Engineering mathematics: Reflections on the dialectics of *in* and *for*

Jana Peters[1] and Reinhard Hochmuth[2]

[1,2]Leibniz University Hannover, Institute of Mathematics and Physics Education

[1] https://orcid.org/0000-0003-0628-7105

Mathematics *in* Engineering and Mathematics *for* Engineering are two important strands of research in engineering mathematics education. In a research project [2-7] we reconstructed mathematical practices in a Signal Transmission course with a view to taking into account the subject-specific particularities of engineering mathematics. To achieve this, we introduced mathematical discourses (e.g. [5])as a methodological tool of the Anthropological Theory of the Didactic (ATD, e.g. [1]).
We reconstructed two different but interrelated institutional mathematical discourses in engineering mathematical practices:

- A mathematical discourse that refers to mathematics taught as an integrated part *in* electrical engineering (ET in Figure 1).
- A mathematical discourse that refers to mathematics taught as a service subject *for* engineering (HM in Figure 1).

The knowledge taught in an electrical engineering course contains both aspects of mathematical practices, *for* and *in* engineering (see Figure 2).

Elements of ATD: Institutional discourses as a methodological tool

Praxeologies and institutional discourses

Theoretical perspective of ATD [1]: Institution, taught knowledge.
Basic model to describe mathematical knowledge:

Praxeology [Praxis ; Logos]
- Praxis (know-how)
  T: task, aim of the action
  τ: relevant solution techniques
- Logos (know-why)
  θ: technology, reasoning discourse, explains and justifies techniques
  Θ: theory; 2nd level reasoning discourse to justify technology

4T-Model: [T, τ ; θ, Θ]

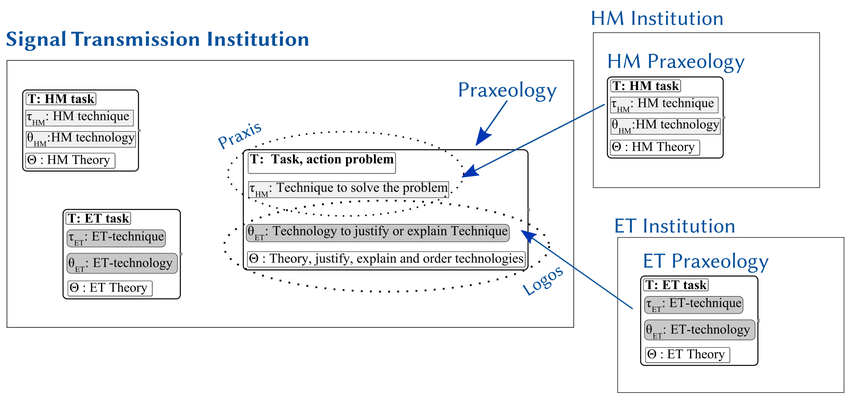

Figure 1: Praxeological configurations with different institutional discourses

HM Institution: Mathematics service course
ET Institution: Electrical Engineering courses

Reasoning discourses (Logos) and therefore also tasks/techniques (Praxis) depend on institutions. With the methodological concept of ***institutional discourse*** [e.g. 5], these aspects can be analytically distinguished from each other when several institutions play a role.

Institutional discourses are characterised via (e.g. Figure 1):
- subject specific rationales (Logos)
- specific ways of doing (Praxis)
- specific disciplinary purposes (what-for aspects, raison d'être)

Results from previous studies [2-7]

Context: Lecturer sample solution of amplitude modulation (AM) task
Method/Methodology: Praxeological analysis and reconstruction of inst. mathem. discourses
Mathematical topic: Complex numbers (in HM and ET)

HM discourse on complex numbers: useful for solving polynomial equations and important objects of calculation. Arrows are used to illustrate calculation rules and properties.
- Internal mathematical conception of concepts
- No concrete references to reality,
  orientation towards academic mathematicsa
- Concentration on calculation rules

ET discourse on complex numbers: oscillating signals can be described algebraically in a very suitable way, visualised graphically as phasors. Important analysis tools, e.g. for AC circuits or AM.
- Specific references to reality
- Degree of reference to reality varies greatly, along with the degree of formalisation
- Typical way of "system-thinking", "linguistic shift"

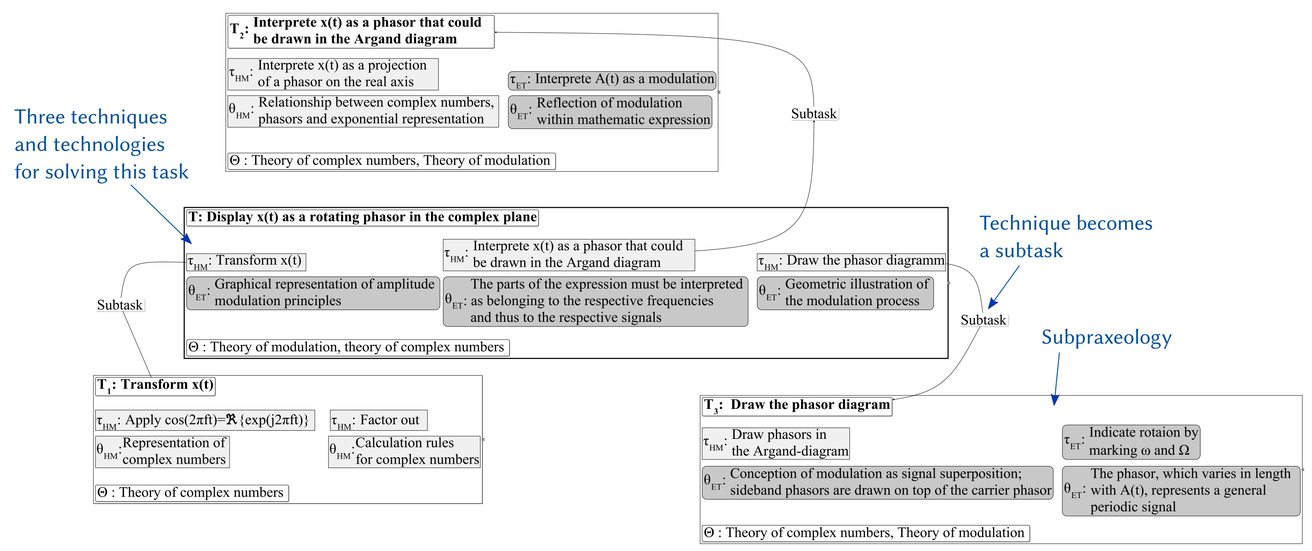

Figure 2: Discourses in an exercise on amplitude modulation [3]

A dialectical view on the relationship between mathematics and engineering

The two mathematical discourses (HM and ET) can be seen as contradictory orientations within the Signal Transmission course that must be considered together.
- Mathematics *for* Engineering (HM discourse): associated with mathematical rigour and formalism, also deviations therefrom (with references to it)
- Mathematics *in* Engineering (ET discourse): pragmatism [4], mathematical practices have developed as historic-specific manifestations addressing specific engineering needs

Both contradictory orientations have to be integrated and transformed into new mathematical practices in Signal Transmission.

Application and modelling
Resulting mathematical practices are neither applications of rigorous mathematics in an extra-mathemathematical engineering context, nor understandable with standard modelling cycles.

- How can we conceptualise modelling or the usage of application examples that take the dialectics of *in/for* into account?

The disconnectedness of mathematics courses in/for engineering
The disconnectedness is considered a problem and research and teaching design projects aim at fostering connections.

- How can we promote interconnections without over-emphasising one side of the dialectics and ignoring the other?
- Can disconnectedness be a productive learning opportunity if seen as dialectical?

References

[1] Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114. https://doi.org/10.24529/hjme.1205
[2] Hochmuth, R., & Peters, J. (2020). About the "Mixture" of Discourses in the Use of Mathematics in Signal Theory. *Educação Matemática Pesquisa: Revista do Programa de Estudos Pós-Graduados em Educação Matemática*, *22*(4), 454–471. https://doi.org/10.23925/1983-3156.2020v22i4p454-471
[3] Hochmuth, R., & Peters, J. (2021). On the Analysis of Mathematical Practices in Signal Theory Courses. *International Journal of Research in Undergraduate Mathematics Education*, *7*(2), 235–260. https://doi.org/10.1007/s40753-021-00138-9
[4] Hochmuth, R., & Peters, J. (2022). About Two Epistemological Related Aspects in Mathematical Practices of Empirical Sciences. In Y. Chevallard, B. B. Farràs, M. Bosch, I. Florensa, J. Gascón, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Advances in the Anthropological Theory of the Didactic* (pp. 327–342). Birkhäuser Basel. https://doi.org/10.1007/978-3-030-76791-4_26
[5] Peters, J. (2022). Modifying Exercises in Mathematics Service Courses for Engineers Based on Subject-Specific Analyses of Engineering Mathematical Practices. In R. Biehler, G. Gueduet, M. Liebendörfer, C. Rasmussen, & C. Winsløw (Eds.), *Practice-Oriented Research in Tertiary Mathematics Education*. (pp. 581–601). Springer. https://doi.org/10.1007/978-3-031-14175-1_28
[6] Peters, J., & Hochmuth, R. (2021). Praxeologische Analysen mathematischer Praktiken in der Signaltheorie. In R. Biehler, A. Eichler, R. Hochmuth, S. Rach & N. Schaper (Hrsg.), *Lehrinnovationen in der Hochschulmathematik: praxisrelevant – didaktisch fundiert – forschungsbasiert* (pp. 109–139). Springer. https://doi.org/10.1007/978-3-662-62854-6_6
[7] Peters, J., & Hochmuth, R. (2022). Sometimes mathematics is different in electrical engineering. *Hiroshima Journal of Mathematics Education*, (15), 115–127. https://doi.org/10.24529/hjme.1510

Studies [2-7] are collected in Peters, J. (2023). *Sometimes Mathematics is Different - Studies on Mathematical Practices in Electrical Engineering*. Hannover: Gottfried Wilhelm Leibniz Universität, Diss., 202 S. http://dx.doi.org/10.15488/14094 (scan QR code for download)