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# Constraining Lorentz Violation in Electroweak Physics

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**Abstract.** For practical reasons, the majority of past Lorentz tests has involved stable or quasistable particles, such as photons, neutrinos, electrons, protons, and neutrons. Similar efforts in the electroweak sector have only recently taken shape. Within this context, Lorentz-violation searches in the Standard-Model Extension's  $Z$ -Boson sector will be discussed. It is argued that existing precision data on polarized electron–electron scattering can be employed to extract the first conservative two-sided limits on Lorentz breakdown in this sector at the level of  $10^{-7}$ .

## 1. Introduction

Investigations of Lorentz symmetry may provide experimental access to physics beyond the Standard Model [1, 2, 3, 4, 5]. The effective-field-theory description of such effects is provided by the Standard-Model Extension (SME) framework [6, 7], which contains the usual Standard Model and General Relativity as special cases, but also incorporates Lorentz- and CPT-violating operators of arbitrary mass dimension. To date, the SME framework has not only been employed for numerous experimental [8, 9, 10, 11, 12, 13, 14] and theoretical [15, 16, 17, 18, 19, 20] analyses of Lorentz breakdown, but also for phenomenological studies of spacetime torsion [21] and nonmetricity [22].

Previous Lorentz-violation searches have primarily focused on stable or quasistable particles. Only a few studies have been performed in the context of weak-interaction physics [23, 24, 25]; only some of these studies have constrained the SME coefficients for the massive gauge bosons [24, 25]. The present work reports on recent progress in this field. In particular, we discuss the idea that polarized electron–electron scattering is affected by the Lorentz-breaking SME coefficients  $k_{\phi\phi}$  and  $k_W$  associated with the  $Z$  boson. Both of these coefficients are CPT even and part of the minimal SME (mSME), which restricts attention to power-counting renormalizable Lorentz-breaking operators.

The presentation of this report is divided into two parts. The first part reviews the theoretical aspects of tree-level polarized Møller scattering in the mSME and derives a general expression for the corresponding cross section. The second part focuses on a special case of this cross section, in which the structure of the Lorentz violation takes a simplified form. The goal of this second part is to identify generic experimental signals and estimate the sensitivities that could be reached. Throughout, natural units  $\hbar = c = 1$  are used, and the convention for the Minkowski metric are  $\eta^{\mu\nu} = \text{diag}(+, -, -, -)$ .



## 2. Theory

The mSME is set up to describe general power-counting renormalizable Lorentz breakdown while retaining other fundamental physics properties including the ordinary  $SU(3) \times SU(2) \times U(1)$  gauge structure. The mSME's electroweak parameter space is therefore most transparently introduced before  $SU(2) \times U(1)$  symmetry violation. Spontaneous gauge-symmetry breaking then mixes the mSME coefficients  $k_B$ ,  $k_W$ , and  $k_{\phi\phi}$  of the original  $U(1)$ ,  $SU(2)$ , and Higgs sectors [6]. This leads to the following quadratic contributions to the mSME sector containing the photon and the  $Z$  boson [6, 26]:

$$\begin{aligned} \delta\mathcal{L}_{A,Z}^{(2)} = & -\frac{1}{4}(k_B \cos^2 \theta_W + k_W \sin^2 \theta_W)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} \\ & -\frac{1}{4}(k_W \cos^2 \theta_W + k_B \sin^2 \theta_W)_{\kappa\lambda\mu\nu} Z^{\kappa\lambda} Z^{\mu\nu} \\ & -\frac{1}{4} \sin 2\theta_W (k_W - k_B)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} Z^{\mu\nu} \\ & +\frac{1}{2} M_Z^2 \text{Re}(k_{\phi\phi})_{\mu\nu} Z^\mu Z^\nu, \end{aligned} \quad (1)$$

where  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$  is now the photon gauge field corresponding to the remaining unbroken  $U(1)_\gamma$  symmetry, and is  $Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$  the massive gauge field corresponding to the  $Z$  boson. The usual weak angle is denoted by  $\theta_W$  and the mass of the  $Z$  by  $M_Z$ .

Disregarding Chern–Simons-type Lorentz breakdown, the Lagrangian corrections (1) are general in a purely theoretical sense. However, previous observations may be employed to eliminate those coefficients whose experimental limits are well beyond the sensitivity reach of present-day and near-future Møller-scattering measurements. This arguments applies in particular to the mSME coefficient

$$(k_F)_{\kappa\lambda\mu\nu} \equiv (k_B \cos^2 \theta_W + k_W \sin^2 \theta_W)_{\kappa\lambda\mu\nu} \quad (2)$$

appearing in Eq. (1), which governs the photon: existing bounds effectively render  $k_F$  zero for our present purposes. Then,  $k_B$  may be expressed in terms of  $k_W$  as

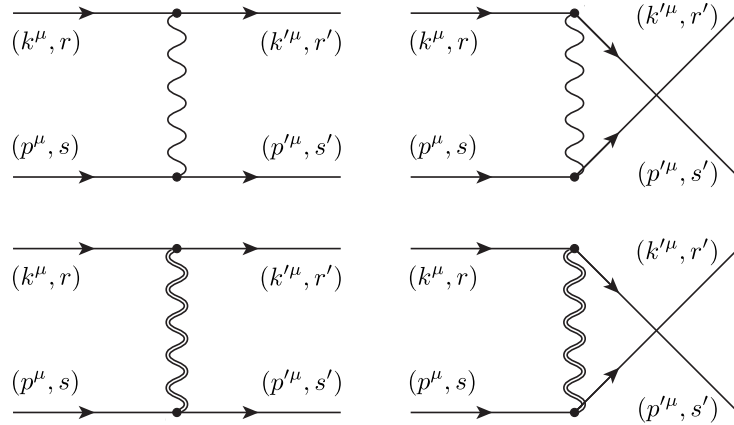
$$(k_B)_{\kappa\lambda\mu\nu} = -\tan^2 \theta_W (k_W)_{\kappa\lambda\mu\nu}. \quad (3)$$

This phenomenological simplification together with Eq. (1) determines the Feynman rules given in Fig. 1. These rules govern the dominant Lorentz-violating effects in electron–electron scattering.

$$\begin{aligned} \lambda \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \nu & = -2i(1 - \tan^2 \theta_W) (k_W)_{\kappa\lambda\mu\nu} p^\kappa p^\mu \\ & \quad + iM_Z^2 \text{Re}(k_{\phi\phi})_{\lambda\nu} \\ \lambda \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \nu & = -2i \tan \theta_W (k_W)_{\kappa\lambda\mu\nu} p^\kappa p^\mu \end{aligned}$$

**Figure 1.** Feynman rules for Lorentz-breaking corrections to the  $Z$  boson. The corrections relevant for the present report take the form of propagator insertions. The single and double wavy lines represent the conventional Lorentz-symmetric photon and  $Z$ -boson propagators, respectively. Taken from Ref. [26].

With the Feynman rules in place, the Lorentz-violating corrections to polarized Møller scattering can be determined. This process is dominated by the electromagnetic interaction via photon exchange. However, Lorentz-invariant subdominant contributions arise through diagrams in which the internal photon line is replaced by an internal  $Z$ -boson line. Although



**Figure 2.** Leading conventional tree-level contributions to Møller scattering. Solid lines denote electrons. They are labeled by one of the four external 4-momenta  $k^\mu, k'^\mu, p^\mu, p'^\mu$  as well as by one of the helicity observables  $r, r', s, s'$ . A single wavy line represents the ordinary Lorentz-symmetric photon propagator, and a double wavy line the usual  $Z$ -boson propagator. All vertices are the conventional ones. Taken from Ref. [26].

small, the associated effects have previously been measured [27] for the purpose of investigating the usual weak charge of the electron

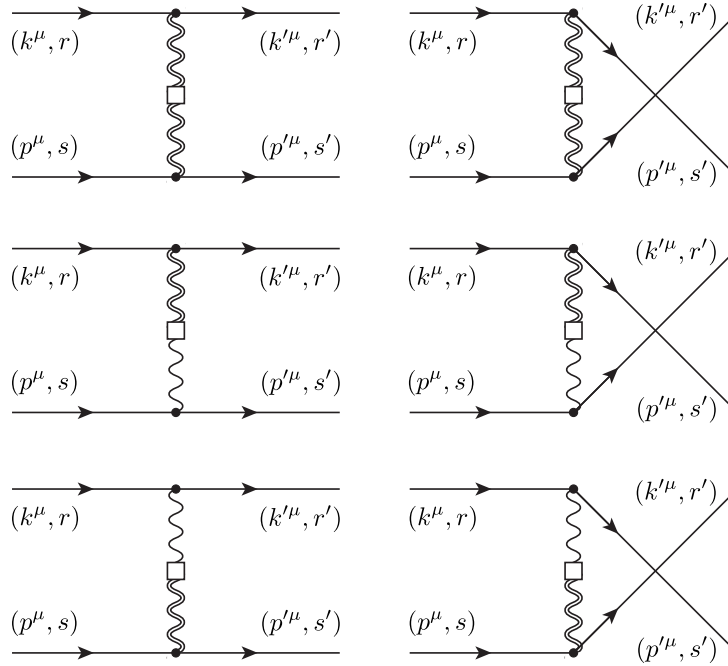
$$Q_W^e = 4 \sin^2 \theta_W - 1, \quad (4)$$

and future, more sensitive measurements are planned [28]. But the precision of such measurements and their dependence on  $Z$ -boson physics make them also an excellent candidate for the study of the Lorentz-violating effects represented in Fig. 1.

To calculate these Lorentz-breaking  $Z$ -boson effects in polarized Møller scattering within the mSME, we make the reasonable assumption that mSME coefficients are small, and leading-order result are therefore sufficient. We have previously argued that the photon's mSME coefficients are known to be too small to contribute to observable effects in the present context and can hence be disregarded here. An analogous reasoning holds for the electron's mSME coefficients, so that we may take the leading Lorentz-breaking effects in our Møller process to be entirely due to the propagator insertions shown in Fig 1. At tree level, this process is thus described by the conventional diagrams depicted in Fig. 2 and the Lorentz-violating corrections displayed in Fig. 3.

To streamline the calculation of the diagrams, it is helpful to tailor the external-leg polarizations to the actual experimental set-up. The measurements we have in mind, such as the E158 experiment at the Stanford Linear Accelerator Center (SLAC) [27], typically focus a beam of incoming, longitudinally polarized, relativistic electrons on a fixed unpolarized target. After the scattering process, outgoing Møller electrons in a finite range of scattering angles are detected and counted. It follows that we have to perform a calculation for incoming states of definite helicity, average over the spin states of the fixed-target electrons, and sum over outgoing electron spins. This gives two squared matrix elements  $|\overline{\mathcal{M}}_R|^2$  and  $|\overline{\mathcal{M}}_L|^2$  for incident right-handed and left-handed beam electrons, respectively. The general structure of these is determined by

$$\begin{aligned} |\overline{\mathcal{M}}_R|^2 &= |\overline{\mathcal{M}}_R^0|^2 + \delta |\overline{\mathcal{M}}_R|^2, \\ |\overline{\mathcal{M}}_L|^2 &= |\overline{\mathcal{M}}_L^0|^2 + \delta |\overline{\mathcal{M}}_L|^2, \end{aligned} \quad (5)$$



**Figure 3.** Dominant Lorentz-breaking corrections to Møller scattering. The notation is the same as before. In particular, the 3-point vertices are the conventional Lorentz-invariant ones, and the insertions are those shown in Fig. 1. Taken from Ref. [26].

where  $|\overline{\mathcal{M}}_R^0|^2$  and  $|\overline{\mathcal{M}}_L^0|^2$  are the ordinary Lorentz-symmetric contributions, which can be found in the literature [29]. The Lorentz-breaking effects in this Møller process are contained in the corrections  $\delta|\overline{\mathcal{M}}_R|^2$  and  $\delta|\overline{\mathcal{M}}_L|^2$ . Our present interest is directed at the calculation of explicit expressions for these corrections as functions of the relevant kinematical quantities and the  $k_W$  and  $k_{\phi\phi}$  coefficients.

To cast the result for  $\delta|\overline{\mathcal{M}}_R|^2$  and  $\delta|\overline{\mathcal{M}}_L|^2$  into a relatively compact form, we note that the four conventional external momenta  $k^\mu$ ,  $k'^\mu$ ,  $p^\mu$ , and  $p'^\mu$  obey energy–momentum conservation, as usual. This leaves three independent momenta, which we choose to parametrize as

$$S_\mu = k_\mu + p_\mu, \quad (6)$$

$$T_\mu = k'_\mu - k_\mu, \quad (7)$$

$$U_\mu = p_\mu - k'_\mu, \quad (8)$$

with a notation inspired by the Mandelstam variables. With these definition, one can show

that [26]

$$\begin{aligned} \delta|\overline{\mathcal{M}_R}|^2 = & \frac{2e^4(k_W)^{\kappa\mu\lambda\nu}}{sM_Z^2 y^2(1-y)^2 \cos^2 \theta_W} \left\{ (y-1) [(2-4y+y^2)Q_W^e + 1 - 2y] S_\kappa T_\mu S_\lambda T_\nu \right. \\ & + y [(1-2y-y^2)Q_W^e + 1 - 2y] S_\kappa U_\mu S_\lambda U_\nu + [(2-y+y^2)Q_W^e + 1] T_\kappa U_\mu T_\lambda U_\nu \\ & - s y (1-y) [y(2-y)Q_W^e + 1] \eta_{\mu\nu} T_\kappa T_\lambda - s y (1-y) [(1-y^2)Q_W^e + 1] \eta_{\mu\nu} U_\kappa U_\lambda \left. \right\} \\ & - \frac{e^4(Q_W^e + 1) \operatorname{Re}(k_{\phi\phi})^{\mu\nu}}{2M_Z^2 y(1-y) \sin^2 2\theta_W} \left\{ (Q_W^e - 1) [(1-2y+2y^2) S_\mu S_\nu - T_\mu T_\nu - U_\mu U_\nu] \right. \\ & \left. + s [(1-y+y^2)Q_W^e + 1 + y - y^2] \eta_{\mu\nu} \right\}, \end{aligned} \quad (9)$$

and

$$\begin{aligned} \delta|\overline{\mathcal{M}_L}|^2 = & \frac{2e^4(k_W)^{\kappa\mu\lambda\nu}}{sM_Z^2 y^2(1-y)^2 \cos^2 \theta_W} \left\{ (y-1) [(2-4y+y^2)Q_W^e - 1 + 2y] S_\kappa T_\mu S_\lambda T_\nu \right. \\ & + y [(1-2y-y^2)Q_W^e - 1 + 2y] S_\kappa U_\mu S_\lambda U_\nu + [(2-y+y^2)Q_W^e - 1] T_\kappa U_\mu T_\lambda U_\nu \\ & - s y (1-y) [y(2-y)Q_W^e - 1] \eta_{\mu\nu} T_\kappa T_\lambda - s y (1-y) [(1-y^2)Q_W^e - 1] \eta_{\mu\nu} U_\kappa U_\lambda \left. \right\} \\ & - \frac{e^4(Q_W^e - 1) \operatorname{Re}(k_{\phi\phi})^{\mu\nu}}{2M_Z^2 y(1-y) \sin^2 2\theta_W} \left\{ (Q_W^e + 1) [(1-2y+2y^2) S_\mu S_\nu - T_\mu T_\nu - U_\mu U_\nu] \right. \\ & \left. + s [(1-y+y^2)Q_W^e - 1 - y + y^2] \eta_{\mu\nu} \right\}. \end{aligned} \quad (10)$$

In these expressions,  $s = S^2 = (k+p)^2$  denotes the usual Mandelstam variable corresponding to the center-of-mass energy of the system. Employing the notation of Ref. [29], we have defined  $y \equiv -s^{-1}T^2 = -s^{-1}(k' - k)^2$ , which provides a measure of the scattering angle. In the above results, terms of order  $M_Z^{-3}$  and higher have been dropped, and the ultrarelativistic limit for the external electron momenta has been adopted by setting explicitly appearing  $m$  to zero. Equations (9) and (10) are the primary theoretical result of this report. We remark that the last lines of both Eqs. (9) and (10) contain the Lorentz-symmetric trace part of  $k_{\phi\phi}$ , which is independently unobservable and can thus be disregarded.

Actual measurements of  $|\overline{\mathcal{M}_R}|^2$  and  $|\overline{\mathcal{M}_L}|^2$  are often combined into the following asymmetry observable:

$$A \equiv \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = \frac{|\overline{\mathcal{M}_R}|^2 - |\overline{\mathcal{M}_L}|^2}{|\overline{\mathcal{M}_R}|^2 + |\overline{\mathcal{M}_L}|^2}. \quad (11)$$

Just as the individual cross sections (5), the asymmetry  $A = A_0 + \delta A$  consists of a Lorentz-invariant contribution [29]

$$A_0 = \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{y(1-y)}{(y^2 - y + 1)^2} s Q_W^e. \quad (12)$$

and a Lorentz-breaking correction [26]

$$\begin{aligned} \delta A = & \frac{G_F}{\sqrt{2}\pi\alpha} \frac{(k_W)^{\kappa\mu\lambda\nu}}{(y^2 - y + 1)^2} \frac{\sin^2 \theta_W}{s} [(1-2y)y S_\kappa U_\mu S_\lambda U_\nu - (2y^2 - 3y + 1) S_\kappa T_\mu S_\lambda T_\nu \\ & + T_\kappa U_\mu T_\lambda U_\nu - s(1-y)y \eta_{\mu\nu} (T_\kappa T_\lambda + U_\kappa U_\lambda)]. \end{aligned} \quad (13)$$

These expressions are again correct at leading order in  $M_Z^{-2}$  and in the ultrarelativistic limit for the external momenta. For convenience, we have expressed  $M_Z$  in terms of the fine-structure constant  $\alpha$  and the Fermi constant  $G_F$ . It turns out that the asymmetry correction  $\delta A$  depends on  $k_{\phi\phi}$  only through the unobservable Lorentz-invariant trace of  $k_{\phi\phi}$ , which has been dropped from Eq. (13).

### 3. Experimental tests

In principle, the general mSME prediction (13) for the dominant Lorentz-breaking effects in Møller scattering allows the extraction of experimental constraints on the full  $k_W$ . However, in what follows we assume a special structure for  $k_W$  for simplicity:

$$(k_W)^{\mu\nu\rho\sigma} = \frac{1}{2} \left[ \eta^{\mu\rho} \zeta^{\{\nu\xi\sigma\}} - \eta^{\mu\sigma} \zeta^{\{\nu\xi\rho\}} + \eta^{\nu\sigma} \zeta^{\{\mu\xi\rho\}} - \eta^{\nu\rho} \zeta^{\{\mu\xi\sigma\}} \right], \quad (14)$$

where

$$\zeta^{\{\mu\xi\nu\}} \equiv \frac{1}{2} (\zeta^\mu \xi^\nu + \zeta^\nu \xi^\mu), \quad \zeta^\mu = (1, \vec{0}), \quad \xi^\mu = (0, \vec{\xi}). \quad (15)$$

This assumption reduces the number of Lorentz-breaking coefficients from 19 for the general  $k_W$  to the three components of  $\vec{\xi}$ . Note that the analogous three coefficients in the photon sector are constrained by the weakest experimental limits and have thus also been investigated separately as a special case [30, 13]. With this assumption,  $\delta A$  simplifies to

$$\delta A = \frac{G_F}{\sqrt{2}\pi\alpha} \frac{\zeta^{\{\mu\xi\nu\}} \sin^2 \theta_W}{(y^2 - y + 1)^2} [k'_\mu k'_\nu + y p_\mu p_\nu + (1 - y) k_\mu k_\nu - 2(1 - y) k_\mu k'_\nu - 2y p_\mu k'_\nu]. \quad (16)$$

Here, we have reverted to the original momentum variables with  $p'_\mu$  eliminated by energy-momentum conservation, and we have again implemented the ultrarelativistic limit  $m \rightarrow 0$ .

In the mSME, in flat spacetime, the Lorentz-breaking  $k_W$ , and hence  $\zeta$  and  $\xi$ , are usually considered to be position independent. Their components in cartesian inertial coordinates are then constant. The standard frame in which to express these and other SME coefficients is the Sun-centered celestial equatorial frame (SCCEF) [8]. It is therefore natural (but not necessary) to continue our analysis in the SCCEF. To this end, we need to transform all momenta appearing in Eq. (16) from the terrestrial laboratory frame, where they are typically measured, to the SCCEF. The dominant motion of the laboratory with respect to the SCCEF is the rotation of the Earth about its axis. Other effects, such as the associated boosts or the motion around the Sun can be neglected for our present purposes.

The explicit transformation between laboratory and Sun-centered frames requires a definite laboratory frame. Our choice for a laboratory at colatitude  $\chi$  is characterized by an  $xy$ -plane parallel to the local surface of the Earth with the  $x$ -axis pointing South and the  $y$ -axis East. Right-handed cartesian coordinates then have the  $z$ -axis pointing vertically upward. This yields the following rotation matrix converting between the laboratory frame and the SCCEF [10]:

$$R^{Jj}(t) = \begin{pmatrix} \cos \chi \cos \Omega_\oplus t & -\sin \Omega_\oplus t & \sin \chi \cos \Omega_\oplus t \\ \cos \chi \sin \Omega_\oplus t & \cos \Omega_\oplus t & \sin \chi \sin \Omega_\oplus t \\ -\sin \chi & 0 & \cos \chi \end{pmatrix}. \quad (17)$$

Here,  $\Omega_\oplus = 2\pi/(23 \text{ h } 56 \text{ min})$  is the Earth's sidereal angular frequency,  $J = X, Y, Z$  are the spatial SCCEF components, and  $j = x, y, z$  the spatial laboratory-frame components. The time dependence of  $R^{Jj}(t)$  implies that the spatial components of various momenta that appear constant in the laboratory frame are in general time dependent in the SCCEF. In particular, SCCEF momenta for the beam electrons, the target electrons, and the collected scattered electrons take the form  $k^\mu = (E_k, \vec{k}(t))$ ,  $p^\mu = (m, \vec{0})$ , and  $k'^\mu = (E_{k'}, \vec{k}'(t))$ , respectively.

Expressing the magnitude of all 3-momenta in terms of the beam energy  $E_k$  and the previously introduced variable  $y$ ,<sup>1</sup> and considering an incoming-beam direction that is parallel to the local surface of the Earth and points in a direction  $\alpha$  East of South, we find

$$\begin{aligned}\delta A(t) &= \frac{G_F}{\sqrt{2}\pi\alpha} \frac{E_k y (1-y) \sin^2 \theta_W}{(y^2 - y + 1)^2} \vec{k}(t) \cdot \vec{\xi} \\ &= \frac{G_F}{\sqrt{2}\pi\alpha} \frac{E_k^2 y (1-y) \sin^2 \theta_W}{(y^2 - y + 1)^2} \times \\ &\quad \left[ \sqrt{\xi_X^2 + \xi_Y^2} \sqrt{1 - \cos^2 \alpha \sin^2 \chi} \cos \Omega_{\oplus} t + c_0 \right],\end{aligned}\quad (18)$$

where we have implemented the ultrarelativistic limit, as before. We have also absorbed an irrelevant phase into the definition of the origin of the time variable  $t$ . Moreover, we have dropped a constant shift in the asymmetry correction because it is difficult to disentangle from Lorentz-invariant effects. Our primary result is the square-root term; it predicts sidereal oscillation of the asymmetry  $\delta A$  with an amplitude determined by the Lorentz-breaking coefficients  $\xi_X$  and  $\xi_Y$ .

A measurement of this kind was carried out by the E158 experiment at SLAC [27]. The colatitude of this laboratory is  $\chi = 53^\circ$ , and the incoming-beam direction points  $\alpha = 123^\circ$  East of South. Incoming polarized electrons with an energy of roughly  $E_k = 50$  GeV were scattered off the atomic electrons of a stationary liquid-hydrogen target. The outgoing Møller electrons were detected and counted for angles  $\frac{1}{2} < y < \frac{3}{4}$ . This data has permitted a measurement of the asymmetry  $A$  with a statistical uncertainty of  $1.4 \times 10^{-8}$ .

This result may be used to estimate possible limits on mSME coefficients  $\xi_X$  and  $\xi_Y$  that may be extracted from the full E158 data set. We begin with the assumption that the Lorentz-violating amplitude

$$a_{\oplus} \equiv \frac{\sqrt{1 - \cos^2 \alpha \sin^2 \chi} E_k^2 y (1-y)}{2 M_Z^2 (y^2 - y + 1)^2 \cos^2 \theta_W} \sqrt{\xi_X^2 + \xi_Y^2} \quad (19)$$

of the predicted sidereal oscillations cannot be much larger than the statistical uncertainty of the measurement  $a_{\oplus} \lesssim 1.4 \times 10^{-8}$ . This yields

$$\sqrt{\xi_X^2 + \xi_Y^2} \lesssim 3.4 \times 10^{-7}. \quad (20)$$

To obtain this estimate, the following additional consideration has been implemented: from the range  $\frac{1}{2} < y < \frac{3}{4}$  in the collected data, we have taken  $y = \frac{3}{4}$ . At this value, the prediction for  $a_{\oplus}$  is smallest giving the most conservative estimate. Relative to previous studies performed in other systems and with different methods, the result (20) represents an improvement by two orders of magnitude.

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<sup>1</sup> The quantity  $y$  is understood to be entirely different from the coordinate component  $y$ .



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