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# Testing The Limits: A Robustness Analysis Of Logistic Growth Models For Life Cycle Estimation During The COVID-19 Pandemic

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## Abstract

The semiconductor industry operates in a dynamic environment characterized by rapid technological advancements, extensive research and development investments, long planning horizons, and cyclical market behavior. Consequently, staying vigilant to technological disruptions and shifting trends is crucial. This is especially challenging when external shocks seriously affect supply chain processes and demand patterns. Particularly, recent events, such as the COVID-19 pandemic, the ongoing Russian invasion of Ukraine, and high consumer price inflation impacting the semiconductor cycle emphasize the need to account for these influences.

In this context, we analyze growth patterns and life cycles of various technologies within the semiconductor industry by estimating logistic growth models. The logistic growth model was originally formulated to describe population dynamics. However, many processes outside the discipline of ecology share the fundamental characteristics of natural growth: self-proportionality and a self-regulating mechanism. Out of the different applications, two are of particular interest in the context of strategic business decisions: (1) modeling innovation diffusion and technological change to predict the mid- to long-term growth of a market, and (2) modeling of product life cycles. To obtain market growth and life cycle predictions, we apply the logistic growth model to forecast cumulative revenues by technology over time.

This model treats the analyzed technology as a closed system. However, in practice, external shocks are the norm. To analyze the robustness to such external shocks, we compare technology life cycle estimates derived from logistic growth models before and after the effects of COVID-19 became evident for a wide array of semiconductor technologies. We find that the impact of COVID-19 on these life cycle estimates is mixed, but the median change is low. Our findings have implications for the application of logistic growth models in strategic decision-making, helping stakeholders navigate the complexities of technological innovation, diffusion, and market growth.

# Keywords

Innovation Management; Business Strategy; Technology; Forecasting; S-Curve; Sigmoidal Growth; Robustness; External Shocks; Supply-Chain Disruption; Semiconductor; Trustworthiness

## 1. The semiconductor industry

The semiconductor industry is characterized by long lead times for expanding fabrication capacity, shortening life cycles, and rapid technological advances [1–3]. Consequently, strategic decisions are often long range and high impact, especially when involving R&D and fabrication capacities. Missing out on an important development can cost months or years to catch up market share. Capital-intensive investments in fabrication and high R&D costs raise the stakes further [4,5]. This highlights the need for forecasting methods that provide support to managers [4–6]. Life cycle modelling can give an indication of technologies prone to stagnation and disruption [7], providing managers with important information.

Apart from its importance to the wider economy [8], the semiconductor industry is a great test case for forecasting methods, which include technology diffusion and life cycle models [7,9,10], due to the challenges involved. Rapid technological advancements and shortening product life cycles imply that demand is volatile and difficult to predict [4,11,12]. Furthermore, the semiconductor industry does not only have complicated supply chains but also lies upstream in the supply chain for many consumer products. Consequently, it is exposed to the bullwhip effect. This effect refers to the phenomenon where small fluctuations in consumer demand can result in amplified variations in ordering patterns along the supply chain [13,14].

The above-mentioned challenges suggest that a closed-system view might be simplistic. In fact, the semiconductor industry has experienced several external shocks with severe consequences over the last few years: From geographic risks, such as earth quakes damaging sensitive fabrication equipment [15], to the impact of the COVID-19 pandemic, including subsequent government responses, and trade frictions on the global semiconductor supply chain [16] or the ongoing Russian invasion of Ukraine with its effects on inflation and consumer sentiment [17]. Therefore, the consideration of external disruptions in forecasts and technology life cycle analysis is particularly relevant in this industry, which motivates this study.

**The main objective** of this paper is the assessment of the trustworthiness of technology life cycle estimates derived from the logistic growth model under extreme events. Hence, a case study involving a significant external shock - the onset of the COVID-19 pandemic – is presented and its effects on these life cycle estimates are examined.

**Structure**: the next section provides an overview of the history of the logistic growth model and its application to innovation management and technology life cycle analysis. The methodology of this paper is presented in Section 3 with an emphasis on the estimation of the logistic curve and the derivation of technology life cycles. Section 4 completes the paper by applying the methodology to a technology portfolio of a leading semiconductor company and discussing the findings of this case study.

## 2. The logistic growth model

The logistic growth model, also referred to as S-shaped or sigmoidal curves, is characterized by the logistic differential equation  $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$ . Here, *N* represents the population size, *K* the carrying capacity, and *r* the growth rate. It was first derived by Verhulst in the early to mid-nineteenth century to describe population dynamics [18,19] and plays an important role in the Lotka-Volterra equations [20,21]. Since then, they have become popular to quantify natural growth more broadly. Growth patterns outside the discipline of ecology resemble a similar dynamic and their application across diverse disciplines has been studied by several authors [7,22–24]. For example, logistic growth models have been employed in studying the adoption of renewable energies [25], the development of prostatic hyperplasia [26], production forecasting in extremely low permeability oil and gas reservoirs [27], performance analysis of technologies [28], modelling bacterial growth [29], forecasting long-term country GDP development [30,31], and more recently, modelling the development of COVID-19 cases [32–34]. Furthermore, S-curves are popular in corporate

strategic decision-making revolving around innovation, such as anticipating disruptive attacks on one's business [35] or life cycles and investments in the adoption of new technologies as they diffuse through the marketplace [7,36,37]. This technological progression is exemplified by the evolution of mobile phones in Figure 1. The lower left logistic curve represents classical cell phones, starting from their introduction in the 1970's. Despite their vast technological improvements over the decades, their design was a limiting factor to the value they could offer to consumers: the small screen and buttons meant that they were primarily used for voice calls, messaging, and short emails. The utility of the mobile phone was radically redefined with the introduction of the iPhone, the first commercially viable smart phone. This marked the launch of the second logistic curve, which subsequently disrupted the classical cell phone market (including the decline of its former champions, Nokia and Blackberry).



Figure 1: Technological progression exemplified by the evolution of mobile phones.

This motivates the application of logistic curves in the context of technological progress and life cycle analysis. To study the impact of the technology life cycle on strategic business decisions and corporate structure, the S-curve is often partitioned into 5 phases, as illustrated in Figure 2 [23,38,39]:

- 1. Birth /winter (1.4% 6.3%): the technology is barely known or explored. Growth is slow and a large degree of effort is needed to progress. Entrepreneurship and a decentralized company structure are common.
- 2. Growth / spring (6.3%-30%): the technology is slowly getting adopted as "the next big thing". Growth remains nearly exponential. This phase is characterized by learning, product innovation, and continuous improvement.
- 3. Maturity / summer (30%-70%): the technology is being adopted and growing at its maximum rate. However, there are first signs of costs of complexity as the rate is approximately constant and departs from earlier near-exponential growth. Processes are driven by vertical integration, refinement, and bureaucratization.
- 4. Decline / autumn (70%-92.7%): the technology nears its limit. The growth rate remains positive but accelerates its decline. Managers should be on the lookout for the next "next big thing". This is the ear of process innovations and face lifts.
- 5. Death / winter (92.7%-98.6%): the technology has nearly reached its peak. There is little growth left to achieve, which requires increasingly higher investments. This means that technological progress stalls and the technology becomes prone to disruptions. Managers are looking to transition to alternative technologies while the current one phases out (compare the transition from cell to smartphones in Figure 1).



Figure 2: Partitioning the S-curve into life cycle stages.

This highlights the interpretability and usability of the simple logistic model, which played a pivotal role in formalizing the study of life cycles, on a business level. However, various extensions and generalizations of the simple logistic curve have been proposed [10, 19,40–44], such as the Richards' curve [45] or Gompertz curve. Ex post, these extended models may yield better fits [29]. However, the generalized models usually require more parameters. This can lead to identifiability issues [46,47]. Furthermore, the trajectory is seldomly observed completely, which can exacerbate the issue. Therefore, we focus on the simple logistic model in our analysis.

### 3. Methodology

We introduce the notation and explain the interpretation and meaning of the different parameters of the logistic growth model in 3.1. Subsection 3.2 provides details of the assumed data generating process and the estimation of model parameters.

#### 3.1 Notation

As described in Section 2, the logistic growth model is described by the differential equation  $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$ . For consistency with common statistical notation, we rewrite the solution of this differential equation as

$$Y = \frac{M}{1 + e^{aX + b}} + C,\tag{1}$$

where Y is the response variable (in place of N), M is the carrying capacity or maximum cumulative market size, a corresponds to the growth rate (the maximum growth rate of aM/4 is reached at the inflection point), b determines the location of the inflection point (at x = -b/a), C is introduced to allow for vertical offsets of the logistic curve, and X denotes the dependent variable. The logistic curve with its parameters and their



Figure 3: Parametrization and interpretation of parameters of the logistic curve.

respective interpretations is illustrated in Figure 3. This parametrization implies that the life cycle stage can be estimated as (max(Y) - C)/(M - C).

#### **3.2 Estimation**

We assume the data generating process

$$Y = \frac{M}{1+e^{aX+b+\varepsilon}} + C,$$
(2)

where  $\varepsilon$  is an identically and independently distributed error variable with existing first and second moments. This model formulation allows us to estimate the parameters *M*, *C*, *a*, *b* in an iterative two-step process:

- i. Estimate *M* and *C*. At initialization, start with two reasonable first guesses  $\hat{M} > max(Y)$ ,  $\hat{C} < min(Y)$ . For later iterations minimize the mean squared error of the step ii. regression with regard to *M* and *C*.
- ii. Given M and C, the remaining parameters can be obtained via a simple linear regression

$$\log\left(\frac{\hat{M}-Y+\hat{C}}{Y-\hat{C}}\right) = aX + b + \varepsilon.$$
(3)

The mean squared error is given by  $\frac{1}{N}\sum_{i=1}^{N} \left[ log\left(\frac{\hat{M}-Y_i+\hat{c}}{Y_i-\hat{c}}\right) - \hat{a}X_i - \hat{b} \right]^2$ , where N is the sample size.

The optimization in step i. can be performed with any conventional optimization routine, such as Nelder and Mead's Simplex algorithm [48] or quasi-Newton methods such as BFGS or L-BFGS-B [49]. However, these local optimization methods tend to struggle with local optima [50]. This is particularly problematic when the error surface is rough. We used Generalized Simulated Annealing, which is implemented in the "GenSA" R package [51], as this yielded the most reliable results.

#### 4. Empirical Analysis

Subsection 4.1 covers the introduction to the data and includes all pre-processing steps in. Subsection 4.2 presents the results of the analysis and discusses the implications.

## 4.1 Data

We obtained revenues for a diverse portfolio of products ranging from October 2006 to January 2023 from a leading semiconductor company. However, it is not reasonable to assume that one Euro today has the same value it had 17 years ago. Therefore, it is important to adjust for the general price level. This was done with the Harmonized Index of Consumer Inflation, which is published by the European Central Bank<sup>1</sup> [52]. These revenues were mapped to technology categorizations on the aggregation levels: Level 1, containing 18 technologies and Level 2, containing 10. Here, Level 1 (component) technology groups are more granular than Level 2 technology groups, see Figure 4 (the technologies and their corresponding technology groups, which are covered in this paper, can be seen in Table 1 in the appendix).



Figure 4: Example of a technology hierarchy.

We consider both, Level 1 and Level 2 technologies, because Level 1 technologies are more homogeneous but Level 2 technologies include more sub-technologies that would need to be excluded due to lack of data history. By aggregating several similar technologies, we further hope to reduce the effects of substitution and similar interactions. Furthermore, the logistic model should be applicable on either level because of its fractal property.



Figure 5: Harmonic F-test statistic over the frequency domain. The dashed red line corresponds to a point wise 95% confidence threshold, the dotted red line to a global 95% confidence threshold.

Further, we analyzed cyclicality by means of spectral analysis, particularly using a periodogram [53]. However, no frequency surpassed the significance threshold of the harmonic F-test [54] provided in the "multitaper" R package. As Figure 5 shows, there is no statistically significant cyclicality at any frequency when adjusting for multiple testing.



Figure 6: Periodograms before (left) and after (right) seasonal adjustment.

<sup>&</sup>lt;sup>1</sup> Information on the Harmonized Index of Consumer Inflation and the corresponding time series can be found at www.ecb.europa.eu/stats/macroeconomic\_and\_sectoral/hicp

Additionally, we performed a time series decomposition into trend, seasonal effects, and a random component [55] using the "decompose" function in R. However, the time series post seasonal adjustment was not noticeably less noisy than the original time series, nor did the periodogram change much (see Figure 6). Therefore, we proceeded with the original data without seasonal adjustments.

#### 4.2 Result

Logistic growth models were fit to cumulative technology revenues using the algorithm described in Section 3.2. We visually checked the models for consistency with the logistic curve by observing the quality of the fit and producing residual plots. These residual plots were helpful in checking if the models converged as expected and to identify residual patterns, which indicate a poor model fit.



Figure 7: Example residual plots

Figure 7 indicates a residual plot of a good logistic fit for Tech. 14, whereas the residual plot for Tech. 17 raises questions. Residuals are indicated by red circles. The drawn blue line represents the linear fit of the data in step ii. of section 3.2. If this line differs from the horizontal line at Y = 0, it indicates that the algorithm did not converge as expected. The dashed and dotted blue lines represent point-wise confidence intervals and confidence bands at 95%, respectively. Hence, a residual lying outside the 95% point-wise confidence interval would be expected to be observed every 20 data points, whereas a residual lying outside the 95% confidence band would be expected every 20 residual plots. These plots can help identify external shocks or patterns that are not captured by the model. In case of the left plot, the residuals are well dispersed and no clear trend is identifiable. This indicates a good fit. On the right, there is a clear pattern, which indicates a poor model fit.



Figure 8: Example logistic fits. Tech. 14 (left) is an example of a good fit, Tech. 17 (right) one of a questionable one.

Additionally, we validated the logistic life cycle estimates by expert opinion. We excluded poor fits and technologies that were clearly driven by external structural effects (the semiconductor industry is largely a B2B business, where sales are often conducted through direct relationships – in segments where revenues are overwhelmingly driven by a few large customers, patterns in the data may be dominated by individual decisions at the level of a single customer and not always be reflective of the technological potential of the product). Overall, we excluded seven of the eighteen Level 1 technologies and four Level 2 technology from further analysis (see Table 1 in the Appendix).

Figure 8 illustrates logistic fits for Tech. 14 (left), which fits well, and Tech. 17 (right), which does not. Tech. 17 is an example of a technology that is driven by individual projects (also observe that the data in the right plot appears to be consisting of two logistic curves, not one). The blue curve indicates the estimated logistic model, the red dots correspond to the observed cumulative revenues, and the grey dotted lines

correspond to the estimated lower bound, C, and the estimated upper bound, M + C. Generally, we did not estimate lower bounds unless we had reason to believe that we were missing previous revenues.

These logistic curves were estimated based on the complete history (from October 2006 to January 2023) and on the basis of the historical data preceding the COVID-19 Pandemic. After the curves were obtained and validated for the various Level 1 and Level 2 Technologies, we compared the life cycle estimates of the pre- and post-COVID models.



Figure 9: Boxplots of life cycle estimates before and after COVID.

Figure 9 illustrates the distributions of the Pre- (red) and Post- (cyan) COVID life cycle estimates on technology Level 1 (left) and 2 (right), respectively. Given the strong demand for consumer electronics and the subsequent rejuvenation of sales numbers, we expected lower life cycle estimates as an impact of the pandemic. Meanwhile, a moderate increase is expected without the interference of external shocks, given that two years elapsed and the technology has aged. On Level 1, the median life cycle estimate remained relatively stable, though more concentrated in the lower range. Thus, the logistic fits for the Level 1 technologies seem relatively unaffected by the pandemic. This is confirmed by Figure 10, which shows that the median life cycle estimate has increased slightly after COVID, which is consistent with our expectation. This result is slightly different for Level 2 technologies. As Figure 9 indicates, the distribution of life cycle estimates has noticeably shifted downwards. This observation is confirmed by Figure 10, which indicates that the median life cycle estimates have decreased by 2% after the revenues during the pandemic are included. This seems to confirm our hypothesis that COVID has had an impact on the logistic growth models. On the other hand, this decrease could be due to the inclusion of new emerging sub-technologies in the broader technology group. These would have been omitted during the analysis of the Level 1 technologies, due to small volumes and an insufficient amount of historical data.



Figure 10: Boxplots of the change in life cycle estimates before and after COVID.

We conclude that logistic growth models are a valuable tool for managers in assessing product life cycles if the model is applicable. This is particularly useful in understanding technological limitations and guarding against the risk of disruption. The robustness to external effects could be further improved by incorporating them into the logistic growth model. For example, researchers have modelled the upper bound dynamically [56,57]. Given the dynamic nature of the semiconductor industry, the complexity of supply chain dependencies, and the exposure to other external factors, this highlights a potential for future research in the field.

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#### Appendix

 Table 1: Technology table of technology groups and the corresponding technologies. Technologies marked with an

 (X) were excluded from further analysis.

Technology Group (Level 2)	Technology (Level 1)
Tech. Group 1	Tech. 7, Tech. 3 (X), Tech. 1 (X)
Tech. Group 2	Tech. 2, Tech. 4
Tech. Group 3	Tech. 17 (X), Tech. 14, Tech. 5
Tech. Group 4 (X)	Tech. 11, Tech. 9 (X), Tech. 6
Tech. Group 5 (X)	Tech. 12, Tech. 10 (X)
Tech. Group 6	Tech. 8
Tech. Group 7	Tech. 13
Tech. Group 8	Tech. 15
Tech. Group 9 (X)	Tech. 16 (X)
Tech. Group 10 (X)	Tech. 18 (X)

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## Biography



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**Markus Pauly** received his diploma degree in mathematics from the Heinrich-Heine University Düsseldorf (2005), where he also completed his Ph.D (2008) and his habilitation (2013). From 2014 to 2019 he was full professor for Statistics at Ulm University. Since 2019 he is full professor at the Statistics Department at TU Dortmund University, where he investigates a wide range of statistical and machine learning topics.