Study of an optical gradiometer for future satellite gravitational missions

Karim Douch[1], Jürgen Müller[1], Phillip Brieden[1], Akbar Shabanloui[1]

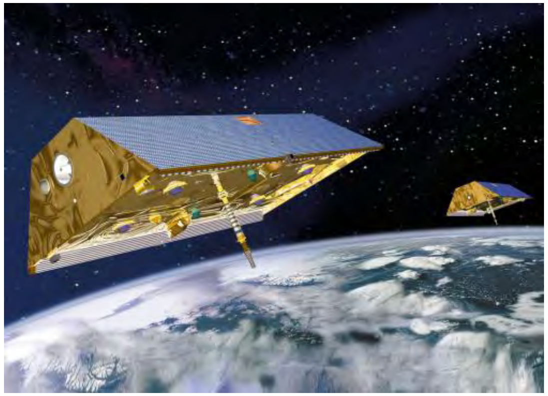
[1]Institut für Erdmessung, LUH, Hannover

Outlines

- **Monitoring the time-variable gravitational field**
- **Principle of an optical gradiometer**
- **Specifications for a drift mode gradiometer**
- **Specifications for a suspension mode gradiometer**
- **Conclusion**

Monitoring the time-variable gravitational field

- GRACE (2002-now)
  - Based on satellite-to-satellite ranging
  - Altitude: 500km



GRACE

- Requirements for future gravity missions
  - Different studies: esa NGGM, esa NGA, **e.motion**, e.motion²…
  - Main objectives:
    - Time-variable gravity @ **200km spatial resolution or better**
    - Temporal resolution of **1month or better**
    - **Increase amplitude sensitivity by 10**
    - **Global coverage**

Objective of the study:
investigate the opportunity to use **gravitational gradients as an observable of the time-variable gravitational field**

Monitoring the time-variable gravitational field

- Gravitational gradients

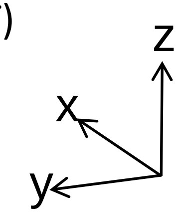
Gravitational potential

V

$$\frac{\partial^2}{\partial i \partial j}$$

Gravitational Gradient Tensor (GGT)

$$\mathbf{V} = \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}$$

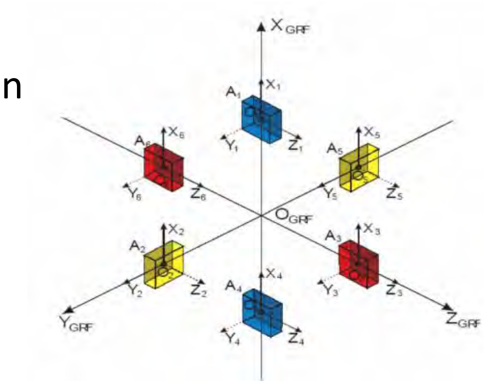


Symmetric and trace-free

Unit: 1E (Eötvös) = 10^{-9} s^{-2}

- GOCE (2009-2013): determination of the static gravitational field
  - 3 pairs of electrostatic accelerometers
  - Altitude ≈ 250km
  - Drag compensation

GOCE (ESA)



Scheme of the gradiometer

Acceleration Gradient Tensor $= \boldsymbol{\Gamma} = \mathbf{V} - \frac{d\boldsymbol{\Omega}}{dt} - \boldsymbol{\Omega}.\boldsymbol{\Omega}$

where: $\boldsymbol{\Omega} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$

Monitoring the time-variable gravitational field

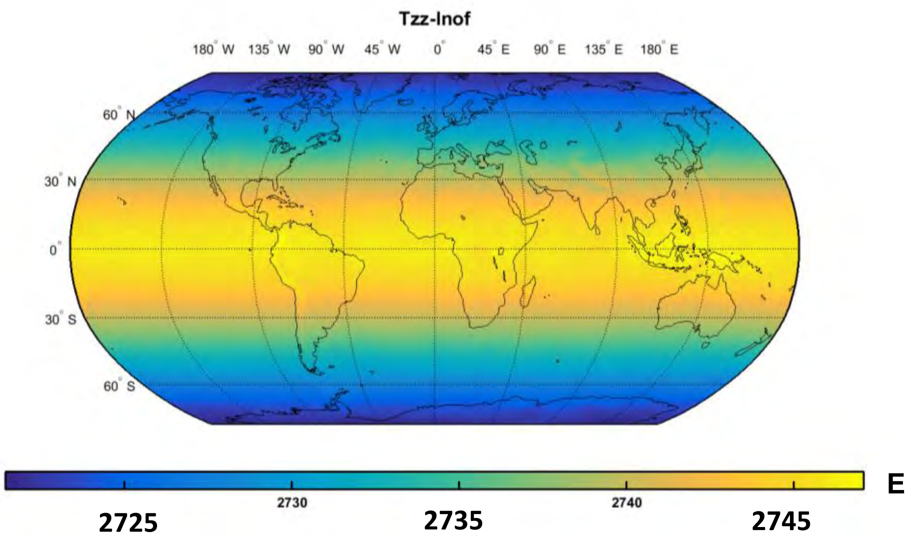


Fig 1. V_{zz} at altitude of 255km (model:Eigen 6c4)

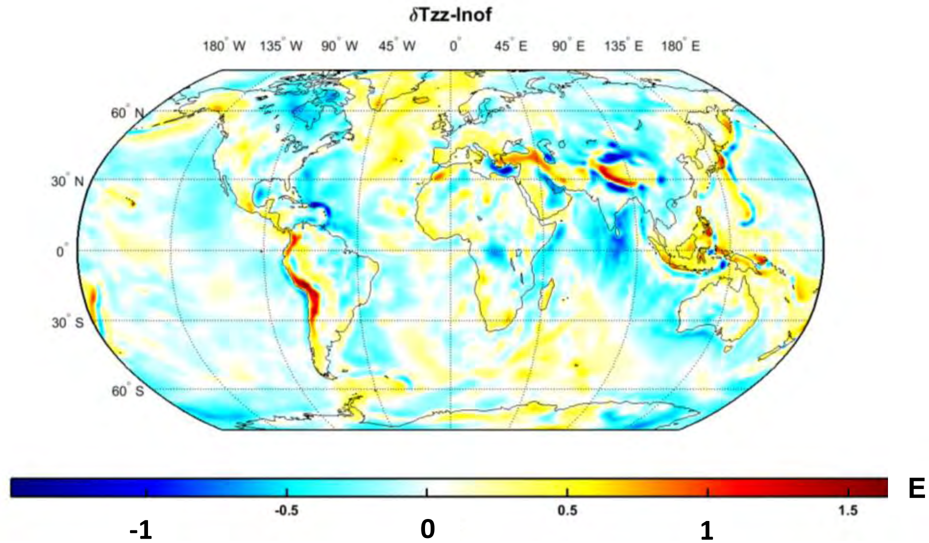


Fig 2. V_{zz} anomaly at altitude of 255km (model:Eigen 6c4)

- <u>GOCE objectives</u>:
  - Geoid:
    - 1cm error @ 100km resolution
  - Gravity anomaly
    - 1-2mGal @ 100km resolution

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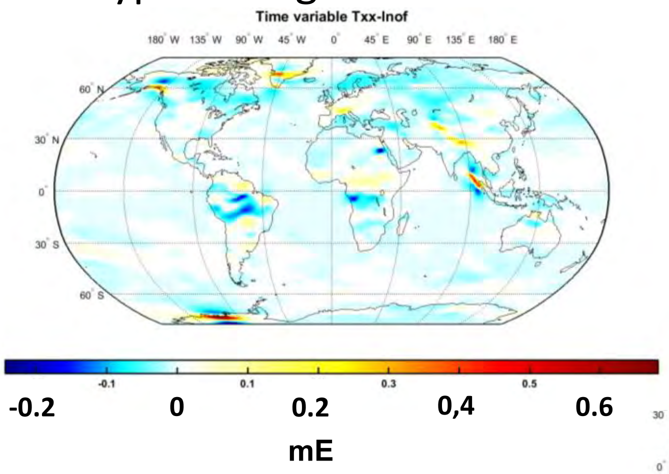
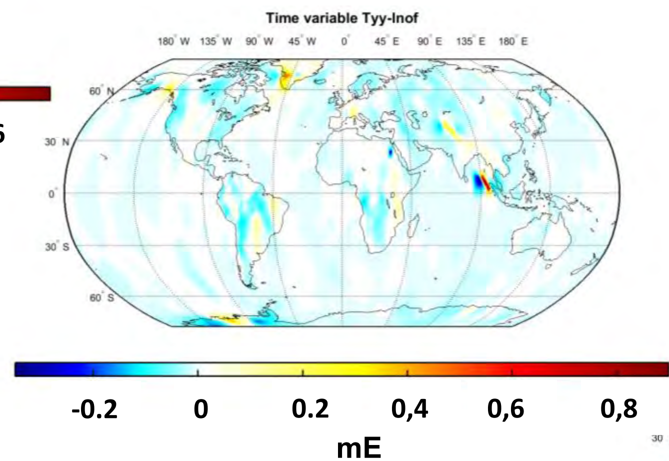
Monitoring the time-variable gravitational field

Typical magnitude of a time-variable gravitational gradients at GOCE altitude

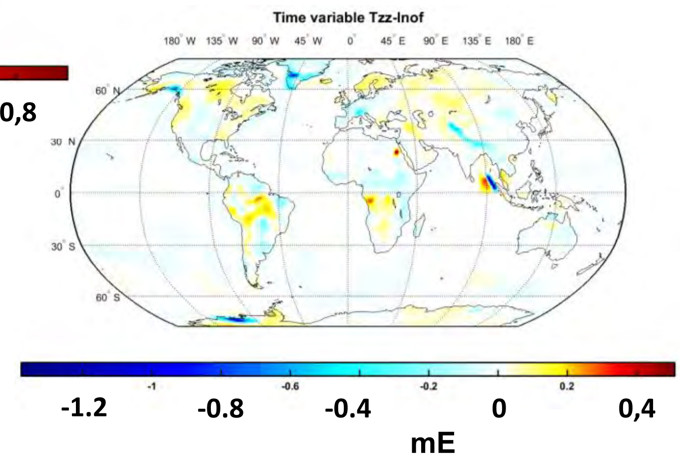


Order of magnitude: < 1mE



Altitude: 255km

Model : The updated ESA Earth system model*
(date:16th of February 2006)



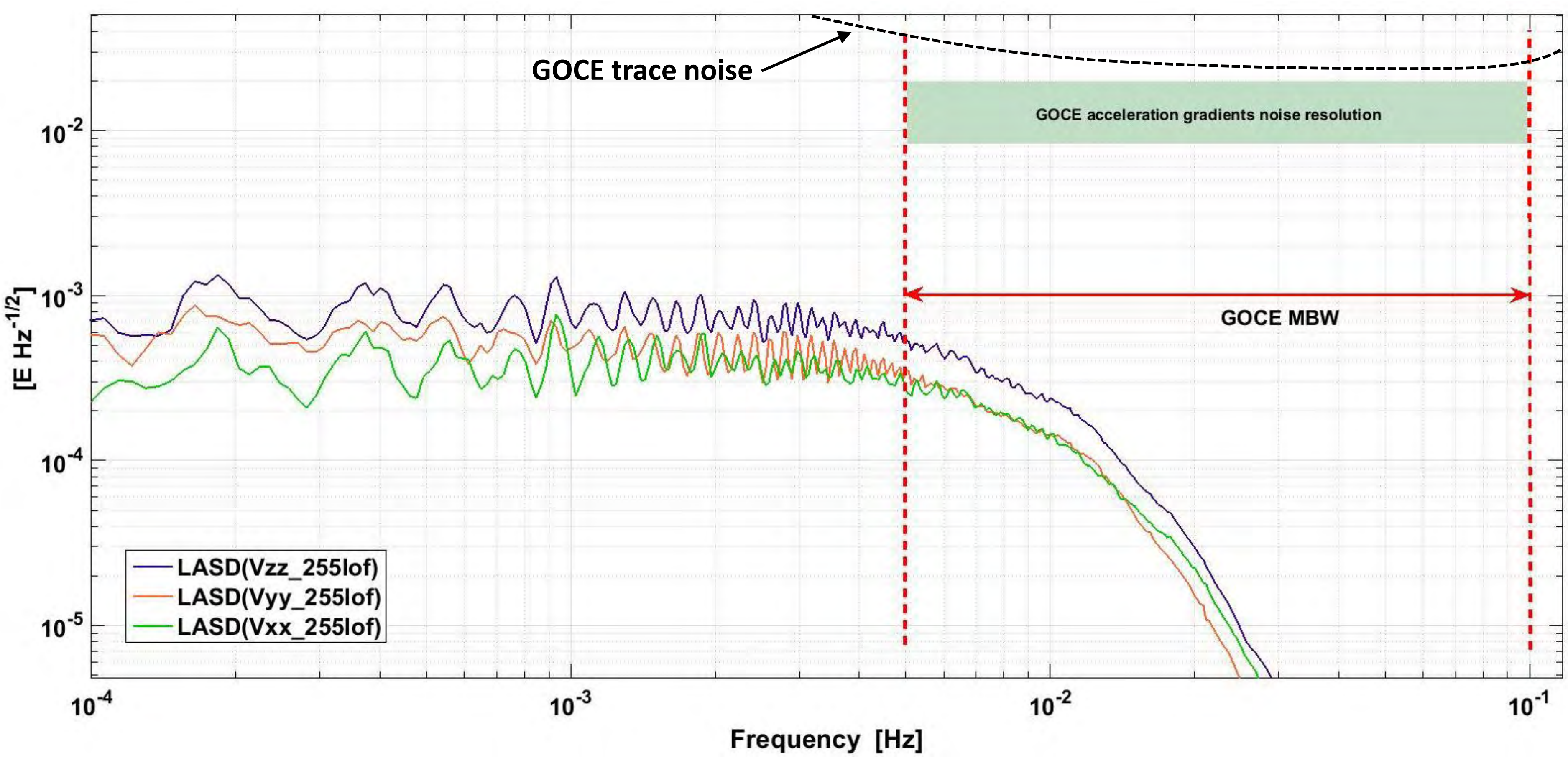
→ 3 quantities to determine with a higher precision:

Γ Ω measurement frame attitude

*Dobslaw et al. 2015

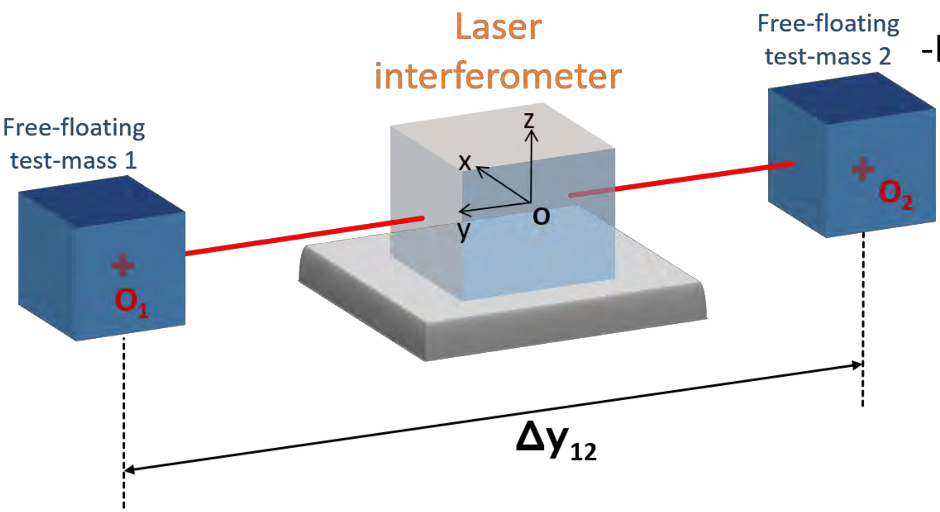
Monitoring the time-variable gravitational field

Comparison of time-variable gravitational gradient signals with GOCE performances



Principle of an optical gradiometer

- Scheme, describtion and explanation (+cf eLISA)

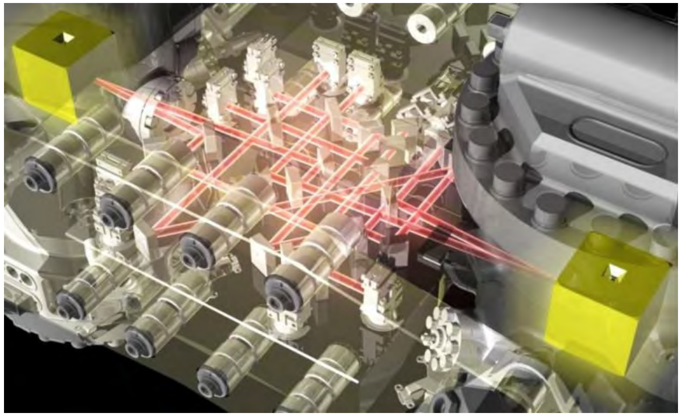


-Readout: **distance variation along one axis**

-LISA Pathfinder technology

- Heterodyne Mach-Zehnder Laser interferometer
- **pm/√Hz precision**

- TMs attitude measured along 2-axes
- **nrad/√Hz**



For 2 inertial test-masses (no non-gravitational forces) we have:

$$\Delta\ddot{y}_{12} = \ddot{y}_1 - \ddot{y}_2 = \left[\left(\mathbf{V} - \mathbf{\Omega\Omega} - \dot{\mathbf{\Omega}}\right).\,\boldsymbol{O_2O_1} - 2\mathbf{\Omega}.\frac{d\boldsymbol{O_2O_1}}{dt}\right].\,\boldsymbol{u}_y$$

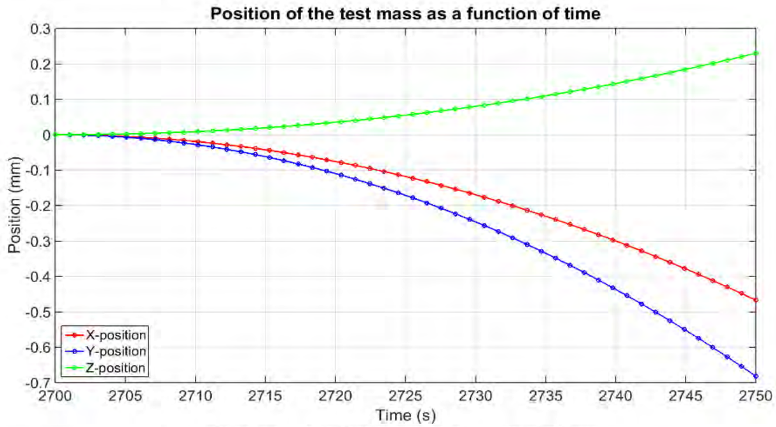
$\Delta\ddot{y}_{12}$: differential acceleration

V : GGT

Ω : Matrix of angular rates of gradiometer frame w.r.t an inertial frame

$\boldsymbol{u}_y$: unit vector in the y direction

Principle of an optical gradiometer



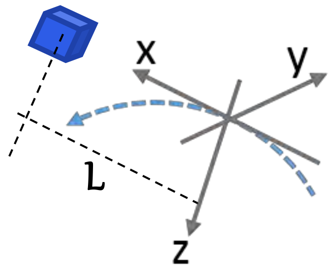
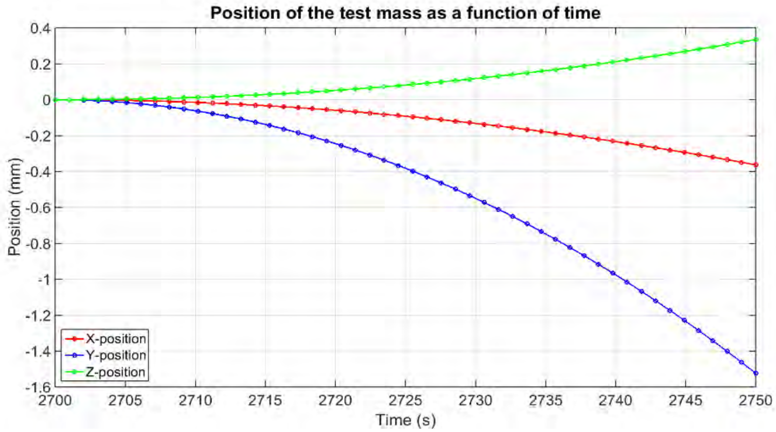
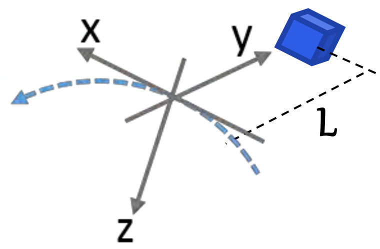


Fig 1. Example of Test-mass motion w.r.t the spacecraft, simulated over 50s for GOCE conditions

-L=50cm

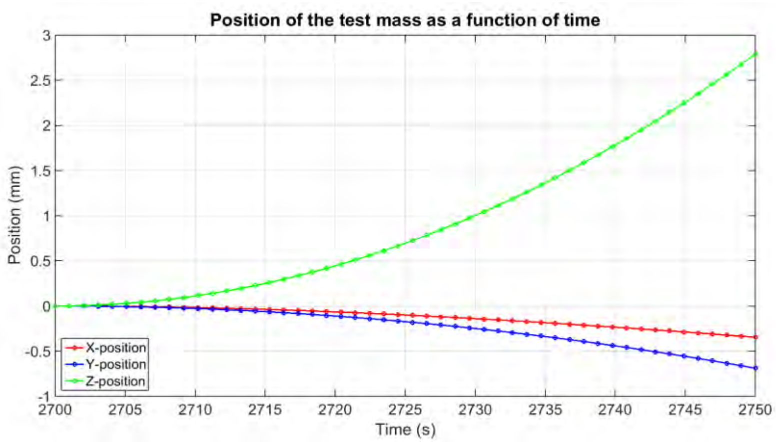
-initial velocity = 0m/s

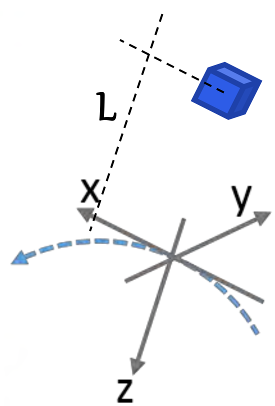




Equation of motion for the test-mass centre of mass:

$$\ddot{\overrightarrow{OO_1}} = (\boldsymbol{V} - \boldsymbol{\Omega}\boldsymbol{\Omega} - \dot{\boldsymbol{\Omega}}).\overrightarrow{OO_1} - 2\boldsymbol{\Omega}.\dot{\overrightarrow{OO_1}} - \boldsymbol{a}_{ng}$$

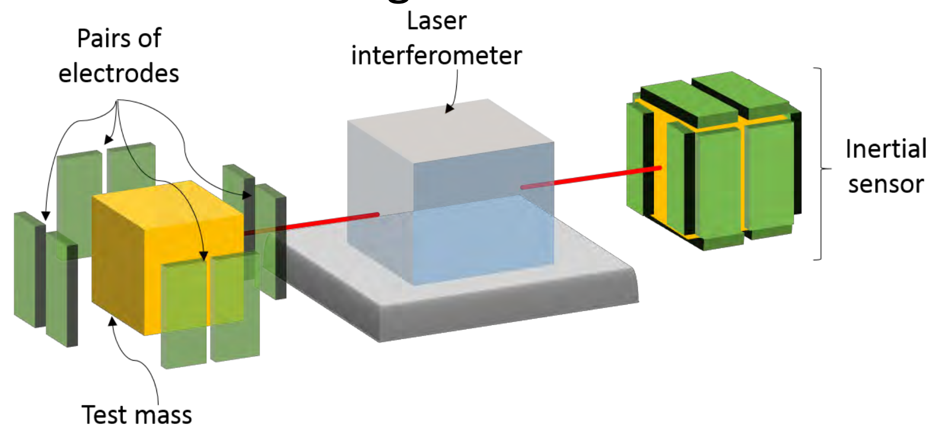




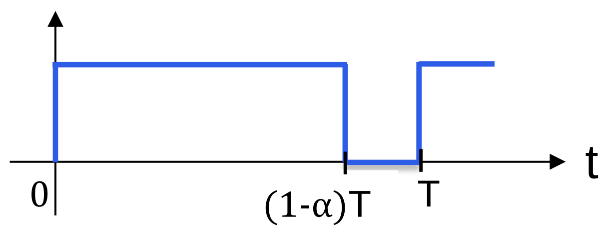
We have to deal with a ≈ 1mm/min drift of the test-masses

Principle of an optical gradiometer

- Test-masses must be in an electrode housing in order to control all 6 degrees of freedom



- 2 different modes:
  - **Drift mode**
    - 0 to (1-α)T: TMs are purely free-falling ⇨ **measurement (absolute value)**
    - (1-α)T to T: suspension forces are applied to **reset the TMs to its initial conditions**



    - <u>Advantages</u>: no electrostatic actuation noise during the measurement phase
    - <u>Disadvantages</u>: dead-time, high- accuracy required on the initial relative position and velocity of the TMs, rotation of TMs must be controlled anyway

Principle of an optical gradiometer

- **Suspension mode**
  - The TMs are controlled in position through a control loop (≈ GOCE)
  - The actuation only filters out the TMs accelerations for frequencies below the MBW on the sensitive axis

    → TMs are **free-falling only in the MBW**
  - MBW = [0.5-7]mHz

  - Advantages: the 6 degrees of freedom of the TMs are controlled, no dead-time
  - Disadvantages: increase of cross-talk electrostatic actuations

- Study of operability of both mode in a worst case: GOCE satellite dynamics

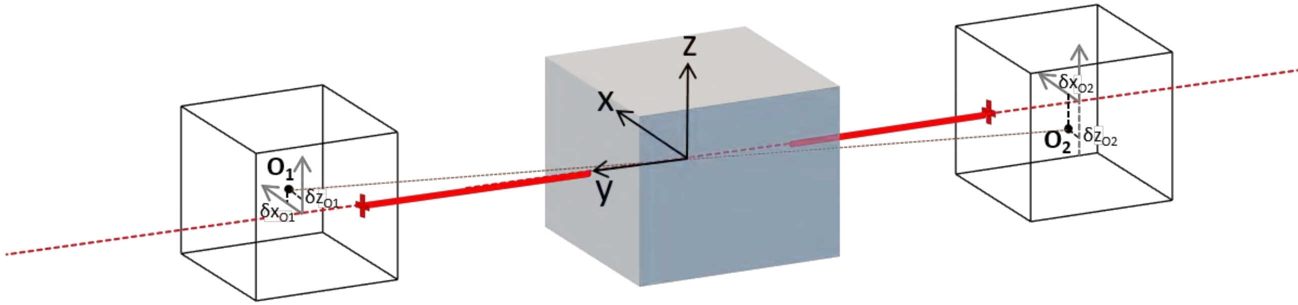
→ **Conditions or requirements to operate the gradiometer**

Specifications for a drift mode gradiometer

- Solution of the equation of motion (2nd order approx.):

$$\Delta y_{12} = \frac{t^2}{2}\left(\Gamma_{yy}.y_0 + \Gamma_{yx}.\delta x_0 + \Gamma_{yz}.\delta z_0 - 2\delta\dot{x}_0.\omega_z + 2\delta\dot{z}_0.\omega_y\right) + \delta\dot{y}_0.\,t$$

For initial conditions: $\Delta y_{012} = (\delta x_0, y_0, \delta z_0)^t$ and $\Delta\dot{y}_{012} = (\delta\dot{x}_0, \delta\dot{y}_0, \delta\dot{z}_0)^t$



3-axes optical interferometer: determination of $\mathbf{V}_{xx}$, $\mathbf{V}_{yy}$ and $\mathbf{V}_{zz}$ only

- Unknown contribution terms (yellow) << precision on $\Gamma_{yy}.y_0$

→ Specifications on TMs position (x_0=0.5m):

	Misalignement (nm)		
	δx_0	δy_0	δz_0
X-axis	20	1.6	9.7
Y-axis	3.5	7.2	9
Z-axis	9.3	8	2.5

Specifications for a suspension mode gradiometer

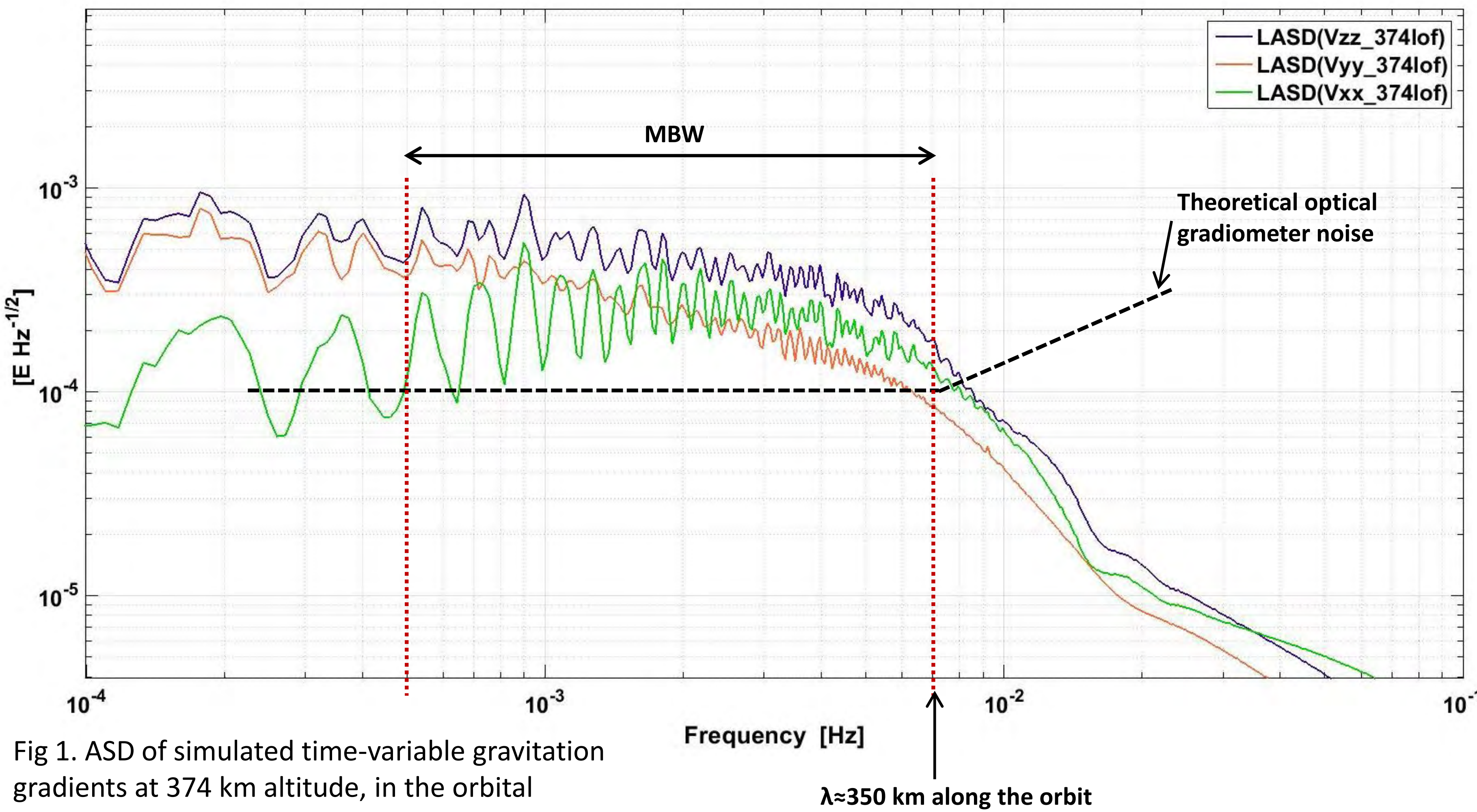


Fig 1. ASD of simulated time-variable gravitation gradients at 374 km altitude, in the orbital frame

Specifications for a suspension mode gradiometer

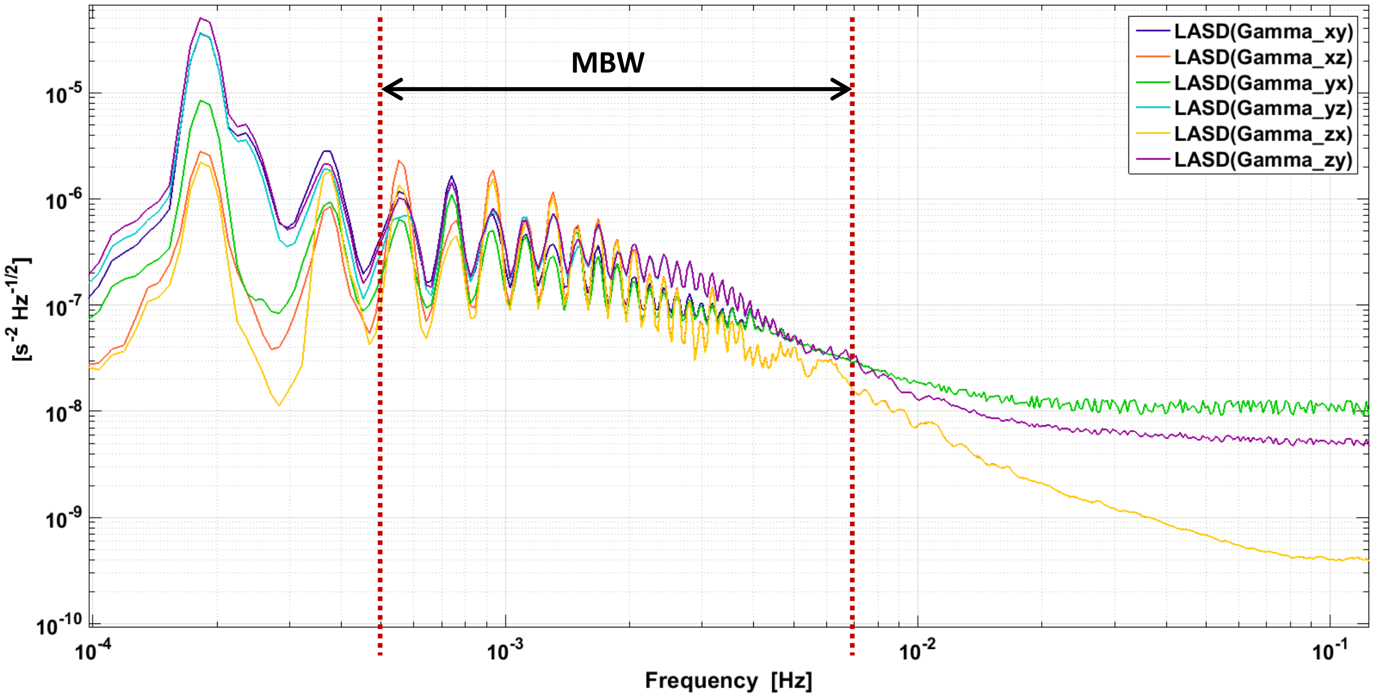


Fig 1. ASD of off-diagonal elements of the acceleration gradient tensor reconstructed from GOCE data

	Misalignement (nm)		
	δx_0	δy_0	δz_0
X-axis	-	50	50
Y-axis	50	-	50
Z-axis	50	50	-

Angular velocity determination

$$\begin{cases} V_{xx} = \Gamma_{xx} - \omega_z^2 - \omega_y^2 \\ V_{yy} = \Gamma_{yy} - \omega_z^2 - \omega_x^2 \\ V_{zz} = \Gamma_{zz} - \omega_x^2 - \omega_y^2 \end{cases}$$

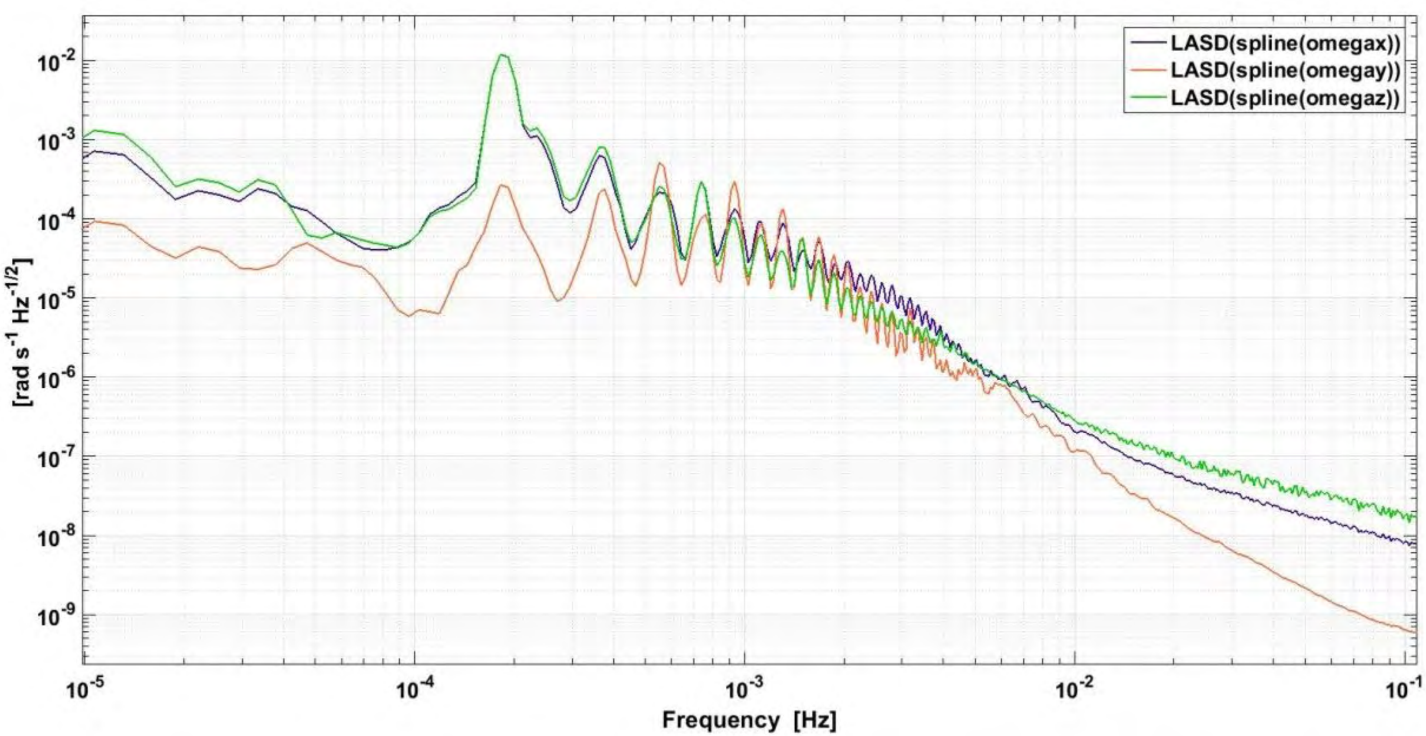


Fig 1. ASD of angular velocity of GOCE w.r.t an inertial frame

Angular velocity must be determined with an accuracy ≈ 200 times better than for GOCE!

Conclusion and outlooks

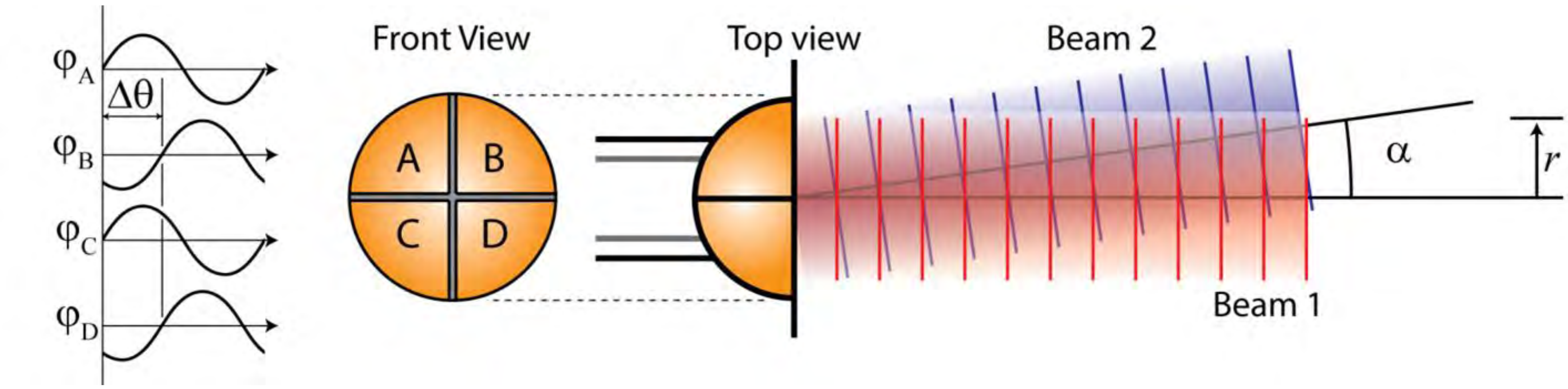
- Envisioned concept: GOCE gradiometer enhanced by laser interferometry in the MBW [0.5-7]mHz
- **Suspension mode** is favored
- So far, spatial resolution is estimated at 350km
- Determination of Γ_{xx}, Γ_{yy} and Γ_{zz} improved by a factor of 100 compared to GOCE
  - Possibility to detect time-variable gravitational signals

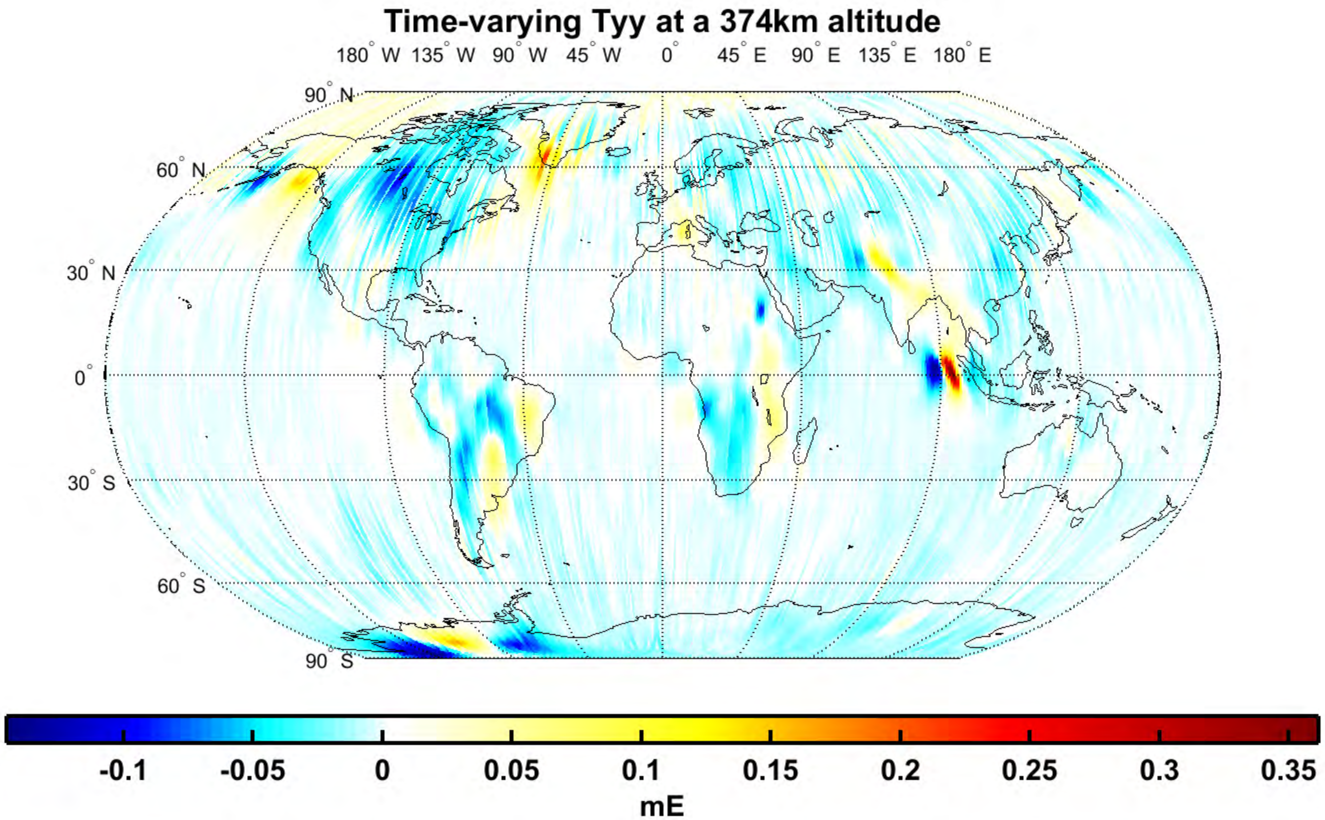
- Questions and issues to address:
  - How to determine the angular velocities with a commensurate precision?
    - Instruments (phasemter, gyrometers)
    - Optimal combination of sensors data
  - Technical aspect: electrostatic accelerometer performances with a lower control on one axis...
  - Requirements on spacecraft attitude determination and feasibility

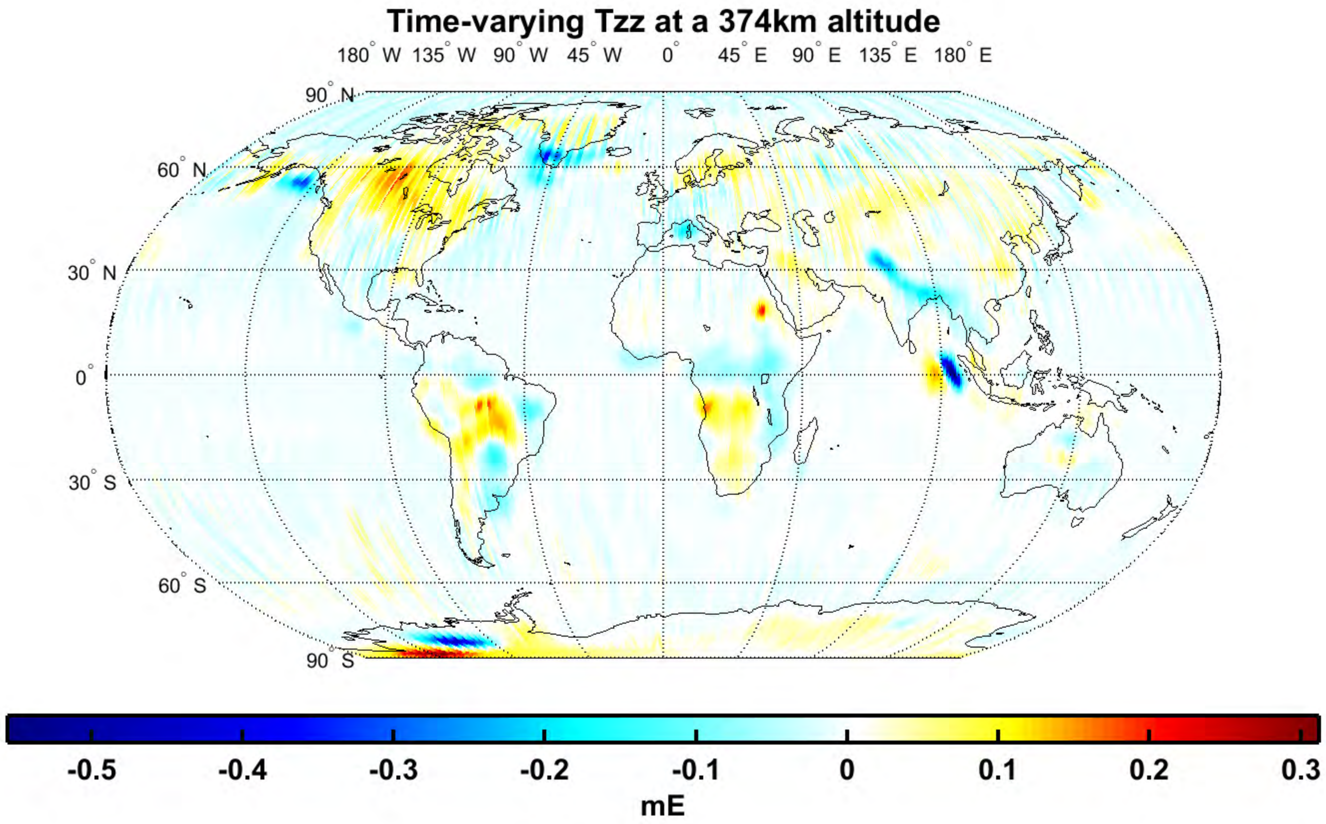
Thank you for your attention!

Sonderforschungsbereich (SFB) 1128 „Relativistische Geodäsie und Gravimetrie mit Quantensensoren (*geo*-Q)"

- Principle of the phasemeter




Time-varying Tyy at a 374km altitude
180° W 135° W 90° W 45° W 0° 45° E 90° E 135° E 180° E
90° N
60° N
30° N
0°
30° S
60° S
90° S
-0.1 -0.05 0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
mE



Time-varying Tzz at a 374km altitude
180° W 135° W 90° W 45° W 0° 45° E 90° E 135° E 180° E
90° N
60° N
30° N
0°
30° S
60° S
90° S
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3
mE