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A classical to quantum transition via key experiments

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Abstract

This paper presents an educational concept for promoting quantum teaching and learning via an educational structured quantum optical experiment. The experiment is designed to demonstrate the striking differences between classical physics and quantum physics, for example *quantum interference of unbreakable photons* (the ability of probabilities to interfere due to a phase sensitive superposition of states) and *quantum nonlocality* (there is no way to locate photonic states without a fundamental loss of information about the characteristics and a complete change of the state). For this proposal, we developed an experimental setup straightforward enough to be used in advanced physics courses even in secondary school student labs. To explain, or in a more quantum-semantic way, to *interpret* the experimental results quantitatively, we provide an appropriately rigorous quantum optical theory. Our model combines Laplace statistics (to access the statistical behaviour of photon counting) and basic vector calculus to calculate probabilities from the phase sensitivity of probability amplitudes. This article aims to contribute to further discussion and empirical research into novel teaching strategies for a more deeply conceptual approach to quantum theory.

Keywords: quantum physics, quantum optics, conceptual change, knowledge in pieces, Key Experiment, single-photon experiment, interferometry

S Supplementary material for this article is available online

(Some figures may appear in colour only in the online journal)

1. Introduction

Headlined 'The Future is Quantum', the website of the European Quantum Flagship initiative QT (https://qt.eu/) presents a view of the second quantum revolution and a wide variety of applications of the new quantum physics. Indeed, nowadays it seems superfluous to discuss the overwhelming influence of quantum physics on our daily life. To ensure, therefore, that the basic knowledge of quantum phenomena is not reserved exclusively for professional physicists, educating young people in quantum physics (QP) is necessary for social participation. What is stopping us from doing this? The strong epistemological position of QP has been gained by a deep paradigmatic change in our view of nature, deep enough that the foundations of QP have been hotly debated, even in the scientific community, right up to the present day (Schlosshauer *et al* 2013). There is no need for further empirical justification; teaching QP is one of most challenging educational projects in the field of advanced physics.

The recent situation may be described in the following way:

- QP has secured a leading position in physics research and is expected to have an increasing importance as some of that research leads to quantum technology. Educators, therefore, should invest in developing and practising an operative and effective introduction to QP.
- Beyond a so-called 'minimal interpretation', we have no consistent visualization of quantum phenomena (Stadermann *et al* 2019). Existing visualizations are derived from a rather mystical and strange image of QP.
- We have known for many years that there are significant barriers to learning QP. On the other hand, the debate on how best to teach QP is controversial. One line of discussion moves between two poles: (1) teaching QP rigorously based on concrete applications of quantum theory (QT) (Alonso 2002) versus (2) underlining particular characteristics of quantum theory and connecting them with apparent quantum phenomena to get a deeper insight into the meaning of quantum theory (Wesenszuege = specific traits by Müller and Wiesner (2002b)).

For students, the process of learning QP is related to a necessary change of mental pathway from the well-taught and deeply internalized classical path to a new quantum pathway, which is counter-intuitive to a large extent (Ireson 2000). Obviously, from a somewhat rigorous epistemic perspective there is no smooth connection between these pathways. It seems plausible that learning QP requires more than a careful readjustment of our way of thinking about physics: partly like a knowledge reboot, partly like a change of mental pathway. Physics education, research and practical teaching find the provision of appropriate educational settings challenging.

QP is currently implemented in the secondary school curricula of many countries and the vast majority follow an application-oriented approach: physics of **a**toms, evolution of **m**icrosystems and wave-particle-**d**uality (AMD) (Krijtenburg-Lewerissa *et al* 2017). The educational advantage of the AMD introduction to quantum physics is a good conceptual linkage to classical physics (CP), that readily leads to applications of quantum theory to solve concrete physical problems. The background to this curricular status is the enormous influence of the semi-classical theory of the light-matter interaction (e.g. Fermi's golden rule). Though the quantization of the energy of the radiation field was central in the early beginnings of quantum physics, the semiclassical theory, treating the quantities E and B as classical variables, worked well.

Understanding the conceptual core of quantum theory requires readiness for a fundamental change of how reality is perceived and constructed (Krijtenburg-Lewerissa *et al* 2017). Although school education should enable pupils to explain scientific phenomena and solve problems (NRC, 2012), students are often only able to reproduce factual knowledge learned by rote (Baumert *et al* 1998). One reason for this is a low degree of internal coherence within educational disciplines (ibid.); to explain and understand phenomena, it is necessary to be endowed with a cross-linked knowledge base (Bransford *et al* 2000). For QP, this cross-linked knowledge base can be realised by the use of key ideas that then enable the linking of as many sub-networks as possible (diSessa 2013). There is consensus that a knowledge element only acquires its meaning from being embedded in a knowledge system and becoming a concept (networked knowledge). The introduction of quantum probability, nonlocality, and superposition will force students to shift to concepts that are counterintuitive and that conflict with nearly everything that was previously understood. In their international study cited above, the authors analysed different contemporary approaches to quantum education (Stadermann *et al* 2019). They found that counterintuitive quantum concepts should be taught with the utmost care, as a conceptual change is required.

We have evidence that easily comprehensible simulations dealing with photonic radiation fields and their quantum optical interpretation can provide a deeper insight into fundamental quantum theoretical principles (Müller and Wiesner 2002a). Here we propose a short sequence of real experiments aimed at instigating the previously-mentioned 'jump' of mental pathways. The idea presented here is strongly influenced by one of the statements of Feynman in his lectures, to '*examine a phenomenon which is impossible, **absolutely** impossible, to explain in any classical way, and which has in it the heart of quantum mechanics'* (Feynman *et al* 1965). Inasmuch as the experimental sequence cannot be explained by any classical path, it contrasts classical physical realism with quantum physical constructs, which is why these experiments may function as key experiments in teaching quantum physics. In order to avoid, or at least minimise, the confusion of CP with QP models, the experiments are provided with QP explanations containing, hopefully, no spurious influences from CP.

Far from presenting a complete quantum physics course (as presented by Küblbeck, Müller and Wiesner (Müller and Wiesner 2002a, Küblbeck and Müller 2003) and by Bronner (Bronner 2010)), we focus solely on the specific aspect of real key experiments instead of simulations and interactive screen experiments (ISEs). German physics teachers have a strong preference for real experiments for their physics lessons (Weber 2018), especially in quantum physical teaching. Similarly to the web presentation of quantum optics given by the physics education group of the university of Erlangen-Nuremberg (quantumLab: an interactive approach to the fascinating world of quantum physics 2010; http://didaktik.physik.uni-erlangen.de/quantumlab/index.html), our experiment is accompanied by distinct fully quantum theoretical explanations for a true QP experiment.

In section 2, we demonstrate how we aim to reduce comprehension barriers using the specifics of the experiment. Section 3 is dedicated to the details of the key experiment, followed by a full quantum theoretical explanation at an upper high school level in section 4. The basics of a scientific educational classification are given in section 5. Experimental and apparatus details provided in the appendix should remove any doubts regarding the true quantum character of the experiment.

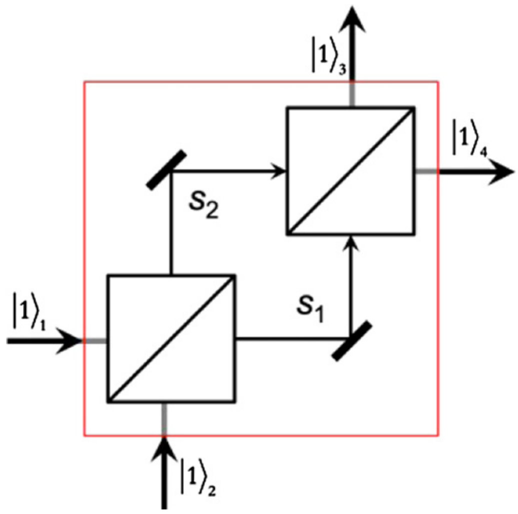

Figure 1. Single photons in a Mach–Zehnder-interferometer; optical pathways s_1 and s_2. Dirac's kets describe the quantum states at the OBS-ports.

2. Comprehension barriers

Single photon interference, as shown here, is one of the most mystifying quantum phenomena. Every attempt to explain this effect without rigorously referring to quantum theory will fail. As a consequence, single photon interference is effective as a prompt for a jump from an explanation pathway in the classical domain to a quantum pathway, but at the same time, it produces specific difficulties for the students trying to access the relevant quantum concepts (figure 1; Marshman and Singh 2016).

Students trying to learn QP have faced difficulties linked to a whole spectrum of different barriers. For the interference experiments shown in this paper, however, studies are shedding some light on some basic problems (ibid.):

- **The classical particle barrier (i.e. photons do not interfere)**: a beam of light resembles a stream of photons. There is no interference in the classical case, because at each optical beam splitter (OBS) about one half of the huge number of photons will be transmitted and the other half will be reflected. So, we will have half the irradiance ($I/2$) at each output detector.
- **The localising barrier (I)**: the basic idea of a radiation field consisting of a large number of single photons and how they interact with the interferometer may imply a locality for each photon.
- **The localising barrier (II)**: students try to localize the photon in one of the output paths of the beam splitter and thus ignore the superposition of the two output states.
- **Hybrid models (the breakable photon)**: students struggle with the idea of a photon as an undividable quantum (non-correlated photons at the OBS) and claim that the single photon can split and the two halves can interfere.
- **Refusing complexity**: phase shifts have no meaning for these students.

The stumbling blocks listed here hinder students from understanding quantum physics phenomena and clearly reveal the relevance of the following:

(a) Experiments that demonstrate the existence of photons as unbreakable quanta (see section 3.2, 'part (I): unbreakable photons').
(b) Experiments that demonstrate the existence of single photon interference in an interferometer (see section 3.2, 'part (II): the quantum enigma: quantum interference').
(c) Exclusively quantum physical explanations that are obviously and unequivocally free of internal contradictions and are accessible even for early beginners—at least in principle.

Single photon interference experiments, as used here, are standard even for undergraduate labs at university, to illustrate the quantum nature of light (Galvez and Beck 2015). With the help of two relevant studies, we are trying to evaluate the mind-changing potential of the key experiment. In a first study (Roesler 2018), electromagnetic waves as a model for light are shown to be an extremely strong attractor, a leading paradigm for everything that follows in the physics learning career.

A second study, currently underway, will help to find out how physics novices build up their physical models and how modelling details from the quantum domain may be integrated into the thought process. A list of 34 items has been developed for a pre/post-test. Based on a Rasch sample Wright Map, we checked the items against a person's ability to discern likely problematic items (Boone *et al* 2014). There are 33 items left that will now be used in a survey with trainee teachers, with the results yet to be published. These items can be found in the supplementary file on the foeXlab website[3] or they can be requested by email (r.scholz@iqo.uni-hannover.de). Unfortunately, they are only available in German.

In the next step, we will study whether students can benefit from an experimental approach to quantum optical phenomena. A straightforward experimental setup for direct observation of single photon phenomena derived from a school physics course is suitable for use in a university students' early practical experiment. The central research question is: whether the classical to quantum learning barrier can be reduced by doing so; and if so, to what extent.

3. A quantum optical key experiment

3.1. Justification

'What's behind all this?' This question, from students trying to understand quantum physics, recalls the question of Alice in *Through the Looking-Glass, and What Alice Found There* (Carroll 1872). On the one side of the glass, we live in our real world characterized by commonplace experience, known as the classical domain. The other side, known as the quantum domain, is governed by a set of new rules that differ substantially from the rules guiding the classical domain. Just as Alice is able to jump through the glass, thereby moving to the other the side, experimental quantum physics may be interpreted as a jump through the looking-glass. What we see is a special manifestation of a quantum phenomenon in the classical domain leaving the question unanswered: 'What's behind all this?' From multiple specific traits ('Wesenszuege') of the quantum domain behind the mirror we single out *Born's principle* for the calculation of probabilities, the *principle of superposition* (POS) and *quantum theoretical nonlocality*. We will demonstrate that quantum interference patterns occur immediately when quantum states with stable phases are superposed. Phase differences lead to maxima or minima in the measured photon numbers.

[3] https://praktikumphysik.uni-hannover.de/fileadmin/praktikumphysik/Bilder/foeXlab/DL/Quantum_survey.zip.

Perhaps the reader would have preferred to include complementarity in the list of selected traits? Undoubtedly, complementarity is one of the most challenging concepts of QP. At the very least, it can be blamed for a dazzling variety of facets:

- Textbooks on quantum mechanics tell the story that Niels Bohr, remarking on a persistent 'doubleness' of quantum mechanics, attributed it to a so-called 'complementarity' as a specific trait of the quantum domain.
- The particle nature of quanta is often described as being complementary to a wave-like character.
- Complementarity describes a limiting uncertainty condition for the preparation of quantum states. Complementary operators obey an uncertainty relation.
- Experimental complementarity describes mutually exclusive experimental conditions: no experiment exists that can simultaneously reveal interference and path information.

Though the discussion about complementarity has surely been a contributory element for the development of quantum theory, we will not rely on it for educational purposes. Dualistic arguments have been used for the introductory phase of teaching quantum mechanics for at least forty years. Our teaching experience shows that young novice learners, in the worst case, learn from dualism/complementarity arguments that quantum physics is nothing more than classical physics without clarity—we can do much better.

As explained further in this section, the small two-step series of experiments presented in this paper is designed to work as a key experiment. In contrast to some historical meaning and following a notation given in Laumann *et al* (2019), the experiment is designed to support a modelling approach based on the concept of: (1) identifying elements to be changed, (2) constructing a new knowledge pattern with a very careful integration of preconceptions and new pieces of knowledge into one novel concept, closer to a scientific understanding.

3.2. The experiments

Part (I): unbreakable photons. In the first part of the key experiment (figure 2(a)), single photons from a true single photon source are incident on the OBS. The students' ideas of what will happen may be distilled into three distinct basic choices:

(a) The photon behaves just like a billiard ball. The '50/50 beamsplitter' just means that the particle will burst and we will always have the same counting probability at the detectors $P(\mathrm{D}_3) = P(\mathrm{D}_4) = P(\mathrm{D}_3\ \&\ \mathrm{D}_4)$, where $P(\mathrm{D}_3\ \&\ \mathrm{D}_4)$ is the probability for coincident clicks.

(b) The photons behave like classical amplitudes of their electromagnetic fields. The amplitudes will simply split at the OBS leading to identical intensity at the detectors but independent counting events at the beam splitter. In this case, the counting probabilities at D_3 and D_4 are independent, coincidences are purely accidental with a probability given by the product of the single event probability, $P(\mathrm{D}_3\ \&\ \mathrm{D}_4) = P(\mathrm{D}_3) \cdot P(\mathrm{D}_4)$ (see equation (A.2)).

(c) The photon is elementary, a quantum. Thus, it is unbreakable by the OBS. These quanta will completely lose their energy in one single detection process. It follows that detection at D_3 versus D_4 is exclusive and the probability for a coincidence vanishes. Either D_3 or D_4 is incrementing, never both.

In the experiment, we measured the probabilities $P(\mathrm{D}_3\ \&\ \mathrm{D}_4)$, $P(\mathrm{D}_3)$ and $P(\mathrm{D}_4)$ and calculated the so-called anti-correlation coefficient $\alpha = P(\mathrm{D}_3\ \&\ \mathrm{D}_4)/(P(\mathrm{D}_3) \cdot P(\mathrm{D}_4)$

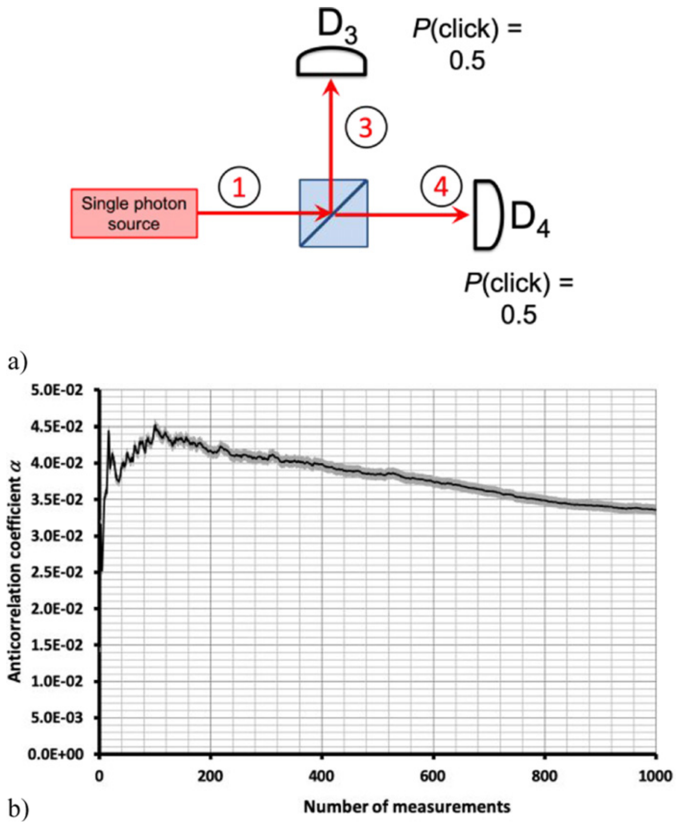

Figure 2. (a) Unbreakable photons: photons are true quanta: (a) the principal setup; (b) experimental results; the mean value of $\alpha = P(\mathrm{D}_3\ \&\ \mathrm{D}_4)/(P(\mathrm{D}_3)) \cdot P(\mathrm{D}_4)$ is approaching zero.

(see appendix A). α is well suited to distinguish different cases of event correlation. Case (a) would lead to $\alpha = 1/P(\mathrm{D}_3) = 1/P(\mathrm{D}_4) \geqslant 1$. In the case of classical fields, [case (b)] we would expect $\alpha = 1$.

Figure 2(b) shows the evolution of the mean value $\langle\alpha\rangle$ along the number M of measurements, revealing

$$\langle\alpha\rangle \rightarrow 3.36 \times 10^{-2} \quad \text{for } M \rightarrow 1000.$$

This means that the number of coincidences from detector D_3 and D_4 and thus the probability of a single photon break vanishes. In reality, we will not find $\langle\alpha\rangle = 0$, due to residual light in the lab and dark noise activations by the uncooled detectors. Instead we always will have $0 < \langle\alpha\rangle \ll 1$. Figure 2 shows huge fluctuations of $\langle\alpha\rangle$ for approximately the first hundred measurements. The standard deviation decreases with an M^{-1} power law. For small M the relative uncertainty is greater than 30%, approaching 2% for $M = 1000$.

This result, $P(\mathrm{D}_3) = P(\mathrm{D}_4)$ and $\langle\alpha\rangle \approx 0$ is exactly what should be expected from the classical theory of Bernoulli experiments. For a 50/50 OBS we find $P(\mathrm{D}_3) = P(\mathrm{D}_4) = 0.5$ (with some

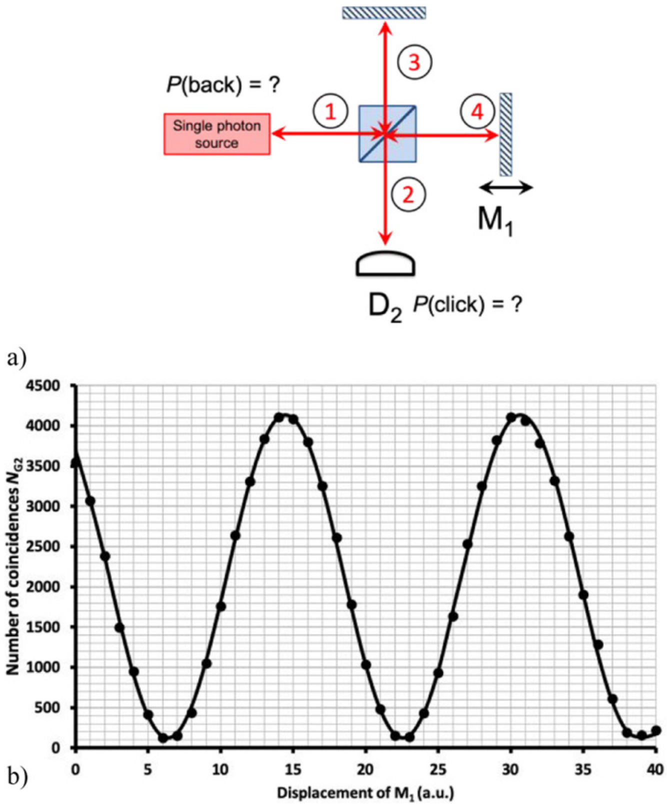

Figure 3. The number of single photon clicks from D_2: (a) setup; (b) quantum interference produced by single photons; visibility $V = 94\%$; relative standard deviation 0.3%.

lost counts due to a detector efficiency smaller than 1). Students are familiar with this result and it fits their real-world experience. Fortunately, quantum theory gives the same result (section 4.4).

Part (II): quantum interference of an unbreakable photon. Our second experiment belongs to the puzzles R. P. Feynman referred to (see above, Feynman *et al* (1965)): '*examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way...*'

The straightforward setup is shown in figure 3. The single photon, identified as an unbreakable particle in the first part, is incident on the 50/50 OBS of a Michelson interferometer. Following the classical unbreakability argument (similar to the one used in experiment part (I)), we expect no coincidence of the outputs (3) and (4) and thus a constant probability of 0.5 for D_2 clicking (and a probability of 0.5 for the photon to returning back to the source). The experiment shows a completely different result: depending on the position of the mirror M_1, interference fringes are obtained as shown in figure 3(b). The number N_{G2} of coincidence clicks of detector D_2 is shown in figure 3(b). To minimize noise, a trigger detector D_G was used (D_G is not shown in figure 3, see appendix A for experimental details). The visibility of the interference pattern is convincing. Inserting experimental data, we get for the visibility:

$$V = \frac{N_{G2}(\max) - N_{G2}(\min)}{N_{G2}(\max) + N_{G2}(\min)} = \frac{3982}{4234} = 0.94. \tag{3.1}$$

Students trying to interpret this result sometimes use a dualistic hybrid model: '*The photon saves a bit of the wave-like structure of the electromagnetic field*.'

4. Key phenomena: the need for a quantum theory

4.1. Historical impact

In the very first chapter of his famous textbook *The Principles of Quantum Mechanics*, Paul Dirac substantiated the '*necessity for a departure from classical mechanics*' (Dirac 1957). His arguments rely on the overarching argument that there is a new non-classical theory, well suited to explain the experimental results, but obviously clashing with classical physics. The list of key phenomena demonstrating failures of the classical physical theory left its mark on educational approaches to quantum physics:

- Classical electrodynamics is inadequate to explain the stability of atoms.
- Internal degrees of freedom of atoms do not contribute to the specific heat according to classical quota.
- The existence of zero point energy.
- The breakdown of causality due to an unavoidable indeterminacy of the quantum state. The preparation of a quantum state must be carefully discriminated from the measurement.
- Light phenomena are contradictory: interference and diffraction of light can only be explained on the basis of a wave theory; Einstein's theory of photo-electric emission on the other hand gave a strong hint that light was composed of particles able to exchange momentum with other particles. These light particles, called photons, were found to have definite energy and momentum given by the wavelength.
- The scattering of photons by free electrons (the Compton effect) can easily be explained by combining the relativistic term for the kinetic energy, Planck's quantum hypothesis and de Broglie's wave length $p = W/c = h \cdot f/c = h/\lambda_B$.

To construct a quantitative basis for the new physical theory, '*a new set of accurate laws of nature is required*', however '*the changes which the scheme involves being of a very profound character*' (ibid.). There is a set of various fundamental quantum principles which, although sounding a bit like classical terms, is quite not in line with the rules of classical physics. Two of these principles, paving the way towards entanglement as one of the most striking quantum specialities, form the basic framework for our approach: the *principle of superposition of states* and the *nonlocality of quantum states*, leading to *quantum interference* as an undoubtedly quantum phenomenon.

We will now give a very rough sketch of the theoretical basis. The considerations presented here are guided by a strong commitment to using real experiments to engage students to leave their classical way of thinking (for a careful study of model based reasoning in the physics laboratory; see Zwickl *et al* (2015)):

- Dirac's line of argument may be viewed as a conversion of the general epistemological principle of physical science: the perpetual search for all possible observations demonstrating the relevant properties, especially of abstract theories like quantum theory, is an unambiguous mandatory requirement.
- Moreover, a particularly educational aspect appears: we have empirical evidence that experimenting may be conductive to success in teaching science (Hopf *et al* 2007).

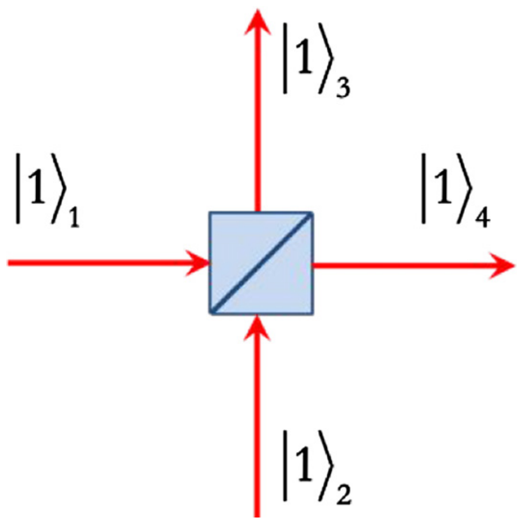

Figure 4. Single photon states at the beam splitter.

This applies in particular to the need for experiments suitable for arousing interest in quantum physics and giving clear proof that quantum physics is real, especially when strange phenomena are involved.

4.2. Single photons at the optical beam splitter: quantum states and probability

Experiments with single photons at the OBS and the quantum theoretical analysis of these experiments illustrate two fundamental quantum phenomena: quantum randomness and the superposition of nonlocalized quantum states. Because some of these features are very basic and sometimes no general definition or axiom exists, we should start the analysis with the explanation of basic quantum theoretical terms (for more details see modern undergraduate introductions to quantum physics, e.g. Ballentine (1998), Lvovski (2018), Cohen-Tannoudji *et al* (2009)). Keeping in mind the pedagogical purpose of this article, it seems clear that the discussion may omit many of the formal details if they are not absolutely necessary to understand what happens, although they are important for the general quantum theory. Apart from this, the details presented should be a good fit for quantum theory.

Physical system (PS). Quantum theory tells, more as a recipe for calculations than as an explanation, that a physical system is associated with an abstract Hilbert space $\mathcal{H}$ in such a way that the system at a given time is completely described by a set of vectors in (or better: a ray in) $\mathcal{H}$. These vectors are solutions of quantum mechanical equations of motion, e.g. the Schrödinger equation for nonrelativistic systems, or they are the result of physical intuition. As may be proven by quantum theory (but also immediately plausible), the physical system behind the single photon experiments of this paper can be described within a four dimensional discrete Hilbert space: one dimension for each mode of the OBS. The four basic states are given by the possible OBS-ports (figure 4) and can be written as

'one photon at port (1) and none at the other ones': $\hat{e}_1$; 'one photon at port (2) and none at the other ones': $\hat{e}_2$;

'one photon at port (3) and none at the other ones': $\hat{e}_3$; 'one photon at port (4) and none at the other ones': $\hat{e}_4$.

At this stage, we may introduce Dirac's ket-representation to facilitate a proper connection to the standard representation of quantum theory. Alternatively, one can use a column/row-vector representation, because students are much more familiar with it:

$$|1\rangle_1 = \hat{e}_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix};\ |1\rangle_2 = \hat{e}_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix};\ \ldots. \tag{4.1}$$

These vectors provide an orthonormal basis of the Hilbert space. Using the standard presentation of the inner product of the vectors and introducing $\hat{e}^-$ for the transformed basic vector, one sees ($\delta_{j,k}$ is the Kronecker symbol):

$$\begin{aligned}(1\ \ 0\ \ 0\ \ 0)\cdot\begin{pmatrix}1\\0\\0\\0\end{pmatrix} &= e_1^-\cdot e_1 = 1;\ e_2^-\cdot e_2 = 1;\ e_3^-\cdot e_3 = 1;\ldots;\\ &\quad e_4^-\cdot e_4 = 1;\ e_1^-\cdot e_2 = 0;\ldots;\ e_j^-\cdot e_k = \delta_{j,k}.\end{aligned} \tag{4.2}$$

Quantum state (QS). Quantum states are a core concept of the probabilistic nature of the quantum domain. Physically a QS differs substantially from a classical PS. In contrast to a classical state, the QS in general does not define distinct values of physical variables. Physicists agree that experiments governed by the rules of QP are intrinsically statistical in a sense that, in general, it is not possible to define all characteristics of a PS exactly. The so called *preparation* of a QS will not lead to well-determined real outcomes but to well-determined probability distributions of different possible outcomes. Positioned against this background and apart from any formal definition, a QS is viewed as a set of instructions to construct probability distributions for a specific physical measurement (of a specific observable).

To avoid confusion with any other physical concept or variable, Dirac's bra-ket notion $|\psi\rangle$ is used as a formula symbol for quantum states. In this representation, the inner product between two state vectors $|\psi_1\rangle$ and $|\psi_2\rangle$ is written $\langle\psi_1|\psi_2\rangle$. Each vector of the Hilbert space can be expanded into a linear combination of the basic state equation (4.1). In this sense, the basic states deliver a complete set of elementary QSs: physically, each basic state represents a possible photonic state of the system and clearly, the vector set equation (4.1) covers all possibilities.

Superposition principle. It might be helpful to memorize a general principle of quantum theory at this point: with state vectors $|X_1\rangle$, $|X_2\rangle$, $|X_3\rangle$, … of a specific QS, it is always possible to find factors C_1, C_2, C_3, … so that $|\psi\rangle = C_1 \cdot |X_1\rangle + C_2 \cdot |X_2\rangle + C_3 \cdot |X_3\rangle + \ldots$ is a state vector of the PS as well; a sum such as this is called a superposition of states.

Preparation of states. If the preparation process of the QS is strongly and carefully discriminated from the measurement, this facilitates the QP-analysis of physical experiments and produces more transparency. As much as the QS $|\psi\rangle$ may be viewed as an instruction to construct probability distributions, physical experimentation shows itself to be a repeatable process of creating well-defined relative frequencies for the observation. In quantum theory (QT) this process is called the *preparation of a quantum state*. Mathematically, the preparation defines a state function as a product of a number and a basic vector or as a sum of such constructions. A state preparation with an OBS demonstrates how this works.

In terms of quantum theory, the effect of the OBS is to transform the input state $|\psi\rangle_{\text{in}}$ = 'one photon at port (1) & none else' into an output state which is a specific superposition of the two output basic states with appropriate numbers c_3 and c_4:

$$|\psi\rangle_{\text{in}} = \hat{e}_1 \rightarrow |\psi\rangle_{\text{out}} = c_3\hat{e}_3 + c_4\hat{e}_4. \tag{4.3}$$

To address the c-numbers, an important quantum theoretical phase concept must be introduced. If the c are real, they always can be written as $c = \text{sign}(c) \cdot |c|$, with $\text{sign}(c) = \pm 1$. As a first generalization, one introduces a phase angle φ, arbitrarily assigning $\varphi = 0$ for positive numbers (the right side of the real axis) and $\varphi = \pi$ for negative ones (the left side of the real axis) and defining a factor $q(\varphi)$ by $q(0) = 1$ and $q(\pi) = -1$: $c = q(\varphi) \cdot |c|$ with $\varphi = 0$ or $\varphi = \pi$. In quantum physics, one has to generalize this concept and allow each angle $0 \leqslant \varphi \leqslant 2\pi$. These phases will play an important role for the preparation of states. Thus, the output state in equation (4.3) will take the form:

$$|\psi\rangle_{\text{out}} = |c_3| \cdot q\,(\varphi_3) \cdot \hat{e}_3 + |c_4| \cdot q\,(\varphi_4) \cdot \hat{e}_4. \tag{4.4}$$

Measurement and Born's rule. The physical meaning of the absolute value $|c_i|$ will become clear in the context of Born's rule for calculating the probability of the result of a particular measurement. At this point, it is worth mentioning that even now, after more than a hundred years of thinking about the interpretation of quantum theory, there seems to be an unsolved problem of how the probabilistic distributions of all possible outcomes of a measurement can be extracted, on the basis of quantum theory (Friebe *et al* 2015). In any case, the final registration (a click of the counter unit or the position of a dial instrument) is purely classical.

The specific procedure can be illustrated using the output function equation (4.4) to calculate the probability of a click signal from D_3. Following the minimum interpretation, the procedure to realize results of a measurement relies on Born's *quantum measurement postulate* (Born 1926): consider a PS prepared to be in the QS $|\psi\rangle$ and let $|O\rangle$ be an eigenstate describing the observation/measurement.

The absolute values of the c-numbers $|c_1|, |c_2|, |c_3|, |c_4|, \ldots$ are equal to the square root of the probability for the system to be in the basic QS $\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4$. It follows that the sum over all squared values $|c_1|^2, |c_2|^2, |c_3|^2, |c_4|^2$ must equal one: $|c_1|^2 + |c_2|^2 + |c_3|^2 + |c_4|^2 = 1$. For a 50/50-beam splitter reflecting half of the incoming light and transmitting the other half, one clearly gets: $|c_3|^2 = |c_4|^2 = 1/2 \Rightarrow |c_3| = |c_4| = 1/\sqrt{2}$.

For those who are familiar with complex valued numbers, it should be noted that the product $|c| \cdot q(\varphi)$ is just the polar representation of complex numbers $z = |c| \cdot q(\varphi)$ with $q(\varphi) = \exp(\mathrm{i}\varphi)$ and $\tan\varphi = \text{Im}(z)/\text{Re}(z)$.

Nonlocality. Nonlocality is unquestionably one of the most unsettling quantum phenomena. A famous consequence is the phenomenon of quantum entanglement. Albert Einstein and his friends Boris Podolsky and Nathanial Rosen clearely laid out the consequences of nonlocality in their famous 1935 EPR paper (Einstein *et al* 1935). Today, we can demonstrate entanglement by violating the Bell/CHSH limit for local hidden variables using entangled photon states (Dür and Heusler 2014). Though entanglement would be one of the most striking effects for revealing pure quantum physics, we will elucidate quantum phenomena using quantum interference alone, for educational reasons: the experimental and theoretical analyses are less sophisticated when compared to entanglement.

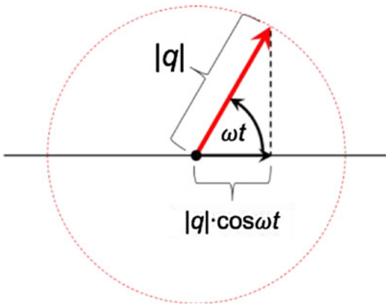

Figure 5. Phasor representation of the oscillation term $q \cdot \cos \omega t$; amplitude q, circular frequency ω.

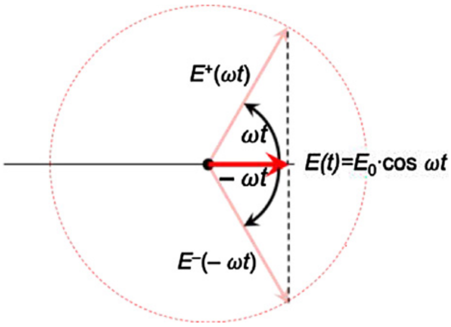

Figure 6. Phasor representation of an oscillating electrical field (field strength $E(t)$, circular frequency ω).

4.3. Geometrical interpretation of the phasor q(φ)

The so-called phasors occurring as a q-factor in equation (4.4) constitute a concept of invaluable importance, even in classical physics. They are well-suited to give a geometrical representation of oscillation by a rotating pointer. To facilitate the solution of problems in the physics of oscillations and waves, phasors are used instead of trigonometrical functions.

As the simplest case, figure 5 demonstrates the phasor representation of a harmonic oscillation often used in upper level high school.

The more advanced figure 6 demonstrates how to decompose the usual trigonometrical term $E_0 \cdot \cos \omega t$ for an oscillating field (field amplitude E_0, circular frequency ω) into the addition of two rotating phasors E^+ and E^-, rotating in opposite directions:

$$E\left(t\right) = E_0 \cdot \cos\ \omega t = \frac{1}{2}\left(E^+\left(\omega t\right) + E^-\left(-\omega t\right)\right). \tag{4.5}$$

A straightforward generalization of this idea draws phasors of length $|q| = 1$ and a phase angle φ to get $\cos \varphi = (q(\varphi) + q(-\varphi))/2$ (figure 7). From this we derive calculation rules for $q(\varphi)$:

In the quantum domain, the portentous factor $q(\varphi)$ carries the complete phase information of the QS. For example, the reflection of light by dielectric surfaces leads to a phase jump of $\pi/2$ and thus to the phase factor $q(\pi/2)$.

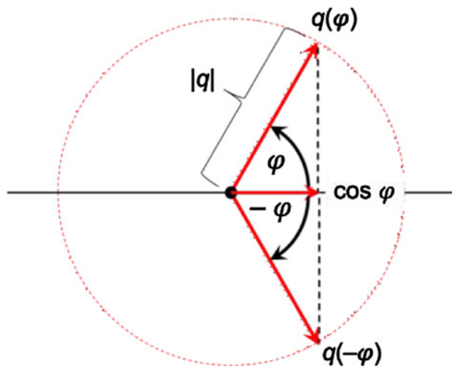

Figure 7. Generalized phasors: arbitrary phase angles φ.

Table 1. Born's method to calculate the probability of a distinct observation.

Firstly we have to determine the state $\|O\rangle$ describing the observation. In the case chosen here, $\|O\rangle$ is the state responsible for a click in detector D_3: a photon at D_3:	$\|O\rangle = \hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$
Usually, the observation eigenstate $\|O\rangle$ will not cover the complete QS but just a subpart. To determine the ' size' of this subpart with respect to the total state $\|\psi\rangle$, one has to project $\|O\rangle$ on $\|\psi\rangle$. This procedure is well-known from standard vector calculus: one has to calculate the inner product $\langle O\|\psi\rangle$:	$\hat{e}_3^- \cdot \|\psi\rangle = \hat{e}_3^- \cdot \left[\|c_3\|\, q(\varphi_3)\, \hat{e}_3 + \|c_4\|\, q(\varphi_4)\, \hat{e}_3\right]$ $= \|c_3\|\, q(\varphi_3)\, \underbrace{\hat{e}_3^- \cdot \hat{e}_3}_{=1} + \|c_4\|\, q(\varphi_4)\, \underbrace{\hat{e}_3^- \cdot \hat{e}_4}_{=0}$ $= \|c_3\|\, q(\varphi_3)\,.$
The third step is given by Born's quantum measurement postulate: one has to take the absolute square of the inner product. The probability of measuring a click from $D_3 = P$(PS is in the state $\hat{e}_3$), is:	$P(D_3) = \left\|\hat{e}_3^- \cdot \|\psi\rangle\right\|^2 = \|c_3\|^2 \underbrace{\|g(\varphi_3)\|^2}_{=1}.$ $P(D_3) = \|c_3\|^2$ (4.6)

As stated above, we have $q(\varphi) = \exp(i\varphi)$ and thus $q(\varphi)$ can be viewed as the algebraic version of the pointer representation of $\exp(i\varphi) = \cos\varphi + i{\cdot}\sin\varphi$. In this sense, equation (4.4) is very close to Feynman's pointer representation of quantum electrodynamics (Feynman 1985).

4.4. Full quantum theory of the key experiment

Now that the quantum theoretical toolbox is sufficiently filled, it is possible to analyse the key experiment. The formal QP analysis is straightforward. The positioning of the detectors discriminate the different output states $\hat{e}_3$ and $\hat{e}_4$.

Part (I): unbreakable photons.

Preparation of the QS.

Here the use of a 'phase angle budget' helps to provide an overview. A phase jump $\varphi = \pi/2$ for the reflection at dielectic surfaces $\varphi = 0$ for transmission is compatible with conservation

Table 2. Calculation rules for the phasors $q(\varphi)$.

RI	Multiplication of $q(\varphi)$ (the geometric interpretation is rotation of the pointer)	$q(\varphi_1)\cdot q(\varphi_2)=q(\varphi_1+\varphi_2)$; $\quad q^2(\varphi)=q(2\varphi)$
RII	Some special values of $q(\varphi)$ for real numbers	$q(0)=1,\quad q(\pi)=-1$
RIII	The absolute squared value	$\lvert q(\varphi)\rvert^2=q(\varphi)\cdot q(-\varphi)=q(0)=1$
RIV	The values for negative phase angles	$q(-\varphi)=q(\varphi)^{-1}$
RV	Addition of $q(\varphi)$	$q(\varphi_1)+q(\varphi_2)=2\cdot q\left((\varphi_1+\varphi_2)/2\right)\cdot\cos\left((\varphi_1-\varphi_2)/2\right)\Rightarrow$ $q(\varphi)+q(-\varphi)=2\ \cos(\varphi)$ and $q(\varphi)-q(-\varphi)=2\cdot q\left(\pi/2\right)\cdot\sin(\varphi)$

of energy and the optical path to the detector contributes an additional phase φ_3. Thus the total phase budget is $\varphi_{\text{total}}=\pi/2+\varphi_3$:

$$|\psi_{\text{in}}\rangle=\hat{e}_1;\ |\psi_{\text{out}}\rangle=\frac{1}{\sqrt{2}}\left(q\left(\frac{\pi}{2}+\varphi_3\right)\hat{e}_3+\hat{e}_4\right). \tag{4.7}$$

Construction of the measurement.

$\hat{e}_3$ is the QS to find the photon at D_3; $\hat{e}_4$ is the QS to find the photon at D_4. Thus we find, using the rules from tables 1 and 2:

$$\begin{aligned} P(D_3)&=\left|\hat{e}_3^-\cdot|\psi_{\text{out}}\rangle\right|^2=\frac{1}{2}\left|\left(q\left(\frac{\pi}{2}+\varphi_3\right)\hat{e}_3^-\cdot\hat{e}_3+\hat{e}_3^-\cdot\hat{e}_4\right)\right|^2=\frac{1}{2}\left|q\left(\frac{\pi}{2}+\varphi_3\right)\right|^2=0.5\\ P(D_4)&=\left|\hat{e}_4^-\cdot|\psi_{\text{out}}\rangle\right|^2=\frac{1}{2}\left|\left(q\left(\frac{\pi}{2}+\varphi_3\right)\hat{e}_4^-\cdot\hat{e}_3+\hat{e}_4^-\cdot\hat{e}_4\right)\right|^2=0.5.\end{aligned} \tag{4.8}$$

The interpretation is clear: the input photon at port (1) will be either transmitted to reach D_4 or reflected to reach D_3, with equal probability. This explains why no coincident counts are measured. This result, a lack of coincident counts as shown in figure 2, indicates that the source is producing single-photon states: argument (c) above. Due to the positions of the detectors, the superposition of states constructed by the beam splitter cannot be observed.

Part (II): quantum interference of an unbreakable photon.

Preparation of the QS.

Again we start with the phase angle budget to construct the output-PS:

Preparation of the output state:
$$|\psi_{\text{out}}\rangle=\frac{1}{2}\left(\underbrace{\left(q\left(\pi+\frac{\pi}{2}+2\varphi_3\right)+q\left(\frac{\pi}{2}+2\varphi_4\right)\right)\hat{e}_1}_{\text{process (4)}}+\underbrace{\left(q(\pi+2\varphi_3)+q(\pi+2\varphi_4)\right)\hat{e}_2}_{\text{process (3)}}\right).$$

1. Reflection and transmission	from port (3): $\varphi = \pi/2$; from port (4): $\varphi = 0$
2. Optical pathway to the mirrors and back to the OBS	from port (3): $\varphi = \pi/2 + 2 \cdot \varphi_3 + \pi/2$; from port (4): $\varphi = 0 + 2 \cdot \varphi_4 + \pi/2$
3. Reflection and transmission to port (2) (= detector D_2)	from port (3): $\varphi = \pi/2 + 2\varphi_3 + \pi/2 + 0 = \pi + 2\varphi_3$; from port (4): $\varphi = 0 + 2\varphi_4 + \pi/2 + \pi/2 = \pi + 2\varphi_4$
4. Reflection and transmission to port (1)	from port (3): $\varphi = \pi/2 + 2\varphi_3 + \pi/2 + \pi/2 = \pi + \pi/2 + 2\varphi_3$ from port (4): $\varphi = 0 + 2\varphi_4 + \pi/2 + 0 = \pi/2 + 2\varphi_4$

Construction of the measurement.
Final registration is a click from D_2, thus the observation vector is $|1\rangle_2 = \hat{e}_2$:

$$\begin{aligned}
P(D_2) &= \left|\hat{e}_2^- \cdot \psi_{\text{out}}\right|^2 = \left|\frac{1}{2} \cdot \hat{e}_2^- \left\{ \left(q\left(\pi + \frac{\pi}{2} + 2\varphi_3\right) + q\left(\frac{\pi}{2} + 2\varphi_4\right)\right) \cdot \hat{e}_1 \right.\right. \\
&\quad \left.\left. + \left(q\left(\pi + 2\varphi_3\right) + q\left(\pi + 2\varphi_4\right)\right) \cdot \hat{e}_2 \right\} \right|^2 \\
&= \left|\frac{1}{2} \cdot \left\{ \left(q\left(\pi + \frac{\pi}{2} + 2\varphi_3\right) + q\left(\frac{\pi}{2} + 2\varphi_4\right)\right) \cdot \underbrace{\hat{e}_2^- \cdot \hat{e}_1}_{=0} \right.\right. \\
&\quad \left.\left. + \left(q\left(\pi + 2\varphi_3\right) + q\left(\pi + 2\varphi_4\right)\right) \cdot \underbrace{\hat{e}_2^- \cdot \hat{e}_2}_{1} \right\} \right| \\
&= \frac{1}{4}\left|\left(q\left(\pi + 2\varphi_3\right) + q\left(\pi + 2\varphi_4\right)\right)\right|^2 = \frac{1}{4}\left(q\left(-\pi - 2\varphi_3\right)\right. \\
&\quad \left. + q\left(-\pi - 2\varphi_4\right)\right)\left(q\left(\pi + 2\varphi_3\right) + q\left(\pi + 2\varphi_4\right)\right) \\
&= \frac{1}{4}\left(q(0) + q(0) + q\left(2\left(\varphi_3 - \varphi_4\right)\right) + q\left(-2\left(\varphi_3 - \varphi_4\right)\right)\right) \\
&= \frac{1}{2}\left(1 + \cos\left(2\left(\varphi_3 - \varphi_4\right)\right)\right) = \cos^2\left(\varphi_3 - \varphi_4\right). \qquad (4.9a)
\end{aligned}$$

It is instructive to calculate the probability of the photon going back into the light source:

$$\begin{aligned}
P(\text{port }(1)) &= \left|\hat{e}_1^- \cdot \psi_{\text{out}}\right|^2 = \left|\frac{1}{2} \cdot \hat{e}_1^- \left\{ \left(q\left(\pi + \frac{\pi}{2} + 2\varphi_3\right) + q\left(\frac{\pi}{2} + 2\varphi_4\right)\right) \cdot \hat{e}_1 \right.\right. \\
&\quad \left.\left. + \left(q\left(\pi + 2\varphi_3\right) + q\left(\pi + 2\varphi_4\right)\right) \cdot \hat{e}_2 \right\} \right|^2 \\
&= \left|\frac{1}{2} \cdot \left\{ \left(q\left(\pi + \frac{\pi}{2} + 2\varphi_3\right) + q\left(\frac{\pi}{2} + 2\varphi_4\right)\right) \cdot \underbrace{\hat{e}_1^- \cdot \hat{e}_1}_{=1} \right.\right. \\
&\quad \left.\left. + \left(q\left(\pi + 2\varphi_3\right) + q\left(\pi + 2\varphi_4\right)\right) \cdot \underbrace{\hat{e}_1^- \cdot \hat{e}_2}_{0} \right\} \right|
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}\left|\left(q\left(\pi+\frac{\pi}{2}+2\varphi_3\right)+q\left(\frac{\pi}{2}+2\varphi_4\right)\right)\right|^2 = \frac{1}{4}\left(q\left(-\pi-\frac{\pi}{2}-2\varphi_3\right)\right.\\
&\quad\left.+q\left(-\frac{\pi}{2}-2\varphi_4\right)\right)\left(q\left(\pi+\frac{\pi}{2}+2\varphi_3\right)+q\left(\frac{\pi}{2}+2\varphi_4\right)\right)\\
&= \frac{1}{4}\left(q\left(0\right)+q\left(0\right)+q\left(\pi+2\left(\varphi_3-\varphi_4\right)\right)+q\left(-\pi-2\left(\varphi_3-\varphi_4\right)\right)\right)\\
&= \frac{1}{2}\left(1+\cos\left(\pi+2\left(\varphi_3-\varphi_4\right)\right)\right) = \frac{1}{2}\left(1-\cos\left(2\left(\varphi_3-\varphi_4\right)\right)\right)\\
&= \sin^2\left(\varphi_3-\varphi_4\right).
\end{aligned}
\tag{4.9b}
$$

The interference fringes, very clearly visible in figure 3(b) (visibility $> 94\%$), are given by the cos term respectively. Moving the mirror M_1 will alter the phase φ_4.

The effect caused by the cos/sin terms is called *quantum interference*. There is no classical physical explanation, because there is neither an electromagnetic wave to produce interference nor more than one photon to allow for some ghostly inter-photonic interaction. Quantum theory is well suited to resolve this conflict. The quantum interference phenomenon shown experimentally is a consequence of the *interplay of superposition and nonlocality*. The coherent superposition of the basic quantum states, one for the transmission and one for the reflection, leads to interference fringes in the final probability. Again we find $P(D_1) + P(D_2) = 1$. This equality mathematically demonstrates the rules of statistical theory, moreover it ensures conservation of energy: no photon is lost. This effect proves the *nonlocality of photons*: the photon ‘belongs’ to the whole interferometer. Nonlocality is characteristic of quantum physics.

A dualistic interpretation relies on the wave-like character of the photons. However, this will hamper a switch from the wave model of light to a full quantum theoretical photon model (see section 5). Without referring to any wave theory, it is possible to obtain interference patterns of quanta solely using a coherent superposition of quantum states and Born’s measurement postulate. Coherence says nothing more than ‘a stable phase difference between the superposed states’. Decoherence due to contact with the environment yields stochastic phase shifts: the interference pattern vanishes.

5. Summarizing remarks: jumping the pathway to quantum physics

Contemporary dualistic light models discuss the wave/particle problem in terms of a ‘neither/nor’ solution. It follows that the dualism is, in principle, disposable, yet it is part of most of the current classroom introductions to quantum physics. However, there are at least two arguments against this introduction to quantum physics:

- Firstly, as shown here, there are straightforward experiments that force us to jump off the well-trodden pathways of classical physics and
- Secondly, students’ difficulties when trying to learn quantum physics give clear support for a more rigorous avoidance of classical arguments within quantum teaching strategies.

Our aim is that students gain a foundation of factual knowledge. We want to help them to replace their initial cognitive structures with the scientific content to eventually develop scientific competence (Bransford *et al* 2000). Because it is important that people take control of their own learning, it is important to recognize what kinds of evidence people need in order to believe unusual claims (Bransford *et al* 2000).

To summarize the broad scope of this paper, we now present some basic pillars of ideas for a learning process closely supporting a conceptual change that will foster mental pathway jumps.

Knowledge in pieces. In order to achieve this, our intention is to engage students with the help of key phenomena. With these phenomena, we aim to provide 'knowledge in pieces' (diSessa *et al* 2016)—like *what is happening at a beam splitter?* or *preparation of a quantum state*—and make it comprehensible even for students. It is noticeable that the key experiment strikingly isolates key phenomena. Together with fruitful and logically accessible explanations, these phenomena will be converted into pieces of the mosaic to be part of a sustainable science-oriented knowledge pattern. The specification of the KiP-concept presented here resembles the concept of the 'Wesenszüge' presented much earlier (Müller and Wiesner 2002a).

Factual knowledge. Students learning QP often find it challenging to adapt their individual view of the real world, since QP is often counterintuitive to a commonplace mechanistic view (Ireson 2000). Regarding our ambition—a classical to quantum transition via key experiments—students have to gain factual knowledge (Bransford *et al* 2000) about photons as undividable quanta from a key experiment that shows single photon interference and a simultaneous awareness of nonlocality and the phenomenon of superposition.

Learning QP. The approach to QP proposed here is based on the idea of learning as a search for meaning. We want students to scrutinize what they observe when doing the key experiment. Therefore, students have to predict what they think they will observe regarding the key experiment, they have to observe the experiment and they have to understand that the key experiment cannot be explained by classical physics approaches (when using the predict/observe/explain approach, at the very least step 3, explaining the key experiment, will fail; see White and Gunstone (1992)). All this is a first step on the path of building the QP concept.

Conceptual change. For learning and understanding quantum physics, a comparably radical conceptual change is required (Kalkanis *et al* 2003). It is likely that the standard model for a conceptual change (Posner *et al* 1982) is not perfectly suited to describe the successful learning of quantum physics, mainly because of lack of plausibility of quantum theory. In principle, adoption of an alternative physical model can be achieved through a shift in basic understanding and new explanatory patterns (diSessa *et al* 2016).

Preconceptions and initial structures. It is crucial to know the ideas that students will bring to the mutual learning process (Kattmann *et al* 1997). Empirical studies exemplify details of students' preconceptions regarding quantum physical phenomena (Müller and Wiesner 2002a) as part of their explanatory contextual thinking about the world.

Special traits and key ideas. Several teaching concepts have already taken up the challenge of demonstrating the failure of classical physical explanations of quantum physical phenomena in the classroom. A special feature of the 'Münchener Unterrichtskonzept' is the distinction of key ideas, such as 'Wesenszüge' (Küblbeck and Müller 2003): *stochastic predictability, ability to interfere, possible measurement results, complementarity and entanglement* (Küblbeck and Müller 2003, Müller and Wiesner 2002a). In the approach presented here, similar key ideas are used in order to explain a key experiment: *nonlocality* and *the principle of superposition*. The limitations are the quantum objects themselves. Outside QP these terms are without prominent roles.

Quantum interference	Regarding the key experiment, students are familiar with the interference of waves. In their physics classes they dealt with this topic in detail. However, the model of electromagnetic waves produces a strong attractor: Interference is consistently connected with waves (Roesler 2018). Here one finds a good justification to avoid wave-like illustrations in QP-explanations.
Nonlocality	As far as we know, nonlocality is not part of the typical set of preconceptions students bring to the learning process and there are no everyday connections to this term. Nonlocality will therefore be noticed as a quantum theoretical curiosity that has to be accepted.
Quantum superposition	The preconception connected with this term strongly depends on the topics covered in classroom physics students prior to the introduction to QP. A superposition principle has frequently been cited in the context of a superposition of motions (rolling as a superposition of rotation and translation) and a superposition of waves. This last case will produce the typical effect of an attractor towards electromagnetic light theory giving rise to a juxtaposition or even a merging of CP- and QP-models.

6. Conclusion

We have reported on a single photon experiment designed to work as a key experiment to introduce the quantum concept of light. On the one hand, there is no classical physical explanation of the results (interference without waves), on the other hand, the results are an excellent fit for the quantum physical description. For this description, we developed a representation of a reduced quantum theory based completely on mathematics and formal arguments accessible to upper secondary school students. The experiments are very similar to the experiments published by (Thorn *et al* 2004) for the university undergraduate level.

The light field is prepared in a single photon Fock state from a spontaneous parametric down-conversion process, incident on an optical beam splitter and a Michelson interferometer. This setup has been used in a first step to give a proof of the existence of photons (intended to demonstrate unbreakable quanta) and immediately after that, the 'same' photons were used to produce interference fringes in a single photon Michelson interferometer, thus showing a realization of quantum interference.

For the theoretical description, we restricted the analysis to pure state quantum physics. As a consequence, the concept of quantum states has been governed by basic ideas of probability distributions. The superposition of states reveal themselves as a sum of basic statistical possibilities. Inverting Born's rule to find the probability of an event, we introduced a phasor $q(\varphi)$ as an algebraic version of Feynman's pointers. A rigorous but careful separation of the preparation of a quantum state versus the measurement simplified the notorious problems of the discussion of the measurement in quantum physics.

This physical approach was motivated by a discussion of typical students' difficulties in learning quantum physics. Careful studies show severe learning barriers, for example: viewing photons as hard spheres, interaction of photons with mirrors as a local effect, interference as an effect of split photons (split by the beam splitter), localizing the photons instead of thinking about the superposition of states, completely ignoring the possibility of a phase shift.

The efficiency of key experiments and the associated framework of explanations as a successful system for quantum teaching is currently under investigation (Moritz Waitzmann, PhD project). A list of Rasch analysed items can be found in the supplementary file via the students' lab website[4] or directly via email (r.scholz@iqo.uni-hannover.de). Unfortunately they are only available in German.

Acknowledgments

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Appendix A. The single photon test of the equipment

A.1. Statistics of a single photon state

To ensure the true quantum physical character of the phenomenon we decided to experiment with true single photon states. For our experiments, we use a setup now standard in many contemporary optics laboratories, even ones for undergraduate students (figure A1; for details of the experiment we refer to Scholz *et al* (2018)). In a spontaneous parametric fluorescence a so-called pump photon, with high photon energy

$$W_{\text{photon}} = h \cdot c/\lambda = 4.9 \ \times \ 10^{-19}\ \text{J} = 3.06\ \text{eV}; \quad \lambda_{\text{P}} = 405\ \text{nm},$$

is nonlinearly scattered into a pair of correlated low energy photons (usually called the idler photon and the signal photon). Equality of the photon energy of the idler photon and the signal photon would be in accordance with the law of conservation of energy if

$$W_{\text{i;s}} = h \cdot c/\lambda = 2.45 \ \times \ 10^{-19}\ \text{J} = 1.53\ \text{eV}; \quad \lambda_{\text{i;s}} = 810\ \text{nm}.$$

To be additionally compliant with the law of conservation of momentum within the crystal, the output photon states are characterized by $\vec{p}_{\text{pump}} = \vec{p}_{\text{signal}} + \vec{p}_{\text{idler}}$ (the so called phase matching condition) leading to different main directions of propagation of the output photons. In our experiment, phase matching could be achieved with a 3° angle between the directions of propagation of the idler and the signal photons. The idler and signal photons created in one single quantum electrodynamic process are strongly correlated to give a two-photon-state with zero variance of the photon number (it may be written as a complex superposition of two single photon states). Using one of these two photons as a herald (in our setup via the detector D_G) and counting only events coincident with a click in detector D_G (coincidence experiments), we can prepare the other photon in a single photon state.

Quantum optics is a statistical optics relying on counting the number N of voltage pulses from high sensitivity binary photodetectors running in the Geiger mode (single-photon avalanche diode, SPAD). The detectors in figure A1 feed single and coincidence counters. These N-numbers had to be averaged over up to 10 000 independent measurements: N_G, N_3, N_4 denote the singles of each detector, the number of mutual twofold coincidences N_{G3}, N_{G4} and N_{34} reveal the correlation of D_G, D_3 and D_4 and finally N_{G34} counts the number of triple coincidences.

Click events are defined to be coincident if the clicks occur within a small temporal coincidence window w_c. In the experiment, w_c could be adjusted in steps $\Delta w_c = 4.9$ ns between 6.6 ns and 26.3 ns. It follows that w_c is small enough that at a maximum, one single event

[4] https://praktikumphysik.uni-hannover.de/fileadmin/praktikumphysik/Bilder/foeXlab/DL/Quantum_survey.zip.

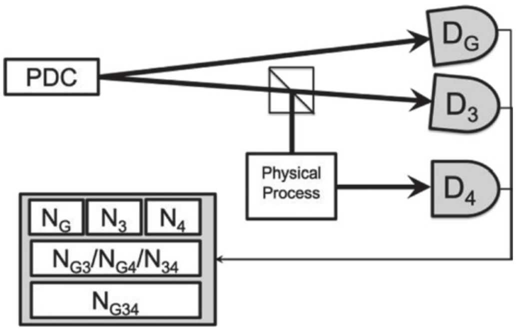


Figure A1. The setup to experiment with correlated photon pairs generated in parametric down-conversion.

will be counted during w_c. The total sampling time, denoted by T, had a typical value $100\,\mathrm{ms} < T < 1\,\mathrm{s}$, thus $3.8 \times 10^6 < T/w_c < 1.15 \times 10^8$ gives the possible maximum of the number of counts during T. From elementary Laplacian statistics we get the relevant probabilities of events:

$$P\,(\mathrm{D}_i) = \frac{\text{favored events}}{\text{all possible events}} = \frac{N_i}{N_{\max}} = N_i \frac{w_c}{T}. \tag{A.1}$$

If the events at D_3 and D_4 are independent, coincident events are purely accidental and the probability of registering coincidences from detectors D_3 and D_4 is

$$P_{\mathrm{acc}}\,(\mathrm{D}_3\,\&\,\mathrm{D}_4) = P\,(\mathrm{D}_3) \cdot P\,(\mathrm{D}_4) = \frac{N_3 N_4}{N_{\max}^2} = N_3 N_4 \left(\frac{w_c}{T}\right)^2. \tag{A.2}$$

Depending on the nature of the light, the probability P_c to measure a coincidence may be equal to P_{acc} for accidental coincidences (as for Poisson distributed photons in a laser field) or not ($P_c = 0$ for the single photon state and $P_c > P_{\mathrm{acc}}$ for thermal light). This phenomenon can be well depicted by the correlation parameter α:

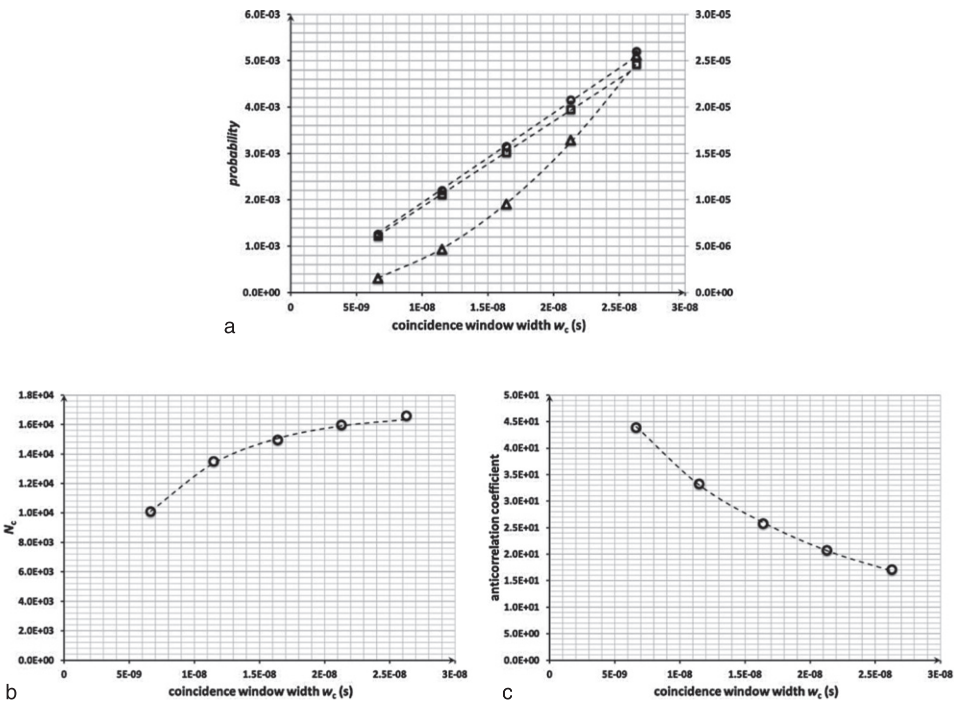
$$\alpha = \frac{P_c\,(\mathrm{D}_3\,\&\,\mathrm{D}_4)}{P_{\mathrm{acc}}\,(\mathrm{D}_3\,\&\,\mathrm{D}_4)} = \frac{P_c\,(\mathrm{D}_3\,\&\,\mathrm{D}_4)}{P\,(\mathrm{D}_3) \cdot P\,(\mathrm{D}_4)} = \frac{N_c}{N_3 N_4} N_{\max} = \frac{N_c}{N_3 N_4} \left(\frac{T}{w_c}\right). \tag{A.3}$$

A.2. Statistical test of the setup

Now equations (A.2)/(A.3) revealing the dependence of the different counting probabilities on the value w_c will be used to prove the single photon character of the photonic state experimentally.

Figure A2(a) demonstrates the dependence of the probabilities $P(\mathrm{G})$ and $P(3)$ of measuring the singles from detector D_G and D_3, and $P_{\mathrm{acc}}(\mathrm{D_G}\ \&\ \mathrm{D}_3)$ for the accidental coincidences of D_G and D_3 on the window width w_c. The dotted lines give the theoretical values calculated from equations (A.1) and (A.2).

Figure A2(b) shows the measured numbers N_c of twofold coincidences. As can be seen, N_c reduces with $w_c \to 0$, as should be expected. The cause of the saturation effect for greater values of w_c is not quite clear, since the production rate of correlated photons cannot depend on w_c. Most likely, it is due to a saturation effect of the AND gate in the counter electronics. The dotted line shows a fit with $A \cdot (1 - \exp(-w_c/B))$ with $A = 1.68 \times 10^4$ and $B = 7.19$ ns.



a b c

Figure A2. Results of the counting experiments (sampling time $T = 1000$ ms; mean values from averaging over 1.000 measurements); (a) the probabilities $P(D_G)$ (circles); $P(D_{3)}$ (squares) and $P_{acc}(D_G\ \&\ D_3)$ (triangles; right scale). (b) Number of twofold coincidences N (c) anticorrelation coefficient α.

Finally, figure A2(c) shows the experimental values of the correlation coefficient α as a function of w_c. The circles are determined from $\alpha = \frac{N_c}{N_3 N_4}\left(\frac{T}{w_c}\right)$ with the measured values for N_c, N_G and N_3. The dotted line has been calculated from (equation (5.3)) with the fitted values for N_c from figure A2(b). Values of α larger than one show the signals from the detectors to be correlated. This fact, alongside a small temporal window width w_c provides good evidence that the equipment generates single photon states for our quantum optical experiments.

Appendix B. Apparatus notes

Figure B1 shows a photo of the experimental setup. Many technical details are published elsewhere (Scholz *et al* 2018).

The pump-laser

We used a GaN based laser-diode (Sanyo DL-4146-101S) without external cavity. The laser can be stabilized to $\Delta\lambda = \pm 0.1$ pm. The optical power P can be controlled via a stabilized forward current in the diode in an optical power range $0.1\ \text{mW} \leqslant P \leqslant 15\ \text{mW}$. From a Gaussian fit, we found an oval shaped beam cross-section with diameters $\Delta x/\Delta y = (1.032 \pm 0.012)\ \text{mm}/(1.260 \pm 0.012)\ \text{mm}$ at the location of the BBO-crystal (see figure B2).

Linear polarization of the laser radiation was ensured via external polarizing filters.

The nonlinear crystal

For the type-1 parametric down-conversion we used a coated BBO-crystal sized $W = H = 6$ mm, $L = 3$ mm, cut at an angle of 29.2°, input wavelength 405 nm, output wavelength 810 nm.

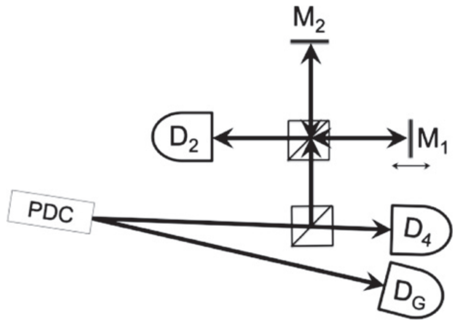


Figure B1. Experimental set up for the single photon experiment; the mirror M_1 has been mounted on a piezo linear actuator.

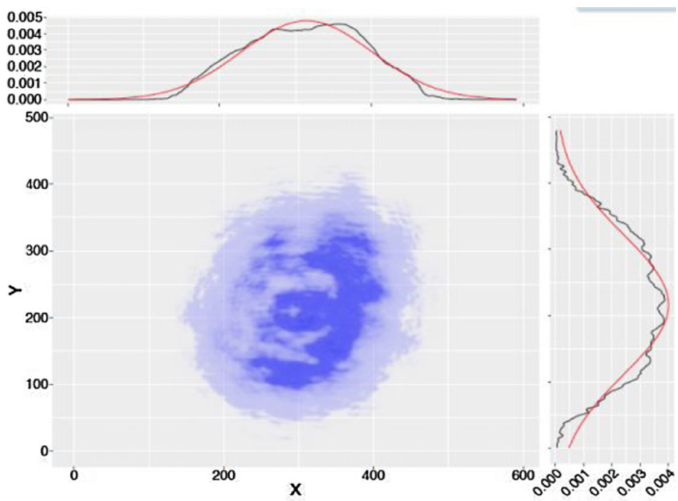


Figure B2. Measurement of the beam cross section; *X* and *Y* are the relative perpendicular coordinates of the photodiode used to measure the intensity; from the Gaussian fit, we get absolute values $\Delta x/\Delta y = (1.032 \pm 0.012)$ mm/(1.260 ± 0.012) mm.

Detector system

Summarizing the technical demands for our high speed low light level APDs:

- Not fibre based (for didactical reasons)
- Quantum efficiency at 810 nm about 80%
- The APD-dead time of $\tau_D = 50$ ns is given by the length of the quenching pulse; it follows the upper limit of the photon counting rates, much lower than $1/\tau_D = 20 \times 10^6$ s^{-1}
- Educational use imposes high safety requirements. The detection system follows CE-standards (e.g. a housing connected to PE).

Counting unit

In our setup, we had to meet the following technical requirements:

- At least three channels to measure 3-fold coincidence

- A 24 bit counter with sampling rate well above $10^6\ s^{-1}$
- Precise control of the sampling time T
- A variably-adjustable coincidence window width w_c, minimum value $w_c < 7$ ns.

In contrast to other approaches, this setup is based on a synchronous pulse shortener using a phase-locked loop (pll) implemented on an FPGA Altera Cyclone IV to multiply the driving system clock by a factor of eight instead of an asynchronous design. This leads to a reproducible behaviour of the unit but led to a principal 5 ns limit of the coincidence window width on this FPGA. Due to signal processing and FPGA internal behaviour the system actually reaches 6.6 ns, measured with an uncorrelated light source. This still meets the design goal.

Appendix C. Further reading

For a detailed classical and quantum optical description of the interaction between light and optical beam splitters:

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For phenomena and ideas of scientific historical significance:

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