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Tackling excess noise from bilinear and nonlinear couplings in gravitational-wave interferometers

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Abstract.

We describe a tool we improved to detect excess noise in the gravitational wave (GW) channel arising from its bilinear or nonlinear coupling with fluctuations of various components of a GW interferometer and its environment. We also describe a higher-order statistics tool we developed to characterize these couplings, e.g., by unraveling the frequencies of the fluctuations contributing to such noise, and demonstrate its utility by applying it to understand nonlinear couplings in Advanced LIGO engineering data. Once such noise is detected, it is highly desirable to remove it or correct for it. Such action in the past has been shown to improve the sensitivity of the instrument in searches of astrophysical signals. If this is not possible, then steps must be taken to mitigate its influence, e.g., by characterizing its effect on astrophysical searches. We illustrate this through a study of the effect of transient sine-Gaussian noise artifacts on a compact binary coalescence template bank.

1. Introduction

The advanced detector era was ushered in by the two Advanced LIGO (aLIGO) detectors [1] in September 2016. With plans afoot for the Advanced Virgo (AdV) detector [2] to join it this decade, we expect the era of gravitational wave (GW) astronomy to begin in earnest [3]. The sensitivity of an interferometer in the detection band (which in aLIGO is currently between a few tens of Hertz to a few kiloHertz) [1], however, can be affected adversely due to unwanted noise arising from various sources. The effect of noise transients (see, e.g., Ref. [4] and references therein) on astrophysical searches was studied in Ref. [5] for binary neutron signals, in Refs. [6, 7] for intermediate mass black hole signals, in Ref. [8] for binary black hole signals, and in Ref. [9] for unmodeled transient signals. Detector characterization efforts that have been invested toward improving the quality of GW data in the recent past are described in Refs. [10, 11, 12]. In this work we improve upon a subset of those efforts, specifically, in regards to identifying the presence of certain types of noise that appear in the detection band owing to bilinear or nonlinear coupling of fluctuations at lower frequencies. We also describe a higher-order statistics tool we developed to characterize these couplings by finding the frequencies of the fluctuations that contribute to such noise. We demonstrate its utility by applying it to aLIGO engineering data.



One of the simplest methods used for identifying the source of excess noise in the GW strain data of a detector is to measure its coherence with data recorded by, say, a sensor monitoring an environmental disturbance or a fluctuation in an interferometer component. Sensors record such data as time-series in multiple channels. For instance, time variations in the pitch, yaw, and roll (angular) degrees of freedom of each test mass can be extracted from the time-series data acquired by the Optical Sensors and Electromagnetic Actuators (OSEMs) attached to the multiple stages and levels of their suspension and recorded in various channels. The power-line harmonics can be extracted from the data recorded by the magnetometers. On the other hand, an example of an environmental disturbance is ground motion, which is measured by seismometers and accelerometers.

The coherence statistic is useful in identifying noise sources, be they internal to the interferometer or environmental, that couple linearly with the GW channel. There are various tools available to find them effectively and, therefore, will not be studied here (see, e.g., Ref. [14] and references therein). Unwanted noise can also arise in the GW channel owing to higher order couplings among multiple channels. For instance, when a servo-control system drifts near the edge of its actuation range, a further small fluctuation can drive the actuator into a nonlinear regime, causing elevated rate of noise transients in the GW channel. This can involve multiple channels in a nonlinear fashion, such as when a large alignment drift reduces the actuation range for additional length fluctuations. Another example would be a bilinear combination of a slow angular drift in alignment that causes a measured light beam to drift off its detector, thus amplifying the effect of any further higher-frequency jitter of the light-beam alignment. This type of behavior increases the noise background of searches for short-duration astrophysical signals, e.g., from binary black hole coalescences, hence, reducing the detector's sensitivity to them [10].

The Bilinear Coupling Veto (BCV) tool [16] has been shown to be useful in identifying the presence of non-Gaussian, bilinear component of the noise to a certain degree. While there are many tools that can identify the presence of component frequencies in a given time series, those typically employed are not able to provide any definitive evidence of nonlinear couplings that may be present (see, however, Ref. [17]). For example, the noise power spectral density (PSD) of the GW channel can show frequencies at which excess noise due to such couplings is present (see, e.g., the side-bands of the violin mode of a LIGO test mass in Fig. 1). However, there is no phase information present here, and one needs to employ higher-order statistics, in general, for diagnosing the presence of non-Gaussianity and nonlinear couplings [18]. BCV addresses this problem, partially, by finding strong coherence between the GW time-series, on the one hand, and the product of the time-series of two interferometer channels, on the other hand. Reference [16] termed this product as a *pseudo* channel. (Note that the pseudo channel can be constructed from environmental channels as well.) Typically, one of the time-series in that product is taken from a channel that records slow variation, e.g., the changing pitch-angle of an end-test mass, and the other time-series is taken from one that records fast variation, e.g., the signal controlling the length of the power-recycling cavity. We call this coherence the BCV statistic. In this work, we use the higher-order statistics tool to unearth some characteristics of the noise couplings that BCV finds.

2. Detecting bilinear and nonlinear noise couplings

The last published work on BCV [16] presented results obtained from its application to LIGO's sixth Science Run "S6" data, when the wavelet-based Kleine Welle [19] excess-power detection tool was used to identify noise transients. In preparation for aLIGO observations, we have upgraded BCV to work (a) additionally with transients found by "Omicron", which uses the Q -transform on a sine-Gaussian basis [20, 19], and (b) with aLIGO channels, which use a different nomenclature and are more numerous than LIGO channels.

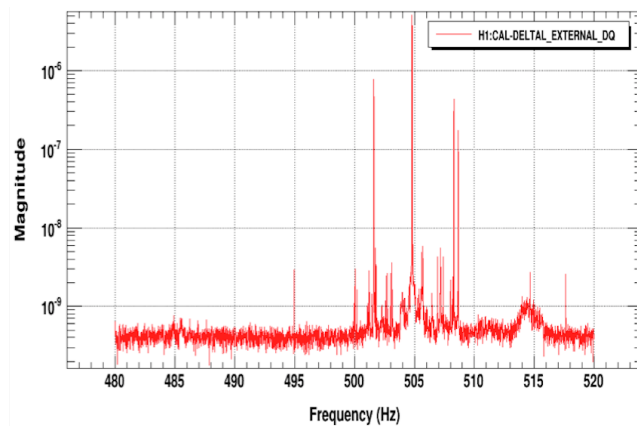


Figure 1. Excess noise in the form of side-bands of a violin mode, at approximately 505 Hz, suggests the possible existence of nonlinear coupling between the strain data in the GW channel (which is represented here by “H1:CAL-DELTA_EXTERNAL_DQ”) and noise in a host of other channels. (The discussion around Eq. (3) explains the origin of such side-bands.)

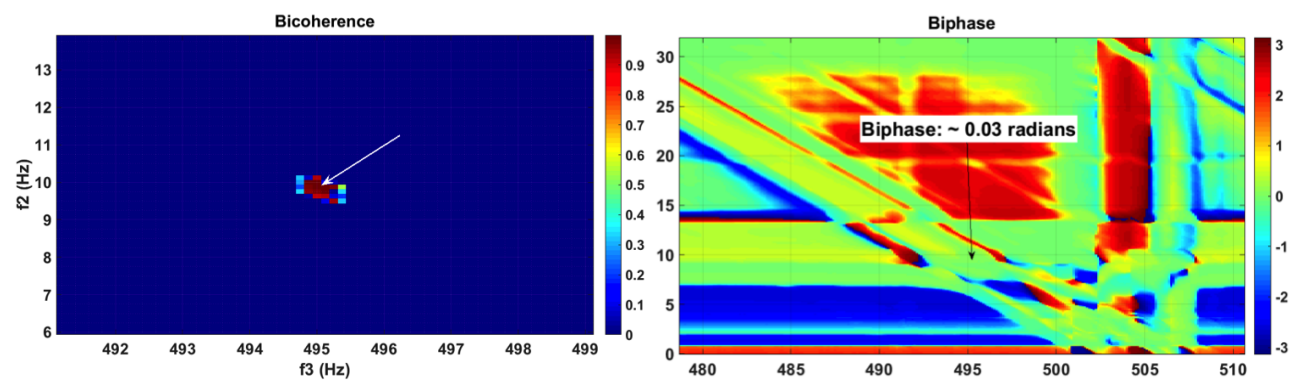


Figure 2. The bounce mode ($f_2 \approx 10$ Hz) of one of the test masses couples with its violin mode ($f_1 \approx 505$ Hz) to create a side-band at ($f_3 \approx 495$ Hz). The color-bar shows the bicoherence and the biphase (in radians) in the left and right figures, respectively, computed on the GW strain data.

We use BCV to identify episodes of bilinear coupling as follows. First, the occurrence times and other characteristics of excess noise transients or “glitchiness” are identified by Omicron or Kleine Welle in the GW channel and multiple instrumental and environmental channels that record fast variations. Second, time-coincidences of transients in that latter set of channels and the GW channel are identified for possible correlations between them. Third, during times of coincidences the BCV statistic is computed on the GW time-series and a pseudo channel, which is a product of the time-series with fast variations and a time-series picked from channels recording slow variations. For every coincidence, the last step is repeated over multiple channels that record slow variations. A strong value of the BCV statistic indicates the presence of bilinear coupling. To assess how large the value of this statistic can be in the absence of a coupling, the BCV statistic is computed also for acausally large relative time-shifts between the GW channel and the pseudo channel of interest. (Reference [16] used the bilinear coherence statistic for identifying bilinear coupling, but it is possible to replace it with the *bicoherence* statistic defined below for diagnosing the presence of nonlinear couplings.) When the BCV statistic is found

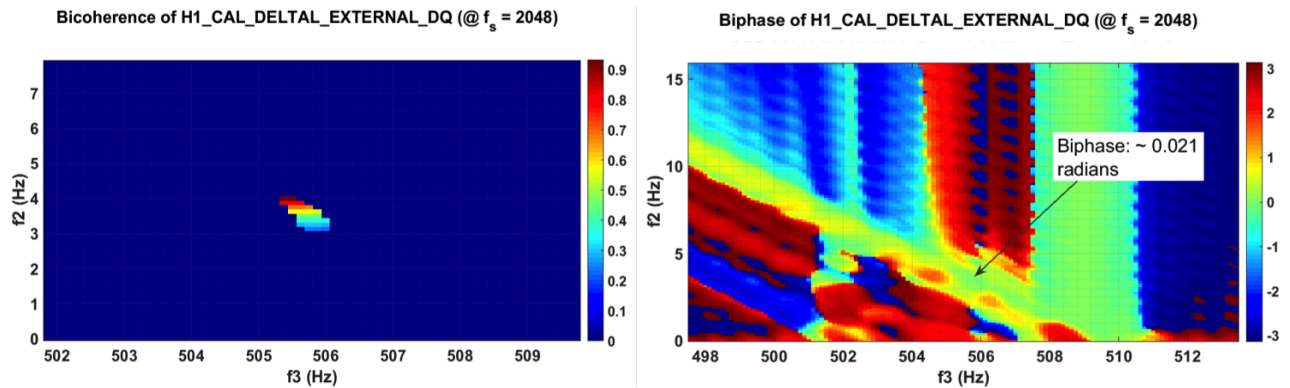


Figure 3. A mechanical resonance at ($f_2 \approx 3.8$ Hz) of the suspension of one of the test masses couples with its violin mode ($f_1 \approx 509$ Hz) to create a side-band at ($f_3 \approx 505.2$ Hz). The color-bar shows the bicoherence and the biphase (in radians) in the left and right figures, respectively, computed on the GW strain data.

to be significantly larger than the background, the GW data is flagged for further data quality studies and for setting vetoes for astrophysical searches. However, to search for the sources of these transients, one must unearth more information about their characteristics than just what channels are involved in producing such bilinear / multi-linear or nonlinear couplings. For example, the frequencies of the disturbances that couple provide valuable information about the source and pathways of such couplings. Below we describe a few of those statistics that we use to find such information.

3. Characterizing noise couplings by employing higher order statistics

One possible way of probing the nature of beyond-linear-order couplings is to compute a higher order statistic, e.g., the bicoherence, of the disturbances that we suspect can couple with one another. Strong bicoherence between channels can suggest couplings of their respective subsystems. For instance, mechanical resonances of structures (e.g., periscopes) show up at higher frequencies in the GW and auxiliary channels as well as some sensor channels.

Mathematically, a set of N disturbances or “signals” x_k is said to combine linearly when they superimpose to form a new signal, z_{lin} , as follows:

$$z_{\text{lin}}(t) = \sum_{k=1}^N x_k(t), \quad (1)$$

In the special case where the x_k are sinusoidal, one gets

$$z_{\text{lin}}(t) = \sum_{k=1}^N a_k \cos(2\pi f_k t + \phi_k), \quad (2)$$

where a_k and f_k are the amplitudes and frequencies of the linearly combining signals, and ϕ_k are their initial phases, respectively. When $N = 2$, we will identify $x_1(t)$ as $x(t)$ and $x_2(t)$ as $y(t)$, and write $z_{\text{lin}}(t) = x(t) + y(t)$.

Though there are different means by which signals may combine nonlinearly, we will consider one of the simplest yet ubiquitous coupling, namely, a type of coupling known as the quadratic phase coupling (QPC) [21]. The reason for this choice is that it is the most significant term

in many nonlinear couplings and is also straightforward to model. In this case, two (or more) signals combine to form a new signal, $z(t)$. For a pair of coupled signals one gets,

$$\begin{aligned} z(t) &= p[x(t) + y(t)]^2 + q[x(t) + y(t)] \\ &= p[a_1 \cos(2\pi f_1 t + \phi_1) + a_2 \cos(2\pi f_2 t + \phi_2)]^2 \\ &\quad + q[a_1 \cos(2\pi f_1 t + \phi_1) + a_2 \cos(2\pi f_2 t + \phi_2)] , \end{aligned} \quad (3)$$

where p and q are real numbers, and the pair of frequencies, (f_1, f_2) , is known as a bifrequency. Heretofore, t will denote a time index. Since the right-hand side has terms that denote oscillations (and power) at frequencies $(f_1 \pm f_2)$, $2f_1$, and $2f_2$, in order to detect QPC it is necessary to use a statistic that can aid the detection of these frequency components. In some of the examples studied in this paper, f_2 is much smaller than f_1 . In such an event, excess noise will appear in the sidebands $(f_1 \pm f_2)$ of f_1 .

To obtain one such statistic, we first divide the time-series, $x(t)$, $y(t)$, and $z(t)$ into M segments $x_\alpha(t)$, $y_\alpha(t)$, and $z_\alpha(t)$, respectively, where α is the segment index. Next the Fourier transforms, $X_\alpha(f)$, $Y_\alpha(f)$, $Z_\alpha(f)$, of each of these segments is taken, respectively, followed by the computation of the average of the quantity $X_\alpha(f_1)Y_\alpha(f_2)Z_\alpha^*(f_1 + f_2)$ over those segments:

$$B(f_1, f_2) = \frac{1}{M} \sum_{\alpha=1}^M X_\alpha(f_1)Y_\alpha(f_2)Z_\alpha^*(f_1 + f_2) . \quad (4)$$

A normalized and real function derived from $B(f_1, f_2)$ is defined as

$$b(f_1, f_2) = |B(f_1, f_2)| \left[\frac{1}{M} \left| \sum_{\alpha=1}^M X_\alpha(f_1)Y_\alpha(f_2) \right| \frac{1}{M} \left| \sum_{\alpha=1}^M Z_\alpha^*(f_1 + f_2) \right| \right]^{-1} , \quad (5)$$

which takes values in the range $[0, 1]$. It attains large values when p is large, i.e., when a strong QPC is present. Another related statistic, defined for individual segments, is

$$\Delta\phi_\alpha(f_1, f_2) = \arg [X_\alpha(f_1)Y_\alpha(f_2)Z_\alpha^*(f_1 + f_2)] . \quad (6)$$

Its average, $\Delta\phi$, for a large enough M , should be at the level of the noise when a QPC is present. Otherwise it can be larger, depending on the type of oscillations present. The statistic $B(f_1, f_2)$ is related to the bispectrum, $B_Z(f_1, f_2)$, which is obtained by replacing $X_\alpha(f_1)$ and $Y_\alpha(f_2)$ in $B(f_1, f_2)$ by $Z_\alpha(f_1)$ and $Z_\alpha(f_2)$, respectively. The same replacements in $b(f_1, f_2)$ and $\Delta\phi$ yield the bicoherence, $b_Z(f_1, f_2)$ and biphas $\Delta\phi_Z$. Thus, for finding QPCs we search for frequency triplets, (f_1, f_2, f_3) for which the bicoherence is high (i.e., close to unity) and the biphas is small (i.e., close to noise level).

In Fig. 2 we show the bicoherence $b_Z(f_1, f_2)$ and the biphas $\Delta\phi_Z$, where Z is the Fourier transform of the GW strain data, in terms of (f_3, f_2) for a stretch of time that was flagged by BCV as a likely candidate for the presence of bilinear or nonlinear coupling noise. Here the bicoherence is strong at $(f_3, f_2) = (495, 10)$ Hz. This indicates that excess noise is being created by a QPC-like nonlinear coupling between $f_2 \approx 10$ Hz, which is known to be a bounce mode of a test mass, and $f_1 = f_2 + f_3 \approx 505$ Hz, which is the violin mode of the same test mass. In Fig. 3 we show plots of the same two quantities on a stretch of data where the bicoherence is strong at $(f_3, f_2) \approx (505.2, 3.8)$ Hz. This is strong evidence for nonlinear coupling noise between $f_2 \approx 3.8$ Hz, which is known to be a mechanical resonance of a test mass, and $f_1 = f_2 + f_3 \approx 509$ Hz, which is the violin mode of that test mass. (Note that different test mass suspensions have somewhat different violin mode frequencies.)

4. Effect of noise artifacts on astrophysical searches

Once a strong indication of bilinear or nonlinear coupling is found as the potential source of a noise artifact, steps must be taken to establish the causal nature of the effect, find its origin and then either remove it or correct its behavior, as the case may be. (More details on this procedure will be reported in a future work, where the improvement in the sensitivity of specific astrophysical searches in aLIGO data brought about by using BCV will be provided.) When it is not possible to completely remove it or correct it steps must be taken to mitigate its effect on the GW channel or on astrophysical searches. A brief example of one such step is presented in this section.

Increased noise, e.g, in the form of side-bands of the violin modes or line harmonics, that is reasonably stationary can hurt the sensitivity of a search for continuous-wave signals from spinning neutron stars [13] or, possibly, a stochastic GW background [14, 15]. On the other hand, transient noise artifacts can adversely affect the sensitivity of astrophysical burst or compact binary coalescence (CBC) searches by increasing their background [10, 22, 23]. For CBC searches, signal discriminatory tests, such as the chi-square test [24], help in keeping the background under control, especially for low-mass searches. For high-mass searches, where the search templates [8] are short in duration and have very few cycles, much like some of the noise transients themselves, these tests are known to be less powerful, and other tests must be devised to supplement them. One such idea was explored in Ref. [25], which found that for a class of these noise artifacts that can be modelled as sine-Gaussian bursts, the time-lag of the CBC templates that get triggered by a burst depends in a (semi-analytically) predictable way on the burst's central frequency and the template's chirp mass. (The chirp mass of a binary with component masses m_1 and m_2 is defined as $(m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$.) As shown in Fig. 4, the SNR of a trigger from such a glitch falls off with decreasing template chirp-mass (and increasing time-lag). For a CBC signal, the SNR typically falls much faster with time-lag (see, e.g., Ref. [26]). These characteristics can be capitalized upon by machine-learning algorithms to complement the ability of traditional chi-square tests to discriminate such glitches from real CBC signals [27, 28].

5. Conclusion

Coupling of disturbances at low frequencies, such as the bounce and roll modes, and those at higher frequencies, e.g., the violin modes or power-line harmonics, can adversely affect the detector sensitivity in the band where we expect to find astrophysical signals. Noise of this type can create a stable background or one that ebbs and flows slowly with changing anthropogenic, environmental (e.g., wind-speed variation, microseismic, etc.) or operating conditions, unless it is recognized and removed. Here we demonstrated how BCV can find the presence of such noise couplings and showed how higher order statistics can be used to diagnose certain aspects of those couplings. By way of their power for finding bilinear and nonlinear couplings and diagnosing some of their properties these statistics make useful additions to the set of detector characterization tools that are already being employed for improving aLIGO and AdV data quality. Characterization of interferometer subsystems and environmental disturbances, and reduction of low-level correlated noise, are critical to ensuring good sensitivity to astrophysical searches. With the advent of the first observation run in the advanced detector era, its need has never been more important.

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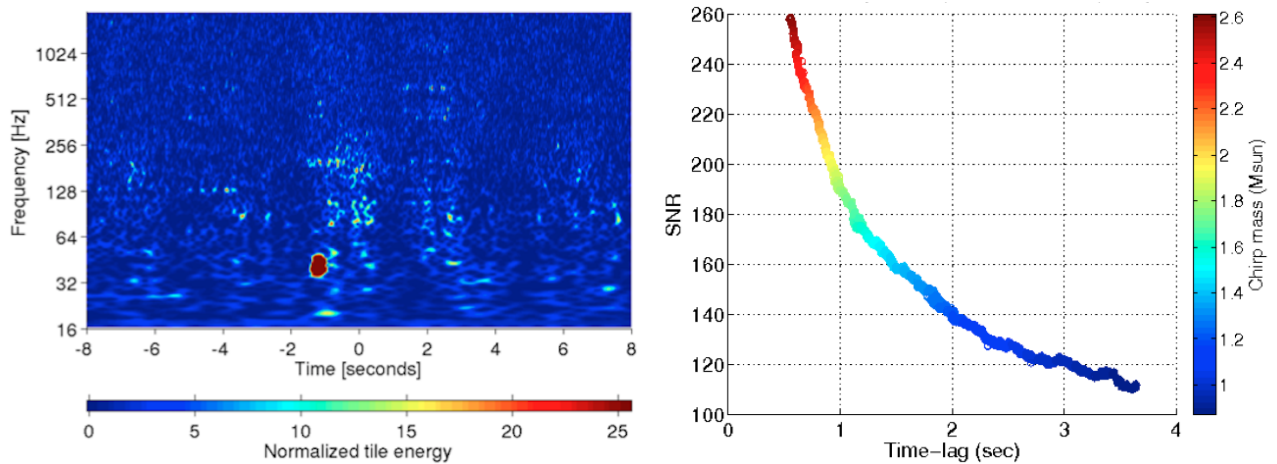


Figure 4. A noise glitch found by Omicron (left figure) and the effect of one such glitch, as modeled by a sine-Gaussian [25] with quality factor, $Q = 10$ and $f_0 = 100$ Hz, on a low-mass CBC template bank (right figure). The time-lag in the latter figure is the time duration between the central time of the glitch and the end-time of the CBC template triggered by that glitch. The signal-to-noise ratio (SNR) of a glitch is maximum when the glitch overlaps with that part of the template where the instantaneous frequency of the latter equals f_0 . As the chirp-mass decreases the time-lag increases because the duration of the template after an instantaneous frequency of f_0 increases.

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