# IfE monthly gravity field solutions using the variational equations

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# Contents:

## 1) Introduction

- GRACE (2002 - 2017) and GRACE FO (from 2018) for temporal mass variations
- About 12 organizations and academic centers are providing monthly solutions.

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- About 12 organizations and academic centers are providing monthly solutions.

  - ❑ CSR, GFZ, JPL, CNES
  - ❑ Graz, Bonn, Bern, Delft and EGSIEM in Europe
  - ❑ Tongji, Huazhong and Wuhan universities in China

## 1) Introduction

- GRACE (2002 - 2017) and GRACE FO (from 2018) for temporal mass variations
- About 12 organizations and academic centers are providing monthly solutions.
- The gravity field modeling from GRACE at IfE started in mid 2016.

  - ❑ IfE is active since many years in sensor analysis, including GRACE (and in the future GRACE FO) sensors.
  - ❑ Monthly solutions from integrated GRACE sensors can explicitly address the sensors' performance and sensors' interaction.
  - ❑ (almost) All the results of sensor analysis should be interpreted (or validated) in the context of the mission end-product.

- Our primary goal is to evaluate the results of our sensor analysis.
- and to contribute to combined solutions such as EGSIEM.

## 2) Gravity field from KBR data: the variational equations

- The variational equations (for range-rate observables)

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- The variational equations (for range-rate observables)

$$\dot{\rho}_o = \dot{\rho}_{ref} + \frac{\partial \dot{\rho}_{ref}}{\partial u} \delta u \quad \Rightarrow \quad \dot{\rho}_o - \dot{\rho}_{ref} = \frac{\partial \dot{\rho}_{ref}}{\partial u} \delta u$$

$$\Delta \dot{\rho} = \mathbf{A} \delta u$$

- ✓ Least-square adjustment
- ✓ Iteration
- ✓ Quality control

## 2) Gravity field from KBR data: the variational equations

- The variational equations (for range-rate observables)

$$\dot{\rho}_o = \dot{\rho}_{ref} + \frac{\partial \dot{\rho}_{ref}}{\partial u} \delta u \quad \Rightarrow \quad \Delta\dot{\rho} = \mathbf{A}\delta u$$

- The reference range rates must be modeled (based on numerical integration) ***as close as possible*** to the real observed range rates

  - **Implementation of force models.**

- The elements of $\mathbf{A}$ have to be numerically integrated.

  - **An efficient, flexible and vectorized numerical integrator.**

## 3) Processing details

- **Implementation of force models.**
- **An efficient, flexible and vectorized numerical integrator.**
- **Parameterization**

## 3) Processing details

- **Implementation of force models.**

- **Mean Earth's gravity,** GIF48, d/o 200
- **Direct tides of the Moon and the Sun**, JPL ephemeris DE405
- **Solid Earth tides** (IERS 2010)
- **Permanent tide correction** (IERS 2010)
- **Ocean tides** (EOT11a , d/o 80)
- **Ocean pole tides** (Desai model, d/o 60)
- **Solid Earth pole tides** (IERS 2010)
- **Relativistic effects** (IERS 2010, Schwarzschild, de Sitter and Lense-Thirring)
- **Non-tidal atm. and ocn. variations** (AOD1B products RL05, d/o 100)
- **Non-gravitational forces** (GRACE L1B accelerometer data and SCA1B quaternions)

# 3) Processing details

➢ **Implementation of force models.**

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Details of implementation and validation: **Today in poster session**

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**Assessment of force models in the context of gravity field recovery at IfE**

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## 3) Processing details

✓ Implementation of force models.

➢ An efficient, flexible and vectorized numerical integrator.

- We developed a modified Gauss-Jackson integration technique which is suitable for simultaneous integration of the state, state transition and parameter sensitivity matrices.

$$\begin{cases} \mathbf{r}_{i+1} = \mathbf{r}_i + h\dot{\mathbf{r}}_i + h^2\mathbf{pBa}_i \\ \dot{\mathbf{r}}_{i+1} = \dot{\mathbf{r}}_i + h\mathbf{qBa}_i \end{cases}, \quad \begin{cases} \mathbf{p} = [p_1 \quad p_2 \quad \cdots \quad p_m]_{1\times m} \\ \mathbf{q} = [q_1 \quad q_2 \quad \cdots \quad q_m]_{1\times m} \end{cases}$$

| j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p_j$ | 0 | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{8}$ | $\frac{19}{180}$ | $\frac{3}{32}$ | $\frac{863}{10080}$ | $\frac{275}{3456}$ | $\frac{33953}{453600}$ | $\frac{8183}{115200}$ | $\frac{3250433}{47900160}$ | $\frac{4671}{71680}$ |
| $q_j$ | 0 | 1 | $\frac{1}{2}$ | $\frac{5}{12}$ | $\frac{3}{8}$ | $\frac{251}{720}$ | $\frac{665}{2016}$ | $\frac{19087}{60480}$ | $\frac{110397}{362880}$ | $\frac{1070017}{3628800}$ | $\frac{137461698}{479001600}$ | $\frac{26842253}{95800320}$ |

## 3) Processing details

✓ Implementation of force models.

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- We developed a modified Gauss-Jackson integration technique which is suitable for simultaneous integration of the state, state transition and parameter sensitivity matrices.

- Details of this integration technique: **Today in poster session**.

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**A modified Gauss-Jackson method for the numerical integration of the variational equations.**

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## 3) Processing details

- ✓ **Implementation of force models.**
- ✓ **An efficient, flexible and vectorized numerical integrator.**
- ➢ **Parameterization**

**Step 1: Orbit pre-adjustment, independent for both satellites**

- Arc-length: 6 hours
- Initial state vector of both satellites
- Accelerometer 3-D bias for both satellites

**Step 2: Orbit and gravity field combined adjustment**

**Global dynamic parameters**:

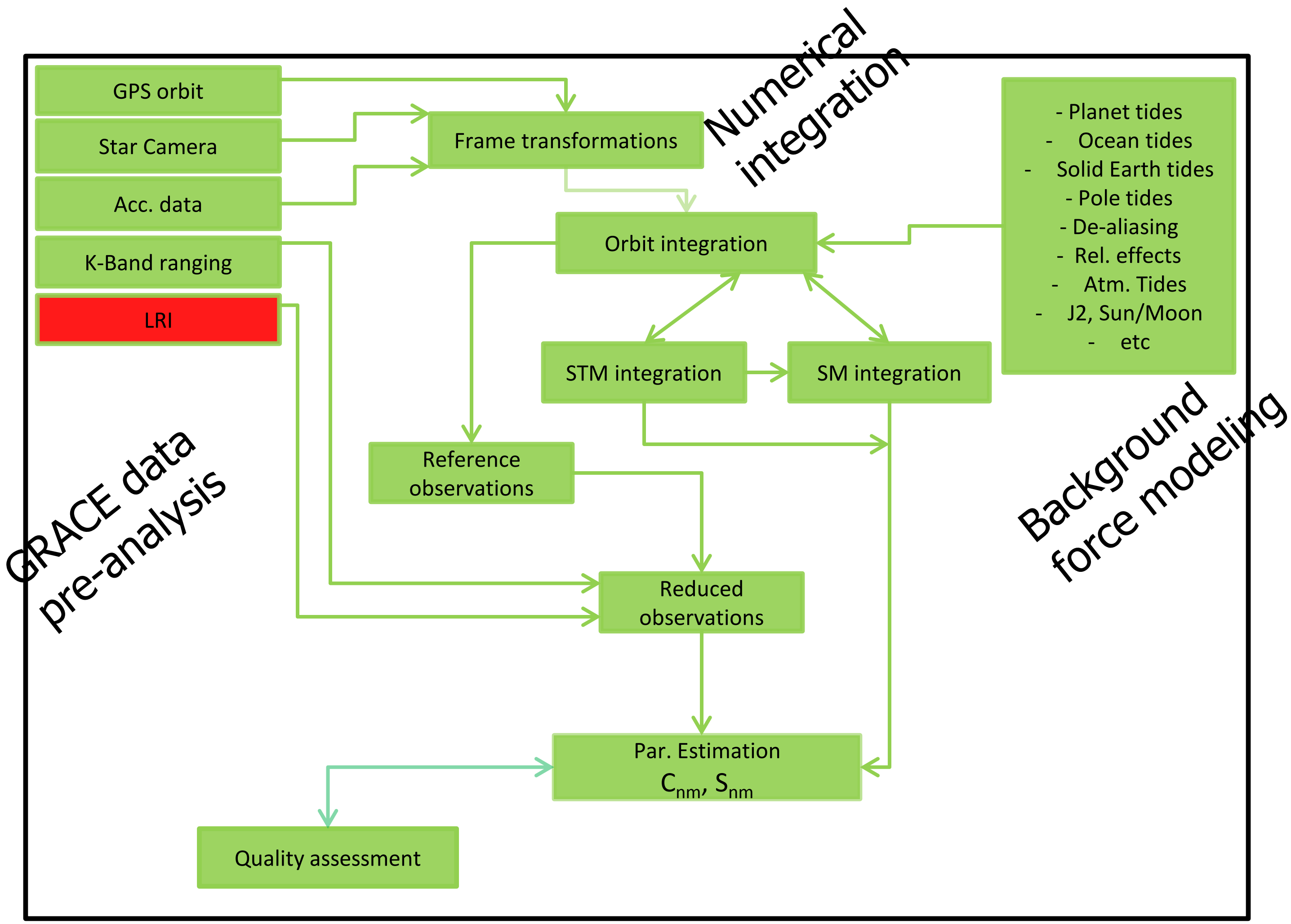
- Spherical harmonic coefficients up to degree and order 80 (total 6557 coef.)

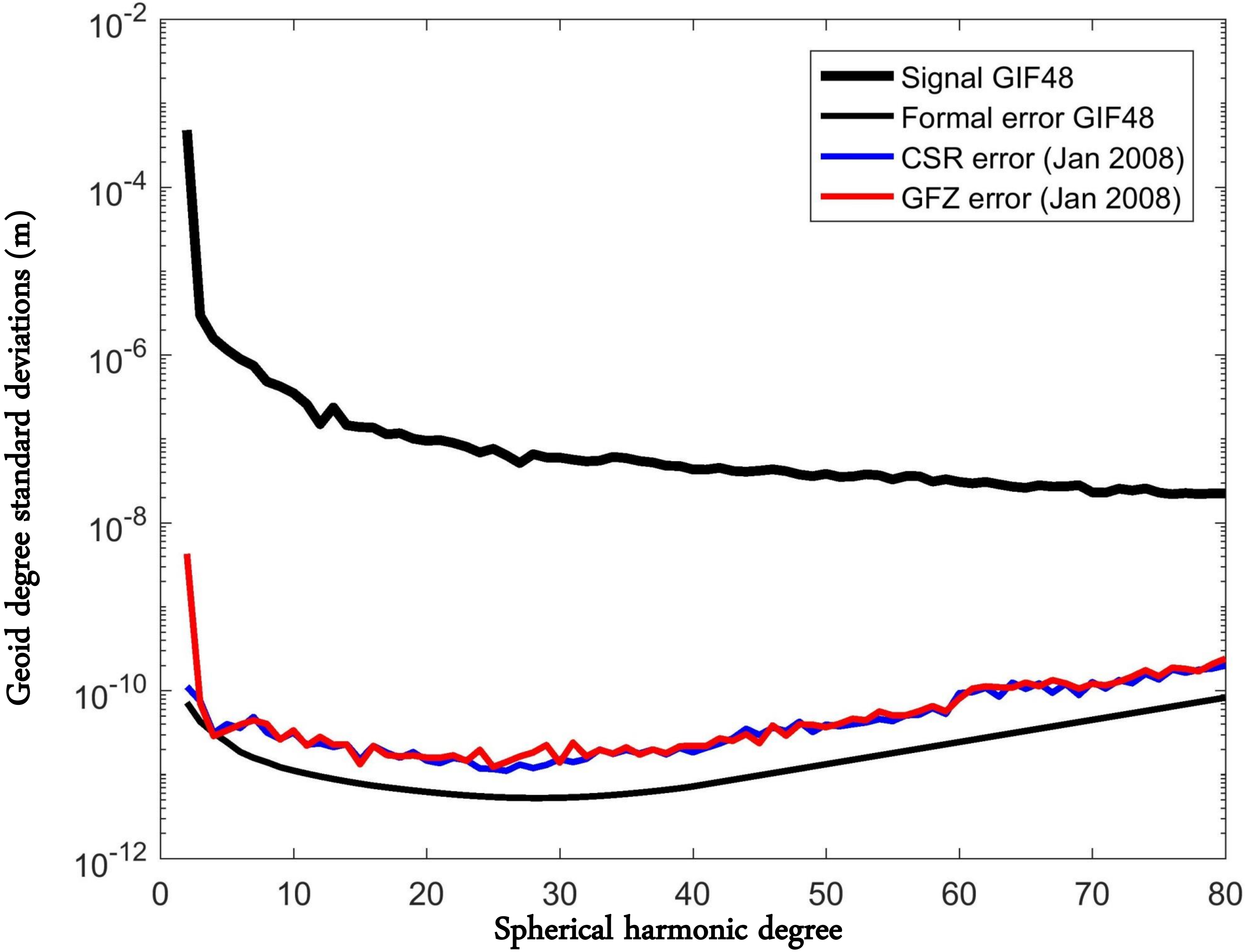
**Local arc-dependent parameters**:

- Arc-length: 6 hours
- Initial state vector of both satellites
- Accelerometer 3-D bias for both satellites
- KBR empirical parameters (linear and periodic bias)

**No constraint / regularization is applied.**

# 3) Processing details





Signal GIF48
Formal error GIF48
CSR error (Jan 2008)
GFZ error (Jan 2008)
$10^{-2}$
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^{-12}$
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20
30
40
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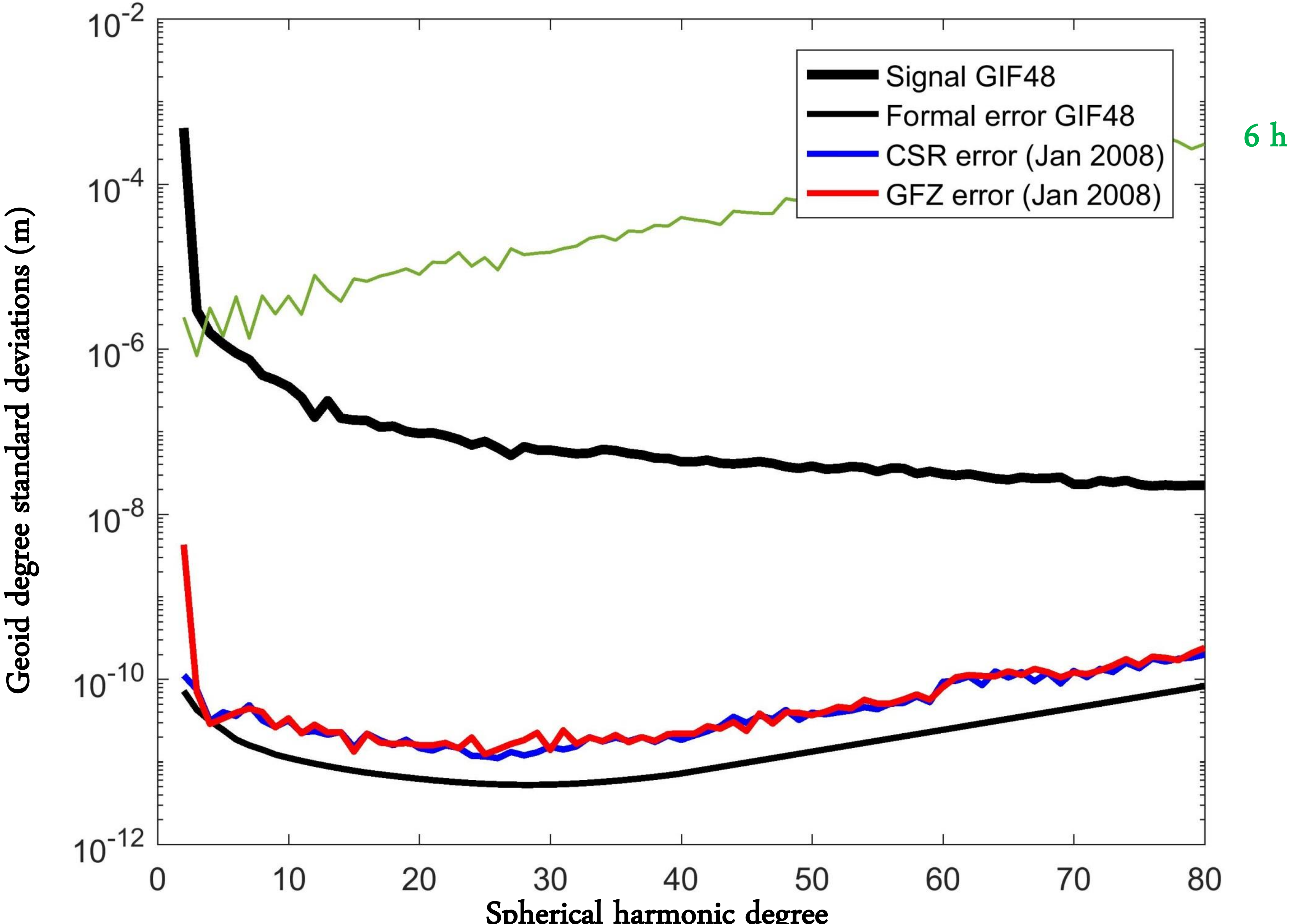
60
70
80
Geoid degree standard deviations (m)
Spherical harmonic degree



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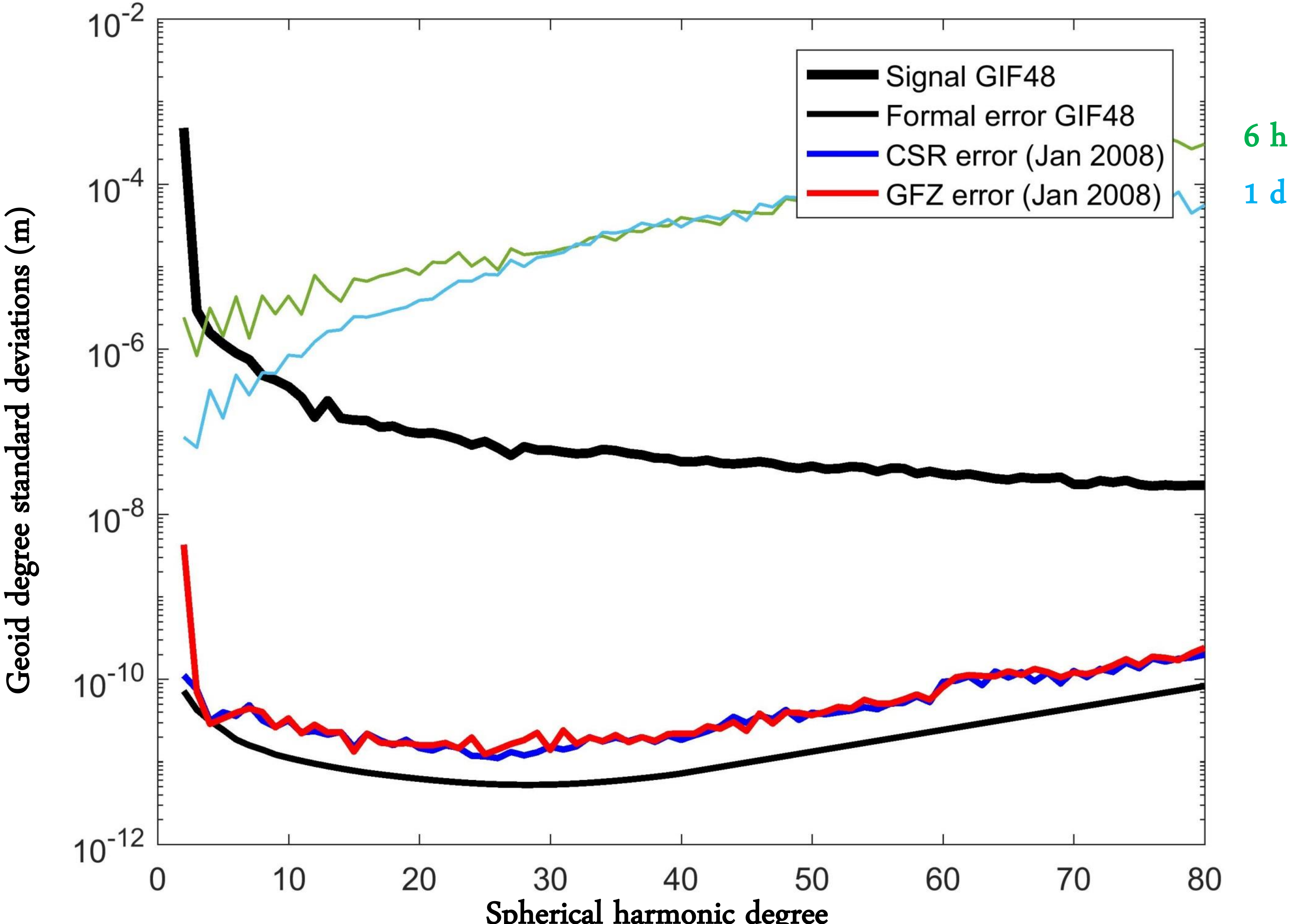

Signal GIF48
Formal error GIF48
CSR error (Jan 2008)
GFZ error (Jan 2008)
6 h
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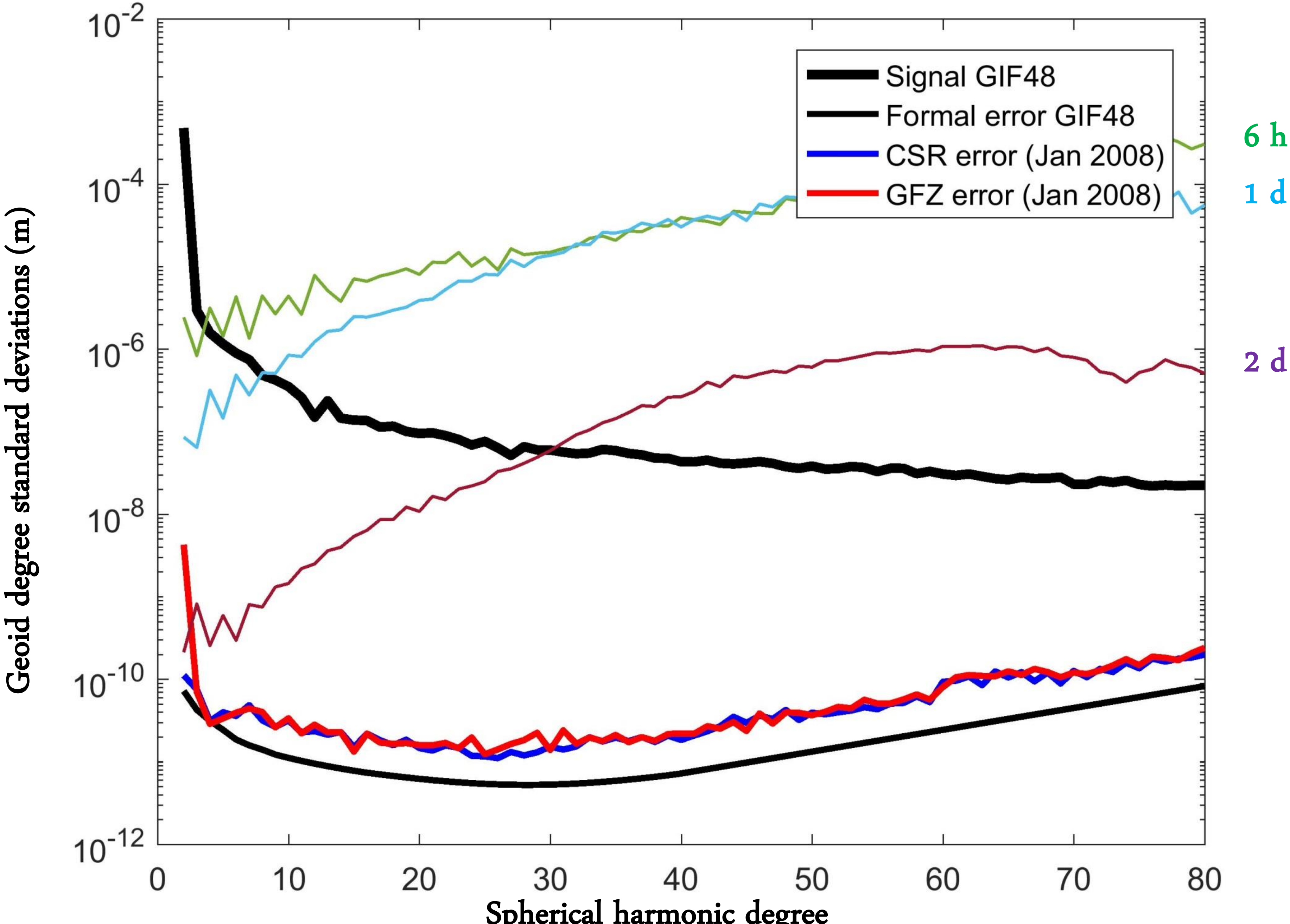
Geoid degree standard deviations (m)
Spherical harmonic degree



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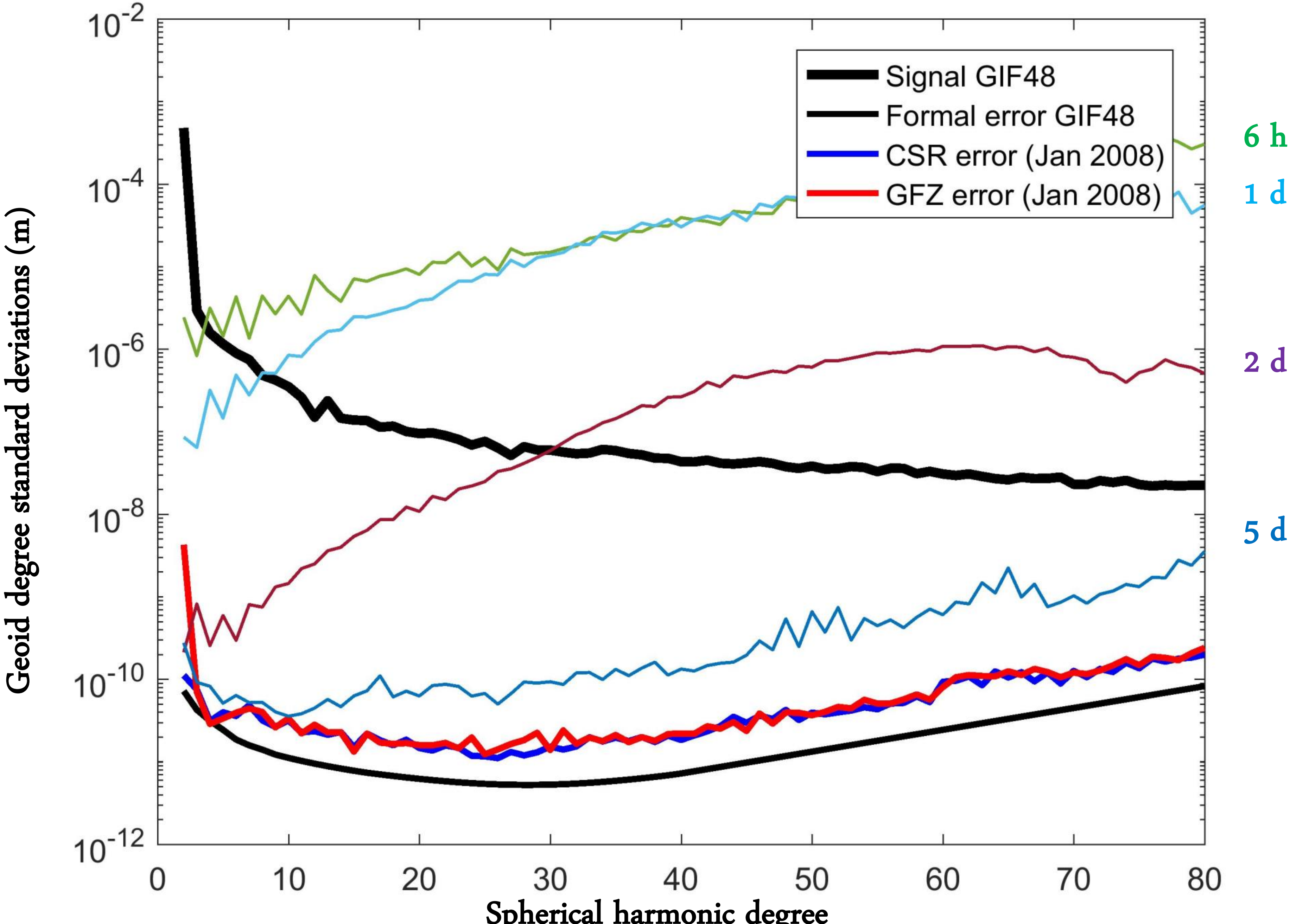

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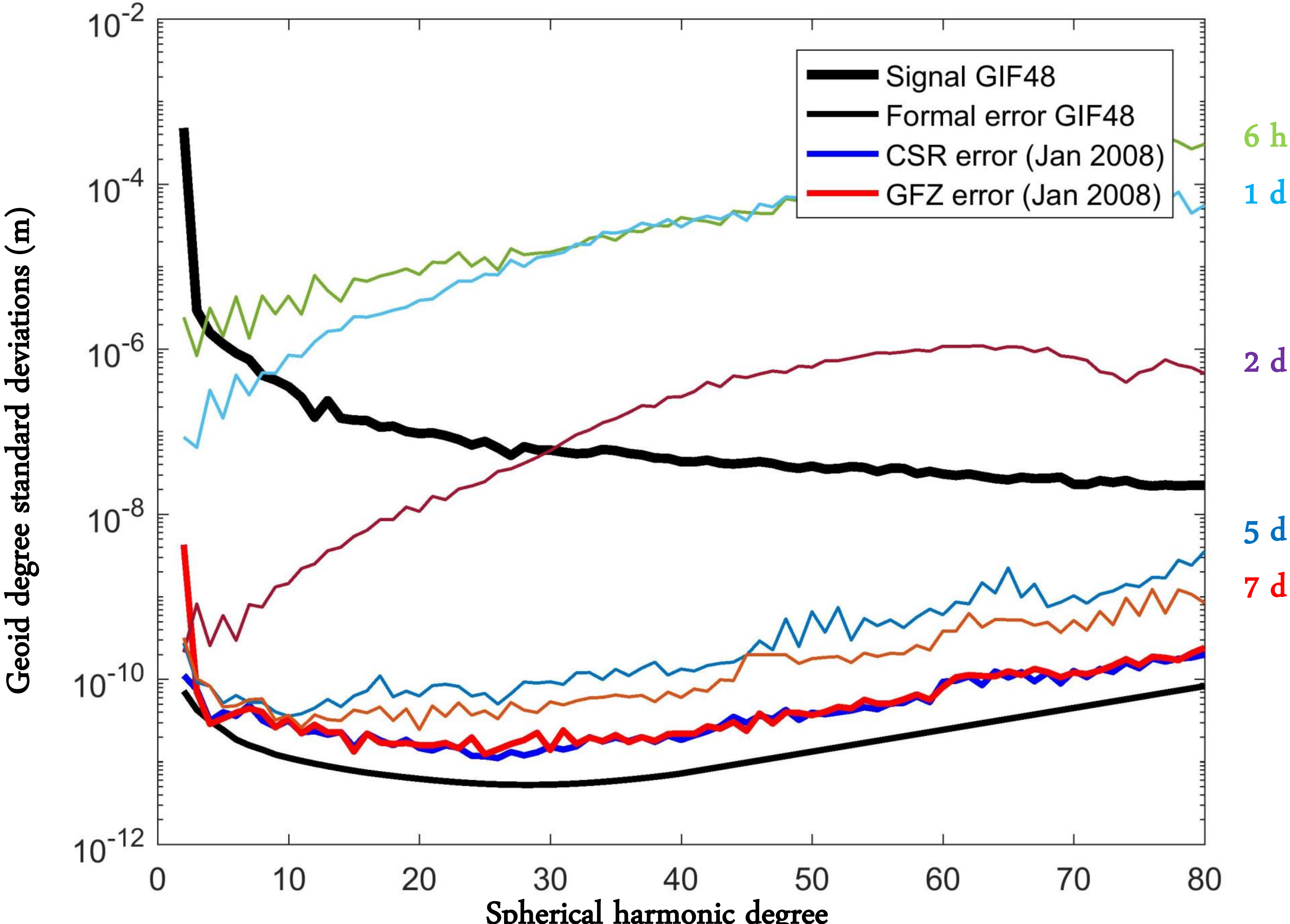

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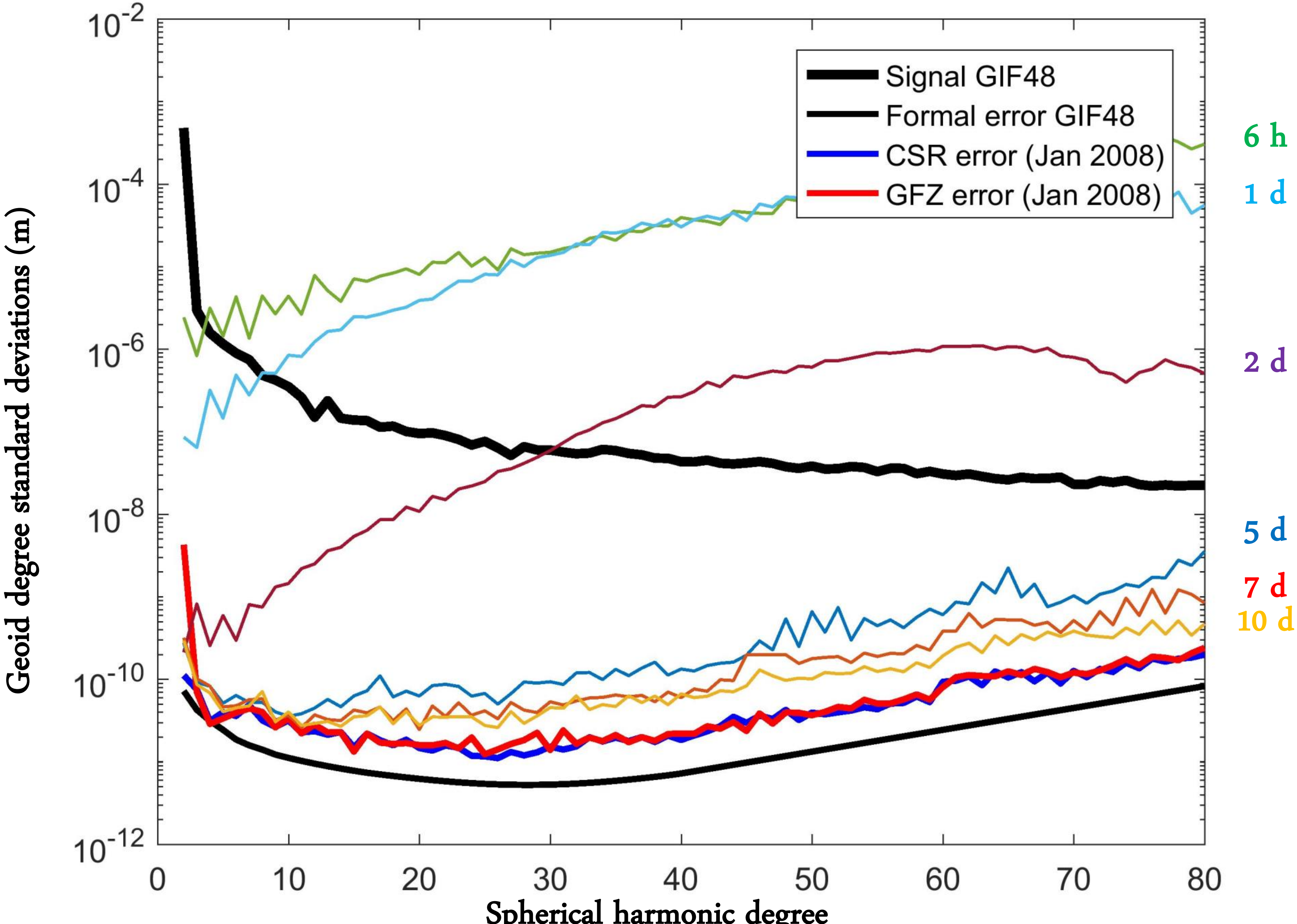

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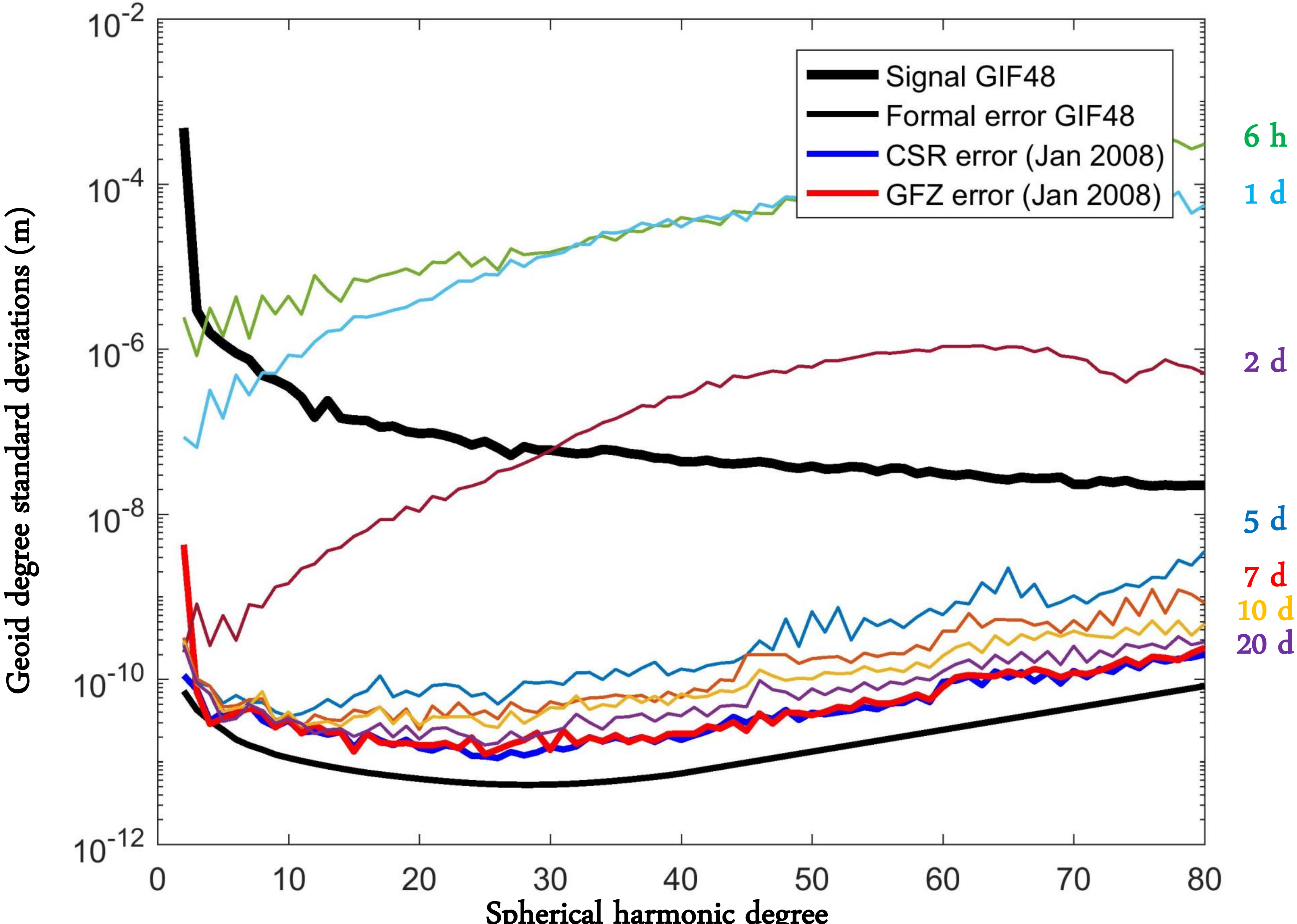

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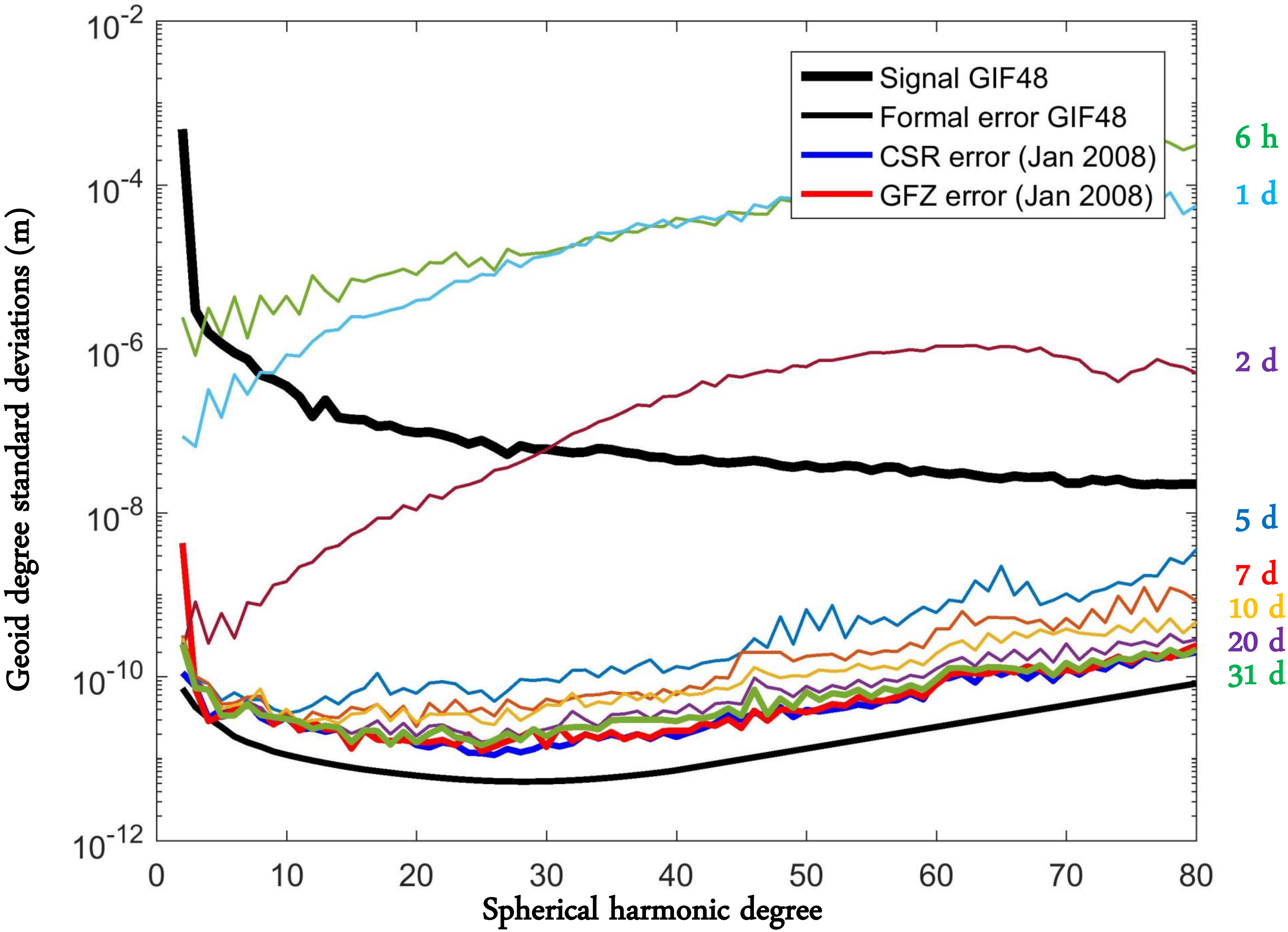

Signal GIF48
Formal error GIF48
CSR error (Jan 2008)
GFZ error (Jan 2008)
6 h
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2 d
5 d
7 d
10 d
20 d
31 d
Geoid degree standard deviations (m)
Spherical harmonic degree
$10^{-2}$
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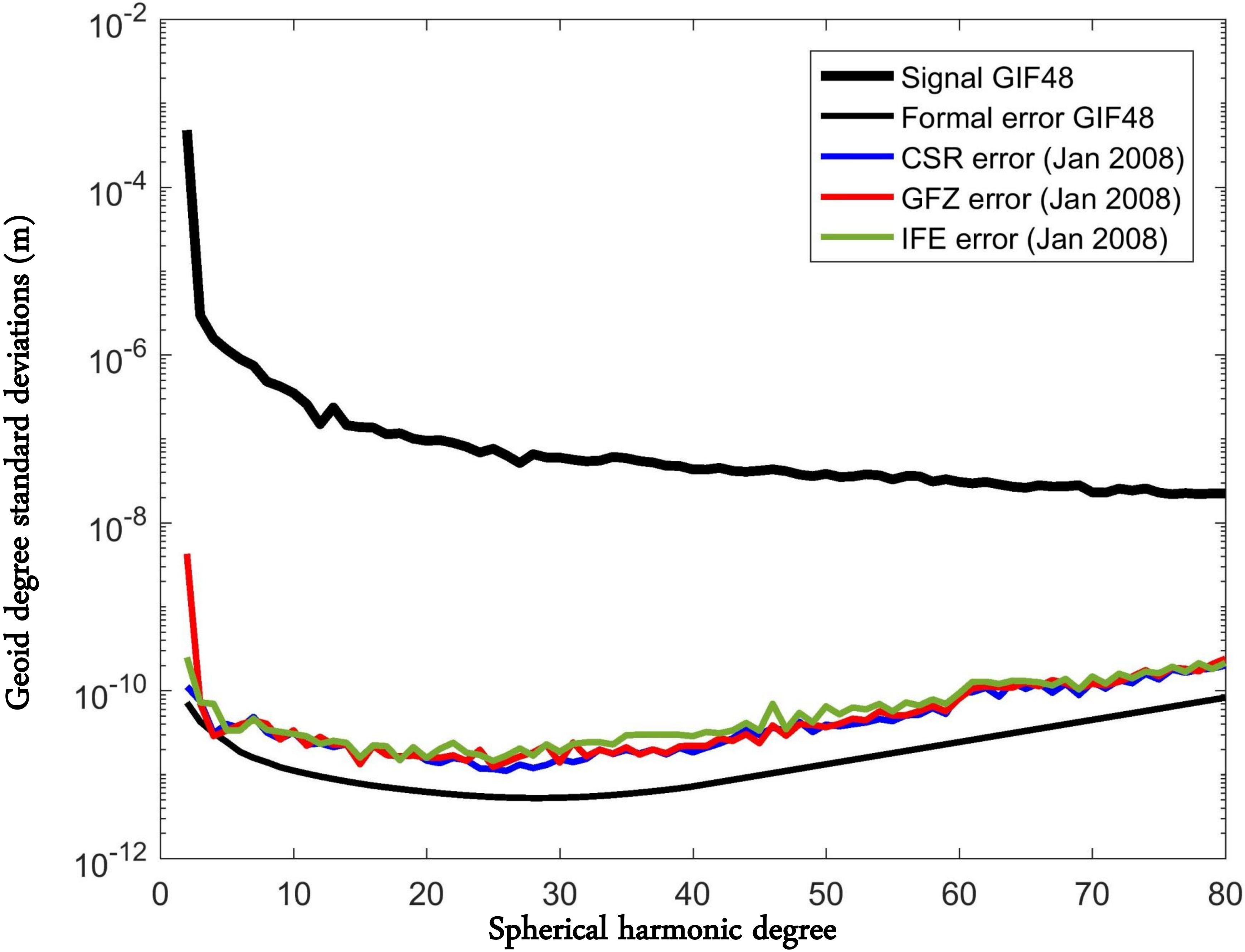

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Formal error GIF48
CSR error (Jan 2008)
GFZ error (Jan 2008)
IFE error (Jan 2008)
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Spherical harmonic degree
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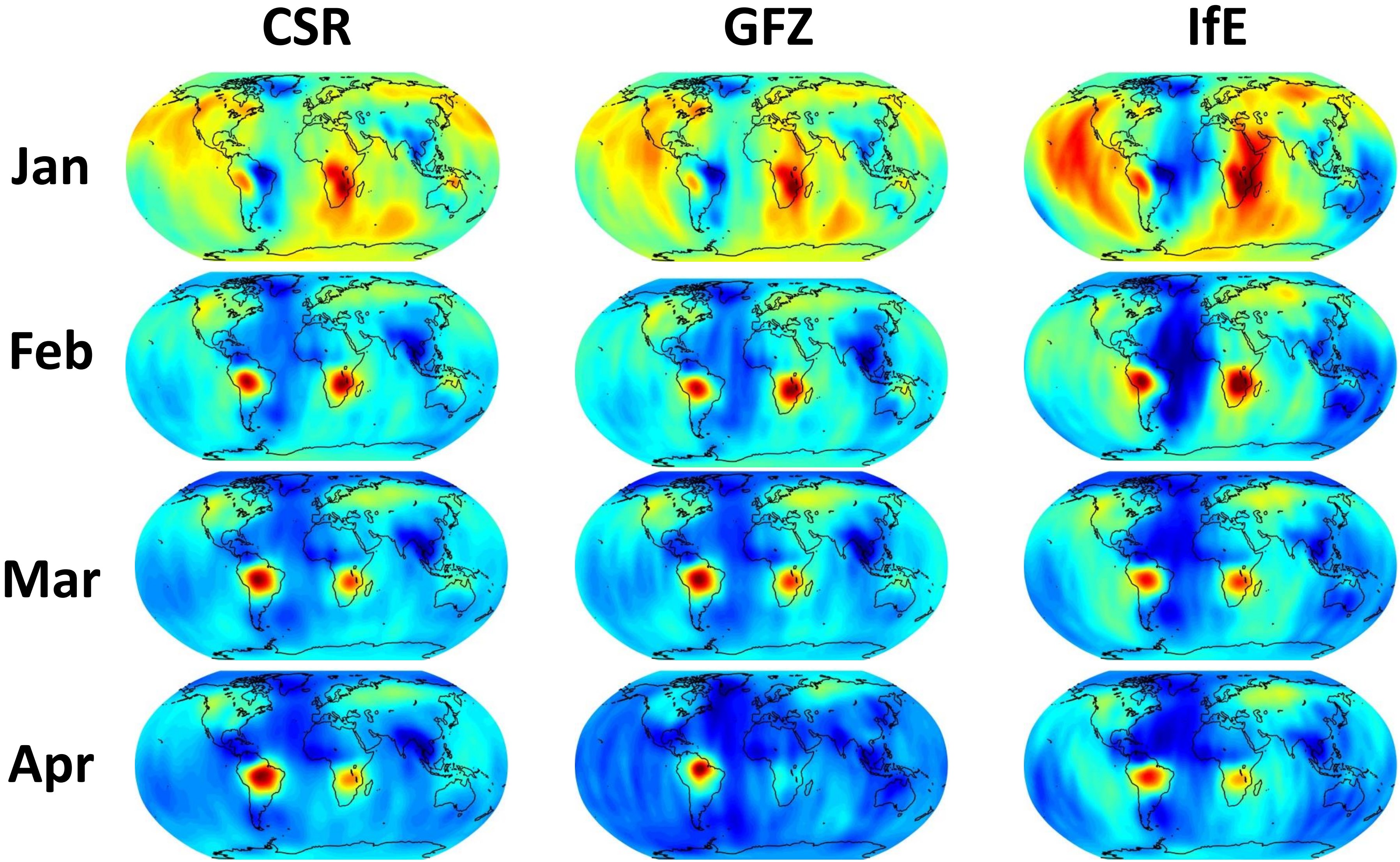
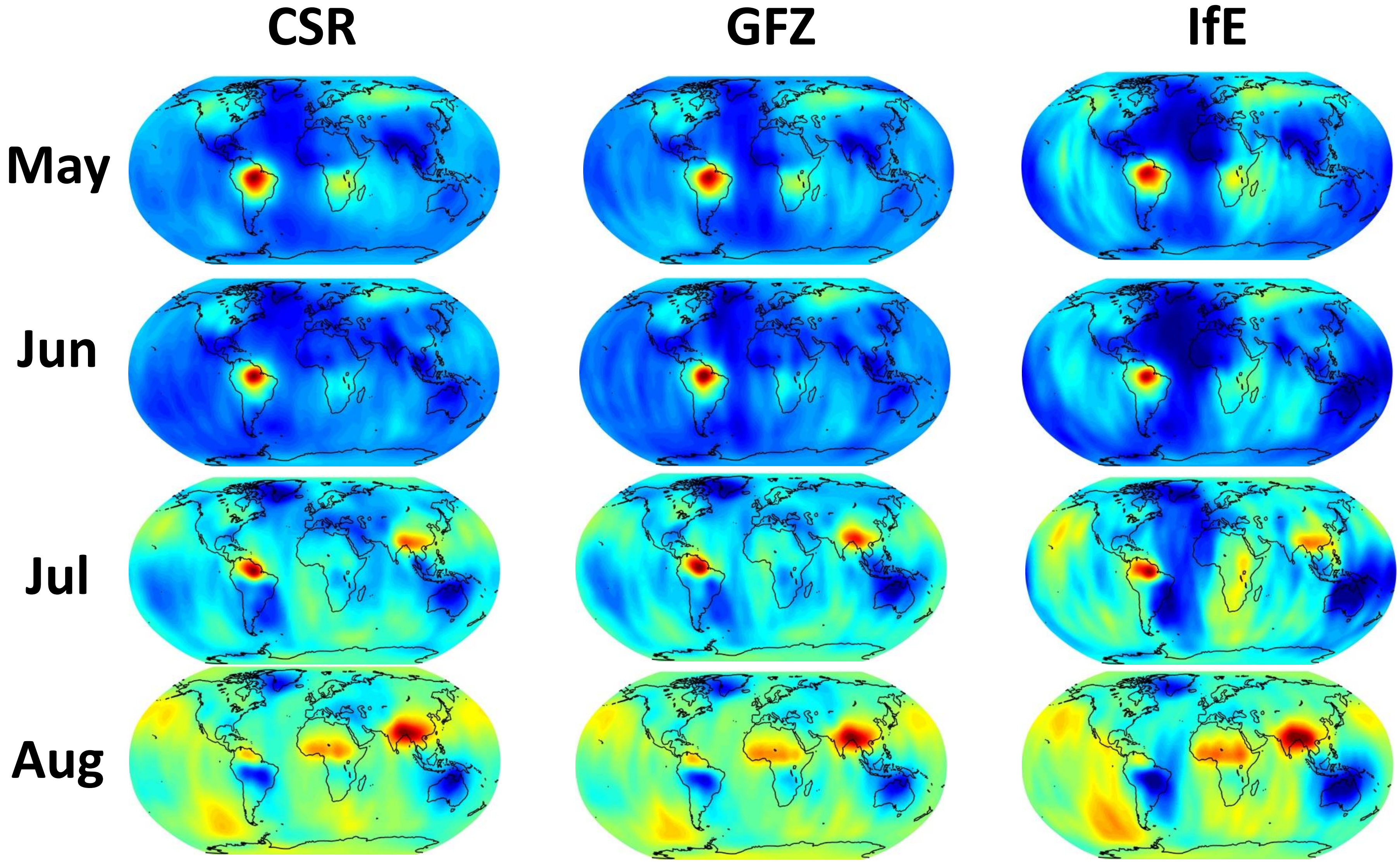
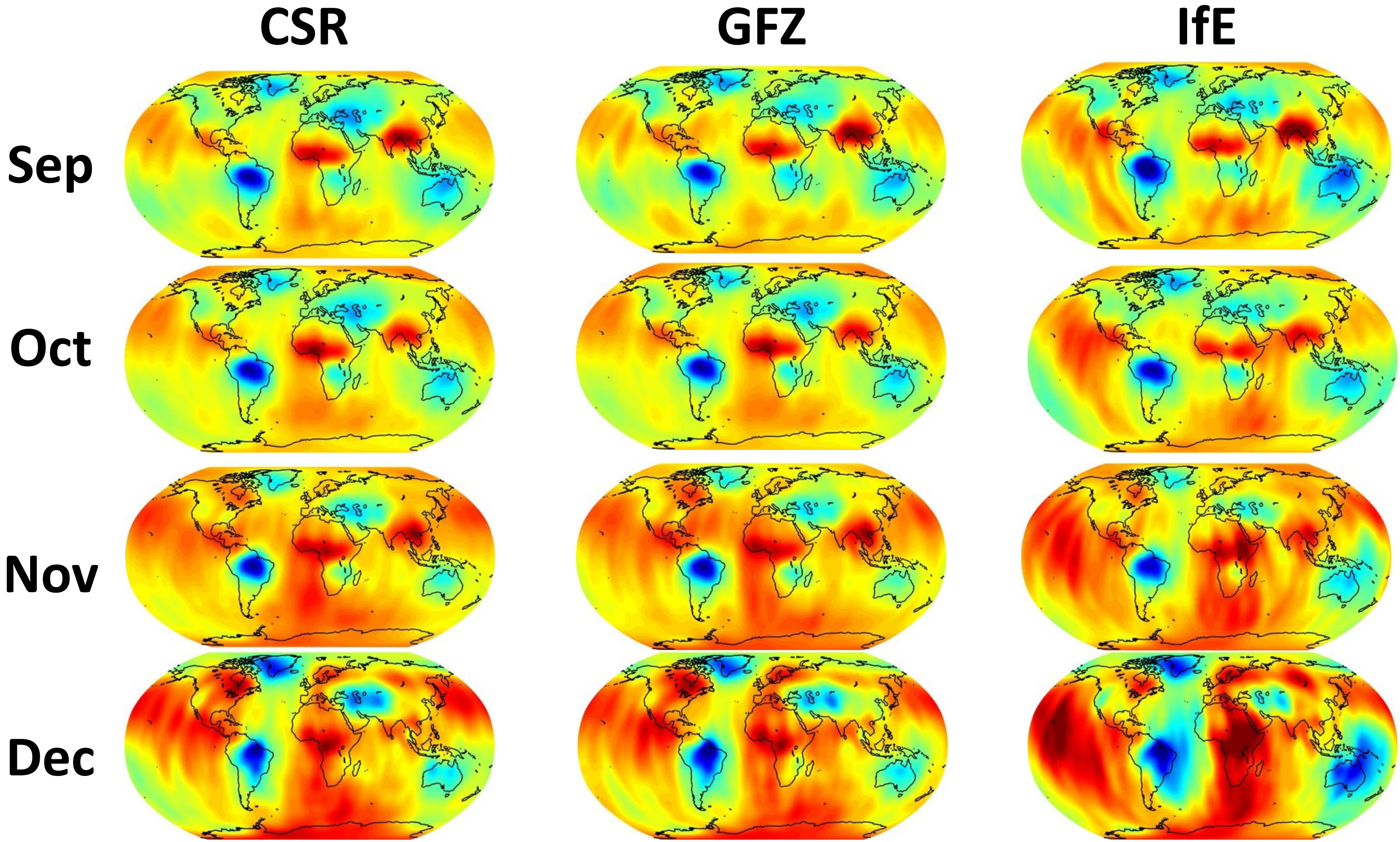

CSR
GFZ
IfE
Jan
Feb
Mar
Apr


CSR
GFZ
IfE
May
Jun
Jul
Aug


CSR
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Sep
Oct
Nov
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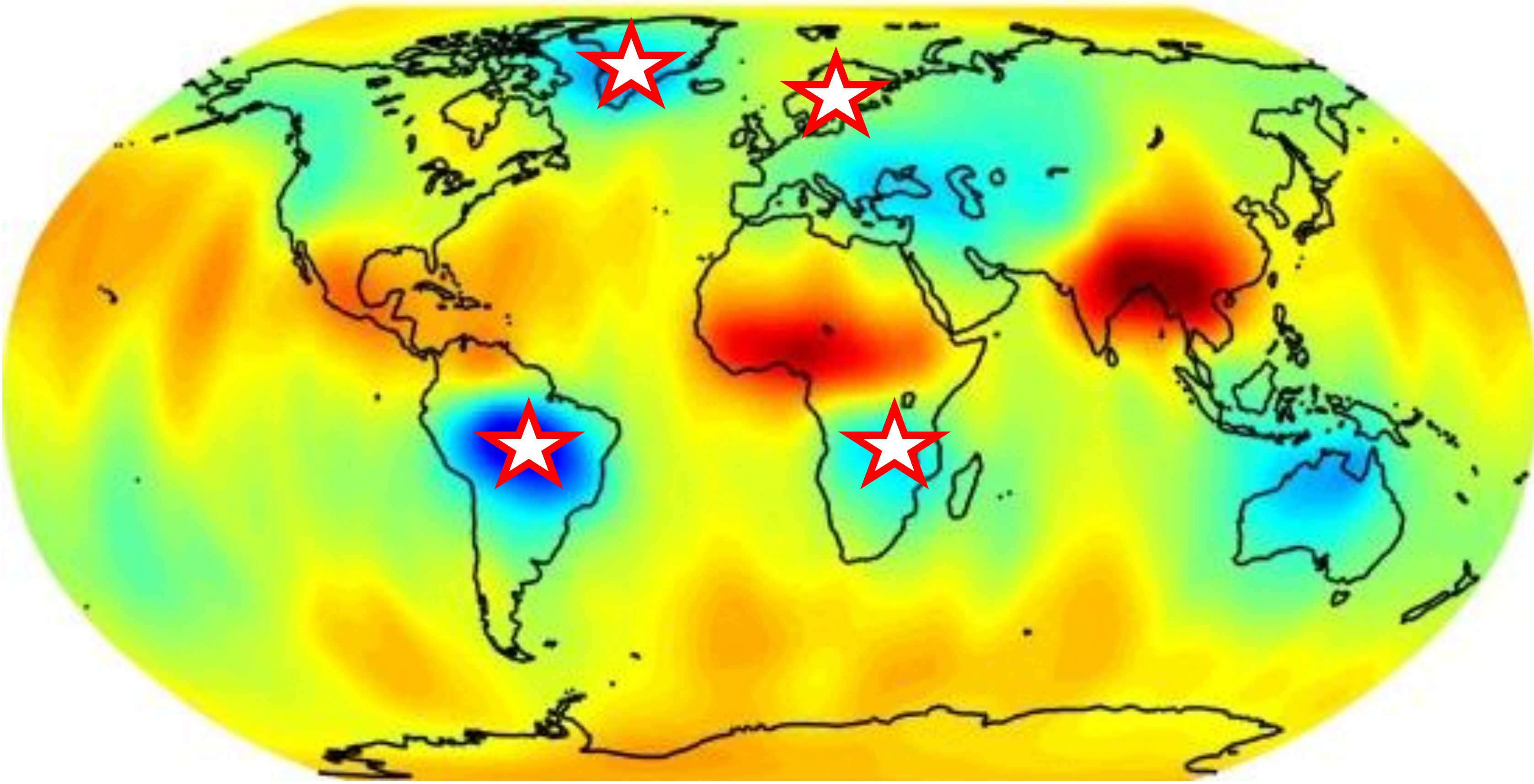
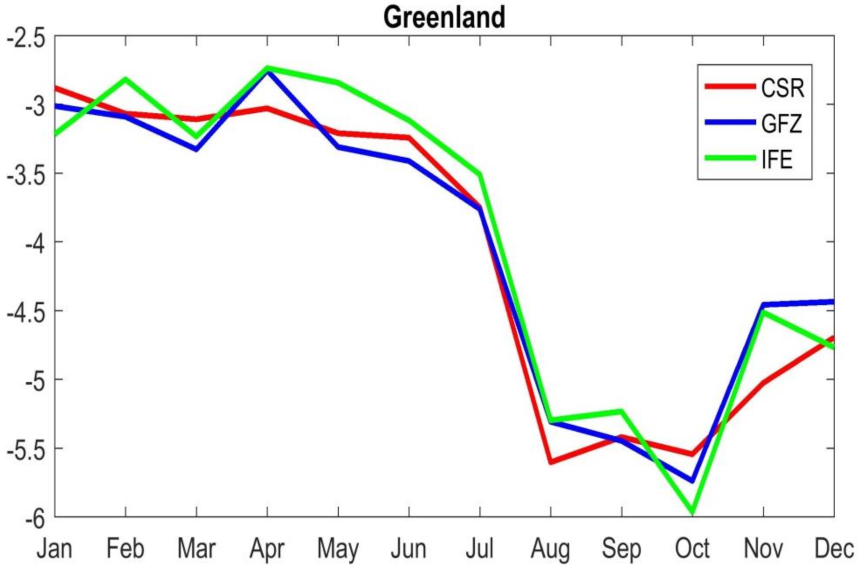
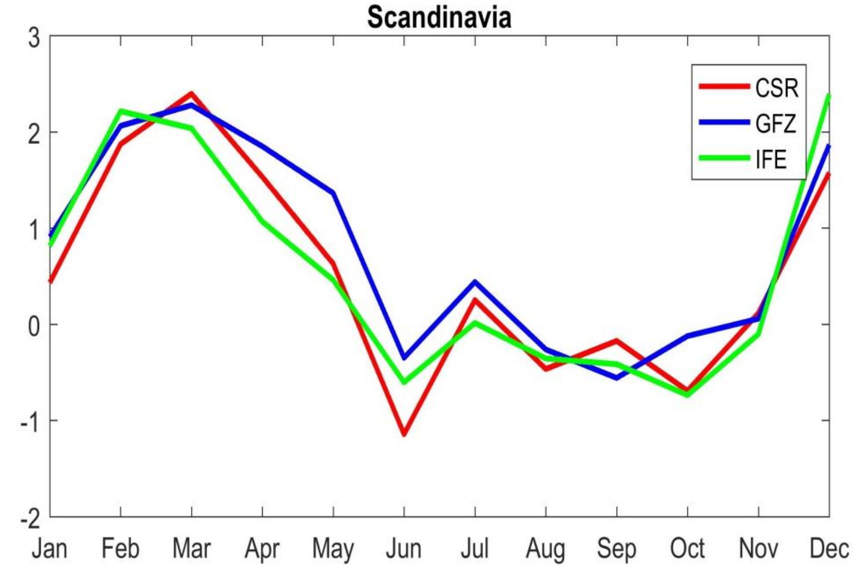
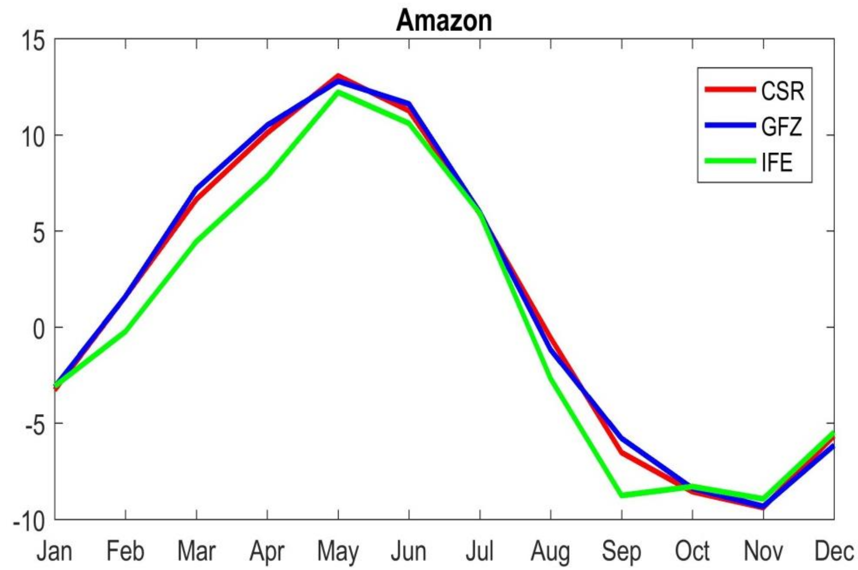
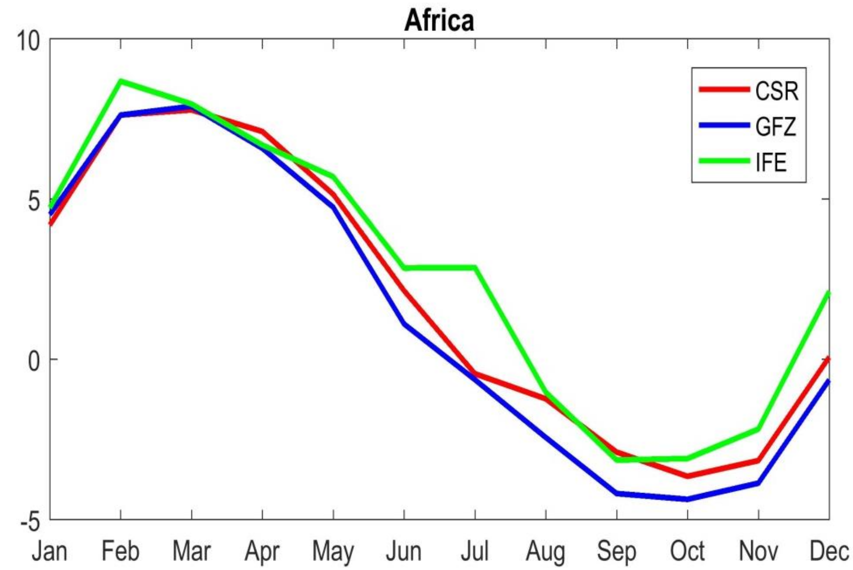





# Geoid changes in mm



## Summary:

- ➢ Successful implementation of the variational equations for GRACE/GRACE FO data at IfE, University of Hannover
- ➢ GRACE Monthly solutions for 2008.
- ➢ Very good agreement with official GRACE processing centers.
- ➢ An efficient user friendly MATLAB-based software, can be used for delivering monthly solutions and also for educational purposes.

## Further plans:

- ❑ Analysis of range-rate residuals and covariance modeling.
- ❑ Using AOD1B RL06 products.
- ❑ Publishing a new time series of GRACE monthly solutions.
- ❑ Adopt the code for GRACE follow-on data, in particular for LRI data.