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Model reduction of parasitic coupling networks of mixed-signal VLSI circuits

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Abstract

Purpose – This paper aims to present a method for the efficient reduction of networks modelling parasitic couplings in very-large-scale integration (VLSI) circuits.

Design/methodology/approach – The parasitic effects are modelled by large RLC networks and current sources for the digital switching currents. Based on the determined behaviour of the digital modules, an efficient description of these networks is proposed, which allows for a more efficient model reduction than standard methods.

Findings – The proposed method enables a fast and efficient simulation of the parasitic effects. Additionally, an extension of the reduction method to elements, which incorporate some supply voltage dependence to model the internal currents more precisely than independent current sources is presented.

Practical implications – The presented method can be applied to large electrical networks, used in the modelling of parasitic effects, for reducing their size. A reduced model is created which can be used in investigations with circuit simulators requiring a lowered computational effort.

Originality/value – Contrary to existing methods, the presented method includes the knowledge of the behaviour of the sources in the model to enhance the model reduction process.

Keywords Modelling, Circuits

Paper type Research paper

Introduction

Smaller technologies require the investigation of parasitic coupling effects in early integrated circuit (IC) design stages. Especially, in mixed-signal very-large-scale integration (VLSI) circuits the influence of the digital switching currents on the analogue part of the circuit is of increasing importance. Analogue and digital part in mixed-signal circuits are mainly coupled through the substrate, the package and interconnects. These parasitic couplings can be modelled by large passive networks composed of resistors, capacitors and inductors as shown in Figure 1 (Verghese *et al.*, 2004; Stanisic *et al.*, 1994; Chen and Neely, 1998; Steinecke *et al.*, 2007). The digital switching currents act as distributed sources inside the IC and can be modelled with a large number of independent current sources (Figure 1; Verghese *et al.*, 2004; Steinecke *et al.*, 2007; Gstottner *et al.*, 2007).

Since the resulting networks are too large to be handled efficiently with circuit simulators, model order reduction (MOR) techniques (Antoulas, 2005; Gugercin and Antoulas, 2004; Phillips and Silveira, 2005) can be used to reduce the size of the network while preserving the behaviour at selected nodes (Silveira, 1995; Freund, 2008; Ionutiu *et al.*, 2007). The large number of independent sources of the networks is a limitation for the reduction, as only the RLC-part can be reduced, and the independent sources have to be extracted and connected by ports. A method for the description of the independent sources modelling the switching currents is presented in this paper, which enables

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an efficient reduction of the parasitic coupling network. In our former paper, Ludwig *et al.* (2008) presented a method which is limited to groups of independent sources with equal or proportional waveforms. In this paper, the method of Ludwig *et al.* (2008) is generalized to a wider class of sources, where the waveforms do not need to be grouped. In addition, the reduction method can incorporate supply voltage dependent internal currents. This paper is structured as follows. In the second section, the standard model reduction flow as well as the proposed extension is presented. In the third section, illustrative examples are given to show the validity of the proposed method while the paper is concluded in the fourth section.

Model reduction

Standard reduction flow

In the standard model reduction of linear networks, all non-linear and time-variant elements are extracted from the reducible RLC-part of the network (Figure 2). With the help of modified nodal analysis (MNA) of the linear network, a differential algebraic system in the form:

$$\begin{aligned}(s\mathbf{C} + \mathbf{G})\mathbf{x} &= \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{L}^T\mathbf{x}\end{aligned} \tag{1}$$

is generated. The number of equations N is defined as the order of the system while p is the number of inputs and outputs of the system. The system matrices $\mathbf{C}, \mathbf{G} \in \mathbb{R}^{N \times N}$ contain the stamps for capacitors, inductors and resistors. The input and output vectors $\mathbf{u}, \mathbf{y}$ are related with the system vector $\mathbf{x}$ with the help of the matrices $\mathbf{B}, \mathbf{L} \in \mathbb{R}^{N \times p}$. The electrical currents or voltages, depending on the choice of impedance or admittance description at the analogue pins are defined as inputs $\boldsymbol{u}$ and outputs $\mathbf{y}$ of the system. In addition the electrical values generated by the extracted independent sources are defined as inputs of

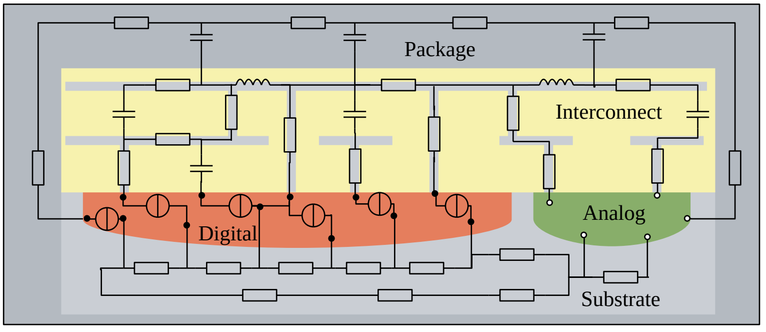


Figure 1.
Schematic of parasitic couplings in mixed-signal circuits

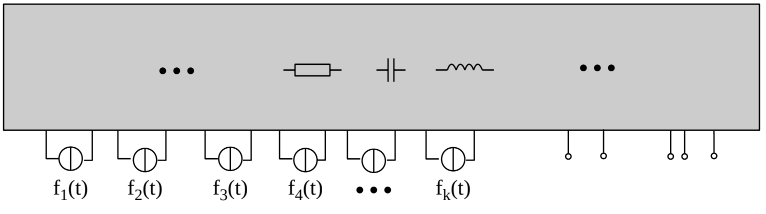


Figure 2.
Reducible part of the network (grey box) with extracted independent sources

the system, and are, therefore, also a part of $\mathbf{u}, \mathbf{y}$. Overall, the number of ports p is now given by the sum of the number pins for the connection with the analogue part p_{pins} and the number of independent sources k:

$$p = p_{extr} = p_{pins} + k \tag{2}$$

The matrix transfer function $\mathbf{H} \in \mathbb{C}^{p_{extr} \times p_{extr}}$ of the reducible linear system is given by:

$$\mathbf{H}(s) = \mathbf{L}^T (s\mathbf{C} + \mathbf{G})^{-1} \mathbf{B} \tag{3}$$

Owing to the typically large number of RLC-elements, the system of equation (1) has a high order N. The goal of MOR is to reduce this system to a smaller order $n \ll N$. The reduced order system should generate for the same inputs $\mathbf{u}$ outputs $\tilde{\mathbf{y}} \approx \mathbf{y}$, who are similar in magnitude and phase for a specified frequency range. The system with a smaller order n replaces the high-order system and enables faster simulations. For the reduction of the system, most common projection methods based on Krylov subspaces are used (Odabasioglu *et al.*, 1998; Elfadel and Ling, 1997). The block moments $\boldsymbol{M}$, capable of approximating the transfer function around a frequency s_0:

$$\mathbf{H}(s) + \mathbf{M}_0 + s\mathbf{M}_1 = s^2 \mathbf{M}_2 \ldots \tag{4}$$

can be computed with:

$$\mathbf{M}_i = \mathbf{L}^T \mathbf{A}^i \mathbf{R} \tag{5}$$

where:

$$\begin{aligned} \mathbf{A} &\equiv (s_0 \mathbf{C} + \mathbf{G})^{-1} \mathbf{C} \\ \mathbf{R} &\equiv (s_0 \mathbf{C} + \mathbf{G})^{-1} \mathbf{B} \end{aligned} \tag{6}$$

By generating a Krylov subspace with:

$$Kr(\mathbf{A}, \mathbf{R}, q \cdot p_{extr}) \equiv colsp[\mathbf{R}, \mathbf{AR}, \ldots, \mathbf{A}^q \mathbf{R}] \tag{7}$$

of size $N \times q \cdot p_{extr}$, it can be seen that for a given number of Krylov iterations, q the size of the Krylov subspace depends directly on the number of ports p_{extr} of the system. With a QR-factorization of the Krylov subspace:

$$[\mathbf{X}, \mathbf{T}] = QR(Kr(\mathbf{A}, \mathbf{R}, q \cdot p_{extr})) \tag{8}$$

a projection matrix $\mathbf{X} \in \mathbb{R}^{N \times n_{extr}}$ with:

$$n_{extr} = q \cdot p_{extr} \ll N \tag{9}$$

as the reduced order is found. With the projection as in Odabasioglu *et al.* (1998):

$$\begin{aligned} \tilde{\mathbf{C}} &= \mathbf{X}^T \mathbf{C} \mathbf{X} \\ \tilde{\mathbf{G}} &= \mathbf{X}^T \mathbf{G} \mathbf{X} \\ \tilde{\mathbf{B}} &= \mathbf{X}^T \mathbf{B} \\ \tilde{\mathbf{L}} &= \mathbf{X}^T \mathbf{L} \end{aligned} \tag{10}$$

a reduced order system:

$$\begin{aligned} (s\tilde{\mathbf{C}} + \tilde{\mathbf{G}})\tilde{\mathbf{x}} &= \tilde{\mathbf{B}} \mathbf{u} \\ \tilde{\mathbf{y}} &= \tilde{\mathbf{L}}^T \tilde{\mathbf{x}} \end{aligned} \tag{11}$$

is now generated. The size of the reduced order system matrices is $\tilde{\mathbf{C}}, \tilde{\mathbf{G}} \in \mathbb{R}^{n_{extr} \times n_{extr}}; \tilde{\mathbf{L}}, \tilde{\mathbf{B}} \in \mathbb{R}^{n_{extr} \times p_{extr}}$. As the first q moments of the transfer function are matched (Odabasioglu *et al.*, 1998), the reduced order transfer function:

$$\tilde{\mathbf{H}}(s) = \tilde{\mathbf{L}}^T (s\tilde{\mathbf{C}} + \tilde{\mathbf{G}})^{-1} \tilde{\mathbf{B}} \tag{12}$$

is similar in magnitude and phase to the high order transfer function $\tilde{\mathbf{H}}(s) \approx \mathbf{H}(s)$ in a frequency range around s_0. In addition to the preservation of moments in s_0, also passivity is preserved with this method, if the expansion point s_0 in equation (6) is chosen as a real frequency (Odabasioglu *et al.*, 1998). An extension for this reduction is given in Elfadel and Ling (1997) where different Krylov subspaces at different frequency points s_0 are calculated and the projection matrix is generated by composing all Krylov subspaces. This leads to more accurate models in a wider frequency band. As last step, the reduced system is synthesized as electrical network to enable simulations with circuit simulators (Ludwig *et al.*, 2008; Yang *et al.*, 2007; Palenius and Roos, 2004). The complete standard reduction flow is divided into the following four steps:

Listing 1. Standard reduction flow with extraction of the sources:

(1) extraction of the large number of current sources modelling the digital modules currents;
(2) generation of a system of high order with a large number of ports describing the network;
(3) reduction of the number of equations of the system with MOR methods; and
(4) network synthesis of the reduced system of equations.

In the standard reduction flow for a large number of extracted sources, a large number of ports is generated. The size of the Krylov subspace depends directly on the number of ports (equation (7)). As the system is projected on the Krylov subspace, the size of the reduced order system does directly depend on the number of ports. Thus, a large number of ports are a strong limitation for the reduction, as for example also investigated in Ludwig *et al.* (2008), Silva *et al.* (2007), and Silva and Silveira (2007). In other MOR-algorithms like for example (Gugercin and Antoulas (2004); Phillips and Silveira (2005), the order of the reduced systems does not directly depend on the number of ports. But there is an indirect dependence of the size of the reduced system on the number of ports, due to the fact that a large number of input-output relations has to be approximated in the reduced system (Silva *et al.*, 2007; Silva and Silveira, 2007). Some adapted methods use the correlation of the ports (Feldmann, 2004; Liu *et al.*, 2007), which is only efficient if the models are regularly structured. Another method to deal with a large number of ports is presented in (Phillips and Silveira, 2005) with input correlated token bucket regulator (TBR). The assumption of statistically distributed input signals leads to a more efficient model reduction. In the systems under consideration in this paper, the waveforms of the sources are fully determined, which is not taken into account in the state of the art MOR techniques. In Wang and Nguyen (2000), an analysis method to include the determined behaviour of sources is presented. This method is restricted to the case where all inputs are determined, which is more likely for a simulation method and cannot be used for the coupling networks handled in this paper, as the signals at the nodes for the analogue circuit are not determined in the modelling and model reduction process.

Proposed reduction flow
As shown in above section, a large number of ports is a limitation for the possible model reduction. In the proposed reduction flow, described in this section, the number of ports for the current sources is to be reduced. The reduced number of ports enables a more efficient model reduction of the parasitic coupling network. For the proposed reduction flow, the waveforms of the current sources modelling the switching currents in the IC are used. Based on the k waveforms described by $f_i(t)$of the sources a small subset of $r \ll k$ basis functions $g_j(t)$:

$$f_i(t) \approx \sum_{j=1}^{r} w_{i,j} g_j(t) \qquad 1 \leq i \leq k \tag{13}$$

has to be found. This smaller subset can be found for example by grouping proportional waveforms by Fourier decomposition of periodic functions, by decomposition of piece-wise-linear functions into ramp functions or any other kind of exact or approximate decomposition with methods of approximation theory (Hornik *et al.*, 1989). With the smaller set of basis functions $g_j(t)$, the complete function space of the sources functions $f_i(t)$ can be described. The basis functions $g_j(t)$ are realized in the network model as additional current sources. The sources modelling the activity in the digital part are replaced by controlled sources, controlled by the additional basis function sources. The factors $w_{i,j}$ in equation (13) are used as the gains of the controlled sources. With these steps, the number of independent current sources in the network model is lowered. The behaviour at the nodes of interest, namely the nodes for the connection with the analogue part is not changed by this method if equation 13 is exact and is nearly the same if equation 13 is approximate. For ensuring stability and passivity of the complete network, controlled sources are connected with the additional basis function sources. The gains of the additional controlled sources are chosen with the transpose of $w_{i,j}$. With this, the energy generated (dissipated) by the controlled sources inside the network is the same as the energy dissipated (generated) by the controlled sources connected with the additional sources. The resulting network is shown in Figure 3. As only the r additional independent sources have to be extracted from the reducible coupling network, a network with a lower number of ports is generated. This network with replaced sources, containing the parasitic RLC-elements as well as the controlled sources is described with MNA, resulting in a system of equations as in equation 1, which is to be reduced with MOR techniques. The number of ports of the system is now p_{pins} plus the number of additional sources r:

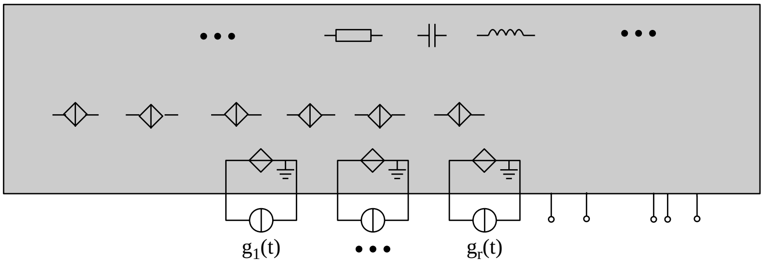


Figure 3.
Reducible part of the network (grey box) with replaced sources for the digital modules and extracted additional sources with the basis waveforms

$$p = p_{repl} = p_{pins} + r \tag{14}$$

which is less than p_{extr} as $r \ll k$ holds. For the system of the network with replaced sources a Krylov subspace with:

$$Kr(\mathbf{A}, \mathbf{R}, q \cdot p_{repl}) \equiv colsp[\mathbf{R}, \mathbf{AR}, \ldots, \mathbf{A}^q\mathbf{R}] \tag{15}$$

of size $N \times q \cdot p_{repl}$ is generated. As for a given q the size of the Krylov subspace depends directly on the number of ports of the system, the Krylov subspace is now smaller than in the standard reduction flow.

As in the standard reduction flow with a QR-factorization of the Krylov subspace:

$$[\mathbf{X}, \mathbf{T}] = QR(Kr(\mathbf{A}, \mathbf{R}, q \cdot p_{repl})) \tag{16}$$

a projection matrix $\mathbf{X} \in \mathbb{R}^{N \times n_{repl}}$ with:

$$n_{repl} = q \cdot p_{repl} < n_{extr} \ll N \tag{17}$$

as the reduced order is found. For a given number of Krylov subspace iterations q the reduced order is now smaller than for the standard reduction flow, as the number of ports is lower. With the projection as in equation 10, a reduced order system is now calculated. The size of the matrices is now $\tilde{\mathbf{C}}, \tilde{\mathbf{G}} \in \mathbb{R}^{n_{repl} \times n_{repl}}; \tilde{\mathbf{L}}, \tilde{\mathbf{B}} \in \mathbb{R}^{n_{repl} \times p_{repl}}$. As the first q moments of the transfer function are matched (Odabasioglu *et al.*, 1998), the reduced order transfer function is similar in magnitude and phase to the high-order transfer function $\tilde{\mathbf{H}}(s) \approx \mathbf{H}(s)$ in a frequency range. The reduced system is now synthesized as electrical network. With the proposed reduction flow, the number of ports for the sources of the system is reduced before the model reduction. The complete proposed reduction flow is divided into the following five steps:

Listing 2. Proposed reduction flow with replacing the sources:

(1) Find basis functions for the waveforms of the current sources modelling the digital modules switching currents:
   - add sources with the basis waveforms; and
   - replace digital module current sources with controlled sources.

(2) Extraction of the low number of sources with the basis waveforms.

(3) Generation of a linear system of high order with a low number of ports describing the network.

(4) Reduction of the number of equations with MOR methods.

(5) Network synthesis of the reduced system of equations.

As there is a dependence of the size of the reduced system on the number of ports, this reduction flow leads to smaller and/or more accurate reduced models.

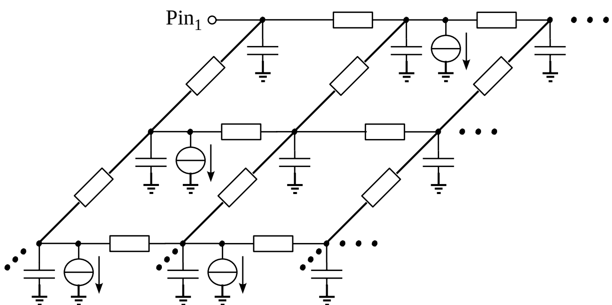
Illustrative examples

Example 1: network with independent current sources

For the validation of the reduction method, an example network with a large number of independent current sources, modelling the power grid of an IC is used. The example network is built of ≈7,500 passive RC-elements, 55 independent current sources and two terminals for the connection with the analogue IC part. The structure of the example is shown in Figure 4.

The waveforms of the independent sources, representing the currents of the digital modules are modelled with 55 piece-wise-linear functions $f_i(t), 1 \leq i \leq k = 55$. Some examples of the current waveforms are shown in Figure 5. Using decomposition of the waveforms, only three basis waveforms $g_j(t), 1 \leq j \leq r = 3$ (Figure 6) are necessary to describe all 55 current sources waveforms $f_i(t)$. The three basis waveforms are realized with independent current sources and the internal sources are replaced with controlled sources, as described in the former section.

The behaviour at the pins of the example network for the connection with the analogue part is not changed by the replacing of the sources. Both networks, the network with extracted as well as the network with replaced sources, are described as a differential algebraic system of equations. The order of the systems is $N = 2,500$ for the standard as well as for the proposed method. The main difference of both systems is the number of ports, which is $p_{extr} = 57$ for the extracted sources network and $p_{repl} = 5$ for the replaced sources network. Both systems are to be reduced with MOR methods. First, both systems are reduced to a comparable accuracy. For the standard reduction flow two expansion points and $q = 2$ Krylov subspace iterations are used. The resulting



Note: RC-mesh with independent current sources

Figure 4.
Example network

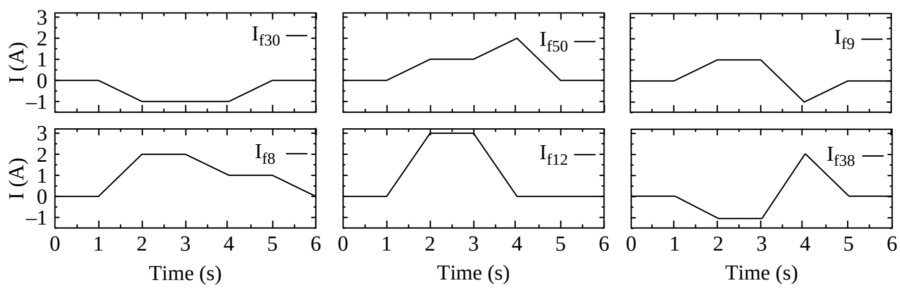


Figure 5.
Some waveforms of the PWL-sources in the example network

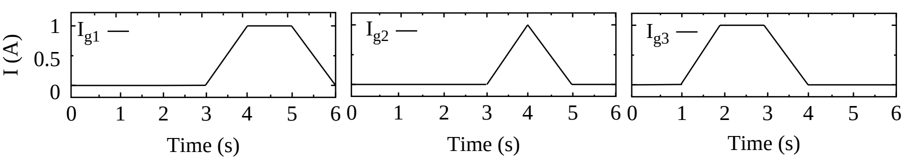


Figure 6.
The three basis waveforms of the PWL-sources

reduced system has an order of $n_{extr} = 228$. For a comparable accuracy for the proposed reduction flow with three expansion points and $q = 3$ a larger number of expansion points and Krylov subspace iterations is necessary. Nevertheless, due to the lower number of ports, the reduced system has a low order of $n_{repl} = 45$, which is much less than in the standard reduction flow. The results of the reduction are shown for the impedance transfer function at one pin for the connection with the analogue part in Figure 7 and the approximation error of the reduced systems is shown in Figure 8. The accuracy of both models is quite comparable. Nevertheless, the proposed reduction flow produces a much smaller model.

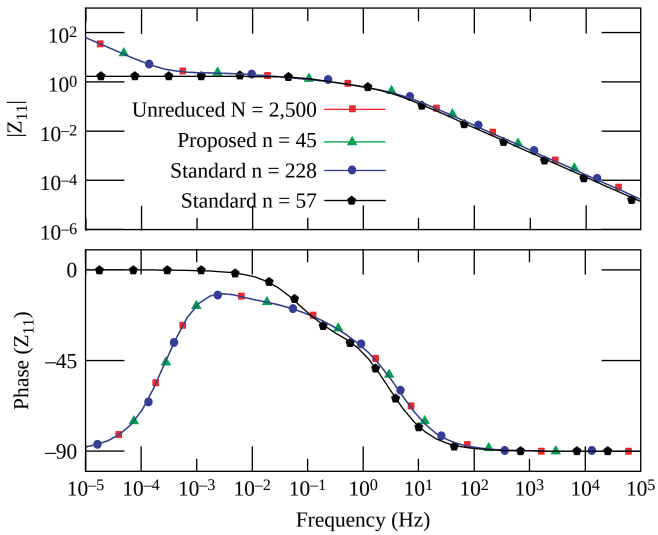


Figure 7.
Magnitude and phase of Z_{11} of RCI-mesh with independent current sources

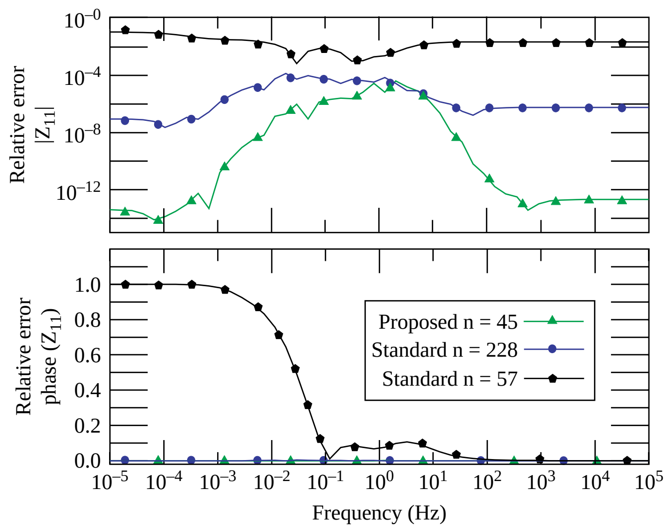


Figure 8.
Approximation error of magnitude and phase of Z_{11} of RCI-mesh with independent current sources

Second, both systems are reduced to a comparable size. Again for the proposed reduction flow three expansion points and $q = 3$ Krylov subspace iterations are used, resulting in an order of $n_{repl} = 45$. For a comparable size with the standard reduction flow, only one expansion point and $q = 1$ Krylov subspace iteration can be used, resulting in an order of $n_{extr} = 57$. The accuracy of the model with the proposed reduction flow is better than of the comparable size model reduced with the standard reduction flow (Figures 7 and 8).

Example 2: network with V_{dd}-dependent current sources

To show the incorporation of supply voltage-dependent currents of the digital modules in the modelling and subsequent model reduction another example network is used. Again the RC-mesh is used as shown in Figure 4. The parasitic network is divided into three parts (Figure 9), an analogue part with 25 terminals for the connection with the analogue IC part and two digital parts.

The example network is built of ≈30,000 RC-elements. For a more precise modelling of the internal switching currents an element is used, which weights the time-variant functions $f_i(t)$ with a function $h(V_{dd})$ of the supply voltage of the digital blocks:

$$\begin{gathered} I_i(t, V_{dd}) = f_i(t) \cdot h(V_{dd}) \\ 1 \leq i \leq 80 \end{gathered} \tag{18}$$

Each function $h(V_{dd})$ is possible, as long as it is the same for all elements. This element acts like a non-linear voltage controlled current source with a time varying gain. At 80 nodes in the digital parts elements described by equation (18) are connected. Again, the waveforms $f_i(t)$ are piece-wise-linear waveforms (some are shown in Figure 5) like in the first example. By decomposition of the functions $I_i(t, V_{dd})$ three basis waveforms $g_j(t)$ (Figure 6) are necessary, which results with the two different supply voltages in overall six basis functions. The networks are described as a system of equations with an order of $N = 10,000$. The number of ports is $p_{extr} = 107$ for the extracted sources network (25 for the analogue part, 80 for the currents and two for the supply voltages). For the replaced sources network the number of ports is only $p_{repl} = 33$ (25 for the analogue part, six for the currents and two for the supply voltages). With MOR, both systems are reduced to a comparable accuracy and to a comparable size. The results of

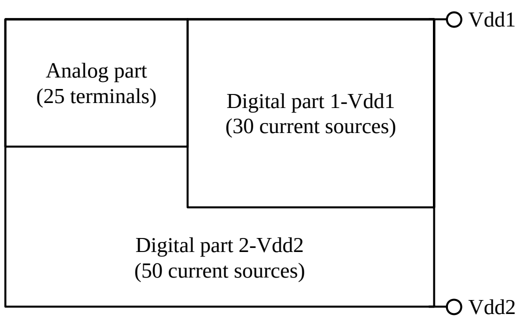


Figure 9.
Top view of model

the reduction are shown at one analogue terminal in Figure 10 and the approximation error of the reduced systems is shown in Figure 11.

For comparable accuracy the standard reduction flow with two expansion points and $q = 2$ Krylov subspace iterations is used. The resulting reduced system has an order of $n_{extr} = 428$. For the proposed reduction flow two expansion points and $q = 2$ is used resulting in $n_{repl} = 132$, which is much less than in the standard reduction flow. For a comparable size with the standard reduction flow only one expansion point and $q = 1$ Krylov subspace iteration can be used, resulting in an order of $n_{extr} = 107$. The reduced model with a comparable size in the standard reduction shows worse accuracy than the reduced model generated with the proposed reduction flow (Figures 10 and 11).

Model reduction results

The results of model reduction and the achieved MATLAB simulation speed-up are summarized in Table I. With the reduction of the example networks, it is shown that for a comparable accuracy of the reduced models the standard reduction flow with extraction of the current sources enables a lower reduction, and, therefore, a lower speed-up, than using the proposed reduction flow with replacing of the independent sources. Also with these examples it is shown that with the proposed reduction flow the reduced model is much more accurate than a model of a comparable size reduced with the standard reduction flow.

Conclusion

In this paper, an efficient method for the reduction of networks modelling the parasitic couplings between digital and analogue part of mixed-signal VLSI ICs is presented. The standard reduction flow is extended by a preceding step to reduce the number of ports for the sources modelling the switching currents of the digital modules. With this reduction flow, smaller and more accurate reduced models can be generated with MOR methods. The improved performance of the reduction process is shown by reducing example networks.

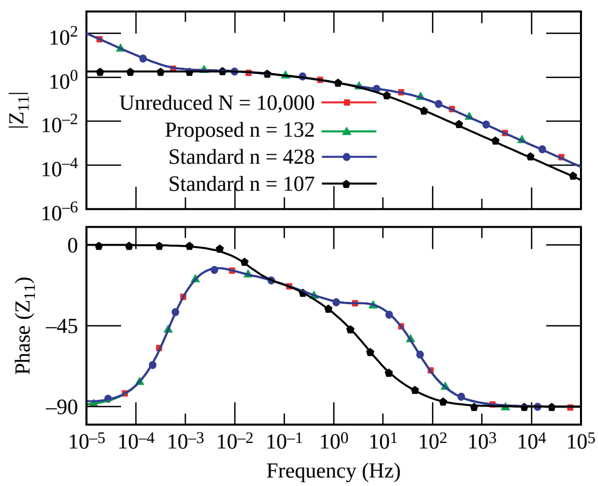


Figure 10.
Magnitude and phase of Z_{11} of RCI-mesh with V_{dd}-dependent currents

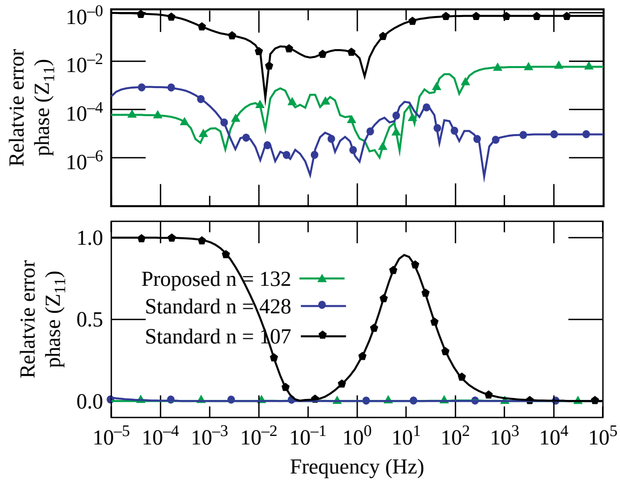


Figure 11.
Approximation error of magnitude and phase of Z_{11} of RCI-mesh with V_{dd}-dependent currents

	Ports	Order	Accuracy	Speed-up
Example 1		$N = 2{,}500$		
Standard reduction	$p = 57$	$n = 228$	Good	≈5
Standard reduction	$p = 57$	$n = 57$	Bad	≈30
Proposed reduction	$p = 5$	$n = 45$	Good	≈40
Example 2		$N = 10{,}000$		
Standard reduction	$p = 107$	$n = 428$	Good	≈20
Standard reduction	$p = 107$	$n = 107$	Bad	≈250
Proposed reduction	$p = 33$	$n = 132$	Good	≈250

Table I.
Results of model reduction

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