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Novel algorithms for the characterization of n-port networks by using a two-port network analyzer

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Abstract. The measurement of the scattering matrices of n-port networks is an important task. For this purpose two ports of the n-port network are connected with the network analyzer and the remaining ports are connected to reflecting terminations. In order to specify the scattering matrix of a n-port network with the multi-port method (Rolfes et al., 2005), n reflecting terminations are required from which at least one reflection factor needs to be known.

There are some cases, in which the multi-port method shows weak convergence properties. For example, a T-junction cannot be identified if the reflecting terminations used are short circuits and if the line length is equivalent to a multiple of a half wavelength. This is due to the fact that the two ports connected to the network analyzer become isolated.

Two new algorithms, named the sub-determinant method and the wave-identification method, respectively, which employ a second set of reflection terminations that have to differ from the first set, allow to identify every n-port network without the necessity to distinguish different cases. Both methods are based on least square algorithms and allow to determine all scattering parameters of a n-port-network directly and uniquely.

1 Introduction

The measurement of multi-ports with more than two ports gets more and more important. Modern circuits are often complex. Additionally the number of ports may be higher than the number of ports of the network analyzer. Therefore, methods for the measurement of multi-ports with a two-port network analyzer are needed, independent of the number of ports. The principle of these methods is to combine all possible two-port measurements of the multi-port so that the scattering parameters of the multi-port can be identified. An easy method is to connect all ports of the multi-port which are not connected with the network analyzer to a match. In this case the scattering parameters can directly be determined by the two-port measurements. But in fact, it is often not possible to connect all ports to a match, for example if non-contacting measurements are performed. Furthermore the available matches might not be accurate enough, especially in the range of higher frequencies. Thus, another method which is independent of the external reflections has to be developed. The following article describes different methods to evaluate a multi-port by using a two-port network analyzer and unknown external reflections. Finally a very general method to evaluate any multi-port is described, which so far has proven to be successful in all cases considered.

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2 Multi-port methods

An already known method (Tippet et al., 1982) offers the possibility to characterize almost every multi-port by connecting each port to one external reflection Γ_i, but does not allow reflection coefficients of ±1 or values in the vicinity of ±1. An improvement was made with the multi-port method (Rolfes et al., 2005), which allows to use external reflections independent of their value and additionally offers the possibility to obtain the value of all reflection coefficients as unknowns, except for one. But there are some multi-ports, which cannot be characterized with only one external reflection on every port, because this reflection might e. g. isolate the measurement ports. One example for such a problem is a tee junction as shown in Fig. 1, connected to short circuits or opens as external reflections. If one port of this 3-port is connected to an open circuit and the length l of the lines is an odd multiple of a quarter of a wavelength, the two other ports which are connected to the network analyzer become

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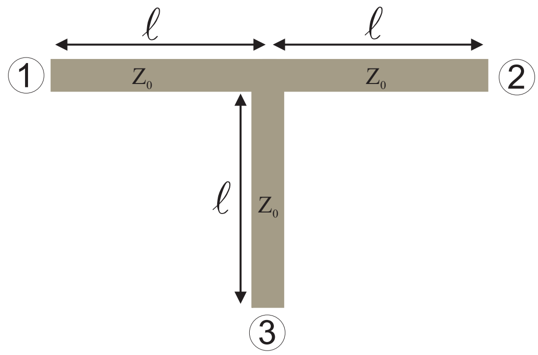

Fig. 1. Setup of a tee junction.

isolated. Then, it is not possible to characterize this 3-port with the multi-port method, if at least two of the external reflections are opens.

A solution of this problem is to use a second set of external reflections which have to differ from the first set. Thus, the different ports of the tee junction are coupled at every frequency point at least once because an isolation only occurs for one of the two external reflections.

The following methods are all described for the identification of a 3-port. In fact, the methods can be used for the identification of multi-ports with any number of ports because every multi-port with n ports can be subdivided into n 3-ports. This subdivision (Rolfes et al., 2005) offers the possibility to characterize all multi-ports with a method which is defined for 3-ports. Although it is possible to extend the following method also for the calculation of a 4-port, this yields a very complex system of equations and furthermore a subdivision remains necessary for all multi-ports with more than four ports. Thus, the following methods are described for the calculation of a 3-port only because it is the simplest structure.

3 The wave-identification method

In general a multi-port can be described by its scattering parameters in the following way

$$\boldsymbol{b} = \mathbf{S} \cdot \boldsymbol{a} \quad , \tag{1}$$

where $\boldsymbol{a}, \boldsymbol{b}$ are the vectors of incident and reflected waves, respectively, and $\mathbf{S}$ is the scattering matrix. If the port i is connected to the external reflection Γ_i it holds

$$a_i = \Gamma_i \cdot b_i \quad . \tag{2}$$

Thus, it follows for a 3-port, if port 2 and port 3 are connected to the network analyzer and port 1 is connected to the external reflection Γ_i,

$$\begin{pmatrix} b_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} \Gamma_1 \cdot b_1 \\ a_2 \\ a_3 \end{pmatrix} \quad . \tag{3}$$

Here m_2 and m_3 are the waves measured by the network analyzer and b_1 is the unknown wave at port 3. The measurement between port 2 and port 3 yields a two-port scattering matrix. With the help of this matrix it is possible to choose the incident waves and calculate the corresponding reflected waves m_2 and m_3. Thus, the incident waves a_2 and a_3 can be chosen as two linearly independent vectors

$$\begin{pmatrix} 0 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ \tilde{a}_2 \\ \tilde{a}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad . \tag{4}$$

With these two different excitations, one two-port measurement yields six equations

$$\begin{aligned} b_1^{(1)} &= S_{11}\Gamma_1 b_1^{(1)} + S_{12} & \tilde{b}_1^{(1)} &= S_{11}\Gamma_1 \tilde{b}_1^{(1)} + S_{13} \\ m_2^{(1)} &= S_{21}\Gamma_1 b_1^{(1)} + S_{22} & \tilde{m}_2^{(1)} &= S_{21}\Gamma_1 \tilde{b}_1^{(1)} + S_{23} \\ m_3^{(1)} &= S_{31}\Gamma_1 b_1^{(1)} + S_{32} & \tilde{m}_3^{(1)} &= S_{31}\Gamma_1 \tilde{b}_1^{(1)} + S_{33} \quad . \end{aligned} \tag{5}$$

The unknown variables b_1 and $\tilde{b}_1$ can be eliminated and one gets four non-linear equations for the identification of the scattering parameters S_{ij} of the 3-port

$$\begin{aligned} m_2^{(1)} &= S_{22} + \frac{\Gamma_1 S_{21} S_{12}}{1-\Gamma_1 S_{11}} & \tilde{m}_2^{(1)} &= S_{23} + \frac{\Gamma_1 S_{21} S_{13}}{1-\Gamma_1 S_{11}} \\ m_3^{(1)} &= S_{32} + \frac{\Gamma_1 S_{31} S_{12}}{1-\Gamma_1 S_{11}} & \tilde{m}_3^{(1)} &= S_{33} + \frac{\Gamma_1 S_{31} S_{13}}{1-\Gamma_1 S_{11}} \quad . \end{aligned} \tag{6}$$

Thus, the three necessary two-port measurements yield twelve non-linear equations for the identification of a 3-port

$$\begin{aligned} S_{22} &= m_2^{(1)} - \frac{\Gamma_1 S_{21} S_{12}}{1-\Gamma_1 S_{11}} & S_{23} &= \tilde{m}_2^{(1)} - \frac{\Gamma_1 S_{13} S_{21}}{1-\Gamma_1 S_{11}} \\ S_{32} &= m_3^{(1)} - \frac{\Gamma_1 S_{31} S_{12}}{1-\Gamma_1 S_{11}} & S_{33} &= \tilde{m}_3^{(1)} - \frac{\Gamma_1 S_{31} S_{13}}{1-\Gamma_1 S_{11}} \\ S_{11} &= m_1^{(2)} - \frac{\Gamma_2 S_{21} S_{12}}{1-\Gamma_2 S_{22}} & S_{13} &= \tilde{m}_1^{(2)} - \frac{\Gamma_2 S_{12} S_{23}}{1-\Gamma_2 S_{22}} \\ S_{31} &= m_3^{(2)} - \frac{\Gamma_2 S_{32} S_{21}}{1-\Gamma_2 S_{22}} & S_{33} &= \tilde{m}_3^{(2)} - \frac{\Gamma_2 S_{32} S_{23}}{1-\Gamma_2 S_{22}} \\ S_{11} &= m_1^{(3)} - \frac{\Gamma_3 S_{31} S_{13}}{1-\Gamma_3 S_{33}} & S_{12} &= \tilde{m}_1^{(3)} - \frac{\Gamma_3 S_{13} S_{32}}{1-\Gamma_3 S_{33}} \\ S_{21} &= m_2^{(3)} - \frac{\Gamma_3 S_{31} S_{23}}{1-\Gamma_3 S_{33}} & S_{22} &= \tilde{m}_2^{(3)} - \frac{\Gamma_3 S_{32} S_{23}}{1-\Gamma_3 S_{33}} \quad , \end{aligned} \tag{7}$$

where the upper index (i) indicates an external reflection Γ_i at port i. These equations can either be solved with numerical methods as the least squares method or they can be linearized for small variations of the parameters, i.e. applying Newton's method.

Therefore, the scattering parameters are defined as a constant $\hat{S}_{ij}$ and a variation ε_{ij}:

$$S_{ij} = \hat{S}_{ij} + \varepsilon_{ij} \quad . \tag{8}$$

The products of the variables $\varepsilon_{ij} \cdot \varepsilon_{nm}$ are neglected. These assumptions yield a linear system of equations for the nine new variables ε_{ij}, if the scattering parameters in the system of Eqs. (7) are replaced by the new expressions defined in Eq. (8). The new system of equations with the coefficient matrix $\mathbf{A}_\varepsilon$ and the solution vector of the right hand side $\boldsymbol{m}_\varepsilon$

$$\mathbf{A}_\varepsilon \cdot \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{32} \\ \varepsilon_{33} \end{pmatrix} = \boldsymbol{m}_\varepsilon \tag{9}$$

is linear concerning the variables ε_{ij} and can easily be extended for a second set of external reflections. This system of equations is over-determined and can be solved by using e.g. a linear regression.

For a good convergence the initial value $\hat{S}_{ij}$ should be close to the solution vector. Good starting values can e.g. be obtained by the sub-determinant method, which is described below. In fact, several simulations and measurements have shown, that there is a very good convergence with only a few iterations, even if the initial vector is taken as the zero vector. Thus, the wave-identification method is a very robust method for the identification of the scattering parameters of multi-ports.

4 The sub-determinant method

The sub-determinant method is another method to identify a multi-port, if a second set of external reflections is used. A transformation of each Eq. (7) yields the following system of equations

$$\begin{aligned}
m_2^{(1)} &= m_2^{(1)}\Gamma_1 S_{11} + S_{22} + \Gamma_1(S_{12}S_{21} - S_{11}S_{22}) \\
\tilde{m}_2^{(1)} &= \tilde{m}_2^{(1)}\Gamma_1 S_{11} + S_{23} + \Gamma_1(S_{13}S_{21} - S_{11}S_{23}) \\
m_3^{(1)} &= m_3^{(1)}\Gamma_1 S_{11} + S_{32} + \Gamma_1(S_{31}S_{12} - S_{11}S_{32}) \\
\tilde{m}_3^{(1)} &= \tilde{m}_3^{(1)}\Gamma_1 S_{11} + S_{33} + \Gamma_1(S_{13}S_{31} - S_{11}S_{33}) \\
m_1^{(2)} &= m_1^{(2)}\Gamma_2 S_{22} + S_{11} + \Gamma_2(S_{12}S_{21} - S_{11}S_{22}) \\
\tilde{m}_1^{(2)} &= \tilde{m}_1^{(2)}\Gamma_2 S_{22} + S_{13} + \Gamma_2(S_{12}S_{23} - S_{13}S_{22}) \\
m_3^{(2)} &= m_3^{(2)}\Gamma_2 S_{22} + S_{31} + \Gamma_2(S_{21}S_{32} - S_{31}S_{22}) \\
\tilde{m}_3^{(2)} &= \tilde{m}_3^{(2)}\Gamma_2 S_{22} + S_{33} + \Gamma_2(S_{23}S_{32} - S_{22}S_{33}) \\
m_1^{(3)} &= m_1^{(3)}\Gamma_3 S_{33} + S_{11} + \Gamma_3(S_{13}S_{31} - S_{11}S_{33}) \\
\tilde{m}_1^{(3)} &= \tilde{m}_1^{(3)}\Gamma_3 S_{33} + S_{12} + \Gamma_3(S_{13}S_{32} - S_{12}S_{33}) \\
m_2^{(3)} &= m_2^{(3)}\Gamma_3 S_{33} + S_{21} + \Gamma_3(S_{31}S_{23} - S_{21}S_{33}) \\
\tilde{m}_2^{(3)} &= \tilde{m}_2^{(3)}\Gamma_3 S_{33} + S_{22} + \Gamma_3(S_{23}S_{32} - S_{22}S_{33})
\end{aligned} \tag{10}$$

for the first set of external reflections and similarly for the second set. A closer look to the equations shows that all terms which are non-linear in S_{ij} are sub-determinants of the scattering matrix $\mathbf{S}$ of a 3-port. This system of equations becomes linear if these sub-determinants are assumed as further variables Δ_k. Thus, one obtains 24 linear equations with 18 variables. Twelve equations result from the measurements with the first set of external reflections

$$\begin{aligned}
m_2^{(1)} &= m_2^{(1)}\Gamma_1 S_{11} + S_{22} + \Gamma_1\Delta_1 \\
\tilde{m}_2^{(1)} &= \tilde{m}_2^{(1)}\Gamma_1 S_{11} + S_{23} + \Gamma_1\Delta_2 \\
m_3^{(1)} &= m_3^{(1)}\Gamma_1 S_{11} + S_{32} + \Gamma_1\Delta_3 \\
\tilde{m}_3^{(1)} &= \tilde{m}_3^{(1)}\Gamma_1 S_{11} + S_{33} + \Gamma_1\Delta_4 \\
m_1^{(2)} &= m_1^{(2)}\Gamma_2 S_{22} + S_{11} + \Gamma_2\Delta_1 \\
\tilde{m}_1^{(2)} &= \tilde{m}_1^{(2)}\Gamma_2 S_{22} + S_{13} + \Gamma_2\Delta_5 \\
m_3^{(2)} &= m_3^{(2)}\Gamma_2 S_{22} + S_{31} + \Gamma_2\Delta_6 \\
\tilde{m}_3^{(2)} &= \tilde{m}_3^{(2)}\Gamma_2 S_{22} + S_{33} + \Gamma_2\Delta_7 \\
m_1^{(3)} &= m_1^{(3)}\Gamma_3 S_{33} + S_{11} + \Gamma_3\Delta_4 \\
\tilde{m}_1^{(3)} &= \tilde{m}_1^{(3)}\Gamma_3 S_{33} + S_{12} + \Gamma_3\Delta_8 \\
m_2^{(3)} &= m_2^{(3)}\Gamma_3 S_{33} + S_{21} + \Gamma_3\Delta_9 \\
\tilde{m}_2^{(3)} &= \tilde{m}_2^{(3)}\Gamma_3 S_{33} + S_{22} + \Gamma_3\Delta_7 \ ,
\end{aligned} \tag{11}$$

and twelve further equations result from the measurements with the second set of external reflections $\tilde{\Gamma}_i$ and waves $n_i^{(j)}$ measured by the VNA

$$\begin{aligned}
n_2^{(1)} &= n_2^{(1)}\tilde{\Gamma}_1 S_{11} + S_{22} + \tilde{\Gamma}_1\Delta_1 \\
\tilde{n}_2^{(1)} &= \tilde{n}_2^{(1)}\tilde{\Gamma}_1 S_{11} + S_{23} + \tilde{\Gamma}_1\Delta_2 \\
n_3^{(1)} &= n_3^{(1)}\tilde{\Gamma}_1 S_{11} + S_{32} + \tilde{\Gamma}_1\Delta_3 \\
\tilde{n}_3^{(1)} &= \tilde{n}_3^{(1)}\tilde{\Gamma}_1 S_{11} + S_{33} + \tilde{\Gamma}_1\Delta_4 \\
n_1^{(2)} &= n_1^{(2)}\tilde{\Gamma}_2 S_{22} + S_{11} + \tilde{\Gamma}_2\Delta_1 \\
\tilde{n}_1^{(2)} &= \tilde{n}_1^{(2)}\tilde{\Gamma}_2 S_{22} + S_{13} + \tilde{\Gamma}_2\Delta_5 \\
n_3^{(2)} &= n_3^{(2)}\tilde{\Gamma}_2 S_{22} + S_{31} + \tilde{\Gamma}_2\Delta_6 \\
\tilde{n}_3^{(2)} &= \tilde{n}_3^{(2)}\tilde{\Gamma}_2 S_{22} + S_{33} + \tilde{\Gamma}_2\Delta_7 \\
n_1^{(3)} &= n_1^{(3)}\tilde{\Gamma}_3 S_{33} + S_{11} + \tilde{\Gamma}_3\Delta_4 \\
\tilde{n}_1^{(3)} &= \tilde{n}_1^{(3)}\tilde{\Gamma}_3 S_{33} + S_{12} + \tilde{\Gamma}_3\Delta_8 \\
n_2^{(3)} &= n_2^{(3)}\tilde{\Gamma}_3 S_{33} + S_{21} + \tilde{\Gamma}_3\Delta_9 \\
\tilde{n}_2^{(3)} &= \tilde{n}_2^{(3)}\tilde{\Gamma}_3 S_{33} + S_{22} + \tilde{\Gamma}_3\Delta_7 \ .
\end{aligned} \tag{12}$$

Thus, a linear system of equations for the calculation of the 3-port scattering parameters is obtained, which can be written in a compact matrix notation with the coefficient matrix $\mathbf{A}_\Delta$, which includes the measured waves $m_i^{(j)}$, $n_i^{(j)}$ and the reflections Γ_i, and the solution vector of the right hand side $\boldsymbol{m}_\Delta$, which includes the measured waves $m_i^{(j)}$ and $n_i^{(j)}$, as

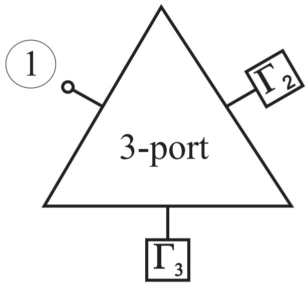

Fig. 2. One-port measurement.

follows:

$$\mathbf{A}_\Delta \cdot \mathbf{\Delta}_S = \boldsymbol{m}_{\mathbf{\Delta}} \quad . \tag{13}$$

Here the vector $\mathbf{\Delta}_S$ consists of the nine unknown scattering parameters $S_{11},\ S_{12}, \cdots,\ S_{33}$ and additionally the nine unknown sub-determinants $\Delta_1,\ \Delta_2,\ \cdots,\ \Delta_9$. This system of equations is over-determined and can be solved with the help of a linear regression. Numerical experiments have shown that the sub-determinant method is less robust against disturbances or measurement errors as compared to the wave-identification method. This can be explained by the number of variables. While the wave-identification method yields a solution concernig nine variables the sub-determinant method is based on 18 variables, although the number of equations is the same in both cases. It is therefore a very successful strategy to use the sub-determinant method to create good starting values for the wave-identification method in order to reduce the number of necessary iterations, although the wave-identification method shows a very good convergence for each starting vector, for example the null vector.

5 The one-port method

For the described methods the assumption was made that the external reflections are known. In fact both methods presented above offer the possibility to obtain the value of all external reflections except for one which has to be known. Alternatively, the one-port method can be used for the characterization of the external reflections. For this purpose, additionally to the necessary two-port measurements two one-port measurements yield the values of all external reflections. This so-called one-port method is performed by connecting one port of the multi-port to the network analyzer and all other ports to their external reflections. The resulting input reflection can be linked with the corresponding two-port measurement for the characterization of the external reflections, also known as de-embedding (Bauer et al., 1974). Figure 2 shows the one-port method applied to port 1. The input reflection ϱ_{11} given by the one-port measurement can be inserted into the equation of the scattering matrix $\hat{\mathbf{S}}$ of the two-port between port 1 and port 2 in the following way

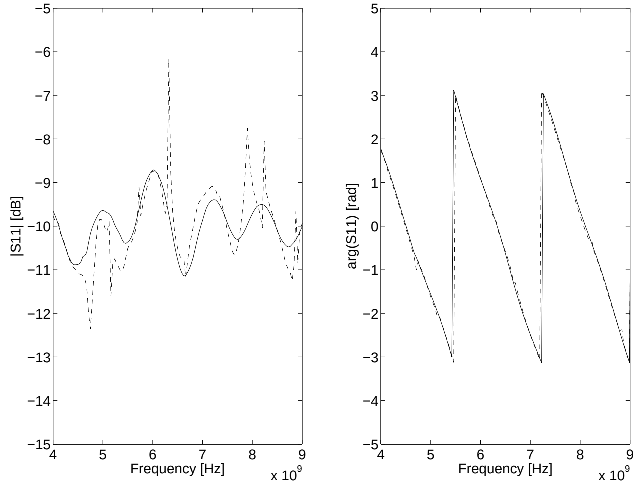

Fig. 3. Measurement results with a single set of external reflections with opens (−−) and with matches (—).

$$\begin{pmatrix} \varrho_{11} \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} \hat{S}_{11} & \hat{S}_{12} \\ \hat{S}_{21} & \hat{S}_{22} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \Gamma_2 \hat{b}_2 \end{pmatrix} \quad . \tag{14}$$

Thus, it holds for the unknown external reflection Γ_2 after the elimination of $\hat{b}_2$

$$\Gamma_2 = \frac{\varrho_{11} - \hat{S}_{11}}{\varrho_{11}\hat{S}_{22} + (\hat{S}_{12}\hat{S}_{21} - \hat{S}_{11}\hat{S}_{22})} \quad . \tag{15}$$

Furthermore this one-port measurement can be linked to the two-port measurement between port 1 and port 3 which offers the possibility to define the unknown external reflection Γ_3. Thus, at least two one-port measurements are necessary to identify all external reflections independently of the number of ports.

6 Results

The measured device which was used to verify the different methods is a 3-port signal divider, consisting of three lines connected in the form of a T-junction, with open circuits as external reflections. The results produced with the multi-port method show some singularities. This can be explained by the isolation of the measurement ports if the length of the third line connected to an open circuit is equal to an odd multiple of a quarter wavelength. Figure 3 shows the results for the scattering parameter S_{11} of the multi-port method and additionally the results achieved with matched terminations as external reflections. It can clearly be seen that it is not possible to characterize this 3-port with a single set of unknown external reflections, because the multi-port method shows measurement singularities in contrast to the results produced with matched terminations as external reflections which have a smooth behaviour versus frequency.

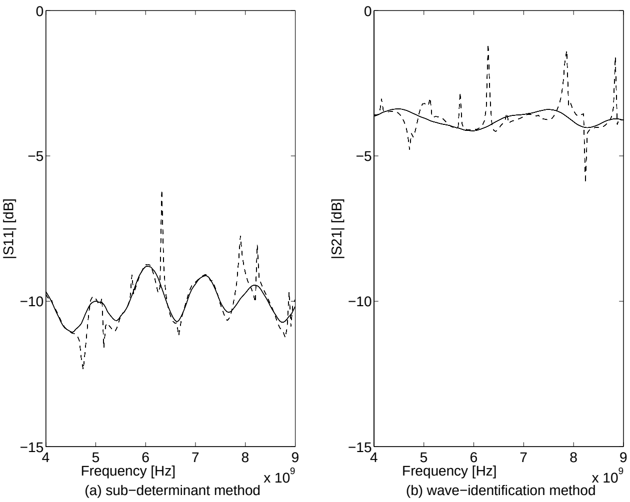

Fig. 4. Measurement results with a single set (−−) and a double set (—) of external reflections.

A further measurement performed with two sets of external reflections, namely three open circuits and three short circuits, is shown in Fig. 4.

In Fig. 4a the wave-identification method with a double set of external reflections is compared to the results achieved with a single set of external reflections, in this case three open circuits. While a single set of external reflections yields several measurement singularities as expected the wave-identification method offers a very smooth behaviour. Part (b) of Fig. 4 shows a comparison of the sub-determinant method and with the results for a single set of external reflections. Similarly to the wave-identification method the sub-determinant method shows a very smooth behaviour in contrast to the singularities of the measurements with a single set of external reflections.

7 Conclusions

In conclusion it can be stated that both described methods offer the possibility to characterize every multi-port, if a double set of external reflections is used. In fact the use of a second set of external reflections doubles the number of necessary measurements, but it yields a solution for the identification of every multi-port independent of its structure and of the values of the external reflections. Thus, no distinction of different cases is needed even if several ports become isolated. Furthermore the one-port method allows to deal with all external reflections as unknowns with only two further measurements. A very robust method is to combine both presented methods in such a manner that the sub-determinant method provides the starting values for the wave-identification method. This approach allows to apply the wave-identification method with just one iteration.

References

Bauer, R. F. and Penfield, P.: De-Embedding and Unterminating, in: IEEE Trans. Microw. Theory Tech., vol. 22, pp. 282–288, March, 1974.

Engen, G. F. and Hoer, C. A.: Thru-Reflect-Line: An improved technique for calibrating the dual six port automatic network analyzer, in: IEEE Trans. Microw. Theory Tech., vol. 27, pp. 987–993, December, 1979.

Eul, H.-J. and Schiek, B.: A Generalized Theory and New Calibration Procedures for Network Analyzer Self-Calibration, in: IEEE Trans. Microw. Theory Tech., vol. 39, pp. 724–731, April, 1991.

Lu, H.-C. and Chu, T.-H.: Multiport Scattering Matrix Measurement Using a Reduced-Port Network Analyzer, in: IEEE Trans. Microw. Theory Tech., vol. MTT-51, pp. 1525–1533, May, 2003.

Rolfes, I. and Schiek, B.: Multiport Method for the Measurement of the Scattering Parameters of N-Ports, in: IEEE Trans. Microw. Theory Tech., vol. 53, pp. 1990–1996, June, 2005.

Tippet, J. C. and Speciale, R. A.: A Rigorous Technique for Measuring the Scattering Matrix of a Multiport Device with a 2-Port Network Analyzer, in: IEEE Trans. Microw. Theory Tech., vol. MTT-30, pp. 661–666, May, 1982.