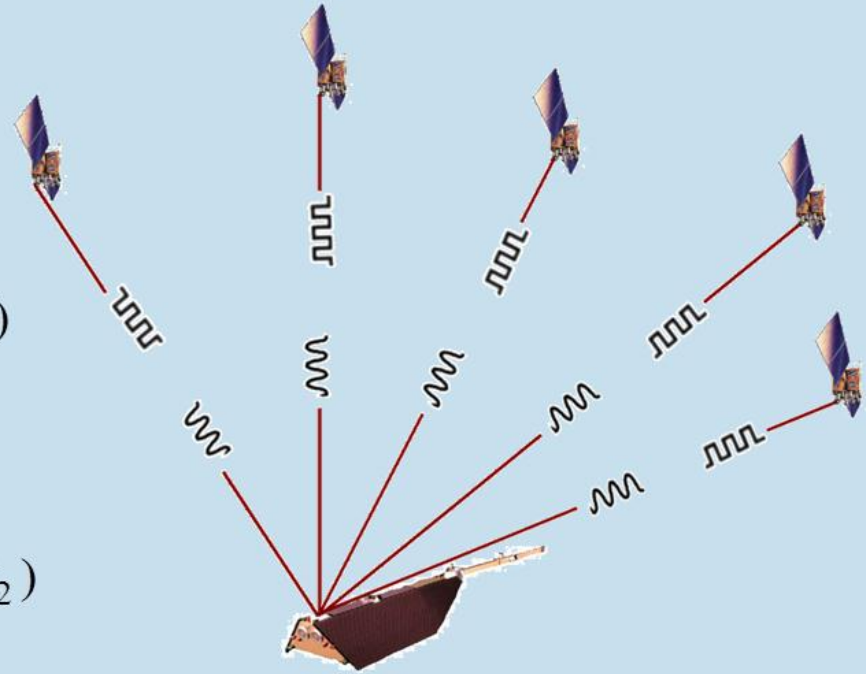
$$\Delta\boldsymbol{\Phi}(t_1) = \mathbf{A}_r(t_1)\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t_1)}$$



Geometrical POD – hl-SST

$$\Delta\boldsymbol{\Phi}(t_1) = \mathbf{A}_r(t_1)\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t_1)}$$

$$\Delta\boldsymbol{\Phi}(t_2) = \mathbf{A}_r(t_2)\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t_2)}$$



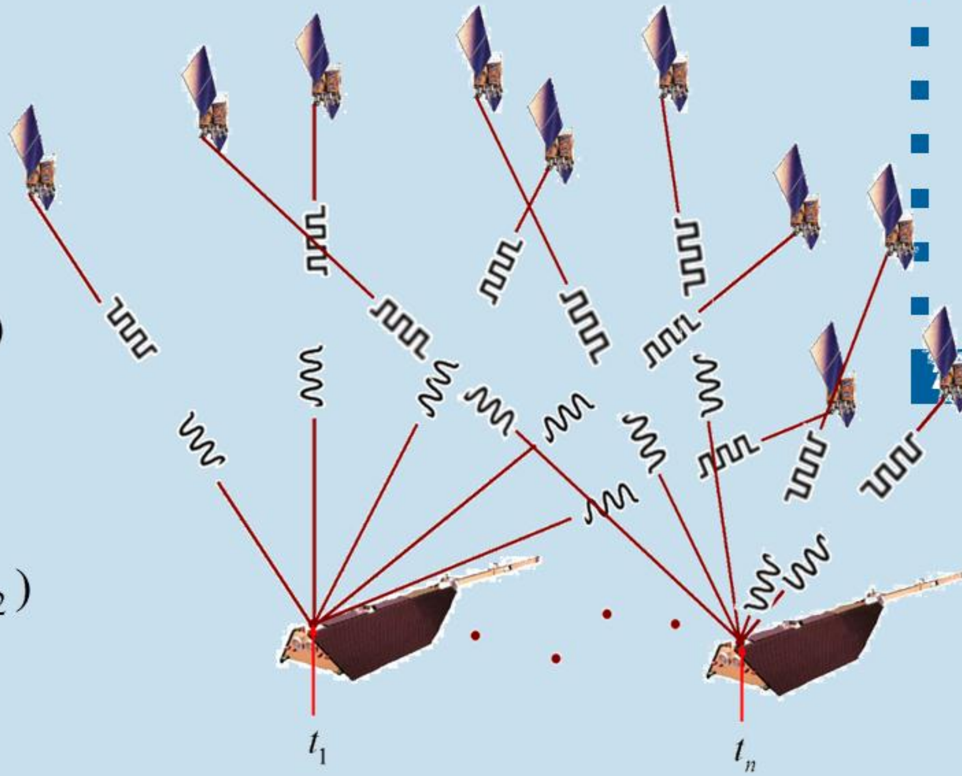
Geometrical POD – hl-SST

$$\Delta\boldsymbol{\Phi}(t_1) = \mathbf{A}_r(t_1)\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t_1)}$$

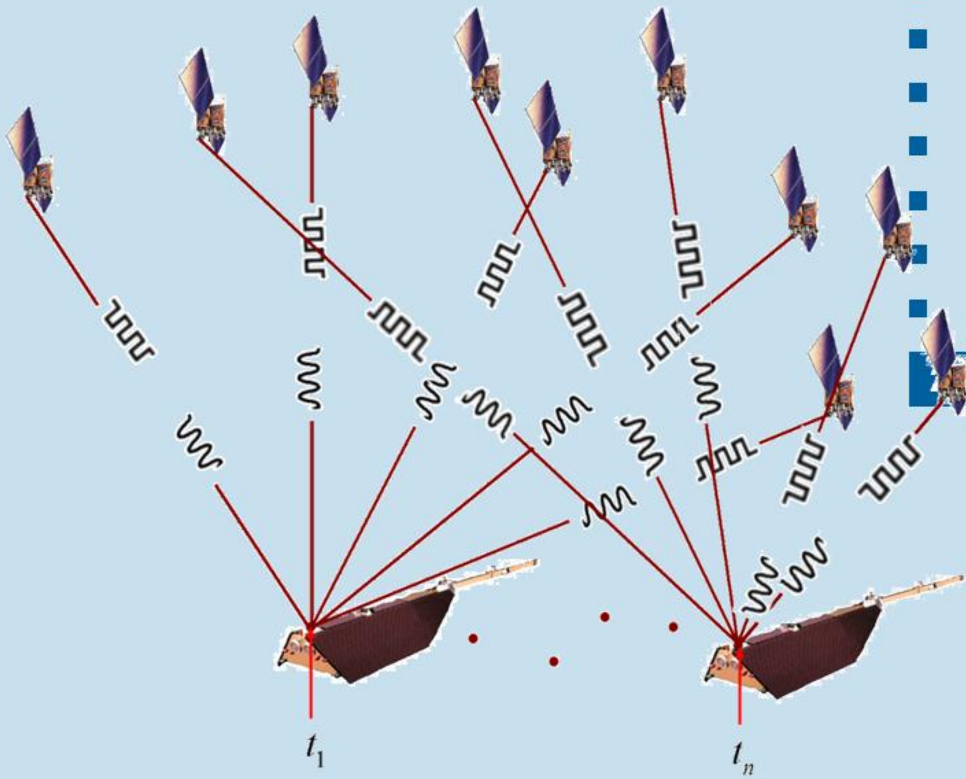
$$\Delta\boldsymbol{\Phi}(t_2) = \mathbf{A}_r(t_2)\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t_2)}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\Delta\boldsymbol{\Phi}(t_n) = \mathbf{A}_r(t_n)\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi(t_n)}$$

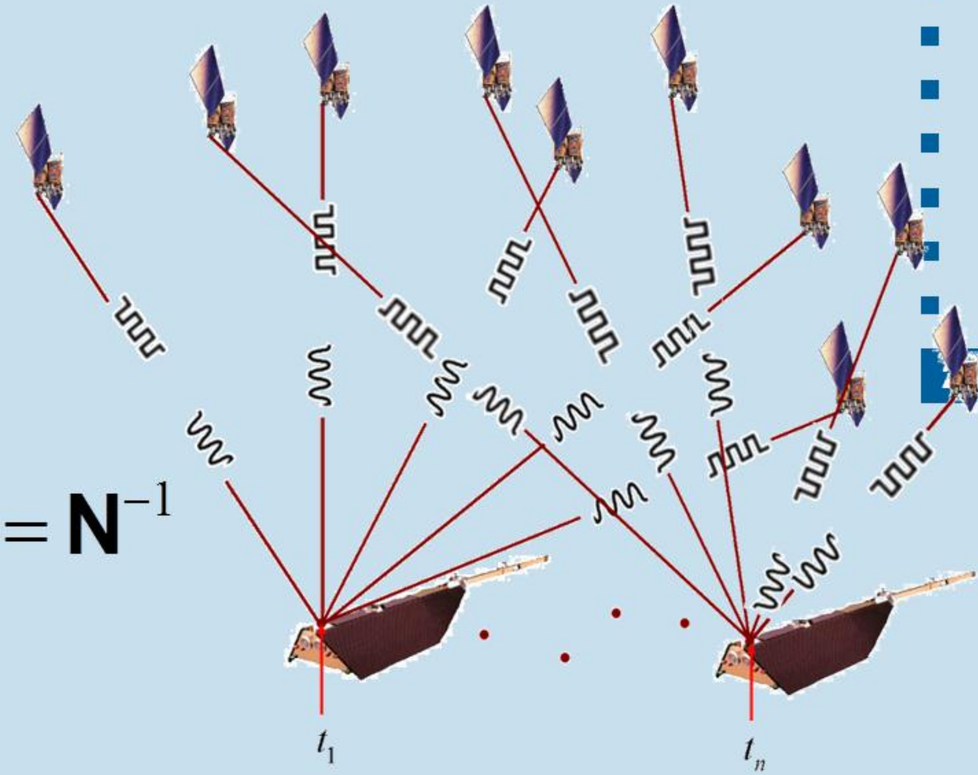


$$\Delta \mathbf{\Phi} = \mathbf{A} \Delta \mathbf{x}, \quad \mathbf{C}_{\Phi}$$



Geometrical POD – hl-SST

$$\Delta\boldsymbol{\Phi} = \mathbf{A}\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi}$$

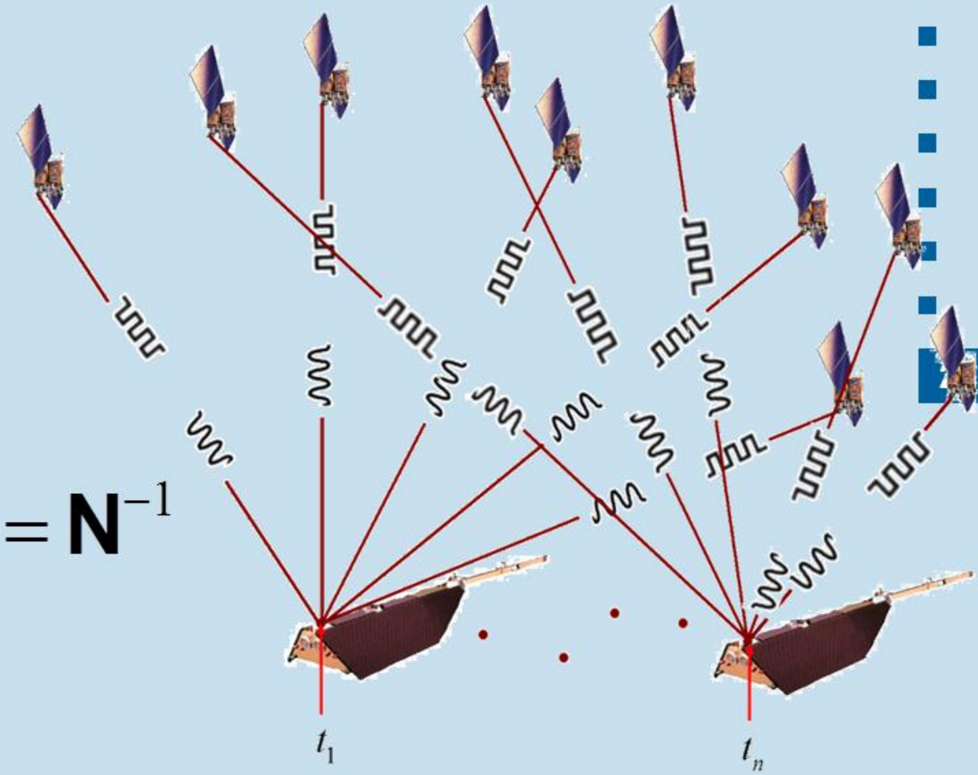


$$\Delta\hat{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{A}^{T}\mathbf{C}_{\Phi}^{-1}\Delta\boldsymbol{\Phi}, \quad \mathbf{C}_{\Delta\hat{\mathbf{x}}} = \mathbf{N}^{-1}$$

$$\mathbf{N} = \left(\mathbf{A}^{T}\mathbf{C}_{\Phi}^{-1}\mathbf{A}\right)$$

Geometrical POD – hl-SST

$$\Delta\boldsymbol{\Phi} = \mathbf{A}\Delta\mathbf{x}, \quad \mathbf{C}_{\Phi}$$

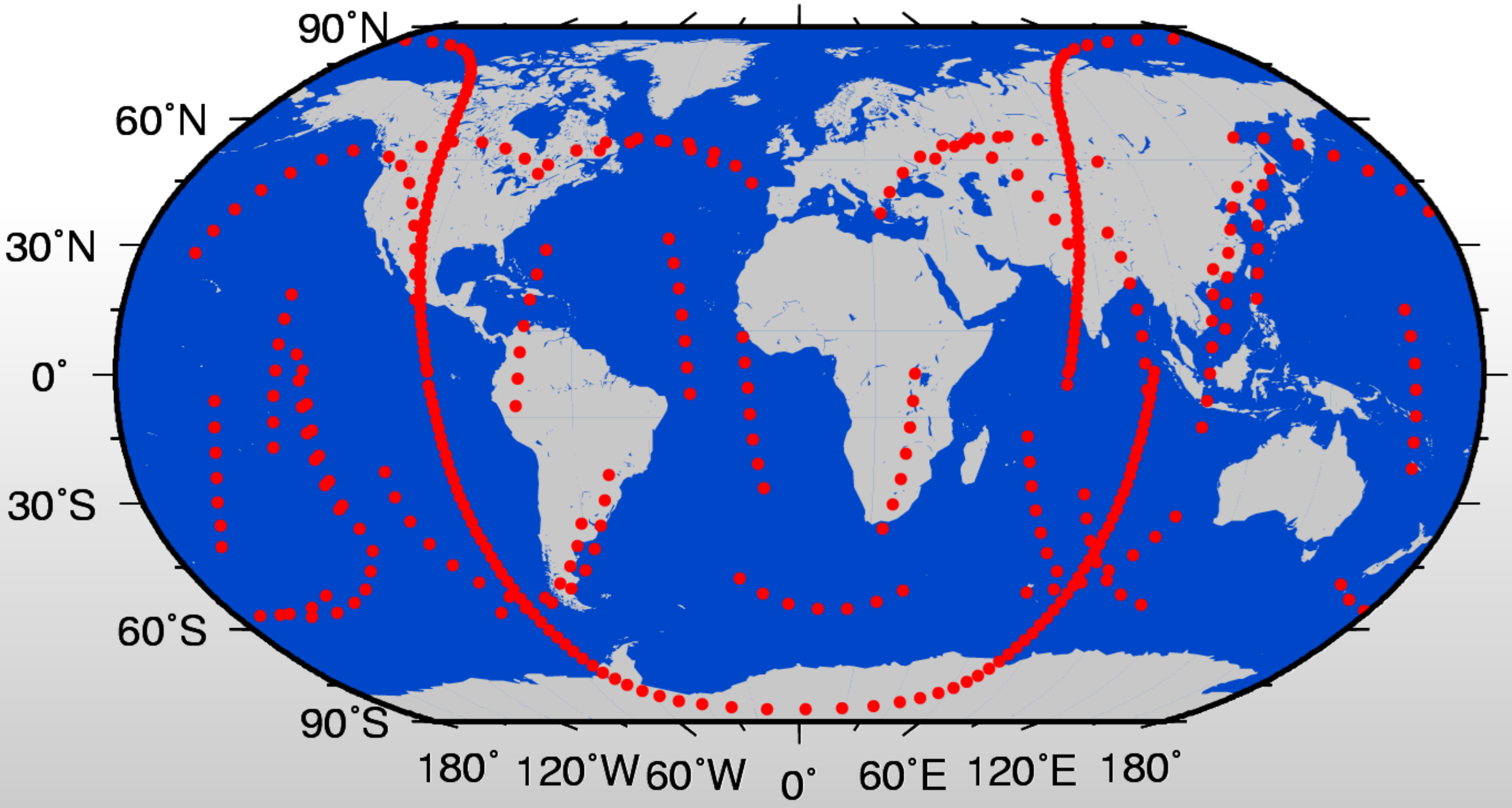


$$\Delta\hat{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{A}^{T}\mathbf{C}_{\Phi}^{-1}\Delta\boldsymbol{\Phi}, \quad \mathbf{C}_{\Delta\hat{\mathbf{x}}} = \mathbf{N}^{-1}$$

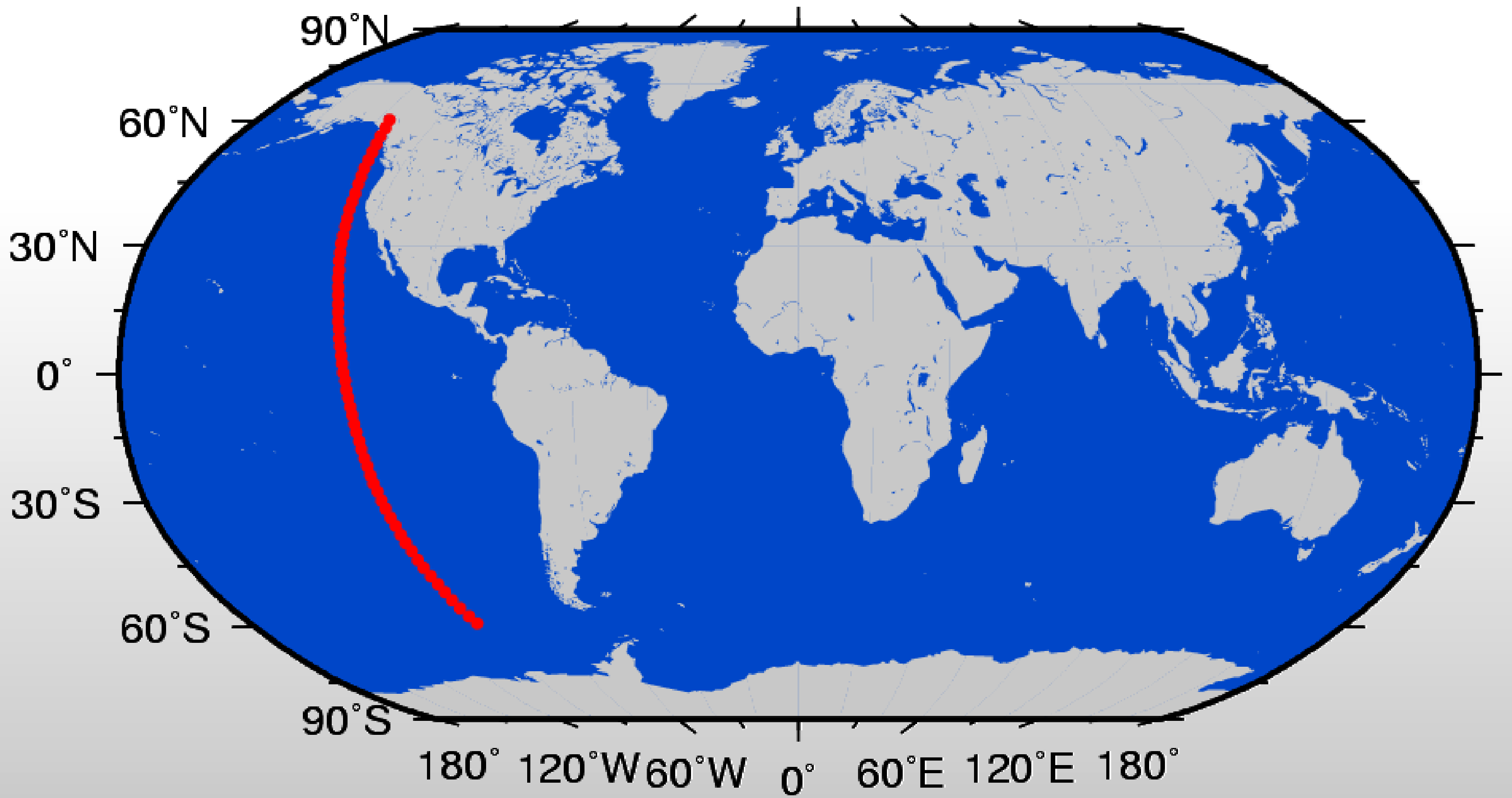
$$\mathbf{N} = \left(\mathbf{A}^{T}\mathbf{C}_{\Phi}^{-1}\mathbf{A}\right)$$

$$\hat{\mathbf{x}} = \mathbf{x}_0 + \Delta\mathbf{x}, \quad \mathbf{C}_{\hat{\mathbf{x}}} = \mathbf{N}^{-1}$$

Ground tracks of GPS, GOCE (2 Nov. 1 Rev.)

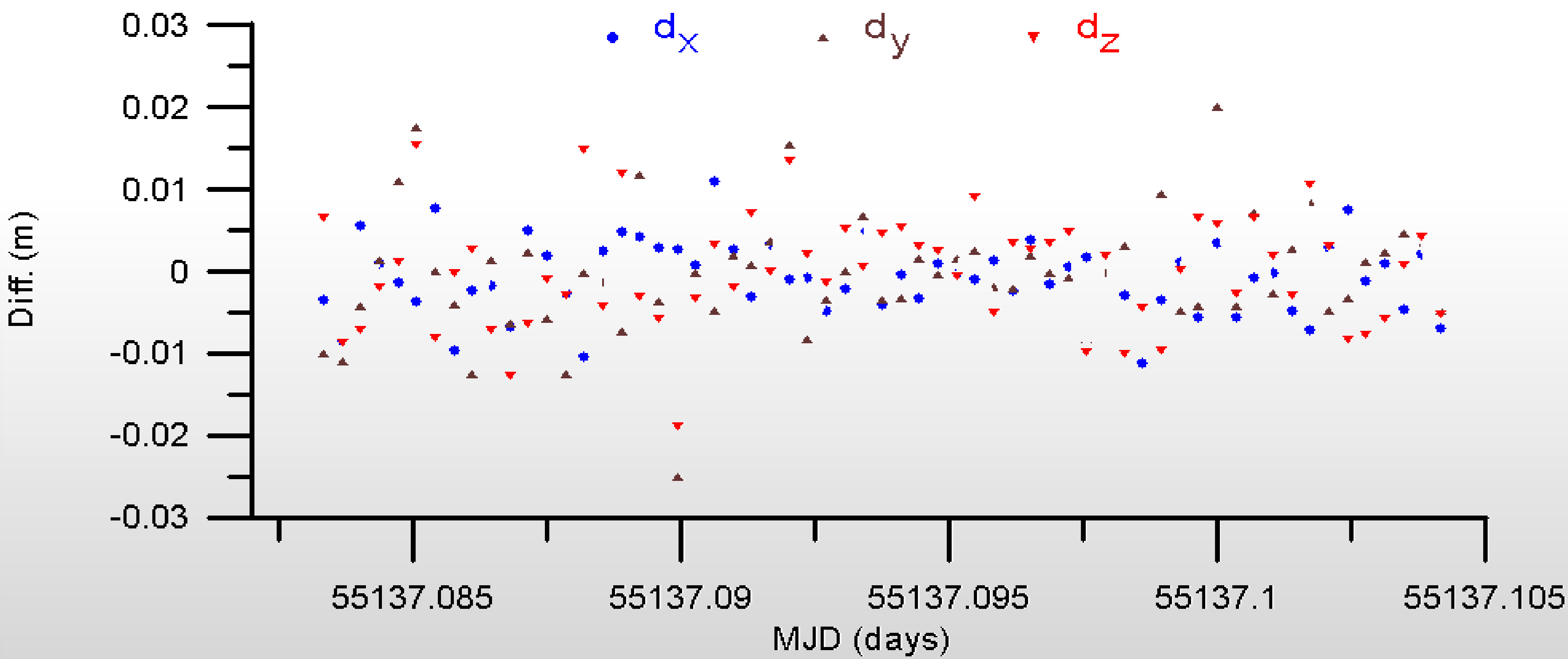


Short arc of GOCE (02 Nov. 2 Rev.)



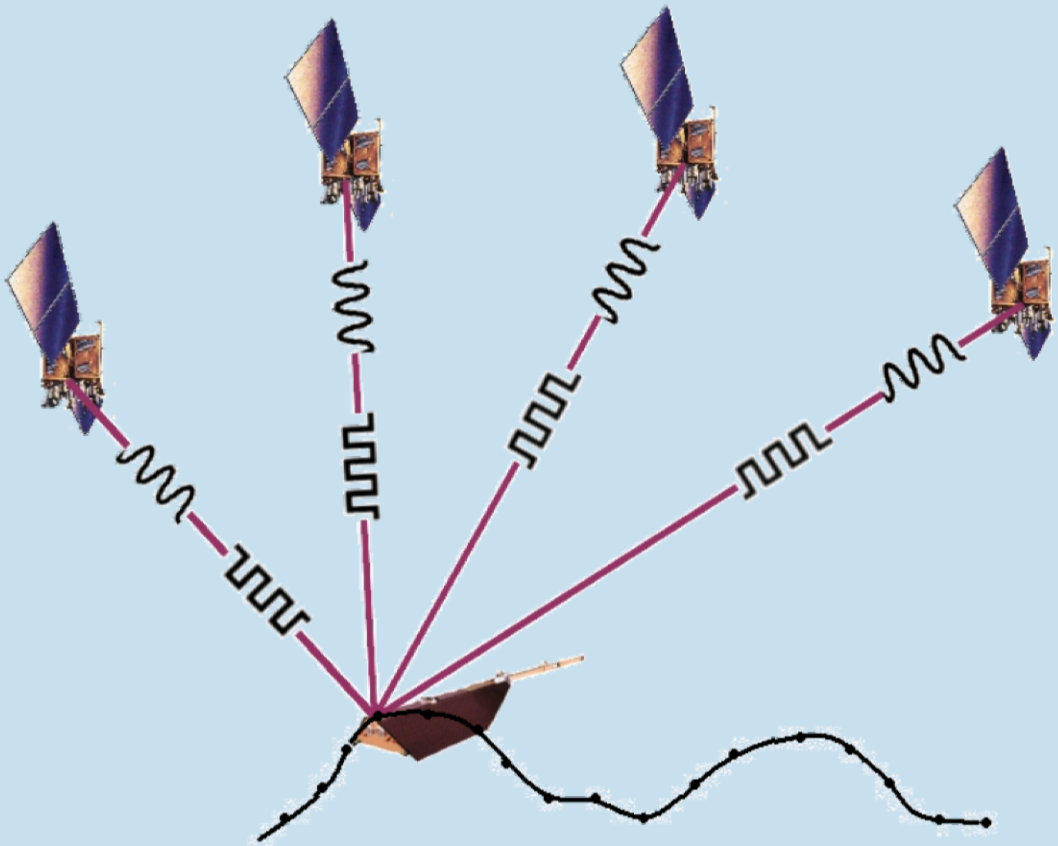
30 minutes short arc (2009-11-02 02 00 00 - 02 30 00)

GPOD - GOCE results



Estimated geometrical 3D Pos. - PSO

Kinematical POD



Short arc representation

Gibbs effect!

LEO orbit can be represented as:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{n} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

Precision!

or

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$

$\mathbf{r}(\tau)$

Solution?

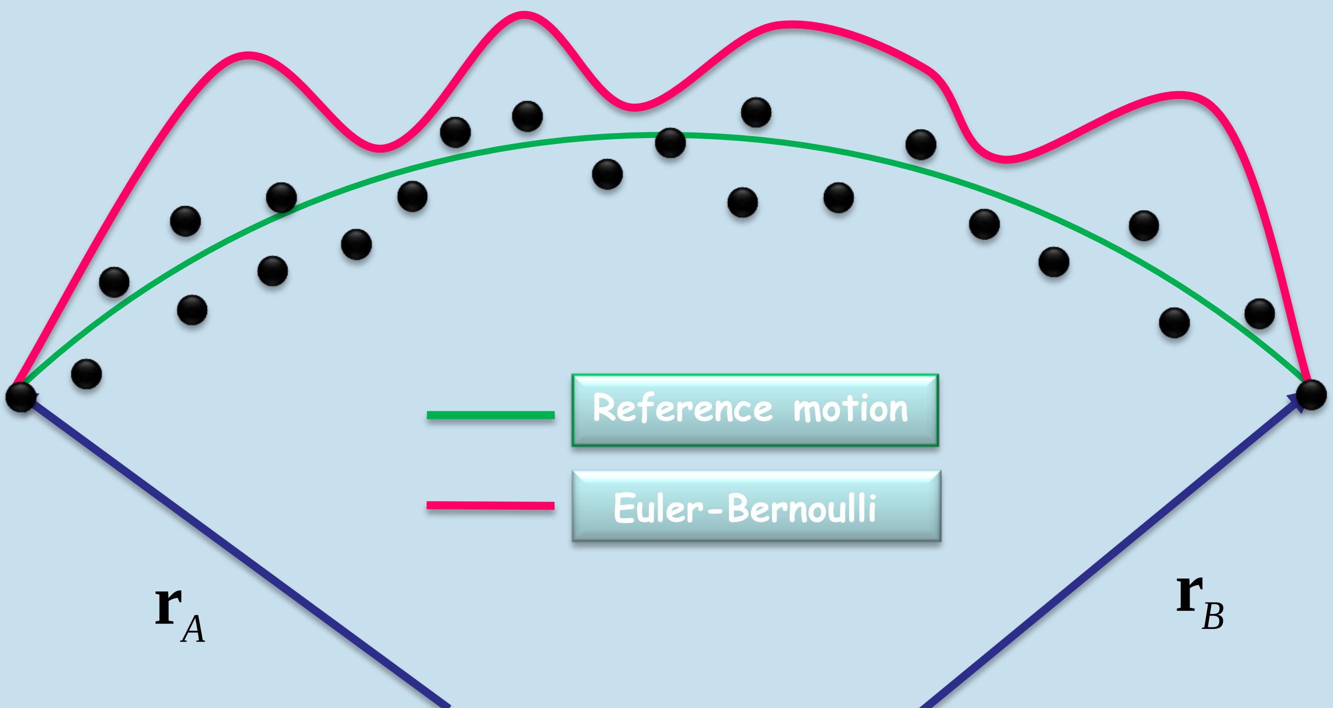
fast convergence!

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau) + \sum_{\upsilon=1}^{\bar{n}} \bar{\mathbf{d}}_{\upsilon} \sin(\upsilon\pi\tau)$$

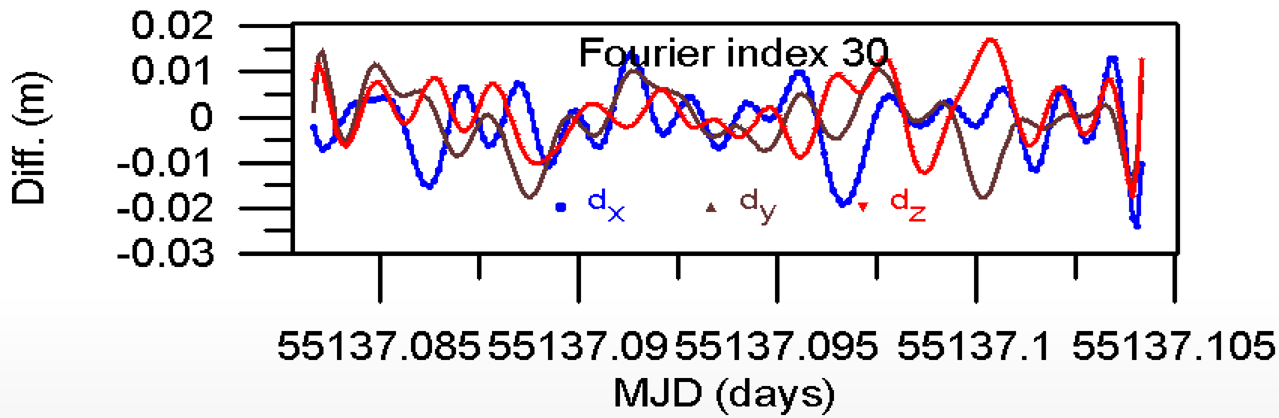
Determination of Euler-Bernoulli coefficients

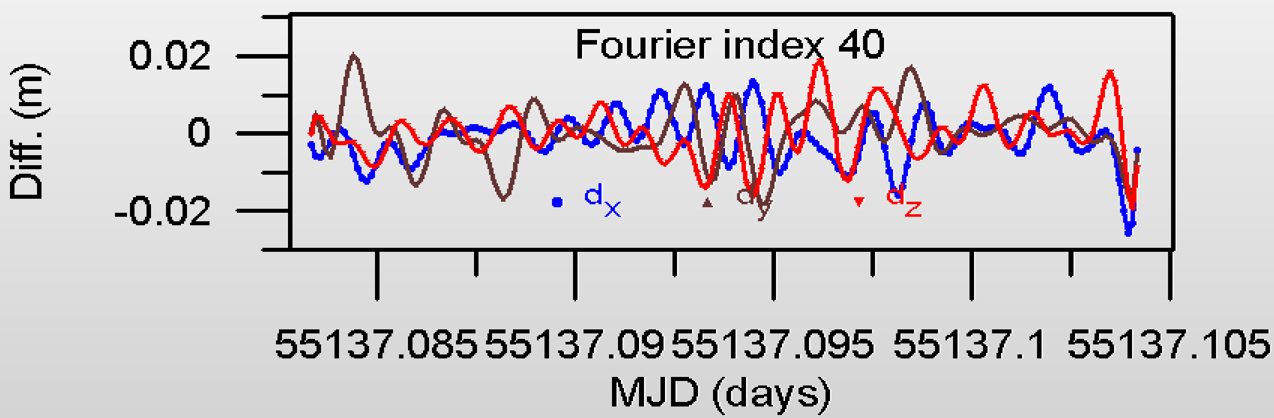
A satellite short arc can be represented with the Euler-Bernoulli term up to degree J as:

$$\mathbf{r}(\tau) - \bar{\mathbf{r}}(\tau) = \mathbf{d}(\tau) \approx \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$



KPOD – GOCE results





Estimated kinematical Pos. (J=4) – PSO (Fourier index 30 and 40)

Conclusions and recommendations

- GNSS-GOCE satellites configuration and geometrical strength play an important role in POD.
- Estimated Geometrical Precise Orbit can be used to estimate kinematical POD of GOCE.
- Kinematical POD can be used to recover the Earth's gravity field model based on the hl-SST methods, (GOCE SST model).
- No gravity field and no force models have been used in the Geometrical and Kinematical modes (advantage).
- Empirical PCV results should improve POD of GOCE!

Thank you
for your attention!