How important is the dynamical information in determination of LEO orbits

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Abstract

The interest in a precise orbit determination of Low Earth Orbiters (LEOs) using GNSS observations, in order to recover the Earth's gravity field, has rapidly grown. Based on the advent of precise orbit and clock products by the IGS analysis centres and geometrical high-low Satellite to Satellite Tracking (hl-SST) observations, the point-wise Geometrical Precise Orbit Determination (GPOD) of LEOs can be introduced with only a single GNSS receiver onboard LEOs. Based on a new proposed Kinematical Precise Orbit Determination (KPOD) method, the orbit is represented by a number of approximation parameters including boundary values of the LEO arc. This kind of orbit representation not only allows to determine an arbitrary functional (e.g. velocity and acceleration) of the satellite arcs, it is also possible to use dynamical information for the determination of the orbit parameters. In the geometrical and kinematical POD procedures, no dynamical information is used at all. Because of the close relation of the estimated kinematical parameters with the force function model, orbit determination can be designed as a pure KPOD on the one hand and a pure Dynamical Precise Orbit Determination (DPOD) on the other hand. If only weak dynamical restrictions or full dynamical information are introduced to the estimation procedure, then a Reduced-Kinematical Precise Orbit Determination (RKPOD) is introduced. In this poster, the new concept, the various possibilities and the effect of the dynamical information in POD based on simulated data are presented for the GOCE mission.

Kinematical Precise Orbit Determination (KPOD)

In *Shabanloui (2008)*, it was demonstrated that a LEO short arc can be kinematically represented as,

$$\mathbf{r}(\tau)=\bar{\mathbf{r}}(\tau)+\sum_{j=1}^{J_{max}}\left(\mathbf{e}_{2j}E_{2j}(\tau)+\mathbf{b}_{2j+1}B_{2j+1}(\tau)\right)+\sum_{\upsilon=1}^{\bar{n}}\mathbf{d}_{\upsilon}\sin(\upsilon\pi\tau) \qquad (1)$$

τ is the normalized time at the time t from the LEO starting time t_A, end time t_B and the arc length T as

$$\tau=\frac{t-t_A}{T}, \quad with \quad t\in[t_A,t_B], \quad T=t_B-t_A \qquad (2)$$

$\mathbf{r}(\tau)$ is the satellite position at the normalized time τ and $\bar{\mathbf{r}}(\tau)$ is the Keplerian orbit which connects the arc's boundaries $\mathbf{r}_A, \mathbf{r}_B$. In Eq. (1), $\mathbf{e}_{2j}$ and $\mathbf{b}_{2j+1}$ denote the Euler-Bernoulli coefficients and $E_{2j}(\tau)$ and $B_{2j+1}(\tau)$ are the absolutely and uniformly continuous series expansions of the Euler polynomial of degree $2j$ and Bernoulli polynomial of degree $2j+1$ at the normalized time τ, respectively. In Eq. (1), the vector $\mathbf{d}_{\upsilon}$ and $\bar{n}$ denote the residual Fourier coefficients and the residual Fourier series upper index, respectively.

In *Ilk (1977)*, the force function acting on the satellite can be determined as,

$$\tilde{\mathbf{d}}_{\upsilon}=\frac{2T^2}{\upsilon^2\pi^2}\int_{\tau'=0}^{1}\sin(\upsilon\pi\tau')\mathbf{a}(\tau;\mathbf{r},\dot{\mathbf{r}})d\tau' \qquad (3)$$

where $\tilde{\mathbf{d}}_{\upsilon}$, υ, $\dot{\mathbf{r}}$ and $\mathbf{a}$ are the dynamical information at the index υ, the Fourier index, the LEO velocity and the force acting on LEO, respectively. Now if the dynamical information contained in the orbit coefficients $\tilde{\mathbf{d}}_{\upsilon}$ is introduced to orbit estimation procedure, then the Reduced Kinematical Precise Determination can be realized. If the quantities $\tilde{\mathbf{d}}_i$ to $\tilde{\mathbf{d}}_j$ (i and j as start and end indices) from Eq. (3) are considered as a-priori dynamical information with the corresponding variance-covariances $\mathbf{C}(\tilde{\mathbf{d}}_i)$ and $\mathbf{C}(\tilde{\mathbf{d}}_j)$, the observation equation reads as,

$$\begin{pmatrix}\mathbf{l}_1\\ \mathbf{l}_2\end{pmatrix}=\begin{pmatrix}\mathbf{A}_1 & \mathbf{A}_2\\ \mathbf{0} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix}, \quad with \quad \mathbf{l}_1=\left(\mathbf{r}(\tau_1) \ \cdots \ \mathbf{r}(\tau_K)\right)^T, \ \mathbf{l}_2=\left(\tilde{\mathbf{d}}_i \ \cdots \ \tilde{\mathbf{d}}_j\right)^T \qquad (4)$$

$$\mathbf{C}=\begin{pmatrix}\mathbf{C}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_2\end{pmatrix}, with \quad \mathbf{C}_1=diag\left(\mathbf{C}(\tau_1) \ \cdots \ \mathbf{C}(\tau_K)\right)^T, \mathbf{C}_2=diag\left(\mathbf{C}(\tilde{\mathbf{d}}_i) \ \cdots \ \mathbf{C}(\tilde{\mathbf{d}}_j)\right)^T$$

where $\mathbf{l}_1$, $\mathbf{C}_1$, $\mathbf{l}_2$ and $\mathbf{C}_2$ denote geometrically determined LEO positions, LEO position variance-covariance matrix, dynamical restrictions and variance-covariance of dynamical restrictions, respectively.

Data processing

To verify the proposed orbit determination procedure, a 30 minutes short arc of GOCE orbit (Fig. 1) with a sampling rate of 30 s, a 10°cut-off angle, a white noise of $\sigma=2cm$ are simulated. In the first step, the GPOD orbit is estimated based on hl-SST observations. The differences between estimated point-wise geometrical orbit and reference orbit, the corresponding observation residuals are shown in Fig. (2). Based on the Euler-Bernoulli coefficients up to degree $J_{max}=4$, the kinematical parameters including boundary positions and residual Fourier coefficients are estimated. The corresponding position, velocity and acceleration differences at every 10 s and the residual Fourier coefficients are shown in Fig. (3a). After introducing the dynamical restrictions $\tilde{\mathbf{d}}_{\upsilon}, \upsilon=1,\ldots,10$, $\tilde{\mathbf{d}}_{\upsilon}, \upsilon=1,\ldots,30$ from EGM96 to the observation equations, the corresponding RKPOD position, velocity and acceleration differences are shown in Figs. (3b) and (3c), respectively. The dynamical restrictions improve the differences w.r.t. KPOD results. Fig. (3d) shows the differences in position, velocity and acceleration for the full dynamical restrictions. In Table 1, a summary of the RMS values for the different POD methods are presented.

Numerical results

Fig. 1: The GOCE ground track of four 30 minutes short arc for the time 2000 07 17 08h 30m 0.0s - 08h 30m 0.0s.

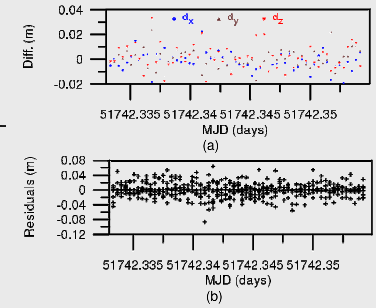

Fig. 2: a) Absolute position differences between estimated geometrically orbit and dynamical GOCE orbit, b) Carrier phase residuals.

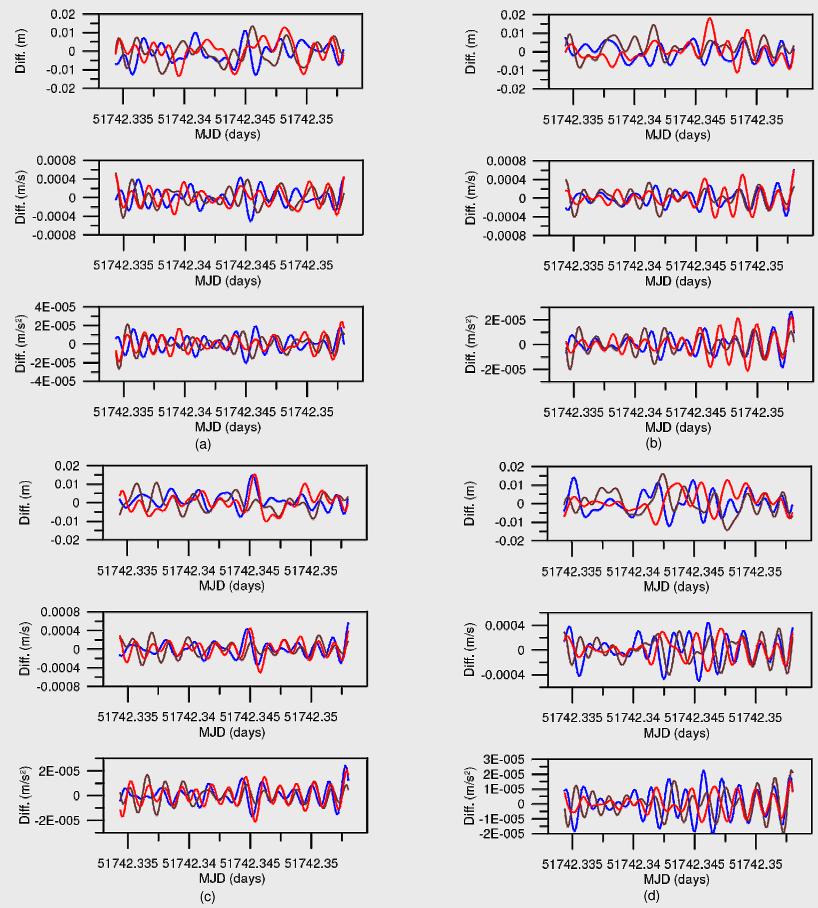

Fig. 3: **a)** Kinematical position, velocity and acceleration differences for the Euler-Bernoulli polynomial $J_{max}=4$ and the Fourier index $\upsilon=30$ **b)** position, velocity and acceleration differences for RKPOD with the dynamical constraints $\tilde{\mathbf{d}}_{\upsilon}, \upsilon=1,\ldots,10$ **c)** position, velocity and acceleration differences for RKPOD with the dynamical constraints $\tilde{\mathbf{d}}_{\upsilon}, \upsilon=1,\ldots,30$ **d)** position, velocity and acceleration differences for RKPOD with the dynamical constraints $\tilde{\mathbf{d}}_{\upsilon}, \upsilon=1,\ldots,59$ (DPOD).

Conclusions

- ✓Estimated absolute positions based on the GPOD techniques are purely geometric and there is no connection between subsequent positions. The geometrical configuration of GNSS satellites and the LEO plays a key role in the estimation of geometrical, point-wise positions. No dynamical information is used in the GPOD procedure.
- ✓Estimated kinematical precise orbits are continuous. Consequently, the LEO velocities, accelerations and other kinematical parameters can be derived directly in the orbit determination procedure. As GPOD, no dynamical information used in the KPOD.
- ✓Based on a new proposed approach, different spectra of the Earth's gravity field as dynamical information can be used to smooth the estimated kinematical precise orbit of GOCE.
- ✓A smooth transition is possible from KPOD to RKPOD and finally from RKPOD to DPOD.
- ✓The new proposed kinematical and reduced-kinematical POD procedures open a wide window to represent low-flying orbits.

Table 1: RMS values of estimated GOCE orbits w.r.t reference GOCE for GPOD, KPOD, RKPOD (1-10), RKPOD(1-30) and DPOD (full spectrum).

Case	Dyn. Info. ($\mathbf{d}_{\upsilon}$)	Pos. (m)	Vel. (m/s)	Acc. (m/s^2)
GPOD	0	0.011810	-	-
KPOD	0	0.009641	0.000313	0.000014
RKPOD	1-10	0.008536	0.000291	0.000013
RKPOD	1-30	0.007707	0.000261	0.000012
DPOD	1-59	0.010035	0.000314	0.000014

Acknowledgement

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References

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