Leibniz Universität Hannover

Rate-Distortion Theory for Affine (Global) Motion Compensation in Video Coding

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Motivation

- Motion compensated prediction (MCP) as one key element in hybrid video coding
- High dependence between accuracy of motion estimation (ME) and prediction error (PE)
- Inaccurate displacement estimation

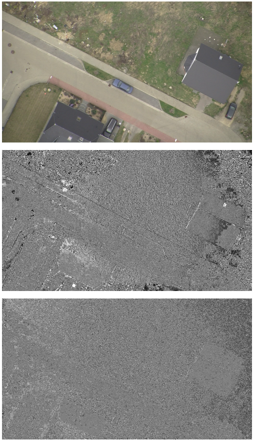
  $\Rightarrow$ High prediction error

  $\Rightarrow$ High entropy

  $\Rightarrow$ High bit rate

Goal:

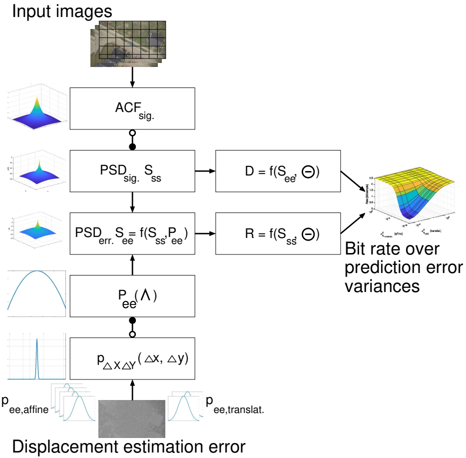
Model the prediction error bit rate as a function of the displacement estimation error for an **affine motion model**



Original aerial frame (top)

Overview[2]

- Model of power spectral density (P.S.D.) of signal
- Model of probability density function (p.d.f.) of displacement estimation error
- Derivation of P.S.D. of displacement estimation error $S_{ee}(\Lambda)$
- Application of rate-distortion theory ⇒ bit rate



[2] Modeling based on B. Girod, "The Efficiency of Motion-Compensating Prediction for Hybrid Coding of Video Sequences," in IEEE Journal on Selected Areas in Communicat., vol. 5, no. 7, pp. 1140–1154, 1987

Outline

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Affine Motion Model and Error Model

$$x = a_{11} \cdot x' + a_{12} \cdot y' + a_{13} \tag{1}$$

$$y = a_{21} \cdot x' + a_{22} \cdot y' + a_{23} \tag{2}$$

- a_{13} and a_{23} translational parameters
- $a_{11,12,21,22}$ "purely affine" parameters (rotation, scaling and shearing)
- Perturbation (indicated by $\hat{\cdot}$) by error terms e_{ij}, $i = \{1,2\}$, $j = \{1,2,3\}$ caused by inaccurate estimation

$$\Delta x = \hat{x} - x = \underbrace{(\hat{a}_{11} - a_{11})}_{e_{11}} \cdot x' + \underbrace{(\hat{a}_{12} - a_{12})}_{e_{12}} \cdot y' + \underbrace{(\hat{a}_{13} - a_{13})}_{e_{13}}$$

$$\Delta x = e_{11} \cdot x' + e_{12} \cdot y' + e_{13} \tag{3}$$

$$\Delta y = e_{21} \cdot x' + e_{22} \cdot y' + e_{23} \tag{4}$$

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Probability Density Function (p.d.f.) of the Displacement Estimation Error

Assumption: e_{ij} zero-mean Gaussian distributed with p.d.f.:

$$p(e_{ij}) = \frac{1}{\sqrt{2\pi\sigma_{e_{ij}}^2}} \cdot \exp\left(-\frac{e_{ij}^2}{2\sigma_{e_{ij}}^2}\right) \tag{5}$$

with $i = \{1,2\}$ and $j = \{1,2,3\}$

Joint p.d.f. for independent e_{ij}:

$$p_{E_{11},\ldots,E_{23}}(e_{11},\ldots,e_{23}) = p(e_{11}) \cdot \ \ldots \ \cdot p(e_{23}) \tag{6}$$

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Probability Density Functions Conversion

- Given now: joint p.d.f. $p_{E_{11},\ldots,E_{23}}(e_{11},\ldots,e_{23})$
- Wanted: p.d.f. $p_{\Delta X,\Delta Y}(\Delta x,\Delta y)$ of displacement estimation errors Δx, Δy

With transformation theorem for p.d.f.s:

$$p_{\Delta X,\Delta Y}(\Delta x,\Delta y)=\int_{\mathbb{R}^6} p_{E_{11},\ldots,E_{23}}(e_{11},\ldots,e_{23}) \cdot\delta\big(\Delta x-(x'e_{11}+y'e_{12}+e_{13})\big) \cdot\delta\big(\Delta y-(x'e_{21}+y'e_{22}+e_{23})\big)\,\mathrm{d}e_{11}\ldots\mathrm{d}e_{23} \qquad (7)$$

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Probability Density Function of the Displacement Estimation Error

$$p_{\Delta X,\Delta Y}(\Delta x, \Delta y) = \frac{1}{2\pi\sigma_{\Delta x}\sigma_{\Delta y}} \cdot \exp\left(-\frac{\Delta x^2}{2\sigma^2_{\Delta x}}\right) \cdot \exp\left(-\frac{\Delta y^2}{2\sigma^2_{\Delta y}}\right) \quad (8)$$

$$\text{with } \sigma^2_{\Delta x} = \sigma^2_{e_{11}} x'^2 + \sigma^2_{e_{12}} y'^2 + \sigma^2_{e_{13}} \quad (9)$$

$$\text{and } \sigma^2_{\Delta y} = \sigma^2_{e_{21}} x'^2 + \sigma^2_{e_{22}} y'^2 + \sigma^2_{e_{23}} \quad (10)$$

Variances $\sigma^2_{\Delta x}$ and $\sigma^2_{\Delta y}$ depend on locations x', y'!

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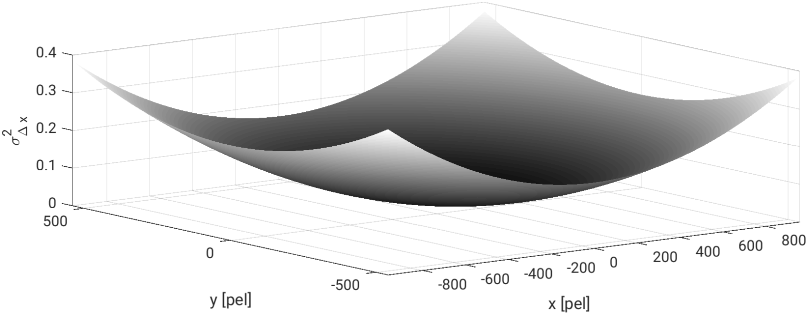
Location Dependent Variance $\sigma^2_{\Delta x}$ of Gaussian Distributed Displacement Estimation Error p.d.f.s for a Full HD Image



$\sigma^2_{e_{11}} = 2.3\text{e}-7$, $\sigma^2_{e_{12}} = 4.6\text{e}-7$ (measured from TAVT[3]) and $\sigma^2_{e_{13}} = 0.04$ (like Girod[2])

[3] *TNT Aerial Video Testset (TAVT)*, Institut für Informationsverarbeitung (TNT), Leibniz Universität Hannover, 2014, online: `https://www.tnt.uni-hannover.de/project/TAVT/`

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Signal and Error Power Spectral Density Functions

- Assumption of isotropic autocorrelation function[4]:

$$R_{ss}(\Delta x, \Delta y) = E\left[s(x,y) \cdot s(x - \Delta x,\ y - \Delta y)\right]$$
$$:= \exp\left(-\alpha\sqrt{\Delta x^2 + \Delta y^2}\right) \qquad (11)$$

- Determination of power spectral density of the video signal (Wiener–Khinchin theorem):

$$S_{ss}(\Lambda) = \mathcal{F}\left(R_{SS}(\Delta x, \Delta y)\right) \qquad (12)$$

- Power spectral density of displacement estimation error[2]:

$$S_{ee}(\Lambda) = 2\, S_{ss}(\Lambda)\left[1 - \mathrm{Re}(P(\Lambda)\right] + \Theta \qquad (13)$$

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[2] B. Girod, “The Efficiency of Motion-Compensating Prediction for Hybrid Coding of Video Sequences,” in IEEE Journal on Selected Areas in Communicat., vol. 5, no. 7, pp. 1140–1154, Aug 1987

[4] J. O’Neal and T. Natarajan, “Coding Isotropic Images”, IEEE Transact. on Inform. Theory, vol. 23, no. 6, pp. 697–707, Nov. 1977

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Rate-Distortion Theory[5]

$$D = \frac{1}{4\pi^2} \iint\limits_{\Lambda} \min \left[\Theta, S_{ss}(\Lambda)\right] \mathrm{d}\Lambda \,, \tag{14}$$

$$R(D) = \frac{1}{8\pi^2} \iint\limits_{\substack{\Lambda:\left(S_{ss}(\Lambda)>\Theta \right.\\ \left.\text{and } S_{ee}(\Lambda)>\Theta\right)}} \log_2 \left[\frac{S_{ee}(\Lambda)}{\Theta}\right] \mathrm{d}\Lambda \text{ bit} \tag{15}$$

with Θ being a parameter that generates the function $R(D)$ by taking on all positive real values

[5] based on Toby Berger, "Rate Distortion Theory: A Mathematical Basis for Data Compression", Prentice-Hall electrical eng. series, Prentice-Hall, 1971

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Pixel Correlations

Sequence	Corr. ρ_x	Corr. ρ_y
Values from Girod	0.928	0.934
BasketballDrive* (HD)	0.9782	0.9488
BQTerrace* (HD)	0.9680	0.9659
Cactus* (HD)	0.9741	0.9812
Kimono* (HD)	0.9883	0.9900
ParkScene* (HD)	0.9634	0.9518
Mean of HD sequences*	**0.9744**	**0.9677**

Measured horizontal and vertical correlations between adjacent pixels for typical test sequences (*: 100 frames each).

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Variances of Affine Transformation Parameters

	$\sigma^2_{e_{11}}$	$\sigma^2_{e_{12}}$	$\sigma^2_{e_{21}}$	$\sigma^2_{e_{22}}$	mean $(\sigma^2_{e_{11}}, \sigma^2_{e_{22}})$	mean $(\sigma^2_{e_{12}}, \sigma^2_{e_{21}})$
350m seq.	2.03e−7	6.03e−7	6.59e−7	2.24e−7	**2.13e−7**	**6.31e−7**
500m seq.	1.94e−7	5.09e−7	3.63e−7	1.94e−7	**1.94e−7**	**4.35e−7**
1000m seq.	1.74e−7	4.05e−7	4.13e−7	2.12e−7	**1.93e−7**	**4.09e−7**
1500m seq.	3.19e−7	3.80e−7	3.69e−7	3.46e−7	**3.33e−7**	**3.75e−7**
Mean	**2.23e−7**	**4.74e−7**	**4.51e−7**	**2.44e−7**	**2.33e−7**	**4.63e−7**

Measured variances $\sigma^2_{e_{ij}}$ of affine transformation parameters of aerial videos from the TAVT data set[3].

Note: $\sigma^2_{e_{11}}$ and $\sigma^2_{e_{22}}$ as well as $\sigma^2_{e_{12}}$ and $\sigma^2_{e_{21}}$ are pairwise similar.

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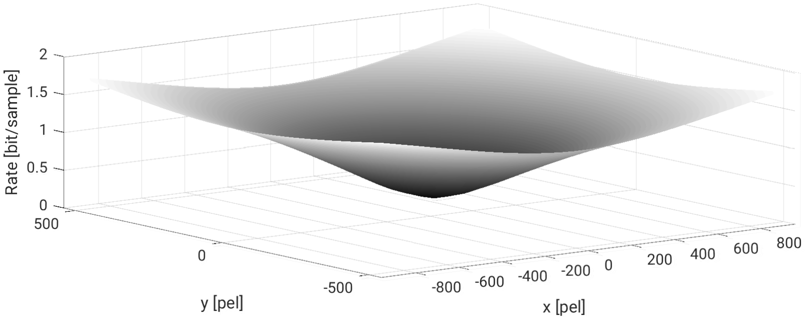
Simulated Location Dependent Bit Rate



Bit rate

Simulation for Gaussian distributed displacement estimation error p.d.f.s for full HD image and variances $\sigma^2_{e_{11}} = \sigma^2_{e_{22}} = 2.3\text{e}-7$, $\sigma^2_{e_{12}} = \sigma^2_{e_{21}} = 4.6\text{e}-7$.

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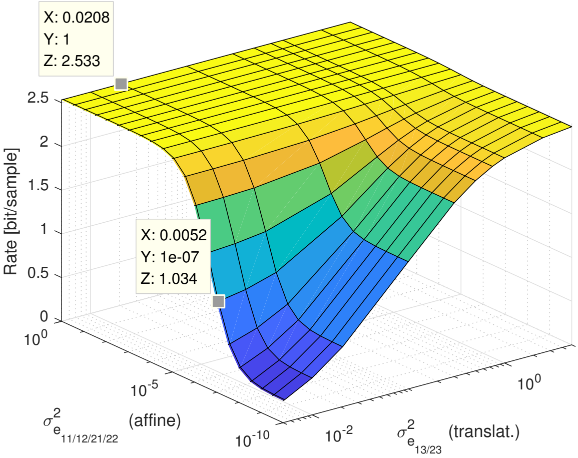
Minimum Required Bit Rate for Prediction Error Coding



Distortion SNR = 30 dB, $\sigma^2_{e_{11}} = \sigma^2_{e_{12}} = \sigma^2_{e_{21}} = \sigma^2_{e_{22}}$ and $\sigma^2_{e_{13}} = \sigma^2_{e_{23}}$, full HD resolution.
Datatips indicate isolines for translational quarter- (0.0052) and half-pel resolution.

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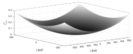
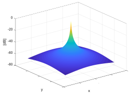
Summary: RD Theory for Affine MCP in Video Coding

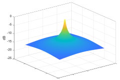
Model for affine motion compensation in video coding:

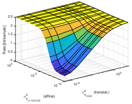
- Model the displacement estimation error as a function of the motion estimation accuracy:
- Model the p.d.f. of displacement estimation error $p_{\Delta X,\Delta Y}(\Delta x, \Delta y)$
- Model the P.S.D. of video signal S_{ss}
- Derivation of power spectral density of displacement estimation error S_{ee}
- Application of rate-distortion function

$\Rightarrow$ **Model for minimum required bit rate for prediction error coding**









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