Hopping Conductivity in the Quantum Hall Effect: Revival of Universal Scaling

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We have measured the temperature dependence of the conductivity $\sigma_{xx}$ of a two-dimensional electron system deep into the localized regime of the quantum Hall plateau transition. Using variable-range hopping theory we extract directly the localization length $\xi$ from this experiment. We use our results to study the scaling behavior of $\xi$ as a function of the filling factor distance $|\delta \nu|$ to the critical point of the transition. We find for all samples a power-law behavior $\xi \propto |\delta \nu|^{-\gamma}$ in agreement with the theoretically proposed universal exponent $\gamma = 2.35$.

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In order to avoid such complications, we follow a different approach to scaling. We directly evaluate the localization length in the well understood regime of variable-range hopping (VRH) conductivity [24]. We have recently shown this method to be reliable for frequency dependent measurements [25]. VRH dominates the conductivity at low temperatures, when the localization length becomes much smaller than the effective temperature length $L_{\text{eff}}$. In the QHE regime the VRH conductivity is given as [24, 26, 27]

$$\sigma_{xx}(T) = \sigma_0 \exp(-\sqrt{T_0/T}), \quad k_B T_0 = C \frac{e^2}{4\pi \varepsilon \varepsilon_0 \xi},$$

with a temperature dependent prefactor $\sigma_0 \propto 1/T$. The characteristic temperature $T_0$ is determined by the Coulomb energy at a length scale given by the localization length $\xi$, the dimensionless constant $C$ being on the order of unity. With this direct access to $\xi \propto 1/T_0$, it is possible to test for scaling behavior of $\xi$ at the edges of the plateau as long as we are careful enough to stay within the localized regime.

Earlier experiments confirmed the expected temperature dependence of $\sigma_{xx}(T)$ and extracted $\xi$ in the QHE...
regime [28,29]. In a recent experiment on the QH plateau-insulator transition VRH conductivity following Eq. (1) was also established to that regime and was used to determine $\xi$, finding a rough agreement with the prediction of universal scaling [30].

In this paper we show that when analyzing the conductivity deep into the localized regime of the quantum Hall plateau transition we find a clear universal scaling behavior of the localization length $\xi \propto |\delta \nu|^{-\gamma}$ as a function of the filling factor distance $\delta \nu = \nu - \nu_c$ to the critical point at $\nu_c$ with a universal $\gamma = 2.35$. Such a universality is observed even in samples where nonuniversally exponents extracted from temperature dependent peak-width scaling are found. Going even further, we show that the conductivities $\sigma_{xx}(T, \delta \nu)$ in the VRH regime for $\delta \nu < 0.3$ can be scaled to a single parameter function of $|\delta \nu|^2/T$ as predicted by scaling theory for the critical regime.

The samples used in this work are based on modulation doped GaAs/AlGaAs heterostructures with additional scatterers in the active region of the 2DES [31,32]. The scatterers are provided by doping the GaAs close to the heterojunction with Si or Be, either as a $\delta$ layer or a weak homogeneous background. This results in relatively low mobilities of a few $10^5$ V/s (see Table I). In order to allow a highly sensitive two-point measurement of very low conductivities the samples were patterned into Corbino geometry using contacts fabricated by standard Ni/Au/Ge alloy annealing. Here the conductivity is given by $\sigma_{xx} = (e^2/h) \ln(r_2/r_1)$ where $r_1 = 500 \mu m$ and $r_2 = 550 \mu m$ are the inner and the outer radii of the Corbino disk. The samples were mounted onto the cold finger of a dilution refrigerator with a base temperature below 20 mK and positioned into the center of a superconducting solenoid.

The sample conductivity $\sigma_{xx}(B, T)$ as a function of magnetic field $B$ and temperature $T$ was extracted from individually measured $I$-$V$ characteristics by numerical differentiation in the linear regime close to zero bias. This method allowed us to measure the conductivity accurately in a huge range between $10^{-13} \Omega^{-1}$ in the plateau center up to $10^{-5} \Omega^{-1}$ in the maximum of the quantum Hall transition.

In Fig. 1(a) the conductivity $\sigma_{xx}(B, T)$ of sample S1 is shown for the transition between the QHE plateaux at $\nu = 1$ and $\nu = 2$. We concentrate our studies on this transition where the spin gap is sufficiently large to exclude activated transport. The critical point $\nu_c$ of this transition, listed in Table I, is given by the position of the maximum of $\sigma_{xx}(\nu)$.

Before analyzing the VRH transport we tested the conventional scaling behavior of the plateau transition width. Data for the full width at half maximum $\Delta \nu$ of the peak in $\sigma_{xx}$ as a function of filling factor $\nu = h\nu/eB$ are shown in Fig. 1(b) for sample S1. For all investigated samples $\Delta \nu$ follows a power-law behavior $\Delta \nu \propto T^\kappa$ down to temperatures below 60 mK. However, as already reported in earlier work on similar samples [17] the critical exponents $\kappa = p/2\gamma$ as presented in Table I deviate considerably from the proposed universal value $\kappa = 0.43$ [8].

Let us now turn to a closer analysis of the VRH regime. As shown in Fig. 1(c) for sample S1 the data fit well the predicted temperature behavior of Eq. (1). We also tested for activated behavior, Coulomb gap with constant $\sigma_c$, and Mott hopping $\sigma_{xx} \propto T^{-m} \exp[(T_0/T)^{1/3}]$ for various $m$, all matching our data worse. From these fits to Eq. (1) we are able to extract the characteristic temperature $T_0$ in the VRH regime. To stay well inside this localized regime we take into account only filling factors where the low temperature conductivity is at least 2 orders of magnitude below the critical conductivity $\sigma_c$.

In Fig. 2 the characteristic temperature $T_0$ is plotted against the distance $|\delta \nu| = |\nu - \nu_c|$ to the critical point $\nu_c$. The data follows a power-law behavior $T_0 \propto |\delta \nu|^\gamma$.

### Table I. Sample characteristics: Type of extra doping, electron density $n_e$, and mobility $\mu_e$ of the 2DES, the critical point $\nu_c$ [33], and the width scaling exponent $\kappa$ for the $\nu = 1 \rightarrow 2$ transition.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Doping</th>
<th>$n_e$ ($10^{15}$ m$^{-2}$)</th>
<th>$\mu_e$ ($10^2$ V s^{-1} m^{-1}$)</th>
<th>$\nu_c$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>$\delta$-Be</td>
<td>2.1</td>
<td>2</td>
<td>1.29</td>
<td>0.66 ± 0.02</td>
</tr>
<tr>
<td>S2</td>
<td>$\delta$ Si</td>
<td>3.2</td>
<td>4</td>
<td>1.62</td>
<td>0.60 ± 0.02</td>
</tr>
<tr>
<td>S3</td>
<td>hom. Be</td>
<td>2.4</td>
<td>12</td>
<td>1.52</td>
<td>0.62 ± 0.03</td>
</tr>
</tbody>
</table>

![Fig. 1.](image-url)
represented by straight lines in this plot, up to distances as large as $|\delta \nu| = 0.3$. This demonstrates that the localization length $\xi \approx 1/T_0$ follows a scaling behavior deep into the localized regime. The lines are drawn for the proposed universal scaling exponent $\gamma = 2.35$ and show a nice agreement to the data within the experimental uncertainty. This result strongly supports the proposed universality, being consistent with numerical calculations ([1] and references therein) as well as with size dependent scaling experiments [7] and with a VRH analysis of the frequency dependence of the conductivity [25]. It underlines the fact that our direct scaling analysis in the VRH regime suits much better as an access to the localization length and its scaling behavior than temperature dependent peak-width scaling where no universality of the exponent $\kappa = p/2\gamma$ is observed. As $\gamma$ is shown to be universal we have to attribute this to the temperature exponent $p$ in $L_T \propto T^{-p/2}$. In fact, for any predictions on $p$ we are still lacking a sufficient knowledge of the temperature dependent transport mechanisms in the metallic regime of the QHE plateau transition.

Now we will estimate the precision of our measurement of the scaling exponent. Linear fits of $\ln(T_0) = \gamma \ln(\delta \nu) + \text{const}$ scatter within 10% around the proposed universal exponent $\gamma = 2.35$. The fitted values are $2.36 \pm 0.09$ ($\nu > \nu_c$) and $2.20 \pm 0.04$ ($\nu < \nu_c$) for S1, $2.10 \pm 0.03$ ($\nu < \nu_c$), and $2.08 \pm 0.04$ ($\nu < \nu_c$) for S2, and $2.46 \pm 0.06$ ($\nu > \nu_c$) and $2.50 \pm 0.04$ ($\nu < \nu_c$) for S3. The given error bars are the pure statistical 1σ intervals resulting from the fit. The total experimental error is presumably larger due to additional systematic errors: The scaling with filling factor distance $\delta \nu$ results from the more fundamental scaling with energy distance $\delta E = E - E_c$ from the critical point. As neither $E$ nor $E_c$ is known, the relation $\delta \nu \propto \delta E$ is used to translate the prediction to measurable quantities. This relation is exactly valid in the case of a flat density of states (DOS) $D(E) = D(E_c)$. Assuming a more realistic Gaussian DOS, a significant deviation from the linear relation $\delta \nu \propto \delta E$ is observed, e.g., 10% for $\delta \nu > 0.3$. Additionally, the extra doping with different scatterers and different spatial dopant distributions leads to a more complicated individual asymmetric DOS for each sample [32]. This introduces a further error in $\delta \nu \propto \delta E$ even for smaller $\delta \nu$. The asymmetric DOS also leads to a temperature dependence of the maximum in $\sigma_{xx}$, leading to an uncertainty of approximately $\pm 0.005$ in $\nu_c$. Summing up we estimate the uncertainty of the relation $\delta \nu \propto \delta E$ to 5%–10% for $\delta \nu = 0.05, \ldots, 0.3$. Thus the deviation between the data points and the lines in Fig. 2 lies within the experimental uncertainty.

Until now we used the term scaling in a rather reduced sense as a synonym for a power-law behavior of the localization length $\xi$. Of course, scaling includes much more, namely the existence of a single parameter scaling function $\sigma_{xx} = f(x)$ with a parameter $x(T, \delta \nu)$. Using our above results in the VRH regime, an appropriate definition is $x = |\delta \nu|^\gamma / T \propto T_0 / T$ with the above deduced exponents $\gamma$. The postulation of single parameter scaling together with the finding of $\sigma_0 \propto 1/T$ then fixes the prefactor in Eq. (1) to $\sigma_0(\delta \nu, T) = \sigma^*|\delta \nu|^\gamma / T$ with a constant $\sigma^*$ and yields a scaling of the conductivity

$$\sigma_{xx}(\delta \nu, T) = \sigma^* \left( |\delta \nu|^\gamma / T \right) \exp\left( -\sqrt{T^*/|\delta \nu|^\gamma / T} \right),$$

where $T^*$ is constant.

In Fig. 3 we have plotted all the conductivity data $\sigma_{xx}(\nu, T)$ for $\nu > \nu_c$ as a function of a single parameter $|\delta \nu|^\gamma / T$. The values for $\gamma$ are taken from the power-law fits of $T_0$. Rescaling the axes in a proper way indeed shows that all experimental points fall onto straight lines as represented by the scaling function in Eq. (2). Therefore, the conductivity in the VRH regime for $\delta \nu < 0.3$ and $\nu > \nu_c$ follows the stringent postulation of single parameter scaling and thus demonstrates the large validity range of the universal scaling phenomenon in the transition between QHE plateaus.

For $\nu < \nu_c$, i.e., approaching the spin gap induced plateau at $\nu = 1$, the single parameter rescaling does not look as good as for approaching the Landau gap at $\nu = 2$, although Fig. 2 showed the same quality of $T_0$ scaling for both sides of the transition. This discrepancy probably mirrors a more complicated behavior of the prefactor $\sigma_0$ in Eq. (1). Obviously, it can no more be written as a simple function $|\delta \nu|^\gamma / T \propto T_0 / T$. Such a complicated behavior in this regime is no surprise, taking into account the redistribution of electrons between spin states and the emerging of spin textures as a function of $\nu$ at this filling range [34].
In addition to the temperature dependence of $\sigma_{xx}$ in the VRH regime, we have also investigated its dependence on the electric field strength $F$. In the nonlinear regime at high voltages the effect of $F$ can be interpreted as an effective temperature $T_F = eF\xi/2k_B$ [24]. Using Eq. (1) the conductivity is then rewritten to $\sigma_{xx} = \sigma_{xx}^0(F) \exp(-\sqrt{F_0/F})$ with $F_0 \propto T_0/\xi \propto 1/\xi^2$. Analogous to the single parameter scaling with temperature in the VRH regime, the natural reduced parameter for scaling with electric field is defined by $y(F, \nu) = |\delta \nu|^{2/\gamma}/F \propto T_0/T_F$. Applying such an analysis to our data without any additional fit parameter [i.e., using the exponents $\gamma$ as determined from $T_0(\nu)$ we find for $\nu > \nu_c$ a single parameter scaling for all data in the range $|\delta \nu|^{2/\gamma}/F < 4 \mu$m/V, which is equivalent to the condition $T_F/T_0 > 0.01$ for the ratio of the effective temperature $T_F \propto F\xi$ and the characteristic energy scale $T_0 \propto 1/\xi$. An example of single parameter scaling for sample S2 is shown in the inset of Fig. 3.

In conclusion, we have investigated the temperature and electric field dependence of the conductivity in the QHE plateau transition for samples where no universal behavior is found in the conventional temperature dependent peak-width scaling experiments. We find a revival of universality in the VRH regime where the localization length $\xi$ scales as $\xi \propto |\delta \nu|^{-\gamma}$ with an experimentally deduced scaling exponent close to the theoretically expected value $\gamma = 2.35 \pm 0.03$. Even further, we have shown that all the data on the Landau gap side of the transition can be rescaled on a single parameter function over more than 5 orders of magnitude in the conductivity.

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[33] Because of the strong $\delta$ doping with Be (Si) the density of states becomes asymmetric and the critical filling factor is shifted from the ideal $\nu_c = 1.5$ to lower (higher) filling factors [32].