Optical Amplification of Spin Noise Spectroscopy via Homodyne Detection

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Spin noise (SN) spectroscopy measurements on delicate semiconductor spin systems, like single (In,Ga)As quantum dots, are currently not limited by optical shot noise but rather by the electronic noise of the detection system. We report on a realization of homodyne SN spectroscopy enabling shot-noise-limited SN measurements. The proof-of-principle measurements on impurities in an isotopically enriched rubidium atom vapor show that homodyne SN spectroscopy can be utilized even in the low-frequency spectrum, which facilitates advanced semiconductor spin research like higher order SN measurements on spin qubits.

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I. INTRODUCTION

Traditional optical experiments study the spin dynamics in semiconductors by the optical excitation of spin-polarized carriers, such as in polarization-resolved photoluminescence or pump-probe Faraday rotation experiments [1,2]. However, these techniques are problematic in spin systems where the perturbation by optical excitation changes the intrinsic carrier spin dynamics. This adverse effect applies to weakly interacting and few-particle spins [3–5], as well as spin systems where an additional interaction is induced by optically injected carriers [6,7]. For such fragile semiconductor spin systems, spin noise (SN) spectroscopy has been established as the method of choice to avoid unnecessary optical excitation [8]. The experimental technique was transferred from quantum optics to semiconductor physics in 2005 [9] and is now used to study, e.g., the spin dynamics of single carriers and trions in single (In,Ga)As quantum dots (QD) [5,10] or the complex interaction of localized electrons with nuclear spins [11]. In this context, SN spectroscopy is used either as a weakly disturbing measurement method at thermal equilibrium [12] or to study quantum spin systems which are strongly driven by a resonant light field [10,13]. In the first case, very low light intensities are required [4,5] and, in the second case, SN at high frequencies [14,15] and high sensitivity [10] is of special interest. In both scenarios, the dominant experimental noise source is not SN or optical shot noise but rather extrinsic electronic noise from the electro-optical detectors entailing long measuring times. This constraint, *inter alia*, impedes fundamental measurements like high-frequency single spin dynamics or the efficient measurement of higher correlation SN.

The impact of electronic noise can be diminished by optical amplification. The technique is known in quantum optics as homodyne detection, and we show in this paper that homodyne detection [16–18] can be transferred smoothly to a standard SN spectroscopy setup even in the regime of low noise frequencies. For high frequencies, Cronenberger and Scalbert recently demonstrated quantum-limited heterodyne detection of SN in GaAs [18] in a nonstandard SN setup. Here, we demonstrate the optical amplification of the SN signal on $^{85}\text{Rb}$ impurities in an isotopically enriched $^{87}\text{Rb}$ vapor, which is a well-characterized reference system [13,19,20]. The same technique will also be applicable to delicate, low-frequency semiconductor spin systems like a single, charged (In,Ga)As quantum dot at low temperatures, where the electronic noise is a dominating obstacle in fast and precise measurements of the spin dynamics. The setup and data presented in Fig. 1 demonstrate the typical dominance of electronic noise in current state-of-the-art QD SN measurements. Such low-dimensional semiconductor spin systems with long spin lifetimes are of special interest due to their prospective potential as spin qubits in quantum information processing [21–23].

II. METHODS

A. Spin noise spectroscopy

Spin noise spectroscopy in semiconductors maps the stochastic spin fluctuations of carriers at thermal equilibrium onto the polarization of laser light that is quasiresonant to an optical transition [8]. Figure 1(a) depicts a typical schematic experimental setup for SN measurements on a single semiconductor QD in reflection. Here, linearly polarized laser light is focused on a single (In,Ga)As QD which is embedded in an asymmetric AlAs/GaAs Bragg resonator and charged by a single hole. The stochastic spin polarization of the hole yields a Faraday rotation of the linear polarization of the laser light [8] which is measured by a polarization bridge and a balanced
Electronic noise (\(\hat{p}\)) suppresses the corresponding noise contributions of the averaged background spectrum (\(\hat{I}\)). In fact, orders of magnitude lower laser by the light field despite the rather low probe laser Nevertheless, the depicted SN spectrum is strongly influenced dependent measurements (not shown) reveal that, never-theless, the depicted SN spectrum is strongly influenced by the light field despite the rather low probe laser intensity [5]. In fact, orders of magnitude lower laser intensities are desirable for such SN experiments whereat optical amplification by homodyne detection becomes essential.

### B. Homodyne detection

In this section, we briefly introduce the principle of homodyne amplification in the context of standard SN spectroscopy. Optical homodyne detection utilizes two phase-locked, monochromatic laser beams which can be described as classical electromagnetic plane waves. For SN spectroscopy, only the electric fields and the polarization states of the two beams are relevant, both of which can be represented by two Jones vectors in a linearly polarized basis [24]. The vertically polarized signal beam travels through the sample and experiences a small stochastic Faraday rotation by an angle \(\theta_F\). The resulting electric-field vector is

\[
E_{\text{sig}} = E_{\text{sig},0} R(\theta_F) \cdot e_{\psi} = E_{\text{sig},0} \left( \begin{array}{c} \sin(\theta_F) \\ \cos(\theta_F) \end{array} \right),
\]

where \(E_{\text{sig},0}\) is the complex amplitude, \(R(\theta_F)\) is the regular Euclidean rotation matrix, and \(e_{\psi}\) is the polarization vector. The second beam yields the local oscillator field and acts as an unchanged polarization reference,

\[
E_{\text{lo}} = \sqrt{2I_{\text{lo}}/c} \cdot e_{\psi} = \sqrt{2I_{\text{lo}}/c} \cdot \left( \begin{array}{c} 0 \\ 1 \end{array} \right),
\]

where \(I_{\text{lo}} = p c_0 c |E_{\text{sig},0}|^2 / 2 = p I_{\text{sig}}\) is the intensity of the local oscillator in terms of the intensity of the signal beam and \(p\) is the corresponding proportionality factor. For SN spectroscopy, the interference of both beams \(E_{\text{ld}}\) is decomposed into a linearly polarized basis rotated by \(\pi/4\) (diagonal \(e_d\) and antidiagonal \(e_A\)), giving the components \(E_{\text{ld},d}\) and \(E_{\text{ld},A}\). The balanced receiver measures the difference of the intensities of these two components:

\[
\Delta I = |E_{\text{ld},d}|^2 - |E_{\text{ld},A}|^2 \\
= 2I_{\text{sig}} \sin(\theta_F) [\cos(\theta_F) + \sqrt{p} \cos(\varphi_{\text{sig,lo}})] \\
= 2I_{\text{sig}} \theta_F [1 + \sqrt{p} \cos(\varphi_{\text{sig,lo}})] + O(\theta_F^2),
\]

where \(\varphi_{\text{sig,lo}}\) is the phase between \(E_{\text{sig}}\) and \(E_{\text{lo}}\) which is typically set to a multiple of \(2\pi\) for optimal amplification. Typical SN measurements involve very small Faraday rotation angles and, for this reason, the use of small angle approximation is justified. In this case, the intensity difference is linear in \(\theta_F\) and \((1 + \sqrt{p})\). Finally, the SN signal is proportional to \(\Delta I^2\) and consequently scales by the factor \(\eta_p = (1 + \sqrt{p})^2\). This dependence allows the signal-to-noise ratio of the SN signal to be increased without disturbing the measured system by increasing the local oscillator intensity up to the point where optical shot noise dominates over electronic noise. In practice, this kind of amplification is limited only by the maximally allowed optical power incident on the photodiodes of the balanced receiver.

### C. Noise power and signal-to-noise ratio

Spin noise is typically evaluated by the SN power density spectrum \(S_{\text{sn}}(f)\), which follows, for
monoexponential spin-relaxation dynamics, a normalized Lorentzian profile of area 1 multiplied by the SN power amplitude \( A \). The Lorentzian is shifted in a transverse magnetic field by the Larmor spin-precession frequency \( f_L \) and has a width \( \gamma_s \) given by the spin dephasing rate:

\[
L_{f_L; \gamma_s}(f) = \frac{2\gamma_s}{\gamma_s^2 + 4\pi^2 (f - f_L)^2}.
\]

The measured SN power, i.e., the integral over \( S_{\text{sn}} \), scales with the square of the probe laser power [8], and thus the SN power density scales with the same factor if \( \gamma_s \) is constant. Homodyne detection amplifies \( S_{\text{sn}} \) by approximately \( \eta_p \). Laser photon shot noise yields an additional broadband white-noise contribution that scales linearly with the laser power on the photodiodes. The total laser power at the homodyne operation point of constructive interference (\( \varphi_{\text{sig}, \text{lo}} = 0 \)) reads

\[
P_L = (\sqrt{P_{\text{sig}}} + \sqrt{P_{\text{sig}}}^2) = \eta_p P_{\text{sig}}.
\]

The electronic noise is, in general, independent of the incident laser power, resulting in the total noise power density spectrum

\[
S(f) = AL_{f_L; \gamma_s} \alpha^2 P_{\text{sig}}^2 \eta_p \quad \text{(spin noise } S_{\text{sn}} \text{)} \quad (5a)
\]

\[
+ W \hbar \omega_L \alpha P_{\text{sig}} \eta_p \quad \text{(shot noise } S_{\text{wn}} \text{)} \quad (5b)
\]

\[
+ S_{\text{en}} \quad \text{(electronic noise)}, \quad (5c)
\]

where \( W \) is a constant, \( \hbar \omega_L \) is the laser photon energy, and \( \alpha(f) \) is the (unitless) detector response function.

The signal-to-noise ratio is given by the spin noise signal \( S_{\text{sn}} \) [Eq. (5a)] divided by the nonsignal contributions \( S_{\text{wn}} \) [Eq. (5b)] and \( S_{\text{en}} \) [Eq. (5c)]. Here, however, in order to consistently analyze the performance of homodyne detection, we define the signal-to-noise ratio \( r_{\text{SNR}}^* \) as the ratio of the SN amplitude and the spectral mean of the remaining broadband noise contributions:

\[
r_{\text{SNR}}^* = \frac{\max(S_{\text{sn}})/P_{\text{sig}}}{(S_{\text{en}}) + (S_{\text{sn}})} = \frac{AL_{f_L; \gamma_s}(f_L)\alpha^2(f_L)P_L}{(S_{\text{sn}}) + W \hbar \omega_L (\alpha)P_L},
\]

where the SN amplitude is divided by the probe laser power \( P_{\text{sig}} \) such that \( r_{\text{SNR}}^* \) depends directly on the total laser power \( P_L \) detected on the photodiodes. This definition makes \( r_{\text{SNR}}^* \) robust against fluctuations in \( P_{\text{sig}} \), and the auxiliary parameters \( P_{\text{sig}} \) and \( P_L \) can be easily recorded during the measurement. The denominator of \( r_{\text{SNR}}^* \) is the mean of the background spectrum \( S_{bg} \) that is measured by shifting the spin noise outside of the spectral window considered, while the numerator is recovered from the measured difference spectrum \( S_{\text{sn}} \approx S_{\text{diff}} = S_{\text{fg}} - S_{\text{bg}} \) divided by the measured laser probe power \( P_{\text{sig}} \). With this procedure \( r_{\text{SNR}}^* \) can be easily estimated in the experiment as

\[
r_{\text{SNR}}^* = \max[(S_{\text{fg}} - S_{\text{bg}})/P_{\text{sig}}]/(S_{bg}).
\]

### III. SETUP

#### A. Interferometer

Figure 2 depicts the homodyne SN spectroscopy setup, which is derived from a Mach-Zehnder interferometer. A half-wave retarder and a polarizing beam splitter (PBS) split the incoming laser beam with a power ratio between the transmitted and reflected beams of approximately 5:1. In this configuration, the transmitted beam forms the local oscillator (LO) path, and the reflected beam the signal path. A neutral density filter reduces the intensity in the signal path by 3 orders of magnitude so that the probe intensity at the \( ^{87}\text{Rb} \) vapor cell is adjustable, transverse magnetic field \( B \) in order to shift the Larmor frequency (the center frequency of \( S_{\text{sn}} \)).
piezoelectric actuator allows us to adjust the phase by up to 8 rad and is used to stabilize the interferometer at the desired operating point. A liquid-crystal retarder and an additional polarizer $P_{\text{comp}}$ allow for the adjustment of the laser power in the LO arm. The signal and the LO path overlap on a wedged nonpolarizing beam splitter (BS) and proceed jointly to the SN measurement port (the top port in Fig. 2) and the control port (the right port in Fig. 2). The power in the control port is sampled by a photodetector (PD) and demodulated by a lock-in amplifier which effectively detects the phase between $E_{\text{sig}}$ and $E_{\text{ps}}$. Using $M_1$ the phase is steered by a proportional-integral-derivative loop towards the operating point, i.e., constructive interference in the measurement port with an accuracy of $\delta \phi \approx 0.13$ rad. Power measurements on both paths are realized by blocking the other path by one of two mechanical shutters, $S_1$ or $S_2$, and sampling the remaining power on the photodetector. The polarization alterations in the measurement port are detected by a polarization bridge consisting of a half-wave retarder, a Wollaston prism, and a high-speed balanced photoreceiver. The output of the balanced photoreceiver [25] is postamplified and low-pass filtered to a bandwidth of 5 MHz (3 dB). The resulting voltage signal is sent to an analyzing computer that samples the voltage with a fast, 12-bit digital-to-analog converter operating at 20 MHz and performs a discrete fast Fourier transform. Please note that only electronic noise from the transimpedance amplifier in the first stage of the balanced photoreceiver has a significant influence on the experiment, whereas the electronic noise contribution of the intermediate voltage amplifier, $2.5 \, \text{nV/} \sqrt{\text{Hz}}$ ("post-amplification"), and the digitizer, $80 \, \text{nV/} \sqrt{\text{Hz}}$, can be neglected. We selected at the balanced photoreceiver two different transimpedance gains of $1 \times 10^4 \, \text{V} / \text{A}$ and $1 \times 10^5 \, \text{V} / \text{A}$ corresponding to noise-equivalent-power densities of 10 and 2.5 pW/\sqrt{Hz}, respectively, in order to change the ratio of electronic versus optical shot noise present in the detection. Both amplifications have a bandwidth larger than the auxiliary low-pass filter of 5 MHz. Thus, we alter via a diametrically set amplification by the subsequent voltage amplifier the relative contribution of electronic noise with respect to spin and shot noise.

B. Sample and experimental details

The sample is a $^{87}$Rb vapor cell with helium buffer gas and a purity of the $^{87}$Rb isotope of $> 98\%$. The goal of the presented setup is the optical amplification of extremely small SN signals. Therefore, the weak SN signature of the remaining $^{85}$Rb isotope is analyzed and the measurement is carried out at room temperature, where the Rb vapor partial pressure is very low. The vapor density of $n \approx 1.2 \times 10^{10} \text{cm}^{-2}$ and optical probe power of $8 \, \mu \text{W}$ used in the presented measurements are significantly lower than the optimal values reported on in Ref. [26]. These parameters have been chosen purposely, in order to simulate the unfavorable conditions present in the single-quantum-dot setups that are the main scope of our technique. The laser source used in this setup is an external cavity diode laser stabilized by a Fizeau interferometer and the laser photon energy is blueshifted from the $^{85}$Rb $D_2$ resonance by about 0.3 GHz. A solenoid below the vapor cell produces a transverse magnetic field which modulates the stochastic spin orientation by the Larmor frequency $f_L$. In order to separate the weak SN signature from other noise contributions, a foreground and a background SN spectrum are measured at a magnetic field of $B_{\text{fg}} = 120 \, \mu \text{T}$ and $B_{\text{bg}} = 800 \, \mu \text{T}$, respectively, and the two spectra are subtracted from each other. All other noise contributions are independent of the magnetic field and average out in the difference of the foreground and background spectra, whereas only the SN contribution remains.

IV. RESULTS

Figure 3 depicts a high-quality SN spectrum which is obtained by averaging over SN spectra [27] measured at different amplifications factors $\eta_p$ and normalized by $P_{\text{sig}}^2$:

$$S_{\text{sn}}' = \frac{S(B = B_{\text{fg}}) - S(B = B_{\text{bg}})}{P_{\text{sig}}^2\eta_p}.$$  

The solid (red) line is a fit to the spectrum consisting of a positive (foreground; the blue line) and a negative (background; the green line) Lorentzian, whereas part of the foreground Lorentzian is folded back at zero frequency (the
amplitude at the amplitude of the Lorentzian SN peak normalized to the $\gamma$ widths at half maximum signal-to-noise ratio. This is a direct experimental proof of optical amplification but does not yet prove to be an improvement on the line). Here, spin noise spectroscopy on fragile spin systems—particularly on single (In,Ga)As quantum dots—which is usually limited by the finite electronic noise of the available balanced receivers, can be made feasible. The experiments were successfully carried out with a non-commercial diode laser down to SN frequencies of tenths of kilohertz, which is sufficient for most spin noise experiments. Nevertheless, even lower spin noise frequencies should be easily accessible utilizing stabilized ultranarrow linewidth lasers.

V. CONCLUSIONS

In this paper, we show in a proof-of-principle experiment that the resulting constraints by electronic noise can be successfully circumvented by optical homodyne amplification of spin noise in a standard spin noise spectroscopy setup—even at low frequencies. This low-frequency limit is especially important for semiconductor systems with long spin coherence times, which will be relevant in the framework of future semiconductor spin information processing. Here, spin noise spectroscopy on fragile spin systems—particularly on single (In,Ga)As quantum dots—which is usually limited by the finite electronic noise of the available balanced receivers, can be made feasible. The experiments were successfully carried out with a non-commercial diode laser down to SN frequencies of tenths of kilohertz, which is sufficient for most spin noise experiments. Nevertheless, even lower spin noise frequencies should be easily accessible utilizing stabilized ultranarrow linewidth lasers.

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APPENDIX: AN ALTERNATIVE TO PHASE STABILIZATION

An active phase stabilization loop significantly increases the setup complexity. Here, we propose an alternative scheme that explicitly averages over a changing phase \( \phi_{\text{sig},\text{lo}} \).

The homodyne amplification has a cosine-type phase dependence which can be averaged out if the phase modulation \( (a) \) acts on a different time scale than the spin dynamics and \( (b) \) is faster than the characteristic drift of the modulation. For a modulation much slower than the rate \( f_{\text{rec}} \) at which spectra are recorded, the phase has to be effectively constant. For a modulation much faster than \( f_{\text{rec}} \), the phase dependence must average out for each individual spectrum. For the measured noise power density [roughly the square of Eq. (3)], it is easy to show that

\[
\Delta I^2 = \int_0^{n2\pi} \Delta I^2 \frac{d\phi}{n2\pi} \propto \Delta I^2 \left( \frac{2 + p}{2} \right) + O(\theta_F^2). \tag{A1}
\]

This kind of phase dithering reduces the effective amplification factor of the spin noise signal to \( \eta_{\text{dith},p} = (2 + p)/2 \), while keeping all other properties of the homodyne detection. For the technical realization of this scheme, the piezoelectrically actuated mirror \( M_1 \) has to be replaced ideally by a phase modulator with a very high modulation depth to ensure an optimal phase averaging.


[27] The SN difference spectra often exhibit a small drift term $S_{\text{drift}}$ which arises from the imperfect cancellation of the different noise sources. This drift term can be easily subtracted from the individual difference spectra.

In order to guarantee the most consistent results, all presented data are corrected accordingly in the same manner.