Project

Characteristics of the Specific Fuel Consumption for Jet Engines

Author: Artur Bensel

Supervisor: Prof. Dr.-Ing. Dieter Scholz, MSME
Delivery Date: 31.08.2018

Faculty of Engineering and Computer Science
Department of Automotive and Aeronautical Engineering

DOI:
https://doi.org/10.15488/4316

URN:
http://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2018-08-31.016
Associated URLs:
http://nbn-resolving.org/html/urn:nbn:de:gbv:18302-aero2018-08-31.016

Any further request may be directed to:
Prof. Dr.-Ing. Dieter Scholz, MSME
E-Mail see: http://www.ProfScholz.de

This work is part of:
Digital Library - Projects & Theses - Prof. Dr. Scholz
http://library.ProfScholz.de

Published by
Aircraft Design and Systems Group (AERO)
Department of Automotive and Aeronautical Engineering
Hamburg University of Applied Science

This report has associated published data in Harvard Dataverse:
https://doi.org/10.7910/DVN/LZNNL1

Abstract

Purpose of this project is a) the evaluation of the Thrust Specific Fuel Consumption (TSFC) of jet engines in cruise as a function of flight altitude, speed and thrust and b) the determination of the optimum cruise speed for maximum range of jet airplanes based on TSFC characteristics from a). Related to a) a literature review shows different models for the influence of altitude and speed on TSFC. A simple model describing the influence of thrust on TSFC seems not to exist in the literature. Here, openly available data was collected and evaluated. TSFC versus thrust is described by the so-called bucket curve with lowest TSFC at the bucket point at a certain thrust setting. A new simple equation was devised approximating the influence of thrust on TSFC. It was found that the influence of thrust as well as of altitude on TSFC is small and can be neglected in cruise conditions in many cases. However, TSFC is roughly a linear function of speed. This follows already from first principles. Related to b) it was found that the academically taught optimum flight speed (1.316 times minimum drag speed) for maximum range of jet airplanes is inaccurate, because the derivation is based on the unrealistic assumption of TSFC being constant with speed. Taking account of the influence of speed on TSFC and on drag, the optimum flight speed is only about 1.05 to 1.11 the minimum drag speed depending on aircraft weight. The amount of actual engine data was extremely limited in this project and the results will, therefore, only be as accurate as the input data. Results may only have a limited universal validity, because only four jet engine types were analyzed. One of the project's original value is the new simple polynomial function to estimate variations in TSFC from variations in thrust while maintaining constant speed and altitude.

Characteristics of the Specific Fuel Consumption for Jet Engines

Task for a *Project*

Background

The Specific Fuel Consumption (SFC) of a jet engine c is defined per thrust and called more precisely also Thrust-specific Fuel Consumption (TSFC). This common definition comes from the fact that thrust (and not power) is measured on jet engine test stands. Engine efficiency is however related to power. It follows from first principles that c must depend on the speed of the aircraft V (or Mach number). The most simple representation of this is a linear function $c = c_a \cdot V + c_b$. Other operating conditions that have an influence on TSFC are thrust (equal to aircraft drag in unaccelerated horizontal cruise flight) as well as all parameters that depend on flight altitude (air temperature, pressure, density, speed of sound). Some of these parameters are included in a model by HERRMANN, which unfortunately does not account for thrust.

Task

The task of this project is to investigate TSFC dependencies especially with respect of thrust. Following subtasks have to be considered:

- Explain the principles of specific fuel consumption. Investigate TSFC dependency on speed and on altitude.
- Do a literature review of models for the estimation of TSFC dependency on thrust.
- Do a literature review of data sources providing TSFC dependency on thrust.
- Analyze the data and find a way to estimate the influence of thrust on TSFC.
- Evaluate the importance of including the thrust dependency on TSFC in calculations.
- Re-evaluate the optimum speed for the maximum range with the investigated TSFC dependencies and compare the result with the "classically" taught optimum aircraft speed.

The report has to be written in English based on German or international standards on report writing.

Table of Contents

Page

List of Figures 6
List of Tables 7
List of Symbols 8
List of Abbreviations 9
List of Definitions 10

1 Introduction 11
1.1 Motivation 11
1.2 Definitions 11
1.3 Objectives 12
1.4 Literature 12
1.5 Structure 13

2 Fundamentals 14
2.1 Definitions of Thrust, Speed, and Altitude 14
2.2 Power Specific Fuel Consumption (PSFC) 15
2.3 Breguet Range Equation 15
2.4 Optimum Speed for Maximum Range 16
2.4.1 Academically Taught Derivation 16
2.4.2 Derivation Using PSFC 16
2.5 Data Extraction with WebPlotDigitizer 18

3 TSFC Dependency on Altitude 20

4 TSFC Dependency on Speed 21
4.1 Linear Models 21
4.2 Non-Linear Models 22

5 TSFC Dependency on Thrust 25
5.1 Sources for Data Extraction 25
5.2 Comparison of Extracted Data 29
5.3 Importance of TSFC Variation Due to Thrust Change 30

6 Numerical Optimum Speed for Max Range 34

7 Summary and Conclusions 36

References 38

List of Figures

Fig. 2.1 WebPlotDigitizer on the example of a graph from **Hill 1992** (p. 197) 18
Fig. 4.1 Estimation of TSFC by different models (**Roux 2002**, p. 38).......................... 21
Fig. 4.2 TSFC characteristics of typical aircraft engines (**Mattingly 1996**, p.29) 22
Fig. 4.3 Comparison between Roux and Herrmann in 11000 m (after **Scholz 2017**) ... 23
Fig. 4.4 Effects of Mach, BPR, Altitude on TSFC (**Roux 2002**, p.31) 24
Fig. 5.1 JT9D-70/-70A turbofan cruise TSFC (**Mattingly 1996**, p. 25) 26
Fig. 5.2 Typical RB211 turbofan performance (**Hill 1992**, p. 197)............................... 27
Fig. 5.3a Performance of the V2527-A5 turbofan (**Risse 2014**, p. 6) 28
Fig. 5.3b SFC bucket curves at cruise design conditions (*M* = 0.85, 35000 ft, ISA), with and without the influence of average off-takes (**Risse 2016**, p. 129)....... 28
Fig. 5.4 Performance of the RB211-524-D4 turbofan (**Scholz 2018a**) 29
Fig. 5.5 Dimensionless TSFC over thrust from extracted data...................................... 30
Fig. 5.6 Dimensionless TSFC over thrust data for cruise flight 32
Fig. 5.7 Typical cruise TSFC versus thrust relationship normalized with respect to the optimum (or lowest TSFC) condition (**Young 2018**)...................................... 33

List of Tables

Table 5.1 Engine Specifications of the four sources (**Meier 2005**)................................. 25
Table 5.2 Aircraft Specifications (**A320 2018**, **A330 2017**, **A350 2018**, **A380 2016**)..... 31
Table 5.3 Thrust variations in cruise flight for multiple aircraft 31
Table 6.1 Optimum speeds according to different models 35

List of Symbols

A	Aspect Ratio
A_1	Parameter
B_1	Parameter
b	Wing Span
C_{D_0}	Zero-lift drag coefficient
c	(Thrust) Specific Fuel Consumption
c_{opt}	Optimum (lowest) Thrust Specific Fuel Consumption
c_a	Parameter
c_b	Parameter
c_P	Power Specific Fuel Consumption
c_T	Thrust Specific Fuel Consumption
$c_{T,ref}$	Reference Thrust Specific Fuel Consumption
D	Drag
E	Gliding Ratio
e	Oswald factor
g	Gravitational Acceleration
h	Altitude
L	Lift
m_1	Starting Mass
m_2	Final Mass
$\dot{m}_F$	Mass Fuel Flow
P	Power
R	Range
S	Wing Area
T	Thrust
T_{ref}	Reference Thrust
T_0	Reference Temperature
$T(h)$	Temperature in Altitude h
V	Velocity
W	Weight
ρ	Density

List of Abbreviations

ATC	Air Traffic Control
BPR	Bypass-Ratio
CeRAS	Central Reference Aircraft data System
CSV	Comma-Separated Values
DOC	Direct Operating Costs
ISA	International Standard Atmosphere
MTOW	Maximum Take-Off Weight
MZFW	Maximum Zero-Fuel Weight
PSFC	Power Specific Fuel Consumption
TSFC	Thrust Specific Fuel Consumption
TAS	True Air Speed
URL	Universal Resource Locator

List of Definitions

CeRAS
"CeRAS (Central Reference Aircraft data System) is a central database hosting reference design data of commercial aircraft. It is intended to be used for research projects dealing with conceptual to preliminary aircraft design studies as well as technology integration and assessment." (**CeRAS 2018**)

Turbomatch
"Over many years Cranfield has developed TURBOMATCH, a gas turbine performance software suite. This has been widely used in aircraft engine research projects and within the Cranfield postgraduate community. It is the only UK based gas turbine performance code external to gas turbine manufacturers that is currently used by a large public. Thousands of Cranfield gas turbine postgraduates have used it." (**Cranfield 2018**)

1 Introduction

1.1 Motivation

This Project is based on **Scholz 2017,** where the author's conclusion is that the academically taught optimum speed for maximum range needs to be reconsidered. In the memo, the author points out, that the Power Specific Fuel Consumption (PSFC) can be considered constant, however, the Thrust Specific Fuel Consumption (TSFC) is dependent on True Air Speed (TAS). The PSFC and TSFC are coupled by the factor of TAS. If the Breguet-Range-Equation is written for both cases it delivers different results for the optimum speed for the maximum range. In case of constant PSFC, the result is the minimum drag speed, in case of constant TSFC, the result is 1,316 times the minimum drag speed, which is the conventional academically taught version. In the frame of this problem, the dependencies of the TSFC on altitude, flight speed and thrust need to be investigated and the resulting optimum speed for the maximum range has to be recalculated.

1.2 Definitions

Thrust Specific Fuel Consumption (TSFC)

To compare the performance and efficiency of different engines, one can divide the mass flow of fuel by the thrust that is produced by the engine. The TSFC, therefore, shows how much fuel the engine needs for a unit of thrust. The smaller the TSFC, the better the efficiency of the jet engine.

$$TSFC = c_T = \frac{\dot{m}_F}{T} \tag{1.1}$$

Jet Engines

Jet engines are air-breathing propulsion systems that are used to power and propel aircraft. A compressor compresses the air; heat is added in the combustion chamber and the air leaves through a turbine which powers the compressor. The excess energy results in thrust. The thermodynamic principle is called the Brayton Cycle. In turbofan engines, the turbine also powers fan blades that accelerate surrounding air masses that bypass the engine. The ratio between the air masses that bypass the engine to the air masses that go through the engine is called bypass ratio. Nowadays the trend goes towards high bypass ratio engines since they are more fuel efficient.

1.3 Objectives

The objectives of this project are to give the reader a good overview of TSFCs dependencies on parameters as found in the Breguet Range Equation. The dependencies on altitude, speed and drag (and hence thrust) have to be investigated including their importance for aircraft design and performance. Other parameters of the engine such as the bypass ratio or the overall pressure ratio will not be investigated since they are basically fixed as a design choice and less related to flight conditions in cruise. Furthermore, the optimum speed for maximum range has to be numerically calculated based on the investigated TSFC dependencies.

1.4 Literature

The basis for this project is **Scholz 2017** and an Excel-File by Prof. Scholz, which compares the TSFC models from **Herrmann 2010** and **Roux 2002**. Both have adjusted previous models. While Herrmann adjusted the model from **Torenbeek 1982** by changing the numerical estimation of efficiencies for single elements in the turbofan engine, Roux adjusted a model from **Mattingly 1996** to fit the SI units instead of the imperial system.

Nowadays the main goal in aviation is to keep down the fuel consumption. This is because of environmental responsibility but mainly because of Direct Operation Costs (DOC). The mathematics behind it is quite simple: The more fuel you burn, the more expensive the flight will be. That is the reason why engine manufacturers protect their most valued data, the TSFC, like nothing else. You can find a lot of data on thrust, geometry or basic assembly; however, it is insanely difficult to find actual data on fuel consumption. Multiple engine manufacturers compete in the global aviation market and do not want their main selling point to be openly available.

In the course of this project a few literature sources were obtained, where a relation between thrust and TSFC is presented. One of these sources is **Hill 1992**, where data from Rolls-Royce can be seen. Another one is **Mattingly 1996**, who published data from Pratt & Whitney. Furthermore, numerical simulations of turbofan engines were obtained from **Risse 2014** and **Scholz 2018a**.

In **Schulz 2007**, the author gives a good overview of the literature about TSFC. It covers different approaches by different authors on how TSFC may be estimated for aircraft design. There are multiple approaches for TSFC change due to speed or altitude. Unfortunately, he does not cover the TSFC dependency on thrust in great detail.

General equations on drag, range, and performance are from **Young 2001**, where everything is collected in form of lecture notes for flight mechanics.

1.5 Structure

The main part of this project is structured as follows:

Chapter 2	explains fundamentals of fuel consumption and range calculation. Furthermore, two different derivations of optimum speed for maximum range, with different results, are shown. Additionally, a program that was used throughout this project is presented. With the program, the user may extract data from a graph whose origin data is not available.
Chapter 3	shows the dependency of TSFC on altitude change, while speed and thrust are being kept constant.
Chapter 4	shows linear and non-linear models to describe TSFC dependency on speed, while maintaining constant altitude and thrust.
Chapter 5	evaluates the influence that thrust change has on TSFC, while speed and altitude are being kept constant. Multiple data sources are identified and analyzed which ultimately results in an equation that allows estimating the TSFC dependency on thrust under cruise conditions.
Chapter 6	re-evaluates the optimum speed that aircraft have to fly to reach their maximum range. The TSFC dependencies from Chapter 3 through 5 are the basis for this numerical optimization.

2 Fundamentals

2.1 Definitions of Thrust, Speed, and Altitude

Thrust

Thrust is the force that moves the aircraft through the air. It is the force that overcomes the drag on the aircraft. In cruise flight, the thrust equals the drag, hence there is no acceleration of the aircraft. Air-breathing engines generate thrust by accelerating masses of gas. According to Newton's third law, the force gets generated in the opposite direction in which the gas is accelerated. In the combustion chamber, fuel is burnt and heat is added to the gas. The gas expands and accelerates out of the rear of the engine, pushing the aircraft forward.

Speed

Often, speed and velocity get confused. While speed is a scalar and consists of distance and time, velocity is a vector and consists of distance, time and direction of motion. For example, negative velocities are possible, while negative speeds are not. The average speed gets calculated as the distance over time, while the average velocity gets calculated by displacement over time. If an Olympic runner, for example, runs exactly one lap, he might have an average speed of 10 m/s while having an average velocity of 0 m/s. In this project, however, the terms speed and velocity are used interchangeably. They are both used in an effort to maintain different conventions at the same time. For example, the flight speed is called true airspeed (TAS), while in several equations from flight mechanics a V for velocity is used. From a physical point of view, the interchangeability of both terms may not be correct. However, in this project velocity and speed will actually turn out to be identical, since the focus is set on level cruise flight. Usually, aircraft travel on a straight line during cruise and therefore distance and displacement are identical. Moreover, there is a difference between TAS and ground speed. TAS is the speed with which aircraft move through the surrounding air, while ground speed is the speed of aircraft over the ground. For example, if an aircraft has a TAS of 100 m/s and there is a headwind of 100 m/s, the ground speed will be 0 m/s. In this project, when speeds or velocities are mentioned they represent the TAS.

Altitude

Altitude is the height above sea level. Generally, changing altitude results in changing ambient conditions. In the troposphere, the ambient pressure, temperature, and density will decrease with increasing altitude. In the stratosphere, the temperature remains constant, while the pressure and density further decrease with increasing altitude. In this project, the International Standard Atmosphere (ISA) is used as a basis for calculating densities and temperatures in different altitudes.

2.2 Power Specific Fuel Consumption (PSFC)

The comparison of efficiency and performance of turboprop engines is usually done with the PSFC. It is the mass flow of fuel divided by the power outtake of the engine. The PSFC, therefore, shows how much fuel the engine needs for a unit of power. The smaller the PSFC, the better the efficiency of the engine.

$$PSFC = c_P = \frac{\dot{m}_F}{P} \tag{2.1}$$

Since the power of an aircraft equals the thrust times velocity ($P = T\ V$), the connection between PSFC and TSFC is

$$TSFC = c_T = c_P V \quad . \tag{2.2}$$

2.3 Breguet Range Equation

In the derivation of the range equation, assumptions must be made. According to **Young 2001,** there are usually three flight schedules that are considered. The Breguet Range Equation considers the flight schedule where velocity V and lift coefficient C_L are being kept constant while the altitude h may vary. Since the altitude has to vary to keep the other parameters constant, this flight schedule is also named cruise-climb. The derivation of this range equation is the easiest and in terms of aircraft design the most elegant version, since you are able to see the influences each parameter has on the range. However, due to Air Traffic Control (ATC), this flight schedule cannot actually be flown. You are assigned a speed and an altitude by the ATC, the C_L, therefore, changes due to reduced weight which is a result of burnt fuel. A common practice is the step climb, where the pilot requests new flight levels as fuel is burnt. This simulates a cruise-climb and that is why the relative error of the Breguet Range Equation can be ignored.

$$R = \frac{E\ V}{c_T\ g} \ln\left(\frac{m_1}{m_2}\right) \tag{2.3}$$

Considering (2.2), the Breguet Range Equation may also be written as:

$$R = \frac{E}{c_P\ g} \ln\left(\frac{m_1}{m_2}\right) \tag{2.4}$$

Where:

E is the *gliding ratio L/D*

g is the *gravitational acceleration*

m_1 is the *starting mass*

m_2 is the *final mass*

V is the *velocity*

2.4 Optimum Speed for Maximum Range

2.4.1 Academically Taught Derivation

The optimum speed for maximum range follows from a maximum *Specific Air Range* (SAR). SAR is defined as the distance travelled per unit fuel mass consumed. The inverse 1/SAR is the fuel consumption, which has to be minimized (**Young 2001**).

$$\left(-\frac{dm_F}{dx}\right)=\frac{\left(-\frac{dm_F}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{Q}{V}=\frac{c\,T}{T}=\frac{c\,D}{V} \tag{2.5}$$

The minus sign in (2.5) stems from the fact that the change in fuel mass dm_F is negative. In the academically taught derivation, **c_T (TSFC) is considered to be constant** (**Young 2001**). Therefore, the range is a maximum when *D/V* gets minimized. D is a function of speed and must, therefore, be estimated by

$$D=\left[\frac{C_{D_0}\,\rho\,S}{2}\right]V^2+\left[\frac{2\,W^2}{\pi\,A\,e\,\rho\,S}\right]\frac{1}{V^2} \tag{2.6}$$

or

$$D=A_1\,V^2+B_1\,\frac{1}{V^2} \tag{2.7}$$

With: $A_1=\left[\frac{C_{D_0}\,\rho\,S}{2}\right]$ and $B_1=\left[\frac{2\,W^2}{\pi\,A\,e\,\rho\,S}\right]$

Where:

D is the *drag*

C_{D0} is the *zero-lift drag coefficient*

ρ is the *density*

S is the *wing area*

V is the *true airspeed*
W is the *weight of the aircraft*
e is the *Oswald factor*
A is the *aspect ratio* that can be calculated by

$$A = \frac{b^2}{S} \tag{2.8}$$

where b is the *wingspan*

With (2.7) then follows:

$$\frac{d}{dV}\left(\frac{D}{V}\right) = A_1\, V + B_1\, \frac{1}{V^3} \overset{!}{=} 0 \tag{2.9}$$

$$V_{opt} = \sqrt[4]{3}\, \sqrt[4]{\frac{B_1}{A_1}} = 1{,}316 \cdot V_{md} \tag{2.10}$$

with $$V_{md} = \sqrt[4]{\frac{B_1}{A_1}} \; .$$

2.4.2 Derivation Using PSFC

According to **Scholz 2017**, there could also be a different derivation of the optimum speed for the maximum range. He states that TSFC is a linear function of speed and because of (2.2) it is a **better assumption to consider c_P (PSFC) to be constant** then considering c_T (TSFC) to be constant. Based on (2.5) we can write

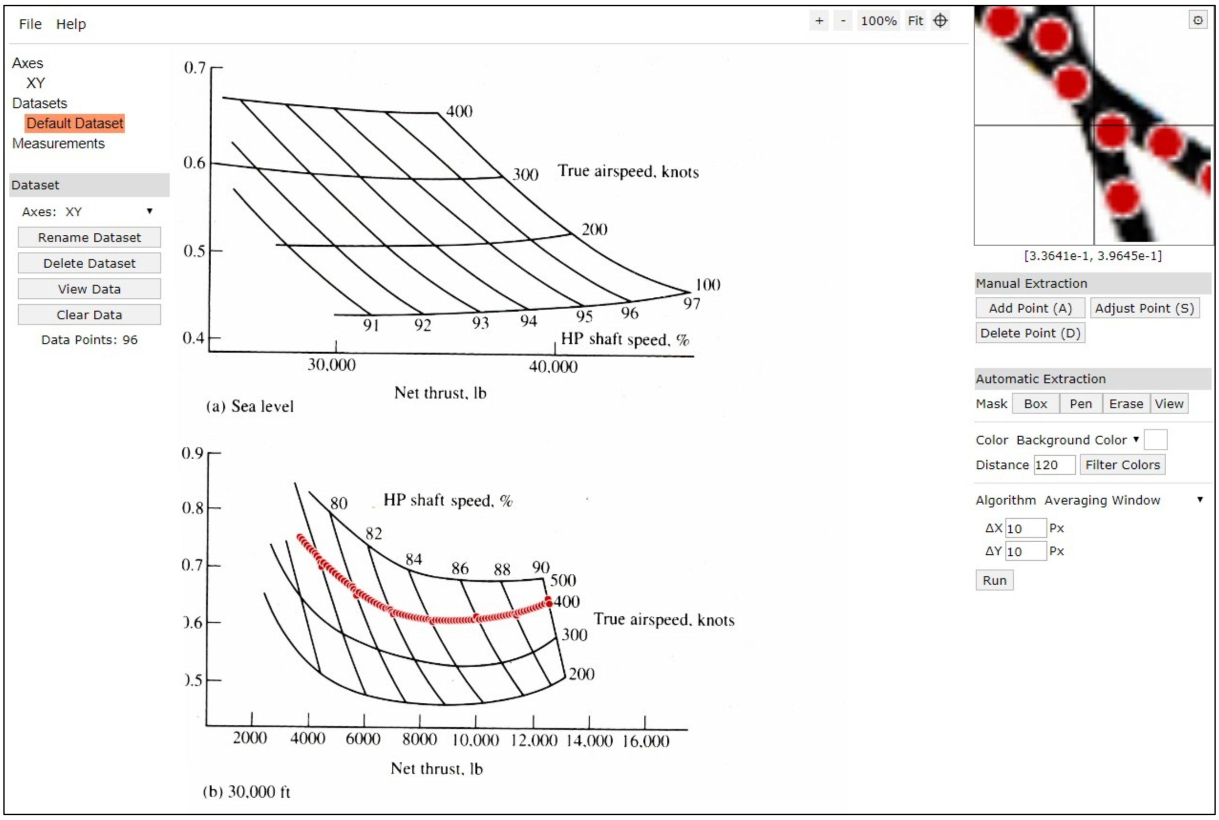
$$\left(-\frac{dm_F}{dx}\right) = \frac{c\, D}{V} = c_P\, D \; . \tag{2.11}$$

In (2.11) only drag D needs to be minimized for a minimum fuel consumption and maximum Specific Air Range (SAR). The speed for minimum drag is simply the minimum drag speed V_{md} and therefore, the optimum speed is in this case

$$V_{opt} = V_{md} = \sqrt[4]{\frac{B_1}{A_1}} \; . \tag{2.12}$$

2.5 Data Extraction with WebPlotDigitizer

For this project, no numerical data on TSFC were available. Hence, diagrams from multiple publications had to be re-engineered into numerical data to be able to perform further analyses on the provided information. In order to do the re-engineering, multiple programs were tested and the WebPlotDigitizer turned out to be the best fit for this project. While other programs might have been more advanced in terms of digitalization of polar plots or comparable difficult plots, they were also more complicated to handle and often not entirely open source. The WebPlotDigitizer is open source, easy to handle and its limited capabilities are more than enough for the plots that needed to be digitalized during this project. As of June 2018, the program may be accessed through a browser under **Rohatgi 2018.**



Axes: XY
Data Points: 96
[3.3641e-1, 3.9645e-1]
Color Background Color
Distance 120
Algorithm Averaging Window
ΔX 10 Px
ΔY 10 Px

Fig. 2.1 WebPlotDigitizer on the example of a graph from **Hill 1992** (p. 197)

In Fig. 2.1 the reader can see a screen capture of the program’s interface. Once an image is uploaded to the program an initial calibration of both axes needs to be done. Two points on each axis have to be selected with the cursor. A dialog box will pop up and values for the selected four points need to be entered, at this stage, the user can enter whether the axes are scaled linearly or logarithmically. After the calibration is done, data may be extracted from the picture.

On the left-hand side, the different datasets can be managed and the calibration of the axes can be adjusted or completely redone. On the right-hand side, the different tools for the data extraction are displayed. One can choose between a box and a pen to highlight the area in which the program is looking for data to extract. The selected area can be adjusted by erasing certain parts of it, this allows the user to select the entire plot at first and only erase the unwanted area. Once an area is selected, either a foreground or background color is selected, so that the program can properly identify the targeted curve. Additionally, the algorithm for data extraction can be chosen. In this project "Averaging Window" was used in each scenario. Values for delta x and delta y have to be entered, the smaller the entered values, the higher the density of points on the curve. Finally, the program may be executed by clicking "Run". Depending on how accurately the area was selected, the generated points will be distributed on surrounding curves as well as on the targeted curve. In the example of Fig. 2.1, one can see that some generated points are slightly off. On the right-hand side, there are tools to add, remove or adjust any given point. The magnifying glass in the top right corner helps with the accuracy of this task. To complete the data extraction, the data viewer can be opened on the left-hand side. The sortation and separation, as well as the formation of the data, can be adjusted and finally, the data may be downloaded as a CSV file.

The program is very well documented. For further information or detailed video tutorials on how to properly use the program, one may click "Help" in the upper left corner.

3 TSFC Dependency on Altitude

Up to the tropopause air temperature drops with altitude. Jet engines benefit from a big temperature difference between maximum internal temperatures (limited by engine material) and outside air temperature. For this reason jet engine efficiency increases with altitude up to the tropopause. Hence, a drop of TSFC with altitude can be expected. However, this was not reflected in the literature review. Furthermore, **since jet transport airplanes usually cruise in the stratosphere where temperature is constant** with altitude **only little variations of TSFC with altitude are expected** in the stratosphere where these aircraft cruise.

According to **Schulz 2007,** there are only a few models to describe the TSFC dependency on altitude most of them can be traced back to one of only two equations. In this project, the focus is set on TSFC during cruise flight, since the aim is to see its influence on the range equation. A common altitude for cruise flight of a civil passenger aircraft is somewhere between 25000 ft and 40000 ft. A further literature review has shown that in between those boundaries the TSFC varies but little with altitude (**Scholz 2017**, **Hill 1992**, **Mattingly 1996**). The relative variations depend on the model or the author but in general they are only about 1% ... 2%. These variations are certainly of interest for airlines, which are operating existing aircraft, however, for preliminary sizing in aircraft design there are so many assumptions being made that the TSFC dependency on altitude may be neglected.

4 TSFC Dependency on Speed

4.1 Linear Models

The model from **Mattingly 1996** that was adjusted by **Roux 2002** and **Scholz 2017** states that the TSFC is linearly dependent on the speed of the aircraft. Models collected by **Schulz 2007** state the same correlation between speed and TSFC. In Fig. 4.1 multiple models for TSFC calculation are compared and the general trend of linearity can be seen.

In Fig. 4.2 one can see different types of engines and their qualitative TSFC dependencies on Mach number. Since we are only interested in subsonic flight, one may say that all types of engines are linearly dependent on the speed of the aircraft. A small exception would be the turboprop; however, turboprops will usually not be operated above Mach 0.6 in which case the curve may still be seen as linear. The linear function that will later be used is from **Scholz 2017**:

$$c_T = c_a V + c_b \tag{4.1}$$

Where:

$c_a = 3.38 \cdot 10^{-8}$ kg/(Ns)

$c_b = 1.04 \cdot 10^{-5} \sqrt{\frac{T_0}{T(h)}}$ kg/(Ns)

$T_0 = 288.15$ K

T(h) is the temperature in altitude h

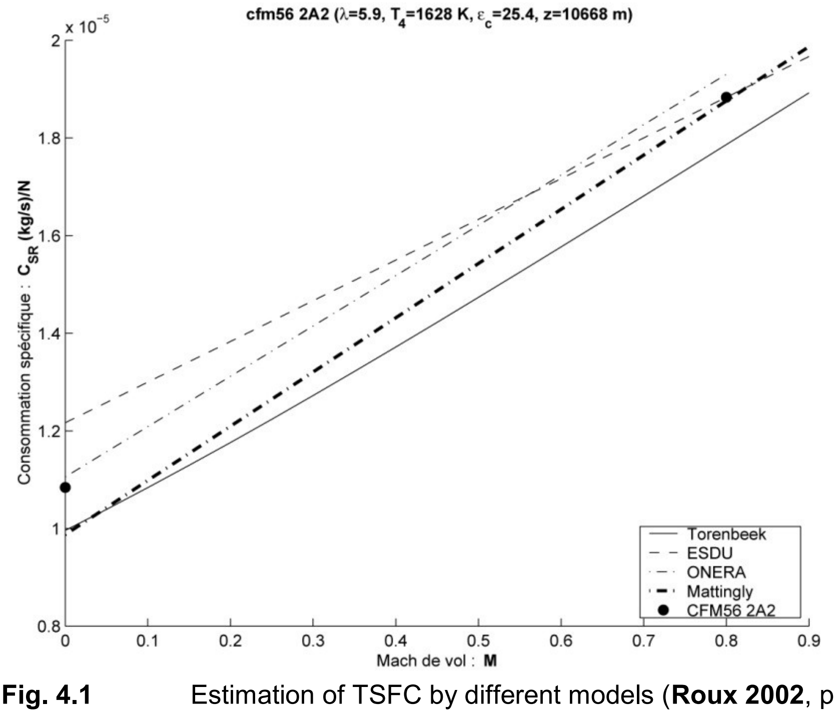


Fig. 4.1 Estimation of TSFC by different models (**Roux 2002**, p. 38)

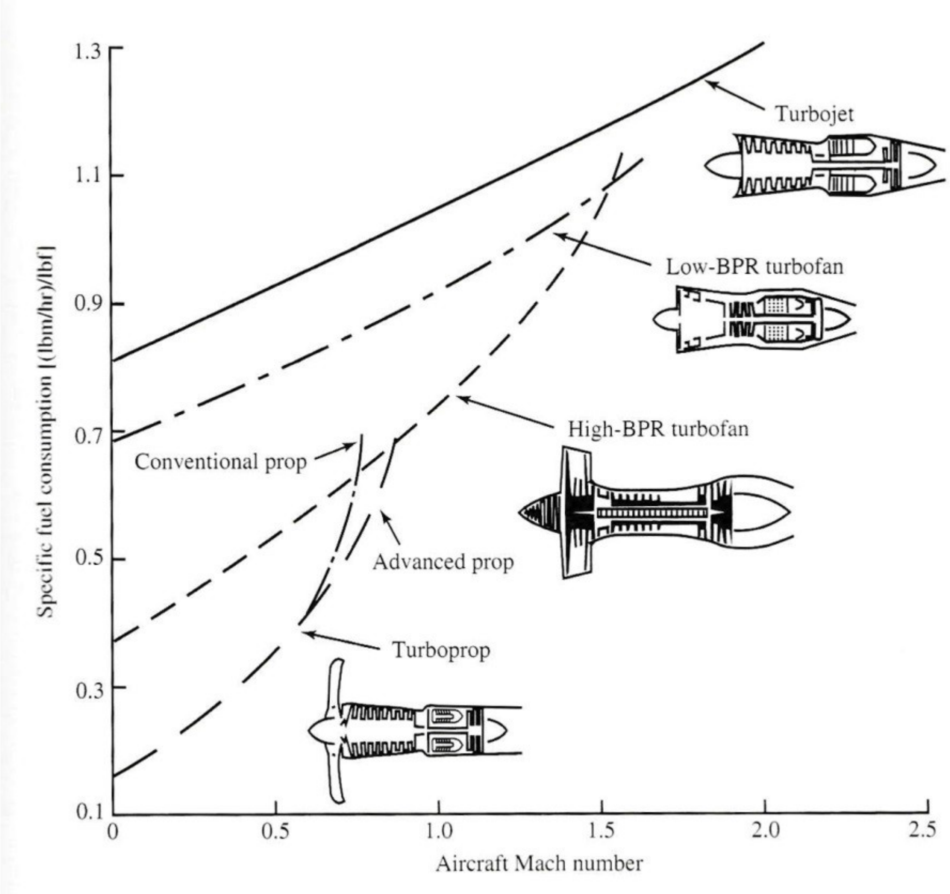


Fig. 4.2 TSFC characteristics of typical aircraft engines (**Mattingly 1996**, p.29)

4.2 Non-Linear Models

The model from **Herrmann 2010** that is based on **Torenbeek 1982** does not resemble a linear function. Nonetheless, in **Scholz 2017** a comparison between **Herrmann 2010** and **Roux 2002** is made in which one can see that the model of **Herrmann 2010** can also roughly be seen as linear, even though it technically is not (Fig. 4.3).

The calculations in Fig. 4.3 were done with a bypass-ratio (BPR) of 5.0, which counts as a High-BPR turbofan. Comparing Fig. 4.2 with Fig 4.3, one can see that **Herrmann 2010** might be the better fit for the actual TSFC calculation. It is almost also linear, but it is slightly bending upwards just as it does for high BPR turbofans in Fig. 4.2.

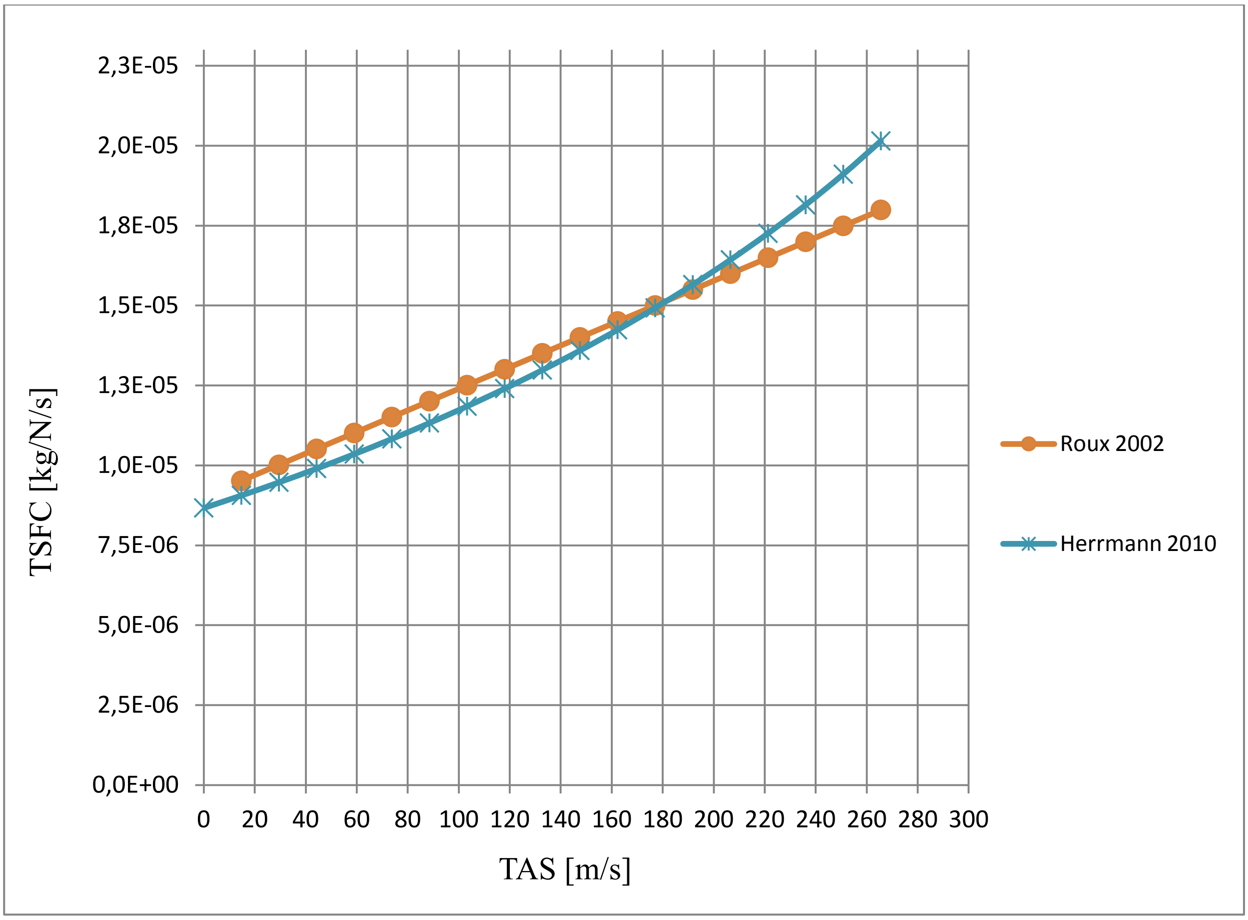


Fig. 4.3 Comparison between Roux and Herrmann in 11000 m (after **Scholz 2017**)

Additionally, it might be easier to find the right input for the model from **Herrmann 2010** than for the linear approach. In **Herrmann 2010**, one has simply to enter the Mach cruise number, the cruise altitude, the BPR of the engine and the take-off thrust. These are data that are usually available in the course of aircraft design. In opposition, for the linear approach, one has to have two TSFC values for two cruise velocities. It is obviously possible to simply use the adjusted values that are provided in **Scholz 2017**; however, in that case, every turbofan would have the same slope for TSFC as well as the same TSFC at 0 m/s. The reducing effects of higher BPR which can be seen in Fig. 4.4 would not be taken into account.

The model from Herrmann was published by **Scholz 2013**. An Excel file with the model is also provided in a dataset at **Harvard Dataverse** that is accompanying this report. The link to the dataset is given on the second page of this report.

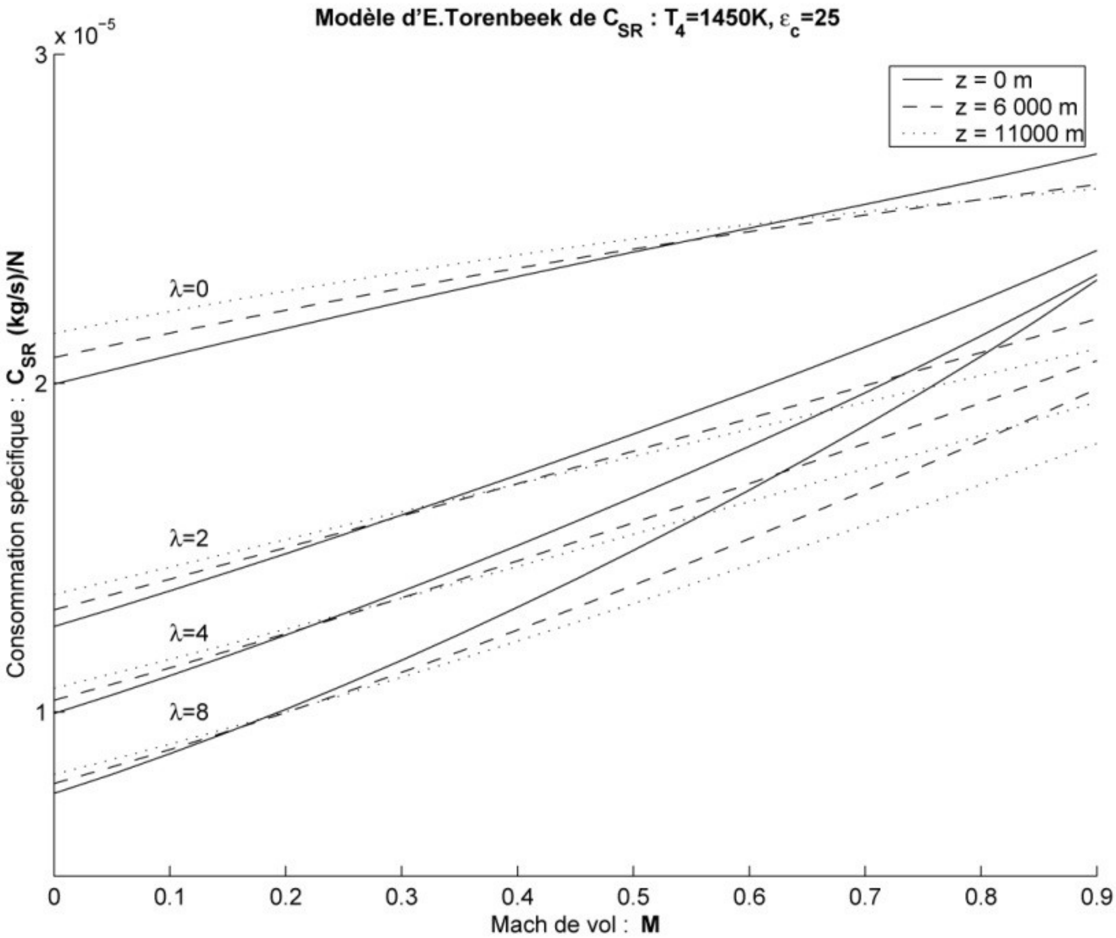


Fig. 4.4 Effects of Mach, BPR, Altitude on TSFC (**Roux 2002**, p.31)

5 TSFC Dependency on Thrust

5.1 Sources for Data Extraction

Four sources were obtained, where diagrams show the relationship between thrust and TSFC while maintaining a constant speed and altitude. Table 5.1 is an overview of the specifications of the different turbofans that are subjects in the sources. Three of them are quite similar in thrust since all of them found application on the Boeing 747. Only one engine is a little bit smaller and found application on the Airbus A320 and A319. All of the four engines are high BPR turbofans.

Table 5.1 Engine Specifications of the four sources (**Meier 2005**)

Engine	Thrust kN	BPR	Manufacturer	Application	Year
JT9D-70/-70A	236	4.9	Pratt & Whitney	747	1969
RB211 family	170-260	4.3-5.0	Rolls-Royce	747, 757, 767	1970s-2000s
V2527-A5	118	4.8	IAE	A319, A320	1993
RB211-524-D4	236	4.4	Rolls-Royce	747	1972

The first source is Fig. 5.1 from **Mattingly 1996.** It shows actual data from the JT9D-70/-70A turbofan from Pratt & Whitney. The diagram shows data for both Mach 0.8 as well as Mach 0.9 for the altitudes 30000 ft and 35000 ft. All four curves were found to be suitable for the project since they are in the range of common cruise altitude and common cruise Mach number.

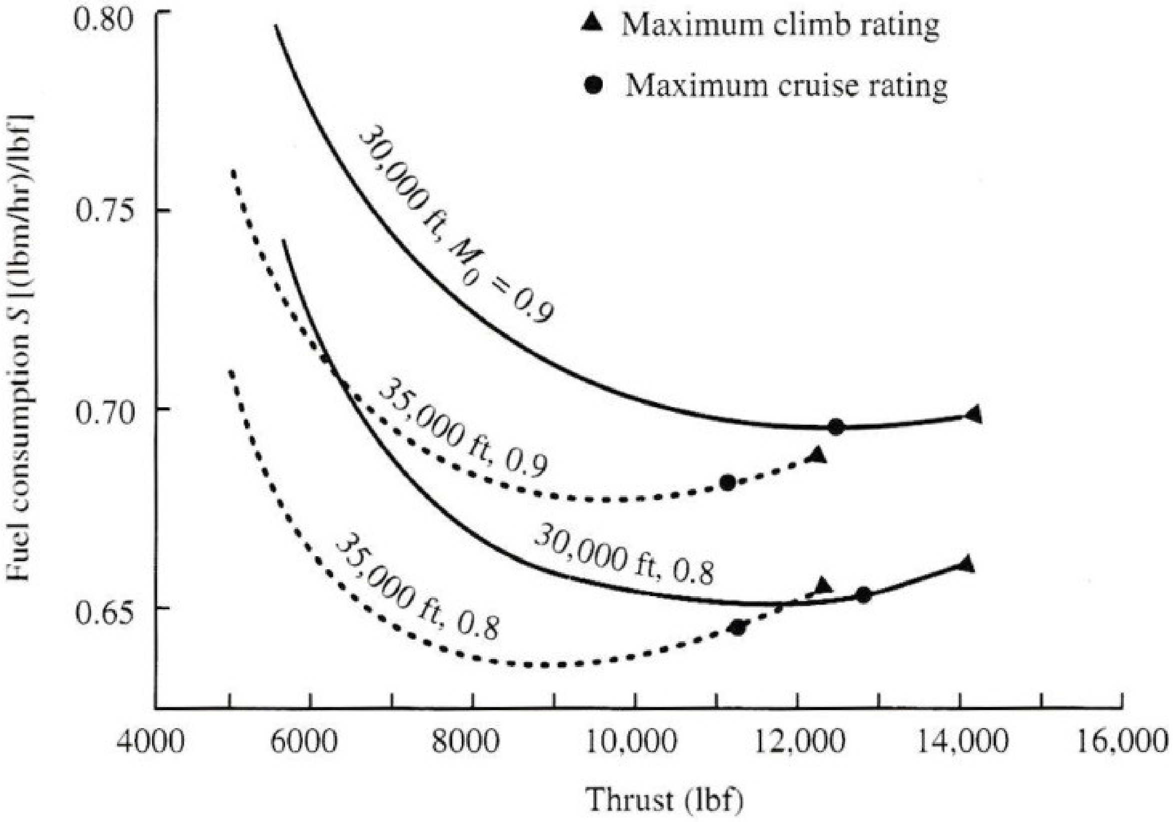


Fig. 5.1 JT9D-70/-70A turbofan cruise TSFC (**Mattingly 1996**, p. 25)

The second source is Fig. 5.2 from **Hill 1992**. It shows typical data from a turbofan of the RB211 family from Rolls-Royce. The diagram shows four curves with constant TAS of 200, 300, 400 and 500 knots. The altitude is being kept constant at 30000 ft for all TAS. All four curves were chosen to be suitable for the project since they are as well in the range of common cruise altitude and common cruise TAS.

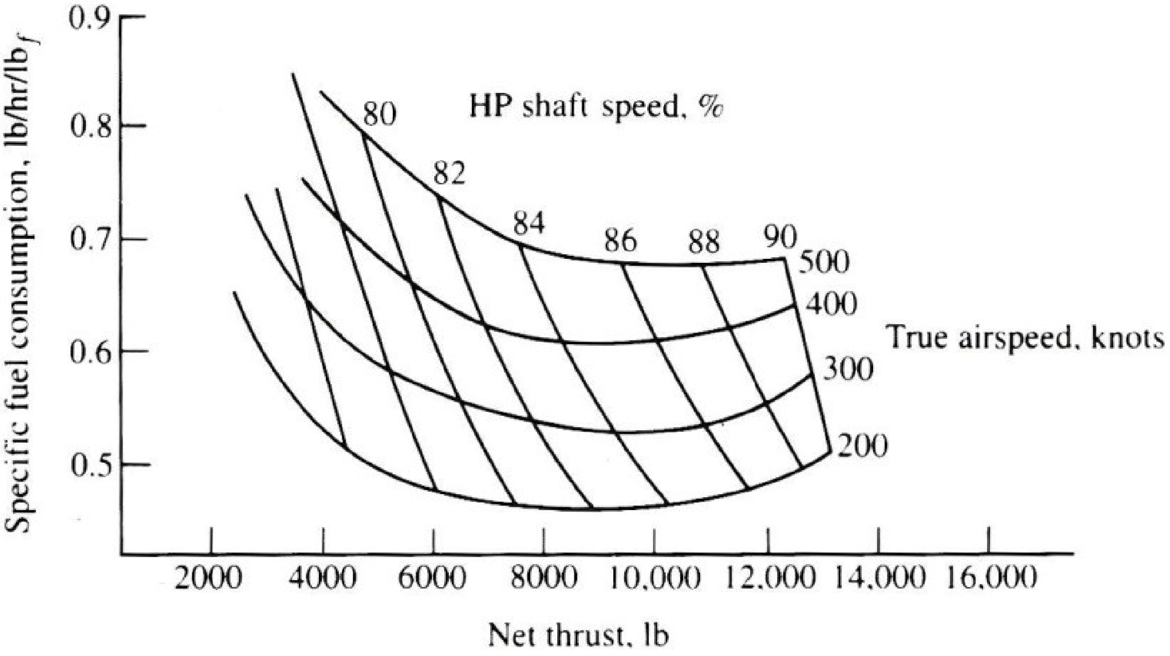


Fig. 5.2 Typical RB211 turbofan performance (**Hill 1992**, p. 197)

The third source is Fig. 5.3a from **Risse 2014**. The reference is a presentation about key data from CeRAS (Central Reference Aircraft data System). In CeRAS the RWTH Aachen University remodeled the A320; hence they also remodeled the fitting V2527-A5 turbofan from the joint venture IAE. The source is therefore not actual data, however, the data is said to be very similar to the original A320 data. In Fig. 5.3a the TSFC was modeled for a cruise Mach number of 0.78 and a cruise altitude of 35000 ft. One can see the performance with or without power off-takes. For this project, only the curve with power off takes was found to be suitable for data extraction, since the data from the other sources resemble actual data, where power off takes are always included. Moreover, the two curves are almost identical; they are only shifted a little bit on the y-axis. In a dimensionless depiction, both curves would look almost identical.

Similar to Fig. 5.3a is Fig. 5.3b from **Risse 2016**. Fig. 5.3b shows a similar shape and was not evaluated here for that reason.

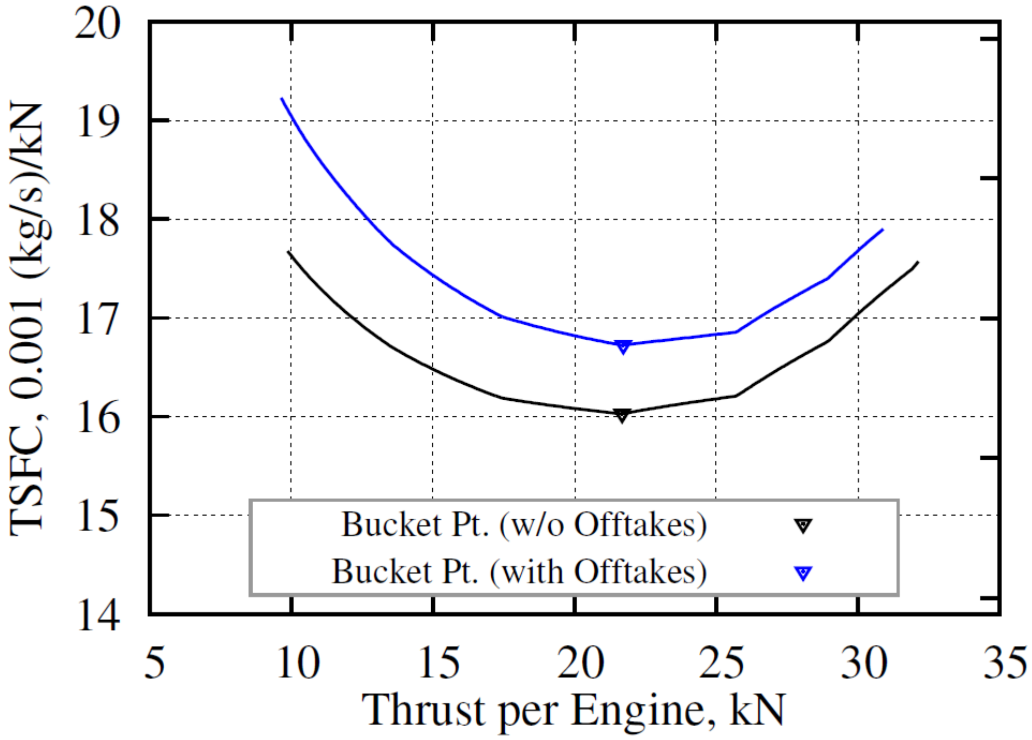


Fig. 5.3a Performance of the V2527-A5 turbofan (**Risse 2014**, p. 6)

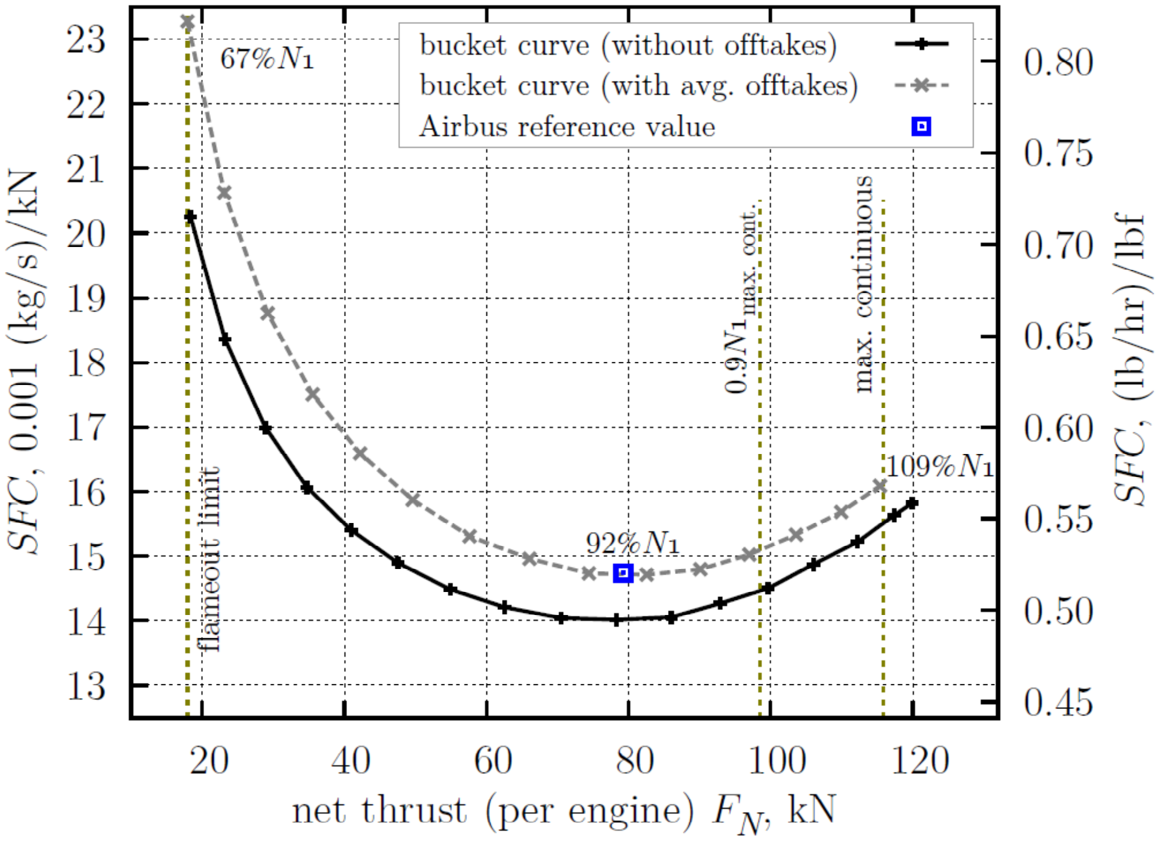


Fig. 5.3b SFC bucket curves at cruise design conditions (M = 0.85, 35000 ft, ISA), with and without the influence of average off-takes (**Risse 2016**, p. 129)

Finally, the fourth source is Fig. 5.4 from **Scholz 2018a**. TSFC of the RB211-524-D4 turbofan from Rolls Royce was remodel at Cranfield University with a program called Turbomatch. The calculations were made for an altitude of 10000 m and Mach numbers from 0.0 to 0.8 – giving nine different curves. For this project, only the curves with a Mach number higher or equal to Mach 0.4 were found suitable for the data extraction. This decision was made because Mach numbers smaller than Mach 0.4 are uncommon for cruise. Even Mach 0.4 might be an unrealistic cruise speed, but it may be helpful as the lower boundary.

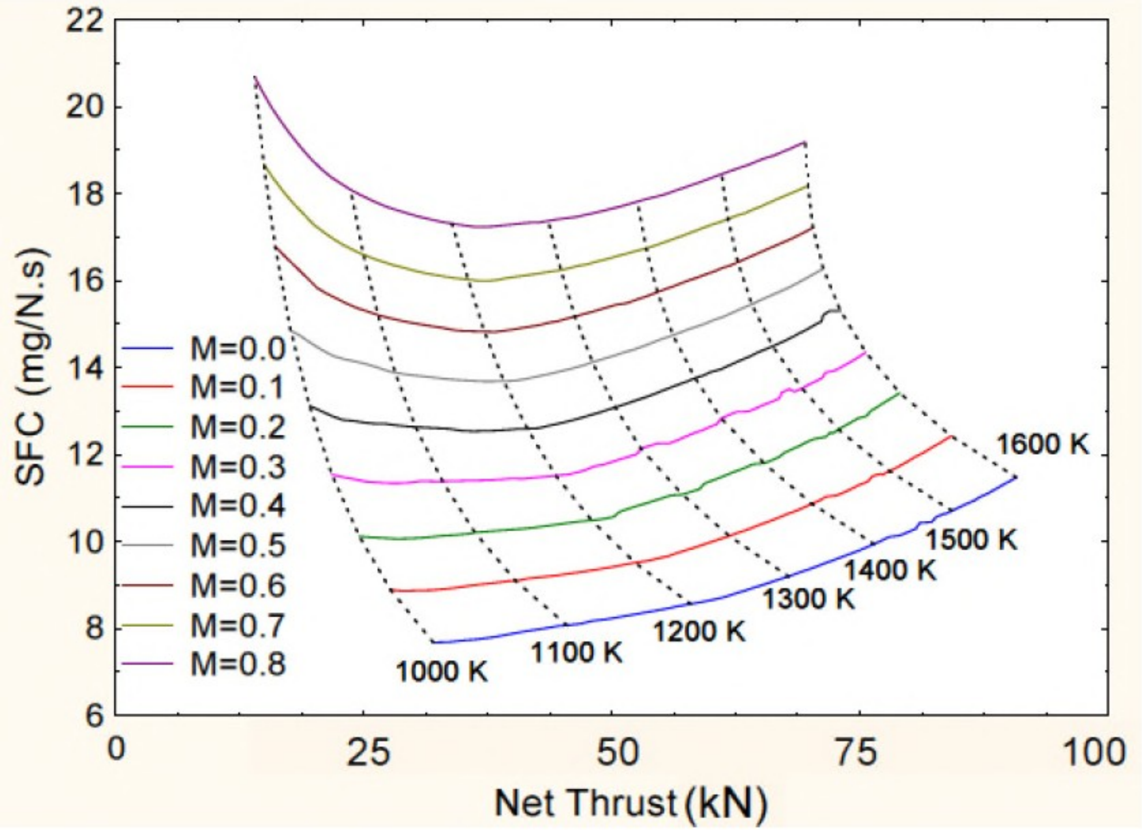


Fig. 5.4 Performance of the RB211-524-D4 turbofan (**Scholz 2018a**)

5.2 Comparison of Extracted Data

To properly compare the data from the four sources they were digitalized with WebPlotDigitizer as explained in Chapter 2.5. Afterwards, the data had to be processed to be dimensionless. The lowest point of each bucket curve was chosen as the design mark. The values of TSFC were divided by the lowest TSFC and the values of the thrust were divided by the thrust at the point with the lowest TSFC. In doing that, it is possible to see the relative values for TSFC of thrust. In Fig. 5.5 all 14 datasets from the four sources are put together. It is clearly visible that the curves share the same tendencies.

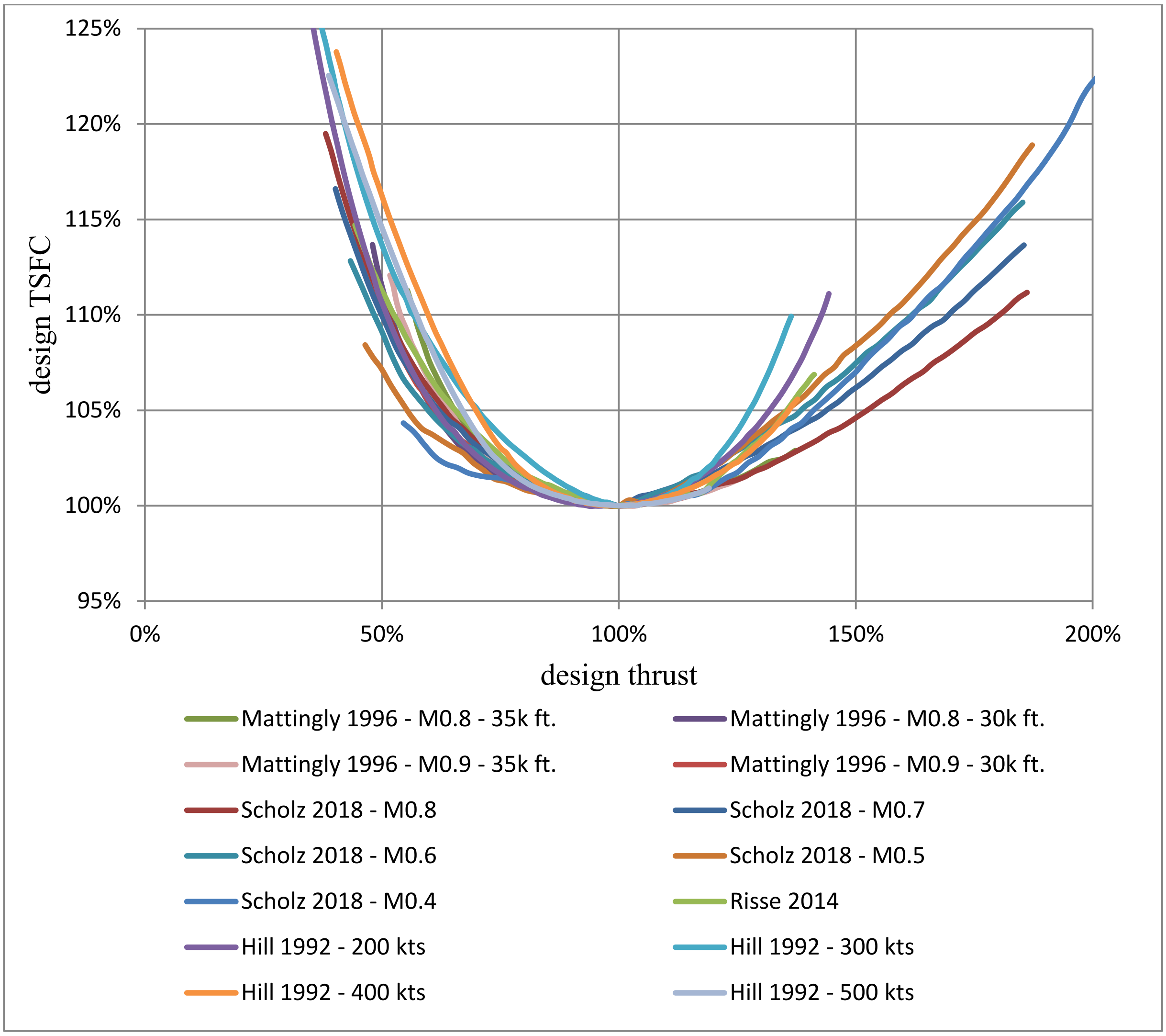


Fig. 5.5 Dimensionless TSFC over thrust from extracted data

5.3 Importance of TSFC Variation Due to Thrust Change

In this project, the focus is set on cruise flight. That is the reason why Fig. 5.5 has to be further evaluated since the thrust during cruise will not be increased or decreased by 50%.

Each aircraft is perfectly matched with its engine. The assumption is made that when TSFC is at its minimum, the aircraft is in the middle of its flight. In level cruise flight the thrust of the engines equals the drag on the aircraft. The drag for the entire aircraft can be estimated by (2.6). The total drag then must be divided by the number of engines on the aircraft to get the thrust of each engine. To get the thrust deviations of a mix of different sized aircraft, the Airbus A320, A330, A350 and A380 were selected as a basis for the calculation. The specifications for each aircraft can be found in Table 5.2. The thrust for each engine was calculated for maximum take-off weight (MTOW) and maximum zero-fuel weight (MZFW) since those two configurations are the heaviest and lightest. The thrust for each engine was also calculated for the mean of MTOW and MZFW so that a deviation from that value could be calculated.

The Oswald factor for each aircraft was estimated to be 0.85. The zero-lift drag coefficient was estimated to be 0.02 for each aircraft. Even though we assume a step-climb, the density was set to 0.3692 kg/m³, which is according to ISA conditions in the tropopause.

Table 5.2 Aircraft Specifications (**A320 2018**, **A330 2017**, **A350 2018**, **A380 2016**)

	A320	A330	A350	A380
Mach Number in Cruise	0,78	0,82	0,85	0,85
TAS in 11 km [m/s]	230	242	250	250
Wing Area [m²]	120	362	442	845
Wing Span [m]	34	60	65	80
MZFW [kg]	55560	175000	196000	369000
Average Mass [kg]	64530	208500	238000	472000
MTOW [kg]	73500	242000	280000	575000
No. of Engines [-]	2	2	2	4
Take-Off Thrust [kN][a]	110	310	370	350

[a] per Engine

The calculated thrust for the light, medium and heavy configurations as well as the deviations from both light to medium and heavy to medium can be seen in Table 5.3. Most of the aircraft have a maximum deviation of around 10%, even the worst case scenario is still under a deviation of 15% which means that the scale of Fig. 5.5 does not help in evaluating the influence of thrust variations.

Table 5.3 Thrust variations in cruise flight for multiple aircraft

Aircraft	Thrust per engine in configuration			Deviation	
	Light (MZFW) kN	Medium (average) kN	Heavy (MTOW) kN	Light to medium %	Heavy to medium %
A320	16.65	18.37	20.34	9.36	9.72
A330	53.31	59.26	66.25	10.04	10.55
A350	65.20	71.93	79.98	9.37	10.06
A380	65.36	75.93	89.09	13.92	14.77

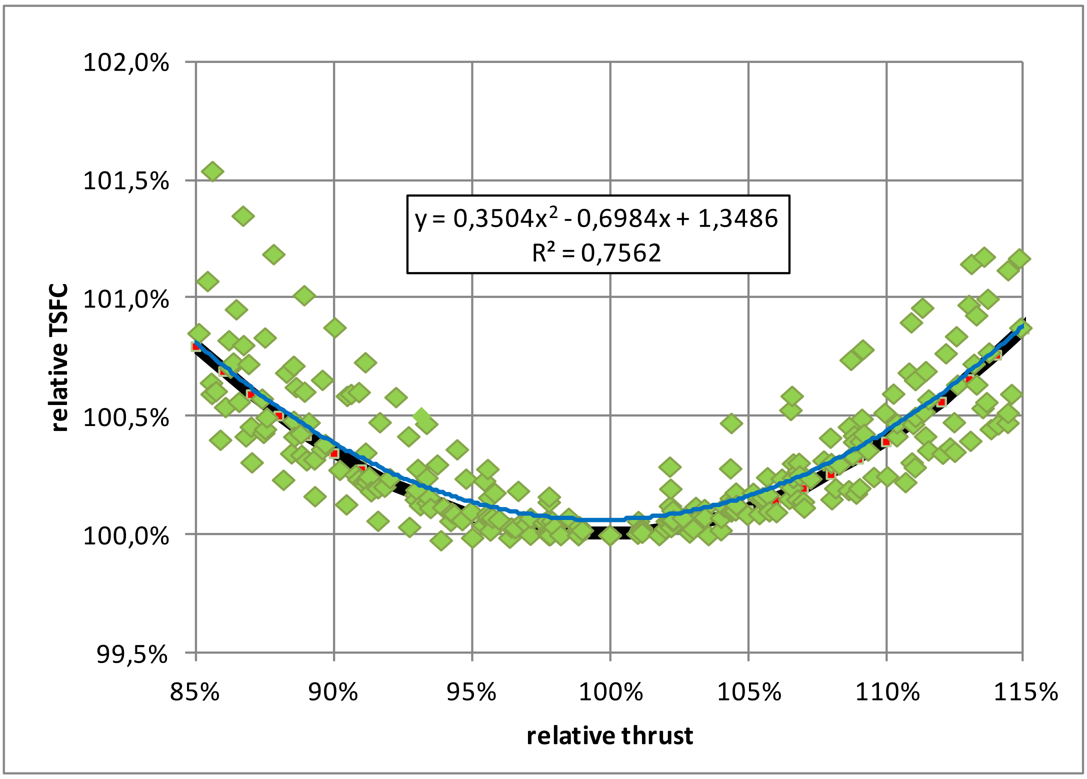


Fig. 5.6 Dimensionless TSFC data (green) over thrust for cruise flight with curve fit (blue) and curve fit through 100% relative thrust and 100% relative TSFC (black with red dots)

In Fig. 5.6 the axes have been scaled according to the expected thrust variations. The data points are no longer separated by their source. A regression for the data was made for a second order polynomial function with free coefficients. The resulting equation is given in Fig. 5.6. Since the curve has to yield 100% relative TSFC for 100% relative thrust the adapted polynomial $y = ax^2+bx+c$ with $c = 1-a-b$ is

$$\frac{c_T}{c_{T,ref}} = 0.3722\left(\frac{T}{T_{ref}}\right)^2 - 0.7420\frac{T}{T_{ref}} + 1.370 \tag{5.3}$$

In (5.3) the reference value for the TSFC is the minimum TSFC value, while the reference value for the thrust is the thrust at the same condition, where the TSFC is at its minimum. The coefficient of determination R^2 drops to 0.58 which shows that there is a good amount of uncertainty. However, if one looks at Fig. 5.6 it can be seen that a common general trend is covered by the function.

The magnitude of the influence can be evaluated by taking the worst-case scenario from Table 5.3 and the worst-case scenario from Fig. 5.6. In that scenario, there would still only be a variation in TSFC of less than 1%. For normal cruise operation of commercial aircraft, the impact of thrust variation would only be a fraction of a percent. **Therefore it is fair to say that there is no need to include the effect of thrust on TSFC into account for aircraft design calculations.**

Fig. 5.6 is in general agreement with a figure published by **Young 2018** (presented here as Fig. 5.7). Since **Young 2018** has only published lately, the information from Fig. 5.7 has not been included in the numerical evaluation.

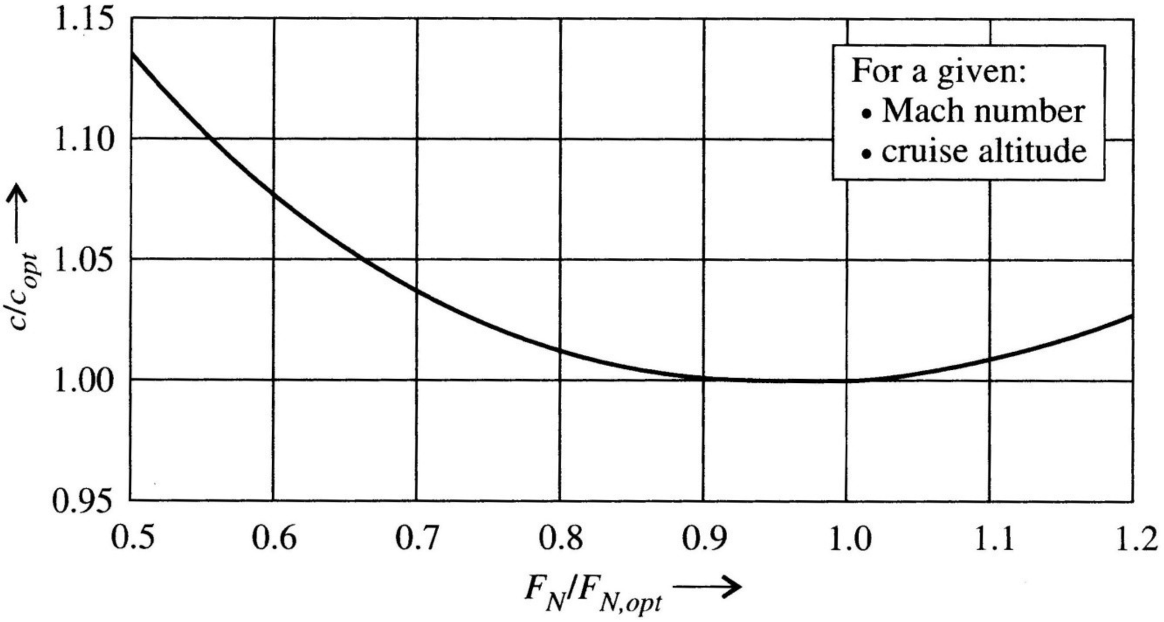


Fig. 5.7 Typical cruise TSFC versus thrust relationship normalized with respect to the optimum (or lowest) TSFC condition (**Young 2018**)

6 Numerical Optimum Speed for Max Range

Cruise of jet airplanes if optimized for **maximum range** by maximizing Specific Air Range (SAR). Details were given in Chapter 2. If SAR is maximized this means also that for a given flight distance we get **minimum fuel consumed**.

In Chapters 3 through 5 it was shown that TSFC = *f (T, h, V)* can be reduced to TSFC = *f (V)*. That information stands in contrast to the academically taught optimum speed from Chapter 2.4.1, where TSFC was considered to be constant.

To numerically optimize the speed for maximum range, (2.5) will once again be reviewed. To maximize the range, the full term

$$\frac{c_T(V)\,D(V,m)}{V} \tag{6.1}$$

will have to be minimized. Everything in (6.1) is dependent on the speed V. Drag, $D = f(V)$ can easily be calculated from the speed polar by (2.6). The speed polar shows the primary influence of speed on drag. Further details as the influence of Mach number on induced drag and the addition of wave drag are ignored in this project.

The calculation was once again done for three different mass configurations, *m*, for each aircraft, given in Table 5.2 with a density of 0.3692 kg/m³ in 11 km altitude. The minimum drag speed was calculated by (2.12).

For $c_T = f(V)$ two models were presented; a linear and a non-linear approach. The linear approach is Equation (4.1) and the non-linear approach is the one from **Herrmann 2010**, published in **Scholz 2013**. For the latter model take-off thrusts are required. In accordance with current engine options, the following were selected: A320: 110 kN, A330: 310 kN, A350: 370 kN, A380: 350 kN. Further parameters for the method from Herrmann included an altitude of 11000 m and a BPR of 5.0. Aim is not a detailed calculation of the named aircraft. These aircraft are only taken to get somewhat realistic numbers for a generic type of calculation.

In Table 6.1 the results of the numerical calculation for optimum speed can be seen as well as the factor between the optimum speed and the minimum drag speed. This allows comparing these results with the academically taught derivation.

The linear model for TSFC has an optimum of V_{opt} = 1.15 V_{md} for almost every aircraft and each weight class. However, there is a tendency to decrease to V_{opt} = 1.14 V_{md} in the heavy configuration. These results can be further improved, once data for an actual engine is available. It would allow reconfiguring c_a and c_b in (4.1) and be more accurate in TSFC estimation. Hence, the optimum speed might shift a little bit and it would fit the aircraft more precisely.

The non-linear model for TSFC has an optimum of V_{opt} = 1.05…1.11 V_{md} depending on the weight configuration. The factors are lower for heavy aircraft and increase towards higher values, once aircraft get lighter. On average V_{opt} = 1.09 V_{md} would be a good estimation.

As it was expected from Fig. 4.3, the results of both models are quite close to one another, the results after Herrmann being somewhat smaller. Both models combined would bring an optimum speed of approximately V_{opt} = 1.12 V_{md}.

Table 6.1 Optimum speeds according to different models

		Herrmann		Roux-Scholz	
Aircraft	V_{md} m/s	V_{opt} m/s	V_{opt}/V_{md} -	V_{opt} m/s	V_{opt}/V_{md} -
A320					
Light	185.2	204.5	1.10	213.5	1.15
Medium	199.6	217.5	1.09	229.0	1.15
Heavy	213.0	229.5	1.08	243.5	1.14
A330					
Light	187.8	207.0	1.10	216.5	1.15
Medium	204.9	222.5	1.09	235.0	1.15
Heavy	220.8	236.5	1.07	252.0	1.14
A350					
Light	181.6	201.0	1.11	209.5	1.15
Medium	200.1	218.5	1.09	229.5	1.15
Heavy	217.1	233.5	1.08	248.0	1.14
A380					
Light	191.0	210.0	1.10	220.0	1.15
Medium	216.0	232.5	1.08	247.0	1.14
Heavy	238.4	251.5	1.05	270.5	1.13

.

7 Summary and Conclusions

In the course of this project, it was found that the TSFC is a function of altitude, speed, and thrust. A further literature review has shown that the **TSFC dependency on altitude** may be neglected since it has only little influence in the range of altitudes where passenger aircraft usually cruise. The **TSFC dependency on speed** was presented in a linear and non-linear model and it was shown that the linear model is similar to the non-linear model. Hence, because of its simplicity, the linear model might preferably be used for a first estimation. Further actual data of an engine would greatly improve the linear model so far that it might fit quite accurately. Since it was clearly visible that the TSFC grows with increasing speed, the idea of a constant TSFC was not further discussed. The third dependency that was investigated is the **TSFC dependency on thrust** while maintaining a constant speed and altitude. No model for this dependency could be found, which is why data was extracted from multiple sources. The sources were filtered to only cover cruise speeds and altitudes. A regression for the data was made and even though it did not fit perfectly, it covered the similarities of the different curves. Based on this data, a new function (Equation 5.3) was found to calculate the TSFC dependency on thrust as

$$\frac{c_T}{c_{T,ref}} = 0.3722\left(\frac{T}{T_{ref}}\right)^2 - 0.7420\frac{T}{T_{ref}} + 1.370 \, .$$

Moreover, it showed that in the worst case scenario, thrust variations result in less than 1% of TSFC change. On that account, it was found that there is no need to take the effect of thrust on TSFC into account. Conclusively, it was found that **for estimations under cruise flight conditions, TSFC can be seen as a function of speed only.**

Two principally different derivations of **optimum speed for maximum range** were shown. If TSFC is considered constant V_{opt} = 1.316 V_{md}. In contrast, if PSFC is considered constant V_{opt} = 1.00 V_{md}. Since the real SFC characteristic is a mixture of these two extremes, the expectation was to find the true factor in the range 1.00…1.316. With both the linear and non-linear model for TSFC dependency on speed the optimum speed for the maximum range was numerically calculated. Indeed, results proved this hypothesis. The linear model showed an optimum speed of V_{opt} = 1.15 V_{md}, while the non-linear model showed different optimum speeds depending on aircraft weight that were between **V_{opt} = 1.05…1.11 V_{md}**.

These results show that jet aircraft have to fly about 9% faster than minimum drag speed to achieve minimum fuel consumption.

This project only considered drag, $D = f(V)$ calculated from the speed polar (2.6). Further details as the influence of Mach number on induced drag and the addition of wave drag were ignored. **It can be recommended to start another study where the aerodynamic effect of**

Mach number on drag is modeled. **Scholz 2018b** has simplified aerodynamic methods for the calculation of the Oswald factor including the Mach-effect and for wave drag estimation that could be applied. Taking these effects into account drag would show a stronger drag increase with speed. This would mean that the **optimum speed is reduced even more** towards the minimum drag speed V_{md}. This means that the effect shown in this project would most probably come out even more pronounced.

In reality, the TSFC dependencies are extremely complicated and an analytical solution is almost impossible. Collecting big amounts of data on engines helps to optimize conditions and develop future generations of engines. That is the reason why the manufacturers are doing just that and also keep their data as secret as possible.

References

A320 2018 Airbus: A320 Aircraft Characteristics – Airport and Maintenance Planning. Toulouse : Airbus, 2018 (Revision No. 34 - Feb 01/18). http://www.airbus.com/aircraft/support-services/airport-operations-and-technical-data/aircraft-characteristics.html (2018-06-28), archived as http://bit.ly/2KmHxOa

A330 2017 Airbus: A330 Aircraft Characteristics – Airport and Maintenance Planning. Toulouse : Airbus, 2017 (Revision No. 24 - Jan 01/17). http://www.airbus.com/aircraft/support-services/airport-operations-and-technical-data/aircraft-characteristics.html (2018-06-28), archived as: http://bit.ly/2yZ0IIJ

A350 2018 Airbus: A350 Aircraft Characteristics – Airport and Maintenance Planning. Toulouse : Airbus, 2018 (Revision No. 4 - Jun 01/18). http://www.airbus.com/aircraft/support-services/airport-operations-and-technical-data/aircraft-characteristics.html (2018-06-28), archived as: http://bit.ly/2KAvGLH

A380 2016 Airbus: A380 Aircraft Characteristics – Airport and Maintenance Planning. Toulouse : Airbus, 2016 (Revision No. 15 - Dec 01/16). http://www.airbus.com/aircraft/support-services/airport-operations-and-technical-data/aircraft-characteristics.html (2018-06-28), archived as: http://bit.ly/2z5y9JM

CeRAS 2018 Risse: CeRAS (Central Reference Aircraft data System), 2018. – URL: http://ceras.ilr.rwth-aachen.de (2018-06-16), archived as: http://bit.ly/2KxJhjz

Cranfield 2018 Cranfield University: Centre for Propulsion Engineering, PERSEUS Theme in Propulsion, TURBOMATCHURL, 2018. – : https://www.cranfield.ac.uk/centres/centre-for-propulsion-engineering/epsrc (2018-06-17), archived as: http://bit.ly/2IJ8mX4

Herrmann 2010 Herrmann, Steffen: *Untersuchung des Einflusses der Motorenzahl auf die Wirtschaftlichkeit eines Verkehrsflugzeuges unter Berücksichtigung eines optimalen Bypassverhältnisses*. Berlin, TU Berlin, Fachgebiet Luftfahrzeugbau und Leichtbau, Diploma Thesis, 2010

Hill 1992 HILL, Philip; PETERSON, Carl: *Mechanics and Thermodynamics of Propulsion*. Reading, Massachusetts : Addison-Wesley Publishing Company, 1992. – URL: https://archive.org/details/MechanicsAndThermodynamicsOfPropulsionHillPeterson (2018-06-28)

Mattingly 1996 MATTINGLY, Jack: *Elements of Gas Turbine Propulsion*. New York : McGraw-Hill, 1996. – Archived as: https://perma.cc/QCG5-APDV

Meier 2005 MEIER, Nathan: *Civil Turbojet/Turbofan Specifications*, 2005. – URL: http://www.jet-engine.net/civtfspec.html (2018-06-12), archived as: http://bit.ly/2tPqVDH

Risse 2014 RISSE, Kristof; SCHÄFER, Katharina: *CeRAS - Central Reference Aircraft data System, CSR01, Propulsion*, Aachen, RWTH Aachen, Institute of Aerospace Systems, 2014. – URL: http://bit.ly/2IFhSKW (2018-06-12), archived as: http://bit.ly/2lP3CXb

Risse 2016 RISSE, Kristof: Preliminary Overall Aircraft Design with Hybrid Laminar Flow Control. Dissertation, RWTH Aachen University, 2016. – URL: http://doi.org/10.18154/RWTH-2017-00974 (2018-06-28)

Rohatgi 2018 ROHATGI, Ankit: *WebPlotDigitizer*. Austin, Texas, USA, URL: https://automeris.io/WebPlotDigitizer (2018-06-16), archieved as: http://bit.ly/2tSZwk3. Softare archived: http://bit.ly/2yZBee4

Roux 2002 ROUX, Élodie: *Modèles Moteur…Réacteurs double flux civils et réacteurs militaires à faible taux de dilution avec PC*. Toulouse, ISAE-SUPAERO, Report, 2002
http://elodieroux.com/ReportFiles/ModelesMoteurVersionPublique.pdf (2018-06-28), archived as: http://bit.ly/2Nb6r1b

Scholz 2013 SCHOLZ, Dieter; SERESINHE, Ravinka, STAACK, Ingo; LAWSON, Craig: Fuel Consumption due to Shaft Power Off-Takes from the Engine. In: ESTORF, Otto von; THIELECKE, Frank (Ed.): *4nd International Workshop on Aircraft System Technologies, AST 2013* (TUHH, Hamburg, 23.-24. April 2013). Aachen : Shaker, 2013, pp. 179 - 179. – ISBN 978-3-8440-1850-9, download CD-Version(!): http://Off-Takes.ProfScholz.de

Scholz 2017 SCHOLZ, Dieter: *Der spezifische Kraftstoffverbrauch von Flugtriebwerken (TSFC und PSFC)*. Hamburg, HAW Hamburg, Department of Automotive and Aeronautical Engineering, Memo, 2017. – URL: http://reports-at-aero.ProfScholz.de (2018-06-28)

Scholz 2018a SCHOLZ, Dieter: *Typical Thrust Specific Fuel Consumption (SFC) for a Jet Engine used on Short-Medium Range Aircraft Simulated with Turbomatch at Cranfield University*. Personal Information, 2018

Scholz 2018b SCHOLZ, Dieter: *Aircraft Design*. Hamburg, HAW Hamburg, Department of Automotive and Aeronautical Engineering, Lecture notes, 2018. – http://HOOU.ProfScholz.de (2018-06-28)

Schulz 2007 SCHULZ, Oliver: *Assessment of Numerical Models for Thrust and Specific Fuel Consumption for Turbofan Engines*. Hamburg, HAW Hamburg, Department of Automotive and Aeronautical Engineering, Diploma Thesis, 2007. – http://library.ProfScholz.de (2018-06-28)

Torenbeek 1982 TORENBEEK, Egbert: *Synthesis of Subsonic Airplane Design*. Delft : Delft University Press, 1982. – http://resolver.tudelft.nl/uuid:229f2817-9be9-49b6-959a-d653b5bac054 (2018-06-28)

Young 2001 YOUNG, Trevor: *Flight Mechanics*. Limerick, University of Limerick, Department of Mechanical & Aeronautical Engineering, Lecture Notes, 2001. – http://FM.ProfScholz.de (password required) (2018-06-28)

Young 2018 Young, Trevor: *Performance of the Jet Transport Airplane*. Chichester, UK : Wiley, 2018. – ISBN: 978-1-118-38486-2