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Abstract. The main goal of LISA Pathfinder (LPF) mission is to estimate the acceleration noise models of the overall LISA Technology Package (LTP) experiment on-board. This will be of crucial importance for the future space-based Gravitational-Wave (GW) detectors, like eLISA. Here, we present the Bayesian analysis framework to process the planned system identification experiments designed for that purpose. In particular, we focus on the analysis strategies to predict the accuracy of the parameters that describe the system in all degrees of freedom. The data sets were generated during the latest operational simulations organised by the data analysis team and this work is part of the LTPDA Matlab toolbox.

1. Introduction

The LISA Pathfinder mission [1] is a joint mission of the European Space Agency (ESA) and the National Aeronautics and Space Administration (NASA). It is aimed to test and validate key technologies for a future space-based Gravitational-Wave (GW) observatory, like the eLISA [2] mission concept. The eLISA is going to directly detect the effects of arriving GWs, via monitoring the relative motion between pairs of test masses (TMs) in free fall conditions. The LPF mission, being a demonstrator mission, consists in a single spacecraft hosting two proof masses following nominal geodesic motion, whose displacements are monitored by means of Mach-Zehnder laser interferometry. The principal requirement of the mission is expressed as

$$S_{\Delta a}^{1/2}(f) = 3\times 10^{-14}\left[1+\left(\frac{f}{3\,\mathrm{mHz}}\right)^2\right]\mathrm{ms}^{-2}\mathrm{Hz}^{-1/2}, \qquad 1\,\mathrm{mHz} \leq f \leq 30\,\mathrm{mHz}, \tag{1}$$

where the $S_{\Delta a}^{1/2}$ term, is the spectral density of the differential acceleration between the TMs. In order to reach the level of Eq. (1), the instrument must be fully characterised during the mission, and the various unknown dynamical parameters need to be estimated. For that reason, a series of system identification experiments have been proposed, each one stimulating the system in a particular manner. In the following sections we will discuss the planned experiments, together with the analysis strategies. Finally, we will present results of the algorithm performance on simulated data sets. This work is fully integrated in the standardised LTPDA toolbox [3], and is also part of the pipeline analysis to be performed during mission operations.

2. The system identification experiments

The planned system identification investigations can be divided into two main categories; those over the sensitive axis, and the cross-talk experiments [4]. The first case consists in exciting the three body system (the two TMs and the space-craft) along the x-axis, which is monitored by the most sensitive instrument, the differential interferometer (or o_{12} for short). In the second case, the system is excited along various degrees of freedom, and it aims at studying any possible cross-coupling effects that “pollute” the o_{12} optical channel. In particular, the TMs are commanded to rotate around their z-axis (ϕ angle), and along their y-axis, and the space-craft only around its z-axis (Φ angle). The commanded injection signals are, in essence, a series of sinusoids that will induce a large (in amplitude) response of the system to be used to calibrate the close-loops transfer functions, and to estimate the dynamic parameters with the desired accuracy.

3. The Bayesian data analysis framework

For the analysis of the aforementioned experiments, a Bayesian framework has been developed [5, 6], where a posterior distribution is sampled via Markov Chain Monte Carlo (MCMC) methods. The posterior distribution $\pi(\vec{\theta}|y)$ can be expressed as

$$\pi(\vec{\theta}|y) = \frac{\pi(y|\vec{\theta})p(\vec{\theta})}{\pi(y)}, \tag{2}$$

where $\vec{\theta}$ the parameter set to be estimated given the data-set y, $\pi(y|\vec{\theta})$ the likelihood function, $p(\vec{\theta})$ the prior Probability Density Functions (PDFs), and $\pi(y)$ is the so-called evidence of the model. Usually, the evidence is omitted in parameter estimation procedures, as it serves only as a normalisation constant. Then, assuming Gaussian properties of the noise and high signal-to-noise ratio (SNR), the likelihood can be written as

$$\pi(y|\vec{\theta}) = C \times e^{-\frac{1}{2}(y - h(\vec{\theta})|y - h(\vec{\theta}))} = C \times e^{-\chi^2/2}, \tag{3}$$

where C a constant, and the $(\cdot|\cdot)$ determines the natural inner product in frequency domain $(a|b) = 2\int\limits_0^\infty \mathrm{d}f[\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)]/\tilde{S}_n(f)$, where $\tilde{S}_n(f)$ is the power spectral density of the noise time-series. A tilde denotes the operations in frequency domain, and an asterisk denotes the complex conjugation. The $h(\vec{\theta})$ is the so-called template, or in other words, the model of the dynamics of the system.

But for the case of the LTP, the noise time-series might be unknown, or simply not available during the system identification experiments. This can be tackled by modelling the curve of the spectral density of the noise as in [7], where the coefficients i of the $\tilde{S}_n(f)$ term are multiplied by a set j of η amplitudes to be also estimated together with the dynamics parameters

$$\tilde{S}_{n,i} \to \eta_j \tilde{S}_{n,i}, \quad i_j < i \leq i_{j+1}. \tag{4}$$

Another method has been proposed, if one can assume zero mean and Gaussian properties of the noise. Then, the noise coefficients can be marginalised out of the posterior distribution. More details can be found in [8].

4. Results from simulated data sets

We can now apply the available techniques on the experiments over the sensitive axis. A simple realisation of a differential acceleration model can be expressed as [6, 8]

$$\alpha_{12} = \left[\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \omega_2^2\right] o_{12}(t-\tau) + (\omega_1^2 - \omega_2^2) o_1(t-\tau) - A_{sus}\frac{g_2(f)}{m_2}, \tag{5}$$

where ω_1^2 and ω_2^2 are the parasitic stiffnesses of the TMs, o_1 the readout of the first interferometer that measures the displacement of the first TM with respect to the space-craft, A_{sus} a control loop gain, and τ a system delay. The $g_2(f)/m_2$ is the (normalised to the mass m_2) applied force in the second TM, that may be frequency dependet. In this simple case, we have modelled a low-pass filter procedure applied to the commanded forces.

Working in the acceleration domain, has certain advantages, like no controller transfer functions appearing in the equations, but one must consider that the acceleration noise spectral density depends on the parameter set, so that $\tilde{S}_n \equiv \tilde{S}_n(\vec{\theta})$ [9]. A first approach to solve this problem is to apply the *iterative* χ^2 scheme [8]. This method is based on a loop of sequential estimations of $\vec{\theta}$ and then $\tilde{S}_n(\vec{\theta})$, then constructing the likelihood function, and maximising it

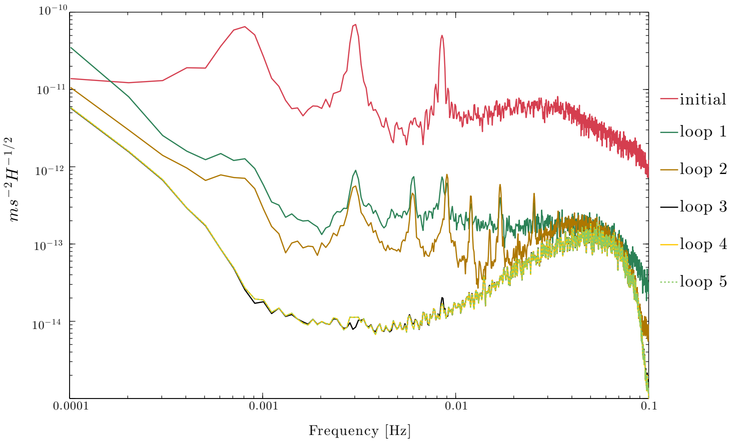

Figure 1. Example of the iterative χ^2 scheme fitting the differential acceleration α_{12}. At the end of each estimation of $\vec{\theta}_i$, with $1 \leq i \leq N_{iter}$ the likelihood function is updated taking into account the $\tilde{S}_n(\vec{\theta}_i)$ and a new estimation procedure takes place, until a convergence criterion is satisfied. These curves correspond to the fitting progress of a high dimensional model on the cross-talk experiments. See text for details.

to apply a new round of corrections on $\vec{\theta}$. An example of the iterative χ^2 scheme can be seen in Fig. 1. For practical reasons, the outer loop of estimations is performed with methods that can be faster than a MCMC run. We use the Nelder-Mead Simplex algorithm [10] until convergence is achieved, and then employ the MCMC algorithm to sample the posterior after assigning prior densities. In Fig. 2, we compare the iterative χ^2 technique to the noise modelling method of Eq. (4). As expected, there is no bias between the two methods, mostly due to the very high SNR of the particular experiments.

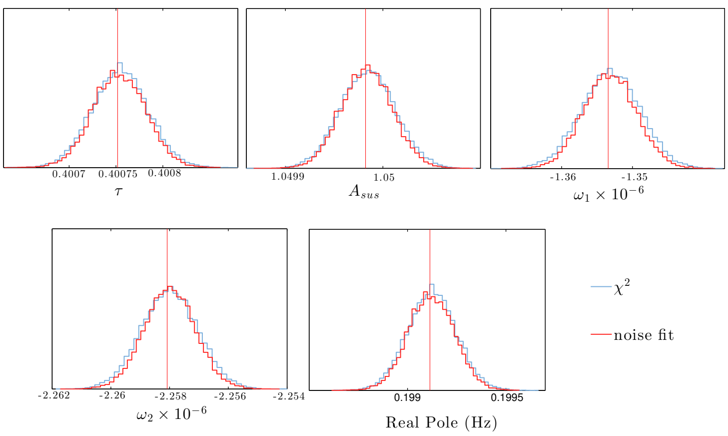

Figure 2. The PDFs of the system parameters of Eq. (5), as estimated by both techniques of the iterative χ^2 and the noise modelling of eq. (4). Since the SNR for the particular experiments is very large, we did not expect to find significant difference.

The developed MCMC methods, can be directly used for the case of the cross-talk experiments, where the SNR is smaller for certain parameters. As already mentioned, the three bodies of the system are commanded in different degrees of freedom in a consecutive manner. The first TM is commanded over ϕ and y, then the same injection signals are applied to the second TM, and finally the space-craft is commanded to rotate around its z-axis. The parameters to be estimated are the various cross-coupling effects that induce signal on the sensitive axis of the experiment. They can be of *(1)* geometrical origin, where the motion on other degrees of freedom is projected on the x-axis, *(2)* electrostatic origin, where distortion of the field lines from the command electrodes project undesired electrostatic forces, *(3)* gravitational origin, where a self-gravity imbalance of the space-craft could affect the TMs motion, and finally *(4)* external effects of magnetic or thermal origin.

We may choose to form an analytical model in the acceleration domain, that includes the complete span of the cross-talk experiments, yielding a total of sixteen parameters to be estimated. A long MCMC search over the parameter space produces satisfactory results for the cross-talk coefficients. In Fig. 3, the residual acceleration during the cross-talk experiment is displayed. It is evident that the estimated parameters yield a residual level identical to the noise measurement, performed in a previous day of the experiment.

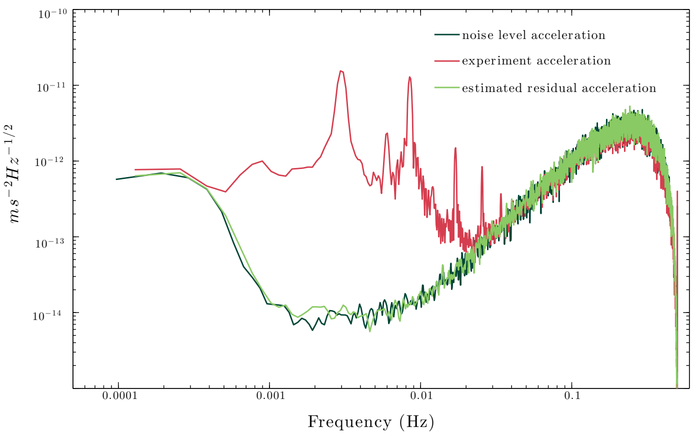

Figure 3. The residuals differential acceleration between the two TMs compared with the noise level measured in a previous day, and the total differential acceleration during the crosstalk-experiments.

The analysis of the cross-talk experiments can be more demanding, mostly due to the high dimensionality of the models, but also due to some cross-coupling coefficients contributing with a very low SNR to the overall signal measured. A suitable model is therefore required for the analysis, to quantitatively assess the physical effects that can explain more efficiently the observations. A Reversible Jump MCMC (RJMCMC) [6] algorithm has already been developed for this case, and already applied to the simulated data. The RJMCMC algorithm can be seen as a generalised MCMC method that is capable of sampling different parameter spaces simultaneously. It directly estimates the so-called Bayes factor B_{XY}, which is defined as the ratio of the evidences of the two competing models X and Y. For the cross-talk case we compared two models, the first one (which we name X) being higher in dimension by one extra parameter, which corresponds to the cross-coupling coefficient of the θ angle to the differential acceleration. It was then proven that the new realisation of the model, which includes the new cross-talk term is more probable by $\log B_{\mathrm{XY}} = 85.0$, where Y is the model without this contribution. It is the same model that produces the residuals in Fig. 3.

5. Discussion

We have developed a Bayesian framework to be applied for the analysis of the LPF mission planned system identification experiments. Different approximations have been employed for modelling of the noise, or the treatment of an unknown level of the noise. In addition, model selection techniques have been developed in order to identify the most probable physical effects contributing to the overall differential acceleration. This is proven to be of major significance for cases where the model is complicated, like in the cross-talk experiments. The designed analysis has been successfully tested and validated in operational exercises for the preparation of the data analysis algorithms. This work is part of the pipeline analysis that is going to be run during operations.

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