

# Precise Orbit Determination of GOCE based on GNSS observations

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#### Abstract

The geometrical point-wise satellite positions can be derived by GNSS (Global Navigation Satellite System) analysis techniques. Based on the advent precise orbit and clock products by the IGS analysis centres and geometrical high-low Satellite to Satellite Tracking (hl-SST) observations, the point-wise Geometrical Precise Orbit Determination (GPOD) of LEOs can be introduced with only a single GNSS receiver on-board LEOs. With only geometrical hl-SST between GNSS satellites and LEO (Low Earth Orbiter), the absolute positions are estimated and defined as Geometrical POD (GPOD). In this poster, the zero differenced estimation procedure is applied to GOCE (Gravity field and steady-state Ocean Circulation Explorer) hl-SST observations. Initial POD results show that the GOCE geometrical point-wise positions can be estimated based on hl-SST with an accuracy of 2-3 cm.

### Methodology

The carrier phase GNSS-SST observations between the GNSS satellite s and the LEO satellite r at frequency i read:

$$\Phi_{r,i}^{s}(t) = \rho_{r}^{s}(t) - cdt_{r}(t) + \lambda_{i}N_{r,i}^{s} + e_{r,\Phi_{i}}^{s}(t) + \mathcal{E}_{\Phi_{i}}(t), \qquad (1)$$

 $\Phi_{r,i}^s$  : Observed carrier phase between the GNSS *s* and the LEO *r* 

- *t* : Observation epoch
- $\rho_r^s$  : The geometrical distance between *s* and *r*
- $N_{r,i}^s$  : Ambiguity parameter
- $\lambda_i$  : GNSS signal wavelength at frequency of *i*

 $e_{r,i}^s$ ,  $\mathcal{E}_{\Phi_i}$ : Summation of all systematic errors and the remaining error The linear ionosphere-free combination of the carrier phase observation at two signal bands L<sub>i</sub> and L<sub>i</sub> reads as

| $\Phi_{r,3}^{s}(t) = \mathbf{R}_{Z}(a)$                              | $\left\  \boldsymbol{\rho}_{\boldsymbol{e}} \boldsymbol{\tau}_{r}^{s} \right\  \mathbf{r}^{s}(t-\boldsymbol{\tau}_{r}^{s}) - \mathbf{r}_{r}(t) \left\  + \lambda_{3} N_{r,3}^{s} - c dt_{r}(t) + \boldsymbol{e}_{r,\Phi_{3}}^{s}(t) + \boldsymbol{\varepsilon}_{\Phi_{3}}(t) \right\ , (2)$ |
|--|---|
| $	au_r^s$ :  | GNSS signal travel time between sender and receiver   |
| $\mathbf{R}_{Z}(\boldsymbol{\omega}_{e}\boldsymbol{\tau}_{r}^{s})$ : | Sagnac effect (rotation matrix around Z axis)   |
| $\mathbf{r}^{s}(t-\tau_{r}^{s})$ :                                   | Position of the GNSS satellite <i>s</i> at the sending time   |
| (1)  |   |

 $\mathbf{r}_r(t)$  : Position of LEO receiver *r* at the receiving time

To estimate geometrical position of LEO, the ionosphere free carrier phase observation equation (2) has to be linearized as,

$$\Phi_{r,3}^{s}(t) = \Phi_{r,3,0}^{s}(t) + \frac{\partial \Phi_{r,3}^{s}(t)}{\partial \mathbf{x}}(\mathbf{x} - \mathbf{x}_{0}),$$

 $\mathbf{x}_0, \mathbf{x}$  : The initial and final value of unknown parameters

or

$$\Delta \Phi_{r,3}^{s}(t) = \mathbf{a}_{r}^{s}(t) \Delta \mathbf{x}(t)$$

The Gauss-Markov model for all observed carrier phase observations with corresponding weight matrix in a batch processing for all desired epochs reads as,

Methodology ...

$$\Delta \mathbf{l} = \mathbf{A} \Delta \mathbf{x}, \quad \mathbf{W}_{1},$$

by a least square adjustment, the unknowns read as

$$\Delta \hat{\mathbf{x}} = \left(\mathbf{A}^{\mathrm{T}} \mathbf{W}_{1} \mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{W}_{1} \mathbf{I}.$$

(4)

(5)

(6)

(7)

The estimated unknowns are the corrections to the LEO absolute positions, the LEO clock offsets and the GNSS ambiguity terms. Because of observation non-linearity, the convergence is achieved after a few iteration; subsequently the unknowns read as,

$$\hat{\mathbf{x}}_i = \hat{\mathbf{x}}_{i-1} + \Delta \hat{\mathbf{x}}_i, \quad \mathbf{C}_{\hat{\mathbf{x}}_i} = \hat{\sigma}_0^2 \left( \mathbf{A}^{\mathrm{T}} \mathbf{W}_{\mathbf{I}} \mathbf{A} \right)^{-1},$$

the estimated unknowns are the LEO geometrical absolute positions, the LEO clock offsets and the float GNSS ambiguity terms at all desired epochs.

## GOCE Data processing strategy and Input data

The geometrical point-wise orbit of GOCE has been estimated based on an inhouse developed software package (*Shabanloui*, 2008). To estimate the geometrical POD of GOCE, IGS post processed GNSS orbit and GNSS clock offset and hl-SST data with an interval of 15 minutes, 30 sec. and sampling rate of 30 s. are used, respectively. To minimize the environment effect e.g. multipath effect, a cut-off angle of 10° is used in the GNSS data processing strategy. In this poster, twenty days of GOCE observations (see Fig. (1)) are used to estimate GPOD. Fig. (2) shows the differences between estimated geometrical precise orbit determination and reduced-dynamical orbit of GOCE (PSO\_PRD sub-products).

#### Conclusions

Estimated absolute positions based on GNSS techniques are purely geometric. No dynamical information is used in the procedure.

✓ GPOD results of GOCE demonstrate that an accuracy of 2-3 cm. is achievable.

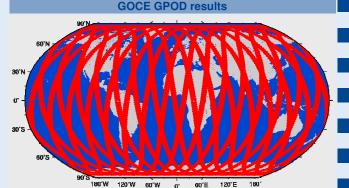
 $\checkmark$  Zero diff.method provides an efficient possibility to estimate GOCE GPOD.

#### Acknowledgement

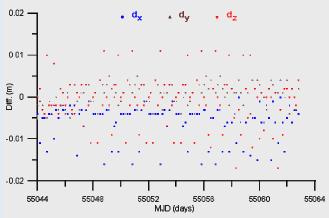
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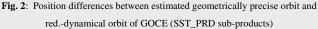
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**Fig. 1**: The GOCE ground tracks of twenty days (2009-08-01 00h 00m 0.0s-2009 08 19 23h 59m 30.0s)





## References

[1] Ilk, K.H. (1977), 'Berechnung von Referenzbahnen durch Loesung selbstadjungierter Randwertaufgaben', Ph.D. Dissertation, DGK, Reihe C, Heft 228, Munich, Germany.

[2] Shabanloui, A. (2008), 'A New Approach for a Kinematic-Dynamic Determination of Low Satellite Orbits Based on GNSS Observations', Ph.D. Dissertation, Institute of Geodesy and Geoinformation (IGG), Bonn University of Bonn, Germany.

