Essays on Persistence and Volatility in Financial Time Series

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Abstract

This thesis contains four essays on persistence change tests and non-stationarity tests. Persistence change tests are analysed under non-standard conditions and a new family of tests to detect changes in persistence and unit roots is proposed that is based on the CUSUM testing principle. These can be applied in economic and financial time series.

Chapter 1 introduces the existence and implications of persistence in time series and structural changes. Furthermore, the impact of asymmetric volatility and different types of outliers is discussed. A new testing principle based on the concept of squared CUSUM of residuals is developed.

Chapter 2 reviews the literature on different methods for persistence change tests including parametric and non-parametric modifications. A family GARCH model is presented to consider different asymmetric conditional volatility models within the persistence change model. The Wild bootstrap approach is introduced and bootstrap analogues of the persistence change tests are derived. The bootstrap procedure is conducted in a comprehensive Monte Carlo study to analyse the behaviour of the tests under asymmetric volatility. The results show that the tests suffer from severe size distortions, while the bootstrap method provides reasonable results in small samples. In an application to the U.S. stock market, asymmetric volatility models are estimated on the return series, where the persistence change tests and the bootstrap analogues are conducted. The main finding is that the tests falsely detect a change in persistence under asymmetric volatility, while the bootstrap analogues assume stationary behaviour.

In chapter 3 the effect of outliers on inference in models with changing persistence is under consideration. We introduce the additive and innovative outlier with different outlier detection and removal methods. In a Monte Carlo study, the performance of the tests is investigated and compared in uncontaminated, outlier contaminated and adjusted series. The main finding is that innovative outliers do not affect the size, while additive outliers deteriorate the performance of the tests if the series exhibits a high degree of persistence. We present a modified outlier detection and removal method which is applied in a simulation study. In an empirical application to inflation data of the G7 countries the tests and the new method are conducted.

Chapter 4 introduces a new approach to test for a unit root based on squared CUSUMs of residuals. The procedure is based on the squared sum of all different consecutive observations of the time series. The limiting distributions of the tests are derived and consistency can be shown. A comprehensive simulation study in ARMA models suggests that the new method provides better properties. In stationary processes the tests show higher power than commonly used unit root tests, while the size is closer to the nominal significance level, when a unit root is present in the data. The empirical application to the historical Nelson-Plosser data provides slightly different results compared to the findings in the literature.

In chapter 5 the same procedure as in chapter 4 is used to develop tests for a change in persistence based on squared CUSUMs of sub-sample residuals. We construct one test for the null hypothesis of a stationary process and another test for the non-stationarity hypothesis. Similar to previous testing procedures a maximum and a ratio based test is constructed for the alternative of a change in persistence in an unknown direction. While common persistence change tests weight the residuals in the partial sums differently or ignore cross-dependencies of the residuals, the presented tests provide squared partial sums of equally weighted observations and exploit the cross-dependencies. The limiting distributions of the tests are derived and consistency against a change in persistence can be shown. The simulation study provides better size and power properties for both developed tests.

Keywords: Asymmetric Volatility · Brownian Bridge · Brownian Motion · Change in Persistence · CUSUM Tests · Monte Carlo · Outlier Detection · Persistence Change · Unit Root · Wild Bootstrap

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Chapter 1

Introduction

Model building, hypotheses testing and forecasting are of major interest in time series analysis and econometrics. Economic and financial time series often exhibit different levels of persistence or show non-stationary behaviour with theoretical implications for model building. Dividing a series into stationary and non-stationary parts and the decision to model the level of a time series or the first differences is of crucial interest for the practitioner as it will lead to different economic interpretations. Limiting distributions and correct inference of hypotheses tests are affected by different order in limits of a series and the optimal forecast is based on the correct model specification. In general, researchers focus on modelling the stochastic part of a time series, for example with an ARMA model in linear models, and determine the deterministic parts including the level, trend, structural change points and the order of integration. In the following the common notation is used and stationary time series are denoted by $I(0)$, while non-stationary processes are denoted by $I(1)$.

Testing for a change in persistence is based on the assumption of constant persistence under the null hypothesis against the alternative of a change in persistence from $I(0)$ to $I(1)$ or vice versa. Inflation rates are the most typical applications for changes in persistence (cf. Kang et al. (2009) and Martins and Rodrigues (2014)), while exchange rates (cf. Gabas et al. (2011)) or the dividend-price ratio (cf. Sollis (2006) and Park (2010)) are further examples. Testing for a unit root is based on a $I(1)$ process under the null hypothesis and is in almost all economic and financial time series of crucial relevance, for example to test the stock markets efficiency, which is one of the main questions in finance (cf. Chan et al. (1997)). Further examples are the GDP (cf. Perron and Phillips (1987)) and the exchange rates (cf. MacDonald (1996)). The famous data set of Nelson and Plosser (1982) contains 14 economic and financial time series typically used in empirical work.

Conrad and Karanasos (2014) point out the stylized fact that series with strong dependencies often exhibit highly persistent volatilities. Modelling heteroskedasticity with conditional volatility models is based on the ARCH model proposed by Engle (1982) and is extended to the GARCH model by Bollerslev (1986). In addition to heteroskedasticity, there is evidence for asymmetric conditional volatility in the empirical literature. The first observing and documenting this phenomenon is Black (1976). The volatility of stock returns is more affected by negative returns than by positive returns (cf. Wu (2001) and

Bekaert and Wu (2000)) which is called *leverage effect*, while higher inflation and growth uncertainty are typically associated with higher volatility (cf. Grier et al. (2004)).

Chapter 2 presents the persistence change model and four different hypotheses within this model. With the family GARCH model, following Hentschel (1995), a wide class of asymmetric volatility models can be considered in this setup. This family GARCH model nests the most asymmetric GARCH models and we analyse the APARCH, GJR-GARCH and EGARCH model in detail and point out the differences. We review the most relevant tests for changes in persistence in time series. The commonly used tests can be categorized into three different groups. The first is based on the ratio of partial sums in the associated sub-sample and is introduced independently by Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004). Regression based tests are the second group, where the GLS de-trending method of Elliott et al. (1992) is employed and the resulting statistic is based on the t-ratio of $\hat{\rho}$, where ρ is estimated in the ADF-regression (cf. Leybourne et al. (2003)). The last group contains CUSUM of squared residuals based tests (cf. Leybourne et al. (2007b)). Several parametric and non-parametric modifications exists (cf. Leybourne and Taylor (2004) and Harvey et al. (2006)). Previous studies on non-standard volatility (cf. Cavaliere and Robert Taylor (2006) and Cavaliere and Taylor (2008)) demonstrate the impact on persistence change tests resulting in non-pivotal limiting distributions due to nuisance parameters. They adopt wild bootstrap procedures which replicate the pattern of heteroskedasticity in the shocks, developed by Liu (1988), and show pivotal limiting distributions. Bootstrap analogues of the presented tests are derived in this paper. In an extensive Monte Carlo study, the three different asymmetric volatility models are simulated with constant and changing persistence. The results suggest that the bootstrap procedure works well in finite samples. In an empirical application this procedure is applied to 20 stocks from the U.S. market. The return series show evidence of asymmetric volatility and the original tests indicate a change in persistence, while the bootstrap analogues display stationarity.

In chapter 3, co-authored with Saskia Rinke, the ratio persistence change tests by Kim (2000), Kim et al. (2002), and Busetti and Taylor (2004) and the CUSUM based test by Leybourne et al. (2007b) are analysed in outlier contaminated time series. We introduce the concept of additive and innovative outliers with the differences in affecting time series. The outlier detection and removal methods by Tsay (1988) and Shin et al. (1996) are demonstrated. While the former requires the specification and estimation of a parametric model and the outlier detection method is based on a ratio of the estimated outlier effect and the standard deviation, the latter is based on the standardised differences of the time series. In a simulation study the effect of outliers on the performance of the tests is evaluated. While the ratio based tests are not seriously affected by outlying observations, the CUSUM test suffers from severe size and power distortions, especially in additive outlier contaminated time series. We modify the detection and removal method proposed by Shin et al. (1996) and provide reasonable results. The empirical application presents the procedure in quarterly inflation rates of the G7 countries and we find different test decisions for the original and the adjusted series for four of the G7 countries applying our method.

In chapter 4, co-authored with Philipp Sibbertsen, we develop a new approach to construct tests for detecting a unit root. Classical unit root testing is mostly based on the Dickey-Fuller test developed by Dickey and Fuller (1979). Parametric modifications for higher order autoregressive models are proposed by Dickey and Fuller (1981), Dickey and Said (1981) and Said and Dickey (1984), whereas the most popular non-parametric extensions, based on the estimation of the long-run variance, are developed by Phillips (1987) and Phillips and Perron (1988). Several authors analyse unit root testing in panel data, time series with structural changes or nonlinear time series (cf. Maddala and Wu (1999), Perron (1990), Kapetanios et al. (2003)). These testing methods have the common feature of estimating the autoregressive term in parametric ARMA specifications and many studies illustrate the poor size and power properties, when the autoregressive parameter is close to unity or a large moving average term is present in the data (cf. DeJong et al. (1992b), Perron and Ng (1996)). Our new method is not based on the OLS estimates or parametric model specification, but exploits the behaviour of a non-stationary time series. The test statistic is based on the sum of squared partial sums of all sequences since a unit root process contains sequences of large positive or negative observations. We derive the limiting distribution and show consistency of the tests. The conducted simulation study shows reasonable results in favor of our tests. In a simulation study the procedure is applied to the historical data set analysed by Nelson and Plosser (1982) and provides slightly different results than previous studies.

In chapter 5, co-authored with Philipp Sibbertsen, we develop new tests for changes in persistence. While the tests presented in chapter 2 are based on partial sum processes of residuals starting with the first observation or on CUSUM of squared residuals, our tests are based on a similar method as described in chapter 4. Analysing the test statistics of Kim (2000), Kim et al. (2002), and Busetti and Taylor (2004) in detail, they are based on weights of squared observations of the time series, while the test proposed by Leybourne et al. (2007b) ignores cross-dependencies. The new presented approach is based on the sum of all squared partial sums in the respective sub-samples with equally weighted residuals including the cross-dependencies. We develop one test for the stationarity hypothesis and another for the non-stationarity hypothesis. The limiting distributions and consistency of the test statistics are derived. In a simulation study these tests are compared to the commonly used tests by Kim (2000), Kim et al. (2002), Busetti and Taylor (2004) and Leybourne et al. (2007b). The results suggest that the new testing approach is preferable.

Chapter 2

Testing for Changes in Persistence in Asymmetric Volatility Models

2.1 Introduction

Testing for changes in persistence in all its variations and combinations has always been a contemporary issue. Following the latest research, the ability to characterize the stationary and non-stationary parts of a time series seems to be very desirable, since this can positively impact effective model building, hypothesis testing and forecasting in economics as well as in finance. In order to discriminate between $I(0)$ and $I(1)$ behaviour, conventional approaches are targeted on simple unit root testing procedures. Rightly, this proceeding is criticized for its low power and size properties in tests (cf. DeJong et al. (1992a), DeJong et al. (1992b) and Haldrup and Jansson (2006).

Therefore, to divide a time series into stationary and non-stationary parts, a number of different testing procedures have been developed to test for a change in the persistence. Banerjee et al. (1992), Kim (2000), Kim et al. (2002), Busetti and Taylor (2004) and others consider the stationary null hypothesis $I(0)$ throughout, while Leybourne et al. (2003) and Leybourne et al. (2007b) consider the non-stationary null hypothesis $I(1)$ throughout.

Many different approaches have been developed to test for a change in persistence. The former authors of the persistence change tests use ratio based test statistics, while the latter are based on DF-GLS de-trending regression or CUSUM of squares based tests. Even more modifications of the mentioned tests exists, mostly based on different standardisations or transformations of the test statistics, see for example Leybourne and Taylor (2004) and Harvey et al. (2006). Leybourne et al. (2007b) state that it is impossible for various tests to adequately differentiate between a change in persistence and constant persistence of the form not fully covered by their respective null hypothesis. Therefore, all of the mentioned tests provide a procedure to test for an unknown change direction.

Further, Conrad and Karanasos (2014) point out that for series with strong dependencies there is evidence that these series also feature GARCH effects with highly persistent volatilities and the ability to distinguish between stationary and non-stationary parts of the time series is more complicated.

Furthermore, in the empirical literature of stock prices and stock price volatility there is a lot of evidence for asymmetric volatility, where negative returns are generally associated with higher conditional volatility than positive returns, see Engle and Ng (1993), Zakoian (1994) and Wu and Xiao (2002). The presence of the asymmetric volatility mostly appears in times of a stock market crash with decreasing returns and high volatility. Black (1976) was one of the first observing and documenting the asymmetric volatility which is called *leverage effect*.

Testing for changes in persistence under non-standard conditions is of several interest in the literature. Cavaliere and Robert Taylor (2006) analyse the behaviour of persistence change tests in the presence of a volatility shift. Cavaliere and Taylor (2008) test for a change in persistence with non-stationary volatility, Chen et al. (2012) test multiple changes for a heavy-tailed sequence, Sibbertsen and Kruse (2009) provide a robust test under long-range dependencies and Martins and Rodrigues (2014) in fractionally integrated models.

The first three mentioned studies use the wild bootstrap approach to correct the size of the tests and to obtain reasonable results. Wild bootstraps are considered for the improvement of many tests in non-standard volatility process. Cavaliere and Taylor (2009) and Cavaliere et al. (2011) apply the wild bootstrap to unit root tests and Cavaliere et al. (2019) make use of them in seasonal unit root tests.

Testing for a change in persistence in conditional volatility models like GARCH models or in asymmetric volatility models is not yet considered. This is why we aim to account for these shortcomings by combining the topic of changes in persistence with asymmetric volatility models which nests as a byproduct the symmetric volatility models. Furthermore, we follow the wide literature and apply the wild bootstrap to several persistence change tests to achieve optimal results.

The rest of the paper is organised as follows. In Section 2.2 we introduce the persistence change model, the different hypotheses and the class of asymmetric volatility models. In Section 2.3 we present the most common tests for a change in persistence and the variations which rejects for an unknown change in persistence to have power against both alternatives. The different types of persistence change tests are ratio based tests, the locally best invariant tests, regression based tests and CUSUM of squares based tests. In Section 2.4 the bootstrap versions of all the presented tests are provided to obtain an asymptotically pivotal limiting distribution in heteroskedastic models. The Monte Carlo study of Section 2.5 analyses the behaviour of the different test statistics and their associated bootstrap analogue in cases of constant stationary and non-stationary persistence and under the alternative of a change in persistence in both directions. In addition to the change, the simulated series include different asymmetric volatility processes with a low or high degree of volatility which can change within the time series. An empirical application to the US stock market in Section 2.6 demonstrates the procedure in practice.

2.2 Modelling Persistence Changes and Asymmetric Volatility

In this paper we analyse the behaviour of tests for a change in persistence in a wide class of symmetric and asymmetric conditional volatility models. In addition to the volatility specification we allow the time series to break in persistence, that is we allow a change from $I(0)$ to $I(1)$ and from $I(1)$ to $I(0)$. In the following section we present the persistence change model, the associated assumptions and the class of volatility models which nests several classical and special volatility processes.

2.2.1 The Persistence Change Model

In the following study we generalise the persistence change model of Kim (2000), inter alia, and consider a stochastic process $\{y_t\}$ that undergoes a shift in persistence. We analyse the following model

$$\begin{aligned} y_t &= d_t + \rho_t y_{t-1} + \epsilon_t, \quad t = 1, \ldots, T \\ d_t &= x_t' \beta, \\ \epsilon_t &= \sigma_t \eta_t, \end{aligned} \tag{2.1}$$

where d_t is the deterministic kernel and modelled as either a constant, $d_t = \beta_0$, or as a constant and a linear time trend, $d_t = \beta_0 + \beta_1 t$. In general x_t is a fixed $(k+1) \times 1$ sequence with $x_{1t} = 1$ throughout with the implication that an intercept term is always included in the model. It is assumed that x_t satisfies the mild regularity conditions of Phillips and Xiao (1998) where it is assumed that a scaling matrix δ_T and a bounded piecewise continuous function $F(\cdot)$ on $[0,1]$ exists such that $\delta_T x_{\lfloor \cdot T \rfloor} \rightarrow x(\cdot)$ uniformly on $[0,1]$. Here $\lfloor \cdot \rfloor$ denotes the integer part of its argument and $\int_0^1 x(s)x(s)'$ is positive definite. The polynomial trend of order k, $x_t = (1, t, \ldots, t^k)$' is the leading example for this condition. With this assumption in our analysis there is no break in the intercept and no break in the intercept and linear trend case allowed.

In the classical assumption the stochastic process $\{\epsilon_t\}$ of the innovation sequence is a zero-mean and strictly stationary mixing process (α-mixing process) with $E|\epsilon|^\gamma < \infty$ for some $\gamma > 2$ with mixing coefficients α_m such that $\sum_{m=1}^{\infty} \alpha_m^{1-2/\gamma} < \infty$. The long-run variance $\sigma_\epsilon^2 = \sum_{j=0}^{\infty} E[\epsilon_{j+1}\epsilon_j]$ is strictly positive and bounded. This assumption of the process $\{\epsilon_t\}$ fulfills the strong α-mixing conditions of Phillips and Perron (1988).

In this study the volatility process $\{\sigma_t\}$ is assumed to fulfill the condition $a_T^{-1}\sigma_{\lfloor sT \rfloor} = \omega(s)$ with some strictly positive and deterministic sequence $\{a_t\}$. Here $\omega(\cdot) \xrightarrow{w} \omega(s)$, where $\xrightarrow{w}$ denotes weak convergence in the space $\mathcal{D}$ with the Skorohod metric and $\omega(\cdot) \in \mathcal{D}$ is a stochastic function, which is independent of the process $\{\eta_t\}$, which is assumed to be independently and identically distributed with mean 0 and variance 1. This assumption allows a wide class of volatility processes for $\{\sigma_t\}$ and we refer to Cavaliere and Taylor (2009) for further discussion. Under this assumption $\epsilon_t = \sigma_t \eta_t$ is heteroskedastic and the process $\{y_t\}$ contains heteroskedasticity. This generalisation of the model reduces to the original persistence change model of Kim (2000) only for a constant conditional volatility, where $\sigma_t = \sigma$, for $t = 1, \ldots, T$.

The autoregressive coefficient ρ satisfies $|\rho| \leq 1$ or $\rho = 1$, which controls the persistence of the process. Within model (2.1), following Kim (2000), we consider four hypotheses.

The first is that y_t is $I(1)$ throughout the sample period which implies a non-stationary process. This is denoted by H_1 with $\rho_t = 1 - \bar{\alpha}/T$, $\bar{\alpha} \geq 0$, $\forall t$, to allow a unit root and local to unit root behaviour. The second hypothesis is that y_t is $I(0)$ throughout the sample period with $\rho_t = \rho$, $|\rho| < 1$, $\forall t$ and is denoted by H_0. In this hypothesis the process y_t exhibit stationary behaviour.

The third hypothesis is a change in persistence from a stationary $I(0)$ process to a non-stationary $I(1)$ process at time $\lfloor \tau T \rfloor$, where τ is the change-point proportion and an unknown point in Λ with $\tau \in \Lambda = [\tau_l, \tau_u]$ a symmetric interval in $[0, 1]$ around 0.5. This hypothesis is denoted by H_{01}

$$\begin{aligned} y_t &= d_t + \rho y_{t-1} + \epsilon_t, && t = 1, \ldots, \lfloor \tau T \rfloor, \\ y_t &= d_t + y_{t-1} + \epsilon_t, && t = \lfloor \tau T \rfloor + 1, \ldots, T, \end{aligned}$$

where $\rho_t = \rho$ with $|\rho| < 1$ for $t \leq \lfloor \tau T \rfloor$ and $\rho_t = 1$ for $t > \lfloor \tau T \rfloor$. The fourth and last hypothesis is a change in persistence from a non-stationary $I(1)$ process to a stationary $I(0)$ process at time $\lfloor \tau T \rfloor$. This hypothesis is denoted by H_{10}

$$\begin{aligned} y_t &= d_t + y_{t-1} + \epsilon_t, && t = 1, \ldots, \lfloor \tau T \rfloor, \\ y_t &= d_t + w_{\lfloor \tau T \rfloor} + \rho y_{t-1} + \epsilon_t, && t = \lfloor \tau T \rfloor + 1, \ldots, T, \end{aligned}$$

where $\rho_t = 1$ for $t \leq \lfloor \tau T \rfloor$, $\rho_t = \rho$ with $|\rho| < 1$ for $t > \lfloor \tau T \rfloor$, $w_{\lfloor \tau T \rfloor} = \sum_{t=1}^{\lfloor \tau T \rfloor} \epsilon_t$ and $y_{\lfloor \tau T \rfloor} = 0$ to avoid a jump to zero at $\lfloor \tau T \rfloor + 1$ as pointed out of Banerjee et al. (1992). It should be noted that a change in persistence from $I(1)$ to $I(0)$ is equivalent to a change in persistence from $I(0)$ to $I(1)$ in the reversed series $z_t = y_{T-t+1}$ and vice versa.

2.2.2 Modelling Asymmetric Conditional Volatility

In the following section we discuss the specification of the volatility process $\{\sigma_t\}$. The volatility function is assumed to be stochastic and allows for a wide class of volatility models. Furthermore, we assume that $\{\sigma_t\}$ and $\{\eta_t\}$ are stochastically independent while η_t is a zero mean, unit variance process, $\eta_t \sim iid(0,1)$. Then σ_t^2 is the conditional variance in Equation (2.1).

In the classical generalised autoregressive conditional heteroskedasticity (GARCH) model proposed by Bollerslev (1986), who extended the ARCH model introduced by Engle (1982), the variance process depends on the past realisation of the shocks ϵ_t and the past realisation of the variance σ_t^2. The volatility process in the GARCH(p,q) model is given by

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

which reduces to the ARCH(p) model when $q = 0$. To simplify, in the following only the GARCH$(1,1)$ versions will be presented as they are used in the further study. Symmetric volatility models are not able to distinguish between the influence of a large and a small shock on the volatility and furthermore, Nelson and Foster (1994) pointed out that an asymmetric volatility model is a more efficient filter of the conditional variance than GARCH models in the presence of leptokurtic error distributions.

Following Hentschel (1995) we consider a parametric family of asymmetric GARCH models that nests the Exponential GARCH (EGARCH) from Nelson and Cao (1992) and the Asymmetric Power ARCH (APARCH) model proposed by Ding et al. (1993) which nests seven different symmetric and asymmetric GARCH models by itself, especially the GJR-GARCH model from Glosten et al. (1993).

Hentschel (1995) derives the family of GARCH models from the asymmetric absolute value GARCH model by Taylor (1986) and Schwert (1989) with $\lambda = 1$ and $v = 1$

$$\begin{aligned} \frac{\sigma_t^{\lambda} - 1}{\lambda} &= \omega' + \alpha \sigma_{t-1}^{\lambda} f^{v}(\epsilon_t) + \beta \frac{\sigma_{t-1}^{\lambda} - 1}{\lambda}, \\ f^{v}(\epsilon_t) &= |\epsilon_t - b|^{v} - c(\epsilon_t - b)^{v}. \end{aligned} \tag{2.2}$$

This is a Box-cox transformation (cf. Box and Cox (1964)) of the conditional standard deviation σ, where the parameter λ determines the shape of the transformation. This is convex for $\lambda > 1$ and concave for $\lambda < 1$. The function $f^{v}(\epsilon_t)$ determines the asymmetry of the volatility model with the parameter v to transform the absolute value function $f(\cdot)$. The three in this familar GARCH nested asymmetric GARCH models under consideration are obtained by appropriately choosing the parameter combination λ, v, b and c.

For the EGARCH model we choose $\lambda = 0$, $v = 1$, $b = 0$ with no restrictions on c. As λ goes to zero we obtain $\lim_{\lambda\to 0}(\sigma_t^\lambda - 1)/\lambda = \ln\sigma_t$. The Box-cox transformation converges to the natural logarithm with l'Hôpital's rule.

The EGARCH model is then obtained by subtracting the unconditional mean of $f(\epsilon_t)$ from $f(\epsilon_t)$ and adding it to the intercept term

$$\ln\sigma_t^2 = 2\omega'' + 2\alpha[|\epsilon_t| - E|\epsilon_t| - c\epsilon_t] + \beta\ln\sigma_{t-1}^2. \quad (2.3)$$

To obtain the GJR-GARCH model the parameters are set to $\lambda = v = 2$, $b = 0$ and c is not restricted

$$\sigma_t^2 = \omega'' + 2\alpha\sigma_t^2[(1+c^2)\epsilon_t^2 - 2c|\epsilon_t|\epsilon_t] + \beta\sigma_{t-1}^2. \quad (2.4)$$

The APARCH model is a generalisation of the nonlinear ARCH model introduced by Higgins and Bera (1992) with $\lambda = v$, $b = c = 0$ and $|c| \leq 1$ to obtain asymmetry

$$\sigma_t^\lambda = \omega'' + \alpha\lambda(|\sigma_{t-1}\epsilon_t| - c\sigma_{t-1}\epsilon_t)^\lambda + \beta\sigma_{t-1}^\lambda. \quad (2.5)$$

Sufficient conditions to ensure that σ_t^2 takes positive values in Equation (2.2) implies that $\omega' > 0$, $\alpha \geq 0$, $\beta \geq 0$, $|c| \leq 1$ and $f^v(\epsilon_t)$ takes positive values. These conditions are not always necessary in all models in the family. To obtain covariance stationary the condition $E[(\alpha\lambda f^v(\epsilon_t) + \beta)^{2\lambda}] < 1$ must hold. For the EGARCH model the condition is $\beta < 1$. Models with $\lambda = v = 2$ like the GJR-GARCH need the restriction $\beta + 2\alpha < 1$, which can be calculated numerically as in the APARCH model with different λ and v. We refer to Hentschel (1995) for a detailled discussion and use the sufficient conditions on the parameters and the covariance stationarity.

To investigate differences in the behaviour of these asymmetric volatility models we use the *news impact curve* introduced by Pagan and Schwert (1990) and compare it with the news impact curve of a standard GARCH model. The news impact curve analyses the impact of shocks of different magnitude on the conditional standard deviation.

The parameter b, which is set to 0 in the chosen asymmetric volatility models, would cause a shift of the volatility in the direction and magnitude of b. The parameter c controls the slope and thus, the rotation of the news impact curve. A positive value in c leads to a clockwise rotation of the curve. The news impact curve is steeper for negative shocks, than it is for positive shocks, corresponding to the stylized facts of stock return volatility that a negative shock has a larger impact on the volatility than a positive shock. A transformation of the news impact curve results from λ.

In Figure 2.1, the news impact curves of the GARCH and the asymmetric volatility models are presented. The GARCH model is symmetric around 0 which results in a symmetric volatility for positive and negative shocks. The transformation parameter λ

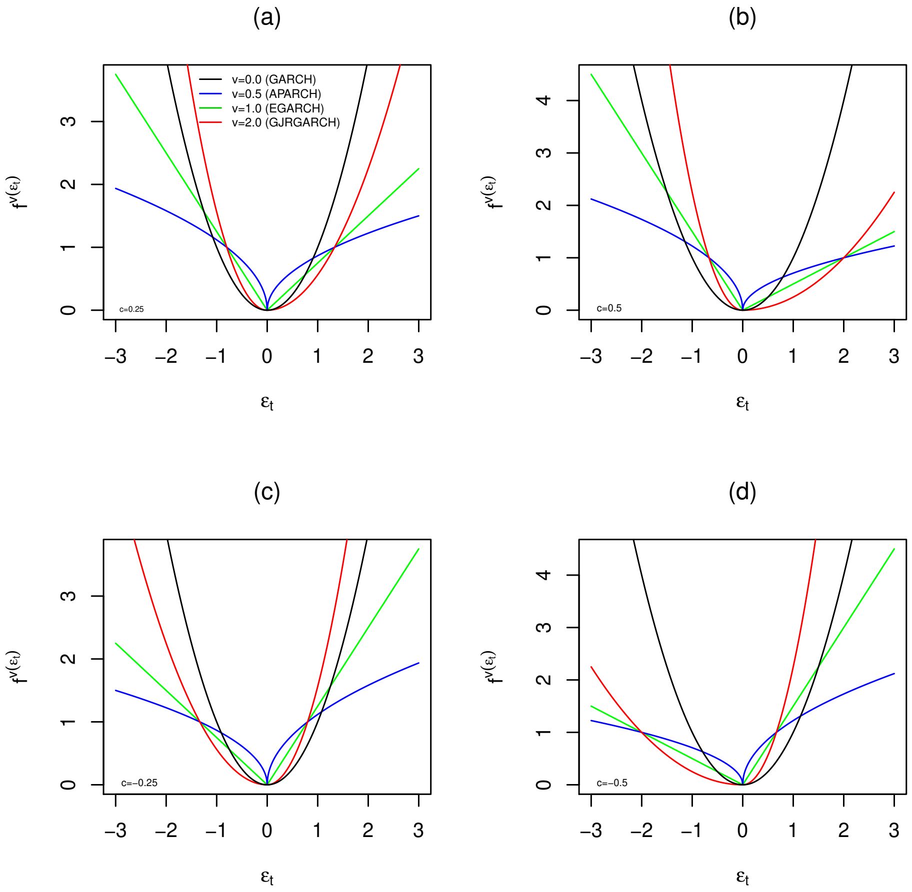

Figure 2.1: The Panels Include the News Impact Curve with the Transformation of $f^v(\epsilon_t)$ in the GARCH, APARCH, EGARCH and GJR-GARCH Model for Different Values of c with $b = 0$.

leads to a rotation of the GJR-GARCH model in direction c which is positive in (a) and (b) and negative in (c) and (d). The EGARCH model results in linear response functions on both sides with a different slope parameter and the APARCH model shows similar rotation asymmetry as the other models but is transformed due to the parameter $\lambda = 0.5$. The news impact curve of the APARCH model could result in different shape as it nests the GJR-GARCH and other GARCH models like the Threshold GARCH (TGARCH) model from Zakoian (1994) for $\lambda = v = 1$, $b = 0$ and $|c| \leq 1$ or the Nonlinear ARCH (NARCH) model from Higgins and Bera (1992) for $\lambda = v$, $b = c = 0$.

2.3 Testing for Changes in Persistence

Testing against a change in persistence behaviour is of practical relevance as pointed out in the introduction. Different procedures exists which include the original ratio based persistence change tests of Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004), several modifications of the ratio based tests, the sub-sample augmented Dickey-Fuller-type test of Leybourne et al. (2003) and the CUSUM based test of Leybourne et al. (2007b). Additional procedures exist for a change in persistence in fractionally integrated series (cf. Sibbertsen and Kruse (2009)) or multiple changes in persistence (cf. Leybourne et al. (2007a)), which are not considered in this analysis.

Some of the presented tests assume a null hypothesis of H_0 throughout, while others assume H_1 under the null throughout. Thus, the alternative hypothesis is a change in persistence H_{01} or H_{10}. The main problem with some of the tests is the possibility that the null hypothesis is not true and they are not able to adequately distinguish between a change in persistence and constant persistence.

To overcome this problem, the maximum of pairwise statistics of the forward and reversed statistic is used to test for a change in unknown direction. In the following chapter the tests for changes in persistence, which are used in the further study, and their consistency rates are presented.

2.3.1 Ratio Tests for Changes in Persistence

Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004) have independently proposed a variance ratio based test for the null hypothesis H_0 against the alternative H_{01}. Several modifications of variance ratio tests exists, which are discussed in the following. These tests are constructed from the sequence of ratios

$$K^f(\tau) = \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \left(\sum_{i=\lfloor \tau T \rfloor + 1}^{t} \hat{\epsilon}_{2,i} \right)^2}{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \left(\sum_{i=1}^{t} \hat{\epsilon}_{1,i} \right)^2}, \tag{2.6}$$

with $\tau \in \Lambda$, where Λ is a given sub-set of $[0, 1]$. In Equation (2.6), $\hat{\epsilon}_{1,t}$ in the denominator are the OLS residuals from the regression of y_t on x_t from Equation (2.1), that is either an intercept or an intercept and a linear trend based on the observations $t = 1, \ldots, \lfloor \tau T \rfloor$. Similarly $\hat{\epsilon}_{2,t}$ in the numerator are the OLS residuals from the regression of y_t on x_t based on the observations $t = \lfloor \tau T \rfloor + 1, \ldots, T$ to obtain exact invariance to an intercept or an intercept and a linear trend. For example, in the constant case the data are de-meaned

$$\hat{\epsilon}_{1,t} = y_t - \bar{y}_1(\tau) \quad \text{with} \quad \bar{y}_1 = \lfloor \tau T \rfloor^{-1} \sum_{t=1}^{\lfloor \tau T \rfloor} y_t,$$

$$\hat{\epsilon}_{2,t} = y_t - \bar{y}_2(\tau) \quad \text{with} \quad \bar{y}_2 = (T - \lfloor \tau T \rfloor)^{-1} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} y_t.$$

Because the true breakpoint τ^* is unknown, Kim (2000) and Busetti and Taylor (2004) make use of three appropriate functions based on the sequence of statistics $\{K^f(\tau), \tau \in \Lambda\}$.

The first is the maximum over the sequence of statistics after Andrews (1993)

$$K_1^f = \mathcal{H}_1(K^f(\cdot)) = \max_{\tau \in \Lambda} K^f(\tau),$$

the second is the mean statistic after Hansen (2002)

$$K_2^f = \mathcal{H}_2(K^f(\cdot)) = \int_{\tau \in \Lambda} K^f(\tau),$$

and the third is the mean exponential statistic after Andrews and Ploberger (1994)

$$K_3^f = \mathcal{H}_3(K^f(\cdot)) = \log \left\{ \int_{\tau \in \Lambda} \exp(K^f(\tau)) \right\}.$$

In each case, the null hypothesis is rejected in favor of the alternative for large values of these statistics and the tests are consistent with rate $O_p(T^2)$ under H_{01}. Suppose now that y_t is generated by a process that includes a change in persistence H_{10}.

In this case Busetti and Taylor (2004) show that the tests which reject for large values of K_j^f are inconsistent and of $O_p(1)$ under H_{10}. In order to test H_0 against H_{10} Busetti and Taylor (2004) demonstrate that tests based on the sequence of reciprocals of $K(\tau)^f$, denoted by K_j^r, are consistent with rate $O_p(T^2)$ in this case, but inconsistent against H_{01}. Furthermore, to test against an unknown direction of change, Busetti and Taylor (2004) suggest a test based on the maximum of sequences K_j^f and K_j^r,

$$K_j = max\{K_j^f, K_j^r\}, \quad j = 1, 2, 3, \tag{2.7}$$

which rejects for large values. Busetti and Taylor (2004) show that the pairwise ratio based tests are of order $O_p(T^2)$ under both H_{01} and H_{10} alternatives, because of the behaviour of the test statistics K_j^f and K_j^r under the wrong alternative. Furthermore, all of the statistics are of $O_p(1)$ under H_0 with pivotal limiting distributions. They show that these test statistics do not depend on the long-run variance of $\{\epsilon_t\}$, even though neither the numerator nor the denominator of any test statistic is scaled by a long-run variance estimator.

The representations for the limiting distributions of the statistics are given in Kim et al. (2002) and Busetti and Taylor (2004), the critical values can be found in Kim et al. (2002).

Non-Parametrically Modified Ratio Test

In their former paper, Busetti and Taylor (2004) discuss the properties of the ratio test and noted that in the presence of serially correlated innovations, they do not require the arguably arbitrary decisions over the lag truncation parameter. In practical situations, the finite sample size properties of the ratio based tests are not satisfactory. Furthermore, the test is oversized in weakly dependent series and the constant I(1) process is neither covered under the null nor the alternative.

Leybourne and Taylor (2004) argued that even if it is not necessary for the purpose of achieving statistics with pivotal limiting null distributions it is feasible to scale both, the numerator and the denominator of Equation (2.6) by an appropriate estimator of the sub-sample long-run variance using m estimated autocovariances. They expect the finite sample distribution of the ratio based tests to better approximate the limiting null distribution by considering an individually modification of the numerator and the denominator. The modified forward test statistic of Equation (2.6) yields to

$$LT^f(\tau, m) = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} K^f(\tau), \tag{2.8}$$

with

$$\hat{\sigma}_1^2 = \lfloor \tau T \rfloor^{-1} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{\epsilon}_{1,t}^2 + 2 \lfloor \tau T \rfloor^{-1} \sum_{i=1}^{m} w(i, m) \sum_{t=i+1}^{\lfloor \tau T \rfloor} \hat{\epsilon}_{1,t} \hat{\epsilon}_{1,t-i},$$

and

$$\hat{\sigma}_2^2 = (T - \lfloor \tau T \rfloor)^{-1} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \hat{\epsilon}_{2,t}^2 + 2 \lfloor \tau T \rfloor^{-1} \sum_{i=1}^{m} w(i, m) \sum_{t=i+\lfloor \tau T \rfloor+1}^{T} \hat{\epsilon}_{2,t} \hat{\epsilon}_{2,t-1},$$

where $w(i,m) = 1 - i/(m+1)$ is the chosen kernel and m is the lag truncation parameter. The procedure can be repeated for the reversed test statistic and applied to the maximum of pairwise sequences. The resulting test statistics are denoted analogue by $LT_j^f(m)$, $LT_j^r(m)$ and $LT_j(m)$, with $j \in \{1, 2, 3\}$.

With this modified test statistics Leybourne and Taylor (2004) follow Kwiatkowski et al. (1992a), because the numerator and the denominator are standard KPSS stationarity test statistics applied to the first and the second sub-sample.

The bandwidth or lag truncation parameter m is chosen to satisfy the conditions that $m \to \infty$ as $T \to \infty$ with the result $m = o(T^{1/2})$ (cf. Kwiatkowski et al. (1992a)). The same procedure is feasible for the modified ratio test $LT_j(m)$, whereby the usual size-power trade-off exists by the choice of m, which results in a possible control over the finite-sample properties in the modified ratio test.

Leybourne and Taylor (2004) show that m can be any non-negative integer value including zero since $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ will converge in probability to the same constant as $T \to \infty$ under the null. The asymptotic critical values provided in Kim et al. (2002) and Busetti and Taylor (2004) are still valid, because the modified test statistics will possess identical limiting null distributions as the unmodified test statistics, whereby they are consistent at rate $O_p(T/m)$, due to the fact that variance estimators constructed from the $I(0)$ process of the sample will converge in probability to some finite constant and variance estimators from the $I(1)$ process of the sample diverge at rate $O_p(Tm)$.

Leybourne and Taylor (2004) show the improvements in the finite-sample size properties of the modified tests and recommend a low-order fixed lag truncation to give useful results in size while keeping the degree of power loss small.

Parametrically Modified Ratio Test

Harvey et al. (2006) propose a parametric modification of Equation (2.6) with the use of variable addition pseudo-statistics as scale factors which have the same critical values regardless of whether the process is H_0 or H_1 throughout. They multiply the original ratio test statistic by a unit root test.

A modified variation of the ratio based test against H_{01}, following the approach of Vogelsang (1998), is the same for all the tests and is given by

$$HLT_j^f = exp(-bJ_{1,T})K_j^f, \quad j = 1, 2, 3,$$

where b is a finite constant and $J_{1,T}$ is T^{-1} times the Wald statistic for testing the joint hypothesis $\gamma_{k+1} = \ldots = \gamma_9 = 0$ in the regression

$$y_t = x_t'\beta + \sum_{i=k+1}^{9} \gamma_i t^i + error, \quad t = 1, \ldots, T.$$

With the standard result that the Wald statistic is $O_p(1)$ it can be followed that $J_{1,T}$ is $O_p(T^{-1})$ and $exp(-bJ_{1,T}) \xrightarrow{p} 1$ and hence, HLT_j^f has the same limiting distribution under H_0 as K_j^f. The correct choice of b make sure that for any given significance level the asymptotic upper-tail critical value of HLT_j^f under H_0 and H_1 remains the same value of K_j^f. Under H_{01} and H_{10} it can be demonstrated that $exp(-bJ_{1,T})$ is of $O_p(1)$ in both cases and retains the same rate of consistency under H_{01} as the original test.

In the case of a change in the other direction, that is H_{10}, the construction of HLT_j^r can be done with the reversed series of y and is analogue to HLT_j^f. Following the approach of Vogelsang (1998) it follows that

$$HLT_j^r = exp(-bJ_{1,T})K_j^r, \quad j = 1, 2, 3,$$

where again b is a finite constant and $J_{1,T}$ is obtained from the regression above. The test statistic HLT_j^r which rejects for large values is consistent at rate $O_p(T^2)$ under H_{10} as the original test statistic K_j^r and is of $O_p(1)$ under H_{01}. The test statistic for an unknown direction of change in persistence can be obtained by the maximum of pairwise statistics

$$HLT_j = exp(-bJ_{1,T})K_j.$$

All of the constructed modified test statistics have the same asymptotic critical values under either H_0 or H_1 as the corresponding unmodified tests. The critical values are still valid but the value of b is test specific and depends on the chosen significance level and can be found in Harvey et al. (2006). Furthermore, the modified test statistics display the same consistency rates under H_{01} or H_{10} as the unmodified tests, whereas the distribution is highly irregular.

Harvey et al. (2006) show that the modified tests can eliminate the spurious over-rejections of the unmodified tests against the constant H_1 alternative, while remaining competitive in terms of power.

LBI Tests

The locally best invariant (LBI) test of H_0 against H_1 introduced by Busetti and Taylor (2004), using King and Hillier (1985), is defined by the critical region

$$LBI^f(\tau) = \hat{\sigma}^{-2}(T - \lfloor \tau T \rfloor)^{-2}\hat{\epsilon}'\mathbf{A}^f(\tau)\hat{\epsilon} \geq l,$$

where $\hat{\epsilon} = (\hat{\epsilon}_1, \dots, \hat{\epsilon}_T)'$, $\hat{\sigma}^2 = T^{-1}\sum_{t=1}^T \hat{\epsilon}_t^2$, and l is a positive constant. The variance covariance matrix is denoted by $\mathbf{A}^f(\tau)$ of $\mu = (\mu_1, \dots, \mu_t)'$ with the (i, j)th element equal to $\min\{i - \lfloor \tau T \rfloor, j - \lfloor \tau T \rfloor\}$, for $i, j = \lfloor \tau T \rfloor + 1, \dots, T$, and all other equal to zero. The test $LBI^f(\tau)$ may be written as

$$LBI^f(\tau) = \hat{\sigma}^{-2}(T - \lfloor T\tau \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \left(\sum_{j=t}^{T} \hat{\epsilon}_j \right)^2.$$

When τ^* is unknown there is no LBI test of H_0 against H_1 and we consider the functions $\mathcal{H}_j(LBI_j^f(\tau))$ for all j applied to the sequence with the resulting test statistics LBI_j^f.

The LBI test of H_1 against H_0 is defined by the critical region

$$LBI^r(\tau) = \hat{\sigma}^{-2}(\lfloor \tau T \rfloor)^{-2}\hat{\epsilon}'\mathbf{A}^r(\tau)\hat{\epsilon} \geq l,$$

where $\mathbf{A}^r(\tau)$ is the variance covariance matrix of $\mu = (\mu_1, \ldots, \mu_T)$' with the (i,j)th element equal to $\min\{i, j, \lfloor \tau T \rfloor\}$, for $i, j = 1, \ldots, \lfloor \tau T \rfloor$ which can be written as

$$LBI^r(\tau) = \hat{\sigma}^{-2}(\lfloor T\tau \rfloor)^{-2}\sum_{t=1}^{\lfloor \tau T \rfloor}\left(\sum_{j=t}^{\tau T}\hat{\epsilon}_j\right)^2.$$

Again, where τ^* is unknown there is no LBI test of H_1 against H_0 and the functions $H_j(LBI^r(\tau))$, $j = 1, 2, 3$, $\tau \in \Lambda$ are considered with the test statistics LBI_j^r.

Busetti and Taylor (2004) show that the finite sample power properties of the LBI test have low power against the wrong alternative hypothesis and the tests are not consistent. Therefore, the same principle to the test statistics as before is applied and we use the maximum pairwise statistics of both, LBI_j^f and LBI_j^r as

$$LBI_j = \max\{H_j(LBI^f(\cdot)), H_j(LBI^r(\cdot))\}, \quad j = 1, 2, 3,$$

which rejects for large values of the test statistics LBI_j. It can be shown that the pairwise LBI test statistics are $O_p(T)$ under H_{01}, H_{10} and constant H_1 alternatives. Busetti and Taylor (2004) show that the LBI pairwise test statistic perform generally better than the standard persistence change tests, when the direction of change under the alternative is not known.

2.3.2 Regression based Tests

In order to test the null hypothesis of a constant H_1 process against the alternative H_{01}, Leybourne et al. (2003) propose a test, where the GLS de-trending method of Elliott et al. (1992) is employed to estimate $\beta = (\beta_0, \beta_1)$. The GLS-transformed series for a given $\tau \in \Lambda$ is defined by

$$\begin{aligned} y_\alpha(\tau) &= [y_1, y_2 - \bar{\alpha}y_1, \ldots, y_{\lfloor \tau T \rfloor} - \bar{\alpha}y_{\lfloor \tau T \rfloor - 1}]', \\ Z_\alpha(\tau) &= [z_1, z_2 - \bar{\alpha}z_1, \ldots, z_{\lfloor \tau T \rfloor} - \bar{\alpha}z_{\lfloor \tau T \rfloor - 1}]', \end{aligned}$$

with $\bar{\alpha} = 1 + \bar{c}T$ for $\bar{c} < 0$. The OLS estimate $\hat{\beta}$ of β is obtained from the locally generalized least square (GLS) de-trending regression of $y_\alpha(\tau)$ on $Z_\alpha(\tau)$. The test statistic $DF^f(\tau)$

is the t-ratio associated with $\hat{\rho}(\tau)$ in the sub-sample augmented Dickey-Fuller (ADF) regression

$$\Delta \hat{y}_t = \hat{\rho}(\tau)\hat{y}_{t-1} + \sum_{i=1}^{p} \hat{\phi}_i(\tau)\Delta \hat{y}_{t-i} + \hat{\varepsilon}_t \quad t = 1, \ldots, \lfloor \tau T \rfloor. \tag{2.9}$$

In practice the true changepoint τ^* is unknown why Leybourne et al. (2003) propose the test which rejects for large negative values of the statistic

$$DF^f = \inf_{\tau \in \Lambda'} DF^f(\tau) \quad \text{for } \Lambda' = [\tau_l, 1].$$

They show that this test has a pivotal limiting null distribution when the standard condition (cf. Berk (1974) and Said and Dickey (1984)), that $1/p + p^3/T \to 0$ as $T \to \infty$, holds on the selected lags p. They also show that the test is consistent against H_{01}, but not against H_{10}. To obtain a consistent test of constant H_1 against a change in persistence H_{10}, they apply the same procedure to the reversed series, that is $z_t = y_{T-t+1}$, with the same condition required on the lag truncation parameter. Therefore, the test which rejects for large negative values of the statistic is denoted by $DF^r = \inf_{\tau \in \Lambda''} DF^r(\tau)$, where $DF^r(\tau)$ is the t-ratio associated with $\hat{\rho}(\tau)$ estimated in the regression (2.9) using z_t in place of y_t throughout with $\Lambda'' = [0, \tau_u]$.

For an unknown direction of change, they also suggest the test which rejects for large negative values of the pairwise minimum statistic, $DF = \min\{DF^f, DF^r\}$ and show that this test is consistent under both H_{01} and H_{10}. Under H_0, they show that DF^f, DF^r, and DF all diverge to $-\infty$ and in large samples the tests reject H_0 with probability one.

2.3.3 CUSUM of Squares based Test

To test against structural changes in the autoregressive parameter, tests based on CUSUM of squared residuals are typically used (cf. Brown et al. (1975)). Leybourne et al. (2007b) propose a test based on the standardised CUSUM of squared sub-sample OLS residuals. They test the null hypothesis of H_1 against H_{10} with the test statistic

$$L^f(\tau) = \frac{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{\epsilon}_{1,t}^2}{\hat{\omega}_f^2(\tau)}, \tag{2.10}$$

where $\hat{\epsilon}_{1,t}$ is defined before and the denominator is an estimator of the long-run variance,

$$\hat{\omega}_f^2(\tau) = \hat{\gamma}_0 + 2\sum_{s=1}^{m} w_{s,m}\hat{\gamma}_s, \quad \hat{\gamma}_s = \lfloor \tau T \rfloor^{-1} \sum_{t=1}^{\lfloor \tau T \rfloor} \Delta\hat{\epsilon}_{1,t}\Delta\hat{\epsilon}_{1,t-s}, \quad w_{s,m} = 1 - sl^{-1},$$

with lag truncation parameter m and associated bandwidth $l = m + 1$. The difference between the test of Leybourne et al. (2007b) and other tests is the usage of cumulative sum of squared residuals $\hat{\epsilon}_{1,t}$ to ensure that the test statistic $L^f(\tau)$ is of different magnitude for $\tau \leq \tau^*$ and $\tau > \tau^*$, although only the residuals from the first sub-sample are used. This property allows the test to consistently identify H_{01}.

To identify H_{10} correctly the analogue of Equation (2.10) can be used for the reversed series z_t and the reversed test statistic is

$$L^r(\tau) = \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=1}^{(T-\lfloor \tau T \rfloor)} \hat{\epsilon}_{2,t}^2}{\tilde{\omega}_r^2(\tau)},$$

with

$$\tilde{\omega}_r^2(\tau) = \tilde{\gamma}_0 + 2\sum_{s=1}^{m} w_{s,m}\tilde{\gamma}_s, \quad \tilde{\gamma}_s = (T - \lfloor \tau T \rfloor)^{-1} \sum_{t=1}^{T-\lfloor \tau T \rfloor} \Delta\hat{\epsilon}_{2,t}\Delta\hat{\epsilon}_{2,t-s}.$$

Leybourne et al. (2007b) show that $L^f(\tau)$ converges in probability to zero under H_{01} for all $\tau \leq \tau^*$ and is $O_p(1)$ under H_{10} for all τ. $L^r(\tau)$ converges in probability to zero under H_{10} for all $\tau > \tau^*$ and is $O_p(1)$ under H_{01} for all τ. If the true direction of change is known, a test of H_1 against the change in persistence, either H_{01} or H_{10}, could be based on the forward or reversed CUSUM of squared statistics respectively. Because a ratio of the minimum of sequences of $L^f(\tau)$ and $L^r(\tau)$ diverge to positive infinity for H_{01} and collapse to zero for H_{10}, Leybourne et al. (2007b) propose the two-tailed test

$$L = \frac{\inf_{\tau \in \Lambda} L^f(\tau)}{\inf_{\tau \in \Lambda} L^r(\tau)} =: \frac{N}{D}.$$

To test either for H_{01} or H_{10} the two-tailed test statistic L can be used and the null hypothesis H_1 throughout can be rejected for small or large values. This is due to the behaviour of the sequences of statistics $K^f(\tau)$ and $K^r(\tau)$ under H_{01} and H_{10} and ensures that a test based on L is consistent against either H_{01} or H_{10}.

Furthermore, if the direction of the possible persistence change is known, separate tests based on N and D can be conducted and the null hypothesis of H_1 can be rejected in favor of H_{01} (H_{10}) for small values of N (D). Leybourne et al. (2007b) derive the limiting distribution of the L statistic under the null and demonstrate that the distribution of L under H_0 is degenerated and therefore, the test is conservative against H_0 processes.

Numerical results show that existing tests tend to reject their constant H_0 (H_1) far too often when applied to H_1 (H_0) series and hence, cannot be of any practical use as tests for a change in persistence. Unlike the latter, the ratio test displays no tendency to reject against processes with constant persistence. Their proposed ratio test appears more useful than existing tests for identifying the direction of change, when a change in persistence occurs.

2.4 Bootstrap Persistence Change Tests

In order to account for the present volatility in the time series and, associated therewith, the size and power distortions of the tests for changes in persistence, we apply a wild bootstrap procedure. The wild bootstrap is developed by Liu (1988) who follow a suggestion of Wu (1986) and Beran (1986). In our special context, the wild bootstrap scheme is required, instead of standard residual or block bootstrap re-sampling schemes, because unlike these schemes the wild bootstrap can replicate the pattern of heteroskedasticity in the shocks.

Cavaliere and Robert Taylor (2006) demonstrate the impact of volatility shifts on persistence change tests and Cavaliere and Taylor (2008) analyse the behaviour of persistence change tests in the presence of a non-stationary volatility. The key result is that the asymptotic null distribution of the test statistics depends on the sample path of the volatility process, which are based on the residuals and therefore, the tests have no longer a pivotal limiting distribution. This leads to severe size distortions of the presented persistence change tests.

In order to overcome the inference problems and to obtain asymptotically pivotal inference under the null hypothesis, we follow Cavaliere and Taylor (2008) and propose wild bootstrap versions of the test statistics. These bootstrap versions are robust to volatility processes from Section 2.2.2 and thus we obtain asymptotic pivotal limiting distributions.

Further, Cavaliere and Taylor (2008) derive consistency properties under H_{01} and H_{10} for the ratio based tests. The proposed wild bootstrap approach constitutes a nonparametric treatment of the heteroskedasticity, since no specification of a parametric model for the volatility process nor any pre-testing is necessary, such as the test of Horváth et al. (2006) in the presence of non-stationary volatility. In order to account for x_t, following Hansen (2000), the tests build on heteroskedastic fixed regressors.

Beran (1988), Davidson and MacKinnon (1999) and Hall (2013) show that bootstrap inference in asymptotically pivotal tests produce more accurate results than inference based on asymptotic theory which results in lower errors.

2.4.1 The Bootstrap Algorithm

In the following section, the Bootstrap versions of the tests for a changes in persistence are presented. The first stage of the bootstrap algorithm for all ratio based tests and the CUSUM of squared residuals test is to compute the full sample residuals $\hat{\varepsilon}_t$ from the regression of y_t on x_t for $t = 1, \ldots, T$. A bootstrap sample is then generated by

$$y_t^b := \hat{\varepsilon}_t \zeta_t, \quad t = 1, \ldots, T, \tag{2.11}$$

where $\{\zeta_t\}_{t=1}^T$ is an independent $N(0,1)$ sequence. Then, under the null hypothesis the bootstrap residuals y_t^b replicate the pattern of heteroskedasticity present in the original shocks since, conditionally on $\hat{\varepsilon}_t$, y_t^b is independent over time with zero mean and variance $\hat{\varepsilon}_t^2$ (cf. Cavaliere and Taylor (2008)).

Note, that Davidson and Flachaire (2008) discusses other distributions of ζ_t to generate the pseudo residuals for an improvement of the results. Mammen (1993) suggests the most popular and asymmetric two-point distribution, where $\zeta_t = -(\sqrt{5}-1)/2$ with probability $p = (\sqrt{5}+1)/(2\sqrt{5})$ and $\zeta_t = (\sqrt{5}+1)/2$ with probability $1-p$. Another two-point symmetric distribution is discussed by Davidson et al. (2007), where ζ_t takes on the value 1 or -1 with equal probabilities. As there are no differences between the small sample properties in the following simulation study, we will only present the results for the gaussian distribution.

In Equation (2.11) define $\hat{\varepsilon}_{1,t}^b$ as the residuals obtained from the OLS regression of y_t^b on x_t for $t = 1, \ldots, \lfloor T\tau \rfloor$ and similarly $\hat{\varepsilon}_{2,t}^b$ as the residuals obtained from the OLS regression of y_t^b on x_t for $t = \lfloor T\tau \rfloor + 1, \ldots, T$.

The bootstrap analogue of the forward ratio based test statistic $K^f(\tau)$ is then given by the bootstrap test statistic

$$K^{f,b}(\tau) = \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor +1}^{T} \left(\sum_{i=\lfloor \tau T \rfloor +1}^{t} \hat{\varepsilon}_{2,i}^b \right)^2}{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \left(\sum_{i=1}^{t} \hat{\varepsilon}_{1,i}^b \right)^2},$$

which corresponds to the statistic in Equation (2.6) except that it is constructed from the pseudo residuals. The associated bootstrap equivalent of the reversed ratio based test statistic $K^r(\tau)$ can be constructed in a similar way and is denoted by $K^{r,b}(\tau)$. When the changepoint is unknown, the bootstrap equivalents of K_j^f and K_j^r based on the sequence of statistics can be constructed for all j as

$$K_j^{f,b} = \mathcal{H}_j(K^{f,b}(\cdot)) \quad \text{and} \quad K_j^{r,b} = \mathcal{H}_j(K^{r,b}(\cdot)).$$

Again, for the test against a change in persistence in unknown direction the bootstrap analogue of Equation (2.7) can be constructed as the maximum of pairwise statistics

$$K_j^b = \max\{K_j^{f,b}, K_j^{r,b}\}, \quad j = 1, 2, 3.$$

The bootstrap p-value can be calculated by $p_T^b(\tau) := 1 - G_b(K_j^b(\tau))$, where $G_b(\cdot)$ denotes the cumulative distribution function (cdf) of K_j^b. As the cdf is unknown in practice, it can be approximated with B conditionally independent bootstrap statistics $K_{j,i}^b$, for $i = 1, \ldots, B$ generated from $y_{t,i}^b = \hat{\varepsilon}_t \zeta_{i,t}$ as above with a doubly independent $N(0,1)$ sequence $\{\{\zeta_{i,t}\}_{t=1}^T\}_{i=1}^B$.

By simulating B boostrap samples, the p-value is approximated as $\hat{p}_{1,T}^b = B^{-1} \sum_{i=1}^B \mathbb{1}(K_{j,i}^b > K_j)$ because it cannot be calculated analytically.

Consistency of $\hat{p}_{1,T}^b$ for $p_{1,T}^b$ is shown by Hansen (1996) as $B \to \infty$. Davidson and MacKinnon (2000) propose a pretest procedure for choosing the number of bootstrap samples. In general the bootstrap size must be chosen that $\alpha(1 + B)$ is an integer, where α is the significance level and 9999 bootstrap samples give in most cases accurate results.

The bootstrap p-values for the further test statistics can be calculated in a similar way and are not reported for each test statistic. As discussed in Hall and Titterington (1989) there will be some loss of power when $B < \infty$.

The modified ratio based test statistics of Leybourne and Taylor (2004), Harvey et al. (2006) and the LBI test of Busetti and Taylor (2004) can be constructed using K_j^b with the bootstrap version of the different standardisations. For Leybourne and Taylor (2004)'s studentized statistic LT_j of Equation (2.8), the bootstrap statistic is

$$LT_j^b = \frac{\hat{\sigma}_1^{2,b}}{\hat{\sigma}_2^{2,b}} K_j^b, \quad j = 1, 2, 3,$$

where $\hat{\sigma}_1^{2,b}$ and $\hat{\sigma}_2^{2,b}$ are the long-run variance estimators of the same form as used in Equation (2.8) with bandwidth m^b applied to the first $\lfloor \tau T \rfloor$ and the last $T - \lfloor \tau T \rfloor$ observations from the bootstrap sample y_t^b.

A bootstrap test statistic of Harvey et al. (2006) can be constructed by

$$HLT_j^b = exp(-bJ_{1,T}^{b^*}) K_j^b, \quad j = 1, 2, 3,$$

where b^* is a finite constant and $J_{1,T}^{b^*}$ is T^{-1} the Wald statistic for testing the joint hypothesis $\gamma_{k+1} = \ldots = \gamma_9 = 0$ in the regression

$$y_t^b = x_t'\beta + \sum_{i=k+1}^{9} \gamma_i t^i + error, \quad t = 1, \ldots, T,$$

with the observations from the bootstrap sample y_t^b.

The LBI bootstrap equivalent to test against a change in persistence from H_0 to H_1 is based on

$$LBI^{f,b}(\tau) = \hat{\sigma}^{-2,b}(T - \lfloor T\tau \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor+1}^{T} \left(\sum_{i=t}^{T} \hat{\varepsilon}_i^b \right)^2 ,$$

where $\hat{\varepsilon}_t^b$ are the full sample residuals from the regression of y_t^b on x_t for $t = 1, \ldots, T$ and $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^{2,b}$. In a similar way $LBI^{r,b}(\tau)$ can be constructed, which leads with $\mathcal{H}_j(\cdot)$ for the unknown direction of change and the pairwise maximum of the test statistics to the LBI bootstrap equivalent for an unknown change in direction LBI_j^b.

The bootstrap residuals for the LKSN test are constructed from the OLS residuals $\tilde{\varepsilon}_t$, from the full sample ADF-type regression (2.9) based on the local GLS de-trended time series $\hat{y}_t$ and are given by $\tilde{\varepsilon}_t^b = \tilde{\varepsilon}_t \zeta_t$. A bootstrap sample is then obtained by constructing the partial sum process

$$y_t^b = y_0^b + \sum_{i=1}^{t} \tilde{\varepsilon}_i^b, \quad t, \ldots, T,$$

for some initial value y_0^b. As long as $E(y_0^b)^2 < \infty$ the asymptotic distributions of the considered tests are not affected by the initial condition and is set to $y_0^b = 0$.

With the bootstrap sample, the associated $DF^{f,b}(\tau)$ bootstrap regression based test statistic is the t-ratio associated with $\hat{\rho}^b(\tau)$ in the sub-sample augmented Dickey-Fuller regression

$$\Delta\hat{y}_t^b = \hat{\rho}^b(\tau)\hat{y}_{t-1}^b + \sum_{j=1}^{p} \hat{\phi}_j^b(\tau)\Delta\hat{y}_{t-j}^b + \hat{\varepsilon}_t \quad t = 1, \ldots, \lfloor \tau T \rfloor.$$

The bootstrap equivalent of an unknown changepoint is then given by

$$DF^{f,b} = \inf_{\tau \in \Lambda'} DF^{f,b}(\tau) \quad \text{for } \Lambda' = [\tau_l, 1].$$

Here $\hat{y}_t^b = y_t^b - z_t'\hat{\beta}$, where $\hat{\beta}$ is the OLS estimate of β obtained from the local generalized least square (GLS) de-trending regression of $y_\alpha^b(\tau) = [y_1^b, y_2^b - \bar{\alpha}y_1^b, \ldots, y_{\lfloor \tau T \rfloor}^b - \bar{\alpha}y_{\lfloor \tau T \rfloor - 1}^b]'$ on $Z_\alpha^b(\tau) = [z_1^b, z_2^b - \bar{\alpha}z_1^b, \ldots, z_{\lfloor \tau T \rfloor}^b - \bar{\alpha}z_{\lfloor \tau T \rfloor - 1}^b]'$ with $\bar{\alpha} = 1 + \bar{c}T$ for some $\bar{c} < 0$. In a similar manner the bootstrap version of DF^r can be constructed and is denoted by $DF^{r,b}$ and the bootstrap regression based test against a change in persistence in an unknown direction follows

$$DF^b = \min\{DF^{f,b}, DF^{r,b}\}.$$

Note, that the bootstrap p-value for the left-sided test is approximated by
$\hat{p}_{1,T}^b = B^{-1} \sum_{i=1}^{B} \mathbb{1}(DF_{j,i}^b < DF_j)$.

The bootstrap analogue of Leybourne et al. (2007b)'s CUSUM of squares based residuals test is based on Equation (2.11) with the bootstrap residuals $\hat{\varepsilon}^{b}_{1,t}$ and $\hat{\varepsilon}^{b}_{2,t}$ defined earlier. The resulting forward and reversed test statistics are given by

$$L^{f,b}(\tau) = \frac{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{\varepsilon}^{2,b}_{1,t}}{\hat{\omega}^{2,b}_{f}(\tau)} \qquad L^{r,b}(\tau) = \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=1}^{(T-\lfloor \tau T \rfloor)} \hat{\varepsilon}^{2,b}_{2,t}}{\hat{\omega}^{2,b}_{r}(\tau)},$$

where $\hat{\omega}^{2,b}_{f}(\tau)$ and $\hat{\omega}^{2,b}_{r}(\tau)$ are the corresponding long-run variance estimators. Finally the bootstrap version against a change in persistence from H_{01} and H_{10} is given by

$$L^{b} = \frac{\inf_{\tau \in \Lambda} L^{f,b}(\tau)}{\inf_{\tau \in \Lambda} L^{r,b}(\tau)}.$$

Simulating the bootstrap p-value for the two-tailed test statistic by Leybourne et al. (2007b) yields to the formula

$$\hat{p}^{b} = 2 \min \left(\frac{1}{B} \sum_{i=1}^{B} \mathbb{1}(L^{b}_{j,i} \leq L_{j}); \frac{1}{B} \sum_{i=1}^{B} \mathbb{1}(L^{b}_{j,i} > L_{j}) \right).$$

2.5 Numerical Results

2.5.1 Simulation Setup

In this section, we use Monte Carlo simulation methods to investigate the size and power behaviour of the tests of Section 2.3 and the corresponding bootstrap versions of Section 2.4, when applied to data generated by the persistence change model of Section 2.2.1 including asymmetric conditional volatility of Section 2.2.2. We consider the following data generating process (DGP)

$$y_t = \rho_t y_{t-1} + \epsilon, \quad t = -200, \dots, T$$
$$\epsilon_t = \sigma_t \eta_t,$$

where $\eta_t \sim N(0,1)$, the parameter ρ_t is defined in the following sections and the volatility process $\{\sigma_t\}$ is generated by $\frac{\sigma_t^\lambda - 1}{\lambda} = \omega' + \alpha\sigma_{t-1}^\lambda f^v(\epsilon_t) + \beta\frac{\sigma_{t-1}^\lambda - 1}{\lambda}$ with $f^v(\epsilon_t) = |\epsilon_t - b|^v - c(\epsilon_t - b)^v$.

Three volatility processes are considered, namely the APARCH ($\lambda = v = 1.5$, $b = c = 0$), the GJR-GARCH ($\lambda = v = 2$, $b = 0$) and the EGARCH ($\lambda = 0$, $v = 1$, $b = 0$) according to Section 2.2.2. The asymmetric volatility is simulated with the representations given in Equations (2.3)-(2.5) with $\omega'' = 0.0005$ throughout.

The specification of the volatility process and therewith the choice of the parameters α, β and c is reported in the tables of the simulation results, meeting the stationary conditions in both sub-samples. Notice, that the volatility process is not necessarily constant. A break in the parameters of the volatility can occur independently of the choice of ρ. That is, a break in $\{\sigma_t\}$ can be present in a constant H_0 or H_1 DGP and in a change in persistence H_{01} or H_{10}. The volatility model can switch from low asymmetric volatility to a higher level of asymmetric volatility and vice versa. The break point of the volatility is assumed to be at the half of the sample in the constant cases H_0 and H_1 and occurs at τ^* where the persistence changes in H_{01} and H_{10}.

In the context of the LT test we set $m = \lfloor 4(T/100)^{1/4} \rfloor$, the values for b in the HLT test are given in Harvey et al. (2006) and for the L test we follow Leybourne et al. (2007b) and choose $m = 0$ yielding OLS variance estimators and use the two-sided test with the half of the nominal significance level in both directions. The lag length p in the LKSN test is chosen using the Modified Akaike Information Criterion rule following Ng and Perron (2001) where the maximum lag length is set to $p_{max} = \lfloor 12(T/100)^{1/4} \rfloor$.

The finite sample critical values of the individual test statistics for $T = 100$ and $T = 250$ were obtained by Monte Carlo simulation using $1,000,000$ Monte Carlo replications and are reported in Table 2.1 and Table 2.2. Note, that the $LKSN$ test rejects for large negative values and the two-sided L test rejects for both large negative and large positive values, while the rest reject for large values. The critical values of LT_j and HLT_j are the same as K_j.

	$T = 100$			$T = 250$		
α	10%	5%	1%	10%	5%	1%
K_1	17.11	21.75	34.33	17.438	22.169	34.897
K_2	4.666	5.914	9.262	4.626	5.824	9.223
K_3	5.232	7.389	13.37	5.114	7.228	13.209
LBI_1	1.561	1.974	2.939	1.561	1.974	2.939
LBI_2	0.913	1.214	1.787	0.913	1.214	1.787
LBI_3	0.473	0.631	0.940	0.473	0.631	0.940
$LKSN$	-3.236	-3.512	-4.147	-2.969	-3.237	-3.842

Table 2.1: Critical Values for the K_j, LT_j, HLT_j, LBI_j, and $LKSN$ Tests for $T = 100$ and $T = 250$.

α	0.5%	2.5%	5%	95%	97.5%	99.5%
$T = 100$	0.111	0.188	0.247	3.853	5.130	9.132
$T = 250$	0.104	0.178	0.236	4.013	5.365	8.834

Table 2.2: Critical Values for the L Test for $T = 100$ and $T = 250$.

All the tables in the simulation study are organised in the same manner. In the first row within the first four DGP's, the volatility process is constant in both sub-samples and increases from left to right with a positive asymmetric volatility parameter $c = 0.5$. The second four DGP's include a change in the volatility process from low magnitude to a higher degree and vice versa again with $c = 0.5$. The last four DGP's in the tables report results for a negative asymmetric volatility parameter $c = -0.5$ with constant and changing volatility. To analyse the small-sample behaviour of the proposed tests we set $T = 100$ and $T = 250$.

For a better overview, the results are reported illustrative for the APARCH model and the results of the GJR-GARCH and EGARCH model can be found in the Appendix 2.8, where Tables 2.15-2.22 include the GJR-GARCH results and Tables 2.23-2.30 the EGARCH results. In general, the results for the different volatility processes are very similar and show the same pattern. The tables report the empirical rejection frequencies with the associated rejection frequencies of the bootstrap version in parentheses and are reported only in the de-meaned case $d_t = \beta_0$, as the differences to the de-trended case $d_t = \beta_0 + \beta_t$ are small and the analysis is transferable.

As typical in the literature we use $\Lambda = [0.2, 0.8]$ and in all simulations we use 1,000 Monte Carlo replications, construct 999 Bootstrap samples and use 200 observations as burn-in period for the DGP. The simulation study is extendable by an additional moving-average parameter or multiple breaks in persistence in the DGP but as the interest is in the behaviour under asymmetric volatility the results should not get distorted.

2.5.2 Properties of the Tests under H_0 and H_1

In the following section we investigate the size of the tests under the null hypothesis using Monte Carlo simulation methods. The tests are applied to data generated by

$$y_t = \rho y_{t-1} + \epsilon, \quad t = -200, \ldots, T,$$
$$\epsilon_t = \sigma_t \eta_t,$$

where $\rho \in \{0, 1\}$. The degree of persistence is constant in this processes throughout the sample and we set $\rho = \rho_t$, $\forall t$. This leads to a constant stationary process H_0 for $\rho = 0$ and a constant non-stationary process for $\rho = 1$, H_1. Furthermore, we set $d_t = 0$ in Equation (2.1).

The analysis is extendable for a higher degree of persistence in the stationary case for $0 < \rho < 1$ where the size is more distorted but again, we are interested in the effects of the asymmetric volatility models analysed in the standard processes.

Note, that the null hypothesis of the tests K_j, LT_j and LBI_j is H_0, the null hypothesis of the $LKSN$ and L tests is H_1 and the null hypothesis of the HLT_j tests is constant persistence, that is H_0 or H_1. Therefore, the size results of the ratio based tests besides the HLT_j tests are obtained in Table 2.3 and Table 2.4 and the size results of the $LKSN$ and the L tests are obtained in Table 2.5 and Table 2.6. The HLT_j tests can be analysed in both cases.

As a key feature of the test by Leybourne et al. (2007b) there is no tendency to spuriously over-reject, when applied to constant stationary processes and the results in Table 2.3 and Table 2.4 are nevertheless interpretable for the L test. Empirical rejection frequencies are reported for the 10%, 5% and 1% significance level.

Analysing the results in the H_0 case provides a clear pattern. The K_j and HLT_j tests suffer most from severe size distortions for the APARCH model in Table 2.3 with $T = 100$. The size distortions are increasing with increasing constant GARCH parameters α and β in the first row of the table. In the case of a volatility break, the size distortions in the next four DGP's are even larger. These are higher for a change in β, than for a change in α. The results remain for a negative asymmetric volatility parameter $c = -0.5$ in the last row of the table. When the volatility breaks from no conditional volatility to high asymmetric volatility in the last DGP, the size distortions are larger than in high constant asymmetric volatility.

The LT_j tests are conservative in the case of low constant asymmetric volatility and are slightly over-sized in high constant asymmetric volatility. Furthermore, the tests are only affected by a break in the β parameter but not by a break in α. The rejection frequencies are close to the nominal size, when α breaks and for a break from no conditional volatility to high asymmetric volatility. The negative asymmetric volatility parameter has no further negative influence. The non-parametrically modification with the long-run variance estimator leads to better results, than the standard and parametric ratio tests.

The LBI_j tests are close to the nominal significance level in the first DGP, when the constant volatility is small and is oversized for the higher volatility processes. Surprisingly, when $\alpha = 0.35$ and $\beta = 0.55$ the tests are undersized. The LBI_j tests are more affected by the β parameter, than by the α parameter. This is true for the next four DGP's. The size of the tests is not negatively affected by a break in α, whereas a break in β results in size distortions. The negative asymmetry show similar results. In general, the LBI_j tests show similar behaviour as the LT_j tests. This is an interesting result, as the LT_j tests are based on long-run variance estimators and the LBI_j tests are divided by the short-run variance.

The L test provides the same results as in the constant H_0 case without conditional variance and never rejects the null hypothesis by construction.

Rejection frequencies obtained from the bootstrap analogues of the test statistics show a desirable result. The bootstrap versions of all the tests are in most cases very close to the nominal significance level. They are conservative in constant asymmetric volatility and slightly over-sized if β changes. The bootstrap versions of the tests retrieve the correct asymptotic null distribution and the results of an asymptotically pivotal limiting distribution of Cavaliere and Taylor (2008) seem to hold. Note, that the LKSN test cannot be analysed in H_0.

The results are highlighted in Table 2.4 with $T = 250$. The size distortions are smaller than in $T = 100$ but still large and increasing in the constant GARCH parameters and larger for a change in β than in α. Again, the bootstrap versions of the tests provide satisfactory results.

Analysing the results in the H_1 case in Table 2.5 with $T = 100$ leads to another interesting pattern.

The HLT_j tests suffer from less size distortions than in the H_0 case but is still clearly over-sized. In H_1 the bootstrap version of the HLT_j is over-sized, if the parameter β of the asymmetric volatility process changes but works well in other DGP's.

The $LKSN$ test behaves similar to the LT_j and LBI_j tests in the H_0 processes and is slightly oversized for low constant GARCH parameters, increasing with α and β. The size is higher when the volatility parameter changes. The bootstrap version of the $LKSN$ test provides better results and is closer to the nominal size level.

The L test show a different pattern. In the first row of the table, this test is just slightly oversized and close to the nominal significance level for $\alpha = 0.35$ and $\beta = 0.55$. For a break in the volatility parameter, the test is oversized if β changes and is close to the size if α changes. In an asymmetric volatility process, where the asymmetric volatility parameter is negative, that is the last row of the table, the test is close to or under the nominal size level. The results of the bootstrap version of L are slightly closer to the nominal size level.

Again, the bootstrap versions of the tests provide better results than the original test statistic, but are slightly oversized for the $LKSN$ and L test and suffer from severe size distortions for the HLT_j tests.

In a similar manner the results obtained with $T = 250$ in Table 2.6 can be interpreted.

Comparing the results in the H_0 and H_1 case to the GJR-GARCH and the EGARCH model lead to no notable difference in the analysis. The results of the GJR-GARCH model can be found in the Tables 2.15 - 2.18 and the results of the EGARCH model in the Tables 2.23 - 2.26.

The main findings of the analysis in constant persistence is that the bootstrap analogues of the test statistics provide the best results. The LT_j tests and the LBI_j tests show the best properties. In general, the size is increasing with increasing asymmetric volatility, a change in β has more impact on the behaviour of the tests, than a change in α. The direction of the asymmetric volatility does not affect the rejection frequencies.

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.135(0.051)	0.067(0.016)	0.015(0.000)	0.131(0.057)	0.070(0.013)	0.008(0.000)	0.343(0.027)	0.253(0.015)	0.118(0.000)	0.426(0.087)	0.313(0.008)	0.166(0.000)
K_2	0.135(0.079)	0.076(0.006)	0.017(0.000)	0.212(0.094)	0.080(0.023)	0.022(0.000)	0.398(0.065)	0.244(0.027)	0.177(0.000)	0.378(0.071)	0.311(0.030)	0.158(0.000)
K_3	0.164(0.037)	0.069(0.022)	0.021(0.000)	0.163(0.057)	0.079(0.010)	0.008(0.000)	0.342(0.032)	0.250(0.016)	0.120(0.000)	0.432(0.092)	0.321(0.006)	0.178(0.000)
LT_1	0.046(0.125)	0.033(0.046)	0.000(0.000)	0.035(0.124)	0.034(0.033)	0.000(0.000)	0.118(0.112)	0.045(0.072)	0.010(0.012)	0.099(0.086)	0.060(0.036)	0.034(0.025)
LT_2	0.082(0.106)	0.030(0.025)	0.000(0.021)	0.079(0.129)	0.031(0.069)	0.000(0.000)	0.112(0.109)	0.040(0.039)	0.014(0.015)	0.112(0.096)	0.046(0.043)	0.022(0.016)
LT_3	0.039(0.127)	0.025(0.026)	0.000(0.000)	0.037(0.131)	0.032(0.025)	0.000(0.000)	0.119(0.105)	0.053(0.070)	0.010(0.005)	0.113(0.088)	0.071(0.040)	0.033(0.031)
HLT_1	0.133(0.036)	0.065(0.017)	0.008(0.000)	0.118(0.043)	0.060(0.014)	0.010(0.000)	0.332(0.032)	0.223(0.016)	0.103(0.000)	0.433(0.091)	0.313(0.006)	0.169(0.000)
HLT_2	0.131(0.075)	0.073(0.011)	0.017(0.000)	0.191(0.093)	0.077(0.000)	0.018(0.000)	0.352(0.072)	0.235(0.020)	0.130(0.000)	0.366(0.072)	0.295(0.033)	0.161(0.000)
HLT_3	0.138(0.036)	0.071(0.022)	0.006(0.000)	0.118(0.050)	0.059(0.007)	0.013(0.000)	0.344(0.026)	0.222(0.019)	0.102(0.000)	0.422(0.087)	0.299(0.013)	0.174(0.000)
LBI_1	0.103(0.115)	0.060(0.053)	0.007(0.022)	0.156(0.183)	0.120(0.089)	0.027(0.044)	0.119(0.059)	0.055(0.032)	0.030(0.010)	0.075(0.044)	0.036(0.012)	0.012(0.000)
LBI_2	0.134(0.093)	0.054(0.057)	0.000(0.012)	0.149(0.127)	0.121(0.096)	0.018(0.033)	0.106(0.097)	0.066(0.037)	0.036(0.012)	0.039(0.036)	0.005(0.015)	0.000(0.010)
LBI_3	0.114(0.086)	0.057(0.056)	0.000(0.007)	0.153(0.144)	0.118(0.099)	0.022(0.041)	0.113(0.101)	0.056(0.043)	0.035(0.014)	0.043(0.050)	0.012(0.023)	0.000(0.010)
$LKSN$	1.000(0.872)	1.000(0.927)	1.000(0.893)	1.000(0.994)	1.000(0.918)	1.000(0.877)	1.000(0.988)	1.000(0.969)	1.000(0.940)	1.000(0.943)	1.000(0.915)	1.000(0.885)
L	0.000(0.082)	0.000(0.033)	0.000(0.000)	0.000(0.172)	0.000(0.055)	0.000(0.011)	0.000(0.143)	0.000(0.070)	0.000(0.032)	0.000(0.058)	0.000(0.005)	0.000(0.014)

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.718(0.083)	0.623(0.053)	0.519(0.010)	0.822(0.062)	0.740(0.041)	0.587(0.000)	0.258(0.030)	0.176(0.032)	0.078(0.000)	0.318(0.084)	0.283(0.043)	0.145(0.012)
K_2	0.744(0.089)	0.677(0.071)	0.543(0.008)	0.830(0.070)	0.758(0.047)	0.651(0.009)	0.199(0.040)	0.148(0.016)	0.087(0.009)	0.339(0.088)	0.287(0.043)	0.135(0.021)
K_3	0.721(0.087)	0.620(0.062)	0.533(0.020)	0.825(0.063)	0.745(0.036)	0.601(0.000)	0.261(0.026)	0.174(0.029)	0.075(0.011)	0.313(0.082)	0.291(0.040)	0.155(0.008)
LT_1	0.203(0.155)	0.130(0.084)	0.031(0.022)	0.236(0.207)	0.154(0.139)	0.018(0.035)	0.083(0.115)	0.017(0.039)	0.009(0.009)	0.080(0.099)	0.058(0.059)	0.026(0.014)
LT_2	0.092(0.121)	0.044(0.024)	0.000(0.000)	0.158(0.137)	0.088(0.103)	0.044(0.029)	0.039(0.056)	0.018(0.024)	0.009(0.007)	0.101(0.087)	0.017(0.033)	0.000(0.000)
LT_3	0.193(0.149)	0.119(0.080)	0.028(0.023)	0.236(0.211)	0.149(0.121)	0.021(0.030)	0.055(0.098)	0.017(0.029)	0.007(0.014)	0.080(0.094)	0.059(0.062)	0.029(0.010)
HLT_1	0.705(0.068)	0.611(0.045)	0.512(0.013)	0.800(0.065)	0.736(0.037)	0.582(0.000)	0.227(0.033)	0.163(0.029)	0.073(0.000)	0.314(0.080)	0.273(0.036)	0.150(0.013)
HLT_2	0.731(0.088)	0.652(0.060)	0.527(0.014)	0.819(0.069)	0.760(0.047)	0.619(0.009)	0.204(0.043)	0.149(0.024)	0.087(0.012)	0.342(0.076)	0.280(0.037)	0.132(0.006)
HLT_3	0.692(0.081)	0.606(0.056)	0.507(0.017)	0.825(0.056)	0.740(0.043)	0.584(0.000)	0.233(0.034)	0.140(0.030)	0.074(0.007)	0.306(0.084)	0.274(0.035)	0.148(0.005)
LBI_1	0.199(0.112)	0.131(0.078)	0.061(0.010)	0.243(0.110)	0.189(0.056)	0.086(0.000)	0.081(0.040)	0.051(0.005)	0.006(0.000)	0.104(0.089)	0.062(0.027)	0.022(0.009)
LBI_2	0.125(0.097)	0.104(0.053)	0.052(0.012)	0.240(0.121)	0.143(0.063)	0.062(0.021)	0.061(0.053)	0.026(0.023)	0.008(0.009)	0.092(0.071)	0.069(0.052)	0.020(0.005)
LBI_3	0.145(0.099)	0.102(0.047)	0.047(0.005)	0.228(0.118)	0.146(0.065)	0.071(0.008)	0.052(0.051)	0.034(0.024)	0.014(0.013)	0.092(0.065)	0.068(0.063)	0.021(0.005)
$LKSN$	1.000(0.990)	1.000(0.946)	1.000(0.907)	1.000(0.985)	1.000(0.935)	1.000(0.896)	1.000(0.992)	1.000(0.957)	1.000(0.892)	1.000(0.987)	1.000(0.979)	1.000(0.934)
L	0.000(0.136)	0.000(0.085)	0.000(0.043)	0.000(0.143)	0.000(0.061)	0.000(0.016)	0.000(0.115)	0.000(0.031)	0.000(0.010)	0.000(0.075)	0.000(0.045)	0.000(0.047)

α	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.257(0.073)	0.189(0.039)	0.111(0.000)	0.479(0.021)	0.404(0.000)	0.208(0.000)	0.446(0.067)	0.369(0.037)	0.184(0.000)	0.643(0.025)	0.497(0.027)	0.364(0.008)
K_2	0.345(0.082)	0.237(0.017)	0.096(0.000)	0.536(0.008)	0.417(0.006)	0.264(0.000)	0.453(0.071)	0.385(0.017)	0.190(0.000)	0.666(0.045)	0.521(0.016)	0.389(0.007)
K_3	0.278(0.065)	0.191(0.042)	0.106(0.000)	0.477(0.021)	0.413(0.000)	0.214(0.000)	0.445(0.066)	0.358(0.044)	0.175(0.000)	0.643(0.033)	0.475(0.028)	0.350(0.019)
LT_1	0.088(0.112)	0.069(0.065)	0.022(0.024)	0.112(0.107)	0.092(0.069)	0.020(0.005)	0.087(0.115)	0.050(0.064)	0.005(0.018)	0.252(0.195)	0.143(0.107)	0.041(0.024)
LT_2	0.072(0.077)	0.062(0.061)	0.038(0.044)	0.044(0.060)	0.026(0.039)	0.014(0.010)	0.088(0.125)	0.026(0.048)	0.000(0.000)	0.153(0.132)	0.045(0.037)	0.023(0.009)
LT_3	0.092(0.101)	0.066(0.073)	0.017(0.015)	0.096(0.108)	0.086(0.072)	0.018(0.006)	0.076(0.141)	0.052(0.059)	0.008(0.016)	0.247(0.198)	0.129(0.101)	0.043(0.018)
HLT_1	0.237(0.072)	0.184(0.041)	0.103(0.000)	0.470(0.020)	0.390(0.000)	0.197(0.000)	0.436(0.068)	0.363(0.025)	0.174(0.000)	0.607(0.026)	0.475(0.026)	0.337(0.008)
HLT_2	0.317(0.104)	0.243(0.030)	0.095(0.000)	0.540(0.006)	0.407(0.000)	0.244(0.000)	0.439(0.058)	0.375(0.014)	0.176(0.000)	0.634(0.044)	0.503(0.034)	0.393(0.007)
HLT_3	0.264(0.065)	0.186(0.038)	0.099(0.000)	0.478(0.021)	0.402(0.000)	0.202(0.000)	0.449(0.072)	0.360(0.030)	0.166(0.000)	0.625(0.031)	0.474(0.028)	0.339(0.021)
LBI_1	0.114(0.097)	0.081(0.047)	0.026(0.011)	0.072(0.095)	0.053(0.046)	0.035(0.014)	0.133(0.115)	0.111(0.050)	0.027(0.011)	0.119(0.066)	0.104(0.041)	0.060(0.012)
LBI_2	0.120(0.062)	0.033(0.027)	0.000(0.000)	0.056(0.053)	0.061(0.034)	0.021(0.007)	0.177(0.137)	0.092(0.076)	0.030(0.011)	0.114(0.072)	0.087(0.044)	0.034(0.022)
LBI_3	0.114(0.065)	0.029(0.016)	0.011(0.000)	0.060(0.055)	0.055(0.033)	0.024(0.007)	0.182(0.154)	0.096(0.067)	0.033(0.005)	0.112(0.071)	0.092(0.038)	0.031(0.022)
$LKSN$	1.000(0.993)	1.000(0.962)	1.000(0.906)	1.000(0.954)	1.000(0.921)	1.000(0.876)	1.000(0.982)	1.000(0.907)	1.000(0.914)	1.000(1.000)	1.000(0.956)	1.000(0.917)
L	0.000(0.081)	0.000(0.022)	0.000(0.017)	0.000(0.060)	0.000(0.026)	0.000(0.000)	0.000(0.102)	0.000(0.055)	0.000(0.022)	0.000(0.103)	0.000(0.072)	0.000(0.006)

Table 2.3: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_0}$ with $\mathbf{T=100}$.

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$ 10%	5%	1%
K_1	0.167(0.019)	0.111(0.000)	0.023(0.000)	0.139(0.100)	0.057(0.046)	0.034(0.000)	0.368(0.025)	0.268(0.054)	0.116(0.000)	0.403(0.091)	0.295(0.014)	0.203(0.000)
K_2	0.130(0.112)	0.081(0.006)	0.000(0.000)	0.216(0.092)	0.117(0.016)	0.000(0.000)	0.435(0.031)	0.252(0.008)	0.226(0.002)	0.365(0.034)	0.356(0.011)	0.203(0.000)
K_3	0.139(0.083)	0.059(0.025)	0.010(0.000)	0.125(0.027)	0.069(0.000)	0.000(0.000)	0.315(0.077)	0.253(0.000)	0.158(0.000)	0.430(0.106)	0.303(0.000)	0.182(0.000)
LT_1	0.042(0.126)	0.000(0.088)	0.000(0.000)	0.035(0.094)	0.000(0.017)	0.016(0.036)	0.124(0.073)	0.084(0.118)	0.032(0.000)	0.143(0.091)	0.066(0.045)	0.060(0.069)
LT_2	0.122(0.124)	0.051(0.000)	0.007(0.014)	0.110(0.102)	0.016(0.052)	0.040(0.039)	0.095(0.132)	0.031(0.053)	0.041(0.019)	0.095(0.088)	0.076(0.086)	0.063(0.000)
LT_3	0.015(0.119)	0.000(0.020)	0.000(0.000)	0.036(0.096)	0.004(0.000)	0.013(0.000)	0.133(0.114)	0.102(0.092)	0.000(0.055)	0.110(0.072)	0.084(0.076)	0.004(0.011)
HLT_1	0.118(0.079)	0.032(0.053)	0.000(0.000)	0.130(0.087)	0.099(0.005)	0.036(0.050)	0.317(0.013)	0.183(0.027)	0.094(0.000)	0.437(0.097)	0.304(0.031)	0.219(0.000)
HLT_2	0.104(0.044)	0.042(0.000)	0.031(0.000)	0.192(0.061)	0.051(0.000)	0.064(0.000)	0.361(0.060)	0.280(0.000)	0.172(0.000)	0.407(0.106)	0.263(0.080)	0.123(0.035)
HLT_3	0.164(0.030)	0.034(0.001)	0.048(0.033)	0.122(0.098)	0.076(0.011)	0.000(0.019)	0.387(0.031)	0.199(0.000)	0.094(0.032)	0.443(0.131)	0.278(0.000)	0.150(0.000)
LBI_1	0.104(0.101)	0.094(0.060)	0.016(0.019)	0.156(0.226)	0.150(0.132)	0.028(0.041)	0.112(0.059)	0.064(0.058)	0.054(0.000)	0.125(0.048)	0.000(0.032)	0.055(0.039)
LBI_2	0.146(0.123)	0.073(0.019)	0.002(0.023)	0.136(0.141)	0.149(0.115)	0.036(0.013)	0.151(0.083)	0.034(0.070)	0.014(0.000)	0.067(0.075)	0.039(0.050)	0.000(0.000)
LBI_3	0.136(0.082)	0.038(0.101)	0.000(0.040)	0.143(0.182)	0.108(0.059)	0.000(0.041)	0.117(0.086)	0.054(0.006)	0.076(0.018)	0.052(0.083)	0.020(0.031)	0.004(0.052)
$LKSN$	1.000(1.000)	1.000(0.956)	0.992(0.858)	0.985(0.992)	1.000(0.947)	0.986(0.882)	1.000(1.000)	0.966(0.989)	0.969(0.907)	0.969(0.960)	1.000(0.914)	0.981(0.883)
L	0.023(0.107)	0.007(0.000)	0.008(0.007)	0.026(0.137)	0.040(0.021)	0.000(0.054)	0.049(0.133)	0.023(0.032)	0.019(0.000)	0.000(0.080)	0.037(0.032)	0.002(0.000)

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$ 10%	5%	1%
K_1	0.715(0.073)	0.627(0.094)	0.500(0.056)	0.850(0.050)	0.783(0.071)	0.598(0.000)	0.270(0.048)	0.190(0.005)	0.089(0.045)	0.359(0.063)	0.308(0.051)	0.105(0.000)
K_2	0.727(0.106)	0.661(0.096)	0.553(0.000)	0.876(0.068)	0.756(0.058)	0.630(0.034)	0.189(0.080)	0.122(0.042)	0.100(0.057)	0.370(0.062)	0.284(0.093)	0.096(0.000)
K_3	0.736(0.053)	0.637(0.071)	0.528(0.031)	0.827(0.037)	0.791(0.070)	0.566(0.000)	0.264(0.025)	0.197(0.030)	0.035(0.000)	0.322(0.043)	0.283(0.028)	0.179(0.001)
LT_1	0.215(0.178)	0.123(0.050)	0.035(0.038)	0.242(0.191)	0.117(0.113)	0.036(0.059)	0.053(0.161)	0.015(0.052)	0.007(0.042)	0.053(0.072)	0.096(0.085)	0.046(0.049)
LT_2	0.060(0.136)	0.007(0.064)	0.000(0.030)	0.124(0.141)	0.109(0.095)	0.016(0.024)	0.009(0.017)	0.055(0.006)	0.000(0.046)	0.081(0.052)	0.000(0.019)	0.011(0.000)
LT_3	0.224(0.145)	0.093(0.093)	0.033(0.000)	0.265(0.241)	0.131(0.119)	0.010(0.046)	0.015(0.086)	0.037(0.015)	0.008(0.026)	0.123(0.118)	0.097(0.102)	0.018(0.009)
HLT_1	0.739(0.038)	0.592(0.019)	0.560(0.000)	0.799(0.055)	0.727(0.009)	0.619(0.000)	0.226(0.052)	0.193(0.015)	0.121(0.033)	0.281(0.102)	0.238(0.037)	0.155(0.000)
HLT_2	0.694(0.096)	0.671(0.020)	0.515(0.045)	0.830(0.087)	0.809(0.086)	0.589(0.048)	0.172(0.047)	0.167(0.061)	0.133(0.035)	0.312(0.088)	0.285(0.048)	0.148(0.038)
HLT_3	0.693(0.083)	0.615(0.056)	0.502(0.028)	0.792(0.073)	0.773(0.041)	0.579(0.000)	0.217(0.017)	0.140(0.076)	0.063(0.044)	0.289(0.080)	0.313(0.063)	0.138(0.047)
LBI_1	0.212(0.099)	0.120(0.051)	0.070(0.041)	0.291(0.126)	0.203(0.042)	0.131(0.016)	0.057(0.068)	0.031(0.001)	0.038(0.017)	0.151(0.107)	0.027(0.017)	0.021(0.004)
LBI_2	0.148(0.092)	0.098(0.089)	0.035(0.000)	0.248(0.154)	0.168(0.027)	0.074(0.025)	0.028(0.079)	0.000(0.069)	0.010(0.000)	0.135(0.034)	0.110(0.092)	0.018(0.025)
LBI_3	0.179(0.147)	0.101(0.082)	0.085(0.029)	0.273(0.094)	0.185(0.068)	0.105(0.000)	0.054(0.011)	0.078(0.071)	0.000(0.000)	0.063(0.096)	0.107(0.070)	0.000(0.033)
$LKSN$	0.988(1.000)	1.000(0.971)	1.000(0.941)	0.962(0.992)	1.000(0.895)	0.986(0.906)	1.000(0.989)	0.997(0.930)	0.987(0.942)	0.960(0.959)	0.979(1.000)	0.972(0.900)
L	0.014(0.123)	0.019(0.076)	0.000(0.019)	0.040(0.182)	0.045(0.070)	0.033(0.001)	0.020(0.077)	0.000(0.000)	0.000(0.043)	0.000(0.077)	0.021(0.077)	0.006(0.068)

α	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$ 10%	5%	1%	$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$ 10%	5%	1%	$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$ 10%	5%	1%	$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$ 10%	5%	1%
K_1	0.305(0.087)	0.203(0.002)	0.116(0.000)	0.472(0.013)	0.382(0.000)	0.207(0.008)	0.417(0.101)	0.334(0.046)	0.231(0.000)	0.660(0.000)	0.477(0.003)	0.410(0.000)
K_2	0.380(0.054)	0.222(0.000)	0.072(0.028)	0.569(0.010)	0.378(0.026)	0.311(0.042)	0.416(0.081)	0.354(0.042)	0.188(0.000)	0.635(0.091)	0.491(0.056)	0.421(0.051)
K_3	0.308(0.095)	0.166(0.022)	0.117(0.000)	0.513(0.000)	0.398(0.050)	0.207(0.024)	0.461(0.041)	0.376(0.086)	0.182(0.000)	0.674(0.067)	0.525(0.056)	0.344(0.062)
LT_1	0.067(0.107)	0.070(0.078)	0.027(0.039)	0.129(0.068)	0.136(0.068)	0.006(0.039)	0.092(0.078)	0.074(0.044)	0.009(0.018)	0.262(0.217)	0.157(0.068)	0.081(0.000)
LT_2	0.036(0.073)	0.083(0.085)	0.029(0.056)	0.075(0.066)	0.051(0.082)	0.012(0.020)	0.098(0.127)	0.000(0.053)	0.026(0.022)	0.202(0.153)	0.040(0.049)	0.071(0.042)
LT_3	0.129(0.141)	0.082(0.094)	0.022(0.037)	0.101(0.154)	0.073(0.080)	0.007(0.030)	0.091(0.151)	0.025(0.061)	0.000(0.000)	0.218(0.182)	0.093(0.113)	0.057(0.017)
HLT_1	0.203(0.091)	0.189(0.039)	0.144(0.000)	0.458(0.059)	0.431(0.046)	0.201(0.049)	0.462(0.041)	0.367(0.069)	0.223(0.000)	0.639(0.038)	0.488(0.064)	0.324(0.004)
HLT_2	0.299(0.135)	0.285(0.060)	0.114(0.007)	0.523(0.000)	0.396(0.030)	0.281(0.039)	0.425(0.068)	0.401(0.057)	0.206(0.000)	0.663(0.041)	0.530(0.074)	0.404(0.018)
HLT_3	0.284(0.060)	0.152(0.038)	0.134(0.037)	0.460(0.000)	0.393(0.000)	0.246(0.000)	0.495(0.113)	0.373(0.062)	0.198(0.000)	0.585(0.046)	0.508(0.051)	0.373(0.048)
LBI_1	0.109(0.133)	0.068(0.065)	0.051(0.000)	0.084(0.095)	0.083(0.041)	0.070(0.033)	0.156(0.105)	0.124(0.099)	0.035(0.040)	0.141(0.086)	0.083(0.045)	0.038(0.000)
LBI_2	0.129(0.068)	0.004(0.017)	0.000(0.025)	0.093(0.038)	0.089(0.015)	0.021(0.005)	0.201(0.106)	0.142(0.078)	0.041(0.000)	0.150(0.120)	0.085(0.082)	0.083(0.068)
LBI_3	0.120(0.072)	0.031(0.065)	0.000(0.000)	0.054(0.054)	0.074(0.028)	0.005(0.053)	0.179(0.170)	0.071(0.057)	0.070(0.051)	0.130(0.040)	0.136(0.019)	0.021(0.000)
$LKSN$	1.000(1.000)	1.000(0.928)	0.985(0.919)	1.000(0.953)	1.000(0.915)	0.996(0.840)	0.968(1.000)	1.000(0.923)	0.984(0.962)	0.973(0.993)	1.000(0.934)	1.000(0.964)
L	0.045(0.084)	0.041(0.033)	0.048(0.000)	0.024(0.069)	0.019(0.059)	0.050(0.013)	0.015(0.143)	0.016(0.024)	0.000(0.000)	0.000(0.107)	0.000(0.086)	0.021(0.045)

Table 2.4: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_0}$ with $\mathbf{T=250}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.877(0.732)	0.832(0.630)	0.726(0.428)	0.829(0.618)	0.791(0.513)	0.662(0.350)	0.819(0.639)	0.781(0.531)	0.670(0.411)	0.811(0.579)	0.761(0.493)	0.673(0.399)
K_2	0.796(0.588)	0.711(0.496)	0.552(0.332)	0.804(0.544)	0.734(0.397)	0.584(0.232)	0.765(0.506)	0.694(0.474)	0.588(0.338)	0.785(0.499)	0.733(0.441)	0.580(0.337)
K_3	0.877(0.735)	0.842(0.632)	0.730(0.433)	0.838(0.618)	0.800(0.506)	0.672(0.353)	0.837(0.644)	0.787(0.545)	0.666(0.414)	0.822(0.579)	0.770(0.492)	0.667(0.398)
LT_1	0.286(0.292)	0.220(0.200)	0.086(0.071)	0.318(0.317)	0.232(0.225)	0.076(0.085)	0.301(0.278)	0.185(0.195)	0.081(0.087)	0.290(0.348)	0.183(0.188)	0.087(0.098)
LT_2	0.221(0.191)	0.131(0.120)	0.067(0.029)	0.227(0.141)	0.098(0.067)	0.016(0.005)	0.166(0.128)	0.106(0.097)	0.042(0.023)	0.230(0.160)	0.134(0.057)	0.015(0.018)
LT_3	0.285(0.293)	0.223(0.196)	0.101(0.084)	0.310(0.321)	0.229(0.192)	0.076(0.089)	0.313(0.273)	0.160(0.201)	0.082(0.102)	0.306(0.290)	0.208(0.192)	0.101(0.092)
HLT_1	0.129(0.034)	0.086(0.022)	0.048(0.012)	0.154(0.056)	0.102(0.044)	0.051(0.000)	0.207(0.099)	0.135(0.087)	0.105(0.062)	0.269(0.099)	0.230(0.076)	0.160(0.035)
HLT_2	0.145(0.040)	0.104(0.027)	0.064(0.008)	0.111(0.045)	0.090(0.032)	0.048(0.000)	0.187(0.081)	0.135(0.074)	0.084(0.025)	0.257(0.067)	0.220(0.071)	0.130(0.026)
HLT_3	0.102(0.017)	0.086(0.024)	0.041(0.006)	0.188(0.095)	0.132(0.079)	0.089(0.053)	0.259(0.169)	0.207(0.164)	0.168(0.129)	0.347(0.193)	0.309(0.158)	0.233(0.140)
LBI_1	0.986(0.990)	0.989(0.986)	0.985(0.970)	1.000(1.000)	0.991(0.987)	0.987(0.986)	1.000(1.000)	1.000(0.985)	0.993(0.989)	0.993(0.993)	0.982(0.985)	0.978(0.957)
LBI_2	0.994(0.985)	0.986(0.990)	0.993(0.993)	1.000(1.000)	0.991(1.000)	0.985(0.983)	1.000(1.000)	1.000(1.000)	0.988(0.969)	0.987(0.990)	0.979(0.985)	0.984(0.965)
LBI_3	0.990(0.994)	0.992(0.993)	0.989(0.980)	1.000(1.000)	0.990(0.987)	0.993(0.978)	1.000(1.000)	1.000(1.000)	0.994(0.993)	0.993(0.991)	0.977(0.989)	0.977(0.970)
$LKSN$	0.129(0.000)	0.093(0.000)	0.021(0.000)	0.096(0.000)	0.043(0.000)	0.010(0.000)	0.146(0.000)	0.057(0.000)	0.032(0.000)	0.272(0.000)	0.137(0.000)	0.082(0.000)
L	0.134(0.092)	0.030(0.041)	0.000(0.023)	0.131(0.096)	0.066(0.049)	0.007(0.009)	0.142(0.060)	0.074(0.034)	0.011(0.021)	0.090(0.091)	0.050(0.050)	0.000(0.022)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.883(0.706)	0.880(0.682)	0.857(0.543)	0.915(0.681)	0.896(0.614)	0.873(0.492)	0.882(0.696)	0.850(0.617)	0.763(0.481)	0.788(0.574)	0.716(0.427)	0.635(0.329)
K_2	0.882(0.673)	0.864(0.649)	0.795(0.511)	0.895(0.662)	0.900(0.573)	0.853(0.450)	0.822(0.633)	0.755(0.501)	0.600(0.361)	0.773(0.506)	0.718(0.432)	0.606(0.320)
K_3	0.880(0.722)	0.883(0.693)	0.858(0.559)	0.910(0.684)	0.902(0.616)	0.865(0.503)	0.897(0.707)	0.846(0.620)	0.745(0.476)	0.793(0.567)	0.719(0.425)	0.657(0.325)
LT_1	0.291(0.277)	0.231(0.229)	0.098(0.099)	0.237(0.285)	0.122(0.132)	0.042(0.031)	0.285(0.336)	0.202(0.202)	0.072(0.046)	0.264(0.270)	0.172(0.183)	0.046(0.068)
LT_2	0.213(0.172)	0.109(0.087)	0.013(0.019)	0.145(0.110)	0.060(0.046)	0.028(0.022)	0.199(0.193)	0.119(0.071)	0.032(0.000)	0.227(0.185)	0.146(0.130)	0.035(0.011)
LT_3	0.290(0.268)	0.229(0.242)	0.102(0.102)	0.257(0.266)	0.125(0.120)	0.037(0.027)	0.318(0.335)	0.185(0.191)	0.068(0.037)	0.258(0.260)	0.164(0.166)	0.045(0.080)
HLT_1	0.417(0.197)	0.391(0.138)	0.295(0.111)	0.436(0.139)	0.374(0.098)	0.259(0.059)	0.178(0.089)	0.146(0.083)	0.093(0.045)	0.182(0.079)	0.151(0.059)	0.092(0.000)
HLT_2	0.438(0.198)	0.393(0.143)	0.280(0.078)	0.518(0.183)	0.431(0.108)	0.314(0.075)	0.209(0.114)	0.189(0.056)	0.113(0.043)	0.262(0.091)	0.207(0.046)	0.141(0.000)
HLT_3	0.539(0.337)	0.500(0.287)	0.453(0.259)	0.506(0.236)	0.418(0.230)	0.320(0.217)	0.248(0.152)	0.199(0.131)	0.132(0.113)	0.242(0.095)	0.188(0.069)	0.125(0.052)
LBI_1	0.990(0.988)	0.983(0.976)	0.969(0.933)	0.987(0.994)	0.988(0.975)	0.988(0.912)	1.000(1.000)	0.988(0.989)	0.980(0.949)	1.000(1.000)	0.993(1.000)	0.985(0.955)
LBI_2	0.987(1.000)	0.982(0.971)	0.945(0.949)	0.994(0.989)	0.987(0.971)	0.989(0.913)	1.000(1.000)	0.992(0.994)	0.981(0.980)	1.000(1.000)	0.987(1.000)	0.986(0.975)
LBI_3	0.987(1.000)	0.982(0.971)	0.952(0.947)	0.993(0.991)	0.985(0.982)	0.986(0.896)	1.000(1.000)	0.992(0.989)	0.976(0.982)	1.000(1.000)	0.989(1.000)	0.988(0.975)
$LKSN$	0.258(0.000)	0.192(0.000)	0.080(0.000)	0.244(0.000)	0.122(0.000)	0.049(0.000)	0.183(0.000)	0.106(0.000)	0.042(0.000)	0.188(0.012)	0.145(0.009)	0.084(0.007)
L	0.137(0.096)	0.038(0.046)	0.013(0.014)	0.177(0.085)	0.126(0.055)	0.042(0.011)	0.116(0.095)	0.052(0.052)	0.012(0.013)	0.107(0.081)	0.020(0.052)	0.000(0.016)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.814(0.660)	0.800(0.549)	0.685(0.375)	0.875(0.689)	0.841(0.642)	0.761(0.505)	0.808(0.636)	0.762(0.573)	0.694(0.364)	0.849(0.713)	0.838(0.659)	0.809(0.497)
K_2	0.804(0.582)	0.748(0.460)	0.629(0.214)	0.829(0.632)	0.786(0.601)	0.678(0.442)	0.794(0.528)	0.735(0.459)	0.578(0.326)	0.864(0.645)	0.820(0.583)	0.752(0.427)
K_3	0.807(0.662)	0.799(0.552)	0.694(0.378)	0.894(0.675)	0.839(0.636)	0.762(0.511)	0.817(0.663)	0.759(0.573)	0.686(0.363)	0.874(0.713)	0.843(0.660)	0.814(0.500)
LT_1	0.243(0.252)	0.186(0.188)	0.027(0.051)	0.311(0.298)	0.221(0.235)	0.118(0.114)	0.222(0.208)	0.143(0.139)	0.089(0.067)	0.166(0.242)	0.143(0.125)	0.089(0.065)
LT_2	0.181(0.119)	0.084(0.046)	0.010(0.000)	0.203(0.167)	0.138(0.100)	0.028(0.011)	0.177(0.139)	0.140(0.091)	0.022(0.030)	0.084(0.062)	0.028(0.021)	0.013(0.016)
LT_3	0.224(0.217)	0.185(0.178)	0.037(0.048)	0.311(0.289)	0.222(0.220)	0.122(0.112)	0.243(0.198)	0.135(0.153)	0.096(0.091)	0.175(0.209)	0.139(0.134)	0.090(0.059)
HLT_1	0.148(0.056)	0.119(0.046)	0.052(0.000)	0.346(0.220)	0.310(0.169)	0.248(0.130)	0.217(0.066)	0.169(0.054)	0.110(0.019)	0.356(0.166)	0.305(0.120)	0.229(0.036)
HLT_2	0.140(0.049)	0.091(0.022)	0.050(0.012)	0.368(0.178)	0.332(0.139)	0.218(0.086)	0.177(0.052)	0.142(0.042)	0.109(0.000)	0.329(0.138)	0.260(0.097)	0.211(0.045)
HLT_3	0.181(0.109)	0.147(0.080)	0.104(0.047)	0.414(0.264)	0.353(0.221)	0.291(0.188)	0.285(0.149)	0.195(0.138)	0.157(0.123)	0.469(0.290)	0.410(0.221)	0.302(0.189)
LBI_1	1.000(1.000)	1.000(0.994)	0.988(0.992)	1.000(1.000)	1.000(1.000)	0.993(0.960)	0.988(0.993)	0.985(0.989)	0.992(0.987)	1.000(1.000)	1.000(1.000)	0.972(0.938)
LBI_2	1.000(1.000)	1.000(1.000)	0.993(0.971)	1.000(1.000)	0.989(0.991)	0.984(0.966)	0.990(0.990)	0.987(0.989)	0.992(0.973)	1.000(1.000)	1.000(1.000)	0.981(0.948)
LBI_3	1.000(1.000)	1.000(1.000)	1.000(0.978)	1.000(1.000)	0.989(0.991)	0.977(0.965)	0.989(0.992)	0.994(0.991)	0.988(0.978)	1.000(1.000)	1.000(1.000)	0.983(0.946)
$LKSN$	0.121(0.000)	0.023(0.000)	0.013(0.000)	0.289(0.000)	0.199(0.000)	0.076(0.000)	0.162(0.000)	0.072(0.000)	0.006(0.000)	0.311(0.006)	0.178(0.000)	0.092(0.000)
L	0.067(0.097)	0.047(0.052)	0.000(0.015)	0.094(0.097)	0.048(0.055)	0.000(0.001)	0.087(0.095)	0.033(0.068)	0.000(0.020)	0.114(0.087)	0.049(0.059)	0.010(0.001)

Table 2.5: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_1}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.918(0.731)	0.799(0.608)	0.696(0.441)	0.819(0.660)	0.827(0.555)	0.671(0.362)	0.819(0.635)	0.780(0.578)	0.681(0.441)	0.820(0.605)	0.765(0.488)	0.714(0.406)
K_2	0.769(0.600)	0.708(0.478)	0.563(0.303)	0.776(0.586)	0.770(0.380)	0.587(0.263)	0.763(0.468)	0.660(0.464)	0.594(0.349)	0.776(0.546)	0.713(0.455)	0.582(0.321)
K_3	0.839(0.740)	0.826(0.674)	0.694(0.472)	0.829(0.592)	0.832(0.524)	0.683(0.346)	0.861(0.687)	0.802(0.539)	0.670(0.413)	0.869(0.590)	0.771(0.540)	0.690(0.408)
LT_1	0.270(0.295)	0.206(0.182)	0.110(0.116)	0.313(0.295)	0.221(0.214)	0.082(0.126)	0.346(0.257)	0.214(0.210)	0.084(0.123)	0.285(0.323)	0.153(0.156)	0.122(0.114)
LT_2	0.242(0.169)	0.137(0.094)	0.113(0.021)	0.221(0.160)	0.072(0.064)	0.015(0.008)	0.154(0.174)	0.093(0.136)	0.055(0.060)	0.270(0.160)	0.101(0.063)	0.000(0.038)
LT_3	0.321(0.269)	0.253(0.174)	0.078(0.049)	0.291(0.324)	0.214(0.169)	0.092(0.099)	0.299(0.307)	0.144(0.201)	0.100(0.098)	0.281(0.281)	0.172(0.154)	0.102(0.097)
HLT_1	0.130(0.075)	0.131(0.017)	0.093(0.019)	0.183(0.022)	0.101(0.004)	0.054(0.000)	0.185(0.119)	0.115(0.097)	0.132(0.067)	0.309(0.088)	0.220(0.110)	0.180(0.049)
HLT_2	0.133(0.087)	0.090(0.014)	0.064(0.000)	0.109(0.094)	0.073(0.000)	0.084(0.000)	0.179(0.082)	0.167(0.064)	0.070(0.075)	0.299(0.088)	0.258(0.119)	0.137(0.031)
HLT_3	0.114(0.046)	0.099(0.042)	0.024(0.000)	0.186(0.136)	0.173(0.053)	0.074(0.020)	0.267(0.200)	0.202(0.201)	0.204(0.122)	0.338(0.196)	0.350(0.187)	0.223(0.183)
LBI_1	1.000(0.956)	1.000(0.993)	0.958(0.941)	1.000(1.000)	0.961(0.966)	0.974(0.974)	1.000(1.000)	1.000(0.991)	1.000(0.956)	0.954(0.974)	1.000(0.951)	1.000(0.919)
LBI_2	0.964(0.985)	1.000(0.954)	0.990(0.973)	0.997(1.000)	1.000(1.000)	0.983(1.000)	1.000(0.967)	1.000(1.000)	0.989(0.930)	0.997(0.982)	1.000(0.972)	1.000(0.945)
LBI_3	1.000(1.000)	1.000(0.979)	0.983(0.971)	1.000(1.000)	1.000(1.000)	0.966(0.943)	0.978(0.993)	0.964(0.963)	1.000(0.966)	1.000(1.000)	1.000(0.982)	0.977(0.978)
$LKSN$	0.127(0.019)	0.133(0.000)	0.024(0.022)	0.093(0.003)	0.087(0.030)	0.044(0.046)	0.140(0.043)	0.092(0.047)	0.055(0.007)	0.292(0.000)	0.143(0.030)	0.127(0.000)
L	0.166(0.088)	0.000(0.081)	0.000(0.025)	0.106(0.132)	0.048(0.066)	0.037(0.000)	0.118(0.077)	0.107(0.053)	0.015(0.071)	0.091(0.136)	0.095(0.048)	0.000(0.021)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.900(0.706)	0.929(0.713)	0.869(0.570)	0.908(0.678)	0.940(0.616)	0.876(0.482)	0.889(0.720)	0.884(0.607)	0.756(0.511)	0.781(0.591)	0.733(0.415)	0.614(0.331)
K_2	0.892(0.639)	0.847(0.675)	0.804(0.534)	0.877(0.698)	0.906(0.543)	0.842(0.437)	0.860(0.644)	0.759(0.539)	0.568(0.356)	0.782(0.537)	0.747(0.440)	0.572(0.359)
K_3	0.853(0.727)	0.931(0.739)	0.900(0.601)	0.923(0.715)	0.911(0.597)	0.850(0.525)	0.934(0.723)	0.828(0.626)	0.736(0.501)	0.806(0.605)	0.766(0.440)	0.683(0.358)
LT_1	0.329(0.263)	0.238(0.255)	0.089(0.119)	0.250(0.313)	0.108(0.153)	0.004(0.043)	0.306(0.366)	0.183(0.187)	0.041(0.050)	0.296(0.318)	0.189(0.218)	0.031(0.103)
LT_2	0.234(0.211)	0.111(0.101)	0.059(0.017)	0.127(0.148)	0.098(0.022)	0.000(0.000)	0.226(0.169)	0.107(0.034)	0.025(0.044)	0.219(0.219)	0.176(0.133)	0.000(0.037)
LT_3	0.327(0.249)	0.223(0.280)	0.140(0.142)	0.272(0.311)	0.109(0.098)	0.083(0.048)	0.349(0.320)	0.178(0.177)	0.041(0.030)	0.222(0.304)	0.156(0.152)	0.064(0.102)
HLT_1	0.377(0.181)	0.391(0.157)	0.270(0.113)	0.467(0.114)	0.406(0.060)	0.250(0.054)	0.153(0.078)	0.115(0.050)	0.092(0.073)	0.181(0.059)	0.160(0.029)	0.077(0.000)
HLT_2	0.417(0.235)	0.432(0.161)	0.316(0.093)	0.496(0.144)	0.424(0.136)	0.303(0.112)	0.179(0.107)	0.190(0.080)	0.124(0.021)	0.243(0.086)	0.238(0.070)	0.147(0.043)
HLT_3	0.561(0.382)	0.535(0.256)	0.497(0.278)	0.481(0.207)	0.468(0.204)	0.281(0.226)	0.260(0.117)	0.166(0.109)	0.144(0.108)	0.273(0.081)	0.151(0.057)	0.108(0.034)
LBI_1	0.979(1.000)	0.958(1.000)	0.955(0.962)	0.956(0.972)	0.991(1.000)	1.000(0.928)	1.000(1.000)	1.000(0.971)	0.992(0.984)	0.961(0.990)	0.964(1.000)	0.976(0.966)
LBI_2	0.973(0.994)	1.000(0.938)	0.960(0.969)	0.968(0.965)	1.000(0.939)	1.000(0.907)	1.000(1.000)	0.976(0.985)	1.000(0.981)	1.000(1.000)	0.953(1.000)	1.000(0.987)
LBI_3	0.954(1.000)	0.996(0.940)	0.933(0.984)	0.959(1.000)	0.965(1.000)	0.955(0.924)	1.000(1.000)	0.991(0.949)	1.000(0.966)	1.000(1.000)	0.968(0.971)	1.000(1.000)
$LKSN$	0.268(0.000)	0.174(0.015)	0.083(0.000)	0.224(0.048)	0.128(0.031)	0.086(0.010)	0.159(0.000)	0.076(0.000)	0.019(0.016)	0.225(0.000)	0.131(0.000)	0.126(0.029)
L	0.101(0.099)	0.084(0.021)	0.001(0.038)	0.142(0.113)	0.087(0.085)	0.020(0.041)	0.128(0.097)	0.068(0.036)	0.007(0.043)	0.080(0.042)	0.031(0.052)	0.000(0.025)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.814(0.709)	0.847(0.566)	0.712(0.399)	0.838(0.654)	0.890(0.643)	0.792(0.528)	0.807(0.633)	0.784(0.565)	0.740(0.363)	0.885(0.675)	0.807(0.689)	0.830(0.541)
K_2	0.838(0.590)	0.795(0.460)	0.647(0.222)	0.854(0.626)	0.756(0.580)	0.657(0.409)	0.778(0.539)	0.706(0.469)	0.564(0.359)	0.832(0.658)	0.838(0.586)	0.762(0.455)
K_3	0.775(0.627)	0.761(0.578)	0.713(0.343)	0.923(0.718)	0.847(0.655)	0.770(0.554)	0.835(0.633)	0.723(0.622)	0.679(0.337)	0.843(0.752)	0.874(0.690)	0.820(0.544)
LT_1	0.256(0.250)	0.185(0.173)	0.022(0.082)	0.282(0.312)	0.269(0.247)	0.106(0.164)	0.204(0.229)	0.184(0.110)	0.108(0.033)	0.177(0.218)	0.144(0.168)	0.126(0.043)
LT_2	0.177(0.096)	0.054(0.074)	0.000(0.010)	0.247(0.158)	0.152(0.127)	0.000(0.034)	0.227(0.139)	0.152(0.073)	0.011(0.044)	0.130(0.050)	0.032(0.000)	0.044(0.003)
LT_3	0.235(0.215)	0.219(0.228)	0.073(0.027)	0.345(0.334)	0.263(0.212)	0.144(0.153)	0.284(0.213)	0.118(0.167)	0.092(0.100)	0.158(0.218)	0.152(0.174)	0.090(0.031)
HLT_1	0.162(0.068)	0.136(0.077)	0.054(0.000)	0.383(0.214)	0.319(0.199)	0.220(0.148)	0.209(0.097)	0.194(0.026)	0.070(0.022)	0.369(0.187)	0.300(0.086)	0.275(0.051)
HLT_2	0.128(0.039)	0.087(0.027)	0.029(0.000)	0.354(0.150)	0.355(0.129)	0.265(0.106)	0.215(0.044)	0.109(0.019)	0.127(0.037)	0.298(0.118)	0.238(0.095)	0.193(0.036)
HLT_3	0.161(0.158)	0.164(0.044)	0.072(0.061)	0.378(0.259)	0.348(0.236)	0.255(0.210)	0.279(0.135)	0.186(0.174)	0.154(0.128)	0.495(0.282)	0.424(0.252)	0.284(0.231)
LBI_1	1.000(1.000)	0.983(0.985)	1.000(1.000)	1.000(1.000)	1.000(1.000)	1.000(0.934)	1.000(1.000)	1.000(1.000)	1.000(0.949)	0.960(0.972)	0.992(0.983)	0.975(0.914)
LBI_2	0.998(0.977)	0.969(0.983)	0.989(0.968)	1.000(1.000)	1.000(1.000)	0.977(1.000)	1.000(1.000)	0.979(0.975)	1.000(1.000)	0.989(1.000)	0.994(1.000)	0.962(0.928)
LBI_3	0.982(1.000)	0.989(0.970)	0.992(1.000)	0.979(1.000)	1.000(0.966)	1.000(0.947)	0.981(0.987)	0.969(0.994)	0.998(0.951)	1.000(1.000)	1.000(0.974)	0.954(0.995)
$LKSN$	0.117(0.000)	0.000(0.000)	0.000(0.007)	0.260(0.038)	0.214(0.000)	0.037(0.037)	0.179(0.000)	0.049(0.030)	0.007(0.000)	0.337(0.000)	0.209(0.046)	0.103(0.000)
L	0.047(0.101)	0.087(0.032)	0.000(0.063)	0.067(0.082)	0.058(0.044)	0.000(0.036)	0.070(0.140)	0.048(0.048)	0.039(0.066)	0.113(0.072)	0.099(0.101)	0.048(0.000)

Table 2.6: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_1}$ with $\mathbf{T=250}$.

2.5.3 Properties of the Tests under H_{01} and H_{10}

In the following section we investigate the power properties of the test statistics under H_{01}, a change in persistence from $I(0)$ to $I(1)$ and under H_{10}, a change in persistence from $I(1)$ to $I(0)$. The tests are applied to data generated by,

$$y_t = \rho_t y_{t-1} + \epsilon, \quad t = -200, \ldots, T, \tag{2.12}$$

$$\epsilon_t = \sigma_t \eta_t, \tag{2.13}$$

with $\rho_t = 0$, $t = -200, \ldots, \lfloor \tau^* T \rfloor$, and $\rho_t = 1$, $t = \lfloor \tau^* T \rfloor + 1, \ldots, T$ for the case H_{01} and with $\rho_t = 1$, $t = -200, \ldots, \lfloor \tau^* T \rfloor$, and $\rho_t = 0$, $t = \lfloor \tau^* T \rfloor + 1, \ldots, T$ for the case H_{10}. Again, we set $d_t = 0$ and consider the true breakpoint $\tau^* \in \{0.3, 0.5, 0.7\}$ and a nominal size level of 5% for a better overview. The study for the power results is extendable for a change in persistence from $I(1)$ to a lower degree of stationary persistence, that is $0 < \rho_t < 1$, and vice versa. We use the test statistics, which are constructed to test for a change in an unknown direction and thus, we expect power in both directions independent of the null hypothesis of the tests.

The power results for a change in persistence H_{01} in the APARCH model are reported in Table 2.7 with $T = 100$ and Table 2.8 with $T = 250$.

In general the power is decreasing with increasing τ^* for the K_j, LT_j and LBI_j tests, which is reasonable, due to the short period of the $I(1)$ process in conjunction with the null hypothesis H_0. In contrast, the power of the HLT_j and $LKSN$ test increases with increasing τ^* with a similar argument. The best results of the L test provides $\tau^* = 0.5$ in most cases.

We obtain in general very high rejection frequencies, compared to optimal conditions without conditional asymmetric volatility even in the small sample size $T = 100$, which is not surprising, due to the large size distortions. The K_j tests are mostly close to one, while the power of the HLT_j tests is very low compared to the size distortions which are similar to the K_j tests.

An impressive result is obtained by the LT_j and LBI_j tests. The power of the LT_j test is comparable large with rejection frequencies around 80%, in context with the small size distortions. The LBI_j tests exceeds the LT_j tests and provide the best results. In almost all cases the tests reach rejection frequencies equal or close to 1, without noteworthy size distortions in the previous section.

The $LKSN$ and L tests perform poor, compared to other tests in terms of power, where the last one at least holds the size.

For all test, the best results are achieved when the volatility breaks from a low degree to a higher degree together with the persistence. In contrast when β changes from high volatility to a lower degree, the tests give the worst results. Both statements are reasonable, as a higher degree of volatility causes larger observations in absolute values in the time series. The direction of the asymmetric volatility parameter leads not to affect the power.

Overall, the rejection frequencies of the bootstrap versions of the tests are only slightly smaller for all the tests which is very acceptable and justifiable with the compensation in the size results. The wild bootstrap obtain a pivotal limiting distribution of the tests, which hold the nominal significance level and obtain respectable power under the alternative even in asymmetric volatility models in addition with breaks in the volatility parameters.

In general the statements remain the same for the larger sample size $T = 250$ with slightly higher power and for the GJR-GARCH model and the EGARCH model, where the results can be found in the appendix. The analysis can be mirrored for a change in persistence H_{10} and the results are reported in Table 2.9 with $T = 100$ and Table 2.10 with $T = 250$.

Note, that the change in persistence H_{01} is a change in persistence H_{10} in the reversed series and we use test statistics to reject both alternatives.

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.989(0.975)	0.964(0.937)	0.877(0.771)	0.962(0.903)	0.971(0.867)	0.932(0.828)	0.984(0.892)	0.937(0.869)	0.910(0.722)	0.970(0.867)	0.942(0.811)	0.875(0.747)
K_2	0.983(0.944)	0.970(0.935)	0.922(0.816)	0.953(0.880)	0.978(0.888)	0.943(0.835)	0.940(0.852)	0.948(0.900)	0.954(0.772)	0.950(0.803)	0.946(0.839)	0.879(0.776)
K_3	0.992(0.982)	0.964(0.937)	0.912(0.765)	0.957(0.910)	0.965(0.873)	0.935(0.832)	0.979(0.885)	0.940(0.871)	0.944(0.724)	0.969(0.881)	0.940(0.824)	0.890(0.752)
LT_1	0.881(0.857)	0.864(0.863)	0.740(0.765)	0.812(0.808)	0.793(0.801)	0.795(0.854)	0.806(0.759)	0.806(0.761)	0.736(0.800)	0.736(0.762)	0.839(0.832)	0.762(0.781)
LT_2	0.795(0.768)	0.890(0.879)	0.835(0.775)	0.705(0.673)	0.818(0.821)	0.857(0.832)	0.744(0.624)	0.833(0.831)	0.859(0.814)	0.533(0.428)	0.849(0.817)	0.838(0.822)
LT_3	0.876(0.857)	0.872(0.848)	0.761(0.765)	0.802(0.812)	0.806(0.803)	0.779(0.839)	0.810(0.768)	0.810(0.771)	0.773(0.814)	0.740(0.753)	0.847(0.830)	0.785(0.778)
HLT_1	0.597(0.443)	0.728(0.491)	0.802(0.489)	0.588(0.316)	0.676(0.426)	0.875(0.532)	0.562(0.343)	0.663(0.478)	0.768(0.464)	0.537(0.288)	0.655(0.431)	0.769(0.499)
HLT_2	0.626(0.481)	0.850(0.624)	0.884(0.653)	0.562(0.273)	0.842(0.502)	0.932(0.669)	0.574(0.396)	0.762(0.596)	0.909(0.548)	0.492(0.289)	0.755(0.535)	0.855(0.610)
HLT_3	0.761(0.618)	0.790(0.570)	0.809(0.470)	0.688(0.461)	0.777(0.525)	0.892(0.522)	0.604(0.406)	0.826(0.631)	0.815(0.468)	0.607(0.394)	0.724(0.501)	0.774(0.562)
LBI_1	1.000(1.000)	0.988(0.976)	0.915(0.876)	0.994(1.000)	0.971(0.970)	0.953(0.902)	1.000(0.983)	0.978(0.982)	0.890(0.875)	0.987(0.955)	0.969(0.946)	0.933(0.846)
LBI_2	1.000(1.000)	0.992(0.994)	0.906(0.872)	0.994(1.000)	0.967(0.974)	0.925(0.893)	0.989(0.994)	0.970(0.974)	0.867(0.857)	0.973(0.956)	0.960(0.965)	0.903(0.858)
LBI_3	1.000(1.000)	0.985(0.990)	0.909(0.883)	0.986(1.000)	0.965(0.965)	0.916(0.909)	0.990(0.984)	0.981(0.970)	0.873(0.863)	0.970(0.964)	0.970(0.957)	0.896(0.863)
$LKSN$	1.000(0.077)	1.000(0.297)	1.000(0.854)	0.986(0.058)	1.000(0.342)	1.000(0.749)	0.990(0.028)	1.000(0.334)	1.000(0.746)	0.933(0.015)	1.000(0.369)	1.000(0.737)
L	0.341(0.979)	0.512(1.000)	0.333(0.972)	0.296(0.993)	0.489(1.000)	0.307(0.946)	0.283(0.987)	0.426(0.992)	0.325(0.964)	0.189(0.934)	0.486(1.000)	0.397(0.950)

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	1.000(0.987)	1.000(1.000)	1.000(0.946)	0.719(0.503)	0.714(0.536)	0.536(0.339)	1.000(0.981)	0.983(0.930)	0.967(0.848)	0.957(0.909)	0.886(0.803)	0.864(0.697)
K_2	1.000(0.994)	1.000(0.989)	1.000(0.972)	0.733(0.462)	0.717(0.500)	0.585(0.398)	1.000(0.946)	0.982(0.909)	0.985(0.882)	0.941(0.860)	0.910(0.810)	0.871(0.769)
K_3	1.000(0.988)	1.000(1.000)	1.000(0.952)	0.744(0.496)	0.711(0.550)	0.548(0.338)	1.000(0.979)	0.981(0.942)	0.966(0.849)	0.964(0.913)	0.887(0.812)	0.855(0.699)
LT_1	0.826(0.826)	0.858(0.843)	0.841(0.849)	0.797(0.833)	0.774(0.783)	0.684(0.721)	0.852(0.818)	0.838(0.785)	0.816(0.842)	0.826(0.849)	0.820(0.818)	0.783(0.774)
LT_2	0.658(0.640)	0.886(0.888)	0.889(0.860)	0.763(0.683)	0.812(0.768)	0.783(0.762)	0.666(0.626)	0.846(0.838)	0.894(0.873)	0.744(0.633)	0.857(0.845)	0.831(0.827)
LT_3	0.821(0.818)	0.891(0.846)	0.847(0.846)	0.823(0.830)	0.779(0.794)	0.675(0.722)	0.850(0.811)	0.860(0.819)	0.846(0.861)	0.838(0.840)	0.832(0.831)	0.775(0.776)
HLT_1	0.770(0.651)	0.777(0.679)	0.808(0.571)	0.483(0.175)	0.653(0.425)	0.500(0.251)	0.678(0.498)	0.798(0.556)	0.851(0.517)	0.536(0.258)	0.691(0.518)	0.826(0.554)
HLT_2	0.813(0.711)	0.839(0.795)	0.885(0.724)	0.482(0.186)	0.628(0.442)	0.581(0.345)	0.713(0.458)	0.849(0.646)	0.897(0.645)	0.518(0.262)	0.813(0.666)	0.855(0.704)
HLT_3	0.975(0.947)	1.000(1.000)	0.992(0.858)	0.471(0.169)	0.626(0.405)	0.526(0.257)	0.873(0.711)	0.919(0.687)	0.961(0.678)	0.611(0.338)	0.694(0.495)	0.815(0.515)
LBI_1	0.990(0.990)	1.000(1.000)	0.988(0.955)	0.981(0.909)	0.834(0.760)	0.744(0.707)	1.000(1.000)	0.982(0.945)	0.990(0.947)	1.000(1.000)	0.933(0.928)	0.931(0.887)
LBI_2	0.990(1.000)	0.991(0.975)	0.993(0.959)	0.982(0.951)	0.809(0.800)	0.696(0.724)	0.985(1.000)	0.984(0.938)	0.965(0.936)	1.000(1.000)	0.917(0.930)	0.883(0.864)
LBI_3	0.993(1.000)	0.986(0.975)	0.994(0.953)	0.979(0.946)	0.814(0.778)	0.724(0.721)	0.986(1.000)	0.975(0.945)	0.983(0.952)	1.000(1.000)	0.931(0.926)	0.878(0.867)
$LKSN$	0.962(0.060)	1.000(0.329)	1.000(0.781)	0.980(0.143)	1.000(0.494)	1.000(0.698)	0.978(0.046)	1.000(0.250)	1.000(0.738)	0.987(0.101)	1.000(0.455)	1.000(0.843)
L	0.364(0.961)	0.581(1.000)	0.839(1.000)	0.222(0.901)	0.083(0.786)	0.023(0.588)	0.280(0.982)	0.491(1.000)	0.469(0.974)	0.376(0.967)	0.363(0.976)	0.165(0.887)

τ^*	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.985(0.960)	0.969(0.873)	0.904(0.765)	0.993(0.975)	1.000(0.971)	1.000(0.918)	0.900(0.829)	0.865(0.769)	0.784(0.585)	1.000(0.991)	1.000(0.976)	1.000(0.958)
K_2	0.962(0.921)	0.984(0.903)	0.927(0.814)	0.989(0.949)	1.000(0.984)	1.000(0.924)	0.846(0.708)	0.867(0.783)	0.805(0.683)	1.000(0.981)	1.000(0.967)	1.000(0.952)
K_3	0.989(0.962)	0.971(0.867)	0.906(0.779)	0.990(0.976)	1.000(0.974)	1.000(0.918)	0.897(0.830)	0.871(0.779)	0.791(0.603)	1.000(0.986)	1.000(0.975)	1.000(0.956)
LT_1	0.877(0.866)	0.778(0.766)	0.769(0.776)	0.791(0.740)	0.875(0.848)	0.826(0.838)	0.785(0.800)	0.886(0.911)	0.663(0.687)	0.827(0.771)	0.855(0.869)	0.798(0.804)
LT_2	0.755(0.726)	0.847(0.827)	0.866(0.833)	0.620(0.566)	0.806(0.815)	0.877(0.856)	0.723(0.741)	0.899(0.875)	0.812(0.776)	0.564(0.544)	0.860(0.832)	0.893(0.865)
LT_3	0.876(0.869)	0.812(0.780)	0.773(0.813)	0.772(0.759)	0.881(0.871)	0.850(0.847)	0.794(0.799)	0.897(0.914)	0.694(0.696)	0.815(0.756)	0.873(0.874)	0.841(0.819)
HLT_1	0.592(0.357)	0.705(0.416)	0.804(0.527)	0.728(0.605)	0.797(0.646)	0.826(0.596)	0.567(0.294)	0.730(0.518)	0.695(0.455)	0.818(0.720)	0.782(0.660)	0.800(0.617)
HLT_2	0.566(0.346)	0.798(0.555)	0.900(0.666)	0.794(0.594)	0.841(0.745)	0.939(0.714)	0.478(0.231)	0.805(0.558)	0.809(0.586)	0.827(0.768)	0.870(0.769)	0.872(0.732)
HLT_3	0.763(0.497)	0.851(0.566)	0.863(0.563)	0.937(0.865)	0.982(0.920)	0.941(0.795)	0.614(0.311)	0.744(0.467)	0.724(0.435)	0.986(0.949)	0.988(0.898)	1.000(0.894)
LBI_1	1.000(1.000)	0.986(0.994)	0.944(0.921)	0.987(1.000)	1.000(0.945)	0.991(0.973)	0.986(0.987)	0.949(0.947)	0.859(0.882)	0.992(1.000)	0.978(0.971)	0.988(0.957)
LBI_2	0.986(1.000)	0.985(0.992)	0.926(0.900)	0.979(0.993)	0.984(0.963)	0.967(0.938)	1.000(0.990)	0.945(0.961)	0.847(0.851)	0.987(1.000)	0.966(0.964)	0.967(0.953)
LBI_3	0.988(1.000)	0.988(0.988)	0.927(0.901)	0.981(0.988)	0.978(0.939)	0.972(0.954)	1.000(0.994)	0.945(0.957)	0.837(0.851)	0.987(1.000)	0.972(0.963)	0.982(0.958)
$LKSN$	0.992(0.049)	1.000(0.343)	1.000(0.649)	0.982(0.102)	1.000(0.315)	1.000(0.765)	1.000(0.049)	1.000(0.414)	1.000(0.796)	0.989(0.074)	1.000(0.423)	1.000(0.730)
L	0.327(1.000)	0.467(0.989)	0.323(0.948)	0.216(0.972)	0.561(1.000)	0.680(1.000)	0.332(0.948)	0.197(0.944)	0.106(0.870)	0.223(0.951)	0.606(1.000)	0.660(1.000)

Table 2.7: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_{01}}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.965(0.982)	1.000(0.934)	0.925(0.812)	0.922(0.886)	0.985(0.855)	0.975(0.820)	1.000(0.875)	0.944(0.840)	0.912(0.685)	0.942(0.863)	0.937(0.816)	0.887(0.744)
K_2	0.962(0.958)	0.960(0.956)	0.917(0.783)	0.972(0.924)	0.951(0.919)	0.929(0.856)	0.969(0.849)	0.997(0.945)	0.978(0.791)	0.932(0.814)	0.941(0.869)	0.917(0.807)
K_3	1.000(0.980)	0.996(0.953)	0.921(0.800)	0.951(0.922)	0.947(0.882)	0.954(0.859)	1.000(0.898)	0.940(0.877)	0.990(0.735)	0.963(0.905)	0.913(0.857)	0.915(0.764)
LT_1	0.878(0.839)	0.884(0.852)	0.770(0.745)	0.772(0.811)	0.795(0.819)	0.761(0.830)	0.803(0.749)	0.813(0.727)	0.751(0.844)	0.739(0.782)	0.868(0.872)	0.811(0.772)
LT_2	0.843(0.734)	0.924(0.889)	0.822(0.802)	0.733(0.677)	0.778(0.852)	0.863(0.837)	0.744(0.606)	0.878(0.845)	0.904(0.842)	0.543(0.437)	0.895(0.829)	0.807(0.791)
LT_3	0.863(0.821)	0.834(0.859)	0.764(0.763)	0.850(0.793)	0.823(0.804)	0.819(0.823)	0.809(0.735)	0.844(0.803)	0.781(0.789)	0.771(0.738)	0.812(0.863)	0.750(0.754)
HLT_1	0.603(0.471)	0.709(0.500)	0.848(0.511)	0.571(0.299)	0.667(0.450)	0.869(0.562)	0.612(0.369)	0.687(0.512)	0.791(0.480)	0.559(0.324)	0.696(0.434)	0.773(0.509)
HLT_2	0.647(0.489)	0.837(0.672)	0.856(0.643)	0.607(0.237)	0.855(0.503)	0.902(0.700)	0.564(0.356)	0.799(0.605)	0.904(0.537)	0.489(0.309)	0.741(0.571)	0.816(0.579)
HLT_3	0.723(0.612)	0.794(0.592)	0.828(0.454)	0.736(0.478)	0.810(0.489)	0.871(0.514)	0.636(0.422)	0.824(0.624)	0.789(0.452)	0.637(0.385)	0.687(0.464)	0.824(0.564)
LBI_1	0.991(1.000)	1.000(1.000)	0.943(0.900)	0.964(0.961)	0.936(0.932)	0.934(0.925)	1.000(1.000)	1.000(0.947)	0.937(0.879)	0.954(0.921)	0.945(0.941)	0.974(0.844)
LBI_2	1.000(0.969)	1.000(0.963)	0.866(0.905)	1.000(1.000)	0.972(1.000)	0.915(0.929)	1.000(1.000)	0.937(1.000)	0.863(0.843)	0.964(0.950)	1.000(0.999)	0.908(0.903)
LBI_3	0.985(0.970)	1.000(0.988)	0.888(0.882)	1.000(0.966)	0.995(0.998)	0.954(0.935)	0.973(0.979)	0.951(1.000)	0.903(0.912)	0.996(0.992)	0.945(0.981)	0.906(0.833)
$LKSN$	1.000(0.040)	0.970(0.288)	1.000(0.876)	1.000(0.030)	1.000(0.324)	0.987(0.776)	0.956(0.032)	1.000(0.353)	1.000(0.778)	0.978(0.029)	1.000(0.333)	1.000(0.698)
L	0.326(0.940)	0.550(1.000)	0.341(0.973)	0.326(1.000)	0.508(0.997)	0.335(0.994)	0.272(1.000)	0.413(0.978)	0.322(0.934)	0.190(0.901)	0.494(1.000)	0.427(0.950)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.981(0.954)	0.973(1.000)	0.987(0.913)	0.723(0.483)	0.715(0.496)	0.535(0.330)	1.000(1.000)	0.950(0.977)	1.000(0.839)	0.938(0.907)	0.852(0.843)	0.844(0.678)
K_2	0.961(1.000)	0.962(0.987)	1.000(0.960)	0.728(0.426)	0.722(0.524)	0.600(0.389)	1.000(0.936)	1.000(0.916)	0.991(0.889)	0.962(0.859)	0.946(0.858)	0.912(0.755)
K_3	1.000(1.000)	1.000(0.975)	1.000(0.972)	0.712(0.483)	0.695(0.539)	0.539(0.383)	0.970(0.974)	0.986(0.942)	0.993(0.853)	0.996(0.912)	0.894(0.791)	0.865(0.722)
LT_1	0.803(0.812)	0.905(0.812)	0.884(0.859)	0.757(0.817)	0.761(0.781)	0.730(0.714)	0.892(0.825)	0.877(0.749)	0.838(0.809)	0.860(0.825)	0.865(0.836)	0.809(0.769)
LT_2	0.707(0.671)	0.893(0.901)	0.890(0.873)	0.757(0.676)	0.812(0.757)	0.749(0.730)	0.707(0.615)	0.813(0.859)	0.937(0.899)	0.739(0.597)	0.891(0.893)	0.822(0.835)
LT_3	0.866(0.863)	0.854(0.883)	0.869(0.881)	0.799(0.831)	0.741(0.829)	0.635(0.689)	0.810(0.776)	0.898(0.817)	0.864(0.844)	0.820(0.877)	0.868(0.873)	0.815(0.801)
HLT_1	0.810(0.677)	0.743(0.674)	0.797(0.557)	0.507(0.225)	0.698(0.469)	0.524(0.248)	0.709(0.515)	0.769(0.568)	0.878(0.552)	0.584(0.302)	0.723(0.488)	0.820(0.524)
HLT_2	0.792(0.706)	0.852(0.760)	0.865(0.688)	0.505(0.184)	0.675(0.455)	0.544(0.346)	0.755(0.503)	0.844(0.693)	0.947(0.675)	0.499(0.243)	0.789(0.631)	0.817(0.698)
HLT_3	1.000(0.944)	0.974(0.988)	1.000(0.859)	0.457(0.131)	0.603(0.395)	0.491(0.276)	0.852(0.673)	0.888(0.715)	0.974(0.666)	0.627(0.304)	0.702(0.512)	0.863(0.555)
LBI_1	1.000(0.954)	0.961(1.000)	1.000(1.000)	0.954(0.888)	0.801(0.797)	0.713(0.689)	1.000(1.000)	1.000(0.976)	1.000(0.977)	0.967(0.981)	0.952(0.952)	0.974(0.901)
LBI_2	0.997(0.993)	0.988(0.990)	1.000(0.956)	0.971(0.930)	0.791(0.798)	0.733(0.731)	0.957(0.969)	0.947(0.939)	1.000(0.912)	0.962(0.964)	0.891(0.970)	0.905(0.872)
LBI_3	1.000(1.000)	1.000(0.940)	1.000(0.993)	0.994(0.908)	0.836(0.756)	0.752(0.712)	0.952(1.000)	0.975(0.941)	0.987(0.989)	1.000(0.975)	0.895(0.903)	0.869(0.884)
$LKSN$	0.981(0.052)	1.000(0.298)	0.967(0.783)	0.975(0.130)	0.989(0.498)	0.992(0.680)	1.000(0.072)	1.000(0.268)	0.993(0.738)	1.000(0.079)	1.000(0.493)	1.000(0.824)
L	0.402(0.932)	0.624(0.964)	0.825(1.000)	0.196(0.945)	0.107(0.792)	0.018(0.579)	0.258(0.953)	0.524(0.990)	0.449(0.989)	0.337(0.952)	0.338(0.996)	0.149(0.890)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.994(0.941)	0.998(0.909)	0.910(0.731)	0.990(0.978)	0.987(0.964)	1.000(0.961)	0.948(0.789)	0.878(0.764)	0.826(0.564)	1.000(1.000)	0.975(1.000)	1.000(0.977)
K_2	0.995(0.935)	1.000(0.890)	0.969(0.786)	0.985(0.939)	1.000(0.993)	1.000(0.952)	0.818(0.732)	0.862(0.766)	0.855(0.644)	1.000(0.951)	1.000(0.972)	0.993(0.923)
K_3	1.000(0.971)	0.948(0.896)	0.905(0.742)	0.983(0.949)	0.980(0.975)	1.000(0.944)	0.938(0.826)	0.871(0.829)	0.754(0.632)	1.000(0.955)	1.000(0.949)	0.984(0.996)
LT_1	0.839(0.911)	0.798(0.760)	0.735(0.783)	0.816(0.726)	0.910(0.810)	0.837(0.886)	0.770(0.765)	0.850(0.899)	0.663(0.704)	0.847(0.761)	0.904(0.880)	0.770(0.848)
LT_2	0.769(0.766)	0.864(0.842)	0.902(0.852)	0.613(0.566)	0.782(0.777)	0.871(0.884)	0.713(0.784)	0.873(0.843)	0.840(0.819)	0.543(0.581)	0.846(0.796)	0.903(0.874)
LT_3	0.919(0.879)	0.803(0.824)	0.743(0.803)	0.811(0.723)	0.898(0.844)	0.876(0.865)	0.817(0.760)	0.911(0.937)	0.713(0.725)	0.811(0.788)	0.838(0.919)	0.849(0.848)
HLT_1	0.595(0.386)	0.695(0.463)	0.791(0.504)	0.729(0.646)	0.771(0.614)	0.853(0.640)	0.528(0.342)	0.718(0.552)	0.709(0.465)	0.823(0.768)	0.814(0.675)	0.826(0.665)
HLT_2	0.591(0.359)	0.838(0.521)	0.876(0.628)	0.844(0.637)	0.835(0.724)	0.915(0.762)	0.457(0.222)	0.765(0.608)	0.783(0.597)	0.850(0.805)	0.878(0.762)	0.895(0.756)
HLT_3	0.755(0.489)	0.890(0.538)	0.902(0.594)	0.971(0.883)	1.000(0.920)	0.983(0.800)	0.601(0.338)	0.785(0.445)	0.744(0.431)	0.988(0.944)	1.000(0.859)	1.000(0.907)
LBI_1	0.992(0.978)	1.000(0.996)	0.936(0.938)	0.971(0.999)	1.000(0.973)	0.954(0.988)	0.983(0.966)	0.917(0.945)	0.820(0.902)	1.000(1.000)	0.958(1.000)	1.000(0.924)
LBI_2	1.000(1.000)	0.957(1.000)	0.910(0.912)	1.000(0.972)	0.999(0.968)	1.000(0.937)	0.973(0.981)	0.933(0.951)	0.860(0.834)	0.974(1.000)	0.961(0.937)	0.979(0.965)
LBI_3	1.000(0.964)	0.982(1.000)	0.898(0.879)	0.954(0.954)	0.963(0.927)	0.941(0.990)	0.966(0.960)	0.992(0.963)	0.847(0.851)	1.000(1.000)	0.941(0.946)	1.000(0.920)
$LKSN$	1.000(0.009)	1.000(0.372)	0.974(0.659)	1.000(0.123)	1.000(0.356)	0.998(0.740)	0.965(0.041)	0.976(0.458)	0.968(0.785)	0.984(0.119)	1.000(0.393)	0.978(0.707)
L	0.305(1.000)	0.486(0.967)	0.287(0.973)	0.208(0.958)	0.609(1.000)	0.673(1.000)	0.334(0.950)	0.186(0.990)	0.156(0.837)	0.199(0.964)	0.637(0.999)	0.667(1.000)

Table 2.8: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests.
APARCH Model in $\mathbf{H_{01}}$ with $\mathbf{T=250}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.936(0.879)	0.965(0.870)	0.970(0.936)	0.886(0.759)	1.000(0.906)	0.975(0.910)	0.865(0.736)	0.967(0.881)	0.993(0.940)	0.843(0.699)	0.930(0.832)	0.965(0.829)
K_2	0.952(0.885)	0.978(0.912)	0.976(0.904)	0.892(0.787)	1.000(0.955)	0.961(0.912)	0.902(0.776)	0.974(0.929)	0.986(0.906)	0.910(0.749)	0.921(0.861)	0.941(0.809)
K_3	0.943(0.882)	0.981(0.870)	0.968(0.942)	0.887(0.759)	1.000(0.911)	0.977(0.905)	0.857(0.729)	0.969(0.879)	0.988(0.948)	0.843(0.714)	0.926(0.840)	0.967(0.832)
LT_1	0.820(0.873)	0.801(0.808)	0.801(0.790)	0.740(0.741)	0.842(0.831)	0.796(0.792)	0.737(0.792)	0.778(0.786)	0.824(0.780)	0.718(0.739)	0.776(0.791)	0.733(0.712)
LT_2	0.889(0.867)	0.826(0.813)	0.713(0.677)	0.802(0.778)	0.904(0.873)	0.700(0.650)	0.849(0.811)	0.862(0.854)	0.675(0.640)	0.838(0.815)	0.803(0.794)	0.673(0.614)
LT_3	0.828(0.871)	0.796(0.796)	0.811(0.824)	0.768(0.732)	0.847(0.820)	0.823(0.786)	0.757(0.800)	0.807(0.801)	0.809(0.786)	0.752(0.749)	0.811(0.781)	0.749(0.726)
HLT_1	0.845(0.523)	0.684(0.414)	0.586(0.421)	0.816(0.485)	0.791(0.469)	0.579(0.384)	0.767(0.485)	0.722(0.438)	0.664(0.384)	0.733(0.476)	0.662(0.462)	0.532(0.303)
HLT_2	0.934(0.692)	0.816(0.548)	0.619(0.416)	0.871(0.599)	0.860(0.656)	0.584(0.335)	0.865(0.637)	0.802(0.621)	0.684(0.444)	0.858(0.633)	0.722(0.554)	0.561(0.303)
HLT_3	0.847(0.477)	0.781(0.505)	0.720(0.543)	0.871(0.526)	0.891(0.637)	0.717(0.515)	0.798(0.442)	0.829(0.615)	0.734(0.451)	0.739(0.495)	0.774(0.572)	0.629(0.404)
LBI_1	0.984(0.936)	0.991(0.979)	1.000(1.000)	0.912(0.903)	0.985(0.980)	0.988(0.993)	0.943(0.934)	0.980(0.961)	1.000(1.000)	0.920(0.876)	0.974(0.962)	0.985(1.000)
LBI_2	0.962(0.937)	0.979(0.981)	1.000(1.000)	0.918(0.914)	0.987(0.981)	0.992(0.982)	0.943(0.897)	0.992(0.966)	0.989(0.986)	0.897(0.869)	0.965(0.958)	0.985(0.992)
LBI_3	0.962(0.940)	0.979(0.978)	1.000(1.000)	0.905(0.908)	0.986(0.984)	0.988(0.993)	0.961(0.907)	0.977(0.966)	0.986(0.987)	0.907(0.884)	0.973(0.964)	0.987(0.986)
$LKSN$	1.000(0.729)	1.000(0.395)	0.990(0.116)	1.000(0.790)	1.000(0.339)	0.989(0.053)	1.000(0.745)	1.000(0.295)	0.976(0.092)	1.000(0.724)	0.988(0.325)	0.939(0.039)
L	0.490(0.977)	0.426(1.000)	0.299(1.000)	0.316(0.941)	0.466(0.993)	0.275(1.000)	0.351(0.957)	0.457(0.994)	0.399(0.990)	0.461(0.958)	0.440(0.972)	0.225(0.944)
	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	1.000(0.946)	1.000(0.989)	1.000(0.984)	0.562(0.349)	0.777(0.500)	0.751(0.511)	0.967(0.883)	1.000(0.937)	1.000(0.977)	0.765(0.659)	0.908(0.829)	0.921(0.808)
K_2	1.000(0.949)	1.000(1.000)	1.000(0.982)	0.563(0.397)	0.753(0.491)	0.698(0.448)	0.976(0.887)	1.000(0.954)	1.000(0.979)	0.828(0.754)	0.905(0.851)	0.877(0.774)
K_3	1.000(0.959)	1.000(0.994)	1.000(0.986)	0.561(0.353)	0.783(0.516)	0.778(0.497)	0.973(0.876)	1.000(0.937)	1.000(0.975)	0.805(0.663)	0.911(0.842)	0.938(0.809)
LT_1	0.905(0.881)	0.927(0.850)	0.782(0.734)	0.599(0.600)	0.859(0.912)	0.778(0.793)	0.800(0.805)	0.820(0.827)	0.833(0.765)	0.706(0.731)	0.842(0.862)	0.804(0.777)
LT_2	0.896(0.894)	0.887(0.845)	0.608(0.590)	0.723(0.745)	0.887(0.866)	0.686(0.638)	0.863(0.831)	0.852(0.819)	0.645(0.613)	0.800(0.789)	0.879(0.860)	0.716(0.678)
LT_3	0.918(0.883)	0.922(0.837)	0.776(0.720)	0.644(0.613)	0.864(0.906)	0.777(0.786)	0.813(0.808)	0.811(0.834)	0.824(0.763)	0.746(0.737)	0.860(0.855)	0.804(0.792)
HLT_1	0.818(0.666)	0.825(0.723)	0.769(0.670)	0.502(0.303)	0.640(0.347)	0.540(0.277)	0.878(0.511)	0.714(0.472)	0.654(0.483)	0.719(0.575)	0.720(0.423)	0.479(0.276)
HLT_2	0.913(0.738)	0.898(0.806)	0.815(0.727)	0.523(0.345)	0.646(0.362)	0.528(0.224)	0.941(0.636)	0.809(0.630)	0.715(0.449)	0.811(0.657)	0.787(0.598)	0.510(0.258)
HLT_3	0.990(0.926)	1.000(0.984)	1.000(0.949)	0.504(0.295)	0.657(0.337)	0.563(0.272)	0.926(0.621)	0.938(0.709)	0.866(0.758)	0.738(0.566)	0.751(0.427)	0.466(0.290)
LBI_1	0.982(0.935)	1.000(1.000)	0.983(0.976)	0.755(0.727)	0.870(0.871)	0.957(0.875)	0.954(0.905)	0.990(0.991)	1.000(1.000)	0.822(0.816)	0.962(0.945)	0.990(0.993)
LBI_2	0.952(0.925)	0.985(1.000)	0.978(0.981)	0.748(0.740)	0.849(0.864)	0.938(0.887)	0.928(0.906)	0.980(0.975)	1.000(1.000)	0.852(0.833)	0.947(0.954)	0.992(0.983)
LBI_3	0.965(0.932)	0.988(1.000)	0.983(0.979)	0.750(0.745)	0.846(0.862)	0.945(0.891)	0.935(0.914)	0.989(0.981)	1.000(1.000)	0.845(0.840)	0.952(0.952)	0.985(0.980)
$LKSN$	1.000(0.747)	1.000(0.333)	0.991(0.113)	1.000(0.822)	1.000(0.446)	1.000(0.253)	1.000(0.666)	1.000(0.336)	0.993(0.059)	1.000(0.786)	1.000(0.407)	0.986(0.060)
L	0.777(1.000)	0.639(1.000)	0.352(0.983)	0.042(0.665)	0.058(0.874)	0.210(0.892)	0.642(0.991)	0.603(1.000)	0.330(0.986)	0.182(0.849)	0.407(0.955)	0.351(0.974)
	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.865(0.706)	0.974(0.899)	0.980(0.914)	0.957(0.874)	0.991(0.943)	1.000(0.991)	0.762(0.653)	0.840(0.789)	0.906(0.814)	0.986(0.966)	1.000(0.966)	1.000(0.986)
K_2	0.913(0.769)	0.978(0.876)	0.965(0.872)	0.968(0.878)	1.000(0.933)	1.000(0.961)	0.828(0.727)	0.870(0.823)	0.893(0.745)	0.987(0.964)	1.000(0.970)	1.000(0.979)
K_3	0.867(0.709)	0.974(0.903)	0.977(0.912)	0.957(0.875)	0.990(0.936)	1.000(0.988)	0.763(0.647)	0.841(0.791)	0.917(0.808)	0.993(0.972)	1.000(0.982)	1.000(0.987)
LT_1	0.715(0.733)	0.806(0.795)	0.769(0.721)	0.812(0.839)	0.849(0.830)	0.829(0.829)	0.727(0.766)	0.808(0.823)	0.781(0.785)	0.828(0.800)	0.863(0.832)	0.809(0.787)
LT_2	0.760(0.771)	0.869(0.846)	0.593(0.578)	0.841(0.822)	0.825(0.808)	0.774(0.688)	0.826(0.823)	0.857(0.839)	0.704(0.663)	0.869(0.852)	0.850(0.826)	0.651(0.654)
LT_3	0.730(0.734)	0.819(0.807)	0.774(0.732)	0.835(0.836)	0.845(0.839)	0.840(0.830)	0.746(0.775)	0.834(0.824)	0.791(0.777)	0.818(0.808)	0.864(0.829)	0.820(0.792)
HLT_1	0.817(0.482)	0.812(0.481)	0.509(0.350)	0.793(0.614)	0.797(0.682)	0.757(0.623)	0.711(0.527)	0.664(0.541)	0.485(0.246)	0.831(0.659)	0.780(0.646)	0.742(0.621)
HLT_2	0.891(0.590)	0.904(0.653)	0.560(0.351)	0.875(0.683)	0.842(0.737)	0.817(0.684)	0.795(0.678)	0.794(0.588)	0.516(0.249)	0.919(0.757)	0.834(0.729)	0.800(0.641)
HLT_3	0.847(0.525)	0.869(0.575)	0.643(0.499)	0.934(0.772)	0.994(0.909)	0.968(0.923)	0.732(0.498)	0.680(0.491)	0.515(0.258)	0.981(0.861)	1.000(0.951)	0.969(0.940)
LBI_1	0.918(0.868)	0.969(0.944)	1.000(1.000)	0.936(0.904)	0.977(0.974)	0.987(0.993)	0.847(0.869)	0.972(0.937)	1.000(0.985)	0.982(0.947)	0.992(0.981)	1.000(0.990)
LBI_2	0.855(0.822)	0.965(0.964)	1.000(1.000)	0.931(0.899)	0.982(0.972)	0.991(0.986)	0.843(0.857)	0.969(0.940)	1.000(1.000)	0.977(0.954)	0.988(0.973)	1.000(1.000)
LBI_3	0.865(0.826)	0.971(0.958)	1.000(1.000)	0.933(0.895)	0.977(0.976)	0.985(0.991)	0.841(0.861)	0.966(0.939)	1.000(0.985)	0.975(0.947)	0.994(0.969)	1.000(1.000)
$LKSN$	1.000(0.744)	1.000(0.410)	0.971(0.045)	1.000(0.767)	1.000(0.378)	0.988(0.094)	1.000(0.758)	1.000(0.386)	0.976(0.032)	1.000(0.727)	1.000(0.236)	0.980(0.041)
L	0.319(0.919)	0.421(0.988)	0.319(1.000)	0.620(1.000)	0.621(1.000)	0.370(0.980)	0.123(0.848)	0.310(0.966)	0.385(0.936)	0.754(1.000)	0.665(1.000)	0.303(0.988)

Table 2.9: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_{10}}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.913(0.867)	0.956(0.895)	0.978(0.960)	0.852(0.797)	0.974(0.931)	1.000(0.919)	0.851(0.754)	0.973(0.908)	1.000(0.937)	0.831(0.669)	0.958(0.881)	0.989(0.836)
K_2	0.959(0.922)	0.994(0.951)	1.000(0.917)	0.860(0.827)	0.961(0.960)	0.936(0.955)	0.867(0.810)	0.941(0.945)	0.997(0.936)	0.932(0.792)	0.923(0.837)	0.906(0.780)
K_3	0.952(0.863)	0.963(0.890)	0.967(0.971)	0.889(0.727)	1.000(0.920)	0.959(0.903)	0.864(0.690)	0.963(0.845)	0.949(0.927)	0.869(0.689)	0.958(0.829)	0.945(0.821)
LT_1	0.780(0.885)	0.817(0.855)	0.769(0.778)	0.720(0.739)	0.878(0.809)	0.764(0.779)	0.710(0.763)	0.796(0.785)	0.818(0.771)	0.691(0.782)	0.824(0.793)	0.752(0.749)
LT_2	0.873(0.896)	0.859(0.778)	0.701(0.713)	0.827(0.775)	0.899(0.896)	0.735(0.614)	0.865(0.783)	0.854(0.858)	0.683(0.641)	0.801(0.817)	0.773(0.782)	0.634(0.597)
LT_3	0.830(0.902)	0.806(0.764)	0.809(0.788)	0.752(0.721)	0.866(0.806)	0.825(0.821)	0.764(0.807)	0.827(0.786)	0.835(0.752)	0.755(0.763)	0.785(0.776)	0.719(0.760)
HLT_1	0.842(0.531)	0.644(0.452)	0.598(0.421)	0.853(0.523)	0.786(0.436)	0.576(0.378)	0.729(0.463)	0.745(0.460)	0.632(0.380)	0.717(0.444)	0.632(0.459)	0.509(0.314)
HLT_2	0.909(0.722)	0.780(0.555)	0.641(0.376)	0.905(0.593)	0.824(0.643)	0.612(0.298)	0.831(0.684)	0.799(0.671)	0.716(0.452)	0.858(0.677)	0.683(0.556)	0.585(0.304)
HLT_3	0.809(0.475)	0.778(0.506)	0.761(0.506)	0.842(0.497)	0.897(0.666)	0.705(0.560)	0.822(0.486)	0.811(0.639)	0.774(0.461)	0.714(0.479)	0.750(0.544)	0.645(0.376)
LBI_1	0.968(0.909)	1.000(1.000)	1.000(1.000)	0.888(0.888)	0.979(1.000)	0.948(1.000)	0.939(0.925)	1.000(0.950)	0.961(1.000)	0.927(0.898)	0.953(1.000)	1.000(0.975)
LBI_2	0.956(0.922)	0.953(0.979)	1.000(1.000)	0.963(0.882)	0.960(0.993)	0.987(0.989)	0.953(0.882)	0.991(0.932)	0.961(1.000)	0.934(0.915)	0.971(1.000)	0.956(0.978)
LBI_3	0.924(0.916)	1.000(1.000)	1.000(1.000)	0.950(0.921)	1.000(0.969)	1.000(0.994)	0.976(0.919)	0.973(0.949)	1.000(0.950)	0.881(0.923)	0.960(1.000)	1.000(0.962)
$LKSN$	1.000(0.772)	1.000(0.369)	0.963(0.147)	0.975(0.825)	1.000(0.341)	0.965(0.056)	0.976(0.795)	1.000(0.345)	1.000(0.053)	0.992(0.690)	0.979(0.322)	0.985(0.074)
L	0.450(1.000)	0.454(0.988)	0.282(1.000)	0.308(0.986)	0.451(0.987)	0.284(1.000)	0.361(1.000)	0.437(0.963)	0.422(1.000)	0.424(0.995)	0.404(0.993)	0.236(0.982)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.962(0.924)	1.000(1.000)	1.000(1.000)	0.564(0.384)	0.803(0.470)	0.762(0.506)	0.955(0.869)	1.000(0.902)	1.000(0.970)	0.812(0.647)	0.879(0.819)	0.945(0.845)
K_2	1.000(0.989)	1.000(1.000)	0.997(0.999)	0.553(0.446)	0.725(0.490)	0.664(0.439)	0.981(0.867)	1.000(0.968)	1.000(0.978)	0.850(0.715)	0.918(0.875)	0.860(0.788)
K_3	1.000(0.975)	1.000(0.963)	1.000(0.959)	0.578(0.380)	0.795(0.495)	0.793(0.523)	0.953(0.901)	0.984(0.912)	1.000(0.936)	0.775(0.698)	0.894(0.842)	0.943(0.825)
LT_1	0.951(0.864)	0.960(0.898)	0.820(0.721)	0.607(0.625)	0.886(0.908)	0.739(0.818)	0.788(0.803)	0.783(0.791)	0.801(0.735)	0.734(0.766)	0.826(0.875)	0.831(0.789)
LT_2	0.891(0.876)	0.868(0.849)	0.586(0.553)	0.685(0.759)	0.860(0.859)	0.668(0.665)	0.829(0.870)	0.822(0.792)	0.643(0.615)	0.827(0.787)	0.872(0.908)	0.747(0.669)
LT_3	0.965(0.904)	0.931(0.801)	0.821(0.754)	0.674(0.657)	0.851(0.898)	0.742(0.782)	0.795(0.838)	0.820(0.816)	0.806(0.765)	0.735(0.724)	0.836(0.852)	0.765(0.835)
HLT_1	0.842(0.626)	0.793(0.722)	0.804(0.634)	0.485(0.294)	0.629(0.359)	0.507(0.281)	0.863(0.502)	0.680(0.439)	0.692(0.456)	0.690(0.622)	0.700(0.429)	0.518(0.237)
HLT_2	0.892(0.753)	0.940(0.831)	0.823(0.712)	0.513(0.334)	0.655(0.404)	0.518(0.239)	0.916(0.603)	0.812(0.625)	0.728(0.461)	0.797(0.695)	0.760(0.573)	0.490(0.279)
HLT_3	1.000(0.897)	0.966(0.979)	1.000(0.923)	0.523(0.274)	0.661(0.304)	0.605(0.288)	0.904(0.611)	0.908(0.748)	0.846(0.730)	0.720(0.526)	0.791(0.391)	0.485(0.319)
LBI_1	0.980(0.970)	0.969(1.000)	1.000(0.953)	0.762(0.687)	0.859(0.901)	1.000(0.904)	0.984(0.935)	0.972(1.000)	1.000(1.000)	0.823(0.778)	0.947(0.981)	0.987(0.954)
LBI_2	0.973(0.929)	1.000(1.000)	0.995(0.990)	0.779(0.786)	0.883(0.830)	0.943(0.871)	0.921(0.938)	0.994(0.952)	1.000(1.000)	0.896(0.857)	0.993(0.936)	1.000(1.000)
LBI_3	1.000(0.927)	0.972(1.000)	0.949(0.977)	0.735(0.732)	0.831(0.901)	0.992(0.863)	0.922(0.961)	0.975(0.969)	0.974(1.000)	0.805(0.812)	1.000(0.969)	0.973(0.997)
$LKSN$	1.000(0.770)	1.000(0.366)	1.000(0.115)	1.000(0.799)	0.970(0.487)	0.997(0.249)	0.974(0.630)	1.000(0.300)	0.963(0.083)	0.983(0.768)	0.985(0.381)	1.000(0.081)
L	0.774(1.000)	0.616(0.967)	0.364(0.959)	0.091(0.713)	0.102(0.874)	0.183(0.855)	0.681(0.998)	0.572(1.000)	0.354(1.000)	0.200(0.888)	0.382(0.994)	0.373(0.988)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.885(0.738)	0.979(0.936)	0.943(0.960)	0.936(0.845)	0.981(0.947)	0.972(0.961)	0.762(0.651)	0.874(0.772)	0.932(0.810)	0.986(0.946)	1.000(0.962)	0.974(1.000)
K_2	0.885(0.792)	0.943(0.894)	1.000(0.897)	0.998(0.913)	1.000(0.919)	1.000(0.921)	0.845(0.760)	0.917(0.810)	0.861(0.712)	1.000(0.964)	1.000(0.974)	1.000(1.000)
K_3	0.878(0.742)	1.000(0.873)	0.941(0.955)	0.981(0.866)	1.000(0.952)	1.000(1.000)	0.733(0.692)	0.834(0.774)	0.930(0.792)	1.000(1.000)	1.000(0.959)	0.979(1.000)
LT_1	0.740(0.774)	0.775(0.815)	0.751(0.710)	0.791(0.872)	0.893(0.796)	0.840(0.811)	0.739(0.775)	0.857(0.843)	0.760(0.805)	0.818(0.779)	0.838(0.832)	0.849(0.767)
LT_2	0.782(0.802)	0.911(0.833)	0.561(0.627)	0.817(0.850)	0.847(0.779)	0.792(0.687)	0.802(0.820)	0.840(0.842)	0.703(0.707)	0.853(0.850)	0.873(0.793)	0.637(0.637)
LT_3	0.761(0.714)	0.847(0.819)	0.741(0.716)	0.817(0.872)	0.848(0.834)	0.805(0.859)	0.790(0.766)	0.858(0.848)	0.774(0.810)	0.786(0.807)	0.827(0.793)	0.785(0.840)
HLT_1	0.793(0.522)	0.817(0.517)	0.476(0.365)	0.819(0.652)	0.806(0.729)	0.788(0.635)	0.683(0.525)	0.687(0.550)	0.535(0.266)	0.829(0.643)	0.791(0.684)	0.729(0.640)
HLT_2	0.878(0.597)	0.944(0.678)	0.538(0.325)	0.835(0.719)	0.805(0.708)	0.824(0.688)	0.841(0.716)	0.766(0.629)	0.524(0.220)	0.940(0.717)	0.800(0.737)	0.816(0.610)
HLT_3	0.850(0.573)	0.897(0.590)	0.633(0.541)	0.977(0.750)	1.000(0.947)	1.000(0.885)	0.733(0.469)	0.667(0.536)	0.536(0.229)	1.000(0.907)	0.979(0.995)	1.000(0.914)
LBI_1	0.884(0.908)	0.995(0.914)	0.967(1.000)	0.964(0.924)	0.971(0.973)	1.000(0.957)	0.880(0.893)	0.998(0.905)	1.000(1.000)	1.000(0.930)	0.952(0.979)	1.000(0.991)
LBI_2	0.819(0.814)	0.942(0.936)	0.993(1.000)	0.926(0.907)	1.000(0.988)	0.996(1.000)	0.818(0.864)	0.970(0.912)	1.000(0.980)	0.970(0.961)	0.974(0.987)	1.000(0.967)
LBI_3	0.868(0.822)	0.972(0.922)	0.975(0.969)	0.938(0.905)	0.939(1.000)	0.957(0.976)	0.827(0.882)	0.960(0.985)	0.973(0.985)	0.981(0.969)	1.000(0.938)	1.000(1.000)
$LKSN$	1.000(0.712)	0.982(0.417)	0.996(0.073)	1.000(0.776)	1.000(0.368)	0.977(0.137)	0.972(0.772)	1.000(0.369)	0.964(0.067)	0.988(0.756)	1.000(0.200)	1.000(0.020)
L	0.325(0.912)	0.382(1.000)	0.315(0.962)	0.659(0.993)	0.655(1.000)	0.399(0.950)	0.088(0.844)	0.322(0.934)	0.358(0.919)	0.780(1.000)	0.669(0.992)	0.273(0.983)

Table 2.10: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **APARCH** Model in $\mathbf{H_{10}}$ with $\mathbf{T=250}$.

2.6 Application to US Stock Data

In this section the proposed procedure is applied to the US stock market to test for a change in persistence in asymmetric volatility models. As already pointed out, there is empirical evidence for asymmetric volatility in stock returns, so that the impact of negative returns on the following volatility is higher than the impact of positive returns.

We analyse the stock returns of 20 different equities from the US stock market. Namely the considered equities are Amazon.com, Inc. (AMZN), Alphabet Inc (GOOG), Apple Inc. (APPL), Berkshire Hathaway Inc. (BRK-A), eBay Inc. (EBAY), Facebook Inc. (FB), Intel Corporation (INTC), Johnson & Johnson (JNJ), JPMorgan Chase & Co. (JPM), Mastercard Incorporated (MA), Microsoft Corporation (MSFT), Nestlé S.A. (NSRGY), Netflix, Inc. (NFLX), NVidia Corporation (NVDA), PayPal Holdings, Inc. (PYPL), Tesla (TSLA), The Procter&Gamble Company (PG), The Walt Disney Company (DIS), UnitedHealth Group Incorporated (UNH) and Visa Inc. (V).

We use daily data from *Bloomberg* from *01-01-2000* to *01-01-2020*. As the daily stock returns in Figure 2.2 show a clear pattern of non-stationarity, we test for a non-stationary process with the augmented Dickey-Fuller (ADF) test which never rejects the null hypothesis of non-stationarity.

Therefore, we calculate the first differences of each series and as we are interested in the returns we calculate the monthly logarithm returns and obtain 240 observations in total for each series which is listed since *01-01-2020*. We get 92 observations for FB, 164 for MA, 54 for $PYPL$, 115 for $TSLA$ and 142 for V.

The monthly logarithm returns for each series are given in Figure 2.3 with two standard deviations.

Similar to the Monte Carlo simulation we can investigate the small sample behaviour of our procedure. In a first step, we estimate the three proposed asymmetric GARCH models, the APARCH, the GJR-GARCH and the EGARCH model together with an ARMA$(1,1)$ model.

Note, that the best ARMA(p,q) model chosen by the AIC or BIC can be fitted but as we are interested to test for a change in persistence which includes asymmetric volatility, there is no difference in the procedure. Furthermore, we restrict the GARCH parameters in the asymmetric volatility model to a GARCH$(1,1)$ model.

The estimated coefficients and significance levels for the ARMA$(1,1)$-APARCH$(1,1)$ model are given in Table 2.11. Results for the ARMA$(1,1)$-GJR-GARCH$(1,1)$ and the ARMA$(1,1)$-EGARCH$(1,1)$ model are reported in Table 2.13 and Table 2.14 in the appendix. The estimated coefficients in Table 2.11 prove the thesis of asymmetric volatilities in stock returns. In almost all equities we find evidence for asymmetric volatility which is highly significant. The exceptions are JNJ and $NVDA$. In addition to that, we find very high GARCH effects with β higher than 0.9 in most cases and $\alpha + \beta$ close to unity. The estimated coefficients for the other GARCH specification provide a similar pattern.

Table 2.12 reports the outcome of the persistence change statistics for the monthly stock returns of the 20 equities. As we find evidence for asymmetric volatility in the return series, a test for a change in persistence in the return series implies a test for a change in persistence in an asymmetric volatility model.

We apply the K_j, LT_j, HLT_j, LBI_j, $LKSN$, the L tests and the associated bootstrap versions of the test statistics to the 20 stock return series. In 55% cases the K_1 test finds a change in persistence at the 5% significance level, whereas the associated bootstrap test rejects the null hypothesis of no change in persistence only one time at this level, that is only 5%.

The results for all K_j, LT_j, and HLT_j tests are very similar. The original test rejects the null hypothesis in favor of a change in persistence in 7 to 10 series, while the wild bootstrap reject the null hypothesis only one or two times.

The results of the LBI_j the $LKSN$ and the L tests are superior. The original test statistics reject the null hypothesis only in the LBI_1 test too often, but works well in the other cases close to the bootstrap versions.

One reason of the potential over-rejections of the tests is the asymmetric volatility. The other reason is the different behaviour of the return series over time. In most of the series, there are volatility cluster and periods of lower and higher volatility, which could include a change in the GARCH parameter. This leads to several over-rejections as shown in the Monte Carlo simulation.

The main finding is that tests for a change in persistence suffer from severe size distortions in asymmetric volatility models, while the bootstrap versions correct the size distortions and provide a pivotal limiting distribution, where the test is close to the nominal size level.

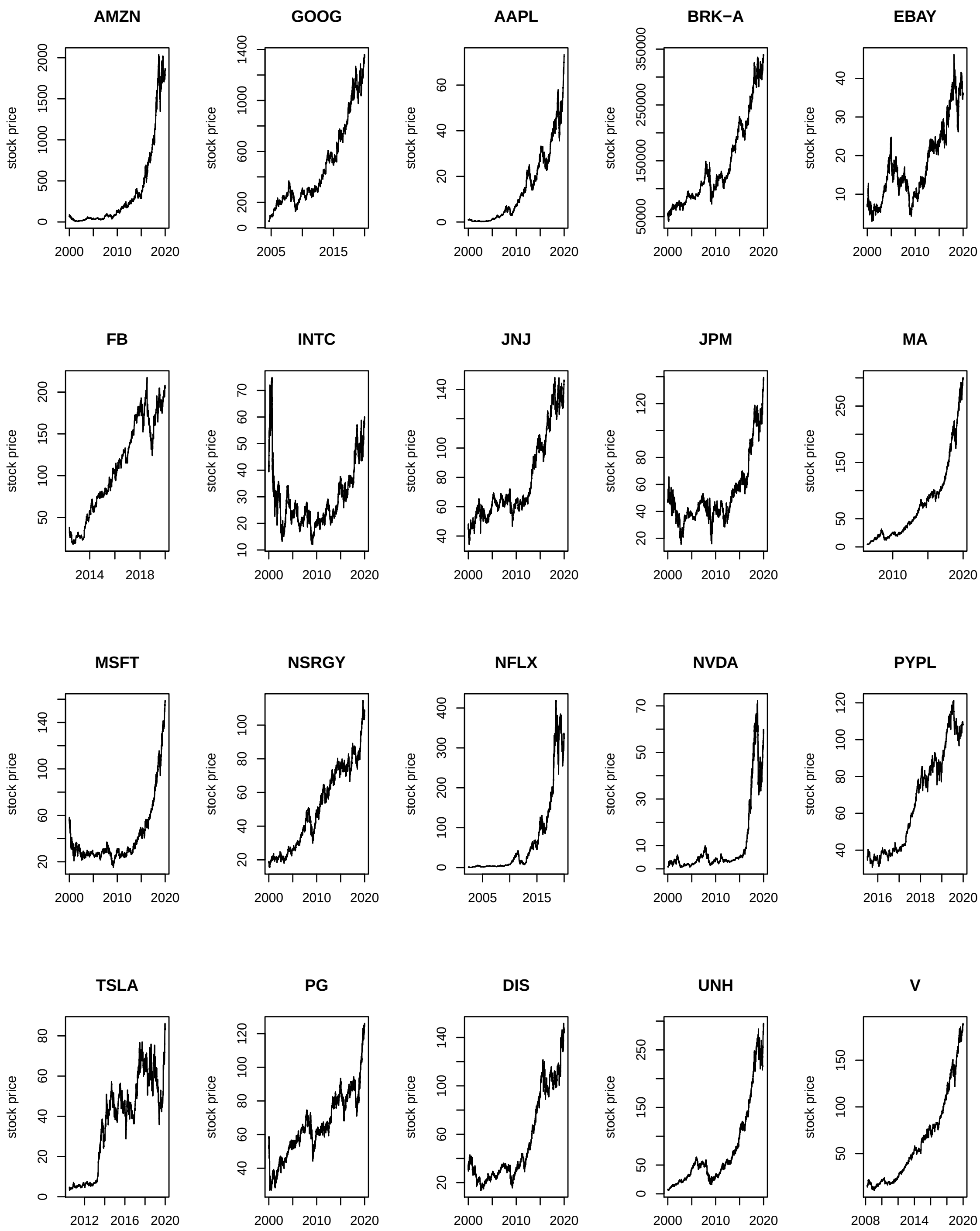

Figure 2.2: The Panel Includes the Daily Stock Prices for 20 US Equities from 2000 to 2020

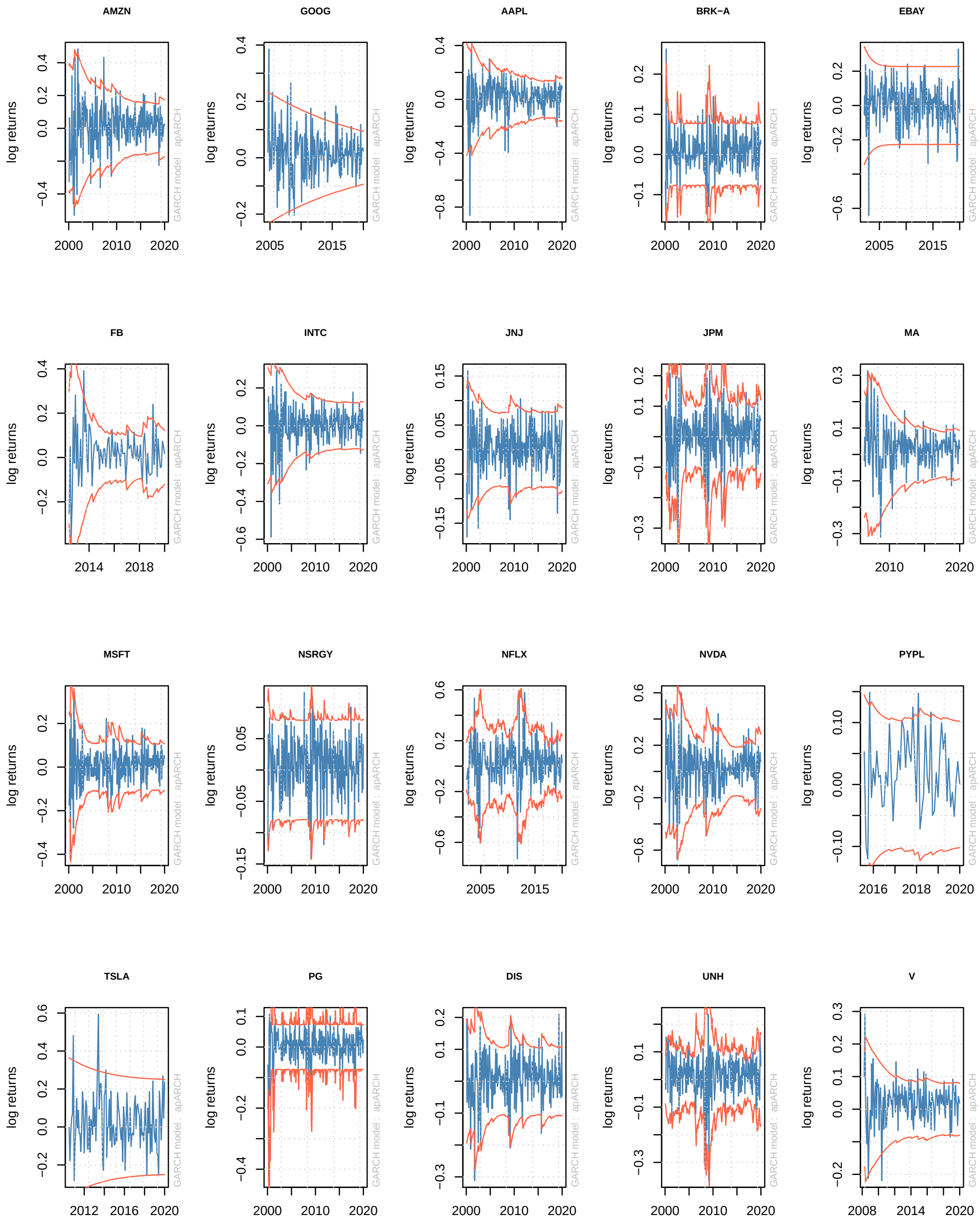

Figure 2.3: The Panel includes the Monthly Logarithmic Returns for 20 US Equities from 2000 to 2020 with 2 Standard Deviations as Red Lines

	β_0	ρ	θ	ω	α	β	c	δ
Amazon.com Inc.	0.0181***	−0.0793	0.8367***	$6.23e^{-06}$	0.0022***	0.9381***	0.9402***	1.2027***
Alphabet Inc.	0.0148***	−0.0645	0.6748***	$2.02e^{-12}$	0.0368	0.9523***	−0.9993***	1.2139***
Apple Inc.	0.0226	0.3763	−0.2914	$4.12e^{-06}$	0.0074	0.9228***	−0.903***	1.4168***
Berkshire Hathaway Inc.	0.0072***	0.0939	−0.9187***	$5.96e^{-06***}$	0.0233	0.4498***	0.9857***	1.3241***
eBay Inc.	0.0235***	−0.0765	0.8536***	$3.73e^{-06***}$	0.0204***	0.9513***	0.3143***	0.6356***
Facebook Inc.	0.0165***	−0.0667	0.8668***	$4.36e^{-06}$	0.0183	0.7764***	0.9976***	0.5249***
Intel Corporation	0.0048	0.0872	−0.9287***	$3.21e^{-06***}$	0.0011***	0.9368***	0.9203***	1.5853***
Johnson & Johnson	0.0049***	0.0598	−0.9272***	$8.52e^{-07}$	0.0118	0.9055***	0.0498	1.3024***
JPMorgan Chase & Co	0.0055	0.0846	−0.9077***	0.0025	0.1308	0.7747***	0.8076***	1.2271***
Mastercard Incorporated	0.0235***	0.4308	−0.5073	$1.15e^{-06}$	0.0058	0.9114***	−0.9732***	0.5350***
Microsoft Corporation	0.0087	0.5524	−0.6208	$6.14e^{-06***}$	0.0357***	0.7997***	0.2819***	1.1614***
Nestlé S.A.	0.0074***	0.1302	−0.9193***	$4.74e^{-06***}$	0.0053	0.6148***	0.9993***	0.7388***
Netflix Inc.	0.0317***	−0.2154	0.9209***	0.0043	0.1004***	0.8913***	0.4004***	0.5019***
NVidia Corporation	0.0220	−0.2319	0.3419	$2.56e^{-05}$	0.0422	0.8719***	0.1426	1.2383***
PayPal Holdings Inc.	0.0221	0.0743	−0.1088	$6.06e^{-06***}$	0.0038	0.7882***	−0.8517***	1.5836***
Tesla	0.0239	0.0117	0.0806	$3.85e^{-05}$	0.0241	0.9676***	0.4357***	1.2156***
The Procter&Gamble Company	0.0039***	0.1021	−0.9736***	$8.08e^{-06***}$	0.0765	0.1236***	0.9690***	1.4091***
The Walt Disney Company	0.0073	−0.0938	0.9563***	$4.38e^{-06***}$	0.0051	0.8609***	0.9235***	0.8195***
UnitedHealth Group Incorporate	0.0166***	−0.0708	−0.0274***	0.0127	0.1747***	0.7850***	0.5054***	0.5947***
Visa Inc.	0.0179***	0.0113	−0.9587***	$7.64e^{-07}$	0.0021	0.9231***	−0.9714***	1.6139***

Table 2.11: Coefficients for Estimated ARMA(1,1)-APARCH(1,1) Model with Significance Levels (*10%, **5%, ***1%)

.

	K_1	K_2	K_3	LT_1	LT_2	LT_3	HLT_1	HLT_2	HLT_3	LBI_1	LBI_2	LBI_3	$LKSN$	L
Amazon.com Inc.	72.111***	31.408**	31.402**	19.088*	7.644***	6.124***	69.829***	30.639**	30.164**	3.622**	1.464	0.809	−1.405	0.197
	(0.012)**	(0.7086)	(0.5175)	(0.0266**)	(0.0484**)	(0.5435)	(0.2783)	(0.0268**)	(0.5785)	(0.6397)	(0.6416)	(0.6014)	(0.5091)	(0.1374)
Alphabet Inc.	116.141	34.649	53.353**	36.586	10.65	13.757	113.076	33.942	51.599**	2.875	1.045	0.601	−1.266	0.697
	(0.964)	(0.3518)	(0.569)	(0.3281)	(0.4467)	(0.3885)	(0.4853)	(0.9342)	(0.3931)	(0.0292**)	(0.1966)	(0.0792*)	(0.7832)	(0.3264)
Apple Inc.	53.98***	17.219***	22.574***	14.869***	4.568***	4.192***	52.782***	16.923***	21.95***	2.912***	0.963	0.561	−1.154	0.418
	(0.1009)	(0.45)	(0.7036)	(0.1348)	(0.3152)	(0.468)	(0.6168)	(0.7565)	(0.6001)	(0.5084)	(0.3361)	(0.1177)	(0.4323)	(0.3663)
Berkshire Hathaway Inc.	8.826***	1.706	1.458	6.204***	1.727	1.052	8.767***	1.697	1.445	0.118	0.081	0.04	−1.082	0.256
	(0.8318)	(0.1732)	(0.7188)	(0.3462)	(0.1665)	(0.9104)	(0.0692*)	(0.8809)	(0.5833)	(0.1752)	(0.116)	(0.6677)	(0.3333)	(0.151)
eBay Inc.	8.333**	3.375	2.111	2.658	1.188	0.632	8.24**	3.346	2.081	0.232	0.129	0.065	−1.728	0.919
	(0.0805*)	(0.3441)	(0.5568)	(0.6663)	(0.6341)	(0.4755)	(0.6882)	(0.8011)	(0.9332)	(0.3832)	(0.5683)	(0.1999)	(0.1203)	(0.3286)
Facebook Inc.	51.919***	10.502	21.923***	7.696***	2.449***	1.545	48.795***	10.012	20.287***	1.385	0.368	0.196	−2.3	2.669***
	(0.8118)	(0.7792)	(0.0322**)	(0.0549*)	(0.3399)	(0.7987)	(0.0887*)	(0.6336)	(0.6781)	(0.3203)	(0.5444)	(0.627)	(0.5131)	(0.3231)
Intel Corporation	8.401***	3.876**	2.304*	2.624*	1.315	0.718	8.26***	3.825**	2.256*	0.766	0.46	0.231	−2.262	0.348
	(0.2709)	(0.5623)	(0.3455)	(0.3023)	(0.1667)	(0.7574)	(0.9179)	(0.0143**)	(0.6212)	(0.2828)	(0.4794)	(0.1556)	(0.8198)	(0.4755)
Johnson & Johnson	5.128*	1.379	0.848	4.993*	2.017	1.211	5.055*	1.364	0.833	0.163	0.104	0.052	−1.958	0.504
	(0.6062)	(0.5918)	(0.2083)	(0.8137)	(0.4075)	(0.1394)	(0.1371)	(0.0758*)	(0.5515)	(0.4251)	(0.3234)	(0.2383)	(0.7295)	(0.4788)
JPMorgan Chase & Co	4.357***	2.059***	1.136	2.29***	1.161	0.591	4.32***	2.045***	1.124	0.69	0.371	0.187	−2.845	0.185
	(0.5735)	(0.3066)	(0.2232)	(0.8789)	(0.9454)	(0.2487)	(0.257)	(0.0524*)	(0.5534)	(0.1228)	(0.8002)	(0.4694)	(0.6289)	(0.364)
Mastercard Incorporated	69.809***	18.465**	30.587***	11.95**	3.685***	3.143***	67.48***	17.988**	29.317***	2.65**	0.784	0.44	−0.882	0.312
	(0.3801)	(0.0944*)	(0.9561)	(0.8934)	(0.2102)	(0.2816)	(0.4123)	(0.6334)	(0.3481)	(0.1898)	(0.5482)	(0.3124)	(0.4217)	(0.6962)
Microsoft Corporation	24.334***	3.637***	7.873***	7.861***	1.607**	1.193*	23.865***	3.583***	7.684***	1.947***	1.303**	0.66	−2.671	0.051
	(0.5518)	(0.3532)	(0.7205)	(0.2416)	(0.3811)	(0.6093)	(0.2382)	(0.0519*)	(0.6128)	(0.5505)	(0.5595)	(0.3752)	(0.5182)	(0.8872)
Nestlé S.A.	4.865***	1.411***	0.887**	6.619***	1.863***	1.34***	4.839***	1.406***	0.881**	0.12	0.062	0.031	−0.726	0.61*
	(0.4808)	(0.6216)	(0.5452)	(0.0421**)	(0.7099)	(0.437)	(0.6917)	(0.3568)	(0.0558*)	(0.7049)	(0.4234)	(0.4605)	(0.2061)	(0.4052)
Netflix Inc.	4.6	1.849	1.054	5.078	1.261	0.787	4.547	1.833	1.039	0.217	0.11	0.055	−1.142	0.317
	(0.7905)	(0.2222)	(0.7184)	(0.7536)	(0.4254)	(0.1152)	(0.9233)	(0.5033)	(0.535)	(0.1059)	(0.7155)	(0.8164)	(0.0736*)	(0.4905)
NVidia Corporation	4.276*	1.297	0.775	8.995**	3.768	2.47	4.188*	1.276	0.755	0.421	0.285	0.143**	−1.988***	0.137**
	(0.669)	(0.5465)	(0.2993)	(0.2497)	(0.7669)	(0.8758)	(0.1413)	(0.5754)	(0.3951)	(0.6028)	(0.387)	(0.2103)	(0.5685)	(0.4139)
PayPal Holdings Inc.	10.649	2.409	2.782	9.892	2.17	2.297	9.898	2.277	2.539	0.521	0.357	0.18	0	0.83
	(0.8992)	(0.9124)	(0.0572*)	(0.2151)	(0.4564)	(0.2535)	(0.3665)	(0.6657)	(0.6235)	(0.4276)	(0.5919)	(0.0755*)	(0.6655)	(0.6476)
Tesla	22.967	3.896	7.707	16.06	2.497	4.55	21.94	3.761	7.278	0.324	0.222	0.112	−2.7	1.168
	(0.596)	(0.8981)	(0.977)	(0.4866)	(0.106)	(0.1951)	(0.4027)	(0.6271)	(0.87)	(0.1825)	(0.2369)	(0.247)	(0.5293)	(0.1539)
The Procter&Gamble Company	14.597	3.485	3.834	3.872	1.364	0.813	13.968	3.369	3.629	1.265	0.398	0.208	−1.516	0.63
	(0.5445)	(0.9842)	(0.9308)	(0.6327)	(0.5207)	(0.6632)	(0.1064)	(0.6589)	(0.5079)	(0.4304)	(0.3156)	(0.609)	(0.8573)	(0.5001)
The Walt Disney Company	9.69	3.305	2.351	5.116	1.781	1.095	9.437	3.239	2.274	1.111	0.602	0.305	−1.786	1.063
	(0.9681)	(0.2007)	(0.374)	(0.3192)	(0.6824)	(0.2897)	(0.1059)	(0.0695*)	(0.6938)	(0.7392)	(0.7459)	(0.8556)	(0.1519)	(0.0784*)
UnitedHealth Group Incorporate	90.755	17.8	40.402	70.403	8.537	30.225	88.061	17.391	38.909	1.288	0.759	0.395	−0.978	0.618
	(0.2699)	(0.897)	(0.6205)	(0.3699)	(0.1791)	(0.3898)	(0.5301)	(0.5414)	(0.6794)	(0.0426**)	(0.7668)	(0.2291)	(0.3606)	(0.2103)
Visa Inc.	13.85	5.701	4.395	4.039	1.722	0.984	13.301	5.525	4.178	0.396	0.211	0.107	−1.783	0.156
	(0.5807)	(0.3259)	(0.7701)	(0.7967)	(0.1845)	(0.5354)	(0.6832)	(0.8091)	(0.6833)	(0.5382)	(0.5392)	(0.5281)	(0.3222)	(0.6122)

Table 2.12: Test Statistics and Bootstrap p-values for K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests with Significance Levels (*10%, **5%, ***1%)

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2.7 Conclusion

In this paper, we analysed the behaviour of different tests for a change in persistence with the null hypothesis of H_0 or H_1 against the alternative of a persistence change H_{01} and H_{10}. We focussed on the versions of the tests, which reject in both directions and have power against both alternatives. In this context we assume an unknown change point direction as it is useful in empirical studies.

We presented the classical model for a change in persistence, following Kim (2000) among others, and presented a class of GARCH models following Hentschel (1995), which nests several classical symmetric GARCH models as well as the asymmetric GARCH models, namely the APARCH, the GJR-GARCH and the EGARCH model, which are under investigation.

After defining different ratio based tests and modifications by Kim (2000), Kim et al. (2002), Busetti and Taylor (2004), Leybourne and Taylor (2004), Harvey et al. (2006), the LBI test by Busetti and Taylor (2004), the regression based test by Leybourne et al. (2003) and the CUSUM of squares based test by Leybourne et al. (2007b), we propose the wild bootstrap version of all the test statistics to replicate the pattern of heteroskecasticity in the bootstrap samples.

As Cavaliere and Taylor (2008) show, that the test statistics of tests for a change in persistence do not follow a pivotal limiting distribution, while the bootstrap versions obtain a pivotal limiting distribution, that is the distribution is independent of unknown parameters.

In a Monte Carlo simulation study, we analysed the behaviour of the tests under constant persistence, H_0 and H_1, and under a change in persistence, H_{01} and H_{10} in small samples, $T = 100$ and $T = 250$.

Our main findings are that the tests suffer from severe size distortions due to the non-pivotal limiting distribution. The rejection frequencies are too large to apply them meaningful in practice. The associated bootstrap versions of the test statistics provide much better results. The rejection frequencies are close to the nominal size and still, they are competitive in terms of power. The bootstrap versions provide the necessary pivotal limiting distribution with rejection frequencies close to the nominal significance level.

In an empirical application to the US stock market, we found evidence for asymmetric volatility in the monthly stock returns of 20 different equities from *01-01-2000* to *01-01-2020*. The different tests for changes in persistence reject the null hypothesis of no change in persistence in favor of the alternative in many cases, while the bootstrap versions, which hold the nominal size level, show other results.

The recommendation in practice is a pretesting for asymmetric volatility before applying a test for a change in persistence to a time series. If there is evidence for asymmetric volatility, the wild bootstrap version of the test statistic is preferable.

As already noted the study is extendable to higher constant persistence than $\rho = 0$ and to a change in persistence from a stationary process with higher persistence than $\rho = 0$ to $\rho = 1$ and vice versa. Further possible extensions are an additional moving-average parameter in the DGP, changing the assumptions of the noise term η which is $N(0, 1)$, and can be generated by a long memory process with parameter d or allow for more breaks in persistence at different time in the series.

2.8 Appendix

	β_0	ρ	θ	ω	α	β	c
Amazon	0.0154	−0.0796	0.848***	0.005858	0.0358***	0.9147***	0.0993
Alphabet	0.0196***	−0.6256	0.6829***	$5.11e-11$	0.0314***	0.9623***	−0.0746***
Apple	0.0253***	0.441	−0.3408	0.009921	0.2745	0.8329***	−0.217***
Berkshire Hathaway	0.0078***	0.4201	−0.4831	0.004812	0.2486	0.5249	0.3167***
eBay	0.0359	0.15111	−0.21438	$3.13e-06$	0.1358	0.81753***	−0.0393
Facebook	0.0136	−0.0709	0.8584***	0.006425	0	0.8236***	0.2959***
Intel Corporation	0.0045	0.0861	−0.9322***	0.003171	0.0875	0.9062***	−0.0662
Johnson Johnson	0.0045***	0.0864	−0.9311***	0.003986	0.0439	0.8787***	0.0329
JPMorgan Chase Co	0.0057	0.0835	−0.9043***	0.004206	0.2589***	0.7518***	0.284***
Mastercard Incorporated	0.001868***	−0.021598	0.1835	0.001836***	0.1198	0.81985***	−0.23183***
Microsoft Corporation	0.0074	0.0927	−0.9793***	0.001444***	0.2237***	0.751***	0.2878***
Nestlé SA	0.008***	0.025	−0.0599	$1.49e-05$***	0.0084***	0.9812***	−0.0281***
Netflix Inc	0.0313	−0.0521	0.8987***	0.006732	0.07	0.7745***	0.0963
NVidia Corporation	0.0204	−0.3575	0.4438	0.006713	0.1004	0.8731***	0.0206
PayPal Holdings Inc	0.0209	0.1123	−0.1622	$2.67e-05$	0.0611**	0.9385***	−0.0224
Tesla	0.0305***	−0.0201	0.1629	0.008325***	0.023	0.9946***	−0.1353***
The Procter and Gamble Company	0.0038***	0.0966	−0.9709***	0.005536	0.1295***	0.861***	0.7209***
The Walt Disney Company	0.0075	−0.0835	0.9504***	0.003293	0.1546***	0.8154***	0.1827***
UnitedHealth Group Incorporate	0.0183***	−0.2	0.0958	0.003227	0.0937	0.7445***	0.1265***
Visa Inc	0.0183***	0.09236	−1***	$1.36e-05$	0.0806***	0.9767***	−0.1166***

Table 2.13: Coefficients for Estimated ARMA(1,1)-GJR-GARCH(1,1) Model with Significance Level (*10%, **5%, ***1%).

	β_0	ρ	θ	ω	α	β	c
Amazon	0.0111	−0.1671	0.1792	−0.11951	−0.2459	0.9627	−0.1859***
Alphabet	0.0219***	−0.6885	0.7278***	−0.44102	0.0605	0.9047***	0.2916***
Apple	0.027***	0.033	−0.226	−0.12735	0.0859	0.9713***	0.3094***
Berkshire Hathaway	0.0071***	0.0939	−1***	−2.05535	−0.2026***	0.6677***	0.2412***
eBay	0.0014***	−0.0509	0.5316***	−0.43518	0.1958	0.9173***	0.4215***
Facebook	0.0047***	−0.0812	0.8832***	−0.00834***	−0.3641***	0.9717***	−0.2521***
Intel Corporation	0.0058	−0.0765	0.8294***	−0.07315	0.0715	0.9922***	0.1534***
Johnson Johnson	0.0043***	0.0891	−0.938***	−0.39173	−0.0337	0.9386***	0.16
JPMorgan Chase Co	0.005	0.0799	−0.8479***	−0.44555***	−0.2156***	0.9181***	0.1303***
Mastercard Incorporated	0.01467***	−0.1641	0.06743	−0.28176	−0.05273	0.91375***	0.35131***
Microsoft Corporation	0.0068***	0.0296	−0.1293	−0.77918***	−0.2114***	0.8659***	0.1882***
Nestlé SA	0.0069	−0.0853	0.0855	−1.75954	−0.1501	0.7231	0.0793
Netflix Inc	0.0297	0.0718	−0.6242***	−0.59379	−0.1537	0.8452***	0.1755***
NVidia Corporation	0.0197***	−0.0317	0.5055***	−0.07934	0.0112	0.979***	0.2252***
PayPal Holdings Inc	0.0155***	0.0966	−0.9913***	−0.10026***	−0.1538***	0.9649***	−0.4503***
Tesla	0.0222	−0.1084	0.1653***	−3.48479***	0.2476	0.1391	0.2371***
The Procter and Gamble Company	0.0041***	0.0909	−0.9557***	−1.61163***	−0.2919***	0.7411***	0.4449***
The Walt Disney Company	0.0065	−0.0888	0.9491***	−0.50842	−0.1655	0.9031***	0.1046***
UnitedHealth Group Incorporate	0.0176***	−0.1731	0.0731	−0.32931	−0.1112	0.9374***	0.2557***
Visa Inc	0.0186***	0.0915	−1***	0.018609***	0.1606***	0.8343***	0.0205

Table 2.14: Coefficients for Estimated ARMA(1,1)-EGARCH(1,1) Model with Significance Levels (*10%, **5%, ***1%).

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$ 10%	5%	1%
K_1	0.097(0.019)	0.063(0.000)	0.044(0.000)	0.120(0.073)	0.030(0.018)	0.045(0.000)	0.329(0.009)	0.299(0.000)	0.123(0.000)	0.469(0.049)	0.289(0.000)	0.213(0.006)
K_2	0.138(0.093)	0.092(0.000)	0.002(0.028)	0.229(0.077)	0.095(0.000)	0.000(0.000)	0.362(0.104)	0.206(0.007)	0.144(0.000)	0.382(0.102)	0.337(0.079)	0.166(0.000)
K_3	0.183(0.006)	0.071(0.038)	0.039(0.009)	0.209(0.038)	0.119(0.000)	0.000(0.029)	0.390(0.066)	0.220(0.025)	0.090(0.000)	0.438(0.100)	0.291(0.010)	0.226(0.038)
LT_1	0.043(0.151)	0.048(0.064)	0.017(0.000)	0.050(0.147)	0.019(0.002)	0.000(0.000)	0.166(0.075)	0.034(0.077)	0.000(0.030)	0.105(0.093)	0.031(0.063)	0.043(0.020)
LT_2	0.067(0.089)	0.010(0.038)	0.000(0.063)	0.070(0.118)	0.027(0.109)	0.000(0.028)	0.124(0.156)	0.017(0.006)	0.027(0.000)	0.119(0.146)	0.058(0.030)	0.049(0.000)
LT_3	0.054(0.155)	0.063(0.004)	0.002(0.025)	0.073(0.141)	0.065(0.007)	0.000(0.000)	0.079(0.155)	0.026(0.055)	0.020(0.000)	0.108(0.117)	0.077(0.046)	0.000(0.013)
HLT_1	0.170(0.077)	0.074(0.026)	0.015(0.013)	0.132(0.055)	0.081(0.050)	0.017(0.000)	0.348(0.040)	0.230(0.016)	0.091(0.035)	0.477(0.120)	0.289(0.051)	0.216(0.033)
HLT_2	0.145(0.125)	0.115(0.000)	0.033(0.000)	0.209(0.074)	0.060(0.000)	0.049(0.031)	0.394(0.043)	0.281(0.054)	0.094(0.000)	0.331(0.035)	0.326(0.044)	0.127(0.000)
HLT_3	0.139(0.064)	0.098(0.022)	0.045(0.013)	0.100(0.031)	0.022(0.000)	0.062(0.018)	0.372(0.018)	0.257(0.025)	0.103(0.021)	0.466(0.122)	0.311(0.018)	0.198(0.016)
LBI_1	0.137(0.110)	0.045(0.088)	0.026(0.063)	0.158(0.175)	0.111(0.131)	0.020(0.057)	0.164(0.081)	0.055(0.064)	0.035(0.000)	0.110(0.055)	0.051(0.023)	0.021(0.000)
LBI_2	0.114(0.059)	0.103(0.029)	0.000(0.052)	0.109(0.134)	0.142(0.107)	0.000(0.060)	0.155(0.119)	0.059(0.047)	0.010(0.000)	0.049(0.028)	0.054(0.028)	0.000(0.000)
LBI_3	0.109(0.069)	0.078(0.018)	0.011(0.000)	0.134(0.139)	0.134(0.085)	0.063(0.030)	0.126(0.095)	0.040(0.038)	0.018(0.028)	0.006(0.085)	0.038(0.050)	0.000(0.000)
$LKSN$	1.000(0.125)	0.966(0.927)	0.981(0.942)	1.000(1.000)	0.995(0.952)	1.000(0.887)	1.000(0.987)	1.000(1.000)	1.000(0.965)	0.999(0.945)	0.985(0.931)	1.000(0.922)
L	0.026(0.120)	0.000(0.056)	0.005(0.021)	0.000(0.167)	0.000(0.053)	0.011(0.048)	0.014(0.186)	0.013(0.061)	0.022(0.000)	0.020(0.105)	0.025(0.000)	0.035(0.054)

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$ 10%	5%	1%	$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$ 10%	5%	1%
K_1	0.684(0.115)	0.624(0.020)	0.509(0.000)	0.829(0.090)	0.753(0.051)	0.617(0.044)	0.308(0.033)	0.159(0.030)	0.079(0.025)	0.338(0.082)	0.299(0.054)	0.195(0.055)
K_2	0.753(0.084)	0.659(0.109)	0.506(0.040)	0.878(0.048)	0.792(0.051)	0.666(0.037)	0.219(0.086)	0.153(0.043)	0.114(0.000)	0.353(0.110)	0.317(0.020)	0.110(0.059)
K_3	0.709(0.116)	0.609(0.097)	0.558(0.054)	0.799(0.074)	0.722(0.067)	0.614(0.023)	0.249(0.012)	0.218(0.023)	0.120(0.016)	0.351(0.128)	0.323(0.060)	0.190(0.000)
LT_1	0.179(0.191)	0.111(0.055)	0.046(0.070)	0.245(0.194)	0.130(0.164)	0.018(0.048)	0.106(0.144)	0.012(0.029)	0.014(0.053)	0.103(0.082)	0.098(0.035)	0.033(0.057)
LT_2	0.113(0.124)	0.034(0.000)	0.000(0.037)	0.195(0.106)	0.082(0.134)	0.062(0.034)	0.062(0.027)	0.029(0.054)	0.010(0.000)	0.122(0.110)	0.051(0.060)	0.000(0.000)
LT_3	0.219(0.155)	0.100(0.102)	0.014(0.059)	0.279(0.175)	0.159(0.125)	0.046(0.000)	0.061(0.063)	0.024(0.011)	0.045(0.037)	0.075(0.116)	0.074(0.026)	0.079(0.000)
HLT_1	0.725(0.095)	0.622(0.071)	0.510(0.000)	0.846(0.030)	0.705(0.014)	0.584(0.000)	0.198(0.071)	0.200(0.062)	0.062(0.000)	0.339(0.078)	0.299(0.052)	0.177(0.047)
HLT_2	0.725(0.055)	0.672(0.097)	0.513(0.039)	0.822(0.116)	0.736(0.039)	0.659(0.000)	0.169(0.055)	0.180(0.012)	0.131(0.000)	0.370(0.082)	0.278(0.012)	0.159(0.045)
HLT_3	0.674(0.048)	0.594(0.042)	0.546(0.047)	0.873(0.020)	0.702(0.020)	0.562(0.020)	0.277(0.040)	0.137(0.000)	0.092(0.013)	0.332(0.120)	0.315(0.051)	0.110(0.016)
LBI_1	0.247(0.141)	0.132(0.100)	0.050(0.010)	0.208(0.128)	0.208(0.106)	0.051(0.032)	0.063(0.000)	0.030(0.000)	0.000(0.028)	0.137(0.136)	0.046(0.048)	0.048(0.023)
LBI_2	0.099(0.057)	0.146(0.021)	0.064(0.029)	0.210(0.156)	0.161(0.093)	0.081(0.055)	0.091(0.064)	0.008(0.030)	0.000(0.023)	0.062(0.046)	0.031(0.069)	0.066(0.006)
LBI_3	0.145(0.091)	0.097(0.014)	0.046(0.000)	0.199(0.147)	0.158(0.099)	0.079(0.010)	0.013(0.014)	0.020(0.014)	0.029(0.055)	0.062(0.028)	0.089(0.099)	0.004(0.000)
$LKSN$	0.984(0.962)	0.980(0.987)	0.961(0.896)	0.961(0.952)	1.000(0.923)	1.000(0.927)	0.972(0.983)	0.992(0.963)	0.969(0.913)	1.000(0.998)	0.995(1.000)	1.000(0.983)
L	0.000(0.155)	0.000(0.051)	0.000(0.034)	0.009(0.115)	0.001(0.023)	0.013(0.037)	0.000(0.098)	0.044(0.071)	0.001(0.035)	0.048(0.116)	0.013(0.070)	0.000(0.026)

α	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$ 10%	5%	1%	$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$ 10%	5%	1%	$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$ 10%	5%	1%	$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$ 10%	5%	1%
K_1	0.295(0.034)	0.231(0.014)	0.130(0.000)	0.455(0.061)	0.376(0.033)	0.214(0.050)	0.448(0.040)	0.387(0.065)	0.150(0.000)	0.650(0.000)	0.536(0.077)	0.358(0.000)
K_2	0.343(0.072)	0.221(0.045)	0.137(0.000)	0.500(0.023)	0.444(0.045)	0.264(0.042)	0.434(0.088)	0.413(0.024)	0.161(0.000)	0.628(0.021)	0.483(0.061)	0.403(0.000)
K_3	0.291(0.089)	0.163(0.014)	0.109(0.000)	0.486(0.005)	0.420(0.040)	0.227(0.000)	0.492(0.069)	0.348(0.074)	0.188(0.007)	0.645(0.071)	0.501(0.012)	0.354(0.034)
LT_1	0.105(0.085)	0.110(0.039)	0.000(0.066)	0.096(0.091)	0.092(0.032)	0.038(0.000)	0.069(0.135)	0.012(0.033)	0.000(0.009)	0.215(0.193)	0.115(0.074)	0.036(0.000)
LT_2	0.057(0.093)	0.070(0.108)	0.061(0.062)	0.080(0.105)	0.038(0.000)	0.004(0.053)	0.055(0.160)	0.003(0.021)	0.000(0.036)	0.170(0.107)	0.013(0.035)	0.015(0.011)
LT_3	0.137(0.132)	0.073(0.104)	0.008(0.023)	0.062(0.088)	0.136(0.089)	0.019(0.020)	0.043(0.180)	0.035(0.067)	0.002(0.000)	0.256(0.194)	0.103(0.089)	0.093(0.000)
HLT_1	0.245(0.039)	0.166(0.057)	0.084(0.000)	0.444(0.000)	0.380(0.000)	0.216(0.037)	0.417(0.107)	0.338(0.031)	0.222(0.030)	0.614(0.002)	0.519(0.049)	0.360(0.021)
HLT_2	0.313(0.099)	0.246(0.063)	0.065(0.000)	0.563(0.000)	0.380(0.000)	0.279(0.000)	0.483(0.087)	0.412(0.000)	0.148(0.000)	0.652(0.070)	0.513(0.021)	0.385(0.049)
HLT_3	0.278(0.042)	0.169(0.035)	0.129(0.001)	0.458(0.059)	0.435(0.024)	0.173(0.000)	0.432(0.032)	0.333(0.026)	0.128(0.000)	0.597(0.033)	0.465(0.064)	0.377(0.023)
LBI_1	0.136(0.094)	0.053(0.032)	0.055(0.061)	0.120(0.141)	0.034(0.023)	0.005(0.000)	0.168(0.165)	0.158(0.044)	0.005(0.039)	0.135(0.083)	0.121(0.076)	0.091(0.044)
LBI_2	0.165(0.044)	0.054(0.000)	0.031(0.050)	0.043(0.027)	0.021(0.029)	0.000(0.052)	0.180(0.145)	0.085(0.085)	0.080(0.055)	0.121(0.063)	0.110(0.094)	0.010(0.000)
LBI_3	0.094(0.033)	0.000(0.000)	0.000(0.000)	0.074(0.056)	0.076(0.002)	0.041(0.056)	0.184(0.186)	0.064(0.049)	0.060(0.010)	0.132(0.034)	0.102(0.066)	0.061(0.060)
$LKSN$	1.000(0.957)	1.000(0.972)	1.000(0.905)	0.961(0.958)	1.000(0.916)	0.976(0.876)	1.000(1.000)	1.000(0.877)	0.975(0.918)	0.984(0.979)	1.000(0.919)	0.986(0.961)
L	0.028(0.065)	0.036(0.043)	0.000(0.055)	0.001(0.084)	0.047(0.068)	0.000(0.000)	0.050(0.087)	0.000(0.067)	0.046(0.000)	0.020(0.069)	0.000(0.087)	0.023(0.019)

Table 2.15: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_0}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.140(0.092)	0.062(0.059)	0.021(0.031)	0.115(0.077)	0.081(0.032)	0.046(0.000)	0.376(0.034)	0.234(0.000)	0.111(0.000)	0.386(0.083)	0.314(0.000)	0.171(0.000)
K_2	0.123(0.124)	0.087(0.000)	0.000(0.023)	0.220(0.068)	0.064(0.014)	0.053(0.017)	0.364(0.042)	0.271(0.056)	0.174(0.013)	0.372(0.119)	0.290(0.020)	0.187(0.000)
K_3	0.154(0.067)	0.036(0.042)	0.011(0.028)	0.129(0.022)	0.048(0.060)	0.012(0.000)	0.376(0.000)	0.298(0.014)	0.124(0.000)	0.406(0.074)	0.341(0.000)	0.212(0.000)
LT_1	0.014(0.107)	0.012(0.052)	0.012(0.000)	0.004(0.124)	0.047(0.004)	0.026(0.000)	0.129(0.127)	0.054(0.116)	0.000(0.000)	0.077(0.131)	0.065(0.078)	0.000(0.050)
LT_2	0.101(0.093)	0.000(0.034)	0.019(0.026)	0.065(0.092)	0.000(0.065)	0.045(0.048)	0.100(0.096)	0.090(0.084)	0.045(0.028)	0.118(0.088)	0.044(0.024)	0.039(0.050)
LT_3	0.059(0.133)	0.002(0.000)	0.000(0.000)	0.048(0.147)	0.009(0.016)	0.000(0.000)	0.085(0.150)	0.070(0.107)	0.041(0.000)	0.100(0.115)	0.067(0.001)	0.064(0.000)
HLT_1	0.153(0.054)	0.081(0.000)	0.000(0.043)	0.105(0.081)	0.071(0.064)	0.016(0.000)	0.382(0.059)	0.259(0.023)	0.084(0.022)	0.449(0.130)	0.362(0.047)	0.154(0.030)
HLT_2	0.139(0.091)	0.089(0.000)	0.031(0.013)	0.161(0.061)	0.056(0.033)	0.000(0.000)	0.323(0.094)	0.214(0.031)	0.159(0.000)	0.345(0.060)	0.268(0.024)	0.168(0.023)
HLT_3	0.140(0.079)	0.044(0.000)	0.048(0.039)	0.115(0.017)	0.033(0.000)	0.050(0.000)	0.363(0.000)	0.256(0.037)	0.074(0.030)	0.401(0.100)	0.341(0.000)	0.137(0.032)
LBI_1	0.147(0.094)	0.046(0.053)	0.055(0.000)	0.117(0.154)	0.155(0.072)	0.075(0.048)	0.128(0.073)	0.090(0.048)	0.007(0.017)	0.084(0.012)	0.000(0.061)	0.057(0.000)
LBI_2	0.127(0.131)	0.035(0.056)	0.000(0.004)	0.137(0.159)	0.091(0.082)	0.036(0.008)	0.153(0.146)	0.037(0.072)	0.076(0.045)	0.033(0.051)	0.010(0.062)	0.000(0.044)
LBI_3	0.081(0.080)	0.033(0.032)	0.034(0.011)	0.113(0.114)	0.088(0.107)	0.057(0.019)	0.102(0.080)	0.024(0.021)	0.078(0.057)	0.053(0.075)	0.000(0.022)	0.000(0.003)
$LKSN$	1.000(0.060)	1.000(0.964)	0.970(0.908)	1.000(1.000)	1.000(0.950)	1.000(0.902)	1.000(1.000)	0.983(0.995)	0.963(0.914)	1.000(0.956)	1.000(0.920)	0.976(0.882)
L	0.000(0.068)	0.005(0.048)	0.000(0.042)	0.045(0.139)	0.000(0.071)	0.000(0.000)	0.034(0.109)	0.007(0.055)	0.000(0.000)	0.000(0.065)	0.000(0.052)	0.000(0.000)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.765(0.049)	0.587(0.026)	0.541(0.048)	0.810(0.027)	0.718(0.001)	0.589(0.000)	0.244(0.046)	0.177(0.000)	0.123(0.000)	0.342(0.100)	0.254(0.063)	0.139(0.019)
K_2	0.774(0.054)	0.660(0.121)	0.537(0.016)	0.853(0.035)	0.761(0.036)	0.618(0.045)	0.225(0.020)	0.124(0.040)	0.082(0.008)	0.384(0.126)	0.249(0.061)	0.113(0.005)
K_3	0.717(0.051)	0.654(0.027)	0.534(0.006)	0.850(0.102)	0.774(0.018)	0.651(0.001)	0.286(0.047)	0.156(0.039)	0.068(0.000)	0.352(0.046)	0.295(0.075)	0.180(0.000)
LT_1	0.185(0.177)	0.123(0.104)	0.062(0.033)	0.278(0.178)	0.124(0.156)	0.027(0.006)	0.124(0.135)	0.060(0.005)	0.000(0.048)	0.101(0.129)	0.104(0.028)	0.076(0.050)
LT_2	0.059(0.123)	0.092(0.033)	0.000(0.000)	0.143(0.184)	0.059(0.073)	0.013(0.077)	0.082(0.065)	0.001(0.030)	0.000(0.052)	0.108(0.091)	0.031(0.020)	0.041(0.005)
LT_3	0.203(0.158)	0.100(0.045)	0.026(0.027)	0.237(0.202)	0.137(0.121)	0.065(0.024)	0.039(0.120)	0.028(0.021)	0.016(0.034)	0.091(0.055)	0.023(0.026)	0.069(0.010)
HLT_1	0.671(0.056)	0.638(0.077)	0.539(0.000)	0.764(0.111)	0.713(0.002)	0.624(0.000)	0.195(0.031)	0.148(0.034)	0.036(0.026)	0.339(0.094)	0.286(0.055)	0.112(0.000)
HLT_2	0.727(0.061)	0.701(0.056)	0.534(0.000)	0.832(0.093)	0.724(0.041)	0.657(0.002)	0.172(0.024)	0.163(0.034)	0.080(0.000)	0.376(0.084)	0.299(0.000)	0.166(0.033)
HLT_3	0.727(0.075)	0.628(0.094)	0.498(0.041)	0.823(0.032)	0.766(0.014)	0.600(0.000)	0.193(0.000)	0.127(0.043)	0.098(0.056)	0.353(0.077)	0.264(0.058)	0.115(0.000)
LBI_1	0.192(0.153)	0.162(0.115)	0.105(0.029)	0.274(0.132)	0.193(0.069)	0.101(0.000)	0.124(0.040)	0.044(0.009)	0.000(0.035)	0.116(0.077)	0.071(0.070)	0.000(0.034)
LBI_2	0.150(0.082)	0.133(0.027)	0.098(0.000)	0.254(0.132)	0.113(0.047)	0.097(0.057)	0.023(0.045)	0.040(0.000)	0.036(0.015)	0.064(0.085)	0.093(0.015)	0.070(0.000)
LBI_3	0.187(0.096)	0.110(0.066)	0.083(0.000)	0.204(0.082)	0.143(0.112)	0.097(0.028)	0.050(0.079)	0.053(0.017)	0.000(0.061)	0.132(0.096)	0.069(0.029)	0.032(0.018)
$LKSN$	1.000(0.972)	0.973(0.943)	0.966(0.896)	0.971(0.993)	0.978(0.899)	1.000(0.903)	1.000(0.999)	0.980(0.945)	1.000(0.918)	1.000(0.993)	1.000(0.945)	0.978(0.983)
L	0.022(0.116)	0.000(0.050)	0.039(0.015)	0.006(0.173)	0.038(0.111)	0.024(0.023)	0.000(0.115)	0.037(0.000)	0.002(0.000)	0.036(0.042)	0.011(0.010)	0.000(0.072)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.299(0.117)	0.213(0.001)	0.123(0.034)	0.511(0.053)	0.403(0.025)	0.181(0.036)	0.433(0.110)	0.403(0.017)	0.186(0.000)	0.685(0.063)	0.530(0.039)	0.324(0.027)
K_2	0.371(0.128)	0.262(0.030)	0.134(0.005)	0.563(0.009)	0.387(0.010)	0.267(0.000)	0.463(0.096)	0.376(0.000)	0.159(0.032)	0.692(0.021)	0.562(0.058)	0.358(0.005)
K_3	0.267(0.046)	0.152(0.047)	0.131(0.000)	0.438(0.001)	0.386(0.000)	0.184(0.000)	0.412(0.033)	0.362(0.041)	0.152(0.000)	0.643(0.035)	0.448(0.008)	0.337(0.065)
LT_1	0.106(0.088)	0.093(0.098)	0.021(0.074)	0.147(0.133)	0.055(0.040)	0.014(0.015)	0.085(0.117)	0.072(0.111)	0.003(0.035)	0.261(0.197)	0.139(0.117)	0.036(0.014)
LT_2	0.063(0.097)	0.099(0.102)	0.030(0.056)	0.036(0.032)	0.000(0.004)	0.059(0.004)	0.061(0.166)	0.065(0.092)	0.000(0.000)	0.162(0.142)	0.016(0.056)	0.040(0.000)
LT_3	0.114(0.118)	0.032(0.060)	0.000(0.000)	0.071(0.129)	0.065(0.083)	0.007(0.042)	0.063(0.101)	0.095(0.076)	0.018(0.004)	0.285(0.191)	0.174(0.100)	0.022(0.017)
HLT_1	0.276(0.113)	0.192(0.085)	0.100(0.046)	0.469(0.000)	0.396(0.000)	0.213(0.000)	0.467(0.067)	0.357(0.067)	0.171(0.000)	0.627(0.070)	0.476(0.031)	0.338(0.000)
HLT_2	0.337(0.139)	0.231(0.011)	0.075(0.000)	0.585(0.000)	0.393(0.000)	0.205(0.048)	0.420(0.026)	0.369(0.000)	0.218(0.014)	0.653(0.061)	0.542(0.062)	0.418(0.000)
HLT_3	0.310(0.032)	0.158(0.039)	0.136(0.032)	0.516(0.000)	0.381(0.048)	0.204(0.001)	0.495(0.096)	0.367(0.002)	0.213(0.000)	0.638(0.031)	0.442(0.052)	0.338(0.009)
LBI_1	0.113(0.138)	0.062(0.075)	0.075(0.000)	0.034(0.102)	0.089(0.079)	0.051(0.000)	0.132(0.088)	0.119(0.030)	0.028(0.023)	0.106(0.071)	0.130(0.081)	0.022(0.000)
LBI_2	0.158(0.093)	0.020(0.010)	0.000(0.000)	0.060(0.042)	0.056(0.043)	0.020(0.044)	0.144(0.165)	0.088(0.049)	0.065(0.030)	0.098(0.060)	0.098(0.056)	0.067(0.029)
LBI_3	0.163(0.113)	0.078(0.004)	0.055(0.025)	0.093(0.059)	0.056(0.075)	0.000(0.000)	0.223(0.164)	0.078(0.035)	0.041(0.021)	0.137(0.055)	0.067(0.029)	0.035(0.009)
$LKSN$	0.971(0.993)	0.965(0.926)	0.964(0.915)	1.000(0.957)	1.000(0.970)	1.000(0.873)	1.000(1.000)	1.000(0.936)	1.000(0.883)	0.979(1.000)	1.000(0.945)	1.000(0.927)
L	0.002(0.078)	0.032(0.065)	0.036(0.053)	0.031(0.072)	0.000(0.001)	0.000(0.008)	0.009(0.132)	0.014(0.051)	0.011(0.025)	0.018(0.124)	0.021(0.087)	0.006(0.019)

Table 2.16: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** model in $\mathbf{H_0}$ with $\mathbf{T=250}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.908(0.755)	0.797(0.651)	0.737(0.448)	0.829(0.609)	0.771(0.486)	0.676(0.342)	0.842(0.613)	0.782(0.496)	0.697(0.396)	0.843(0.588)	0.785(0.475)	0.652(0.381)
K_2	0.792(0.595)	0.687(0.513)	0.579(0.369)	0.773(0.512)	0.697(0.422)	0.556(0.265)	0.813(0.551)	0.739(0.516)	0.565(0.339)	0.778(0.540)	0.749(0.457)	0.609(0.323)
K_3	0.900(0.763)	0.834(0.605)	0.777(0.439)	0.834(0.637)	0.823(0.512)	0.654(0.364)	0.883(0.622)	0.812(0.583)	0.633(0.424)	0.822(0.615)	0.796(0.539)	0.698(0.448)
LT_1	0.294(0.270)	0.205(0.193)	0.107(0.049)	0.328(0.300)	0.262(0.253)	0.061(0.054)	0.349(0.275)	0.152(0.190)	0.084(0.054)	0.281(0.312)	0.155(0.152)	0.070(0.145)
LT_2	0.261(0.236)	0.141(0.137)	0.083(0.074)	0.274(0.113)	0.090(0.103)	0.007(0.000)	0.179(0.116)	0.119(0.090)	0.088(0.021)	0.276(0.197)	0.119(0.043)	0.036(0.020)
LT_3	0.295(0.341)	0.234(0.188)	0.141(0.073)	0.276(0.321)	0.202(0.230)	0.107(0.138)	0.337(0.277)	0.151(0.234)	0.107(0.146)	0.306(0.331)	0.213(0.203)	0.102(0.142)
HLT_1	0.096(0.066)	0.115(0.039)	0.079(0.000)	0.197(0.052)	0.108(0.092)	0.093(0.000)	0.167(0.059)	0.162(0.081)	0.099(0.098)	0.255(0.116)	0.262(0.096)	0.203(0.032)
HLT_2	0.143(0.027)	0.089(0.036)	0.030(0.000)	0.084(0.060)	0.086(0.009)	0.080(0.000)	0.233(0.110)	0.148(0.039)	0.109(0.073)	0.220(0.116)	0.250(0.077)	0.093(0.025)
HLT_3	0.111(0.028)	0.084(0.033)	0.044(0.056)	0.227(0.078)	0.170(0.095)	0.078(0.028)	0.250(0.182)	0.183(0.211)	0.189(0.146)	0.359(0.235)	0.270(0.168)	0.256(0.171)
LBI_1	0.952(0.983)	1.000(0.982)	1.000(1.000)	1.000(1.000)	0.993(0.974)	0.958(0.964)	0.973(1.000)	1.000(0.950)	1.000(1.000)	0.967(0.990)	1.000(1.000)	0.952(0.933)
LBI_2	0.958(0.970)	1.000(1.000)	1.000(1.000)	1.000(1.000)	1.000(0.979)	1.000(0.949)	0.984(0.999)	0.991(0.979)	1.000(1.000)	0.952(0.950)	1.000(0.950)	0.968(0.999)
LBI_3	0.955(0.968)	0.985(0.978)	0.999(1.000)	1.000(0.973)	1.000(0.986)	1.000(0.944)	0.974(1.000)	1.000(1.000)	0.963(0.962)	0.963(1.000)	0.990(0.988)	0.971(0.939)
$LKSN$	0.121(0.046)	0.121(0.020)	0.055(0.000)	0.108(0.000)	0.089(0.000)	0.033(0.022)	0.195(0.000)	0.072(0.025)	0.074(0.000)	0.290(0.000)	0.175(0.026)	0.062(0.048)
L	0.173(0.121)	0.033(0.039)	0.014(0.047)	0.164(0.136)	0.085(0.051)	0.057(0.042)	0.179(0.041)	0.056(0.024)	0.029(0.063)	0.074(0.055)	0.081(0.082)	0.024(0.003)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.911(0.736)	0.899(0.691)	0.847(0.527)	0.929(0.682)	0.900(0.607)	0.834(0.495)	0.890(0.724)	0.813(0.659)	0.760(0.460)	0.757(0.570)	0.763(0.409)	0.685(0.310)
K_2	0.909(0.694)	0.826(0.624)	0.827(0.513)	0.938(0.691)	0.944(0.557)	0.843(0.493)	0.819(0.643)	0.803(0.506)	0.632(0.402)	0.741(0.524)	0.734(0.405)	0.621(0.360)
K_3	0.885(0.768)	0.888(0.659)	0.858(0.551)	0.877(0.684)	0.935(0.604)	0.871(0.527)	0.907(0.740)	0.869(0.607)	0.779(0.500)	0.843(0.575)	0.714(0.397)	0.700(0.328)
LT_1	0.311(0.309)	0.216(0.251)	0.080(0.105)	0.212(0.302)	0.095(0.136)	0.074(0.077)	0.298(0.375)	0.208(0.226)	0.099(0.042)	0.286(0.270)	0.209(0.198)	0.035(0.105)
LT_2	0.254(0.178)	0.135(0.134)	0.050(0.035)	0.163(0.115)	0.088(0.066)	0.002(0.017)	0.249(0.201)	0.116(0.050)	0.000(0.000)	0.210(0.222)	0.117(0.137)	0.067(0.020)
LT_3	0.337(0.282)	0.225(0.231)	0.142(0.141)	0.239(0.273)	0.100(0.137)	0.030(0.050)	0.294(0.326)	0.229(0.156)	0.086(0.024)	0.268(0.245)	0.205(0.179)	0.053(0.042)
HLT_1	0.413(0.160)	0.379(0.170)	0.326(0.083)	0.408(0.180)	0.368(0.145)	0.290(0.101)	0.173(0.066)	0.134(0.120)	0.067(0.025)	0.200(0.044)	0.140(0.033)	0.079(0.049)
HLT_2	0.486(0.178)	0.388(0.192)	0.321(0.047)	0.556(0.165)	0.441(0.138)	0.351(0.123)	0.185(0.103)	0.149(0.083)	0.091(0.033)	0.251(0.080)	0.185(0.086)	0.124(0.000)
HLT_3	0.563(0.370)	0.475(0.288)	0.456(0.240)	0.540(0.241)	0.393(0.210)	0.313(0.243)	0.278(0.192)	0.242(0.148)	0.114(0.107)	0.265(0.059)	0.172(0.082)	0.122(0.026)
LBI_1	1.000(0.986)	0.970(0.938)	0.952(0.967)	0.986(0.982)	1.000(1.000)	0.968(0.962)	1.000(0.985)	0.981(1.000)	0.994(0.971)	0.966(1.000)	0.962(0.965)	0.953(0.951)
LBI_2	0.973(1.000)	0.975(0.942)	0.961(0.928)	0.989(0.950)	0.980(0.939)	1.000(0.963)	0.990(1.000)	0.974(1.000)	0.960(1.000)	1.000(1.000)	1.000(0.991)	0.967(0.950)
LBI_3	0.965(1.000)	1.000(0.985)	0.943(0.961)	0.983(1.000)	1.000(0.946)	1.000(0.895)	0.991(1.000)	0.959(1.000)	0.985(1.000)	1.000(1.000)	1.000(1.000)	0.950(0.952)
$LKSN$	0.231(0.050)	0.234(0.000)	0.053(0.020)	0.272(0.000)	0.103(0.000)	0.029(0.014)	0.171(0.012)	0.119(0.000)	0.013(0.000)	0.188(0.027)	0.145(0.049)	0.128(0.035)
L	0.135(0.073)	0.019(0.021)	0.003(0.043)	0.182(0.109)	0.173(0.069)	0.058(0.002)	0.110(0.111)	0.066(0.090)	0.000(0.000)	0.134(0.101)	0.000(0.100)	0.047(0.043)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.787(0.631)	0.829(0.569)	0.715(0.347)	0.884(0.682)	0.865(0.685)	0.723(0.543)	0.829(0.658)	0.738(0.574)	0.733(0.355)	0.820(0.746)	0.838(0.697)	0.825(0.487)
K_2	0.770(0.572)	0.774(0.438)	0.627(0.243)	0.877(0.665)	0.826(0.633)	0.695(0.463)	0.776(0.543)	0.753(0.463)	0.565(0.328)	0.871(0.671)	0.800(0.596)	0.771(0.474)
K_3	0.834(0.623)	0.803(0.557)	0.741(0.354)	0.861(0.690)	0.843(0.656)	0.789(0.532)	0.777(0.680)	0.735(0.614)	0.724(0.338)	0.913(0.723)	0.854(0.622)	0.864(0.536)
LT_1	0.268(0.278)	0.169(0.194)	0.027(0.031)	0.298(0.288)	0.245(0.227)	0.081(0.135)	0.266(0.174)	0.173(0.148)	0.057(0.055)	0.163(0.285)	0.193(0.137)	0.077(0.079)
LT_2	0.143(0.143)	0.129(0.036)	0.034(0.038)	0.240(0.200)	0.108(0.146)	0.062(0.000)	0.148(0.182)	0.127(0.099)	0.043(0.059)	0.116(0.029)	0.004(0.031)	0.010(0.036)
LT_3	0.196(0.184)	0.230(0.154)	0.026(0.055)	0.281(0.320)	0.267(0.265)	0.088(0.122)	0.285(0.170)	0.122(0.158)	0.141(0.108)	0.156(0.221)	0.151(0.175)	0.124(0.081)
HLT_1	0.165(0.034)	0.131(0.042)	0.053(0.000)	0.312(0.220)	0.318(0.192)	0.273(0.120)	0.197(0.098)	0.172(0.017)	0.081(0.000)	0.390(0.208)	0.329(0.106)	0.204(0.007)
HLT_2	0.116(0.086)	0.093(0.054)	0.084(0.036)	0.351(0.147)	0.325(0.110)	0.242(0.063)	0.164(0.049)	0.133(0.047)	0.133(0.001)	0.370(0.167)	0.249(0.094)	0.174(0.048)
HLT_3	0.189(0.083)	0.196(0.045)	0.094(0.010)	0.443(0.263)	0.326(0.269)	0.276(0.218)	0.332(0.185)	0.196(0.167)	0.169(0.129)	0.476(0.300)	0.380(0.186)	0.264(0.234)
LBI_1	1.000(0.975)	1.000(1.000)	1.000(1.000)	0.976(0.983)	1.000(1.000)	1.000(0.985)	0.994(0.957)	1.000(1.000)	0.982(1.000)	0.979(1.000)	1.000(0.990)	0.932(0.927)
LBI_2	0.973(0.967)	0.988(0.993)	0.958(1.000)	1.000(0.987)	0.971(1.000)	0.951(0.946)	1.000(1.000)	0.987(0.960)	1.000(0.970)	1.000(1.000)	0.975(0.969)	1.000(0.950)
LBI_3	0.976(0.990)	0.964(1.000)	0.986(0.961)	0.979(0.990)	0.950(1.000)	0.974(0.958)	0.975(1.000)	1.000(0.971)	1.000(0.958)	1.000(1.000)	1.000(0.991)	1.000(0.922)
$LKSN$	0.089(0.021)	0.000(0.024)	0.054(0.000)	0.307(0.050)	0.171(0.000)	0.082(0.007)	0.191(0.011)	0.096(0.000)	0.000(0.000)	0.280(0.037)	0.205(0.000)	0.096(0.000)
L	0.092(0.094)	0.045(0.014)	0.039(0.013)	0.069(0.059)	0.082(0.082)	0.035(0.000)	0.070(0.060)	0.000(0.035)	0.000(0.000)	0.076(0.085)	0.015(0.069)	0.000(0.000)

Table 2.17: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_1}$ with $\mathbf{T=100}$.

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.874(0.772)	0.855(0.592)	0.740(0.466)	0.826(0.639)	0.784(0.516)	0.644(0.388)	0.835(0.640)	0.759(0.554)	0.680(0.424)	0.804(0.567)	0.751(0.454)	0.690(0.362)
K_2	0.795(0.553)	0.708(0.488)	0.523(0.350)	0.797(0.522)	0.751(0.386)	0.617(0.220)	0.809(0.532)	0.742(0.499)	0.624(0.366)	0.763(0.497)	0.783(0.405)	0.576(0.359)
K_3	0.889(0.748)	0.824(0.605)	0.706(0.438)	0.868(0.632)	0.792(0.468)	0.693(0.368)	0.851(0.676)	0.774(0.515)	0.680(0.391)	0.822(0.563)	0.809(0.498)	0.666(0.412)
LT_1	0.296(0.322)	0.253(0.196)	0.133(0.092)	0.299(0.277)	0.280(0.234)	0.087(0.125)	0.271(0.265)	0.234(0.159)	0.042(0.058)	0.298(0.341)	0.232(0.210)	0.057(0.103)
LT_2	0.192(0.219)	0.098(0.142)	0.078(0.062)	0.223(0.154)	0.135(0.107)	0.000(0.009)	0.203(0.171)	0.083(0.067)	0.008(0.009)	0.260(0.156)	0.148(0.027)	0.039(0.000)
LT_3	0.249(0.273)	0.269(0.209)	0.104(0.047)	0.321(0.370)	0.265(0.181)	0.071(0.099)	0.297(0.264)	0.183(0.250)	0.098(0.132)	0.350(0.276)	0.201(0.159)	0.085(0.134)
HLT_1	0.174(0.033)	0.084(0.000)	0.074(0.043)	0.201(0.070)	0.082(0.086)	0.060(0.000)	0.249(0.069)	0.162(0.048)	0.119(0.033)	0.229(0.129)	0.237(0.036)	0.208(0.000)
HLT_2	0.154(0.038)	0.069(0.000)	0.112(0.000)	0.113(0.086)	0.115(0.031)	0.025(0.013)	0.229(0.085)	0.164(0.079)	0.122(0.064)	0.223(0.087)	0.228(0.064)	0.163(0.005)
HLT_3	0.080(0.037)	0.083(0.058)	0.083(0.055)	0.154(0.117)	0.117(0.048)	0.058(0.042)	0.240(0.135)	0.250(0.208)	0.167(0.096)	0.316(0.206)	0.346(0.199)	0.269(0.186)
LBI_1	0.983(1.000)	0.960(0.985)	0.997(0.998)	0.975(0.965)	0.953(0.986)	1.000(0.979)	1.000(0.973)	1.000(1.000)	0.960(0.950)	0.998(1.000)	0.953(0.945)	1.000(0.997)
LBI_2	1.000(0.985)	0.994(1.000)	0.958(0.954)	1.000(1.000)	0.984(1.000)	0.964(0.947)	1.000(1.000)	1.000(1.000)	1.000(0.932)	1.000(1.000)	1.000(0.956)	1.000(0.960)
LBI_3	1.000(1.000)	1.000(1.000)	1.000(1.000)	0.982(1.000)	1.000(0.952)	0.988(0.984)	1.000(1.000)	1.000(1.000)	1.000(1.000)	0.980(0.994)	0.989(1.000)	0.964(0.968)
$LKSN$	0.133(0.006)	0.072(0.000)	0.007(0.035)	0.135(0.038)	0.052(0.032)	0.000(0.000)	0.127(0.000)	0.051(0.004)	0.081(0.018)	0.306(0.029)	0.169(0.000)	0.058(0.019)
L	0.141(0.126)	0.068(0.084)	0.017(0.035)	0.108(0.108)	0.082(0.020)	0.000(0.000)	0.120(0.096)	0.049(0.021)	0.000(0.000)	0.071(0.128)	0.014(0.095)	0.009(0.030)

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.890(0.682)	0.872(0.645)	0.845(0.564)	0.886(0.679)	0.900(0.627)	0.847(0.507)	0.849(0.706)	0.846(0.589)	0.744(0.474)	0.809(0.619)	0.691(0.427)	0.624(0.331)
K_2	0.846(0.715)	0.903(0.622)	0.820(0.495)	0.893(0.641)	0.879(0.546)	0.848(0.464)	0.871(0.600)	0.743(0.489)	0.618(0.399)	0.813(0.524)	0.702(0.453)	0.648(0.326)
K_3	0.919(0.683)	0.847(0.694)	0.875(0.565)	0.939(0.704)	0.867(0.614)	0.876(0.499)	0.894(0.719)	0.883(0.596)	0.792(0.516)	0.783(0.608)	0.692(0.457)	0.695(0.365)
LT_1	0.338(0.289)	0.238(0.275)	0.115(0.131)	0.268(0.301)	0.171(0.096)	0.070(0.014)	0.300(0.365)	0.220(0.202)	0.040(0.010)	0.291(0.310)	0.181(0.184)	0.081(0.041)
LT_2	0.243(0.140)	0.112(0.054)	0.016(0.060)	0.131(0.094)	0.034(0.058)	0.057(0.028)	0.235(0.219)	0.090(0.035)	0.047(0.037)	0.194(0.228)	0.184(0.138)	0.023(0.037)
LT_3	0.337(0.257)	0.213(0.211)	0.092(0.107)	0.269(0.280)	0.109(0.155)	0.068(0.032)	0.353(0.297)	0.175(0.241)	0.068(0.074)	0.293(0.272)	0.206(0.188)	0.081(0.088)
HLT_1	0.452(0.219)	0.410(0.101)	0.327(0.132)	0.403(0.153)	0.366(0.090)	0.263(0.040)	0.151(0.084)	0.193(0.062)	0.081(0.023)	0.220(0.101)	0.191(0.084)	0.057(0.000)
HLT_2	0.448(0.199)	0.441(0.160)	0.292(0.121)	0.516(0.200)	0.434(0.137)	0.278(0.049)	0.257(0.107)	0.239(0.042)	0.129(0.045)	0.301(0.138)	0.221(0.064)	0.140(0.015)
HLT_3	0.572(0.314)	0.540(0.270)	0.501(0.267)	0.492(0.248)	0.417(0.231)	0.347(0.215)	0.238(0.177)	0.182(0.120)	0.143(0.139)	0.283(0.117)	0.206(0.055)	0.151(0.012)
LBI_1	0.980(0.972)	0.973(1.000)	0.967(0.980)	0.995(1.000)	1.000(1.000)	1.000(0.898)	0.961(1.000)	0.962(0.949)	0.986(0.984)	0.974(0.964)	0.971(0.980)	1.000(0.958)
LBI_2	0.959(1.000)	0.980(0.944)	0.945(0.922)	0.966(0.951)	0.991(0.993)	1.000(0.891)	1.000(0.969)	0.976(1.000)	1.000(1.000)	0.971(1.000)	1.000(0.974)	0.996(0.987)
LBI_3	1.000(0.978)	0.964(1.000)	0.921(0.918)	0.970(0.961)	1.000(0.991)	0.984(0.917)	0.976(1.000)	0.998(1.000)	0.959(0.969)	1.000(1.000)	0.962(1.000)	1.000(0.994)
$LKSN$	0.265(0.023)	0.176(0.005)	0.089(0.026)	0.221(0.049)	0.119(0.000)	0.077(0.021)	0.186(0.049)	0.099(0.030)	0.035(0.000)	0.237(0.000)	0.180(0.001)	0.069(0.046)
L	0.173(0.136)	0.028(0.040)	0.040(0.028)	0.190(0.082)	0.112(0.058)	0.044(0.000)	0.103(0.134)	0.075(0.018)	0.008(0.037)	0.080(0.089)	0.059(0.079)	0.000(0.018)

α	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.845(0.680)	0.760(0.558)	0.671(0.372)	0.921(0.710)	0.869(0.608)	0.809(0.503)	0.838(0.675)	0.730(0.549)	0.738(0.378)	0.873(0.673)	0.800(0.661)	0.804(0.537)
K_2	0.765(0.559)	0.789(0.422)	0.601(0.222)	0.812(0.638)	0.834(0.606)	0.705(0.477)	0.824(0.510)	0.783(0.503)	0.565(0.331)	0.885(0.651)	0.825(0.570)	0.739(0.422)
K_3	0.799(0.676)	0.813(0.577)	0.731(0.428)	0.896(0.700)	0.831(0.596)	0.731(0.478)	0.779(0.705)	0.806(0.573)	0.700(0.395)	0.891(0.738)	0.875(0.678)	0.822(0.505)
LT_1	0.251(0.236)	0.182(0.224)	0.025(0.050)	0.297(0.348)	0.223(0.235)	0.102(0.104)	0.192(0.223)	0.120(0.116)	0.137(0.028)	0.167(0.220)	0.135(0.112)	0.114(0.032)
LT_2	0.222(0.095)	0.112(0.042)	0.016(0.006)	0.253(0.132)	0.136(0.143)	0.051(0.001)	0.227(0.168)	0.146(0.066)	0.052(0.027)	0.062(0.104)	0.029(0.032)	0.000(0.052)
LT_3	0.273(0.183)	0.178(0.198)	0.026(0.076)	0.305(0.272)	0.205(0.189)	0.090(0.109)	0.272(0.192)	0.117(0.144)	0.119(0.092)	0.183(0.249)	0.187(0.112)	0.125(0.068)
HLT_1	0.159(0.079)	0.079(0.058)	0.101(0.003)	0.362(0.247)	0.279(0.163)	0.248(0.169)	0.200(0.041)	0.150(0.049)	0.138(0.006)	0.384(0.204)	0.298(0.153)	0.205(0.039)
HLT_2	0.168(0.014)	0.079(0.003)	0.058(0.050)	0.417(0.171)	0.341(0.102)	0.246(0.108)	0.223(0.061)	0.172(0.049)	0.153(0.000)	0.350(0.151)	0.286(0.145)	0.245(0.086)
HLT_3	0.231(0.108)	0.131(0.065)	0.137(0.011)	0.398(0.311)	0.343(0.254)	0.253(0.202)	0.322(0.160)	0.173(0.145)	0.177(0.143)	0.502(0.338)	0.453(0.269)	0.266(0.200)
LBI_1	1.000(0.986)	1.000(0.981)	0.955(1.000)	0.967(1.000)	1.000(0.975)	1.000(1.000)	0.978(1.000)	1.000(1.000)	1.000(1.000)	1.000(1.000)	0.987(1.000)	1.000(0.957)
LBI_2	1.000(0.970)	1.000(1.000)	1.000(0.950)	1.000(0.996)	0.984(1.000)	0.947(0.927)	1.000(0.964)	0.962(0.974)	1.000(0.971)	1.000(0.986)	1.000(1.000)	0.950(0.946)
LBI_3	1.000(0.974)	1.000(1.000)	1.000(1.000)	0.975(0.978)	1.000(0.955)	0.977(0.970)	0.967(1.000)	1.000(0.995)	1.000(1.000)	1.000(1.000)	1.000(1.000)	1.000(0.954)
$LKSN$	0.164(0.000)	0.043(0.039)	0.005(0.000)	0.298(0.000)	0.159(0.027)	0.057(0.000)	0.163(0.000)	0.056(0.044)	0.000(0.050)	0.280(0.029)	0.185(0.000)	0.122(0.012)
L	0.062(0.147)	0.037(0.027)	0.000(0.000)	0.086(0.076)	0.018(0.050)	0.000(0.015)	0.107(0.123)	0.071(0.070)	0.007(0.029)	0.131(0.053)	0.039(0.107)	0.000(0.033)

Table 2.18: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_1}$ with $\mathbf{T = 250}$.

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.998(0.986)	0.980(0.902)	0.881(0.743)	0.964(0.893)	0.981(0.870)	0.982(0.808)	0.971(0.873)	0.936(0.909)	0.884(0.707)	0.987(0.913)	0.989(0.836)	0.849(0.780)
K_2	1.000(0.904)	0.968(0.916)	0.936(0.778)	0.927(0.859)	0.990(0.917)	0.911(0.807)	0.973(0.828)	0.990(0.902)	0.966(0.789)	0.993(0.790)	0.994(0.809)	0.887(0.763)
K_3	0.964(0.974)	0.971(0.906)	0.908(0.746)	0.928(0.873)	0.931(0.914)	0.940(0.837)	1.000(0.846)	0.948(0.898)	0.985(0.699)	0.937(0.867)	0.968(0.865)	0.866(0.779)
LT_1	0.905(0.892)	0.850(0.837)	0.743(0.782)	0.784(0.800)	0.774(0.821)	0.770(0.846)	0.854(0.731)	0.837(0.722)	0.697(0.776)	0.765(0.777)	0.847(0.795)	0.746(0.800)
LT_2	0.794(0.770)	0.935(0.895)	0.803(0.814)	0.704(0.685)	0.843(0.850)	0.879(0.843)	0.710(0.649)	0.795(0.793)	0.907(0.811)	0.569(0.439)	0.809(0.801)	0.831(0.822)
LT_3	0.841(0.861)	0.860(0.895)	0.751(0.778)	0.779(0.821)	0.850(0.778)	0.749(0.880)	0.831(0.815)	0.846(0.744)	0.808(0.820)	0.715(0.774)	0.807(0.858)	0.807(0.784)
HLT_1	0.567(0.443)	0.729(0.453)	0.790(0.507)	0.613(0.361)	0.719(0.430)	0.901(0.555)	0.600(0.315)	0.697(0.498)	0.754(0.446)	0.510(0.306)	0.682(0.473)	0.766(0.485)
HLT_2	0.597(0.441)	0.871(0.602)	0.850(0.641)	0.604(0.297)	0.803(0.513)	0.966(0.715)	0.623(0.363)	0.786(0.593)	0.896(0.597)	0.524(0.306)	0.767(0.559)	0.895(0.578)
HLT_3	0.745(0.585)	0.756(0.570)	0.851(0.467)	0.660(0.432)	0.752(0.513)	0.896(0.492)	0.591(0.367)	0.845(0.645)	0.845(0.512)	0.598(0.378)	0.769(0.478)	0.817(0.592)
LBI_1	0.988(1.000)	1.000(0.989)	0.923(0.898)	1.000(0.999)	0.989(0.959)	0.960(0.862)	1.000(1.000)	0.990(0.958)	0.876(0.878)	1.000(0.999)	1.000(0.938)	0.924(0.854)
LBI_2	0.997(0.983)	1.000(1.000)	0.923(0.919)	0.983(0.960)	0.933(0.989)	0.905(0.865)	1.000(1.000)	1.000(0.971)	0.897(0.884)	0.945(0.967)	0.957(1.000)	0.936(0.854)
LBI_3	0.992(0.972)	1.000(1.000)	0.909(0.852)	0.969(1.000)	1.000(0.963)	0.953(0.872)	0.950(1.000)	1.000(0.977)	0.843(0.829)	0.976(0.955)	0.980(0.954)	0.914(0.875)
$LKSN$	0.971(0.064)	0.988(0.345)	1.000(0.870)	1.000(0.040)	1.000(0.344)	1.000(0.798)	1.000(0.067)	0.987(0.336)	1.000(0.719)	0.936(0.050)	0.974(0.404)	0.990(0.776)
L	0.362(1.000)	0.546(1.000)	0.351(0.961)	0.332(1.000)	0.477(1.000)	0.302(0.987)	0.273(1.000)	0.415(1.000)	0.348(0.985)	0.161(0.903)	0.456(0.967)	0.365(0.960)

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	1.000(0.990)	1.000(1.000)	1.000(0.988)	0.734(0.475)	0.693(0.580)	0.586(0.383)	1.000(0.980)	1.000(0.974)	0.998(0.896)	0.920(0.886)	0.870(0.840)	0.890(0.725)
K_2	0.997(1.000)	1.000(1.000)	0.983(0.943)	0.761(0.510)	0.719(0.512)	0.559(0.414)	1.000(0.973)	0.948(0.885)	0.950(0.868)	0.953(0.835)	0.929(0.778)	0.901(0.806)
K_3	0.982(1.000)	0.995(0.972)	1.000(0.971)	0.731(0.506)	0.716(0.515)	0.553(0.337)	0.984(1.000)	0.974(0.921)	0.994(0.891)	0.990(0.886)	0.897(0.834)	0.876(0.666)
LT_1	0.846(0.837)	0.891(0.809)	0.806(0.865)	0.809(0.852)	0.807(0.799)	0.716(0.697)	0.834(0.843)	0.804(0.817)	0.863(0.814)	0.866(0.846)	0.801(0.796)	0.749(0.740)
LT_2	0.635(0.635)	0.935(0.849)	0.851(0.869)	0.754(0.729)	0.837(0.763)	0.754(0.797)	0.689(0.653)	0.848(0.878)	0.907(0.917)	0.715(0.613)	0.893(0.878)	0.837(0.858)
LT_3	0.822(0.806)	0.875(0.869)	0.863(0.861)	0.853(0.839)	0.753(0.782)	0.709(0.691)	0.849(0.801)	0.847(0.808)	0.848(0.823)	0.860(0.852)	0.873(0.830)	0.774(0.778)
HLT_1	0.766(0.665)	0.800(0.690)	0.815(0.567)	0.497(0.138)	0.669(0.391)	0.540(0.224)	0.716(0.484)	0.820(0.603)	0.899(0.552)	0.576(0.266)	0.676(0.499)	0.808(0.568)
HLT_2	0.818(0.714)	0.875(0.825)	0.915(0.766)	0.460(0.166)	0.640(0.466)	0.569(0.318)	0.762(0.449)	0.872(0.627)	0.896(0.685)	0.497(0.296)	0.793(0.666)	0.859(0.748)
HLT_3	0.971(0.928)	0.965(1.000)	1.000(0.835)	0.513(0.138)	0.619(0.401)	0.509(0.295)	0.844(0.692)	0.889(0.733)	0.954(0.666)	0.582(0.387)	0.709(0.535)	0.781(0.543)
LBI_1	1.000(1.000)	0.999(0.960)	0.956(0.964)	1.000(0.903)	0.827(0.779)	0.721(0.732)	1.000(1.000)	0.991(0.971)	0.951(0.924)	1.000(1.000)	0.943(0.903)	0.912(0.934)
LBI_2	0.968(0.978)	1.000(1.000)	1.000(0.955)	0.976(0.920)	0.837(0.762)	0.658(0.708)	0.970(0.968)	0.970(0.936)	0.927(0.896)	0.995(0.976)	0.962(0.928)	0.878(0.849)
LBI_3	0.997(0.971)	0.950(0.991)	0.957(0.913)	1.000(0.959)	0.836(0.793)	0.767(0.740)	0.989(0.973)	0.991(0.989)	0.943(0.978)	1.000(1.000)	0.936(0.928)	0.885(0.904)
$LKSN$	0.931(0.056)	1.000(0.367)	1.000(0.810)	0.982(0.159)	1.000(0.526)	1.000(0.703)	0.952(0.044)	1.000(0.262)	1.000(0.738)	0.950(0.091)	0.965(0.477)	0.963(0.873)
L	0.338(0.955)	0.590(1.000)	0.880(0.994)	0.196(0.936)	0.113(0.768)	0.049(0.571)	0.245(1.000)	0.482(1.000)	0.457(1.000)	0.379(1.000)	0.351(0.978)	0.142(0.892)

τ^*	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.954(0.957)	1.000(0.843)	0.934(0.794)	0.955(1.000)	1.000(0.960)	1.000(0.920)	0.881(0.871)	0.851(0.733)	0.763(0.591)	0.960(0.960)	1.000(0.973)	1.000(0.918)
K_2	0.952(0.953)	0.977(0.942)	0.973(0.863)	0.984(0.975)	1.000(0.969)	1.000(0.950)	0.868(0.676)	0.885(0.805)	0.811(0.724)	0.986(1.000)	1.000(0.988)	1.000(0.975)
K_3	1.000(0.980)	1.000(0.861)	0.943(0.766)	1.000(0.964)	0.987(1.000)	0.990(0.950)	0.892(0.816)	0.839(0.797)	0.804(0.610)	0.989(1.000)	0.973(1.000)	0.997(0.925)
LT_1	0.861(0.911)	0.748(0.756)	0.816(0.755)	0.756(0.756)	0.912(0.832)	0.821(0.864)	0.817(0.796)	0.876(0.950)	0.631(0.723)	0.813(0.791)	0.831(0.858)	0.830(0.779)
LT_2	0.784(0.751)	0.834(0.872)	0.867(0.844)	0.625(0.526)	0.844(0.850)	0.901(0.900)	0.748(0.782)	0.933(0.844)	0.777(0.817)	0.602(0.528)	0.823(0.804)	0.938(0.830)
LT_3	0.903(0.900)	0.782(0.752)	0.769(0.853)	0.765(0.767)	0.917(0.919)	0.841(0.808)	0.766(0.777)	0.923(0.882)	0.731(0.708)	0.827(0.785)	0.919(0.912)	0.821(0.832)
HLT_1	0.636(0.391)	0.724(0.404)	0.814(0.551)	0.729(0.584)	0.831(0.607)	0.813(0.596)	0.603(0.301)	0.775(0.511)	0.742(0.489)	0.795(0.719)	0.768(0.644)	0.765(0.632)
HLT_2	0.556(0.382)	0.802(0.592)	0.902(0.699)	0.808(0.616)	0.834(0.733)	0.915(0.676)	0.499(0.269)	0.824(0.594)	0.842(0.599)	0.844(0.770)	0.895(0.733)	0.877(0.697)
HLT_3	0.759(0.543)	0.812(0.598)	0.876(0.529)	0.928(0.869)	1.000(0.883)	0.983(0.773)	0.626(0.278)	0.764(0.486)	0.722(0.442)	1.000(0.931)	1.000(0.868)	0.999(0.938)
LBI_1	0.971(1.000)	0.955(0.962)	0.921(0.943)	0.951(1.000)	0.966(0.982)	1.000(0.933)	0.990(0.991)	0.930(0.913)	0.846(0.912)	0.965(0.968)	0.984(1.000)	0.987(0.956)
LBI_2	1.000(1.000)	0.945(0.961)	0.962(0.885)	0.971(0.969)	0.958(0.960)	0.939(0.970)	1.000(0.989)	0.942(0.967)	0.814(0.853)	1.000(0.960)	0.929(0.998)	0.977(0.980)
LBI_3	1.000(1.000)	1.000(0.990)	0.947(0.889)	0.980(1.000)	0.998(0.971)	0.984(0.970)	1.000(1.000)	0.953(0.999)	0.803(0.822)	0.966(1.000)	1.000(0.977)	0.959(0.941)
$LKSN$	0.991(0.048)	0.986(0.378)	1.000(0.686)	0.962(0.092)	1.000(0.294)	1.000(0.768)	1.000(0.088)	1.000(0.402)	0.965(0.793)	1.000(0.041)	1.000(0.438)	1.000(0.771)
L	0.291(0.983)	0.430(1.000)	0.346(0.915)	0.241(0.951)	0.593(1.000)	0.728(1.000)	0.314(0.936)	0.190(0.926)	0.115(0.905)	0.265(0.998)	0.572(0.971)	0.623(0.973)

Table 2.19: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_{01}}$ with $\mathbf{T = 100}$.

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$ 0.3	0.5	0.7
K_1	1.000(0.942)	0.928(0.926)	0.885(0.782)	1.000(0.932)	1.000(0.885)	0.907(0.852)	0.965(0.857)	0.916(0.904)	0.884(0.690)	0.946(0.911)	0.954(0.780)	0.925(0.757)
K_2	0.982(0.929)	0.933(0.918)	0.969(0.863)	0.987(0.857)	0.945(0.870)	0.925(0.837)	0.901(0.862)	0.947(0.865)	0.967(0.814)	0.966(0.769)	0.993(0.883)	0.898(0.801)
K_3	1.000(0.951)	0.997(0.910)	0.876(0.764)	0.917(0.951)	0.970(0.868)	0.916(0.835)	0.971(0.894)	0.932(0.840)	0.929(0.715)	0.994(0.928)	0.931(0.810)	0.923(0.720)
LT_1	0.872(0.897)	0.872(0.860)	0.736(0.779)	0.804(0.849)	0.785(0.791)	0.843(0.881)	0.776(0.724)	0.783(0.776)	0.719(0.819)	0.780(0.810)	0.812(0.873)	0.806(0.756)
LT_2	0.806(0.773)	0.852(0.863)	0.811(0.810)	0.749(0.645)	0.811(0.853)	0.870(0.800)	0.766(0.623)	0.876(0.807)	0.885(0.809)	0.522(0.393)	0.828(0.837)	0.837(0.847)
LT_3	0.890(0.897)	0.848(0.812)	0.733(0.739)	0.764(0.780)	0.823(0.846)	0.816(0.854)	0.839(0.805)	0.809(0.735)	0.803(0.804)	0.706(0.719)	0.881(0.793)	0.830(0.807)
HLT_1	0.628(0.413)	0.733(0.528)	0.777(0.450)	0.585(0.307)	0.712(0.400)	0.915(0.529)	0.555(0.338)	0.707(0.462)	0.738(0.427)	0.529(0.273)	0.629(0.410)	0.756(0.503)
HLT_2	0.676(0.531)	0.820(0.623)	0.895(0.635)	0.545(0.303)	0.854(0.495)	0.942(0.654)	0.613(0.373)	0.785(0.605)	0.890(0.563)	0.498(0.256)	0.743(0.529)	0.826(0.608)
HLT_3	0.794(0.583)	0.785(0.549)	0.857(0.437)	0.713(0.468)	0.743(0.572)	0.864(0.572)	0.569(0.438)	0.861(0.646)	0.845(0.508)	0.608(0.355)	0.727(0.524)	0.764(0.594)
LBI_1	1.000(1.000)	0.968(0.987)	0.926(0.914)	0.980(0.967)	1.000(1.000)	0.920(0.888)	0.974(0.999)	0.946(0.990)	0.912(0.843)	0.958(0.952)	0.999(0.917)	0.939(0.809)
LBI_2	1.000(0.990)	0.968(0.995)	0.889(0.846)	0.983(1.000)	0.948(0.997)	0.950(0.914)	0.968(1.000)	1.000(0.971)	0.887(0.888)	0.937(0.947)	0.944(0.954)	0.946(0.879)
LBI_3	0.996(1.000)	0.950(0.988)	0.883(0.855)	0.976(1.000)	0.940(0.997)	0.940(0.941)	0.993(0.979)	0.974(0.954)	0.910(0.842)	0.943(1.000)	0.960(0.940)	0.897(0.870)
$LKSN$	0.976(0.058)	0.967(0.345)	0.985(0.835)	1.000(0.082)	1.000(0.366)	1.000(0.755)	0.970(0.017)	1.000(0.305)	0.973(0.764)	0.939(0.029)	1.000(0.397)	0.994(0.775)
L	0.320(0.992)	0.505(0.997)	0.308(1.000)	0.325(0.997)	0.457(1.000)	0.298(0.965)	0.259(0.983)	0.447(1.000)	0.366(0.988)	0.196(0.974)	0.451(1.000)	0.417(0.972)

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$ 0.3	0.5	0.7
K_1	1.000(0.982)	1.000(0.974)	0.986(0.992)	0.703(0.518)	0.704(0.528)	0.576(0.299)	1.000(0.996)	0.946(0.919)	0.942(0.875)	0.986(0.928)	0.899(0.842)	0.838(0.715)
K_2	0.999(0.961)	0.961(0.978)	1.000(0.969)	0.706(0.487)	0.767(0.520)	0.627(0.404)	1.000(0.928)	0.988(0.900)	0.960(0.890)	0.953(0.889)	0.913(0.829)	0.875(0.770)
K_3	1.000(0.965)	1.000(0.979)	0.969(0.980)	0.776(0.510)	0.752(0.510)	0.544(0.304)	0.974(1.000)	1.000(0.970)	0.933(0.821)	0.969(0.961)	0.892(0.848)	0.903(0.685)
LT_1	0.852(0.838)	0.874(0.852)	0.884(0.875)	0.788(0.836)	0.810(0.797)	0.677(0.691)	0.890(0.823)	0.801(0.826)	0.806(0.836)	0.810(0.832)	0.789(0.821)	0.791(0.817)
LT_2	0.665(0.686)	0.880(0.914)	0.852(0.898)	0.810(0.661)	0.843(0.774)	0.750(0.778)	0.671(0.587)	0.841(0.803)	0.915(0.904)	0.792(0.645)	0.873(0.838)	0.866(0.798)
LT_3	0.818(0.827)	0.863(0.872)	0.810(0.834)	0.848(0.847)	0.797(0.838)	0.656(0.695)	0.870(0.818)	0.844(0.785)	0.883(0.877)	0.841(0.845)	0.856(0.855)	0.790(0.750)
HLT_1	0.769(0.622)	0.764(0.680)	0.826(0.608)	0.519(0.159)	0.660(0.465)	0.498(0.260)	0.683(0.479)	0.804(0.534)	0.898(0.550)	0.583(0.301)	0.688(0.526)	0.804(0.579)
HLT_2	0.845(0.676)	0.800(0.833)	0.877(0.723)	0.490(0.187)	0.678(0.451)	0.543(0.328)	0.681(0.466)	0.840(0.611)	0.870(0.624)	0.480(0.283)	0.846(0.688)	0.845(0.745)
HLT_3	0.975(0.986)	0.990(0.997)	0.980(0.891)	0.455(0.156)	0.603(0.382)	0.549(0.289)	0.908(0.705)	0.901(0.715)	0.945(0.661)	0.590(0.299)	0.696(0.486)	0.826(0.515)
LBI_1	0.966(0.994)	1.000(1.000)	0.987(0.984)	0.953(0.933)	0.871(0.724)	0.711(0.739)	0.970(0.972)	0.989(0.918)	1.000(0.959)	1.000(1.000)	0.978(0.909)	0.956(0.917)
LBI_2	0.996(1.000)	0.978(0.966)	0.985(0.979)	1.000(0.958)	0.825(0.771)	0.729(0.757)	0.966(0.999)	0.956(0.971)	0.944(0.950)	1.000(1.000)	0.933(0.934)	0.875(0.891)
LBI_3	0.984(1.000)	1.000(0.947)	1.000(0.982)	0.957(0.932)	0.794(0.756)	0.772(0.698)	0.972(1.000)	0.994(0.962)	1.000(0.921)	0.991(1.000)	0.903(0.895)	0.893(0.848)
$LKSN$	0.979(0.047)	1.000(0.290)	1.000(0.743)	1.000(0.155)	0.984(0.479)	0.990(0.724)	0.976(0.089)	0.997(0.265)	1.000(0.785)	1.000(0.129)	1.000(0.452)	1.000(0.863)
L	0.405(0.999)	0.583(1.000)	0.850(1.000)	0.247(0.894)	0.098(0.784)	0.004(0.612)	0.283(1.000)	0.485(1.000)	0.467(0.973)	0.370(0.970)	0.338(0.953)	0.183(0.877)

τ^*	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$ 0.3	0.5	0.7	$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$ 0.3	0.5	0.7	$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$ 0.3	0.5	0.7	$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$ 0.3	0.5	0.7
K_1	1.000(0.986)	1.000(0.916)	0.931(0.740)	0.954(0.964)	1.000(0.981)	0.965(0.939)	0.891(0.806)	0.873(0.771)	0.831(0.582)	1.000(0.998)	1.000(0.966)	0.992(0.988)
K_2	0.927(0.938)	1.000(0.948)	0.960(0.783)	0.978(0.928)	0.996(0.962)	0.971(0.939)	0.855(0.739)	0.830(0.783)	0.811(0.657)	0.989(1.000)	1.000(0.949)	0.992(0.915)
K_3	0.960(0.968)	0.970(0.915)	0.901(0.753)	0.977(0.946)	1.000(0.952)	1.000(0.896)	0.943(0.862)	0.873(0.744)	0.777(0.576)	1.000(0.973)	1.000(0.942)	0.981(0.923)
LT_1	0.893(0.877)	0.787(0.752)	0.757(0.795)	0.797(0.721)	0.910(0.880)	0.814(0.866)	0.765(0.767)	0.926(0.944)	0.698(0.720)	0.811(0.819)	0.833(0.896)	0.812(0.787)
LT_2	0.790(0.700)	0.879(0.873)	0.841(0.831)	0.650(0.554)	0.790(0.821)	0.838(0.875)	0.744(0.730)	0.890(0.874)	0.816(0.813)	0.591(0.570)	0.860(0.818)	0.873(0.834)
LT_3	0.861(0.908)	0.841(0.765)	0.738(0.783)	0.736(0.761)	0.905(0.870)	0.822(0.855)	0.780(0.763)	0.863(0.928)	0.664(0.717)	0.825(0.729)	0.904(0.837)	0.883(0.781)
HLT_1	0.594(0.383)	0.740(0.376)	0.792(0.562)	0.692(0.607)	0.818(0.616)	0.852(0.612)	0.609(0.259)	0.714(0.541)	0.670(0.438)	0.838(0.742)	0.788(0.633)	0.805(0.653)
HLT_2	0.539(0.355)	0.776(0.544)	0.887(0.682)	0.758(0.623)	0.817(0.712)	0.974(0.763)	0.467(0.217)	0.846(0.551)	0.787(0.589)	0.829(0.810)	0.837(0.767)	0.877(0.710)
HLT_3	0.768(0.486)	0.861(0.541)	0.869(0.591)	0.915(0.853)	0.970(0.934)	0.973(0.780)	0.616(0.330)	0.744(0.447)	0.701(0.469)	0.988(0.985)	1.000(0.927)	0.993(0.857)
LBI_1	0.974(0.965)	0.979(0.978)	0.927(0.933)	1.000(1.000)	0.991(0.937)	0.955(1.000)	0.997(0.972)	0.984(0.991)	0.822(0.928)	0.973(0.982)	0.947(0.935)	1.000(0.969)
LBI_2	1.000(0.979)	0.967(0.997)	0.889(0.909)	1.000(0.953)	0.997(0.984)	0.997(0.981)	0.976(1.000)	0.948(0.921)	0.896(0.861)	0.960(1.000)	0.990(0.986)	0.991(0.939)
LBI_3	1.000(1.000)	0.971(0.962)	0.942(0.868)	0.958(0.988)	0.939(0.952)	0.979(0.931)	1.000(1.000)	0.953(1.000)	0.834(0.881)	1.000(0.979)	0.954(0.961)	0.951(0.986)
$LKSN$	0.959(0.085)	0.969(0.360)	1.000(0.616)	1.000(0.086)	0.994(0.286)	1.000(0.802)	0.996(0.039)	0.998(0.381)	1.000(0.797)	1.000(0.083)	1.000(0.392)	1.000(0.737)
L	0.373(1.000)	0.476(0.979)	0.298(0.986)	0.247(1.000)	0.537(0.970)	0.677(0.999)	0.317(0.946)	0.177(0.955)	0.073(0.830)	0.244(1.000)	0.589(0.989)	0.701(1.000)

Table 2.20: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_{01}}$ with $\mathbf{T=250}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.936(0.879)	0.969(0.880)	0.961(0.962)	0.866(0.783)	1.000(0.931)	0.957(0.934)	0.888(0.729)	0.949(0.898)	1.000(0.919)	0.845(0.699)	0.937(0.827)	1.000(0.812)
K_2	0.982(0.893)	0.989(0.930)	1.000(0.880)	0.942(0.749)	0.981(0.950)	0.965(0.898)	0.950(0.761)	1.000(0.915)	0.963(0.870)	0.916(0.778)	0.916(0.892)	0.957(0.803)
K_3	0.914(0.855)	1.000(0.873)	0.929(0.908)	0.875(0.760)	0.969(0.915)	1.000(0.870)	0.891(0.747)	0.955(0.897)	1.000(0.917)	0.859(0.695)	0.931(0.867)	0.957(0.800)
LT_1	0.843(0.878)	0.827(0.811)	0.824(0.812)	0.717(0.780)	0.842(0.861)	0.795(0.800)	0.761(0.831)	0.807(0.824)	0.805(0.771)	0.701(0.759)	0.758(0.766)	0.779(0.754)
LT_2	0.932(0.870)	0.829(0.791)	0.727(0.718)	0.841(0.751)	0.908(0.907)	0.736(0.654)	0.838(0.805)	0.862(0.863)	0.637(0.638)	0.830(0.816)	0.763(0.804)	0.686(0.600)
LT_3	0.844(0.906)	0.779(0.794)	0.831(0.792)	0.783(0.731)	0.819(0.870)	0.783(0.768)	0.746(0.796)	0.804(0.848)	0.787(0.764)	0.794(0.731)	0.775(0.826)	0.709(0.730)
HLT_1	0.838(0.488)	0.676(0.410)	0.597(0.443)	0.828(0.535)	0.799(0.501)	0.627(0.429)	0.810(0.519)	0.702(0.424)	0.713(0.417)	0.730(0.488)	0.676(0.456)	0.545(0.293)
HLT_2	0.971(0.711)	0.863(0.527)	0.622(0.397)	0.880(0.615)	0.865(0.622)	0.565(0.385)	0.878(0.676)	0.833(0.628)	0.665(0.417)	0.908(0.630)	0.757(0.560)	0.533(0.312)
HLT_3	0.893(0.443)	0.815(0.494)	0.718(0.573)	0.835(0.512)	0.888(0.622)	0.694(0.563)	0.776(0.465)	0.844(0.597)	0.771(0.476)	0.774(0.490)	0.787(0.611)	0.616(0.376)
LBI_1	0.991(0.968)	1.000(0.948)	1.000(0.973)	0.957(0.891)	0.998(1.000)	0.950(0.999)	0.957(0.906)	1.000(0.973)	1.000(0.978)	0.921(0.848)	0.990(0.930)	1.000(1.000)
LBI_2	0.931(0.919)	0.988(0.949)	1.000(0.977)	0.946(0.921)	0.957(0.954)	0.993(0.960)	0.964(0.891)	0.959(0.999)	0.993(0.951)	0.869(0.896)	0.997(0.981)	0.966(0.957)
LBI_3	0.932(0.962)	0.948(1.000)	1.000(1.000)	0.924(0.906)	1.000(0.956)	1.000(0.979)	0.997(0.871)	0.970(0.967)	0.958(0.959)	0.931(0.851)	1.000(1.000)	1.000(1.000)
$LKSN$	1.000(0.759)	0.997(0.357)	0.986(0.160)	0.971(0.794)	1.000(0.302)	0.985(0.033)	0.982(0.743)	1.000(0.258)	0.950(0.133)	1.000(0.721)	1.000(0.295)	0.932(0.011)
L	0.486(0.990)	0.420(0.983)	0.346(0.997)	0.291(0.970)	0.483(1.000)	0.278(1.000)	0.357(0.948)	0.469(1.000)	0.378(0.967)	0.497(0.955)	0.486(1.000)	0.225(0.912)
	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.969(0.937)	0.986(1.000)	1.000(0.949)	0.562(0.393)	0.782(0.471)	0.744(0.481)	0.939(0.904)	1.000(0.903)	1.000(0.973)	0.805(0.670)	0.922(0.800)	0.963(0.833)
K_2	0.990(0.988)	1.000(1.000)	1.000(0.968)	0.527(0.385)	0.768(0.460)	0.704(0.457)	0.992(0.847)	0.999(0.951)	0.978(1.000)	0.858(0.720)	0.896(0.898)	0.909(0.791)
K_3	1.000(0.951)	0.986(0.977)	0.994(0.993)	0.603(0.351)	0.760(0.487)	0.800(0.502)	0.982(0.841)	1.000(0.907)	0.976(1.000)	0.802(0.703)	0.897(0.884)	0.975(0.805)
LT_1	0.943(0.906)	0.888(0.873)	0.824(0.754)	0.594(0.633)	0.848(0.954)	0.769(0.803)	0.792(0.796)	0.792(0.811)	0.845(0.788)	0.706(0.763)	0.826(0.898)	0.795(0.740)
LT_2	0.888(0.935)	0.861(0.856)	0.654(0.631)	0.717(0.746)	0.914(0.897)	0.720(0.607)	0.834(0.820)	0.843(0.825)	0.674(0.602)	0.774(0.761)	0.928(0.860)	0.727(0.676)
LT_3	0.965(0.861)	0.965(0.848)	0.780(0.745)	0.667(0.610)	0.868(0.868)	0.760(0.773)	0.859(0.803)	0.771(0.802)	0.861(0.763)	0.717(0.756)	0.881(0.870)	0.808(0.799)
HLT_1	0.838(0.647)	0.856(0.705)	0.791(0.703)	0.520(0.325)	0.667(0.345)	0.502(0.290)	0.928(0.507)	0.701(0.441)	0.621(0.449)	0.759(0.621)	0.733(0.464)	0.520(0.299)
HLT_2	0.927(0.759)	0.903(0.782)	0.800(0.747)	0.532(0.382)	0.666(0.347)	0.516(0.188)	0.906(0.686)	0.849(0.613)	0.738(0.420)	0.832(0.700)	0.816(0.586)	0.514(0.243)
HLT_3	1.000(0.945)	0.962(1.000)	1.000(0.938)	0.489(0.294)	0.686(0.371)	0.596(0.297)	0.887(0.639)	0.962(0.731)	0.832(0.794)	0.746(0.550)	0.726(0.454)	0.493(0.261)
LBI_1	0.988(0.948)	1.000(1.000)	1.000(0.937)	0.750(0.748)	0.876(0.921)	0.993(0.878)	0.940(0.894)	1.000(0.954)	1.000(1.000)	0.844(0.810)	0.936(0.935)	0.978(0.983)
LBI_2	0.999(0.965)	1.000(0.999)	0.999(0.977)	0.721(0.746)	0.861(0.889)	0.915(0.878)	0.948(0.929)	1.000(0.999)	0.984(1.000)	0.818(0.846)	0.976(0.927)	0.980(0.990)
LBI_3	1.000(0.967)	0.965(1.000)	0.979(0.947)	0.794(0.726)	0.812(0.825)	0.923(0.861)	0.901(0.906)	1.000(0.997)	1.000(1.000)	0.821(0.810)	0.926(0.941)	0.986(1.000)
$LKSN$	0.978(0.708)	0.969(0.301)	1.000(0.146)	0.972(0.847)	1.000(0.480)	0.984(0.299)	1.000(0.670)	1.000(0.337)	0.957(0.089)	1.000(0.794)	0.970(0.373)	0.949(0.049)
L	0.811(0.989)	0.606(1.000)	0.362(0.972)	0.033(0.708)	0.071(0.866)	0.183(0.900)	0.690(1.000)	0.577(1.000)	0.306(1.000)	0.205(0.867)	0.394(0.931)	0.385(0.980)
	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.892(0.672)	0.971(0.859)	1.000(0.904)	0.934(0.891)	0.992(0.943)	0.999(0.992)	0.760(0.684)	0.884(0.779)	0.955(0.812)	1.000(0.942)	0.996(1.000)	0.976(0.977)
K_2	0.916(0.775)	0.958(0.876)	0.958(0.912)	0.941(0.907)	0.970(0.944)	1.000(0.972)	0.835(0.708)	0.865(0.806)	0.853(0.766)	0.959(1.000)	1.000(1.000)	1.000(0.995)
K_3	0.884(0.733)	1.000(0.946)	0.964(0.927)	0.992(0.917)	0.995(0.976)	1.000(0.994)	0.742(0.638)	0.839(0.818)	0.934(0.856)	1.000(1.000)	0.980(0.988)	0.994(0.972)
LT_1	0.729(0.741)	0.845(0.767)	0.815(0.682)	0.788(0.829)	0.823(0.866)	0.816(0.850)	0.760(0.738)	0.824(0.802)	0.771(0.812)	0.870(0.793)	0.839(0.849)	0.844(0.806)
LT_2	0.785(0.794)	0.868(0.883)	0.582(0.621)	0.809(0.867)	0.803(0.813)	0.770(0.702)	0.830(0.848)	0.826(0.843)	0.734(0.694)	0.831(0.866)	0.850(0.797)	0.641(0.670)
LT_3	0.774(0.747)	0.831(0.842)	0.821(0.695)	0.795(0.847)	0.849(0.865)	0.853(0.824)	0.770(0.749)	0.800(0.819)	0.785(0.821)	0.821(0.833)	0.893(0.864)	0.831(0.802)
HLT_1	0.811(0.447)	0.826(0.483)	0.537(0.349)	0.832(0.587)	0.799(0.701)	0.768(0.639)	0.677(0.571)	0.655(0.505)	0.529(0.240)	0.815(0.665)	0.821(0.650)	0.755(0.654)
HLT_2	0.871(0.566)	0.896(0.650)	0.557(0.345)	0.855(0.681)	0.870(0.769)	0.814(0.649)	0.797(0.648)	0.756(0.606)	0.543(0.245)	0.883(0.760)	0.845(0.747)	0.777(0.611)
HLT_3	0.866(0.533)	0.835(0.586)	0.643(0.465)	0.953(0.733)	1.000(0.941)	0.949(0.931)	0.751(0.479)	0.717(0.513)	0.487(0.281)	1.000(0.865)	0.999(0.967)	0.972(0.966)
LBI_1	0.943(0.905)	0.953(0.956)	0.966(1.000)	0.960(0.880)	1.000(0.998)	0.960(1.000)	0.816(0.864)	0.963(0.972)	0.969(0.962)	0.963(0.935)	0.977(1.000)	0.985(1.000)
LBI_2	0.848(0.861)	0.935(0.955)	0.975(1.000)	0.968(0.890)	0.958(0.977)	0.980(0.970)	0.867(0.848)	0.978(0.909)	0.986(1.000)	0.982(0.942)	0.966(0.961)	1.000(0.995)
LBI_3	0.858(0.846)	0.950(0.991)	1.000(1.000)	0.960(0.907)	0.985(0.947)	0.980(1.000)	0.868(0.895)	0.996(0.958)	1.000(1.000)	0.999(0.923)	0.983(0.970)	0.978(1.000)
$LKSN$	0.997(0.739)	0.973(0.392)	1.000(0.045)	0.998(0.750)	0.977(0.376)	1.000(0.062)	1.000(0.764)	1.000(0.385)	1.000(0.063)	1.000(0.713)	0.987(0.241)	1.000(0.029)
L	0.304(0.892)	0.431(0.983)	0.341(0.979)	0.591(1.000)	0.647(1.000)	0.417(0.998)	0.105(0.859)	0.296(0.939)	0.401(0.985)	0.735(1.000)	0.647(1.000)	0.332(0.989)

Table 2.21: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_{10}}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.944(0.885)	0.956(0.844)	0.997(0.945)	0.933(0.747)	1.000(0.909)	0.952(0.939)	0.898(0.741)	0.983(0.917)	0.997(0.929)	0.819(0.729)	0.907(0.854)	0.983(0.823)
K_2	0.961(0.862)	0.941(0.890)	1.000(0.926)	0.881(0.774)	0.997(0.975)	0.957(0.930)	0.876(0.811)	0.990(0.900)	1.000(0.919)	0.954(0.733)	0.908(0.889)	0.943(0.813)
K_3	0.961(0.899)	0.956(0.833)	0.930(0.987)	0.905(0.787)	0.964(0.929)	0.949(0.933)	0.822(0.720)	1.000(0.898)	0.970(0.997)	0.804(0.753)	0.963(0.814)	1.000(0.839)
LT_1	0.803(0.913)	0.808(0.785)	0.791(0.828)	0.737(0.752)	0.889(0.849)	0.836(0.765)	0.776(0.780)	0.812(0.784)	0.812(0.756)	0.689(0.780)	0.736(0.782)	0.712(0.718)
LT_2	0.860(0.850)	0.817(0.805)	0.713(0.721)	0.818(0.776)	0.884(0.883)	0.667(0.647)	0.879(0.840)	0.860(0.864)	0.677(0.636)	0.867(0.818)	0.778(0.780)	0.655(0.662)
LT_3	0.801(0.903)	0.818(0.810)	0.818(0.835)	0.807(0.708)	0.874(0.862)	0.858(0.804)	0.741(0.823)	0.841(0.785)	0.812(0.780)	0.766(0.728)	0.775(0.761)	0.727(0.745)
HLT_1	0.822(0.494)	0.697(0.398)	0.613(0.470)	0.786(0.517)	0.752(0.504)	0.596(0.382)	0.787(0.482)	0.761(0.450)	0.714(0.428)	0.703(0.486)	0.667(0.512)	0.544(0.295)
HLT_2	0.929(0.706)	0.859(0.511)	0.628(0.390)	0.858(0.597)	0.875(0.637)	0.581(0.380)	0.841(0.684)	0.824(0.646)	0.712(0.426)	0.880(0.597)	0.755(0.540)	0.533(0.275)
HLT_3	0.828(0.457)	0.827(0.500)	0.681(0.593)	0.876(0.545)	0.936(0.640)	0.758(0.490)	0.831(0.427)	0.804(0.592)	0.706(0.441)	0.755(0.469)	0.765(0.572)	0.640(0.442)
LBI_1	0.956(0.940)	0.984(0.996)	1.000(0.998)	0.881(0.938)	1.000(0.951)	1.000(1.000)	0.921(0.975)	0.995(0.937)	0.977(1.000)	0.953(0.861)	0.963(1.000)	0.960(1.000)
LBI_2	0.955(0.921)	1.000(0.957)	1.000(1.000)	0.919(0.944)	1.000(1.000)	1.000(1.000)	0.975(0.873)	1.000(0.959)	1.000(1.000)	0.900(0.897)	0.976(0.931)	1.000(1.000)
LBI_3	0.980(0.946)	0.984(0.945)	0.966(0.992)	0.910(0.904)	0.988(1.000)	0.957(0.985)	1.000(0.869)	1.000(1.000)	1.000(0.988)	0.947(0.864)	1.000(1.000)	1.000(0.988)
$LKSN$	1.000(0.754)	1.000(0.437)	0.979(0.160)	1.000(0.795)	0.983(0.311)	1.000(0.091)	1.000(0.719)	1.000(0.309)	0.975(0.095)	0.976(0.756)	0.966(0.363)	0.974(0.037)
L	0.477(1.000)	0.429(1.000)	0.269(0.967)	0.317(0.924)	0.500(1.000)	0.299(0.960)	0.398(0.952)	0.438(1.000)	0.418(0.950)	0.495(1.000)	0.449(1.000)	0.239(0.993)
	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.993(0.968)	1.000(1.000)	1.000(0.972)	0.541(0.338)	0.747(0.530)	0.715(0.537)	0.974(0.922)	0.996(0.935)	0.978(0.985)	0.725(0.645)	0.914(0.872)	0.885(0.848)
K_2	0.998(0.916)	1.000(1.000)	0.984(1.000)	0.529(0.407)	0.775(0.455)	0.664(0.491)	0.964(0.924)	1.000(0.992)	1.000(0.965)	0.853(0.746)	0.907(0.900)	0.842(0.810)
K_3	0.996(0.944)	1.000(0.986)	0.972(0.989)	0.546(0.402)	0.822(0.497)	0.774(0.469)	0.947(0.839)	0.970(0.949)	1.000(0.941)	0.787(0.687)	0.953(0.862)	0.944(0.850)
LT_1	0.866(0.883)	0.940(0.828)	0.784(0.750)	0.619(0.635)	0.833(0.893)	0.793(0.827)	0.813(0.765)	0.853(0.846)	0.836(0.791)	0.697(0.705)	0.881(0.912)	0.839(0.740)
LT_2	0.879(0.855)	0.852(0.820)	0.577(0.633)	0.706(0.744)	0.859(0.845)	0.728(0.669)	0.873(0.807)	0.826(0.865)	0.661(0.660)	0.761(0.806)	0.846(0.857)	0.708(0.676)
LT_3	0.887(0.909)	0.954(0.861)	0.764(0.682)	0.625(0.650)	0.888(0.903)	0.823(0.790)	0.841(0.818)	0.856(0.868)	0.785(0.772)	0.719(0.741)	0.830(0.829)	0.798(0.802)
HLT_1	0.842(0.644)	0.809(0.702)	0.801(0.711)	0.477(0.295)	0.658(0.382)	0.558(0.326)	0.907(0.536)	0.764(0.445)	0.662(0.507)	0.685(0.560)	0.739(0.448)	0.466(0.298)
HLT_2	0.941(0.706)	0.886(0.773)	0.777(0.723)	0.524(0.367)	0.610(0.353)	0.513(0.230)	0.950(0.632)	0.792(0.653)	0.679(0.454)	0.812(0.689)	0.790(0.610)	0.489(0.303)
HLT_3	0.976(0.931)	0.985(1.000)	1.000(0.971)	0.533(0.311)	0.695(0.323)	0.524(0.267)	0.892(0.645)	0.974(0.746)	0.916(0.794)	0.773(0.531)	0.770(0.445)	0.428(0.338)
LBI_1	0.946(0.950)	1.000(0.972)	0.972(0.992)	0.794(0.761)	0.885(0.844)	0.963(0.918)	0.970(0.923)	0.955(1.000)	0.989(0.966)	0.857(0.779)	0.930(0.992)	1.000(1.000)
LBI_2	0.916(0.953)	1.000(1.000)	0.996(0.968)	0.718(0.751)	0.897(0.855)	0.987(0.923)	0.908(0.929)	0.996(0.977)	1.000(1.000)	0.832(0.880)	0.953(0.932)	0.990(0.959)
LBI_3	0.995(0.901)	0.979(1.000)	0.959(1.000)	0.756(0.732)	0.846(0.904)	0.926(0.883)	0.913(0.888)	0.997(1.000)	0.989(1.000)	0.821(0.841)	0.983(0.918)	1.000(1.000)
$LKSN$	0.975(0.774)	1.000(0.298)	0.952(0.118)	1.000(0.824)	0.990(0.460)	0.978(0.285)	1.000(0.658)	1.000(0.386)	1.000(0.092)	1.000(0.832)	0.974(0.408)	0.948(0.040)
L	0.756(0.995)	0.682(0.987)	0.365(0.973)	0.037(0.697)	0.077(0.858)	0.232(0.913)	0.670(0.975)	0.607(0.996)	0.352(0.982)	0.166(0.893)	0.401(0.944)	0.319(0.974)
	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.868(0.733)	0.956(0.903)	1.000(0.908)	0.970(0.910)	0.966(0.925)	0.967(0.986)	0.775(0.697)	0.833(0.813)	0.942(0.857)	1.000(0.970)	1.000(0.970)	0.963(0.987)
K_2	0.913(0.807)	0.977(0.884)	1.000(0.887)	0.973(0.893)	1.000(0.960)	0.967(0.992)	0.835(0.774)	0.891(0.850)	0.914(0.706)	1.000(0.928)	0.981(0.995)	1.000(1.000)
K_3	0.914(0.694)	1.000(0.910)	1.000(0.946)	0.984(0.866)	1.000(0.940)	0.991(0.954)	0.789(0.612)	0.837(0.834)	0.889(0.778)	1.000(0.989)	0.968(1.000)	1.000(0.979)
LT_1	0.743(0.741)	0.830(0.797)	0.805(0.730)	0.795(0.870)	0.809(0.830)	0.799(0.843)	0.759(0.803)	0.805(0.858)	0.775(0.823)	0.872(0.792)	0.900(0.865)	0.830(0.760)
LT_2	0.760(0.755)	0.866(0.863)	0.592(0.555)	0.854(0.791)	0.797(0.829)	0.792(0.654)	0.851(0.784)	0.905(0.806)	0.708(0.674)	0.867(0.832)	0.871(0.799)	0.621(0.634)
LT_3	0.748(0.694)	0.836(0.816)	0.798(0.703)	0.817(0.867)	0.879(0.879)	0.812(0.828)	0.777(0.799)	0.829(0.813)	0.779(0.737)	0.783(0.814)	0.899(0.850)	0.792(0.757)
HLT_1	0.849(0.502)	0.843(0.505)	0.550(0.400)	0.792(0.591)	0.839(0.727)	0.766(0.598)	0.677(0.533)	0.699(0.534)	0.493(0.217)	0.856(0.674)	0.785(0.634)	0.712(0.630)
HLT_2	0.854(0.605)	0.932(0.665)	0.571(0.392)	0.888(0.730)	0.892(0.723)	0.792(0.676)	0.821(0.682)	0.828(0.551)	0.555(0.247)	0.942(0.772)	0.870(0.736)	0.788(0.690)
HLT_3	0.872(0.497)	0.873(0.562)	0.682(0.541)	0.981(0.790)	1.000(0.926)	0.966(0.966)	0.753(0.506)	0.644(0.492)	0.529(0.299)	0.949(0.884)	0.998(0.924)	0.965(0.928)
LBI_1	0.924(0.905)	0.955(0.957)	1.000(0.969)	0.979(0.912)	0.973(0.955)	1.000(1.000)	0.860(0.899)	1.000(0.929)	0.999(0.974)	0.966(0.945)	0.952(0.986)	1.000(1.000)
LBI_2	0.822(0.865)	0.927(0.957)	0.971(0.998)	0.965(0.929)	1.000(0.996)	0.997(0.947)	0.872(0.900)	0.981(0.983)	1.000(0.972)	0.966(0.979)	1.000(1.000)	0.983(0.994)
LBI_3	0.839(0.797)	1.000(0.995)	1.000(1.000)	0.922(0.897)	0.981(0.973)	1.000(0.978)	0.804(0.821)	0.969(0.953)	1.000(0.961)	0.992(0.910)	1.000(0.929)	1.000(0.984)
$LKSN$	1.000(0.723)	1.000(0.424)	0.955(0.027)	0.986(0.815)	1.000(0.339)	0.982(0.135)	0.972(0.796)	0.979(0.386)	1.000(0.000)	1.000(0.691)	1.000(0.282)	0.954(0.029)
L	0.281(0.962)	0.456(1.000)	0.333(0.991)	0.643(1.000)	0.641(0.978)	0.381(1.000)	0.116(0.873)	0.286(1.000)	0.371(0.908)	0.734(1.000)	0.669(1.000)	0.336(1.000)

Table 2.22: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **GJR-GARCH** Model in $\mathbf{H_{10}}$ with $\mathbf{T=250}$.

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.172(0.063)	0.065(0.020)	0.046(0.037)	0.179(0.029)	0.084(0.037)	0.004(0.031)	0.305(0.009)	0.297(0.000)	0.110(0.044)	0.398(0.064)	0.304(0.043)	0.183(0.003)
K_2	0.105(0.044)	0.112(0.003)	0.000(0.009)	0.234(0.125)	0.104(0.065)	0.000(0.018)	0.426(0.043)	0.237(0.000)	0.226(0.046)	0.421(0.116)	0.277(0.005)	0.131(0.000)
K_3	0.168(0.000)	0.094(0.045)	0.052(0.048)	0.201(0.024)	0.114(0.053)	0.055(0.010)	0.326(0.069)	0.288(0.052)	0.117(0.000)	0.404(0.104)	0.360(0.050)	0.186(0.046)
LT_1	0.053(0.149)	0.036(0.059)	0.000(0.050)	0.012(0.106)	0.025(0.059)	0.023(0.046)	0.112(0.148)	0.089(0.083)	0.018(0.018)	0.106(0.131)	0.051(0.060)	0.015(0.048)
LT_2	0.083(0.137)	0.036(0.000)	0.037(0.052)	0.113(0.098)	0.021(0.045)	0.000(0.000)	0.127(0.079)	0.010(0.086)	0.000(0.061)	0.161(0.088)	0.030(0.084)	0.003(0.054)
LT_3	0.023(0.134)	0.004(0.041)	0.000(0.000)	0.020(0.169)	0.020(0.074)	0.000(0.007)	0.117(0.152)	0.032(0.069)	0.002(0.000)	0.089(0.134)	0.050(0.071)	0.036(0.034)
HLT_1	0.123(0.053)	0.070(0.055)	0.046(0.000)	0.093(0.083)	0.104(0.027)	0.035(0.000)	0.314(0.018)	0.184(0.066)	0.092(0.030)	0.458(0.104)	0.353(0.037)	0.211(0.000)
HLT_2	0.122(0.072)	0.036(0.039)	0.025(0.039)	0.236(0.114)	0.114(0.022)	0.054(0.000)	0.320(0.104)	0.278(0.000)	0.160(0.000)	0.380(0.084)	0.304(0.014)	0.158(0.041)
HLT_3	0.168(0.016)	0.071(0.047)	0.000(0.016)	0.111(0.083)	0.038(0.000)	0.051(0.000)	0.320(0.034)	0.221(0.026)	0.132(0.000)	0.397(0.098)	0.275(0.018)	0.209(0.011)
LBI_1	0.068(0.112)	0.090(0.029)	0.052(0.000)	0.172(0.223)	0.085(0.074)	0.000(0.081)	0.103(0.092)	0.083(0.039)	0.034(0.049)	0.112(0.056)	0.082(0.000)	0.000(0.042)
LBI_2	0.162(0.091)	0.063(0.018)	0.000(0.001)	0.174(0.151)	0.123(0.069)	0.053(0.017)	0.096(0.061)	0.091(0.072)	0.033(0.000)	0.026(0.061)	0.011(0.039)	0.036(0.037)
LBI_3	0.129(0.068)	0.032(0.089)	0.000(0.019)	0.193(0.150)	0.100(0.135)	0.013(0.020)	0.098(0.098)	0.045(0.061)	0.040(0.000)	0.072(0.088)	0.000(0.031)	0.038(0.036)
$LKSN$	1.000(0.047)	0.995(0.916)	1.000(0.901)	0.985(0.957)	1.000(0.888)	1.000(0.865)	1.000(1.000)	1.000(0.988)	1.000(0.951)	1.000(0.908)	0.965(0.953)	1.000(0.868)
L	0.024(0.089)	0.021(0.004)	0.000(0.000)	0.009(0.220)	0.042(0.081)	0.000(0.032)	0.016(0.103)	0.019(0.054)	0.000(0.042)	0.000(0.050)	0.000(0.000)	0.005(0.022)

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.730(0.107)	0.600(0.068)	0.500(0.027)	0.824(0.033)	0.743(0.087)	0.552(0.000)	0.272(0.076)	0.150(0.068)	0.062(0.016)	0.355(0.054)	0.258(0.022)	0.145(0.013)
K_2	0.718(0.127)	0.718(0.083)	0.561(0.036)	0.804(0.042)	0.728(0.018)	0.685(0.001)	0.207(0.017)	0.130(0.064)	0.067(0.033)	0.333(0.066)	0.328(0.078)	0.163(0.016)
K_3	0.741(0.131)	0.665(0.078)	0.555(0.000)	0.872(0.044)	0.770(0.027)	0.605(0.030)	0.281(0.059)	0.137(0.048)	0.122(0.000)	0.282(0.058)	0.329(0.013)	0.177(0.013)
LT_1	0.201(0.118)	0.091(0.088)	0.040(0.053)	0.243(0.207)	0.131(0.169)	0.013(0.049)	0.098(0.081)	0.020(0.057)	0.011(0.000)	0.045(0.098)	0.054(0.058)	0.064(0.017)
LT_2	0.124(0.117)	0.091(0.000)	0.019(0.000)	0.204(0.104)	0.053(0.082)	0.061(0.000)	0.002(0.100)	0.038(0.025)	0.039(0.057)	0.074(0.120)	0.027(0.015)	0.000(0.017)
LT_3	0.163(0.142)	0.080(0.125)	0.056(0.044)	0.212(0.186)	0.194(0.113)	0.000(0.013)	0.044(0.113)	0.005(0.063)	0.000(0.051)	0.064(0.073)	0.027(0.080)	0.042(0.000)
HLT_1	0.705(0.045)	0.611(0.077)	0.538(0.000)	0.836(0.053)	0.768(0.074)	0.575(0.000)	0.203(0.078)	0.153(0.046)	0.116(0.041)	0.307(0.063)	0.236(0.005)	0.141(0.000)
HLT_2	0.763(0.120)	0.628(0.109)	0.534(0.000)	0.866(0.056)	0.783(0.032)	0.651(0.028)	0.218(0.062)	0.198(0.005)	0.082(0.018)	0.302(0.101)	0.303(0.073)	0.105(0.008)
HLT_3	0.742(0.073)	0.567(0.091)	0.531(0.000)	0.803(0.064)	0.739(0.055)	0.600(0.000)	0.267(0.057)	0.145(0.010)	0.045(0.000)	0.277(0.130)	0.239(0.018)	0.181(0.000)
LBI_1	0.195(0.112)	0.173(0.094)	0.036(0.000)	0.235(0.132)	0.178(0.048)	0.081(0.000)	0.093(0.064)	0.067(0.000)	0.000(0.000)	0.100(0.093)	0.066(0.035)	0.040(0.000)
LBI_2	0.166(0.066)	0.064(0.058)	0.095(0.000)	0.236(0.098)	0.165(0.105)	0.095(0.015)	0.063(0.084)	0.076(0.036)	0.000(0.016)	0.109(0.036)	0.037(0.073)	0.014(0.000)
LBI_3	0.148(0.079)	0.116(0.013)	0.046(0.035)	0.215(0.087)	0.122(0.113)	0.048(0.040)	0.071(0.061)	0.010(0.000)	0.000(0.036)	0.058(0.067)	0.105(0.113)	0.048(0.000)
$LKSN$	1.000(0.999)	0.984(0.928)	1.000(0.884)	1.000(0.972)	1.000(0.898)	1.000(0.887)	0.985(1.000)	0.982(0.933)	1.000(0.862)	1.000(0.962)	1.000(0.979)	0.974(0.968)
L	0.000(0.166)	0.000(0.135)	0.031(0.006)	0.010(0.178)	0.000(0.103)	0.000(0.030)	0.036(0.134)	0.000(0.024)	0.025(0.000)	0.046(0.099)	0.032(0.028)	0.026(0.015)

α	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.266(0.059)	0.205(0.043)	0.133(0.034)	0.447(0.026)	0.402(0.000)	0.235(0.000)	0.432(0.096)	0.341(0.029)	0.156(0.000)	0.605(0.060)	0.532(0.062)	0.355(0.026)
K_2	0.389(0.087)	0.235(0.066)	0.115(0.000)	0.533(0.000)	0.421(0.016)	0.240(0.004)	0.483(0.071)	0.375(0.043)	0.172(0.040)	0.634(0.036)	0.544(0.044)	0.406(0.049)
K_3	0.238(0.044)	0.156(0.086)	0.133(0.000)	0.457(0.059)	0.452(0.050)	0.225(0.000)	0.473(0.038)	0.368(0.012)	0.176(0.018)	0.615(0.051)	0.443(0.044)	0.336(0.000)
LT_1	0.079(0.101)	0.064(0.110)	0.039(0.046)	0.126(0.091)	0.104(0.068)	0.001(0.009)	0.055(0.140)	0.087(0.096)	0.029(0.020)	0.261(0.241)	0.155(0.110)	0.010(0.024)
LT_2	0.060(0.113)	0.074(0.108)	0.080(0.060)	0.027(0.080)	0.014(0.044)	0.053(0.031)	0.064(0.170)	0.037(0.019)	0.007(0.015)	0.192(0.113)	0.041(0.031)	0.049(0.019)
LT_3	0.137(0.124)	0.084(0.115)	0.000(0.044)	0.123(0.138)	0.111(0.109)	0.000(0.032)	0.093(0.168)	0.082(0.025)	0.058(0.024)	0.264(0.162)	0.097(0.066)	0.016(0.000)
HLT_1	0.257(0.047)	0.231(0.066)	0.080(0.000)	0.497(0.000)	0.375(0.025)	0.158(0.000)	0.477(0.067)	0.359(0.000)	0.181(0.000)	0.588(0.073)	0.516(0.044)	0.352(0.000)
HLT_2	0.349(0.067)	0.288(0.071)	0.080(0.040)	0.550(0.028)	0.376(0.000)	0.230(0.000)	0.412(0.063)	0.391(0.000)	0.137(0.031)	0.618(0.023)	0.469(0.048)	0.414(0.000)
HLT_3	0.307(0.042)	0.185(0.012)	0.086(0.017)	0.476(0.000)	0.409(0.000)	0.178(0.043)	0.456(0.076)	0.409(0.024)	0.202(0.016)	0.623(0.079)	0.506(0.028)	0.349(0.054)
LBI_1	0.146(0.078)	0.070(0.065)	0.040(0.055)	0.117(0.087)	0.091(0.090)	0.000(0.045)	0.132(0.079)	0.153(0.077)	0.013(0.000)	0.132(0.069)	0.148(0.024)	0.110(0.033)
LBI_2	0.170(0.061)	0.013(0.000)	0.027(0.000)	0.062(0.067)	0.080(0.066)	0.038(0.043)	0.169(0.138)	0.087(0.050)	0.067(0.061)	0.134(0.114)	0.111(0.076)	0.057(0.031)
LBI_3	0.085(0.057)	0.015(0.000)	0.012(0.046)	0.043(0.057)	0.068(0.000)	0.004(0.000)	0.231(0.177)	0.129(0.041)	0.046(0.000)	0.124(0.083)	0.067(0.015)	0.000(0.000)
$LKSN$	0.960(0.988)	1.000(0.969)	1.000(0.875)	1.000(0.928)	1.000(0.901)	1.000(0.889)	0.978(0.970)	1.000(0.949)	1.000(0.891)	1.000(0.994)	1.000(0.951)	1.000(0.909)
L	0.000(0.119)	0.000(0.069)	0.002(0.018)	0.016(0.026)	0.000(0.005)	0.035(0.033)	0.015(0.067)	0.000(0.071)	0.000(0.000)	0.041(0.151)	0.005(0.037)	0.000(0.000)

Table 2.23: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_0}$ with $\mathbf{T=100}$.

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.137(0.064)	0.109(0.062)	0.052(0.024)	0.144(0.073)	0.080(0.020)	0.000(0.000)	0.385(0.005)	0.281(0.008)	0.156(0.000)	0.437(0.062)	0.333(0.056)	0.161(0.000)
K_2	0.124(0.112)	0.109(0.001)	0.000(0.005)	0.227(0.124)	0.098(0.024)	0.000(0.033)	0.447(0.058)	0.283(0.063)	0.219(0.005)	0.353(0.077)	0.344(0.038)	0.124(0.004)
K_3	0.132(0.065)	0.087(0.033)	0.007(0.048)	0.181(0.029)	0.111(0.000)	0.001(0.012)	0.319(0.082)	0.289(0.041)	0.130(0.028)	0.425(0.132)	0.339(0.055)	0.164(0.014)
LT_1	0.065(0.141)	0.040(0.084)	0.027(0.007)	0.084(0.137)	0.075(0.069)	0.042(0.026)	0.135(0.072)	0.015(0.116)	0.027(0.001)	0.117(0.129)	0.110(0.000)	0.061(0.000)
LT_2	0.082(0.108)	0.001(0.001)	0.033(0.056)	0.097(0.159)	0.000(0.115)	0.000(0.006)	0.134(0.111)	0.007(0.034)	0.012(0.029)	0.084(0.105)	0.053(0.060)	0.006(0.005)
LT_3	0.053(0.127)	0.054(0.073)	0.000(0.027)	0.059(0.119)	0.003(0.008)	0.000(0.000)	0.085(0.083)	0.063(0.072)	0.008(0.004)	0.106(0.048)	0.089(0.071)	0.031(0.074)
HLT_1	0.098(0.027)	0.031(0.009)	0.039(0.042)	0.104(0.031)	0.105(0.012)	0.000(0.013)	0.302(0.004)	0.258(0.047)	0.120(0.000)	0.462(0.079)	0.357(0.000)	0.173(0.006)
HLT_2	0.170(0.106)	0.072(0.026)	0.013(0.008)	0.221(0.128)	0.090(0.030)	0.027(0.000)	0.386(0.047)	0.214(0.064)	0.168(0.004)	0.337(0.064)	0.294(0.016)	0.166(0.003)
HLT_3	0.133(0.056)	0.055(0.046)	0.000(0.010)	0.135(0.020)	0.064(0.000)	0.058(0.037)	0.335(0.067)	0.258(0.020)	0.110(0.017)	0.417(0.055)	0.281(0.000)	0.204(0.015)
LBI_1	0.126(0.096)	0.025(0.063)	0.000(0.005)	0.155(0.147)	0.128(0.130)	0.050(0.092)	0.114(0.099)	0.023(0.070)	0.000(0.000)	0.058(0.035)	0.026(0.000)	0.000(0.000)
LBI_2	0.154(0.085)	0.049(0.089)	0.011(0.005)	0.113(0.149)	0.088(0.102)	0.011(0.065)	0.102(0.147)	0.067(0.048)	0.025(0.000)	0.076(0.024)	0.053(0.000)	0.047(0.058)
LBI_3	0.117(0.121)	0.064(0.070)	0.009(0.039)	0.134(0.112)	0.168(0.105)	0.008(0.053)	0.094(0.139)	0.046(0.082)	0.070(0.012)	0.020(0.099)	0.045(0.027)	0.000(0.057)
$LKSN$	1.000(0.088)	1.000(0.910)	1.000(0.882)	0.992(1.000)	0.990(0.957)	0.961(0.890)	0.973(0.987)	1.000(0.970)	0.970(0.943)	1.000(0.963)	0.974(0.890)	0.967(0.900)
L	0.044(0.126)	0.028(0.047)	0.010(0.031)	0.000(0.198)	0.000(0.017)	0.024(0.032)	0.034(0.177)	0.000(0.097)	0.000(0.046)	0.000(0.035)	0.027(0.000)	0.010(0.031)

α	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.718(0.043)	0.640(0.017)	0.524(0.004)	0.863(0.032)	0.754(0.070)	0.549(0.010)	0.253(0.068)	0.175(0.074)	0.092(0.000)	0.287(0.118)	0.300(0.013)	0.170(0.000)
K_2	0.792(0.130)	0.692(0.077)	0.548(0.000)	0.861(0.031)	0.789(0.044)	0.665(0.005)	0.174(0.076)	0.179(0.000)	0.048(0.011)	0.340(0.065)	0.331(0.074)	0.144(0.059)
K_3	0.701(0.082)	0.663(0.036)	0.495(0.009)	0.840(0.062)	0.777(0.011)	0.569(0.000)	0.301(0.000)	0.169(0.022)	0.109(0.016)	0.361(0.112)	0.256(0.001)	0.126(0.049)
LT_1	0.196(0.128)	0.178(0.070)	0.000(0.004)	0.236(0.208)	0.154(0.139)	0.058(0.068)	0.117(0.095)	0.002(0.086)	0.009(0.013)	0.093(0.129)	0.100(0.090)	0.028(0.000)
LT_2	0.137(0.122)	0.021(0.038)	0.028(0.000)	0.184(0.139)	0.055(0.089)	0.055(0.025)	0.002(0.078)	0.000(0.004)	0.000(0.046)	0.081(0.129)	0.004(0.067)	0.000(0.045)
LT_3	0.173(0.142)	0.098(0.110)	0.000(0.013)	0.216(0.241)	0.141(0.128)	0.060(0.026)	0.049(0.106)	0.016(0.050)	0.053(0.000)	0.073(0.073)	0.074(0.026)	0.038(0.000)
HLT_1	0.725(0.110)	0.634(0.029)	0.476(0.000)	0.809(0.079)	0.781(0.071)	0.592(0.000)	0.272(0.020)	0.198(0.000)	0.060(0.000)	0.346(0.073)	0.262(0.007)	0.121(0.052)
HLT_2	0.753(0.098)	0.664(0.072)	0.532(0.034)	0.795(0.059)	0.785(0.081)	0.608(0.000)	0.211(0.074)	0.156(0.040)	0.117(0.038)	0.335(0.124)	0.309(0.075)	0.143(0.000)
HLT_3	0.704(0.061)	0.612(0.042)	0.524(0.020)	0.849(0.016)	0.742(0.023)	0.562(0.045)	0.246(0.003)	0.164(0.068)	0.057(0.000)	0.297(0.049)	0.261(0.031)	0.182(0.000)
LBI_1	0.207(0.148)	0.148(0.081)	0.023(0.000)	0.241(0.086)	0.156(0.037)	0.077(0.012)	0.127(0.036)	0.027(0.000)	0.019(0.012)	0.068(0.130)	0.093(0.011)	0.035(0.000)
LBI_2	0.160(0.128)	0.119(0.014)	0.042(0.000)	0.234(0.113)	0.138(0.037)	0.069(0.037)	0.061(0.018)	0.047(0.038)	0.038(0.000)	0.116(0.063)	0.084(0.031)	0.062(0.000)
LBI_3	0.164(0.097)	0.144(0.075)	0.073(0.041)	0.241(0.096)	0.108(0.070)	0.086(0.015)	0.093(0.083)	0.077(0.029)	0.000(0.000)	0.071(0.029)	0.100(0.059)	0.034(0.001)
$LKSN$	1.000(1.000)	0.992(0.989)	0.980(0.927)	1.000(0.970)	1.000(0.946)	1.000(0.856)	1.000(1.000)	0.978(0.963)	0.969(0.923)	1.000(1.000)	1.000(1.000)	0.994(0.961)
L	0.000(0.172)	0.042(0.113)	0.009(0.050)	0.000(0.132)	0.000(0.060)	0.032(0.000)	0.027(0.155)	0.045(0.043)	0.000(0.011)	0.000(0.090)	0.000(0.035)	0.014(0.038)

α	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.242(0.080)	0.182(0.038)	0.125(0.000)	0.513(0.034)	0.376(0.000)	0.240(0.030)	0.467(0.105)	0.332(0.060)	0.163(0.009)	0.643(0.045)	0.543(0.007)	0.398(0.017)
K_2	0.345(0.115)	0.213(0.000)	0.073(0.000)	0.533(0.054)	0.458(0.026)	0.234(0.025)	0.474(0.059)	0.431(0.000)	0.193(0.041)	0.627(0.035)	0.518(0.062)	0.412(0.054)
K_3	0.273(0.082)	0.179(0.035)	0.111(0.012)	0.496(0.000)	0.382(0.000)	0.234(0.000)	0.427(0.104)	0.354(0.086)	0.157(0.000)	0.670(0.052)	0.502(0.019)	0.310(0.033)
LT_1	0.137(0.101)	0.109(0.063)	0.000(0.000)	0.145(0.149)	0.054(0.066)	0.026(0.000)	0.129(0.164)	0.072(0.085)	0.040(0.031)	0.215(0.225)	0.162(0.091)	0.026(0.000)
LT_2	0.079(0.064)	0.090(0.083)	0.040(0.080)	0.078(0.080)	0.038(0.016)	0.012(0.043)	0.113(0.095)	0.068(0.071)	0.007(0.000)	0.148(0.122)	0.019(0.062)	0.024(0.042)
LT_3	0.095(0.073)	0.064(0.093)	0.000(0.059)	0.058(0.075)	0.128(0.112)	0.050(0.035)	0.111(0.136)	0.056(0.075)	0.000(0.046)	0.231(0.213)	0.140(0.138)	0.030(0.063)
HLT_1	0.227(0.061)	0.214(0.091)	0.088(0.000)	0.475(0.000)	0.351(0.049)	0.172(0.000)	0.486(0.095)	0.389(0.000)	0.220(0.004)	0.568(0.066)	0.484(0.073)	0.368(0.000)
HLT_2	0.278(0.119)	0.291(0.008)	0.144(0.000)	0.546(0.022)	0.397(0.012)	0.236(0.000)	0.425(0.084)	0.353(0.062)	0.185(0.000)	0.673(0.027)	0.524(0.057)	0.367(0.000)
HLT_3	0.277(0.029)	0.211(0.021)	0.103(0.007)	0.450(0.039)	0.434(0.000)	0.170(0.037)	0.435(0.096)	0.359(0.037)	0.209(0.050)	0.657(0.043)	0.458(0.000)	0.309(0.011)
LBI_1	0.085(0.103)	0.075(0.028)	0.071(0.057)	0.103(0.135)	0.062(0.027)	0.056(0.019)	0.101(0.086)	0.138(0.057)	0.011(0.025)	0.098(0.097)	0.092(0.028)	0.046(0.011)
LBI_2	0.159(0.036)	0.035(0.067)	0.042(0.038)	0.071(0.041)	0.033(0.028)	0.010(0.049)	0.170(0.133)	0.102(0.110)	0.000(0.023)	0.127(0.057)	0.083(0.021)	0.026(0.047)
LBI_3	0.089(0.050)	0.000(0.047)	0.008(0.020)	0.106(0.094)	0.100(0.038)	0.000(0.000)	0.169(0.185)	0.081(0.084)	0.037(0.000)	0.115(0.078)	0.139(0.087)	0.043(0.011)
$LKSN$	0.998(1.000)	1.000(0.943)	1.000(0.934)	1.000(0.944)	1.000(0.903)	1.000(0.895)	1.000(0.975)	1.000(0.898)	0.967(0.910)	0.971(0.963)	0.970(0.989)	0.996(0.914)
L	0.026(0.068)	0.050(0.036)	0.050(0.029)	0.022(0.043)	0.035(0.064)	0.000(0.000)	0.024(0.100)	0.000(0.058)	0.000(0.032)	0.000(0.110)	0.030(0.080)	0.000(0.000)

Table 2.24: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests.
EGARCH Model in $\mathbf{H_0}$ with $\mathbf{T = 250}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.906(0.720)	0.870(0.667)	0.694(0.456)	0.834(0.627)	0.775(0.509)	0.702(0.339)	0.806(0.634)	0.749(0.494)	0.677(0.385)	0.806(0.570)	0.753(0.485)	0.651(0.433)
K_2	0.833(0.590)	0.695(0.497)	0.541(0.307)	0.828(0.568)	0.728(0.409)	0.596(0.244)	0.726(0.484)	0.730(0.493)	0.588(0.335)	0.832(0.547)	0.745(0.476)	0.548(0.360)
K_3	0.845(0.723)	0.862(0.643)	0.769(0.399)	0.880(0.595)	0.823(0.520)	0.679(0.362)	0.860(0.674)	0.754(0.556)	0.709(0.440)	0.845(0.602)	0.807(0.455)	0.662(0.426)
LT_1	0.247(0.311)	0.210(0.195)	0.107(0.038)	0.295(0.315)	0.254(0.209)	0.053(0.114)	0.293(0.328)	0.186(0.185)	0.106(0.134)	0.314(0.347)	0.222(0.215)	0.047(0.089)
LT_2	0.205(0.198)	0.167(0.120)	0.041(0.032)	0.248(0.123)	0.117(0.067)	0.033(0.000)	0.211(0.091)	0.145(0.104)	0.006(0.034)	0.263(0.124)	0.116(0.095)	0.000(0.041)
LT_3	0.315(0.272)	0.273(0.182)	0.121(0.064)	0.270(0.313)	0.240(0.165)	0.111(0.118)	0.278(0.288)	0.184(0.168)	0.059(0.109)	0.338(0.252)	0.251(0.228)	0.109(0.093)
HLT_1	0.153(0.058)	0.109(0.005)	0.040(0.031)	0.183(0.105)	0.094(0.005)	0.012(0.004)	0.208(0.129)	0.123(0.126)	0.100(0.060)	0.287(0.100)	0.207(0.100)	0.184(0.001)
HLT_2	0.143(0.078)	0.086(0.024)	0.028(0.003)	0.149(0.064)	0.098(0.075)	0.031(0.042)	0.160(0.127)	0.119(0.071)	0.097(0.031)	0.251(0.091)	0.222(0.033)	0.092(0.067)
HLT_3	0.138(0.019)	0.066(0.000)	0.041(0.047)	0.178(0.118)	0.126(0.092)	0.096(0.014)	0.278(0.183)	0.193(0.158)	0.148(0.099)	0.308(0.226)	0.274(0.195)	0.239(0.122)
LBI_1	1.000(0.995)	1.000(0.984)	1.000(0.974)	0.983(1.000)	0.958(1.000)	0.982(0.964)	0.969(0.981)	0.979(0.960)	1.000(1.000)	0.971(1.000)	1.000(0.951)	0.956(0.994)
LBI_2	1.000(0.991)	0.950(0.984)	0.977(1.000)	0.977(0.964)	1.000(1.000)	0.977(0.969)	1.000(1.000)	1.000(1.000)	1.000(0.963)	0.999(0.999)	0.962(1.000)	0.988(0.939)
LBI_3	0.980(1.000)	0.953(0.981)	1.000(0.969)	1.000(1.000)	0.952(0.991)	1.000(1.000)	1.000(0.969)	1.000(0.973)	0.997(1.000)	1.000(1.000)	1.000(0.958)	1.000(0.931)
$LKSN$	0.115(0.000)	0.072(0.000)	0.055(0.000)	0.127(0.029)	0.036(0.000)	0.059(0.000)	0.191(0.029)	0.077(0.000)	0.060(0.040)	0.234(0.000)	0.163(0.000)	0.080(0.049)
L	0.177(0.079)	0.064(0.061)	0.038(0.042)	0.110(0.120)	0.103(0.040)	0.000(0.000)	0.181(0.092)	0.096(0.012)	0.004(0.048)	0.100(0.099)	0.036(0.029)	0.030(0.047)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.898(0.736)	0.841(0.689)	0.894(0.589)	0.940(0.653)	0.911(0.609)	0.886(0.533)	0.861(0.715)	0.876(0.587)	0.762(0.446)	0.803(0.598)	0.701(0.400)	0.643(0.365)
K_2	0.885(0.652)	0.861(0.653)	0.793(0.523)	0.929(0.693)	0.950(0.601)	0.867(0.434)	0.812(0.598)	0.749(0.526)	0.585(0.361)	0.745(0.544)	0.694(0.443)	0.611(0.296)
K_3	0.862(0.733)	0.848(0.682)	0.887(0.570)	0.877(0.649)	0.862(0.615)	0.829(0.490)	0.907(0.680)	0.876(0.605)	0.760(0.519)	0.765(0.581)	0.746(0.414)	0.618(0.329)
LT_1	0.321(0.297)	0.216(0.217)	0.087(0.083)	0.272(0.293)	0.170(0.162)	0.052(0.051)	0.273(0.373)	0.170(0.179)	0.079(0.064)	0.288(0.261)	0.163(0.229)	0.067(0.091)
LT_2	0.245(0.208)	0.154(0.137)	0.050(0.015)	0.128(0.127)	0.045(0.021)	0.074(0.030)	0.204(0.221)	0.149(0.081)	0.049(0.039)	0.197(0.195)	0.148(0.091)	0.050(0.000)
LT_3	0.272(0.261)	0.201(0.290)	0.065(0.094)	0.224(0.306)	0.172(0.139)	0.032(0.051)	0.282(0.360)	0.223(0.209)	0.054(0.071)	0.305(0.261)	0.153(0.141)	0.050(0.073)
HLT_1	0.459(0.171)	0.431(0.148)	0.314(0.099)	0.399(0.175)	0.391(0.065)	0.300(0.104)	0.222(0.060)	0.110(0.089)	0.076(0.026)	0.221(0.078)	0.134(0.099)	0.132(0.031)
HLT_2	0.443(0.183)	0.391(0.165)	0.272(0.126)	0.539(0.161)	0.423(0.139)	0.292(0.123)	0.259(0.114)	0.219(0.070)	0.073(0.054)	0.300(0.095)	0.250(0.080)	0.132(0.027)
HLT_3	0.532(0.340)	0.516(0.281)	0.415(0.254)	0.532(0.208)	0.464(0.261)	0.282(0.191)	0.295(0.124)	0.165(0.178)	0.159(0.083)	0.264(0.133)	0.198(0.053)	0.124(0.068)
LBI_1	1.000(0.990)	1.000(0.979)	0.946(0.938)	0.949(1.000)	1.000(1.000)	0.975(0.893)	0.969(0.984)	0.960(0.963)	0.967(0.953)	1.000(1.000)	0.995(1.000)	1.000(0.978)
LBI_2	1.000(0.974)	0.993(0.971)	0.991(0.913)	0.986(0.949)	1.000(0.932)	0.992(0.876)	1.000(0.980)	1.000(1.000)	0.953(0.947)	1.000(1.000)	0.950(1.000)	1.000(0.954)
LBI_3	1.000(1.000)	0.959(1.000)	0.944(0.937)	0.991(0.961)	0.976(1.000)	0.969(0.861)	0.993(0.960)	1.000(1.000)	0.989(0.958)	1.000(0.963)	1.000(0.995)	0.993(0.954)
$LKSN$	0.239(0.000)	0.211(0.000)	0.075(0.000)	0.263(0.000)	0.101(0.000)	0.059(0.017)	0.226(0.014)	0.108(0.045)	0.085(0.000)	0.152(0.000)	0.144(0.020)	0.062(0.053)
L	0.138(0.104)	0.079(0.054)	0.000(0.013)	0.176(0.105)	0.151(0.080)	0.071(0.030)	0.166(0.074)	0.102(0.026)	0.000(0.046)	0.149(0.046)	0.016(0.079)	0.003(0.050)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.848(0.702)	0.780(0.537)	0.714(0.345)	0.878(0.649)	0.824(0.670)	0.794(0.491)	0.828(0.631)	0.763(0.555)	0.690(0.414)	0.894(0.697)	0.853(0.681)	0.779(0.470)
K_2	0.851(0.567)	0.763(0.458)	0.637(0.245)	0.849(0.611)	0.767(0.581)	0.714(0.425)	0.768(0.488)	0.697(0.477)	0.565(0.289)	0.861(0.649)	0.855(0.599)	0.756(0.464)
K_3	0.808(0.655)	0.792(0.544)	0.703(0.422)	0.899(0.712)	0.813(0.635)	0.728(0.538)	0.863(0.647)	0.795(0.554)	0.675(0.368)	0.848(0.676)	0.804(0.708)	0.822(0.511)
LT_1	0.251(0.226)	0.205(0.168)	0.045(0.053)	0.353(0.272)	0.184(0.235)	0.123(0.074)	0.257(0.249)	0.150(0.156)	0.075(0.029)	0.188(0.256)	0.153(0.133)	0.133(0.087)
LT_2	0.172(0.087)	0.093(0.018)	0.014(0.011)	0.203(0.156)	0.151(0.099)	0.007(0.000)	0.168(0.161)	0.144(0.081)	0.053(0.023)	0.107(0.051)	0.017(0.000)	0.006(0.000)
LT_3	0.263(0.234)	0.205(0.182)	0.001(0.028)	0.328(0.296)	0.184(0.223)	0.135(0.105)	0.215(0.197)	0.134(0.164)	0.136(0.134)	0.212(0.189)	0.150(0.145)	0.100(0.075)
HLT_1	0.192(0.074)	0.119(0.026)	0.090(0.011)	0.357(0.228)	0.284(0.155)	0.242(0.114)	0.239(0.046)	0.204(0.049)	0.137(0.038)	0.386(0.148)	0.287(0.127)	0.266(0.023)
HLT_2	0.180(0.017)	0.081(0.019)	0.061(0.057)	0.375(0.198)	0.371(0.130)	0.200(0.135)	0.178(0.067)	0.173(0.037)	0.126(0.033)	0.354(0.173)	0.224(0.066)	0.256(0.090)
HLT_3	0.211(0.138)	0.163(0.123)	0.067(0.070)	0.398(0.224)	0.383(0.247)	0.288(0.182)	0.259(0.148)	0.199(0.173)	0.128(0.116)	0.456(0.253)	0.421(0.255)	0.316(0.235)
LBI_1	1.000(0.986)	1.000(1.000)	0.999(0.964)	1.000(0.962)	1.000(1.000)	1.000(0.951)	1.000(1.000)	1.000(1.000)	1.000(1.000)	1.000(1.000)	0.972(1.000)	0.986(0.943)
LBI_2	0.984(0.971)	1.000(0.980)	0.992(0.993)	0.980(0.971)	0.988(1.000)	0.959(0.936)	1.000(0.968)	1.000(0.994)	0.959(0.996)	1.000(1.000)	1.000(1.000)	0.945(0.982)
LBI_3	1.000(0.962)	0.998(1.000)	1.000(1.000)	1.000(0.996)	0.955(1.000)	1.000(1.000)	0.997(0.978)	0.989(0.981)	1.000(0.982)	1.000(0.994)	1.000(0.976)	1.000(0.982)
$LKSN$	0.103(0.047)	0.019(0.000)	0.000(0.000)	0.258(0.000)	0.223(0.000)	0.060(0.000)	0.142(0.003)	0.113(0.005)	0.011(0.000)	0.340(0.000)	0.177(0.000)	0.053(0.000)
L	0.084(0.103)	0.056(0.065)	0.000(0.052)	0.087(0.069)	0.042(0.064)	0.030(0.000)	0.119(0.075)	0.048(0.038)	0.000(0.048)	0.142(0.059)	0.021(0.072)	0.053(0.000)

Table 2.25: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_1}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.890(0.697)	0.845(0.630)	0.757(0.452)	0.819(0.644)	0.775(0.489)	0.708(0.350)	0.858(0.610)	0.795(0.497)	0.647(0.416)	0.812(0.549)	0.721(0.513)	0.676(0.442)
K_2	0.774(0.601)	0.721(0.459)	0.599(0.335)	0.847(0.521)	0.732(0.421)	0.596(0.253)	0.783(0.555)	0.709(0.464)	0.626(0.330)	0.770(0.528)	0.698(0.413)	0.607(0.375)
K_3	0.892(0.762)	0.884(0.664)	0.712(0.466)	0.838(0.623)	0.836(0.529)	0.700(0.388)	0.805(0.629)	0.780(0.590)	0.690(0.409)	0.813(0.611)	0.806(0.459)	0.645(0.446)
LT_1	0.248(0.318)	0.210(0.171)	0.104(0.059)	0.338(0.317)	0.210(0.248)	0.036(0.096)	0.267(0.316)	0.231(0.174)	0.059(0.050)	0.317(0.351)	0.222(0.178)	0.051(0.105)
LT_2	0.253(0.201)	0.094(0.129)	0.052(0.035)	0.259(0.186)	0.114(0.044)	0.055(0.000)	0.214(0.176)	0.116(0.058)	0.067(0.010)	0.204(0.121)	0.147(0.080)	0.004(0.038)
LT_3	0.264(0.271)	0.250(0.200)	0.100(0.079)	0.300(0.345)	0.259(0.157)	0.118(0.067)	0.335(0.312)	0.191(0.182)	0.130(0.074)	0.328(0.340)	0.222(0.202)	0.138(0.128)
HLT_1	0.090(0.003)	0.119(0.013)	0.027(0.000)	0.175(0.043)	0.097(0.043)	0.033(0.013)	0.168(0.064)	0.147(0.106)	0.115(0.045)	0.313(0.129)	0.253(0.041)	0.193(0.042)
HLT_2	0.125(0.022)	0.098(0.041)	0.086(0.013)	0.096(0.010)	0.086(0.050)	0.045(0.000)	0.177(0.041)	0.139(0.065)	0.123(0.000)	0.278(0.068)	0.202(0.064)	0.144(0.012)
HLT_3	0.144(0.052)	0.091(0.000)	0.017(0.000)	0.182(0.076)	0.092(0.090)	0.062(0.028)	0.288(0.188)	0.251(0.185)	0.133(0.174)	0.368(0.166)	0.292(0.202)	0.280(0.116)
LBI_1	0.958(0.961)	1.000(0.996)	0.977(0.998)	0.960(0.980)	1.000(0.964)	1.000(1.000)	0.982(1.000)	0.960(1.000)	1.000(1.000)	1.000(0.975)	1.000(1.000)	0.987(0.939)
LBI_2	0.961(0.984)	0.983(1.000)	0.957(1.000)	1.000(1.000)	1.000(1.000)	0.982(1.000)	1.000(0.999)	0.974(1.000)	0.983(0.968)	1.000(1.000)	0.983(0.957)	1.000(0.931)
LBI_3	0.983(1.000)	0.995(1.000)	1.000(0.985)	1.000(1.000)	0.956(0.954)	1.000(0.970)	1.000(1.000)	0.964(1.000)	0.960(0.978)	0.996(1.000)	1.000(0.985)	0.949(1.000)
$LKSN$	0.100(0.036)	0.127(0.000)	0.069(0.033)	0.114(0.000)	0.093(0.000)	0.000(0.000)	0.124(0.000)	0.059(0.003)	0.000(0.000)	0.268(0.021)	0.109(0.044)	0.068(0.006)
L	0.103(0.060)	0.059(0.064)	0.025(0.057)	0.099(0.101)	0.110(0.010)	0.000(0.000)	0.125(0.089)	0.088(0.052)	0.039(0.000)	0.078(0.085)	0.091(0.058)	0.000(0.000)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.888(0.683)	0.871(0.677)	0.871(0.526)	0.951(0.647)	0.878(0.663)	0.862(0.507)	0.846(0.745)	0.885(0.644)	0.746(0.498)	0.835(0.623)	0.763(0.429)	0.659(0.340)
K_2	0.895(0.686)	0.861(0.650)	0.815(0.528)	0.899(0.693)	0.920(0.541)	0.873(0.443)	0.799(0.663)	0.772(0.488)	0.589(0.373)	0.794(0.537)	0.761(0.480)	0.622(0.343)
K_3	0.916(0.768)	0.909(0.728)	0.829(0.596)	0.878(0.657)	0.926(0.631)	0.875(0.506)	0.936(0.713)	0.818(0.602)	0.787(0.474)	0.757(0.608)	0.703(0.463)	0.692(0.319)
LT_1	0.308(0.238)	0.242(0.262)	0.110(0.066)	0.248(0.252)	0.096(0.100)	0.041(0.000)	0.289(0.339)	0.225(0.240)	0.063(0.027)	0.242(0.272)	0.210(0.225)	0.063(0.075)
LT_2	0.213(0.163)	0.109(0.115)	0.000(0.029)	0.153(0.108)	0.040(0.031)	0.073(0.000)	0.193(0.171)	0.145(0.032)	0.069(0.000)	0.227(0.162)	0.119(0.168)	0.084(0.006)
LT_3	0.327(0.295)	0.274(0.262)	0.140(0.092)	0.288(0.267)	0.092(0.165)	0.029(0.046)	0.349(0.371)	0.185(0.195)	0.118(0.045)	0.303(0.262)	0.204(0.179)	0.035(0.093)
HLT_1	0.410(0.221)	0.390(0.115)	0.281(0.138)	0.476(0.175)	0.349(0.100)	0.258(0.046)	0.164(0.116)	0.150(0.122)	0.061(0.088)	0.184(0.060)	0.127(0.087)	0.073(0.010)
HLT_2	0.458(0.218)	0.362(0.151)	0.264(0.090)	0.538(0.159)	0.418(0.130)	0.319(0.100)	0.184(0.086)	0.167(0.080)	0.134(0.007)	0.274(0.100)	0.239(0.009)	0.181(0.033)
HLT_3	0.542(0.307)	0.539(0.252)	0.434(0.275)	0.510(0.242)	0.423(0.250)	0.353(0.183)	0.216(0.115)	0.170(0.147)	0.096(0.088)	0.207(0.091)	0.167(0.073)	0.122(0.032)
LBI_1	1.000(0.960)	0.997(1.000)	0.994(0.968)	1.000(0.994)	1.000(0.969)	0.970(0.943)	1.000(1.000)	0.956(0.963)	0.988(0.989)	1.000(0.977)	0.962(1.000)	1.000(1.000)
LBI_2	0.982(0.994)	0.960(0.991)	0.978(0.939)	0.967(1.000)	1.000(0.941)	0.967(0.888)	1.000(1.000)	0.956(1.000)	0.960(0.992)	0.967(1.000)	1.000(1.000)	0.977(0.954)
LBI_3	1.000(0.974)	1.000(1.000)	0.921(0.950)	0.977(1.000)	0.987(0.945)	0.958(0.909)	0.970(0.971)	1.000(0.955)	0.943(1.000)	0.984(1.000)	0.969(0.981)	0.997(1.000)
$LKSN$	0.251(0.000)	0.169(0.023)	0.095(0.037)	0.247(0.000)	0.134(0.000)	0.032(0.038)	0.167(0.023)	0.148(0.032)	0.071(0.000)	0.181(0.047)	0.158(0.000)	0.045(0.000)
L	0.137(0.109)	0.063(0.089)	0.006(0.051)	0.203(0.098)	0.152(0.085)	0.035(0.049)	0.135(0.100)	0.064(0.063)	0.000(0.000)	0.067(0.088)	0.000(0.072)	0.042(0.014)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
α	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
K_1	0.835(0.640)	0.838(0.540)	0.694(0.365)	0.925(0.716)	0.828(0.639)	0.761(0.511)	0.799(0.606)	0.744(0.600)	0.723(0.328)	0.885(0.710)	0.845(0.707)	0.843(0.524)
K_2	0.764(0.576)	0.729(0.421)	0.669(0.222)	0.830(0.615)	0.781(0.577)	0.674(0.482)	0.843(0.578)	0.758(0.442)	0.602(0.328)	0.843(0.685)	0.843(0.565)	0.712(0.392)
K_3	0.769(0.690)	0.820(0.580)	0.709(0.390)	0.921(0.724)	0.859(0.633)	0.782(0.552)	0.834(0.639)	0.727(0.595)	0.683(0.395)	0.835(0.684)	0.849(0.652)	0.802(0.489)
LT_1	0.206(0.297)	0.173(0.173)	0.000(0.089)	0.345(0.292)	0.190(0.204)	0.106(0.128)	0.261(0.240)	0.144(0.146)	0.133(0.077)	0.158(0.251)	0.186(0.118)	0.065(0.064)
LT_2	0.141(0.131)	0.066(0.006)	0.057(0.000)	0.238(0.194)	0.151(0.124)	0.038(0.000)	0.173(0.147)	0.117(0.101)	0.023(0.020)	0.080(0.079)	0.037(0.000)	0.041(0.000)
LT_3	0.191(0.224)	0.228(0.175)	0.062(0.071)	0.360(0.250)	0.194(0.248)	0.126(0.123)	0.257(0.174)	0.135(0.178)	0.134(0.108)	0.164(0.201)	0.147(0.131)	0.064(0.060)
HLT_1	0.179(0.068)	0.166(0.042)	0.091(0.004)	0.348(0.183)	0.341(0.142)	0.257(0.159)	0.202(0.098)	0.136(0.053)	0.132(0.000)	0.378(0.179)	0.297(0.115)	0.259(0.079)
HLT_2	0.120(0.082)	0.083(0.026)	0.053(0.040)	0.405(0.173)	0.327(0.183)	0.179(0.086)	0.188(0.059)	0.109(0.039)	0.124(0.000)	0.325(0.106)	0.261(0.071)	0.251(0.047)
HLT_3	0.146(0.124)	0.135(0.125)	0.127(0.050)	0.458(0.238)	0.364(0.206)	0.287(0.165)	0.284(0.180)	0.201(0.129)	0.130(0.121)	0.434(0.270)	0.420(0.242)	0.339(0.184)
LBI_1	1.000(0.998)	1.000(1.000)	1.000(1.000)	0.970(0.975)	0.999(0.982)	1.000(0.994)	1.000(1.000)	1.000(1.000)	0.984(0.984)	1.000(1.000)	1.000(1.000)	0.976(0.972)
LBI_2	0.984(1.000)	0.994(0.979)	1.000(0.957)	0.988(0.998)	0.977(0.960)	0.991(0.994)	1.000(0.953)	0.997(0.969)	1.000(0.941)	1.000(1.000)	1.000(0.998)	0.948(0.955)
LBI_3	1.000(1.000)	1.000(1.000)	0.972(1.000)	0.986(1.000)	1.000(0.958)	0.953(0.971)	1.000(1.000)	1.000(0.982)	1.000(1.000)	1.000(1.000)	1.000(1.000)	0.972(0.929)
$LKSN$	0.120(0.000)	0.070(0.000)	0.026(0.000)	0.253(0.000)	0.173(0.000)	0.092(0.029)	0.197(0.034)	0.080(0.000)	0.030(0.000)	0.292(0.000)	0.186(0.000)	0.065(0.017)
L	0.075(0.121)	0.071(0.052)	0.024(0.000)	0.070(0.059)	0.088(0.094)	0.050(0.006)	0.114(0.086)	0.034(0.117)	0.002(0.022)	0.162(0.090)	0.093(0.054)	0.013(0.035)

Table 2.26: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_1}$ with $\mathbf{T=250}$.

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$ 0.3	0.5	0.7
K_1	0.956(0.999)	0.946(0.983)	0.844(0.756)	0.972(0.950)	0.958(0.852)	0.934(0.856)	0.982(0.896)	0.979(0.846)	0.871(0.738)	0.946(0.889)	0.922(0.806)	0.914(0.708)
K_2	1.000(0.986)	0.952(0.902)	0.949(0.800)	0.962(0.895)	1.000(0.913)	0.984(0.857)	0.977(0.883)	0.995(0.865)	0.925(0.760)	0.965(0.779)	0.933(0.872)	0.851(0.775)
K_3	1.000(1.000)	0.945(0.931)	0.878(0.735)	0.934(0.904)	0.928(0.854)	0.928(0.816)	0.950(0.875)	0.946(0.835)	0.983(0.696)	0.960(0.891)	0.950(0.862)	0.868(0.770)
LT_1	0.900(0.818)	0.905(0.880)	0.712(0.761)	0.814(0.820)	0.820(0.814)	0.808(0.885)	0.814(0.734)	0.793(0.800)	0.779(0.834)	0.696(0.769)	0.874(0.809)	0.782(0.761)
LT_2	0.784(0.781)	0.939(0.915)	0.795(0.783)	0.704(0.635)	0.859(0.866)	0.851(0.864)	0.707(0.613)	0.793(0.815)	0.841(0.841)	0.530(0.450)	0.812(0.793)	0.860(0.856)
LT_3	0.848(0.831)	0.866(0.897)	0.766(0.800)	0.799(0.815)	0.836(0.801)	0.786(0.872)	0.790(0.746)	0.842(0.814)	0.741(0.815)	0.749(0.714)	0.822(0.838)	0.763(0.780)
HLT_1	0.612(0.424)	0.689(0.452)	0.782(0.537)	0.560(0.343)	0.718(0.405)	0.870(0.549)	0.586(0.325)	0.625(0.476)	0.804(0.474)	0.564(0.335)	0.701(0.401)	0.732(0.540)
HLT_2	0.588(0.495)	0.862(0.659)	0.887(0.674)	0.589(0.306)	0.859(0.494)	0.952(0.660)	0.573(0.360)	0.792(0.593)	0.899(0.521)	0.468(0.275)	0.731(0.548)	0.897(0.594)
HLT_3	0.769(0.623)	0.774(0.592)	0.836(0.437)	0.716(0.452)	0.810(0.551)	0.935(0.522)	0.591(0.437)	0.875(0.643)	0.837(0.498)	0.652(0.390)	0.698(0.500)	0.788(0.600)
LBI_1	1.000(0.995)	0.987(0.937)	0.885(0.924)	0.955(0.997)	0.983(0.958)	0.933(0.895)	1.000(0.975)	0.967(1.000)	0.920(0.915)	0.950(0.990)	0.966(0.949)	0.931(0.883)
LBI_2	0.970(0.971)	1.000(0.972)	0.882(0.900)	1.000(1.000)	0.987(0.947)	0.906(0.877)	0.989(0.988)	0.964(1.000)	0.836(0.823)	1.000(0.924)	0.961(0.950)	0.927(0.873)
LBI_3	1.000(0.986)	0.953(1.000)	0.945(0.918)	0.971(1.000)	0.933(0.959)	0.950(0.943)	1.000(0.955)	0.952(0.956)	0.843(0.842)	0.969(0.925)	1.000(0.953)	0.869(0.899)
$LKSN$	1.000(0.040)	1.000(0.285)	1.000(0.814)	0.973(0.063)	1.000(0.374)	0.977(0.741)	0.963(0.059)	1.000(0.309)	0.974(0.784)	0.951(0.047)	0.986(0.401)	0.984(0.754)
L	0.387(1.000)	0.525(1.000)	0.304(0.942)	0.284(1.000)	0.510(1.000)	0.290(0.990)	0.264(0.966)	0.424(0.976)	0.326(1.000)	0.171(0.966)	0.449(0.960)	0.420(0.964)

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$ 0.3	0.5	0.7
K_1	0.964(0.997)	0.982(0.971)	1.000(0.926)	0.727(0.463)	0.750(0.555)	0.554(0.319)	0.993(1.000)	0.962(0.919)	0.972(0.854)	1.000(0.945)	0.871(0.799)	0.867(0.679)
K_2	0.960(0.966)	1.000(0.998)	0.998(1.000)	0.717(0.448)	0.713(0.502)	0.566(0.423)	1.000(0.988)	0.981(0.873)	1.000(0.928)	0.957(0.823)	0.888(0.815)	0.856(0.749)
K_3	1.000(0.950)	1.000(1.000)	0.962(1.000)	0.726(0.499)	0.723(0.561)	0.564(0.341)	0.984(1.000)	0.986(0.962)	0.951(0.856)	0.958(0.895)	0.916(0.772)	0.889(0.706)
LT_1	0.838(0.787)	0.888(0.824)	0.890(0.809)	0.788(0.819)	0.762(0.810)	0.677(0.765)	0.823(0.829)	0.823(0.780)	0.791(0.806)	0.869(0.878)	0.849(0.802)	0.766(0.808)
LT_2	0.646(0.665)	0.911(0.851)	0.886(0.877)	0.792(0.664)	0.806(0.755)	0.810(0.780)	0.694(0.640)	0.858(0.875)	0.867(0.855)	0.735(0.602)	0.844(0.827)	0.800(0.835)
LT_3	0.853(0.847)	0.863(0.841)	0.859(0.872)	0.866(0.848)	0.809(0.789)	0.640(0.726)	0.889(0.801)	0.873(0.814)	0.827(0.907)	0.834(0.816)	0.841(0.821)	0.774(0.807)
HLT_1	0.779(0.696)	0.771(0.652)	0.771(0.613)	0.506(0.155)	0.702(0.428)	0.494(0.230)	0.697(0.484)	0.820(0.575)	0.890(0.487)	0.545(0.272)	0.694(0.488)	0.828(0.542)
HLT_2	0.804(0.680)	0.832(0.805)	0.887(0.741)	0.518(0.199)	0.666(0.407)	0.564(0.337)	0.731(0.442)	0.895(0.660)	0.941(0.668)	0.551(0.278)	0.858(0.710)	0.856(0.724)
HLT_3	0.999(0.976)	1.000(1.000)	1.000(0.821)	0.497(0.170)	0.669(0.398)	0.558(0.222)	0.906(0.752)	0.932(0.702)	0.947(0.677)	0.652(0.357)	0.689(0.544)	0.841(0.514)
LBI_1	0.964(1.000)	1.000(1.000)	1.000(0.994)	0.946(0.925)	0.836(0.754)	0.773(0.675)	0.970(0.988)	0.966(0.953)	1.000(0.913)	0.977(0.998)	0.929(0.927)	0.925(0.902)
LBI_2	0.971(1.000)	0.990(0.959)	1.000(0.923)	1.000(0.949)	0.857(0.761)	0.712(0.729)	1.000(0.983)	1.000(0.945)	0.965(0.981)	0.989(0.972)	0.953(0.948)	0.868(0.904)
LBI_3	0.990(1.000)	1.000(0.955)	1.000(0.960)	0.941(0.958)	0.788(0.739)	0.773(0.710)	0.956(1.000)	0.993(0.967)	1.000(0.993)	1.000(0.992)	0.961(0.898)	0.875(0.859)
$LKSN$	0.927(0.063)	1.000(0.328)	0.990(0.781)	0.978(0.146)	1.000(0.490)	1.000(0.701)	1.000(0.085)	1.000(0.274)	0.994(0.716)	1.000(0.064)	1.000(0.444)	0.973(0.850)
L	0.413(0.962)	0.550(1.000)	0.813(0.984)	0.207(0.885)	0.086(0.795)	0.056(0.624)	0.298(1.000)	0.502(1.000)	0.470(0.935)	0.412(0.998)	0.358(0.956)	0.192(0.919)

τ^*	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$ 0.3	0.5	0.7	$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$ 0.3	0.5	0.7	$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$ 0.3	0.5	0.7	$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$ 0.3	0.5	0.7
K_1	0.966(0.934)	0.992(0.864)	0.953(0.726)	1.000(1.000)	1.000(0.957)	0.977(0.939)	0.886(0.796)	0.914(0.815)	0.768(0.612)	0.990(0.993)	1.000(1.000)	1.000(0.952)
K_2	1.000(0.968)	1.000(0.868)	0.903(0.804)	0.988(0.918)	1.000(0.959)	0.977(0.895)	0.848(0.719)	0.830(0.827)	0.792(0.725)	1.000(0.962)	1.000(0.952)	0.979(0.913)
K_3	1.000(1.000)	1.000(0.864)	0.882(0.741)	0.975(1.000)	0.984(0.958)	1.000(0.957)	0.937(0.792)	0.885(0.742)	0.791(0.605)	1.000(0.990)	0.997(0.986)	0.960(0.957)
LT_1	0.859(0.859)	0.765(0.742)	0.761(0.800)	0.755(0.716)	0.895(0.860)	0.787(0.857)	0.826(0.765)	0.927(0.890)	0.665(0.649)	0.844(0.748)	0.891(0.873)	0.820(0.829)
LT_2	0.779(0.729)	0.819(0.867)	0.854(0.829)	0.619(0.566)	0.855(0.801)	0.862(0.871)	0.693(0.753)	0.915(0.850)	0.779(0.745)	0.556(0.537)	0.903(0.811)	0.908(0.891)
LT_3	0.893(0.881)	0.848(0.744)	0.818(0.798)	0.747(0.755)	0.884(0.833)	0.897(0.879)	0.830(0.807)	0.902(0.940)	0.726(0.718)	0.862(0.781)	0.861(0.873)	0.891(0.811)
HLT_1	0.573(0.388)	0.728(0.386)	0.803(0.534)	0.735(0.643)	0.769(0.621)	0.825(0.636)	0.592(0.300)	0.696(0.528)	0.725(0.452)	0.791(0.759)	0.778(0.659)	0.794(0.667)
HLT_2	0.606(0.356)	0.777(0.600)	0.920(0.688)	0.783(0.603)	0.824(0.739)	0.947(0.681)	0.521(0.212)	0.774(0.541)	0.820(0.581)	0.853(0.790)	0.905(0.767)	0.915(0.744)
HLT_3	0.765(0.465)	0.853(0.535)	0.849(0.571)	0.985(0.874)	0.969(0.880)	0.914(0.762)	0.578(0.334)	0.722(0.449)	0.703(0.411)	0.993(0.992)	1.000(0.879)	1.000(0.914)
LBI_1	1.000(0.995)	0.989(0.991)	0.916(0.907)	1.000(1.000)	1.000(0.954)	1.000(0.944)	1.000(0.980)	0.990(0.907)	0.861(0.905)	0.954(1.000)	0.957(0.965)	0.949(0.921)
LBI_2	1.000(0.980)	1.000(1.000)	0.913(0.860)	0.958(0.971)	0.972(0.951)	0.945(0.960)	0.963(0.980)	0.995(0.974)	0.888(0.893)	1.000(0.981)	1.000(0.973)	0.964(0.918)
LBI_3	1.000(1.000)	1.000(0.969)	0.915(0.881)	0.957(0.986)	1.000(0.921)	0.981(0.916)	0.983(1.000)	0.983(0.989)	0.829(0.825)	1.000(1.000)	1.000(0.929)	1.000(0.924)
$LKSN$	0.989(0.082)	1.000(0.368)	0.980(0.687)	1.000(0.094)	0.963(0.361)	0.973(0.815)	1.000(0.062)	0.993(0.451)	0.989(0.813)	0.967(0.092)	1.000(0.440)	0.961(0.760)
L	0.330(1.000)	0.510(1.000)	0.314(0.924)	0.191(1.000)	0.570(1.000)	0.655(0.994)	0.371(0.918)	0.224(0.907)	0.100(0.888)	0.211(0.963)	0.646(1.000)	0.699(1.000)

Table 2.27: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_{01}}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.977(0.991)	0.990(0.925)	0.841(0.782)	1.000(0.898)	0.989(0.888)	0.928(0.790)	0.949(0.934)	0.971(0.903)	0.907(0.766)	0.964(0.847)	0.988(0.831)	0.861(0.748)
K_2	1.000(0.948)	0.997(0.975)	0.915(0.798)	0.952(0.857)	1.000(0.906)	0.986(0.848)	0.906(0.857)	0.968(0.928)	0.948(0.740)	0.927(0.787)	0.983(0.830)	0.878(0.805)
K_3	0.983(0.953)	0.966(0.944)	0.922(0.760)	0.984(0.944)	0.929(0.854)	0.969(0.825)	1.000(0.860)	0.961(0.884)	0.943(0.708)	0.999(0.904)	0.911(0.809)	0.852(0.714)
LT_1	0.893(0.856)	0.883(0.887)	0.776(0.729)	0.849(0.843)	0.818(0.807)	0.807(0.818)	0.791(0.750)	0.782(0.737)	0.765(0.794)	0.719(0.734)	0.866(0.828)	0.794(0.760)
LT_2	0.772(0.799)	0.909(0.839)	0.862(0.808)	0.752(0.716)	0.800(0.795)	0.862(0.862)	0.740(0.617)	0.855(0.858)	0.863(0.862)	0.536(0.422)	0.851(0.783)	0.805(0.852)
LT_3	0.855(0.868)	0.885(0.831)	0.732(0.801)	0.772(0.775)	0.849(0.774)	0.820(0.851)	0.782(0.776)	0.788(0.749)	0.743(0.810)	0.771(0.724)	0.810(0.864)	0.797(0.769)
HLT_1	0.591(0.459)	0.772(0.492)	0.767(0.487)	0.637(0.324)	0.697(0.447)	0.895(0.525)	0.525(0.376)	0.629(0.445)	0.779(0.463)	0.557(0.273)	0.686(0.432)	0.783(0.489)
HLT_2	0.637(0.500)	0.827(0.591)	0.880(0.625)	0.579(0.322)	0.868(0.541)	0.966(0.630)	0.556(0.363)	0.735(0.630)	0.884(0.557)	0.499(0.302)	0.743(0.508)	0.841(0.646)
HLT_3	0.767(0.652)	0.808(0.554)	0.797(0.443)	0.738(0.479)	0.777(0.496)	0.924(0.515)	0.648(0.393)	0.838(0.674)	0.815(0.516)	0.587(0.379)	0.756(0.477)	0.788(0.546)
LBI_1	0.996(1.000)	0.992(0.998)	0.940(0.923)	1.000(1.000)	1.000(0.992)	0.999(0.881)	0.986(0.958)	1.000(1.000)	0.885(0.895)	1.000(0.954)	0.990(0.927)	0.906(0.814)
LBI_2	0.960(1.000)	0.983(1.000)	0.896(0.854)	0.978(0.996)	0.992(1.000)	0.924(0.864)	0.967(1.000)	0.944(1.000)	0.841(0.821)	0.936(0.937)	0.995(0.944)	0.919(0.875)
LBI_3	0.968(0.966)	1.000(1.000)	0.957(0.851)	0.954(1.000)	0.938(0.996)	0.953(0.955)	0.990(0.982)	1.000(0.937)	0.917(0.842)	0.958(0.948)	1.000(0.931)	0.908(0.847)
$LKSN$	0.970(0.085)	1.000(0.288)	1.000(0.826)	1.000(0.093)	1.000(0.370)	1.000(0.749)	0.977(0.061)	1.000(0.295)	0.992(0.712)	0.976(0.054)	1.000(0.361)	0.977(0.787)
L	0.370(1.000)	0.522(0.978)	0.370(1.000)	0.328(0.986)	0.526(0.979)	0.293(0.942)	0.253(1.000)	0.470(1.000)	0.285(1.000)	0.199(0.951)	0.457(1.000)	0.408(0.958)

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	1.000(0.981)	0.989(1.000)	1.000(0.961)	0.728(0.553)	0.702(0.518)	0.580(0.373)	0.974(1.000)	0.964(0.935)	0.932(0.877)	0.993(0.890)	0.878(0.843)	0.827(0.681)
K_2	1.000(1.000)	1.000(0.962)	0.972(0.957)	0.782(0.483)	0.688(0.542)	0.611(0.430)	0.980(0.977)	0.980(0.937)	0.945(0.844)	0.916(0.827)	0.950(0.794)	0.872(0.750)
K_3	1.000(0.983)	0.961(0.999)	0.962(0.922)	0.772(0.520)	0.697(0.540)	0.555(0.384)	1.000(0.958)	1.000(0.971)	0.934(0.865)	0.970(0.885)	0.922(0.783)	0.834(0.747)
LT_1	0.798(0.874)	0.862(0.805)	0.856(0.882)	0.760(0.816)	0.760(0.768)	0.730(0.749)	0.825(0.820)	0.875(0.830)	0.852(0.817)	0.846(0.860)	0.796(0.858)	0.759(0.811)
LT_2	0.679(0.638)	0.928(0.856)	0.914(0.831)	0.779(0.729)	0.812(0.806)	0.799(0.803)	0.672(0.619)	0.837(0.810)	0.873(0.900)	0.743(0.632)	0.878(0.818)	0.852(0.864)
LT_3	0.789(0.861)	0.903(0.848)	0.818(0.888)	0.800(0.802)	0.823(0.835)	0.639(0.746)	0.839(0.838)	0.897(0.816)	0.844(0.876)	0.860(0.863)	0.815(0.856)	0.786(0.762)
HLT_1	0.758(0.691)	0.811(0.727)	0.781(0.607)	0.467(0.196)	0.681(0.395)	0.543(0.288)	0.678(0.501)	0.780(0.552)	0.825(0.528)	0.566(0.304)	0.678(0.523)	0.832(0.563)
HLT_2	0.824(0.718)	0.842(0.798)	0.922(0.734)	0.477(0.208)	0.592(0.433)	0.554(0.314)	0.709(0.435)	0.845(0.624)	0.923(0.628)	0.546(0.223)	0.829(0.667)	0.847(0.706)
HLT_3	0.988(0.927)	0.993(1.000)	0.964(0.860)	0.504(0.146)	0.630(0.454)	0.494(0.227)	0.863(0.729)	0.967(0.661)	0.946(0.665)	0.629(0.332)	0.676(0.525)	0.833(0.508)
LBI_1	0.975(0.990)	1.000(0.969)	1.000(0.982)	1.000(0.940)	0.875(0.740)	0.737(0.690)	0.986(1.000)	0.994(0.922)	1.000(0.972)	1.000(0.969)	0.909(0.933)	0.943(0.873)
LBI_2	1.000(0.998)	0.965(1.000)	1.000(0.986)	1.000(0.978)	0.791(0.766)	0.682(0.718)	1.000(1.000)	1.000(0.899)	0.963(0.959)	0.999(1.000)	0.955(0.928)	0.925(0.853)
LBI_3	1.000(0.995)	0.982(0.991)	1.000(0.924)	0.990(0.911)	0.817(0.742)	0.706(0.721)	0.967(1.000)	1.000(0.933)	1.000(0.972)	1.000(1.000)	0.970(0.907)	0.840(0.844)
$LKSN$	1.000(0.083)	0.984(0.379)	1.000(0.782)	0.981(0.189)	0.988(0.480)	1.000(0.695)	0.971(0.052)	0.971(0.212)	0.967(0.777)	0.979(0.102)	1.000(0.456)	0.966(0.869)
L	0.330(0.974)	0.611(0.998)	0.848(1.000)	0.272(0.886)	0.107(0.749)	0.019(0.610)	0.285(0.955)	0.461(1.000)	0.502(0.977)	0.422(0.967)	0.381(0.951)	0.190(0.852)

	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.972(0.925)	0.971(0.866)	0.916(0.786)	1.000(0.988)	0.984(0.984)	0.977(0.895)	0.935(0.794)	0.857(0.737)	0.779(0.600)	1.000(0.975)	0.998(0.937)	0.993(1.000)
K_2	1.000(0.906)	0.987(0.879)	0.977(0.859)	0.982(0.963)	0.983(0.997)	1.000(0.884)	0.833(0.700)	0.853(0.781)	0.798(0.706)	1.000(0.957)	0.965(0.949)	0.960(0.943)
K_3	0.996(0.997)	0.986(0.856)	0.871(0.785)	1.000(1.000)	1.000(0.957)	1.000(0.882)	0.927(0.837)	0.850(0.750)	0.790(0.640)	0.998(1.000)	1.000(0.985)	0.968(0.985)
LT_1	0.915(0.889)	0.803(0.763)	0.788(0.767)	0.836(0.764)	0.895(0.890)	0.812(0.888)	0.752(0.819)	0.882(0.909)	0.685(0.711)	0.796(0.769)	0.850(0.896)	0.794(0.827)
LT_2	0.804(0.760)	0.874(0.873)	0.831(0.794)	0.586(0.550)	0.828(0.852)	0.890(0.831)	0.689(0.707)	0.931(0.851)	0.812(0.783)	0.524(0.545)	0.831(0.802)	0.898(0.876)
LT_3	0.846(0.839)	0.849(0.820)	0.796(0.841)	0.788(0.788)	0.848(0.861)	0.824(0.828)	0.803(0.785)	0.881(0.941)	0.716(0.671)	0.847(0.728)	0.890(0.883)	0.832(0.861)
HLT_1	0.642(0.349)	0.689(0.397)	0.785(0.565)	0.778(0.616)	0.793(0.667)	0.874(0.567)	0.584(0.316)	0.755(0.491)	0.739(0.488)	0.786(0.681)	0.781(0.706)	0.825(0.663)
HLT_2	0.615(0.362)	0.838(0.577)	0.893(0.706)	0.809(0.619)	0.819(0.724)	0.915(0.704)	0.467(0.198)	0.797(0.594)	0.839(0.567)	0.830(0.739)	0.860(0.737)	0.919(0.757)
HLT_3	0.740(0.527)	0.820(0.557)	0.863(0.599)	0.939(0.886)	0.980(0.895)	0.967(0.806)	0.639(0.305)	0.728(0.439)	0.772(0.441)	0.990(0.998)	0.988(0.921)	0.991(0.913)
LBI_1	1.000(0.973)	0.972(0.983)	0.928(0.968)	0.962(1.000)	0.966(0.956)	0.951(1.000)	0.947(0.947)	0.915(0.972)	0.890(0.878)	0.986(0.969)	1.000(0.936)	1.000(0.956)
LBI_2	0.958(0.972)	1.000(0.998)	0.935(0.949)	1.000(0.986)	1.000(0.958)	0.969(0.939)	1.000(0.999)	0.979(0.962)	0.843(0.857)	0.951(0.982)	0.967(0.946)	0.998(1.000)
LBI_3	1.000(1.000)	1.000(1.000)	0.921(0.865)	0.986(0.961)	0.983(0.922)	1.000(1.000)	0.971(0.962)	0.978(0.950)	0.869(0.821)	0.993(1.000)	0.989(0.959)	1.000(1.000)
$LKSN$	0.983(0.064)	0.974(0.308)	1.000(0.614)	1.000(0.139)	1.000(0.285)	1.000(0.795)	1.000(0.010)	1.000(0.460)	1.000(0.783)	0.999(0.071)	1.000(0.461)	1.000(0.756)
L	0.375(0.994)	0.433(1.000)	0.371(0.931)	0.189(0.952)	0.521(0.994)	0.649(0.991)	0.304(0.982)	0.241(0.994)	0.067(0.883)	0.272(0.916)	0.566(1.000)	0.620(1.000)

Table 2.28: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_{01}}$ with $\mathbf{T=250}$.

τ^*	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$ 0.3	0.5	0.7	$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$ 0.3	0.5	0.7
K_1	0.955(0.845)	1.000(0.856)	0.956(0.901)	0.870(0.750)	0.963(0.944)	0.982(0.875)	0.841(0.707)	1.000(0.884)	0.975(0.919)	0.862(0.667)	0.936(0.879)	0.968(0.855)
K_2	0.987(0.933)	0.989(0.953)	0.989(0.934)	0.871(0.753)	1.000(0.932)	0.991(0.890)	0.931(0.795)	0.991(0.949)	0.995(0.950)	0.894(0.772)	0.940(0.869)	0.988(0.807)
K_3	0.934(0.912)	0.954(0.876)	1.000(0.974)	0.887(0.803)	1.000(0.949)	1.000(0.938)	0.863(0.736)	0.991(0.849)	0.965(0.935)	0.822(0.756)	0.931(0.850)	1.000(0.834)
LT_1	0.869(0.919)	0.790(0.833)	0.772(0.760)	0.757(0.725)	0.841(0.856)	0.782(0.781)	0.771(0.779)	0.825(0.773)	0.842(0.770)	0.704(0.728)	0.817(0.753)	0.756(0.707)
LT_2	0.889(0.878)	0.794(0.801)	0.755(0.663)	0.821(0.777)	0.925(0.855)	0.736(0.646)	0.888(0.859)	0.834(0.820)	0.699(0.628)	0.865(0.779)	0.785(0.827)	0.662(0.641)
LT_3	0.856(0.877)	0.782(0.815)	0.851(0.850)	0.809(0.740)	0.875(0.852)	0.785(0.822)	0.765(0.805)	0.816(0.848)	0.852(0.820)	0.737(0.729)	0.801(0.826)	0.737(0.759)
HLT_1	0.874(0.537)	0.693(0.398)	0.581(0.440)	0.794(0.518)	0.781(0.484)	0.580(0.425)	0.785(0.501)	0.740(0.407)	0.694(0.396)	0.735(0.462)	0.669(0.498)	0.560(0.334)
HLT_2	0.955(0.703)	0.786(0.579)	0.597(0.395)	0.909(0.619)	0.863(0.697)	0.550(0.313)	0.910(0.606)	0.845(0.663)	0.720(0.474)	0.882(0.646)	0.721(0.529)	0.542(0.318)
HLT_3	0.854(0.527)	0.817(0.540)	0.753(0.587)	0.878(0.567)	0.870(0.685)	0.693(0.487)	0.789(0.468)	0.845(0.626)	0.765(0.488)	0.780(0.538)	0.755(0.540)	0.635(0.426)
LBI_1	0.990(0.951)	0.998(1.000)	1.000(1.000)	0.923(0.941)	0.962(1.000)	1.000(0.962)	0.932(0.942)	1.000(0.974)	0.967(0.982)	0.962(0.864)	0.977(0.955)	0.947(0.987)
LBI_2	0.958(0.925)	0.971(0.962)	1.000(1.000)	0.952(0.914)	1.000(0.987)	1.000(1.000)	0.993(0.879)	1.000(0.945)	0.984(0.984)	0.913(0.833)	1.000(0.937)	0.969(0.981)
LBI_3	0.955(0.956)	0.962(1.000)	1.000(1.000)	0.884(0.909)	1.000(1.000)	0.980(0.971)	0.980(0.875)	0.977(0.958)	1.000(0.989)	0.928(0.904)	1.000(0.960)	0.978(0.946)
$LKSN$	0.970(0.765)	1.000(0.442)	0.965(0.142)	0.960(0.818)	1.000(0.299)	0.973(0.078)	0.992(0.758)	0.992(0.323)	0.945(0.125)	1.000(0.742)	1.000(0.344)	0.944(0.078)
L	0.501(1.000)	0.434(0.962)	0.319(1.000)	0.299(0.947)	0.496(0.959)	0.280(1.000)	0.350(0.955)	0.440(1.000)	0.446(1.000)	0.422(0.928)	0.465(1.000)	0.223(0.927)
	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	1.000(0.968)	0.989(0.987)	0.972(0.994)	0.542(0.374)	0.770(0.502)	0.774(0.483)	0.932(0.848)	1.000(0.911)	0.997(0.986)	0.758(0.635)	0.933(0.791)	0.966(0.826)
K_2	0.990(0.981)	1.000(0.989)	1.000(0.950)	0.580(0.370)	0.735(0.484)	0.701(0.467)	0.952(0.886)	0.976(0.917)	0.972(0.961)	0.874(0.763)	0.951(0.900)	0.916(0.778)
K_3	0.961(0.946)	1.000(1.000)	1.000(1.000)	0.578(0.328)	0.766(0.544)	0.771(0.503)	1.000(0.894)	1.000(0.898)	1.000(0.978)	0.833(0.637)	0.946(0.843)	0.987(0.797)
LT_1	0.877(0.874)	0.966(0.842)	0.761(0.762)	0.614(0.582)	0.892(0.885)	0.754(0.830)	0.826(0.809)	0.806(0.874)	0.857(0.797)	0.696(0.720)	0.835(0.842)	0.819(0.810)
LT_2	0.928(0.908)	0.912(0.852)	0.645(0.625)	0.752(0.742)	0.886(0.904)	0.671(0.623)	0.905(0.835)	0.899(0.808)	0.610(0.615)	0.768(0.764)	0.891(0.907)	0.737(0.656)
LT_3	0.908(0.895)	0.958(0.862)	0.795(0.762)	0.663(0.618)	0.854(0.935)	0.769(0.773)	0.785(0.823)	0.853(0.880)	0.791(0.789)	0.718(0.701)	0.882(0.842)	0.799(0.777)
HLT_1	0.812(0.648)	0.836(0.702)	0.812(0.687)	0.462(0.286)	0.600(0.397)	0.554(0.283)	0.926(0.522)	0.719(0.445)	0.658(0.526)	0.768(0.588)	0.681(0.442)	0.503(0.240)
HLT_2	0.904(0.737)	0.940(0.781)	0.781(0.720)	0.550(0.379)	0.689(0.412)	0.519(0.189)	0.942(0.598)	0.856(0.604)	0.724(0.440)	0.815(0.647)	0.816(0.608)	0.497(0.257)
HLT_3	0.950(0.951)	1.000(0.976)	1.000(0.993)	0.514(0.289)	0.640(0.359)	0.584(0.279)	0.938(0.668)	0.955(0.688)	0.908(0.807)	0.726(0.557)	0.776(0.453)	0.440(0.262)
LBI_1	0.973(0.969)	1.000(0.984)	0.959(1.000)	0.802(0.688)	0.890(0.903)	0.929(0.843)	0.988(0.906)	0.965(1.000)	1.000(1.000)	0.801(0.782)	0.980(0.950)	1.000(1.000)
LBI_2	1.000(0.907)	0.982(0.990)	1.000(1.000)	0.723(0.711)	0.838(0.896)	0.941(0.856)	0.961(0.922)	1.000(0.954)	1.000(1.000)	0.843(0.810)	0.925(0.994)	0.982(1.000)
LBI_3	0.989(0.947)	0.978(0.986)	0.955(0.965)	0.721(0.775)	0.823(0.907)	0.987(0.851)	0.979(0.960)	0.955(1.000)	0.994(1.000)	0.878(0.837)	0.913(0.964)	0.991(0.965)
$LKSN$	1.000(0.756)	1.000(0.372)	1.000(0.137)	1.000(0.792)	1.000(0.410)	0.980(0.216)	1.000(0.660)	1.000(0.301)	0.991(0.106)	0.981(0.751)	0.976(0.418)	0.988(0.107)
L	0.790(0.978)	0.657(1.000)	0.399(0.950)	0.082(0.670)	0.068(0.888)	0.205(0.888)	0.639(1.000)	0.582(0.976)	0.352(1.000)	0.212(0.814)	0.428(0.925)	0.352(1.000)
	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.908(0.743)	0.967(0.909)	0.995(0.897)	0.976(0.841)	1.000(0.961)	1.000(1.000)	0.802(0.618)	0.808(0.783)	0.915(0.790)	1.000(0.932)	1.000(1.000)	0.998(0.988)
K_2	0.934(0.804)	0.977(0.881)	0.928(0.902)	1.000(0.899)	1.000(0.967)	1.000(0.987)	0.866(0.700)	0.875(0.861)	0.908(0.772)	1.000(0.935)	0.965(0.946)	0.967(0.953)
K_3	0.844(0.751)	0.947(0.948)	0.965(0.915)	1.000(0.840)	0.968(0.928)	0.968(0.990)	0.753(0.686)	0.859(0.807)	0.905(0.776)	0.986(0.948)	0.981(0.955)	1.000(0.988)
LT_1	0.737(0.781)	0.844(0.761)	0.739(0.751)	0.832(0.846)	0.848(0.872)	0.863(0.864)	0.691(0.732)	0.827(0.813)	0.745(0.830)	0.877(0.794)	0.827(0.879)	0.841(0.749)
LT_2	0.728(0.766)	0.867(0.861)	0.590(0.587)	0.863(0.845)	0.796(0.799)	0.792(0.725)	0.793(0.805)	0.876(0.882)	0.717(0.693)	0.856(0.865)	0.851(0.864)	0.632(0.681)
LT_3	0.780(0.780)	0.864(0.846)	0.785(0.769)	0.843(0.831)	0.840(0.864)	0.810(0.827)	0.757(0.773)	0.864(0.821)	0.819(0.777)	0.840(0.793)	0.907(0.791)	0.841(0.766)
HLT_1	0.866(0.514)	0.836(0.507)	0.475(0.378)	0.840(0.637)	0.820(0.653)	0.732(0.606)	0.757(0.536)	0.705(0.587)	0.458(0.227)	0.831(0.687)	0.810(0.623)	0.755(0.597)
HLT_2	0.916(0.635)	0.883(0.630)	0.595(0.372)	0.838(0.658)	0.834(0.722)	0.803(0.650)	0.839(0.663)	0.837(0.574)	0.522(0.250)	0.883(0.788)	0.854(0.747)	0.800(0.683)
HLT_3	0.839(0.570)	0.918(0.539)	0.629(0.504)	0.913(0.746)	0.998(0.883)	0.953(0.950)	0.741(0.535)	0.699(0.513)	0.479(0.276)	0.959(0.886)	1.000(0.988)	0.981(0.983)
LBI_1	0.890(0.869)	0.960(0.936)	0.994(1.000)	0.934(0.920)	1.000(1.000)	1.000(1.000)	0.842(0.919)	1.000(0.934)	1.000(0.969)	1.000(0.913)	0.988(0.953)	1.000(0.995)
LBI_2	0.864(0.811)	0.988(0.937)	0.984(1.000)	0.966(0.881)	1.000(0.935)	0.993(1.000)	0.816(0.907)	0.988(0.952)	1.000(1.000)	1.000(0.941)	1.000(1.000)	0.965(1.000)
LBI_3	0.859(0.834)	0.988(0.945)	0.969(1.000)	0.950(0.871)	1.000(1.000)	1.000(1.000)	0.866(0.860)	0.984(0.966)	1.000(1.000)	0.982(0.950)	0.960(0.999)	1.000(1.000)
$LKSN$	1.000(0.705)	0.963(0.389)	0.934(0.091)	0.976(0.748)	1.000(0.422)	0.998(0.136)	0.996(0.768)	0.991(0.387)	0.959(0.009)	1.000(0.768)	1.000(0.197)	0.962(0.036)
L	0.322(0.887)	0.429(0.982)	0.303(1.000)	0.599(0.961)	0.657(0.977)	0.341(0.992)	0.139(0.863)	0.286(0.990)	0.363(0.934)	0.719(1.000)	0.647(1.000)	0.352(1.000)

Table 2.29: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_{10}}$ with $\mathbf{T=100}$.

	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.35)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.55,0.55)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.80)$, $c=0.5$			$\alpha=(0.35,0.35)$, $\beta=(0.55,0.55)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.954(0.867)	0.940(0.836)	0.963(0.929)	0.855(0.802)	0.981(0.876)	0.992(0.879)	0.898(0.738)	1.000(0.895)	1.000(0.966)	0.824(0.711)	0.893(0.811)	0.983(0.806)
K_2	0.922(0.912)	0.945(0.929)	0.999(0.903)	0.935(0.760)	1.000(0.960)	0.968(0.897)	0.927(0.815)	0.974(0.947)	1.000(0.914)	0.928(0.761)	0.968(0.830)	0.985(0.825)
K_3	0.940(0.918)	0.954(0.892)	1.000(0.951)	0.906(0.776)	0.982(0.946)	1.000(0.876)	0.847(0.746)	0.999(0.839)	0.971(0.969)	0.889(0.729)	0.900(0.880)	1.000(0.864)
LT_1	0.832(0.893)	0.769(0.852)	0.845(0.811)	0.738(0.779)	0.849(0.844)	0.764(0.803)	0.754(0.803)	0.762(0.768)	0.800(0.811)	0.716(0.711)	0.764(0.796)	0.726(0.682)
LT_2	0.936(0.892)	0.859(0.791)	0.726(0.692)	0.786(0.811)	0.885(0.909)	0.738(0.689)	0.899(0.821)	0.898(0.901)	0.653(0.688)	0.868(0.793)	0.796(0.817)	0.705(0.585)
LT_3	0.824(0.894)	0.831(0.804)	0.838(0.862)	0.761(0.762)	0.865(0.803)	0.873(0.797)	0.738(0.833)	0.826(0.777)	0.770(0.808)	0.733(0.718)	0.789(0.820)	0.797(0.708)
HLT_1	0.888(0.541)	0.715(0.394)	0.629(0.440)	0.807(0.460)	0.793(0.451)	0.625(0.363)	0.804(0.512)	0.736(0.405)	0.629(0.405)	0.720(0.439)	0.637(0.498)	0.560(0.302)
HLT_2	0.984(0.694)	0.831(0.561)	0.600(0.431)	0.851(0.613)	0.850(0.641)	0.576(0.312)	0.826(0.620)	0.790(0.614)	0.645(0.475)	0.902(0.626)	0.708(0.565)	0.583(0.297)
HLT_3	0.877(0.450)	0.803(0.477)	0.734(0.550)	0.854(0.489)	0.895(0.674)	0.760(0.518)	0.786(0.444)	0.809(0.600)	0.755(0.476)	0.788(0.466)	0.766(0.577)	0.625(0.396)
LBI_1	1.000(0.909)	1.000(0.995)	1.000(1.000)	0.928(0.917)	0.975(0.962)	0.962(0.969)	0.950(0.935)	1.000(0.927)	1.000(0.967)	0.917(0.877)	0.978(1.000)	1.000(1.000)
LBI_2	0.977(0.952)	1.000(0.949)	1.000(0.980)	0.941(0.882)	0.995(1.000)	0.953(1.000)	0.984(0.927)	1.000(0.967)	1.000(0.950)	0.919(0.843)	0.962(0.939)	0.970(0.976)
LBI_3	0.974(0.903)	1.000(0.990)	1.000(1.000)	0.865(0.887)	0.960(0.974)	1.000(1.000)	0.929(0.927)	0.940(0.985)	0.984(0.982)	0.947(0.855)	1.000(0.974)	0.948(1.000)
$LKSN$	1.000(0.726)	1.000(0.432)	0.967(0.117)	0.999(0.812)	0.979(0.329)	0.985(0.103)	1.000(0.772)	0.962(0.306)	0.990(0.116)	1.000(0.709)	0.962(0.309)	0.920(0.052)
L	0.489(0.976)	0.404(1.000)	0.327(1.000)	0.290(0.950)	0.484(0.971)	0.258(1.000)	0.339(0.996)	0.429(0.998)	0.437(1.000)	0.483(0.943)	0.404(0.947)	0.253(0.945)
	$\alpha=(0.15,0.15)$, $\beta=(0.35,0.80)$, $c=0.5$			$\alpha=(0.15,0.15)$, $\beta=(0.80,0.35)$, $c=0.5$			$\alpha=(0.10,0.35)$, $\beta=(0.50,0.50)$, $c=0.5$			$\alpha=(0.35,0.10)$, $\beta=(0.50,0.50)$, $c=0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.999(0.915)	0.976(1.000)	0.986(0.993)	0.589(0.364)	0.752(0.498)	0.773(0.487)	1.000(0.863)	0.987(0.924)	1.000(1.000)	0.796(0.695)	0.947(0.859)	0.922(0.855)
K_2	0.988(0.997)	0.991(1.000)	0.961(1.000)	0.564(0.436)	0.764(0.536)	0.733(0.418)	1.000(0.908)	0.962(0.992)	0.977(0.994)	0.877(0.755)	0.881(0.895)	0.925(0.781)
K_3	1.000(0.928)	1.000(1.000)	0.966(0.966)	0.611(0.331)	0.833(0.482)	0.760(0.538)	0.999(0.925)	0.984(0.975)	0.976(1.000)	0.778(0.705)	0.901(0.860)	0.962(0.858)
LT_1	0.884(0.846)	0.908(0.840)	0.798(0.722)	0.620(0.575)	0.903(0.903)	0.759(0.769)	0.841(0.816)	0.852(0.794)	0.858(0.744)	0.754(0.753)	0.835(0.903)	0.796(0.764)
LT_2	0.936(0.931)	0.863(0.809)	0.586(0.555)	0.748(0.726)	0.900(0.875)	0.683(0.627)	0.851(0.820)	0.895(0.864)	0.622(0.615)	0.829(0.818)	0.880(0.833)	0.737(0.639)
LT_3	0.933(0.931)	0.908(0.806)	0.753(0.759)	0.675(0.576)	0.888(0.948)	0.754(0.810)	0.827(0.802)	0.843(0.825)	0.823(0.751)	0.765(0.737)	0.850(0.893)	0.841(0.787)
HLT_1	0.786(0.648)	0.839(0.725)	0.753(0.641)	0.479(0.304)	0.626(0.330)	0.522(0.279)	0.903(0.543)	0.731(0.480)	0.616(0.451)	0.739(0.568)	0.717(0.431)	0.496(0.314)
HLT_2	0.919(0.715)	0.868(0.773)	0.792(0.738)	0.504(0.353)	0.681(0.334)	0.539(0.245)	0.972(0.633)	0.835(0.616)	0.764(0.481)	0.790(0.680)	0.751(0.611)	0.532(0.265)
HLT_3	0.969(0.919)	1.000(0.996)	1.000(0.928)	0.533(0.267)	0.643(0.314)	0.604(0.283)	0.930(0.603)	0.988(0.694)	0.879(0.796)	0.703(0.565)	0.747(0.424)	0.462(0.330)
LBI_1	1.000(0.962)	1.000(1.000)	1.000(1.000)	0.788(0.770)	0.887(0.833)	0.988(0.872)	0.941(0.897)	0.997(0.959)	1.000(0.967)	0.824(0.837)	0.923(0.955)	0.998(0.957)
LBI_2	0.986(0.919)	1.000(0.962)	0.977(0.997)	0.780(0.757)	0.848(0.899)	0.914(0.924)	0.888(0.897)	1.000(1.000)	1.000(1.000)	0.862(0.803)	0.975(0.951)	1.000(0.953)
LBI_3	0.994(0.929)	0.983(0.974)	1.000(1.000)	0.723(0.722)	0.888(0.862)	0.976(0.926)	0.955(0.919)	1.000(1.000)	0.998(0.974)	0.858(0.820)	0.919(0.947)	0.985(0.962)
$LKSN$	1.000(0.791)	0.968(0.371)	0.962(0.085)	0.971(0.857)	1.000(0.424)	1.000(0.280)	0.964(0.695)	0.961(0.352)	0.980(0.020)	0.967(0.772)	0.981(0.385)	0.998(0.064)
L	0.774(0.978)	0.621(1.000)	0.370(1.000)	0.089(0.660)	0.078(0.884)	0.212(0.854)	0.611(0.981)	0.607(1.000)	0.327(1.000)	0.211(0.817)	0.396(0.925)	0.311(1.000)
	$\alpha=(0.20,0.20)$, $\beta=(0.70,0.70)$, $c=-0.5$			$\alpha=(0.10,0.30)$, $\beta=(0.30,0.60)$, $c=-0.5$			$\alpha=(0.20,0.10)$, $\beta=(0.65,0.30)$, $c=-0.5$			$\alpha=(0.00,0.30)$, $\beta=(0.00,0.60)$, $c=-0.5$		
τ^*	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
K_1	0.884(0.718)	0.995(0.900)	1.000(0.952)	0.993(0.840)	1.000(0.978)	1.000(1.000)	0.786(0.630)	0.808(0.767)	0.937(0.800)	0.970(0.978)	0.974(1.000)	0.974(0.976)
K_2	0.884(0.795)	0.955(0.919)	1.000(0.835)	0.929(0.919)	1.000(0.933)	1.000(0.958)	0.788(0.774)	0.873(0.800)	0.902(0.707)	1.000(0.964)	0.998(0.962)	0.985(0.960)
K_3	0.909(0.752)	0.997(0.932)	0.983(0.883)	0.931(0.887)	1.000(0.925)	1.000(0.991)	0.736(0.684)	0.844(0.779)	0.902(0.836)	0.966(0.988)	0.999(0.951)	1.000(0.985)
LT_1	0.700(0.700)	0.804(0.765)	0.767(0.763)	0.812(0.880)	0.836(0.854)	0.827(0.834)	0.704(0.737)	0.790(0.794)	0.786(0.768)	0.869(0.798)	0.895(0.824)	0.808(0.786)
LT_2	0.781(0.773)	0.892(0.812)	0.582(0.588)	0.818(0.847)	0.866(0.819)	0.798(0.700)	0.861(0.849)	0.837(0.826)	0.672(0.702)	0.881(0.833)	0.898(0.876)	0.668(0.697)
LT_3	0.723(0.760)	0.842(0.814)	0.809(0.708)	0.820(0.834)	0.884(0.833)	0.802(0.814)	0.728(0.805)	0.854(0.794)	0.793(0.739)	0.835(0.794)	0.835(0.791)	0.816(0.817)
HLT_1	0.793(0.452)	0.801(0.505)	0.544(0.341)	0.771(0.658)	0.759(0.678)	0.720(0.601)	0.761(0.510)	0.658(0.521)	0.447(0.263)	0.847(0.703)	0.788(0.668)	0.728(0.631)
HLT_2	0.879(0.602)	0.928(0.683)	0.563(0.369)	0.920(0.651)	0.856(0.778)	0.788(0.720)	0.770(0.728)	0.790(0.638)	0.507(0.266)	0.931(0.794)	0.797(0.701)	0.813(0.643)
HLT_3	0.863(0.496)	0.860(0.612)	0.685(0.510)	0.984(0.778)	1.000(0.874)	0.993(0.967)	0.756(0.543)	0.711(0.475)	0.541(0.266)	1.000(0.849)	1.000(0.997)	0.970(0.971)
LBI_1	0.953(0.837)	0.985(0.993)	0.963(0.973)	0.902(0.864)	0.962(1.000)	1.000(0.978)	0.858(0.911)	0.952(0.951)	1.000(0.986)	0.974(0.913)	1.000(0.977)	0.972(0.966)
LBI_2	0.822(0.791)	1.000(0.976)	1.000(1.000)	0.929(0.880)	0.976(0.987)	1.000(0.985)	0.865(0.871)	0.941(0.920)	0.997(1.000)	0.975(0.932)	1.000(0.957)	1.000(1.000)
LBI_3	0.857(0.866)	0.970(0.973)	0.982(1.000)	0.930(0.886)	0.983(0.947)	1.000(0.963)	0.838(0.908)	0.938(0.912)	1.000(0.985)	0.978(0.949)	0.985(0.993)	1.000(0.965)
$LKSN$	1.000(0.716)	1.000(0.445)	0.979(0.090)	0.965(0.744)	1.000(0.372)	0.981(0.123)	0.974(0.768)	1.000(0.346)	0.988(0.052)	1.000(0.724)	1.000(0.252)	1.000(0.023)
L	0.297(0.968)	0.404(0.988)	0.290(1.000)	0.664(0.998)	0.591(0.975)	0.334(0.956)	0.088(0.870)	0.336(1.000)	0.367(0.941)	0.783(0.987)	0.679(1.000)	0.274(1.000)

Table 2.30: Empirical Rejection Frequencies of K_j, LT_j, HLT_j, LBI_j, $LKSN$ and L Tests. **EGARCH** Model in $\mathbf{H_{10}}$ with $\mathbf{T=250}$.

Chapter 3

Changes in Persistence in Outlier Contaminated Time Series

Co-authored with Saskia Rinke.

3.1 Introduction

Since the introduction of additive outliers (AOs) and innovative outliers (IOs) by Fox (1972), the effect of outliers on statistical inference in time series has been investigated. Martin and Yohai (1986) consider the effect of outliers on parameter estimation. They show that isolated outliers induce a downward bias of the AR coefficients, whereas patches of outliers induce an upward bias. Franses and Haldrup (1994) assess the effect of AOs on the Dickey and Fuller (1979) unit-root test and find that the null hypothesis of a random walk is rejected too often (cf. also Shin et al., 1996). Besides, they also consider the Johansen (1991) trace test for cointegration and find cointegration too often. Hence, they conclude that AOs yield spurious stationarity as well as spurious cointegration and expect similar results in case of a temporary change. Also the performance of linearity tests is deteriorated in the presence of outliers and nonlinear models are preferred to linear models. According to van Dijk et al. (2002) this is due to the fact that nonlinear models can generate data resembling an outlier contaminated linear process. So, van Dijk et al. (1999) find that the test for smooth transition nonlinearity of Luukkonen et al. (1988) becomes oversized in the presence of AOs. In extreme scenarios the size distortion improves but power losses occur. In contrast, IOs do not seriously deteriorate the performance of the test. Therefore, they conclude that the influence of AOs is much more severe than the effects of IOs. Ahmad and Donayre (2016) find evidence for size distortions but power improvements due to outliers for the test against threshold autoregressive nonlinearity of Hansen (1996, 1997).

The effect of outliers on tests for a change in persistence has not been assessed yet. Therefore, in this paper we investigate the performance of the ratio-based tests of Kim (2000); Kim et al. (2002) and of Leybourne et al. (2007b) in outlier contaminated processes. Both tests are based on a ratio of the subsample cumulative sum of squared residuals. Outliers influence the test statistic via the residuals and thus can lead to spurious test decisions.

In our simulation studies we vary the outlier magnitude, the sample size, and the change magnitude to assess their individual effects. Furthermore, we apply the outlier detection method of Shin et al. (1996) which is designed for unit-root testing and compare the performance of the tests in the contaminated and in the adjusted series.

The rest of the paper is organised as follows. In Section 3.2 the model framework and the different outlier types are introduced. In Section 3.3 the tests for a change in persistence are explained. Section 3.4 introduces the outlier detection and removal methods. In Section 3.5 the simulation set-up and the simulation results are presented. Section 3.6 contains a real data example of the G7 inflation rates. Finally, Section 3.7 concludes.

3.2 Modeling Outliers and Changes in Persistence

Outliers can only be defined in the context of a certain model under consideration (cf. Davies and Gather, 1993; van Dijk et al., 1999). In our analysis we will focus on autoregressive processes of order 1 with and without a change in persistence,

$$\Phi(L)\, x_t = \varepsilon_t, \qquad\qquad t = 1, \dots, T, \tag{3.1}$$

where T is the sample size, $\Phi(L) = 1 - \phi_1 L\, \mathbf{1}\{t \leq \lfloor \tau \cdot T \rfloor\} - \phi_2\, L\, \mathbf{1}\{t > \lfloor \tau \cdot T \rfloor\}$, L is the lag operator, $\mathbf{1}\{\cdot\}$ is the indicator function, $\lfloor \tau \cdot T \rfloor$ is the change point, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. There is no change in persistence if $\phi_1 = \phi_2$, $\tau = 0$, or $\tau = 1$. A common way to model outliers in the context of linear time series is the general replacement model of Martin and Yohai (1986),

$$y_t = x_t\,(1 - \delta_t) + \zeta_t \delta_t, \qquad\qquad t = 1, \dots, T.$$

The observable contaminated process y_t consists of the unobservable core process x_t and the contaminating process ζ_t. The random variable δ_t takes the values -1 and 1, each with the probability $\pi/2$, and 0 otherwise, where the probability π is the outlier probability. Allowing δ_t to take positive and negative values, enables us to model symmetric contaminations. The core process is the AR(1) model of Eq. (3.1).
Depending on the specification of the contaminating process ζ_t, different types of outliers are generated, i.e. AOs, IOs, level shifts, and temporary changes (cf. Galeano and Peña, 2013). In the context of time series mostly AOs and IOs are considered (cf. Fox, 1972; van Dijk et al., 1999). For AOs the contaminating process ζ_t and the respective contaminated process y_t are given by

$$\zeta_t = x_t + \zeta,$$
$$\text{and} \qquad y_t = x_t + \zeta \delta_t,$$

where ζ is the constant outlier magnitude depending on the standard deviation of the core process, σ_x. An IO contamination ζ_t and its observable process y_t can be modeled as

$$\zeta_t = x_t + \zeta/\Phi(L)$$
$$\text{and} \quad y_t = x_t + \Big(\zeta/\Phi(L)\Big)\,\delta_t.$$

AOs only have a one-time effect on the series since they do not affect the core process x_t. In contrast IOs have a one-time effect on the errors but influence several observations through the dynamics of the core process. Therefore, IOs have different effects in stationary and in nonstationary core processes. In contrast to IOs in stationary processes, the effect of an IO in a unit-root process is permanent and similar to a level shift.

3.3 Tests for a Change in Persistence

Several procedures exist to test for a change in persistence. They include the ratio-based tests of Kim (2000), Kim et al. (2002), Busetti and Taylor (2004), and Leybourne et al. (2007b) among others, the sub-sample augmented Dickey-Fuller-type test of Leybourne et al. (2003), and the variance ratio test of Leybourne (2004). All tests assume a constant persistence under the null hypothesis, either $I(0)$ like in Kim (2000) or $I(1)$ like in Leybourne et al. (2007b). The alternative is a change from $I(0)$ to $I(1)$ ($I(0) \to I(1)$) or a change from $I(1)$ to $I(0)$ ($I(1) \to I(0)$). We will focus on the test of Kim (2000); Kim et al. (2002) (the Kim test) since it is frequently applied and on the test of Leybourne et al. (2007b) (the Leybourne test) due to its good size and power properties. The idea of the tests is to divide the time series into two subsamples and take the ratio of the subsample cumulative sum (CUSUM) of squared residuals. For both tests simulated critical values are tabulated for the relevant sample sizes and significance levels of the simulation study in Section 3.5.

3.3.1 The Kim Test

Kim (2000) and Kim et al. (2002) test the null hypothesis of constant $I(0)$ against a change in persistence $I(0) \to I(1)$ with the test statistic

$$K_{\lfloor \tau T \rfloor} = \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor+1}^{T} \left(\sum_{i=\lfloor \tau T \rfloor+1}^{t} \tilde{v}_{i,\tau} \right)^2}{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \left(\sum_{i=1}^{t} \hat{v}_{i,\tau} \right)^2},$$

where $\hat{v}_{t,\tau}$ are the residuals from the OLS regression of y_t on a constant term for observations up to $\lfloor \tau T \rfloor$ to obtain invariance to a constant. Similarly, $\tilde{v}_{t,\tau}$ are the OLS residuals from the regression of y_t on a constant term for $t = \lfloor \tau T \rfloor + 1, \ldots, T$. Since the true change point τ^* is unknown, Kim (2000), Kim et al. (2002), and Busetti and Taylor (2004) use the sequence of statistics $\{K_{\lfloor \tau T \rfloor}\}$ for $\tau \in \Lambda$, where the change fraction τ^* is assumed to lie in $\Lambda = [\tau_l, \tau_u]$, an interval in $(0, 1)$ which is symmetric around 0.5, typically $[0.2, 0.8]$. Following Leybourne et al. (2007b) we will only consider the maximum test. Then, the test statistic and the estimated change fraction are given by

$$MX = \max_{\tau \in \Lambda} K_{\lfloor \tau T \rfloor},$$
$$\hat{\tau} = \arg\sup_{\tau \in \Lambda} \Xi(\tau),$$

with $\Xi(\tau) = \left((T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor+1}^{T} \tilde{v}_{i,\tau}^2 \right) \left(\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{v}_{i,\tau}^2 \right)^{-1}$. The null hypothesis will be rejected if the value of the test statistic MX is smaller or larger than the lower or upper tail critical value, respectively.

In Table 3.1 simulated upper and lower tail critical values of the Kim test for different sample sizes are given. They are based on 100 000 replications.

	Quantile					
T	0.005	0.025	0.050	0.950	0.975	0.995
50	0.534	0.910	1.185	16.878	21.588	35.050
100	0.594	0.992	1.292	17.047	21.591	34.001
250	0.647	1.087	1.402	17.776	22.425	36.033
500	0.681	1.111	1.438	17.932	22.646	35.489
1000	0.679	1.140	1.475	18.202	23.084	36.036

Table 3.1: Simulated Critical Values of the Kim Test

3.3.2 The Leybourne Test

In contrast to the Kim test, Leybourne et al. (2007b) test the null hypothesis of constant $I(1)$ against a change in persistence from $I(0) \rightarrow I(1)$ or $I(0) \rightarrow I(1)$ with the following two-tailed test statistic

$$R = \frac{K^f(\tau)}{K^r(\tau)} = \frac{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{v}_{t,\tau}^2}{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=1}^{(T-\lfloor \tau T \rfloor)} \tilde{v}_{t,\tau}^2}, \tag{3.2}$$

where $K^f(\tau)$ is the forward test statistic with $\hat{v}_{t,\tau}$ as defined above and $K^r(\tau)$ is the test statistic for the reversed series. Note that a change $I(1) \rightarrow I(0)$ is equivalent to a change $I(0) \rightarrow I(1)$ in the reversed series, $\tilde{y}_t \equiv y_{T-t+1}$, occurring at time $T - \lfloor \tau^* T \rfloor$.

Leybourne et al. (2007b) show that $K^f(\tau)$ converges in probability to zero for a change $I(0) \rightarrow I(1)$ for all $\tau \leq \tau^*$ and is of $O_p(1)$ if the persistence changes from $I(1) \rightarrow I(0)$ for all τ. $K^r(\tau)$ converges in probability to zero if $I(1) \rightarrow I(0)$ for all $\tau > \tau^*$ and is of $O_p(1)$ if $I(0) \rightarrow I(1)$ for all τ. So, if the true change point $\tau^* T$ is known, a test of the null hypothesis $I(1)$ against a change in persistence, either $I(0) \rightarrow I(1)$ or $I(1) \rightarrow I(0)$, can be based on Eq. (3.2), because a ratio of $K^f(\tau^*)$ and $K^r(\tau^*)$ collapses to zero for $I(0) \rightarrow I(1)$ and diverges to positive infinity for $I(1) \rightarrow I(0)$. Because the true change fraction τ^* is unknown, the test is based on the infima of $K^f(\tau)$ and $K^r(\tau)$ for $\tau \in \Lambda$. The null hypothesis of $I(1)$ throughout will be rejected if R exceeds or falls below the upper or the lower tail critical value, respectively. The estimated change fraction $\hat{\tau}$ is given by $\arg\inf_{\tau \in \Lambda} K^f(\tau)$ for a change $I(0) \rightarrow I(1)$ and by $\arg\inf_{\tau \in \Lambda} K^r(\tau)$ for a change $I(1) \rightarrow I(0)$. In Table 3.2 simulated upper and lower tail critical values of the Leybourne test for different sample sizes are given. They are based on 100 000 replications.

	Quantile					
T	0.005	0.025	0.050	0.950	0.975	0.995
50	0.131	0.213	0.276	3.600	4.686	7.616
100	0.117	0.194	0.256	3.950	5.149	8.572
250	0.104	0.180	0.239	4.177	5.502	9.531
500	0.100	0.177	0.234	4.278	5.684	10.017
1000	0.101	0.177	0.234	4.327	5.773	10.152

Table 3.2: Simulated Critical Values of the Leybourne Test

Leybourne et al. (2007b) show that the test is conservative against a constant $I(0)$ process. Thus, in contrast to the Kim test the Leybourne test does not spuriously detect changes in persistence.

3.4 Outlier Detection and Removal Methods

There are several publications emphasizing the deteriorating effect of outliers on the performance of estimation and testing methods (cf. Franses and Haldrup (1994); van Dijk et al. (1999); Ahmad and Donayre (2016) among others). Two strands of procedures exist in order to handle outlier contaminated series. Either the outliers have to be detected and removed before parameters are estimated and tests are conducted, or the approaches have to be robust against outliers (cf. e.g. van Dijk et al., 1999). Several outlier detection methods have been proposed starting with Chang et al. (1988) and Tsay (1988). The approach of Tsay (1988) works under the initial assumption of an uncontaminated series and consists of specification and estimation in an outer loop and detection and removal of outliers in the inner loop (cf. Figure 3.1). In a first step the critical value C as well as the order of an ARMA model have to be selected and the corresponding parameters are estimated. The inner loop starts with the calculation of the residuals and the estimation of the error term variance $\hat{\sigma}_\varepsilon^2$. For each outlier type $j = AO, IO$ and each observation $t = 1, \ldots, T$ the test statistic $\lambda_{j,t} = \hat{\zeta}_{j,t}/\hat{\sigma}_j$, where $\hat{\zeta}_{j,t}$ is the estimated outlier effect and $\hat{\sigma}_j$ is the corresponding standard deviation depending on $\hat{\sigma}_\varepsilon$, is calculated to test the null hypothesis of no outlier of type j at observation t,

$$H_0 : \hat{\zeta}_{j,t} = 0 \qquad\qquad H_1 : \hat{\zeta}_{j,t} \neq 0.$$

Let t_j denote the observation with the highest probability of being an outlier of type j. In order to identify t_j, Tsay (1988) takes the maximum of the test statistics $\lambda_{j,t}$ over all t. The maximum of both $\lambda_{AO,t_{AO}}$ and $\lambda_{IO,t_{IO}}$ denotes the final test statistic λ to determine the outlier type and position. If λ exceeds the critical value C the outlier is removed depending on the type and the inner loop further iterates.

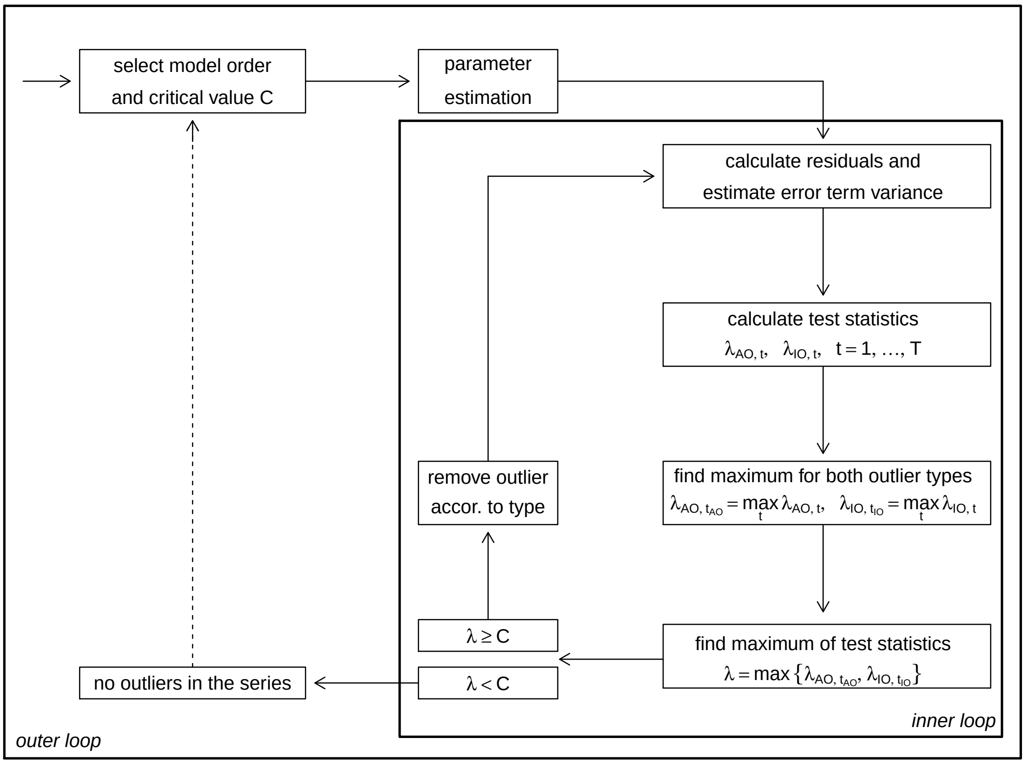


Figure 3.1: The Outlier Detection and Removal Method of Tsay (1988)

If the inner loop is completed after one single iteration, the algorithm stops and the series is uncontaminated. If however the inner loop stops after iterating several times to remove outliers, the outer loop starts again to check a refined model.

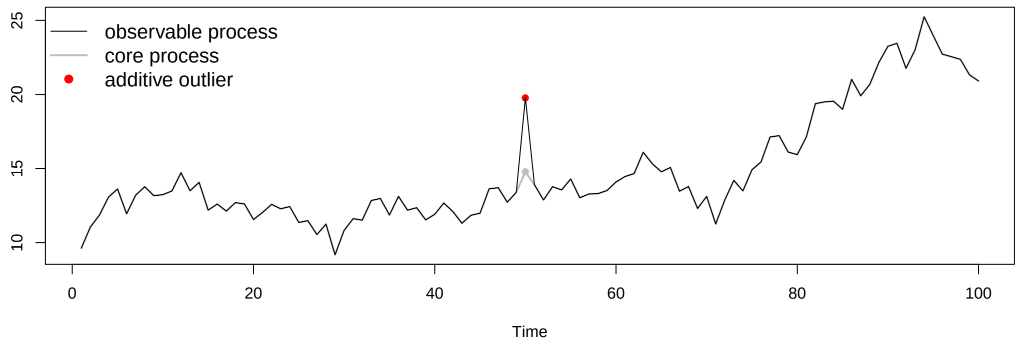
The described algorithm detects outliers sequentially, which is computationally easier and performs well if there exists only a single outlier in the series but can lead to biased estimates if there are multiple outliers (cf. Chen and Liu, 1993). Therefore, Chen and Liu (1993) propose a procedure consisting of three different stages.

In the first stage the algorithm of Tsay (1988) is applied to detect possible outliers. Given the information of the first stage about the estimated time points where outliers occur, the outlier effects can be estimated jointly and the significance of the outliers is assessed. Insignificant outliers are deleted one-by-one until all remaining outlier effects are significant. Finally the model parameters are estimated. Given this information, in the third stage the procedure starts again with the refined parameter estimates.

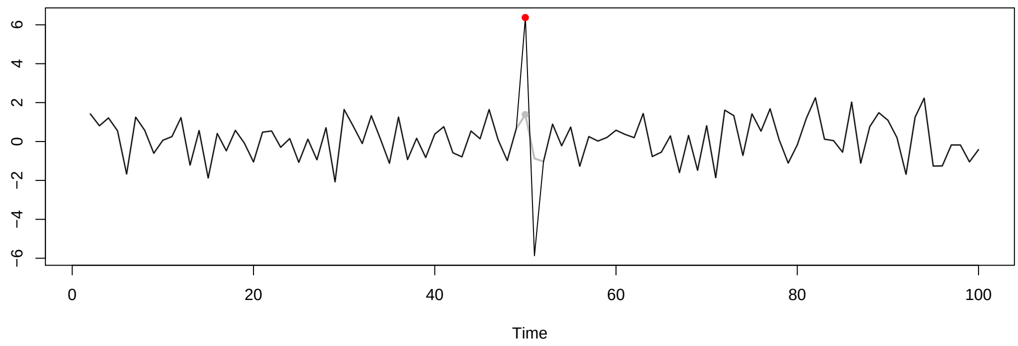
According to Galeano and Peña (2013) the procedure of Chen and Liu (1993) is the standard approach for outlier detection in linear time series. However, it has three major drawbacks, firstly, the type of outlier (IO or level shift) may not be correctly identified which affects the adjustment of the series, secondly, the algorithm depends on initial parameter estimates, may resulting in the break down of the procedure due to biased initial values, and finally, patches of outliers may not be identified due to the masking effect. Sánchez and Peña (2003) further modify the approach in order to solve these problems. For example, they calculate robust initial estimates by eliminating influential points (cf. also Peña, 1991) and use lower critical values C to be able to identify patches of outliers. Although further extensions lead to improved results, the computational burden increases enormously. Moreover, the main aim of the detection algorithms is to obtain unbiased parameter estimates for an ARMA model.

Since we are primarily interested in the demeaned series, we apply the algorithm of Shin et al. (1996) which focuses on outlier detection for unit-root testing and works under the assumption of the series being a random walk. This approach can be valuable in our analysis, since the test by Leybourne et al. (2007b) is $I(1)$ under the null hypothesis. However, the test of Kim (2000); Kim et al. (2002) is $I(0)$ under the null hypothesis and Shin et al. (1996) admit that their outlier detection algorithm does not perform well if the process under consideration exhibits only a small degree of persistence. Nevertheless, our results in the simulation studies show that the performance of the Kim test is not deteriorated by outliers if the process only exhibits a low degree of persistence. Due to the assumption of a random walk, the procedure of Shin et al. (1996) does not need an initial model selection and parameter estimates, thus minimizing the computational effort.

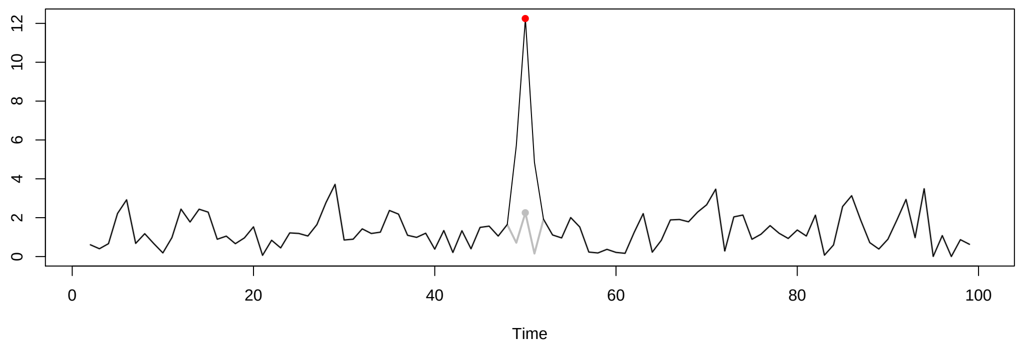
The idea of the Shin et al. (1996) algorithm is illustrated in Figure 3.2. An AO only affects one single observation but two consecutive residuals, i.e. the differences between two consecutive observations, $e_t = y_t - y_{t-1}$. Thus, a test can be based on the difference between the residuals. Since the difference may be negative, the absolute value is considered.



(a) Random Walk with an Additive Outlier at $t = 50$



(b) Residuals as the First Difference of the Random Walk



(c) Absolute value of the First Difference of Residuals

Figure 3.2: Idea of the Shin et al. (1996) Algorithm

Due to the fact that it is not known a priori when an AO occurs, the maximum of the absolute differences is determined. Let $t_{AO} = \arg\max_{2 \leq t \leq T-1} \left| e_{t+1} - e_t \right|$, then t_{AO} is the observation that is most likely to be contaminated by an AO. To test whether there occurs an AO at t_{AO}, $|e_{t_{AO}+1} - e_{t_{AO}}|$ is standardised by the estimated standard deviation of $e_{t_{AO}+1} - e_{t_{AO}}$. The general test statistic is given by

$$\lambda = \frac{1}{\sqrt{2}\hat{\sigma}} \left(\max_{2 \leq t \leq T-1} |e_{t+1} - e_t| \right),$$

where $\hat{\sigma}^2 = (T-3)^{-1} \left((\sum_{t=2}^{T} e_t^2) - e_{t_{AO}}^2 - e_{t_{AO}+1}^2 \right)$ is a robust estimator of σ_ε^2. If the test statistic equals or exceeds a critical value C, an AO is detected. We follow Shin et al. (1996) and use the critical value $C = 3$. A further discussion of the distribution of λ can be found in the appendix.

Shin et al. (1996) recommend to replace an AO contaminated observation with its lagged value to adjust the series. This procedure only takes into account the information up to t_{AO} and leads to constant parts in the time series, resulting in a larger residual $e_{t_{AO}+1}$. Therefore, we suggest to use the full sample information and to replace the outlying observation $y_{t_{AO}}$ by its best full sample prediction, i.e. the mean of the lagged value and the future value, $\hat{y}_{t_{AO}} = (y_{t_{AO}-1} + y_{t_{AO}+1})/2$. The procedure is repeated until no additional outliers are detected, i.e. $\lambda < C$.

The approach can be adjusted to detect IOs (cf. Shin et al., 1996). However, as we will show in the following section, this is not necessary, since IOs do not seriously affect the performance of the tests for a change in persistence.

3.5 Simulation Study

In our simulation study we consider the linear model given in Eq. (3.1) without contaminations ($\zeta = 0$) and with AOs as well as IOs of different outlier magnitudes ζ with an outlier probability of $\pi = 0.05$ (cf. Ahmad and Donayre, 2016). The errors form a Gaussian white noise process. In order to assess the performance of the tests, we apply them to the uncontaminated, contaminated, and adjusted series. To adjust the series we use the modified algorithm of Shin et al. (1996) with a critical value of $C = 3$. We vary the following parameters,

$$\begin{aligned} &\text{sample size} & T &= \{50, 100, 250, 500, 1000\}, \\ &\text{persistence} & \phi_1, \phi_2 &= \{0.00, 0.25, 0.50, 0.75, 0.95, 1.00\}, \\ &\text{outlier magnitude} & \zeta &= \{0\,\sigma_x, 1\,\sigma_x, 2\,\sigma_x, 3\,\sigma_x\}. \end{aligned}$$

For every series 200 additional observations are simulated as a burn-in period to avoid a starting value bias. All initial values are set to zero. The simulation results are based on 1000 replications. The following figures and tables report the simulation results for $\tau = 0.5$. In general we find that the power of the tests is higher if the change point occurs early in the series under the condition that the stationary part of the series is at least as large as the nonstationary part.

3.5.1 Performance in Uncontaminated Series

Table 3.3 tabulates the size properties of the Kim and the Leybourne test in uncontaminated series for different sample sizes T and different levels of significance α.

	significance level α		
T	1%	5%	10%
50	0.009	0.049	0.093
100	0.011	0.052	0.110
250	0.009	0.045	0.094
500	0.008	0.048	0.095
1000	0.010	0.050	0.093

(a) Kim Test $I(0)$

	significance level α		
T	1%	5%	10%
50	0.011	0.046	0.105
100	0.011	0.047	0.097
250	0.010	0.042	0.094
500	0.010	0.052	0.107
1000	0.009	0.046	0.102

(b) Leybourne Test $I(1)$

Table 3.3: Size Properties of the Kim and Leybourne Tests

The size of the Kim and of the Leybourne test coincides with the nominal size. Since the critical values depend on the number of observations, the tests perform well in terms of size for all sample sizes.

Table 3.4 tabulates the power results of the Kim test for $I(0) \rightarrow I(1)$ and of the Leybourne test for both $I(0) \rightarrow I(1)$ and $I(1) \rightarrow I(0)$.

T	significance level α 1%	5%	10%
50	0.779	0.868	0.907
100	0.947	0.978	0.982
250	0.997	0.998	0.999
500	1.000	1.000	1.000
1000	1.000	1.000	1.000

(a) Kim Test $I(0) \rightarrow I(1)$

T	significance level α 1%	5%	10%
50	0.017	0.081	0.156
100	0.084	0.262	0.400
250	0.408	0.690	0.803
500	0.798	0.935	0.977
1000	0.963	0.997	0.999

(b) Leybourne Test $I(1) \rightarrow I(0)$

T	significance level α 1%	5%	10%
50	0.087	0.246	0.393
100	0.238	0.504	0.666
250	0.612	0.858	0.934
500	0.882	0.979	0.995
1000	0.987	1.000	1.000

(c) Leybourne Test $I(0) \rightarrow I(1)$

Table 3.4: Power Properties of the Kim and Leybourne Tests

The power of both tests increases with the sample size. However, in small samples the power of the Kim test is already high and it converges to 1 with an increasing number of observations. In contrast, the power of the Leybourne test crucially depends on the sample size. In very small samples $T = 50$ the power is only slightly higher than its size. Also for $T = 100$ the power is relatively low. For sample sizes of $T \geq 250$ the power increases and the test decision is reliable. With an increasing number of observations the power of the test converges to 1.

All presented results are valid for $\phi_1, \phi_2 = \{0, 1\}$. In general, the size of the Kim test increases if the degree of persistence increases and the power decreases with a decreasing change magnitude $|\phi_1 - \phi_2|$ (cf. Fig. 3.15 and 3.16). For the Leybourne test the size decreases to zero if the process becomes stationary. The power decreases if $|\phi_1 - \phi_2|$ decreases (cf. Fig. 3.17 and 3.18).

3.5.2 Performance in Contaminated Series

Figure 3.3 illustrates the effects of AOs and IOs on the size of the Kim test for different sample sizes, outlier magnitudes ζ, and significance levels. The results show that there is no difference between the effects of AOs and IOs on the size of the Kim test. This is due to the fact that the degree of persistence of the core process is zero under the null hypothesis and an IO can only affect one observation exactly like an AO. The effect of outliers is mostly pronounced for large outlier magnitudes ζ and small to moderate sample sizes. The higher the persistence of the simulated processes, the higher are the size distortions in small samples (cf. Fig. 3.15 and 3.16). However, the size is not deteriorated seriously, but holds the nominal significance level.

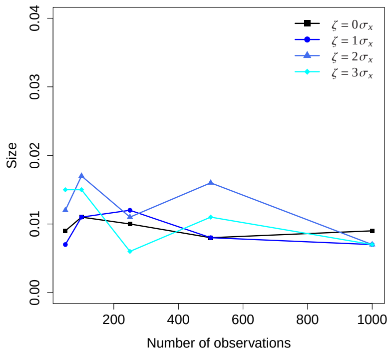
The power of the Kim test is not affected by AO contaminations if ζ is small. Only for large outlier magnitudes $\zeta = 3\sigma_x$ the power of the test decreases in small samples. The power of the test is not affected by IO contaminations (cf. Fig. 3.4).

Figure 3.5 presents the size of the Leybourne test in outlier contaminated series for different sample sizes, outlier magnitudes, and levels of significance. In the left panel the results for AOs can be found. The introduction of AOs decreases the size of the Leybourne test for all sample sizes and all significance levels. This implies that the test becomes undersized. The size distortion increases with the sample size and the outlier magnitude. For large sample sizes combined with large outlier magnitudes the size converges to zero. This is due to the fact that an AO contaminated unit-root process can be confused with a stationary process (cf. Franses and Haldrup, 1994). Since the size of the Leybourne test converges to zero for a constant $I(0)$ process, the size of the Leybourne test decreases to zero in AO contaminated series.

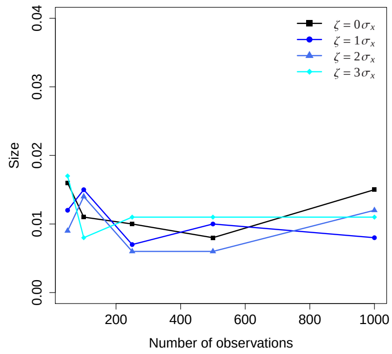
In the right panel of Figure 3.5 the size properties of the Leybourne test in IO contaminated time series are depicted. The size distortions are less severe compared to AO contaminations (cf. also van Dijk et al., 1999). Only in small samples and for large outlier magnitudes the size differs from the nominal significance level.

In terms of size the Leybourne test is more affected by outliers than the Kim test due to the higher degree of persistence under the null hypothesis. The effect of AOs is more serious than the effect of IOs.

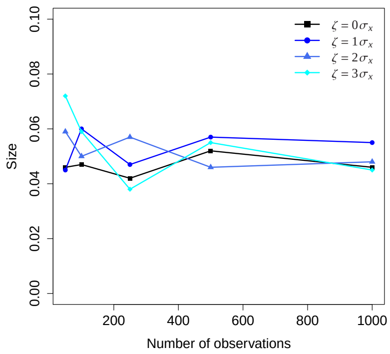
Figures 3.6 and 3.7 illustrate the power properties of the Leybourne test for $I(0) \rightarrow I(1)$ and $I(1) \rightarrow I(0)$, respectively. For both alternatives the results are qualitatively the same. For a change $I(0) \rightarrow I(1)$ the power is slightly higher across sample sizes, significance levels, and outlier magnitudes. This coincides with the findings in the uncontaminated series (cf. Tab. 3.4). In the left panels the effects of AOs on the power properties are depicted. The power decreases and approaches zero for increasing outlier magnitudes because the contaminated series can be confused with a stationary $I(0)$ process. In contrast, IOs do not decrease the power, but lead to power gains since the stationary and the nonstationary part of the series markedly differ (cf. Fig. 3.8a).



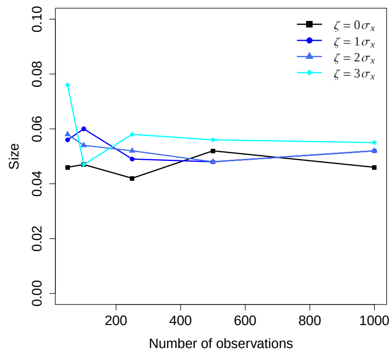
(a) AOs and $\alpha = 1\%$



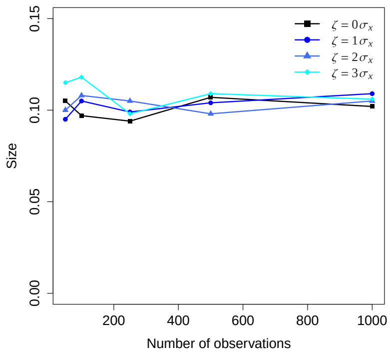
(b) IOs and $\alpha = 1\%$



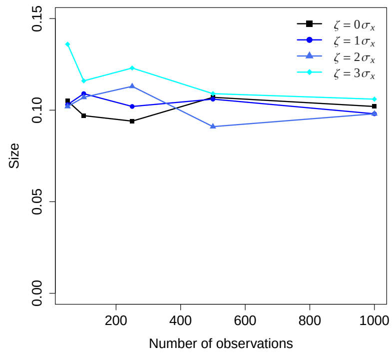
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$

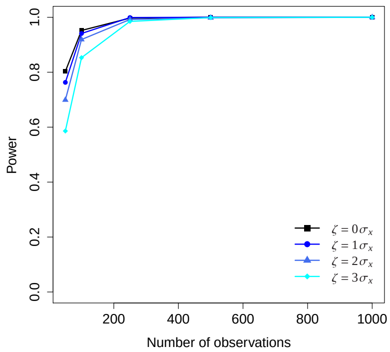


(e) AOs and $\alpha = 10\%$

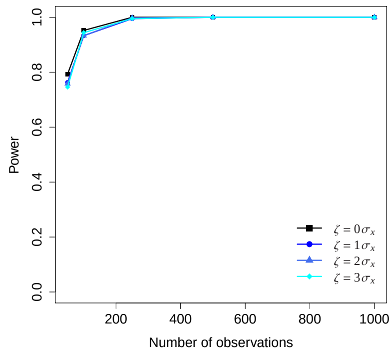


(f) IOs and $\alpha = 10\%$

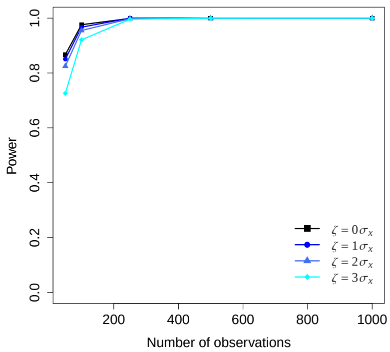
Figure 3.3: Size of the Kim Test ($I(0)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance



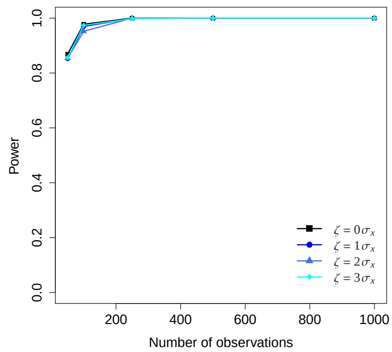
(a) AOs and $\alpha = 1\%$



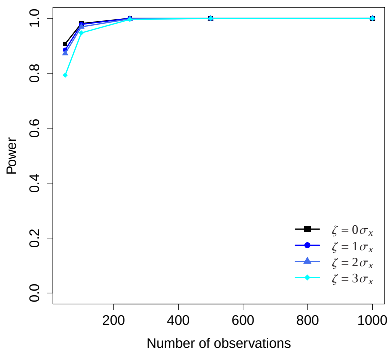
(b) IOs and $\alpha = 1\%$



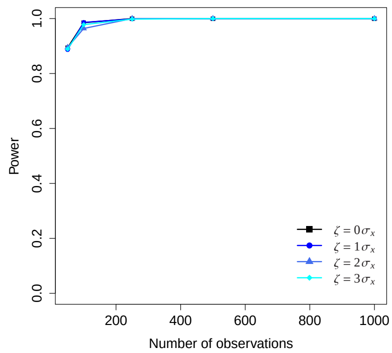
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$

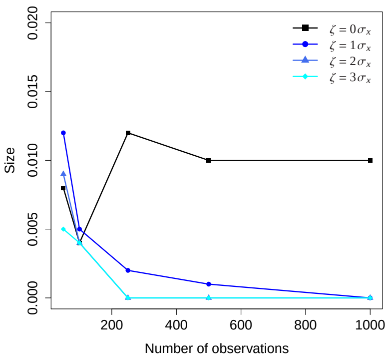


(e) AOs and $\alpha = 10\%$

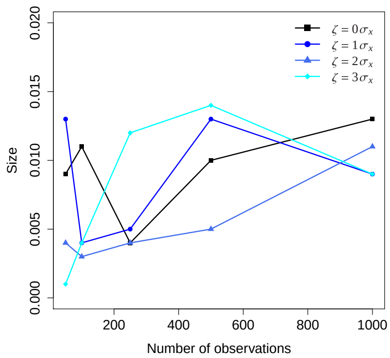


(f) IOs and $\alpha = 10\%$

Figure 3.4: Power of the Kim Test ($I(0) \rightarrow I(1)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance



(a) AOs and $\alpha = 1\%$



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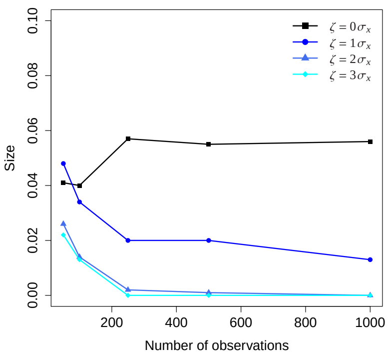
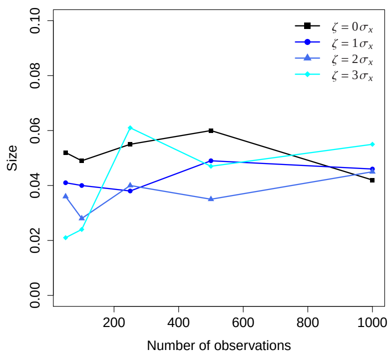
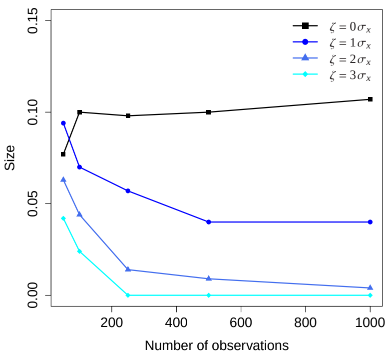


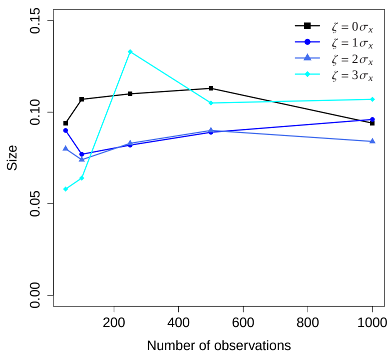
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$

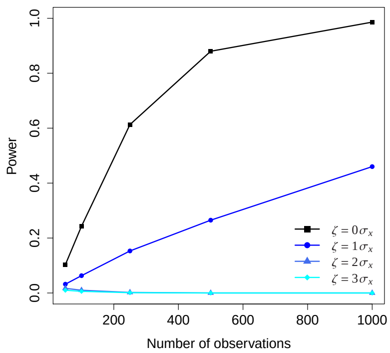


(e) AOs and $\alpha = 10\%$

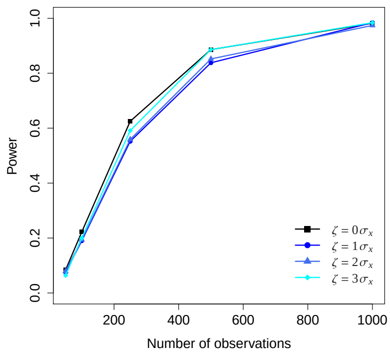


(f) IOs and $\alpha = 10\%$

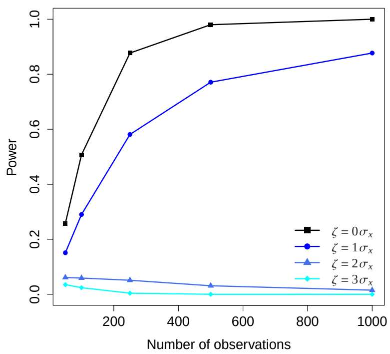
Figure 3.5: Size of the Leybourne Test ($I(1)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance



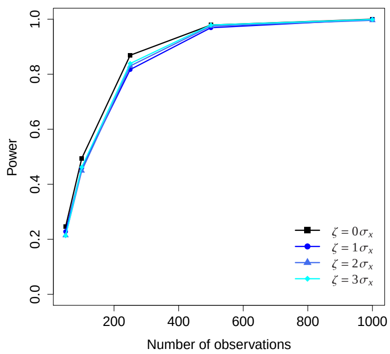
(a) AOs and $\alpha = 1\%$



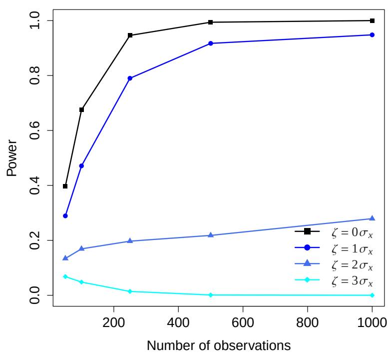
(b) IOs and $\alpha = 1\%$



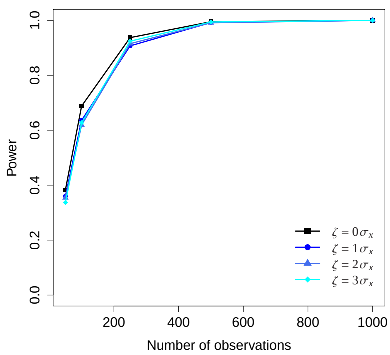
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$

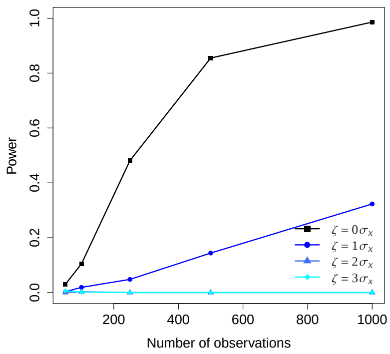


(e) AOs and $\alpha = 10\%$

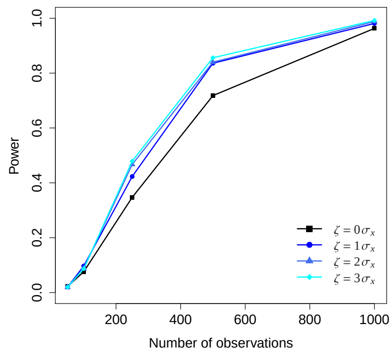


(f) IOs and $\alpha = 10\%$

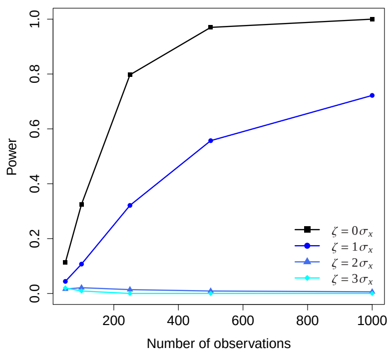
Figure 3.6: Power of the Leybourne Test ($I(0) \rightarrow I(1)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance



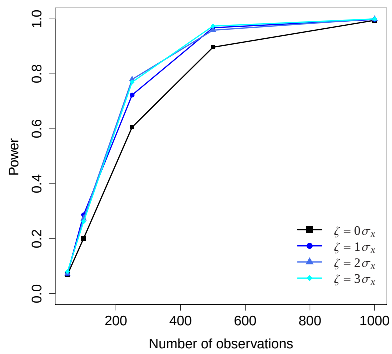
(a) AOs and $\alpha = 1\%$



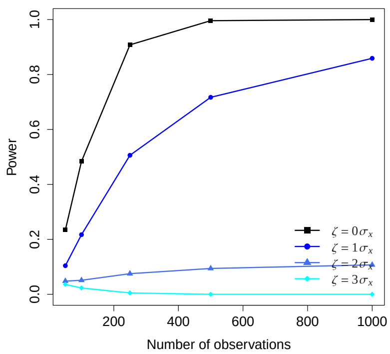
(b) IOs and $\alpha = 1\%$



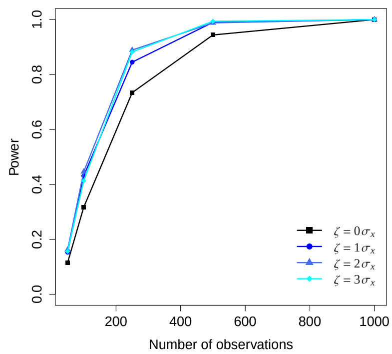
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$



(e) AOs and $\alpha = 10\%$



(f) IOs and $\alpha = 10\%$

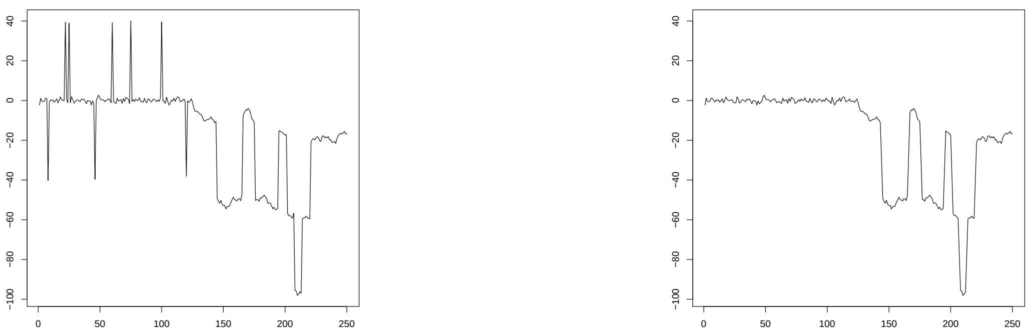
Figure 3.7: Power of the Leybourne Test ($I(1) \rightarrow I(0)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance

3.5.3 Performance in Adjusted Series

The results in Figures 3.3 and 3.4 show that the performance of the Kim test does not suffer from size distortions or power losses due to outliers for low degrees of persistence. Hence, it is not necessary to adjust the series before applying the test. Moreover, the modified algorithm of Shin et al. (1996) is developed for nonstationary time series and thus does not perform well in series with a low degree of persistence. Although the application of the Kim test to the adjusted series results in power gains, it also suffers from an increased size (cf. Fig. 3.15 and 3.16).

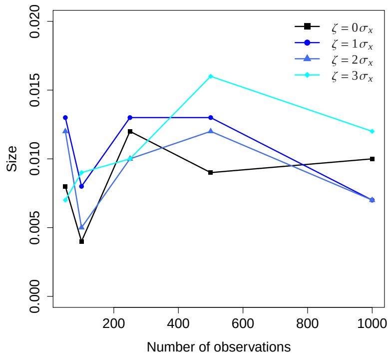
Figure 3.9 shows the size properties of the Leybourne test in the adjusted series. In all uncontaminated series the size is not affected by the adjustment procedure. Therefore, the algorithm does not spuriously detect outliers. Applying the modified algorithm of Shin et al. (1996) to AO contaminated series increases the size of the test back to its nominal significance level in all sample sizes independent of the outlier magnitude. In IO contaminated series the application of the algorithm does not influence the size properties. In fact, the size is not deteriorated by IOs, anyway.

Figures 3.10 and 3.11 present the power properties of the Leybourne test in the adjusted series. In the uncontaminated series the power is not affected by the adjustment of the series. The application of the modified algorithm of Shin et al. (1996) to AO contaminated series increases the power especially in series with large outlier magnitudes and equals the power in the uncontaminated series. In IO contaminated series the power increases and is higher than in the uncontaminated series. This is due to the fact that the algorithm can detect IOs only in the stationary part and thus, the differentiation between the stationary and the nonstationary part becomes easier (cf. Fig. 3.8).

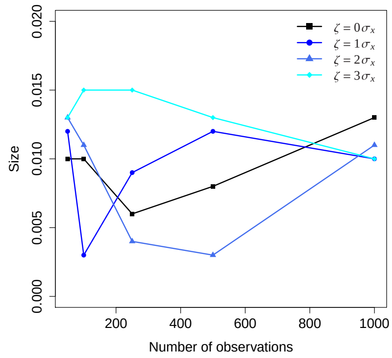


(a) Contaminated Series **(b)** Adjusted Series

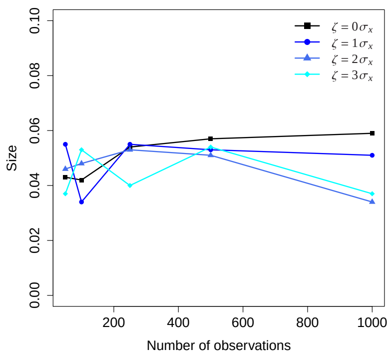
Figure 3.8: Influence of the Adjustment on an IO Contaminated Series with a Change in Persistence ($I(0) \rightarrow I(1)$)



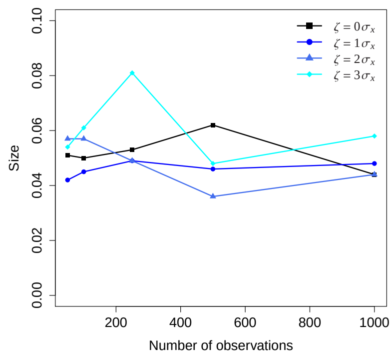
(a) AOs and $\alpha = 1\%$



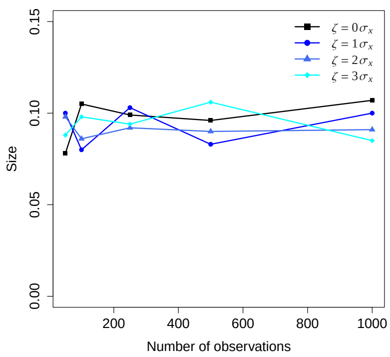
(b) IOs and $\alpha = 1\%$



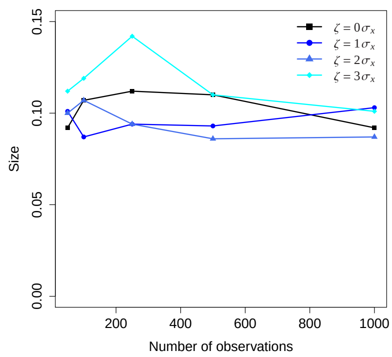
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$

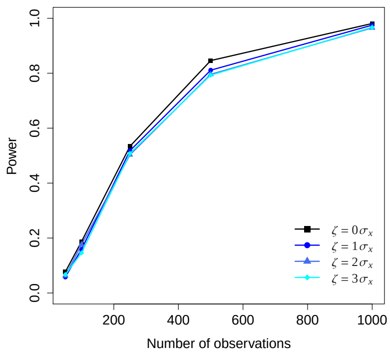


(e) AOs and $\alpha = 10\%$

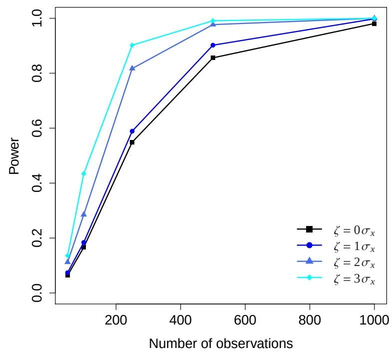


(f) IOs and $\alpha = 10\%$

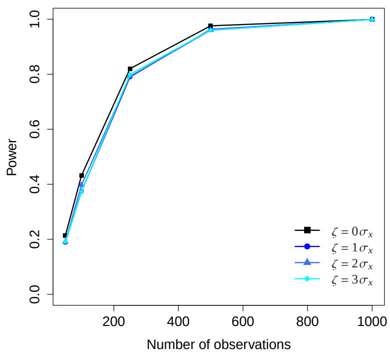
Figure 3.9: Size of the Leybourne Test ($I(1)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance.



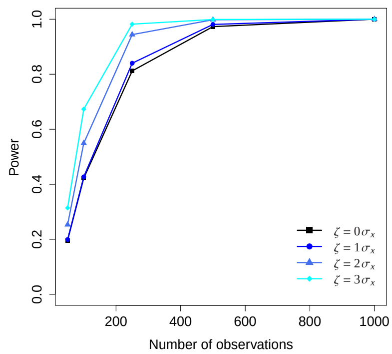
(a) AOs and $\alpha = 1\%$



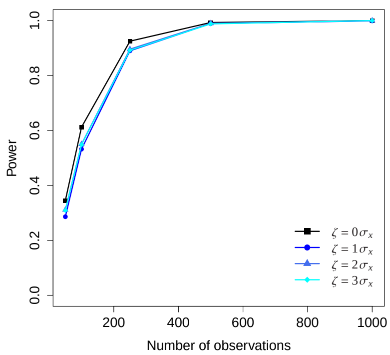
(b) IOs and $\alpha = 1\%$



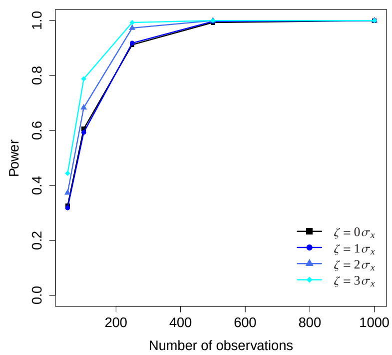
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$

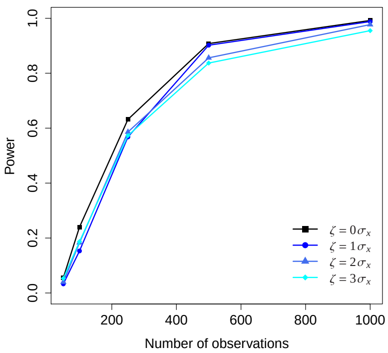


(e) AOs and $\alpha = 10\%$

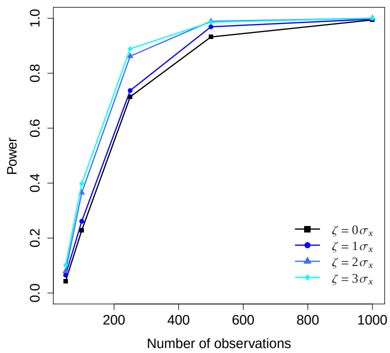


(f) IOs and $\alpha = 10\%$

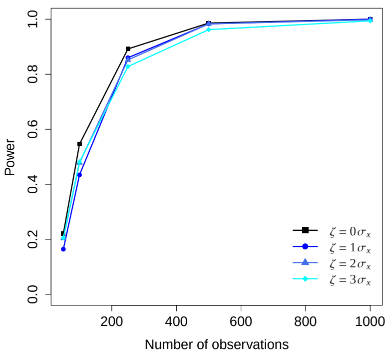
Figure 3.10: Power of the Leybourne Test ($I(0) \rightarrow I(1)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance.



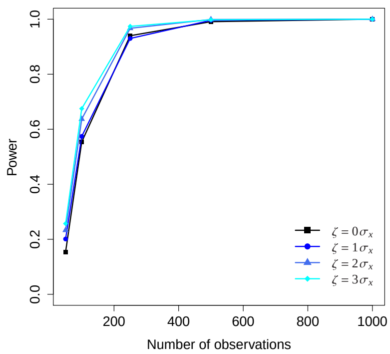
(a) AOs and $\alpha = 1\%$



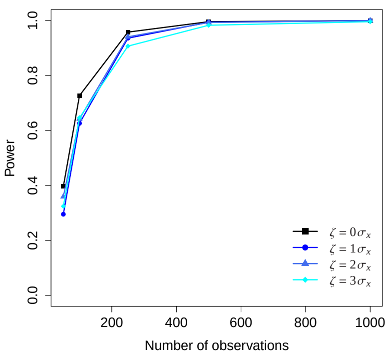
(b) IOs and $\alpha = 1\%$



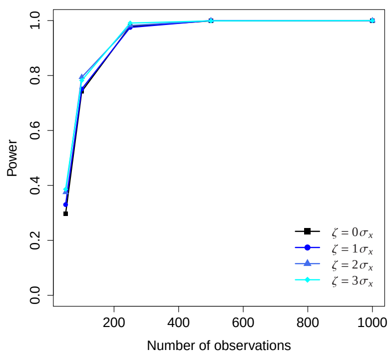
(c) AOs and $\alpha = 5\%$



(d) IOs and $\alpha = 5\%$



(e) AOs and $\alpha = 10\%$



(f) IOs and $\alpha = 10\%$

Figure 3.11: Power of the Leybourne Test ($I(1) \rightarrow I(0)$) for Additive/Innovative Outliers, Different Outlier Magnitudes ζ and Different Levels of Significance.

3.6 Empirical Example

In this section we apply the tests for a change in persistence of Kim (2000); Kim et al. (2002) and of Leybourne et al. (2007b) and the outlier detection method of Shin et al. (1996) to inflation data of the G7 countries. Following Busetti and Taylor (2004), we use quarterly CPI data from the OECD retrieved from FRED from 1970Q1 until 2014Q4 and define the inflation rates as

$$\pi_t = \log(CPI_t) - \log(CPI_{t-1}).$$

Thus, our data set consists of 180 observations for each country. We use the R package $X13$ for seasonal adjustment. The properties of the series change over time. During the Great Inflation in the 1970s and early 1980s inflation rates appear to exhibit a higher degree of persistence. At the beginning of the 1980s there is an overall decrease in the persistence. This period is referred to as the Great Moderation. The transition of the Great Inflation to the Great Moderation could present a change in persistence.

In Table 3.5 the critical values of both tests for a sample size of $T = 180$ are presented.

	Quantile					
	0.005	0.025	0.050	0.950	0.975	0.995
Kim	0.625	1.049	1.358	17.490	22.153	34.887
Leybourne	0.109	0.186	0.246	4.101	5.469	9.382

Table 3.5: Simulated Critical Values for $T = 180$

The test statistics of the Kim and Leybourne test applied to the G7 inflation rates for the original and the adjusted series are given in Table 3.6. Bold numbers indicate the rejection of the null hypothesis. In the original series the Kim test rejects the null hypothesis for Japan at the 10% significance level with an estimated change in 2005Q4. The Leybourne test rejects the null hypothesis in the original series for France at the 10% significance level and for the USA at the 1% level with the estimated changes in 1991Q4 and 1991Q1, respectively. After adjusting the series with the modified algorithm of Shin et al. (1996) the Kim test does not reject the null hypothesis for any country. In contrast, the Leybourne test rejects the null hypothesis for France at the 5% significance level and for Great Britain, Italy and Japan at the 10% significance level with the estimated changes in 1985Q2, 1990Q3, 1996Q1 and 1981Q4.

	CAN	FRA	GBR	GER	ITA	JPN	USA
Kim Test	4.0159	8.9347	3.0654	2.6954	4.5738	**1.1486**	6.9269
Leybourne Test	2.9018	**4.3994**	2.2575	1.7306	1.6157	2.7578	**23.0157**

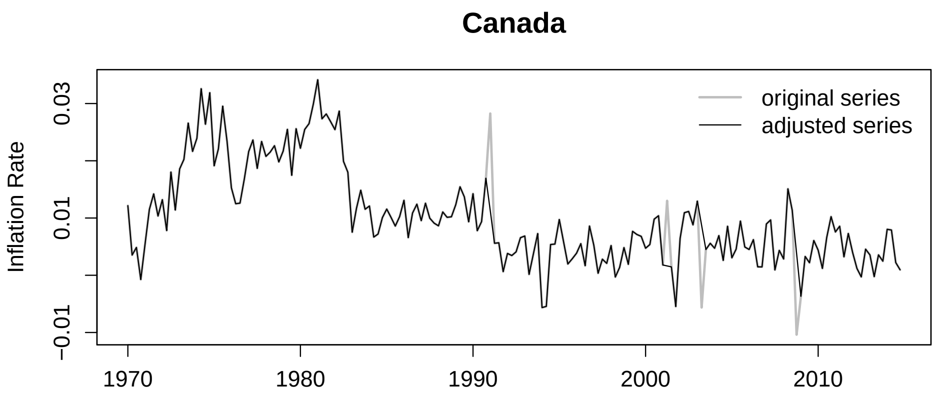
(a) Original Series

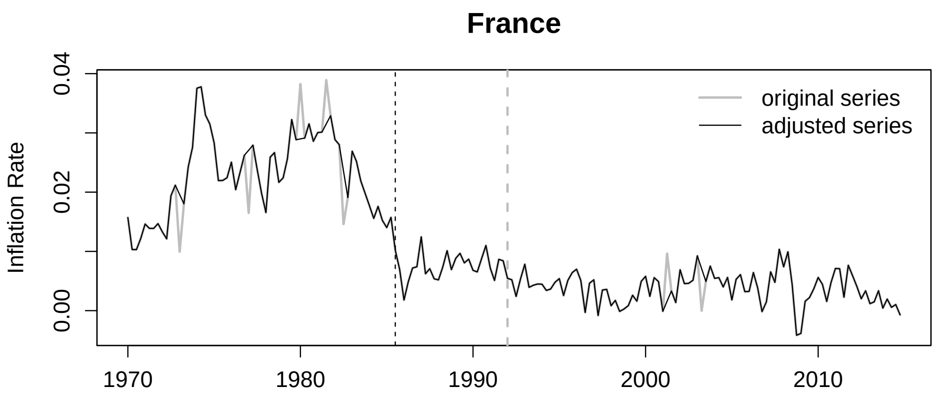
	CAN	FRA	GBR	GER	ITA	JPN	USA
Kim Test	3.7260	8.1539	2.3005	2.5883	6.6409	1.9316	5.8816
Leybourne Test	3.1757	**6.0435**	**5.0082**	1.9686	**4.9083**	**4.5627**	3.8323

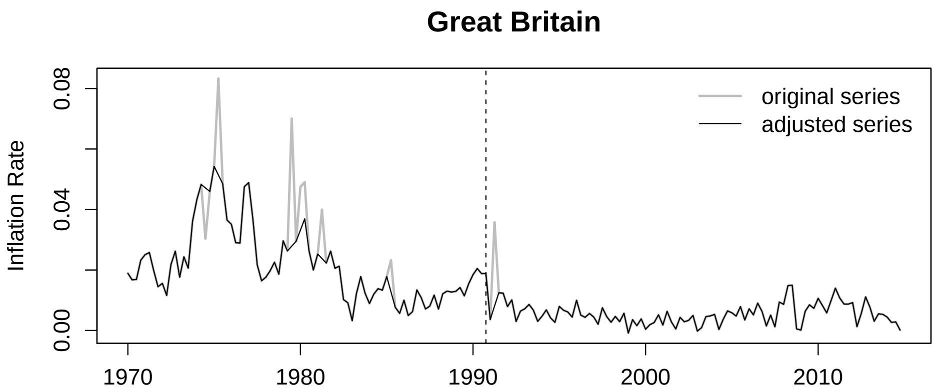
(b) Adjusted Series

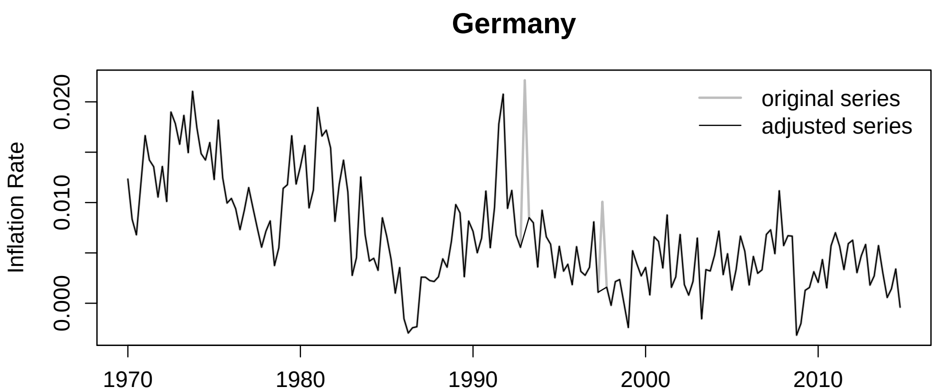
Table 3.6: Test Statistics of the Kim and Leybourne Test

In Figure 3.12 the original and the adjusted series of the G7 inflation rates are presented. The estimated change points are indicated by dashed lines.


Canada
original series
adjusted series
Inflation Rate
−0.01
0.01
0.03
1970
1980
1990
2000
2010


France
original series
adjusted series
Inflation Rate
0.00
0.02
0.04
1970
1980
1990
2000
2010


Great Britain
original series
adjusted series
Inflation Rate
0.00
0.04
0.08
1970
1980
1990
2000
2010


Germany
original series
adjusted series
Inflation Rate
0.000
0.010
0.020
1970
1980
1990
2000
2010

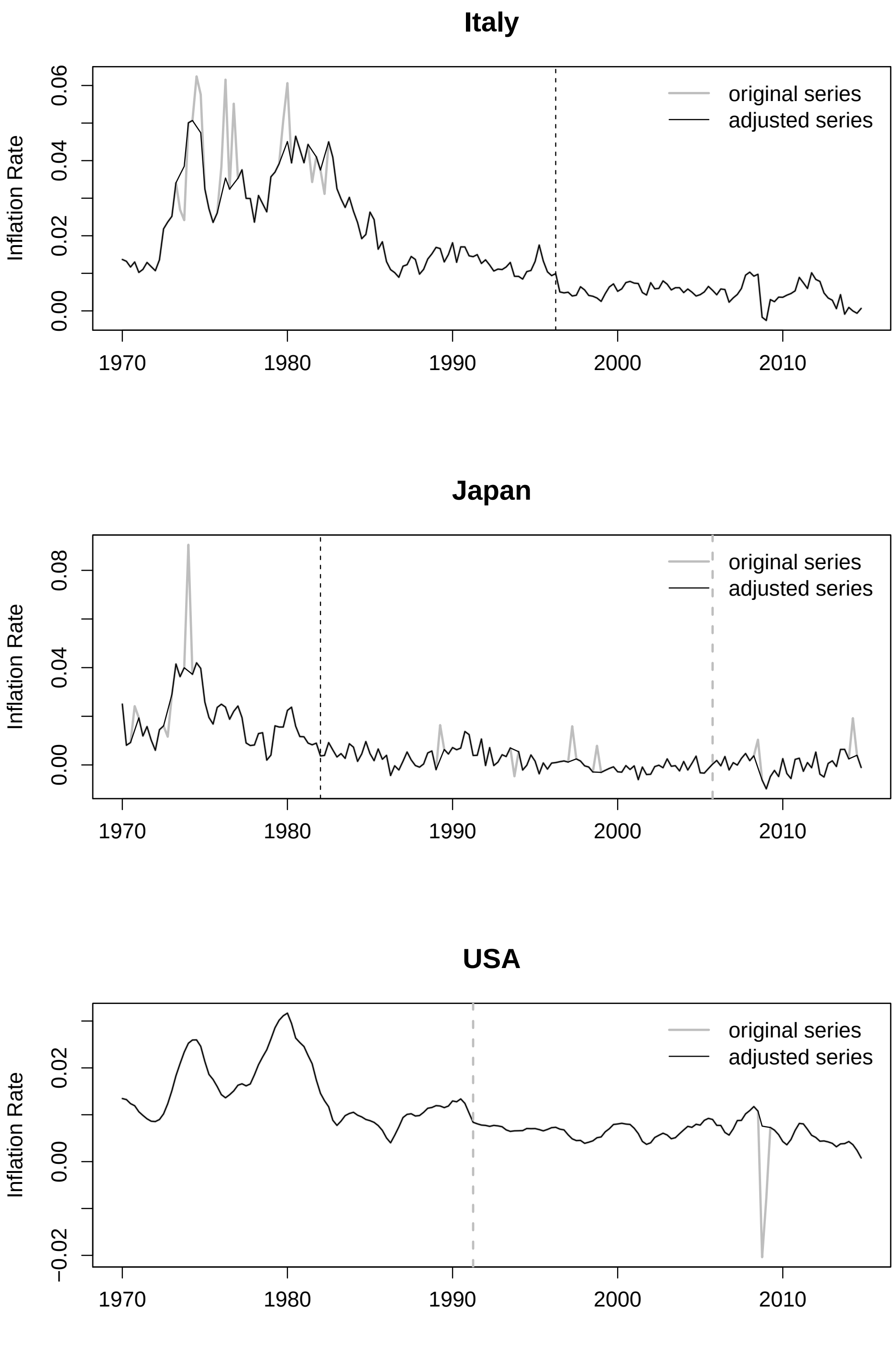


Figure 3.12: Inflation Rates of the G7 Countries

In order to support the test results, we conduct the unit-root test of Dickey and Fuller (1979).

Given the estimated change points the ADF test is conducted for the respective subsamples. The p-values in Table 3.7 confirm the results of the tests for a change in persistence. Except for Japan in the original series and Italy the test detects a unit root in the first subsample and stationarity in the second subsample. This points to a change in persistence from $I(1)$ to $I(0)$ for France (in both series), the USA (in the original series), as well as Great Britain and Japan (in the adjusted series). Although the results for Italy are not as conclusive as for other countries, the p-values differ among the two subsamples. In the first subsample the null hypothesis can be rejected at the 10% level, whereas in the second subsample the p-value falls below 2%. Therefore we conclude that there occurs a change in persistence from $I(1)$ to $I(0)$ in the Italian series, which is also supported by the time series plot in Figure 3.12. In contrast, for Japan in the original series the p-values and the time series plot do not indicate a change in persistence. We deduce that the result of the Kim test is due to a type I error and that there is no change in persistence.

	FRA	JPN	USA
1st subsample	0.2625	0.0817	0.2176
2nd subsample	< 0.01	0.0980	< 0.01

(a) Original Series

	FRA	GBR	ITA	JPN
1st subsample	0.5386	0.2486	0.0741	0.3337
2nd subsample	0.0296	< 0.01	0.0195	0.0442

(b) Adjusted Series

Table 3.7: Subsample p-values of the ADF-Test

Summarizing our results we find different test decisions for the original and the adjusted series for four of the G7 countries. In Great Britain, Italy, and Japan the Leybourne test cannot detect a change in persistence in the original series due to outlier contaminations but confuses the series with a stationary process. After adjusting the series the Leybourne test rejects the null hypothesis in favor of a change in persistence from $I(1)$ to $I(0)$ which is supported by the results of the subsample ADF tests.

3.7 Conclusion

In this paper the effect of two different types of outliers on the performance of the tests for a change in persistence of Kim (2000); Kim et al. (2002) and of Leybourne et al. (2007b) are assessed. We find that the Kim test is not seriously affected by outliers. Especially the size of the test is not deteriorated. Due to the low degree of persistence under the null hypothesis of the test, AOs and IOs have the same effect on the series under the null hypothesis. The contaminated stationary process is identified as a stationary process and thus the size is not affected. Therefore, we conclude that it is not necessary to detect and remove outliers before applying the test. In contrast, the Leybourne test suffers from severe size and power distortions due to AOs. IOs do not affect the size but can even lead to power gains. As a result, we recommend to adjust the contaminated series and remove AOs before applying the test. The modified algorithm of Shin et al. (1996) performs well and is easy to implement. After adjusting the series, the size of the test coincides with the nominal significance levels and the power converges to 1 with an increasing sample size. In the empirical application we use the tests to find changes in persistence in the G7 inflation rates. We detect a change in persistence for France in the original and the adjusted series, and for Great Britain, Italy, and Japan after adjusting the series.

3.8 Appendix

3.8.1 Limiting Distribution

Suppose the core process of the data generating process is a random walk, which coincides with the null hypothesis of the Leybourne test,

$$x_t = x_{t-1} + \varepsilon_t, \qquad t = 1, \ldots, T,$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. The observable series $\{y_t\}$ is contaminated with AOs of magnitude ζ if $\delta_t = \pm 1$,

$$y_t = x_t + \zeta \delta_t.$$

For an AO at $t = s$, we obtain

$$y_{s-1} = x_{s-1} = x_{s-2} + \varepsilon_{s-1} \quad y_s = x_s + \zeta = x_{s-1} + \varepsilon_s + \zeta \quad y_{s+1} = x_{s+1} = x_s + \varepsilon_{s+1}.$$

Under the assumption of $\{y_t\}$ being a random walk, the residuals are given by

$$e_s = \varepsilon_s + \zeta \qquad e_{s+1} = \varepsilon_{s+1} - \zeta,$$

where $e_s \sim N(\zeta, \sigma_\varepsilon^2)$ and $e_{s+1} \sim N(-\zeta, \sigma_\varepsilon^2)$ (cf. Shin et al., 1996). The linear combination $e_{s+1} - e_s$ follows a normal distribution with $\mu = -2\,\zeta$ and $\sigma^2 = 2\sigma_\varepsilon^2$. If the random variable $X \sim N(\mu, \sigma^2)$, then $Z = |X|$ follows a folded normal distribution (cf. Leone et al., 1961), with the density function

$$f(z) = \frac{1}{\sqrt{2\pi\,\sigma^2}} \left[\exp\left(-\frac{(z-\mu)}{2\,\sigma^2} \right) + \exp\left(-\frac{(z+\mu)}{2\,\sigma^2} \right) \right].$$

Under the null hypothesis the series $\{y_t\}$ is uncontaminated and therefore $\zeta = 0$. Thus, $e_s, e_{s+1} \sim N(0, \sigma_\varepsilon^2)$, $e_{s+1} - e_s \sim N(0, 2\,\sigma_\varepsilon^2)$, and $|e_{s+1} - e_s|$ follows a folded normal distribution with density function

$$f(|e_{s+1} - e_s|) = \frac{1}{\sqrt{\pi\,\sigma_\varepsilon^2}} \exp\left(-\frac{(|e_{s+1} - e_s|)^2}{4\sigma_\varepsilon^2} \right).$$

This coincides with twice the right tail of the normal distribution $N(0, 2\sigma_\varepsilon^2)$.

The test statistic

$$\lambda^* = \frac{1}{\sqrt{2\,\hat{\sigma}_\varepsilon^2}} \left(|e_{t+1} - e_t| \right) \qquad t = 2, \ldots, (T-1),$$

where $\hat{\sigma}_{\varepsilon}^2$ is a robust estimator for the error term variance σ_{ε}^2, hence follows a standard folded normal distribution. Critical values can be obtained according to $q_{1-\alpha}^{\lambda^*} = z_{1-\alpha/2}$ for $\alpha \leq 0.5$, where z is a quantile of the standard normal distribution.

Figure 3.13 and Table 3.8 illustrate the convergence of the test statistic λ^* to the standard folded normal distribution.

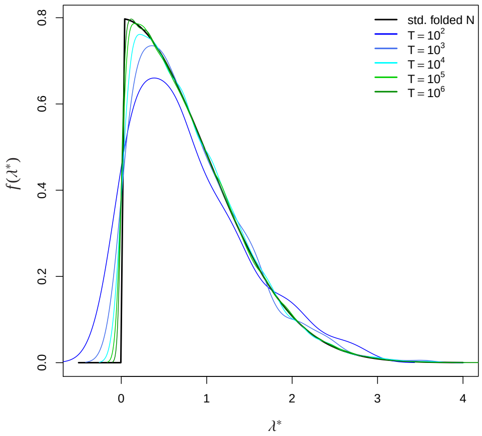


Figure 3.13: Estimated Density of λ^* and the Standard Folded Normal Distribution

	0.01	0.05	0.1	0.9	0.95	0.99
T $= 10^2$	0.0109	0.0412	0.0704	1.8610	2.2003	2.7312
T $= 10^3$	0.0109	0.0641	0.1323	1.6383	2.0642	2.6630
T $= 10^4$	0.0146	0.0681	0.1294	1.6431	1.9664	2.6194
T $= 10^5$	0.0129	0.0613	0.1242	1.6518	1.9667	2.5872
T $= 10^6$	0.0124	0.0629	0.1256	1.6448	1.9623	2.5773
std. folded N	0.0125	0.0627	0.1257	1.6448	1.9600	2.5758

Table 3.8: Quantiles of the Estimated Density of λ^* and of the Standard Folded Normal Distribution

Since it is not known a priori when an AO occurs, the maximum of the absolute difference between two consecutive residuals is taken,

$$\lambda = \frac{1}{\sqrt{2\,\hat{\sigma}_{\varepsilon}^2}} \left(\max_{2 \leq t \leq (T-1)} |e_{t+1} - e_t| \right).$$

According to the extreme value theory, the maximum of random variables from a distribution of the exponential family follows the Gumbel distribution (cf. Gumbel, 1958, pp. 164f, Kotz and Nadarajah, 2000, p. 59) with density function

$$f(x) = \frac{1}{\beta} \exp\left(-\frac{x-\mu}{\beta} + \exp\left(-\frac{x-\mu}{\beta} \right)\right).$$

However, the test statistic λ does not follow a standard Gumbel distribution ($\mu = 0$ and $\beta = 1$) since the standard Gumbel distribution allows for negative realizations (cf. Tab. 3.9), whereas the absolute does not. In order to determine appropriate critical values, we calculate λ for random walks of different sample sizes $T = \{10^2, 10^3, 10^4, 10^5\}$, each with 1000 replications. We find that the distribution of λ crucially depends on the sample size. For an increasing number of observations, the distribution shifts to the right and the quantiles increase (cf. Tab. 3.9).

	0.01	0.05	0.1	0.9	0.95	0.99
T = 10^2	2.1052	2.2251	2.3351	3.3154	3.5183	4.0632
T = 10^3	2.8462	2.9854	3.0545	3.8856	4.0141	4.3855
T = 10^4	3.4985	3.6176	3.6831	4.4230	4.6072	4.8394
T = 10^5	4.0520	4.1789	4.2331	4.8525	4.9963	5.3274
std. Gumbel	-1.5272	-1.0972	-0.8340	2.2504	2.9702	4.6001

Table 3.9: Quantiles of the Estimated Density of λ and of the Standard Gumbel Distribution

In addition to the sample size, the distribution of λ also depends on the number of iterations. If the test is applied more than once to the (adjusted) series, the distribution shifts to the left. For an increasing number of iterations (detection of the maximum in the adjusted series), the distribution asymptotically converges to the standard folded normal distribution. The estimated quantiles of λ based on 10000 replications for different sample sizes $T = \{100, 500, 1000\}$ and different numbers of iterations $\{1, 2, 9, 100\}$ are illustrated in Figure 3.14.

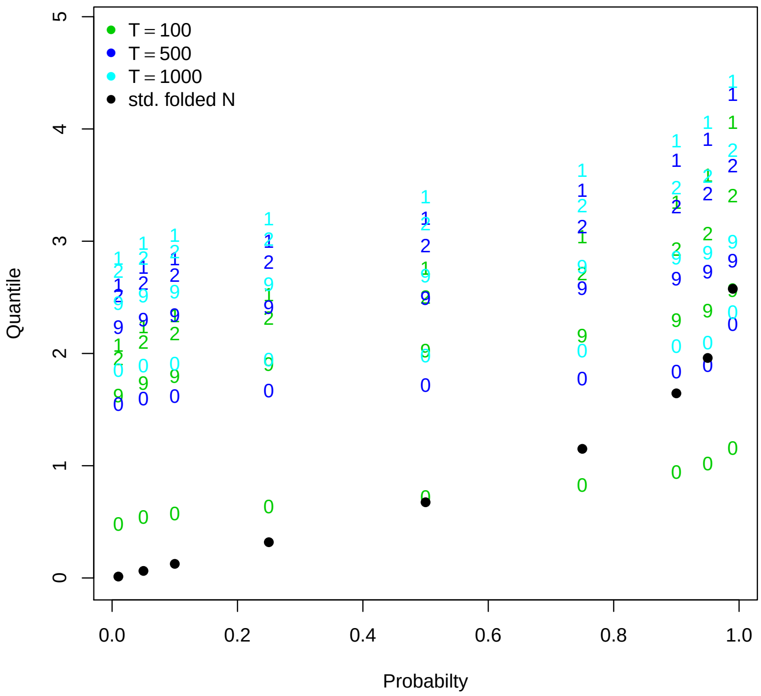
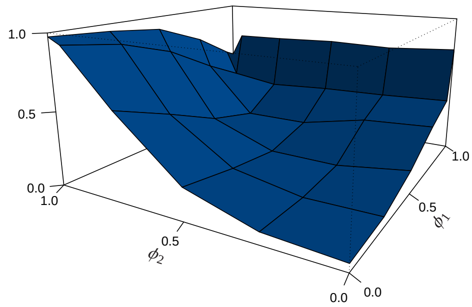


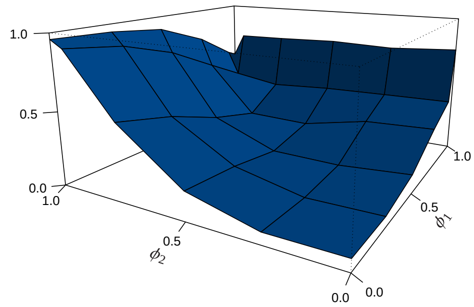
Figure 3.14: Estimated Quantiles of λ after Different Iterations

From Figure 3.14 we conclude that a critical value for λ cannot be derived from a limiting distribution since it is not clear beforehand how many iterations are needed to remove AOs from the series. Applying a large critical value reduces the risk of falsely identifying outliers, but may prevent the algorithm from detecting true outliers. In contrast, using a small critical value guarantees that outliers are correctly identified, but will also lead to spurious detection of outliers. The critical value of $C = 3$ recommended by Shin et al. (1996) seems to balance this trade-off. On the one hand the probability for a standard folded normal distributed random variable to exceed a value of $C = 3$ only amounts to 0.270%. Therefore, we do not expect the algorithm to detect many falsely classified outliers or to get stuck in an endless loop. On the other hand according to Figure 3.14 the critical value of $C = 3$ is small enough for the test not to be conservative.

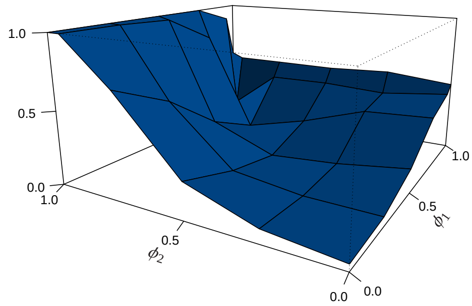
3.8.2 Power Plots



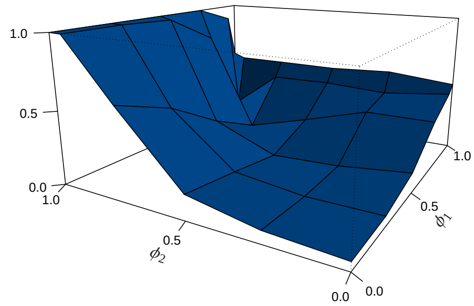
(a) Original Series, $T = 100$, $\zeta = 0$



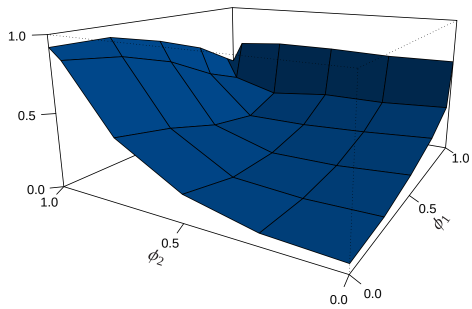
(b) Adjusted Series, $T = 100$, $\zeta = 0$



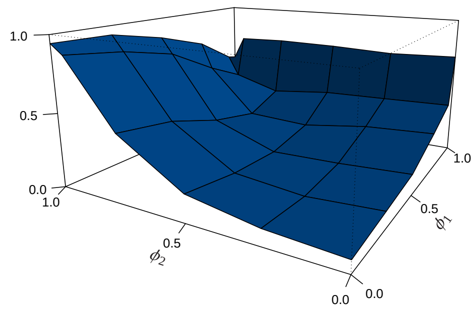
(c) Original Series, $T = 1000$, $\zeta = 0$



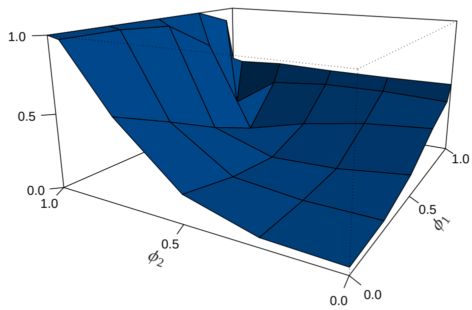
(d) Adjusted Series, $T = 1000$, $\zeta = 0$



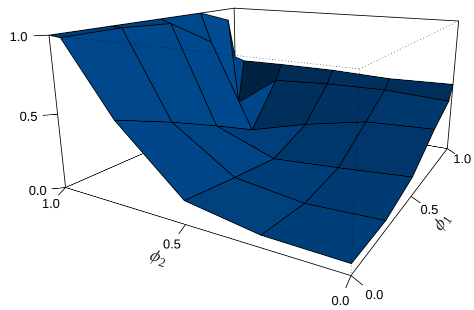
(e) Original Series, $T = 100$, $\zeta = 3$



(f) Adjusted Series, $T = 100$, $\zeta = 3$

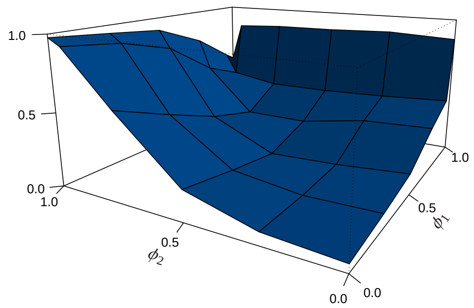


(g) Original Series, $T = 1000$, $\zeta = 3$

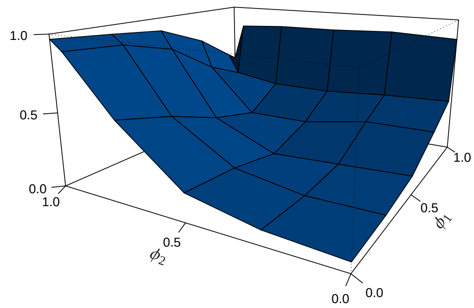


(h) Adjusted Series, $T = 1000$, $\zeta = 3$

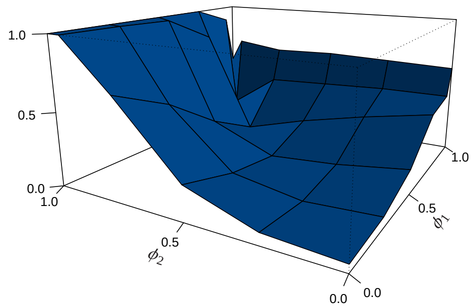
Figure 3.15: Power of the Kim Test for Additive Outliers, Different Degrees of Persistence, and $\alpha = 5\%$.



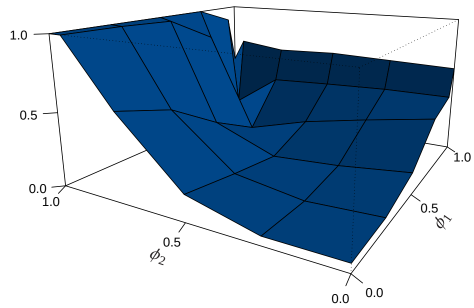
(a) Original Series, $T = 100$, $\zeta = 0$



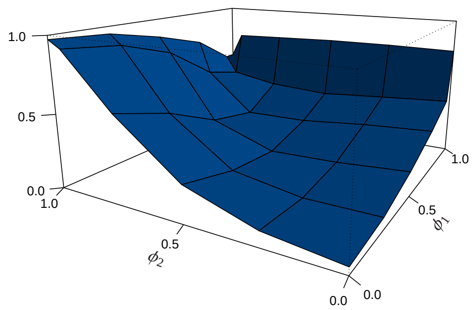
(b) Adjusted Series, $T = 100$, $\zeta = 0$



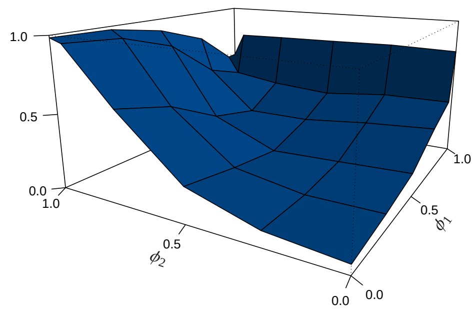
(c) Original Series, $T = 1000$, $\zeta = 0$



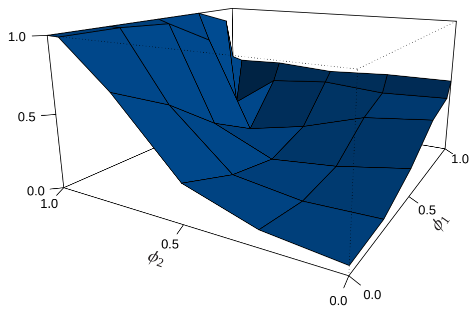
(d) Adjusted Series, $T = 1000$, $\zeta = 0$



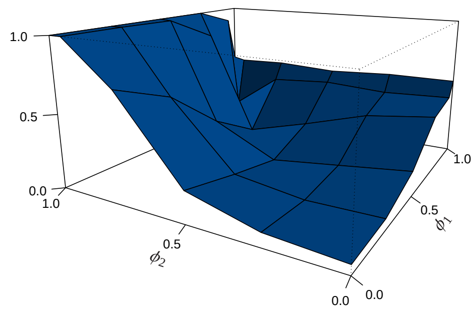
(e) Original Series, $T = 100$, $\zeta = 3$



(f) Adjusted Series, $T = 100$, $\zeta = 3$

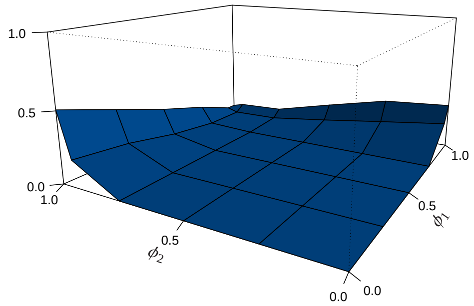


(g) Original Series, $T = 1000$, $\zeta = 3$

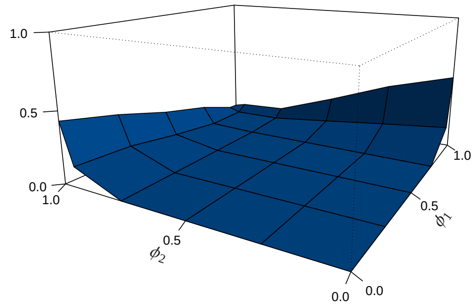


(h) Adjusted Series, $T = 1000$, $\zeta = 3$

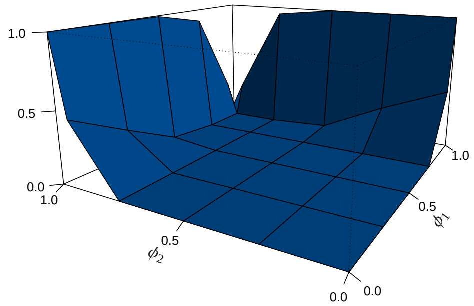
Figure 3.16: Power of the Kim Test for Innovative Outliers, Different Degrees of Persistence, and $\alpha = 5\%$.



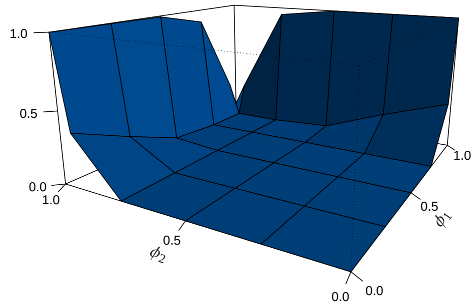
(a) Original Series, $T = 100$, $\zeta = 0$



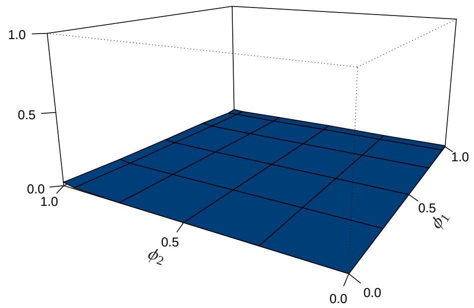
(b) Adjusted Series, $T = 100$, $\zeta = 0$



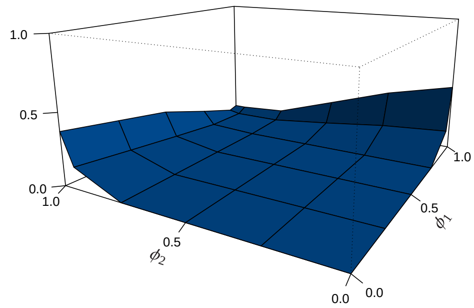
(c) Original Series, $T = 1000$, $\zeta = 0$



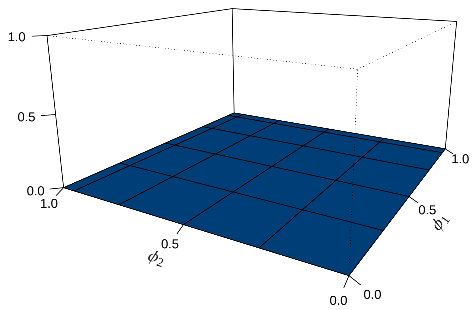
(d) Adjusted Series, $T = 1000$, $\zeta = 0$



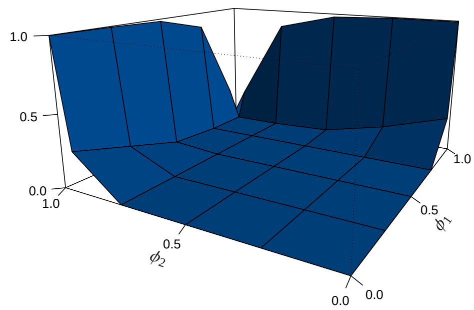
(e) Original Series, $T = 100$, $\zeta = 3$



(f) Adjusted Series, $T = 100$, $\zeta = 3$

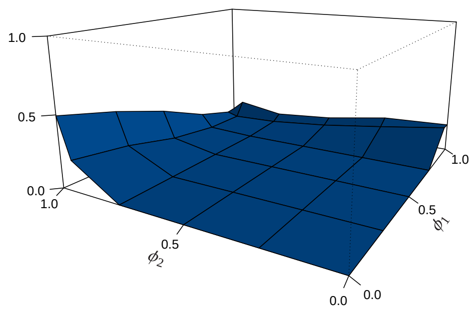


(g) Original Series, $T = 1000$, $\zeta = 3$

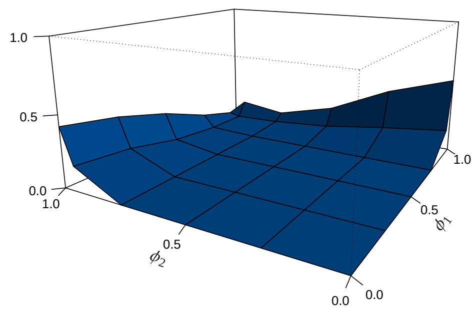


(h) Adjusted Series, $T = 1000$, $\zeta = 3$

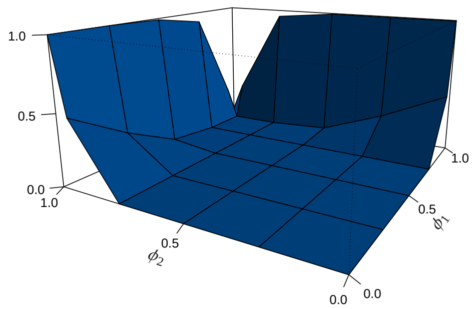
Figure 3.17: Power of the Leybourne Test for Additive Outliers, Different Degrees of Persistence, and $\alpha = 5\%$.



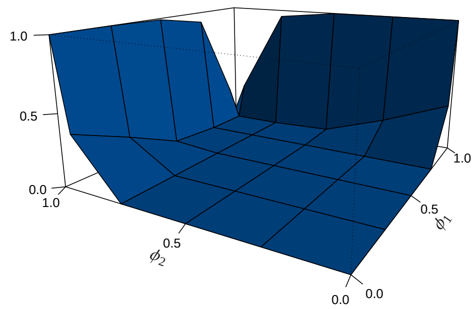
(a) Original Series, $T = 100$, $\zeta = 0$



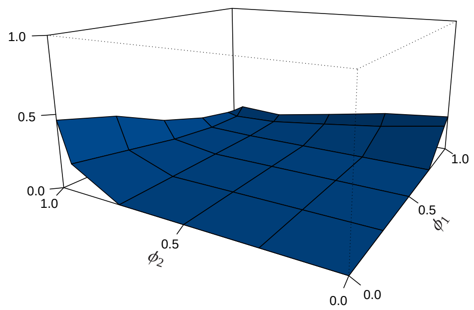
(b) Adjusted Series, $T = 100$, $\zeta = 0$



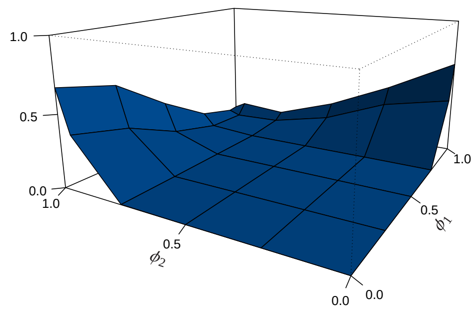
(c) Original Series, $T = 1000$, $\zeta = 0$



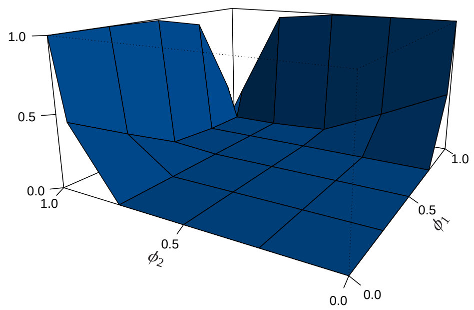
(d) Adjusted Series, $T = 1000$, $\zeta = 0$



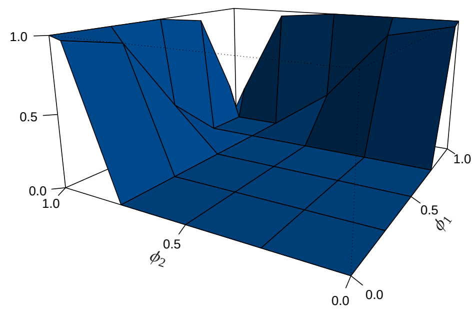
(e) Original Series, $T = 100$, $\zeta = 3$



(f) Adjusted Series, $T = 100$, $\zeta = 3$



(g) Original Series, $T = 1000$, $\zeta = 3$



(h) Adjusted Series, $T = 1000$, $\zeta = 3$

Figure 3.18: Power of the Leybourne Test for Innovative Outliers, Different Degrees of Persistence, and $\alpha = 5\%$.

Chapter 4

Squared CUSUM based Tests for a Unit Root

Co-authored with Philipp Sibbertsen.

4.1 Introduction

The presence of a unit root in economic and financial time series has crucial influence on the behaviour of testing procedures and getting meaningful results when drawing inference. The literature on testing for a unit root in parametric time series models is reviewed by Fuller (1976) and Dickey et al. (1986).

Many theoretical implications follow from the decision to model the level series or the first differences of the time series based on the testing results. Model building, hypotheses testing and forecasting are affected by different orders in limits of a unit root series, that is $O_p(T^{1/2})$, or a stationary process which is $O_p(1)$. Modelling non-stationary series is mostly done by ARIMA models with different orders of integration, while stationary processes are modelled by ARMA models.

In applications, the correct specification of a time series is required for optimal forecasting, conditional expectations or inference. Especially the determination of the integration order is of great interest. From a theoretical point of view it is important since limiting distributions of hypotheses tests differ with the integration order of the time series. From a practical point of view, time series with different orders of integration contain different information about the behaviour of the series.

In the literature, the presence of stationarity in financial and economic time series is questionable and therefore statistical tests for detecting a unit root are typically applied in financial time series such as stock prices (cf. Kasa (1992) and Chan et al. (1997)), real exchange rates (cf. MacDonald (1996) and Smith et al. (2004)), dividends (cf. Campbell and Shiller (1988) and Campbell and Shiller (1989)) or economic time series such as the GNP (cf. Stock and Watson (1986) and Perron and Phillips (1987)), inflation rates (cf. Henry and Shields (2004) and Basher and Westerlund (2008)) or purchasing power parity (cf. Oh (1996) and Coakley and Fuertes (1997)). One of the most popular data set for the comparison of different unit root tests is the historical macroeconomic Nelson-Plosser data from Nelson and Plosser (1982).

The main problem of all unit root tests is the low power when the alternative hypothesis is rather close to the null hypothesis since the autoregressive parameter is very close to but lower than unity. DeJong et al. (1992b) analyse the problems of unit root tests in this setup. Hence, the tests suffer from severe size distortions when a large negative moving-average parameter is present in the time series and are undersized if the process includes a positive moving-average parameter (cf. Perron and Ng (1996) and Schwert (2002)). Hassler and Wolters (1994) investigate the properties of unit root tests when applied to fractionally integrated time series and find that the tests reject the null hypothesis too often. The problem of a unit root in the moving-average component is discussed by Plosser and Schwert (1977) and can be caused by falsely differencing the time series to remove non-stationarity.

The most popular unit root tests are based on the estimation of the autoregressive parameter or the associated t-ratio statistic. These tests are developed by Fuller (1976), Dickey and Fuller (1979) and Dickey and Fuller (1981). The testing procedures are extended in several ways. A non-parametric standardisation of the tests is presented by Phillips (1987) and Phillips and Perron (1988) to allow for a wide class of weakly dependent and heterogeneously distributed data and Said and Dickey (1984) extend the former tests to high-order autoregressive models. Several procedures and modifications exist to test for a unit root in panel data with cross-dependencies (cf. Maddala and Wu (1999) and Pesaran (2007)), in time series with a structural change (cf. Perron (1990)), in nonlinear time series (cf. Kapetanios et al. (2003)), in outlier contaminated processes (cf. Franses and Haldrup (1994)) or fractionally integrated time series (cf. Sowell (1990)).

This paper provides a very new approach to test for a unit root. Two unit root tests are developed based on standardised squared cumulative sums (CUSUMs) of ordinary least squares (OLS) residuals. Instead of estimating the autoregressive parameter or calculating the t-ratio statistic as in published tests, the sum of squares of all partial sums of the time series is calculated. Therefore, the presented tests use the fact that the persistence of a time series is reflected in the squared sum of consecutive observations. This also translates to the squared partial sums of different order. We derive consistency and the limiting distributions of the tests. In an extensive Monte Carlo simulation study we show that the proposed procedure works well in finite samples.

The rest of the paper is organised as follows. Section 4.2 describes the model along with the assumptions. In Section 4.3 the most commonly used unit root tests are presented. Section 4.4 discusses the new squared CUSUMs of residuals based tests for a unit root. The limiting distributions of the tests are derived and consistency is shown. A Monte Carlo study is conducted in Section 4.5 which shows considerable size and power improvements of the new testing approaches compared to the existing ones and Section 4.6 provides an empirical application to the historical Nelson-Plosser data.

4.2 Model and Assumptions

To study a stochastic process $\{y_t\}$, with the null hypothesis $H_0 : y_t \sim I(1)$ that the process is non-stationary against the stationary alternative hypothesis $H_1 : y_t \sim I(0)$, consider the autoregressive model

$$y_t = \rho y_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T. \tag{4.1}$$

In this model the autoregressive parameter ρ is a real number and the process $\{y_t\}$ is called stationary if $|\rho| < 1$ as $t \to \infty$. In the case that $|\rho| = 1$, the time series is not stationary and called a random walk with variance $t\sigma^2$. For $|\rho| > 1$, the process shows explosive behaviour with exponentially growing variance as t increases.

In the following, it is assumed that y_0 has a fixed distribution independent of the sample size T including other proposals by White (1958), where $y_0 = c$ is a constant with probability one and the commonly used case $y_0 = 0$. This assumption provides more flexibility and the consideration of explosive time series.

The considered models in the present paper are generated by a sequence of innovations $\{\varepsilon_t\}$. Following Phillips (1987), Phillips and Perron (1988), Phillips and Solo (1992) and others, the innovation process satisfies the following general conditions:

(a) $E(\varepsilon_t) = 0 \quad \forall t$;

(b) $\sup_t E|\varepsilon_t|^{\gamma+\epsilon} < \infty$ for some $\gamma > 2$ and $\epsilon > 0$;

(c) The long-run variance $\omega^2 = \sum_{j=0}^{\infty} E[\varepsilon_{j+1}\varepsilon_1']$ exists and $\omega^2 > 0$;

(d) $\{\varepsilon_t\}$ is strong mixing with mixing coefficients α_m such that $\sum_{m=1}^{\infty} \alpha_m^{1-2/\gamma} < \infty$.

These conditions include all Gaussian and stationary finite order ARMA models, see Withers (1981), and allow a wide class of weakly dependent and heterogeneously distributed time series, see Phillips (1987).

The first condition (a) ensures that the expectation of the innovation ε_t is zero. The allowable heterogeneity of the process is controlled in condition (b) by restricting unlimited growth in the βth absolute moments of ε_t. Condition (c) is a conventional requirement on the average variance of the partial sums. If $\{\varepsilon_t\}$ is a weakly stationary process, then the result

$$\omega^2 = E(\varepsilon_1^2) + 2\sum_{k=2}^{\infty} E(\varepsilon_1 \varepsilon_k)$$

follows directly from conditions (b) and (d). The mixing decay rate is controlled by condition (d) in relation to the probability of outliers. For a detailed discussion of the assumptions, the reader is referred to Phillips (1987).

These are the common assumptions for the innovation sequence when analysing a non-stationary process, for example a process including a unit root, and are necessary to derive the limiting distributions of the test statistics. Hence, it is assumed in the following, that the innovation sequence $\{\varepsilon\}$ satisfies assumptions $(a) - (d)$.

The class of models under investigation includes the autoregressive model without a mean and trend (4.1), the model

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{4.2}$$

including an autoregressive parameter ρ and the mean parameter μ and the model

$$y_t = \mu + \beta t + \rho y_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{4.3}$$

including an autoregressive term ρ, the mean μ and a linear trend with parameter β.

In the following study, the limiting distributions of the test statistics are derived for the respective model. The analysis is based on model (4.2), where the alternative is a level-stationary process and model (4.3) with the alternative of a trend-stationary process.

4.3 Existing Unit Root Tests

4.3.1 Dickey-Fuller Test

The standard Dickey and Fuller (1979) test (DF-test) is developed in the three univariate autoregressive models of order one. These tests are based on the estimation of the autoregressive parameter ρ and consider the ordinary least squares (OLS) estimator to test the hypothesis of a non-stationary process against the stationary alternative in the three least-squares regression equations for model (4.1) without intercept and trend, the de-meaned model (4.2) and the de-meaned and de-trended model (4.3)

$$y_t = \check{\rho} y_{t-1} + \check{\varepsilon}_t \qquad t = 1, \ldots, T, \tag{4.4}$$

$$y_t = \hat{\mu} + \hat{\rho} y_{t-1} + \hat{\varepsilon}_t \qquad t = 1, \ldots, T, \tag{4.5}$$

$$y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\rho} y_{t-1} + \tilde{\varepsilon}_t \qquad t = 1, \ldots, T, \tag{4.6}$$

where $\check{\rho}$, $(\hat{\mu}, \hat{\rho})$ and $(\tilde{\mu}, \tilde{\beta}, \tilde{\rho})$ are the least squares regression estimates to test the null hypothesis $\check{\rho} = 1$, $(\hat{\mu}, \hat{\rho}) = (0, 1)$ and $(\tilde{\mu}, \tilde{\beta}, \tilde{\rho}) = (0, 0, 1)$.

The least squares estimator of $\check{\rho}$ in (4.4) is the maximum likelihood estimator and is given by

$$\check{\rho} = \left(\sum_{t=1}^{T} y_{t-1}^2 \right)^{-1} \sum_{t=1}^{T} y_t y_{t-1}.$$

White (1958) derives the limiting distribution of the least squares estimator as a ratio of integrals over Wiener processes and Dickey and Fuller (1979) generalize the results to models containing an intercept or an intercept and trend. They consider two different non-stationarity tests. The first one is based on the least squares estimator

$$T(\check{\rho} - 1) = \frac{T^{-1} \sum_{t=1}^{T} y_{t-1} \varepsilon_t}{T^{-2} \sum_{t=1}^{T} y_{t-1}^2},$$

with the corresponding equivalents for model (4.5) and model (4.6) by replacing the maximum likelihood estimator $\check{\rho}$ with $\hat{\rho}$ or $\tilde{\rho}$.

The second one is the associated t-ratio given by

$$t_{\check{\rho}} = \frac{T(\check{\rho} - 1)}{\check{s}},$$

for model (4.4), where $\check{s}$ denotes the standard error of the regression in (4.4). Consistent estimates of the short-run variance, $s^2 = T^{-1} \sum_{t=1}^{T} (y_t - y_{t-1})^2$, are denoted by $\check{s}^2$, $\hat{s}^2$ and $\tilde{s}^2$ within the associated regression.

The t-ratios for model (4.5) and model (4.6) are given by

$$t_{\hat{\rho}} = (\hat{\rho} - 1)\left(\sum_{t=1}^{T}(y_{t-1} - \bar{y})^2\right)^{\frac{1}{2}} / \hat{s},$$
$$t_{\tilde{\rho}} = (\tilde{\rho} - 1)/(\tilde{s}^2 c_3)^{\frac{1}{2}},$$

with the standard error from the corresponding least squares regressions, $\bar{y} = T^{-1}\sum_{t=1}^{T} y_t$ and c_i is the i-th diagonal element of the matrix $(X'X)^{-1}$, where X is the $T \times 3$ matrix of explanatory variables.

Dickey and Fuller (1979) show the non-stationarity of the t-ratio statistics under the null hypothesis. Furthermore, they derive the limiting distribution which is not t-distributed but follows the Dickey-Fuller distribution. The standard DF-test is based on independently and identically distributed (iid) errors. When a time series consists of correlated errors, another regression method should be adopted or the test statistic has to be modified to obtain consistent estimators and statistics.

The augmented Dickey-Fuller test (ADF-test) developed by Dickey and Fuller (1981) extends the approach by changing the regression method with an OLS regression of y_t on l first differences of lags $\Delta y_{t-l} = y_{t-l} - y_{t-1-l}$. They consider different likelihood ratio statistics and derive their limiting distributions. The regression model without constant and linear trend is given by

$$y_t = \rho y_{t-1} + \sum_{j=1}^{l} \phi_j \Delta y_{t-j} + \varepsilon_t, \quad t = 1, \ldots, T \tag{4.7}$$

and new tests can be build with the OLS estimator from Equation (4.7) or the associated t-ratio. Said and Dickey (1984) show that the limiting distribution of the test statistic based on the t-ratio $(t(\hat{\rho}^*))$ is the same as the DF-test when the lag length $l \to \infty$.

A method for testing the hypothesis $d = 1$ in the ARIMA(p,d,q) model when p and q are known is discussed by Dickey and Said (1981). Because the true order of p and q is unknown, Said and Dickey (1984) and Said and Dickey (1985) present a method to approximate an ARIMA(p,d,q) with an autoregression based on the number of observations, where it is possible to test the null hypothesis $d = 1$ without knowing the order of p or q. Additionally, the resulting statistics follow the same limiting distribution as those tabulated by Fuller (1976). The ADF-test corrects for serial correlation in the error terms in the presence of moving-average coefficients. Schwert (2002) shows in a Monte Carlo simulation study that the drawback is the existing tradeoff between power and size in the choice of the number of included lags l. Including too many lags l in the regression results in a loss of power, whereas the probability of not rejecting the null hypothesis of a unit root increases if l is low. Ng and Perron (2001) discussed the optimal lag length selection of l in unit root tests.

4.3.2 Phillips-Perron Test

Non-parametric modifications of the unit root tests to correct for the autocorrelation in the residuals are proposed by Phillips (1987) and Phillips and Perron (1988). The transformations of the test statistics by Dickey and Fuller (1979) and Dickey and Fuller (1981) allow the effect of serial correlation and heterogeneous distribution of the innovation. In their study, they make use of the assumptions (a)-(d). The test statistics for model (4.2) with intercept are the transformations of the Dickey-Fuller tests and are given by

$$Z(\hat{\rho}) = T(\hat{\rho} - 1) - \hat{\lambda}/\bar{m}_{yy},$$
$$Z(t_{\hat{\rho}}) = (\hat{s}/\hat{\omega})t_{\hat{\rho}} - \hat{\lambda}'\hat{\omega}/\bar{m}_{yy}^{\frac{1}{2}},$$

where

$$\bar{m}_{yy} = T^{-2}\sum_{t=1}^{T}(y_t - \bar{y})^2, \quad \hat{\lambda} = \frac{1}{2}(\hat{\omega}^2 - \hat{s}^2), \quad \hat{\lambda}' = \hat{\lambda}/\hat{\omega}^2$$

and

$$\hat{\omega}^2 = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_t^2 + 2T^{-1}\sum_{s=1}^{l}w_{s,l}\sum_{t=1}^{T}\hat{\varepsilon}_t\hat{\varepsilon}_{t-s},$$

where $w_{s,l}$ is an optimal weighting function and l is called lag truncation parameter and is the number of estimated autocovariances. They follow Newey and West (1987) and choose the Bartlett window $w_{s,l} = 1 - s/(l+1)$.

Considering model (4.3) with intercept and trend the corresponding transformations of the test statistics are

$$Z(\tilde{\rho}) = T(\tilde{\rho} - 1) - \tilde{\lambda}/M,$$
$$Z(t_{\tilde{\rho}}) = (\tilde{s}/\tilde{\omega})t_{\tilde{\rho}} - \tilde{\lambda}'\tilde{\omega}/M^{\frac{1}{2}},$$

where

$$m_{yy} = T^{-2}\sum_{t=1}^{T}y_t^2, \quad m_{ty} = T^{-5/2}\sum_{t=1}^{T}y_t, \quad \tilde{\lambda} = \frac{1}{2}(\tilde{\omega}^2 - \tilde{s}^2), \quad \tilde{\lambda}' = \tilde{\lambda}/\tilde{\omega}^2,$$
$$M = (1 - T^{-2})m_{yy} - 12m_{ty}^2 + 12(1 + T^{-1})m_{ty}m_y - (4 + 6T^{-1} + 2T^{-2})m_y^2,$$

with

$$\tilde{\omega}^2 = T^{-1}\sum_{t=1}^{T}\tilde{\varepsilon}_t^2 + 2T^{-1}\sum_{s=1}^{l}w_{s,l}\sum_{t=1}^{T}\tilde{\varepsilon}_t\tilde{\varepsilon}_{t-s}.$$

The standard error of the regression, $\hat{s}$ and $\tilde{s}$, are replaced by estimators of the long-run variance, $\hat{\omega}$ and $\tilde{\omega}$. With this approach, the nuisance parameter dependencies are asymptotically eliminated and the regression statistics are corrected to allow for serial correlation and a heterogeneous distribution of the innovations. The transformation of the Z statistics include the additive correction which depends on the difference between consistent estimators of the long-run and the short-run variances, $\hat{\omega}^2 - \hat{s}^2$ and $\tilde{\omega}^2 - \tilde{s}^2$.

Phillips and Perron (1988) show that the limiting distributions of the Z statistics are invariant within a wide class of weakly dependent and heterogeneously distributed innovations. Furthermore, the limiting distributions of the Z statistics are identical to the limiting distributions of the original untransformed statistics and therefore, the critical values derived by Dickey and Fuller (1979) and Dickey and Fuller (1981) are still valid under the more general conditions. The ADF-test is more sensitive to the lag length selection l compared to the PP-tests to the number of estimated autocovariances l. A guideline for the optimal choice of l is presented by Perron and Ng (1996).

Leybourne and Newbold (1999) compare the DF-test and the different versions of the PP-tests in second-order autoregressive models with the main findings that the probabilities of rejecting the null hypothesis can differ substantially. The tests reported in the present study, $Z(\hat{\rho})$, $Z(t_{\hat{\rho}})$, $Z(\tilde{\rho})$, $Z(t_{\tilde{\rho}})$, are the only variants comparable in terms of rejection frequencies to the DF-test. Many studies analyse the power and size properties of the PP-tests. In general, the findings show that the tests suffer from size distortions with a large MA term. The tests are conservative with a positive MA component and liberal with high rejection frequencies, when a negative MA coefficient is present in the data (cf. Schwert (1987), DeJong et al. (1992b)). Perron and Ng (1996) find evidence for higher size distortions with increasing MA coefficients. They modify the test statistic with a kernel based spectral density estimator with better size properties in ARMA models but a loss in power.

Several modifications of the ADF-test and the PP-tests exist but there is always a tradeoff between power and size and moreover many modifications are only used in specific problems. Still, these tests are the most applied unit root tests in applications with reasonable size and power properties and are therefore considered as the benchmark in our simulation study.

4.4 Squared CUSUM based Unit Root Tests

In the following section, two different test statistics are presented to test the null hypothesis of a non-stationary process against the alternative of a stationary process. The tests are based on standardised squared CUSUMs of OLS residuals. In general, there exist an extensive literature about tests based on CUSUMs of residuals to test for a change in persistence, see Brown et al. (1975) or McCabe and Harrison (1980) for an overview. Standard CUSUM persistence change tests are based on the partial sum process $S_t = \sum_{i=1}^{t} \varepsilon_i$ and the sum of squared S_t for all t in the respective sub-sample.

In contrast, the presented CUSUM tests consider all possible partial sums that can be formed from the time series. Therefore, another partial sum process with other asymptotics is considered not starting with $i = 1$ in S_t.

Specific tests based on squared CUSUMs of residuals to test for a unit root are not yet considered in the literature, although the existing persistence change tests have non-trivial power against constant $I(1)$ alternatives (cf. Leybourne et al. (2007b)). This property motivates the development of the following tests. We discuss the idea of the test statistics, the behaviour of the tests under the null hypothesis and the alternative, derive the limiting distribution and show consistency of the tests.

The approach in this paper is different to common tests for a unit root as it is not based on the least squares regression coefficients $\hat{\rho}$ in model (4.2) or $\tilde{\rho}$ in model (4.3). Furthermore, the tests are not another standardisation of the OLS estimator based test statistics or t-ratio statistics.

Unit Root Test Q_y

The first test statistic is based on the sum of all possible squared partial sums of the de-meaned or de-meaned and de-trended observations. Consider the following test statistic:

$$Q_y = \left(\frac{T^{-5} \sum_{t=1}^{T} \sum_{j=t}^{T} \left(\sum_{i=t}^{j} \hat{y}_i \right)^2}{\hat{\omega}^2} \right)^{-1},$$

where $\hat{y}$ are the residuals from the regression of y_t on a constant ($z_t = 1$) or on a constant and linear time trend ($z_t = (1, t)'$). For example, in the constant case

$$\hat{y}_t = y_t - \bar{y}_t \quad \text{with} \quad \bar{y} = T^{-1} \sum_{t=1}^{T} y_t.$$

In the denominator $\hat{\omega}^2$ is an estimator of the long-run variance ω^2. In the following the estimator

$$\hat{\omega}^2 = \hat{\gamma}_0 + 2\sum_{s=1}^{l} w_{s,l}\hat{\gamma}_s, \quad \hat{\gamma}_s = T^{-1}\sum_{t=1}^{T} \Delta\hat{y}_t\Delta\hat{y}_{t-s}, \quad w_{s,l} = 1 - s/(l+1),$$

is adopted with the lag truncation parameter l and the Bartlett window $w_{s,l}$. This estimator is used to take into account the possible serial correlation in ARMA models following Phillips (1987) and Phillips and Perron (1988).

Define the partial sum process

$$S_{t,j} = \sum_{i=t}^{j} y_i \quad \forall t, j \geq t.$$

Then the test statistic can be rewritten and the behaviour of the test can be analysed in terms of partial sums. The rewritten test statistic is given by

$$Q_y = \left(\frac{T^{-5}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2}{\hat{\omega}^2}\right)^{-1}.$$

Analysing the partial sum process $S_{t,j}$ in detail yields two different interpretations. The interpretations are based on $S_{t,j}^2$ for all t and $j \geq t$. The partial sum processes are given by:

$$\begin{aligned}
S_{1,1}^2 + \ldots + S_{1,T}^2 &= y_1^2 + (y_1 + y_2)^2 + (y_1 + y_2 + y_3)^2 + \ldots + (y_1 + \ldots + y_T)^2 \\
S_{2,2}^2 + \ldots + S_{2,T}^2 &= y_2^2 + (y_2 + y_3)^2 + (y_2 + y_3 + y_4)^2 + \ldots + (y_2 + \ldots + y_T)^2 \\
S_{3,3}^2 + \ldots + S_{3,T}^2 &= y_3^2 + (y_3 + y_4)^2 + (y_3 + y_4 + y_5)^2 + \ldots + (y_3 + \ldots + y_T)^2 \\
&\vdots \\
S_{(T-2),(T-2)}^2 + \ldots + S_{(T-2),T}^2 &= y_{T-2}^2 + (y_{T-2} + y_{T-1})^2 + (y_{T-2} + y_{T-1} + y_T)^2 \\
S_{(T-1),(T-1)}^2 + S_{(T-1),T}^2 &= y_{T-1}^2 + (y_{T-1} + y_T)^2 \\
S_{T,T}^2 &= y_T^2
\end{aligned}$$

The first interpretation is based on the horizontal equations of the matrix. For each t a new part of a Brownian motion is started in $S_{t,T}$ from t to T. While common squared CUSUMs based tests for a change in persistence only consider the first equation, this test calculates the sum of all squared partial sum processes and therefore the sum of all Brownian motions starting at different t.

Another interpretation is based on the vertical equations of the matrix. In this test are not only the first single observation, the first tuple, the first triple, . . . , considered. Instead, each consecutive sequence of observations is considered, that is all single observations, all consecutive two combinations, all triples, and so on.

This exploits the behaviour of a non-stationary process which is expected to have large positive or negative consecutive observations. The squared sum of each single path of the trajectory of the process is calculated, where $S_{t,j}$ is of different magnitude for a non-stationary or a stationary process.

Theorem 4.4.1 provides the limiting distribution of Q_y under the null hypothesis.

Theorem 4.4.1. Let $\{y_t\}$ be generated by model (4.2) or model (4.3) under the null hypothesis H_0 and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let $W(s)$ denote a standard Brownian motion process on $[0,1]$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($z_t = 1$) and the de-meaned and de-trended case ($z_t = (1,t)'$). Then, provided that as $T \to \infty$, it is true that

$$T^{-5}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2 \Rightarrow \omega^2 \int_0^1 \int_b^1 \left(\int_b^a W_\zeta(s)ds \right)^2 dadb,$$

where the symbol $\Rightarrow$ signifies weak convergence of the associated probability measures. In the de-meaned case ($\zeta = 0$)

$$W_0(s) = W(s) - \int_0^1 W(r)dr$$

and in the de-meaned and de-trended case ($\zeta = 1$)

$$W_1(s) = W(s) + (6s-4)\int_0^1 W(r)dr + (-12s+6)\int_0^1 rW(r)dr.$$

Proof of Theorem 4.4.1. The limiting distribution of the test statistic under the null hypothesis, assuming $\rho = 1$, can be obtained with the numerator of the rewritten test statistic

$$T^{-5}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2.$$

Starting with $t = 1$ and the well known results

$$T^{-1/2}y_{[sT]} = T^{-1/2}\sum_{i=1}^{[sT]} \varepsilon_i \Rightarrow \omega W(s),$$
$$T^{-1/2}\bar{y} = T^{-3/2}\sum_{t=1}^{T} y_t \Rightarrow \omega \int_0^1 W(s)ds,$$

where $s \in [0,1]$ and $[\cdot]$ denotes the integer part of its argument, the partial sum process $S_{1,j}$ follows similar to Kwiatkowski et al. (1992b)

$$\begin{aligned} T^{-3/2}S_{1,[aT]} &= T^{-3/2}\sum_{i=1}^{[aT]}(y_i - \bar{y}) \\ &= T^{-1}\sum_{i=1}^{[aT]}T^{-1/2}y_i - ([aT]/T)T^{-1/2}\bar{y} \\ &\Rightarrow \omega\int_0^a W(s)ds - a\omega\int_0^1 W(s)ds = \omega\int_0^a W_\varsigma(s)ds. \end{aligned}$$

With this result the sum of partial sum processes with $t = 1$ follows with the continuous mapping theorem (CMT) following Chan and Wei (1988)

$$T^{-4}\sum_{j=1}^{T}S_{1,j}^2 = T^{-1}\sum_{j=1}^{T}(T^{-3/2}S_{1,j})^2 \Rightarrow \omega^2\int_0^1\left(\int_0^a W_\varsigma(s)ds\right)^2 da.$$

Deriving the limiting distribution of Q_y with the partial sum process $S_{t,j}$ for all t is straightforward. The partial sum process $S_{t,j}$ with $a \in [0,1]$ and $b \in [0,1]$ follows

$$\begin{aligned} T^{-3/2}S_{bT,aT} &= T^{-3/2}\sum_{i=[bT]}^{aT}(y_i - \bar{y}) \\ &= \left(T^{-1}\sum_{i=1}^{[aT]}y_i - ([aT]/T)T^{-1/2}\bar{y}\right) - \left(T^{-1}\sum_{i=1}^{[bT]}y_i - (([bT])/T)T^{-1/2}\bar{y}\right) \\ &\Rightarrow \omega\int_0^a W(s)ds - a\omega\int_0^1 W(s)ds - \omega\int_0^b W(s)ds + b\omega\int_0^1 W(s)ds \\ &= \omega\int_b^a W(s)ds - (a-b)\omega\int_0^1 W(s)ds \\ &= \omega\int_b^a W_\varsigma(s)ds. \end{aligned}$$

Therefore,

$$T^{-5}\sum_{t=1}^{T}\sum_{j=t}^{T}S_{t,j}^2 \Rightarrow \omega^2\int_0^1\int_b^1\left(\int_b^a W_\varsigma(s)ds\right)^2 dadb.$$

□

Critical values of the test statistic Q_y for $z_t = 1$ and $z_t = (1, t)'$ are provided in Table 4.1 and are calculated via a direct simulation for sample sizes of 50, 100, 250, 500 and 1000 with $1,000,000$ replications. The test rejects for large values of the test statistic but can also be used to test the alternative hypothesis of an explosive time series with critical values from the left side of the limiting distribution. The critical values show good behaviour even for the small sample sizes and especially the quartiles of the distribution are very similar in each sample size.

Note that a small sample correction of the test statistic is possible. Assume that the process is generated by $y_t = y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0, 1)$. Then it is easy to show that the expected value of the sum of partial sums under the null hypothesis without mean and trend is given by

$$E\left[\sum_{t=1}^{T}\sum_{j=t}^{T}\left(\sum_{i=t}^{j} y_i\right)^2\right] = (T+1)((T+1)^4 - 1)/30.$$

This small sample correction results in smaller critical values and can lead to an improvement of the behaviour in finite samples but is asymptotically negligible.

z_t	T	0.01%	0.05%	0.10%	0.25%	0.50%	0.75%	0.90%	0.95%	0.99%
1	50	72.88	116.11	156.71	272.21	551.13	1221.99	2604.60	4101.13	9192.17
	100	71.69	115.75	156.65	272.05	551.34	1226.75	2625.06	4130.19	9262.34
	250	70.74	115.71	155.83	271.81	551.03	1228.52	2638.70	4150.09	9303.81
	500	70.53	115.64	155.12	271.62	551.21	1234.58	2653.14	4164.51	9351.20
	1000	70.40	115.42	155.10	271.40	551.22	1240.31	2675.46	4203.88	9380.34
(1,t)'	50	242.74	395.77	533.65	925.79	1867.67	3815.11	7407.23	10798.52	21300.62
	100	230.73	380.14	515.33	910.42	1822.06	3789.23	7317.88	10764.07	21271.39
	250	225.16	371.15	504.49	894.67	1806.98	3772.49	7314.54	10757.42	21259.10
	500	224.45	369.53	501.84	890.78	1793.87	3739.57	7262.33	10704.41	21211.03
	1000	221.77	367.64	499.32	888.12	1789.09	3734.97	7240.48	10670.08	21197.12

Table 4.1: Critical Values of the Q_y Test for $T = 50, 100, 250, 500, 1000$.

Theorem 4.4.2 provides the limiting distribution of Q_y under the alternative and shows consistency of the test statistic.

Theorem 4.4.2. **(i)** Let $\{y_t\}$ be generated by model (4.2) or model (4.3) under the alternative hypothesis H_1, that is $\rho = 0$, and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($z_t = 1$) and the de-meaned and de-trended case ($z_t = (1, t)'$). Then, provided that as $T \to \infty$, it is true that

$$T^{-3}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2 \Rightarrow \omega^2 \int_0^1 \int_a^1 V_\zeta(s)^2 ds da,$$

where $V_0(s)$ is a standard Brownian bridge

$$V_0(s) = W(s) - sW(1)$$

and $V_1(s)$ is a second level Brownian bridge

$$V_1(s) = W(s) + (2s - 3s^2)W(1) + (-6s + 6s^2)\int_0^1 W(r)dr.$$

Therefore, under the alternative hypothesis

$$T^{-5}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2 \Rightarrow O_p(T^2).$$

(ii) Under the alternative hypothesis H_1, when $\{y_t\}$ is generated by model (4.2) or model (4.3), it is true that

$$Q_y \Rightarrow O_p(T^2).$$

Theorem 4.4.2 provides that Q_y converges to integrals of Brownian bridges with standardisation T^{-3}. This shows that the test statistic diverges under the alternative and is of $O_p(T^2)$. Therefore, a consistent test for the unit root hypothesis is derived.

Proof of Theorem 4.4.2. The limiting distribution of the test statistic under the alternative hypothesis, assuming $\rho = 0$, can be obtained with the numerator of the rewritten test statistic

$$T^{-3}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2,$$

with the partial sum process

$$S_{t,j} = \sum_{i=t}^{j} y_i \quad \forall t, j \geq t.$$

When $\rho = 0$, starting with $t = 1$, it follows from Kwiatkowski et al. (1992b) that

$$T^{-2}\sum_{j=1}^{T} S_{1,j}^2 \Rightarrow \omega^2 \int_0^1 V_\zeta(s)^2 ds.$$

This is a special result from McCabe and Leybourne (1988) in the study about regression coefficients in a random walk. The limiting distribution for all t is straightforward and can be obtained with the CMT and is given by

$$T^{-3}\sum_{t=1}^{T}\sum_{j=t}^{T} S_{t,j}^2 \Rightarrow \omega^2 \int_0^1 \int_a^1 V_\zeta(s)^2 ds da.$$

□

Unit Root Test Q_ε

The second test statistic is based on the residuals from the regression of the first differences of the series on intercept ($z_t = 1$) or intercept and linear trend ($z_t = (1,t)'$). Again, the squared sum of all possible partial sums of the residuals is calculated. Consider the following test statistic

$$Q_\varepsilon = \left(\frac{T^{-3} \sum_{t=1}^{T} \sum_{j=t}^{T} \left(\sum_{i=t}^{j} \hat{\varepsilon}_i \right)^2}{\hat{\omega}^2} \right)^{-1}.$$

The first differences of the time series are given by

$$\varepsilon_t^* = y_t - y_{t-1}, \quad \forall t$$

and the regression is conducted with ε_t^*. The denominator $\hat{\omega}^2$ is an estimator of the long-run variance ω^2 with

$$\hat{\omega}^2 = \hat{\gamma}_0 + 2 \sum_{s=1}^{l} w_{s,l} \hat{\gamma}_s, \quad \hat{\gamma}_s = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-s}, \quad w_{s,l} = 1 - s/(l+1).$$

Defining the partial sum process

$$S'_{t,j} = \sum_{i=t}^{j} \varepsilon_i, \quad \forall t, j \geq t,$$

the test statistic can be expressed in terms of $S'_{t,j}$:

$$Q_\varepsilon = \left(\frac{T^{-3} \sum_{t=1}^{T} \sum_{j=t}^{T} \hat{S}'^2_{t,j}}{\hat{\omega}^2} \right)^{-1}.$$

Similar interpretations follow from the rewritten test statistic and are based on $S'^2_{t,j}$ for all t and $j \geq t$. The partial sum processes are given by :

$$\begin{aligned}
S'^2_{1,1} + \ldots + S'^2_{1,T} &= \varepsilon_1^2 + (\varepsilon_1 + \varepsilon_2)^2 + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)^2 + \ldots + (\varepsilon_1 + \ldots + \varepsilon_T)^2 \\
S'^2_{2,2} + \ldots + S'^2_{2,T} &= \varepsilon_2^2 + (\varepsilon_2 + \varepsilon_3)^2 + (\varepsilon_2 + \varepsilon_3 + \varepsilon_4)^2 + \ldots + (\varepsilon_2 + \ldots + \varepsilon_T)^2 \\
S'^2_{3,3} + \ldots + S'^2_{3,T} &= \varepsilon_3^2 + (\varepsilon_3 + \varepsilon_4)^2 + (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)^2 + \ldots + (\varepsilon_3 + \ldots + \varepsilon_T)^2 \\
&\vdots \\
S'^2_{(T-2),(T-2)} + \ldots + S'^2_{(T-2),T} &= \varepsilon_{T-2}^2 + (\varepsilon_{T-2} + \varepsilon_{T-1})^2 + (\varepsilon_{T-2} + \varepsilon_{T-1} + \varepsilon_T)^2 \\
S'^2_{(T-1),(T-1)} + S'^2_{(T-1),T} &= \varepsilon_{T-1}^2 + (\varepsilon_{T-1} + \varepsilon_T)^2 \\
S'^2_{T,T} &= \varepsilon_T^2
\end{aligned}$$

In the differentiated case and under the null hypothesis $\varepsilon \sim N(0, \sigma^2)$. Thus, in each horizontal equation a Brownian bridge starts at zero and is summed and squared.

Again, in the vertical view each consecutive sequence of the differentiated path of the series is considered. Furthermore, each partial sum process $S_{t,j}$ from the first test can be decomposed into $y_j - y_{t-1}$ and therefore sub-samples of the paths of a random walk are considered.

Theorem 4.4.3 provides the limiting distribution of Q_ε under the null hypothesis.

Theorem 4.4.3. Let $\{y_t\}$ be generated by model (4.2) or model (4.3) under the null hypothesis H_0 and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($z_t = 1$) and the de-meaned and de-trended case ($z_t = (1, t)'$). Then, provided that as $T \to \infty$, it is true that

$$T^{-3} \sum_{t=1}^{T} \sum_{j=1}^{T} S_{t,j}'^2 \Rightarrow \omega^2 \int_0^1 \int_a^1 V_\zeta(s-a)^2 ds da.$$

Theorem 4.4.3 shows that the numerator of the test statistic converges to integrals over a Brownian bridge which starts at zero for each t.

Proof of Theorem 4.4.3. The limiting distribution of the test statistic can be obtained with the numerator of the rewritten test statistic

$$T^{-3} \sum_{t=1}^{T} \sum_{j=t}^{T} S_{t,j}'^2$$

and the partial sum process

$$S_{t,j}'^2 = \sum_{i=t}^{j} \varepsilon_i, \quad \forall t, j \geq t.$$

Starting with $t = 1$, it follows directly from Kwiatkowski et al. (1992b) that

$$T^{-2} \sum_{j=1}^{T} S_{1,j}' \Rightarrow \omega^2 \int_0^1 V_\zeta(s)^2 ds.$$

With the result

$$T^{-1/2} S_{1,[aT]}' = T^{-1/2} \sum_{i=1}^{[aT]} \varepsilon_i \Rightarrow \omega V_\zeta(a),$$

where $a \in [0,1]$ and $[aT] \geq i$ it follows with $b \in [0,1]$ and $a \geq b$

$$\begin{aligned} T^{-1/2} S'_{[bT],[aT]} &= T^{-1/2} \sum_{i=[bT]}^{aT} \varepsilon_i \\ &= T^{-1/2} \sum_{i=1}^{[aT]} \varepsilon_i - T^{-1/2} \sum_{i=1}^{[bT]} \varepsilon_i \\ &\Rightarrow \omega V_\zeta(a) - \omega V_\zeta(b) = \omega V_\zeta(a-b) \end{aligned}$$

Therefore,

$$T^{-3} \sum_{t=1}^{T} \sum_{j=t}^{T} S'^2_{t,j} \Rightarrow \omega^2 \int_0^1 \int_a^1 V_\zeta(s-a)^2 ds da.$$

□

Critical values of the test statistic Q_ε are provided in Table 4.2 and are calculated via a direct simulation with sample sizes of 50, 100, 250, 500 and 1000 observations with $1,000,000$ replications. Similar arguments from the test Q_y follow for Q_ε. The test rejects for large values of the test statistic and can be used to test the explosive alternative with the left-tailed critical values. The critical values show reasonable behaviour.

Again, a small sample correction of the test statistic can be applied. Assume that the process is generated by $y_t = y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0,1)$. Then it is easy to show that the expected value of the sum of partial sums of the first differences under the null hypothesis is given by

$$E\left[\sum_{t=1}^{T} \sum_{j=t}^{T} \left(\sum_{i=t}^{j} \varepsilon_i^*\right)^2\right] = (T(T+1)(T+2))/6.$$

This small sample correction is asymptotically negligible but can improve the properties of the test in finite samples.

z	T	0.01%	0.05%	0.10%	0.25%	0.50%	0.75%	0.90%	0.95%	0.99%
1	50	1.411	2.207	2.902	4.756	8.290	13.846	20.961	25.963	37.755
	100	1.379	2.181	2.889	4.778	8.355	14.014	21.194	26.500	38.460
	250	1.359	2.174	2.881	4.766	8.361	14.117	21.516	26.979	39.651
	500	1.349	2.173	2.881	4.778	8.399	14.184	21.613	27.158	39.805
	1000	1.347	2.172	2.880	4.770	8.404	14.257	21.736	27.341	40.291
(1,t)'	50	4.790	6.928	8.522	12.099	17.761	25.454	34.170	40.559	54.241
	100	4.680	6.835	8.439	12.075	17.830	25.751	34.929	41.355	55.155
	250	4.633	6.790	8.410	12.077	17.931	26.050	35.466	42.155	56.823
	500	4.631	6.783	8.394	12.084	17.974	26.126	35.650	42.460	57.381
	1000	4.592	6.763	8.383	12.069	18.009	26.177	35.736	42.518	57.501

Table 4.2: Critical Values of the Q_ε Test for $T = 50, 100, 250, 500, 1000$.

Theorem 4.4.4 provides the limiting distribution of Q_ε under the alternative and shows consistency of the test statistic.

Theorem 4.4.4. **(i)** Let $\{y_t\}$ be generated by model (4.2) or model (4.3) under the alternative null hypothesis H_1, that is $\rho = 0$, and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Then in the de-meaned case and the de-meaned and de-trended case provided that as $T \to \infty$ it is true that

$$\omega^{-2}(T^2 - T)^{-1} \sum_{t=1}^{T} \sum_{j=t}^{T} S'^2_{t,j} \Rightarrow O_p(1)$$

and therefore,

$$T^{-3} \sum_{t=1}^{T} \sum_{j=t}^{T} S'^2_{t,j} \Rightarrow O_p(T)$$

(ii) Under the alternative hypothesis H_1, when $\{y_t\}$ is generated by model (4.2) or model (4.3), it is true that

$$Q_\varepsilon \Rightarrow O_p(T).$$

Theorem 4.4.4 shows that the test statistic diverges under the alternative and is of $O_p(T)$ and hence the test Q_ε is consistent.

Proof of Theorem 4.4.4. Under the alternative hypothesis H_1, with $\rho = 0$, the series is generated by $y_t = \mu + \varepsilon_t$ or $y_t = \mu + \beta t + \varepsilon_t$. For $\mu = \beta = 0$, it is true that $y \sim N(0, \sigma^2)$. After differentiating and de-meaning or de-meaning and de-trending the series the resulting $\hat{\varepsilon}$ are normal distributed with mean 0, variance $2\sigma^2$ and $Cov(\hat{\varepsilon}_i, \hat{\varepsilon}_j) = 0$ for $i \neq j$. Then, it follows

$$
\begin{aligned}
E\left[S'^2_{t,j}\right] &= E\left[\left(\sum_{i=t}^{j} \varepsilon_i\right)^2\right] \\
&= E[\varepsilon_t^2] + \ldots + E[\varepsilon_j^2] \\
&= 2\sigma^2(j - t + 1)
\end{aligned}
$$

and

$$
\begin{aligned}
E\left[\sum_{t=1}^{T}\sum_{j=t}^{T} S'^2_{t,j}\right] &= E\left[\sum_{t=1}^{T}\sum_{j=t}^{T}\left(\sum_{i=t}^{j} \varepsilon_i\right)^2\right] \\
&= TE[\varepsilon_1^2] + 2(T-1)E[\varepsilon_2^2] + 3(T-1)E[\varepsilon_3^2] + \ldots \\
&+ 2(T-1)E[\varepsilon_{T-1}^2] + TE[\varepsilon_T^2] \\
&= 2\sigma^2(T^2 - T).
\end{aligned}
$$

Therefore,

$$
\omega^{-2}(T^2 - T)^{-1} \sum_{t=1}^{T}\sum_{j=t}^{T} S'^2_{t,j} \Rightarrow O_p(1).
$$

□

4.5 Simulation Results

4.5.1 Simulation Setup

In this section, Monte Carlo simulation methods are used to study the behaviour of the tests Q_y and Q_ε of Section 4.4 under the null and the alternative hypothesis, that is the size and power properties of the tests are under investigation in finite samples. Furthermore, the tests are compared to the presented tests of Section 4.3.

The ADF-test t-ratio is denoted by $t(\hat{\rho}^*)$, the non-parametric standardised analogue PP-test is denoted by $Z(t_{\hat{\rho}})$ and the non-parametric standardised PP-test of the DF-test, based on the regression coefficient $\hat{\rho}$, is denoted by $Z(\hat{\rho})$. Note that a test based on the regression coefficient $\hat{\rho}^*$ is not suggested by Said and Dickey (1984) because the limiting distribution of $T\hat{\rho}^*$ depends on nuisance parameters.

The data are generated with $M = 10,000$ replications by the ARIMA process

$$y_t = \rho y_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t, \quad t = -200, \dots, T,$$

where ε_t is independent and identically distributed $N(0,1)$. The autoregressive (AR) parameter is $\rho \in \{0.85, 0.9, 0.95, 0.975, 0.99, 1\}$, the moving-average (MA) parameter $\theta \in \{-0.8, -0.5, -0.2, 0, 0.5, 0.8\}$ and the lag truncation parameter for the included estimated covariances in Q_y, Q_ε, $Z(t_{\hat{\rho}})$ and $Z(\hat{\rho})$ or the included lags in the regression of $t(\hat{\rho}^*)$ is $l \in \{0, 2, 4, 6, 8, 12\}$. The series are simulated with a burn-in period of 200 observations to eliminate the initial effects.

The null hypothesis is true for $\rho = 1$ with different θ and the size properties of the tests are compared. For $\rho < 1$ and different θ, the power properties of the tests are under investigation.

The results are only reported for the tests in the de-meaned case, where y_t is regressed on an intercept ($z_t = 1$) as the de-meaned and de-trended case ($z_t = (1,t)'$) provide similar results. Rejection frequencies are reported for the nominal significance level $\alpha \in \{1\%, 5\%, 10\%\}$ with four decimal places to compare the properties of the tests in detail.

Note that for $l = 0$ the PP-tests $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ include no standardisation and are therefore not reported and $t(\hat{\rho}^*)$ includes no lags and reduces to the DF-test.

4.5.2 Discussion of Results

Table 4.3 and Table 4.4 report the results for $T = 100$ and $T = 250$ with $\theta = 0$ and different ρ and l. In this setup the properties of the tests are under investigation without disturbance due to MA parameters. A clear pattern can be identified for the ARMA model with no MA component. With $\rho = 1$ and $l = 0$ the tests Q_y and Q_ε and the DF-test $t(\hat{\rho}^*)$ are very close to the nominal size level. The size of the tests is slowly decreasing with increasing l and hence, the tests are slightly under its nominal size for $l \geq 4$ and the tests are conservative.

In contrast, the PP-tests $Z(t_{\hat{\rho}})$ and $Z(\hat{\rho})$ are oversized for $l = 2$ increasing with l. The rejection frequencies of the $Z(t_{\hat{\rho}})$ test are more than 10% higher than the nominal size level and the rejection frequencies of the $Z(\hat{\rho})$ test more than 5% for $T = 100$. Therefore, the PP-tests are liberal.

The large sample size $T = 250$ confirms these characteristics, while all tests are closer to the nominal significance level.

With decreasing ρ, all tests have increasing and reasonable power properties with exception of the ADF-test with high l in $T = 100$. In general, Q_y and Q_ε provide higher rejection frequencies than the ADF-test for all ρ and l in both sample sizes.

Comparing the CUSUM test statistics with the PP-tests reveals the next pattern. Although the PP-tests exceed the nominal size, the squared CUSUM based tests provide higher rejection frequencies for $l \in \{2, 4\}$. The results for $l = 6$ are not clear as the tests Q_y and $Z(t_{\hat{\rho}})$ have comparable power properties, whereas $Z(t_{\hat{\rho}})$ is about 20% oversized.

With $l \geq 8$ the rejection frequencies of $Z(t_{\hat{\rho}})$ exceeds the power of the new tests, due to the size distortions increasing with l and the decreasing size of the CUSUM tests. Still, the new approach has reasonable power properties and higher power than $Z(\hat{\rho})$ for all l.

These observations are similar for both sample sizes, whereby the power of all tests is increasing with T and the size of the CUSUM tests are closer to the nominal size.

These results show that the PP-tests are liberal, while the ADF-test and CUSUM based tests are conservative for $l \geq 2$ and the new presented tests are preferable with better size and power properties when there is no MA parameter. These findings can be analysed with size-adjusted critical values, where the Q_y test provide better power properties than the other tests.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0099	0.0496	0.0998	0.0095	0.0480	0.0962	0.0091	0.0477	0.0907	0.0086	0.0469	0.0893	0.0082	0.0439	0.0874	0.0078	0.0403	0.0831
Q_ε	0.0094	0.0485	0.0986	0.0089	0.0474	0.0948	0.0085	0.0452	0.0896	0.0080	0.0442	0.0884	0.0078	0.0412	0.0845	0.0073	0.0390	0.0788
$t(\hat{\rho}^*)$	0.0103	0.0494	0.1013	0.0105	0.0468	0.0954	0.0094	0.0453	0.0922	0.0091	0.0445	0.0918	0.0095	0.0426	0.0884	0.0077	0.0385	0.0790
$Z(\hat{\rho})$				0.0137	0.0558	0.1111	0.0147	0.0589	0.1172	0.0152	0.0605	0.1199	0.0134	0.0605	0.1226	0.0110	0.0559	0.1186
$Z(t_{\hat{\rho}})$				0.0119	0.0519	0.1050	0.0122	0.0558	0.1088	0.0119	0.0557	0.1109	0.0120	0.0551	0.1130	0.0114	0.0528	0.1090
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0147	0.0746	0.1435	0.0136	0.0704	0.1384	0.0130	0.0668	0.1321	0.0128	0.0633	0.1293	0.0122	0.0580	0.1240	0.0115	0.0539	0.1221
Q_ε	0.0141	0.0657	0.1438	0.0130	0.0689	0.1375	0.0127	0.0641	0.1289	0.0121	0.0604	0.1254	0.0117	0.0568	0.1226	0.0107	0.0520	0.1214
$t(\hat{\rho}^*)$	0.0113	0.0562	0.1131	0.0097	0.0547	0.1076	0.0112	0.0514	0.1026	0.0108	0.0498	0.1022	0.0101	0.0465	0.0961	0.0094	0.0468	0.0935
$Z(\hat{\rho})$				0.0119	0.0600	0.1199	0.0125	0.0634	0.1262	0.0118	0.0657	0.1296	0.0111	0.0655	0.1325	0.0104	0.0589	0.1286
$Z(t_{\hat{\rho}})$				0.0128	0.0615	0.1194	0.0131	0.0642	0.1251	0.0131	0.0647	0.1263	0.0130	0.0648	0.1263	0.0124	0.0603	0.1236
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0237	0.1037	0.1909	0.0205	0.0983	0.1865	0.0185	0.0951	0.1822	0.0168	0.0839	0.1687	0.0158	0.0796	0.1605	0.0144	0.0746	0.1569
Q_ε	0.0224	0.1002	0.1836	0.0201	0.0959	0.1818	0.0181	0.0933	0.1782	0.0163	0.0811	0.1640	0.0153	0.0783	0.1583	0.0134	0.0719	0.1508
$t(\hat{\rho}^*)$	0.0137	0.0745	0.1431	0.0139	0.0676	0.1381	0.0111	0.0624	0.1261	0.0101	0.0579	0.1218	0.0105	0.0562	0.1163	0.0083	0.0503	0.1043
$Z(\hat{\rho})$				0.0152	0.0849	0.1693	0.0175	0.0927	0.1780	0.0159	0.0954	0.1812	0.0150	0.0927	0.1842	0.0131	0.0840	0.1795
$Z(t_{\hat{\rho}})$				0.0156	0.0789	0.1523	0.0160	0.0808	0.1601	0.0162	0.0829	0.1628	0.0159	0.0831	0.1639	0.0140	0.0776	0.1570
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0458	0.2020	0.3579	0.0430	0.1958	0.3470	0.0419	0.1830	0.3228	0.0381	0.1772	0.3039	0.0320	0.1648	0.2767	0.0274	0.1566	0.2650
Q_ε	0.0442	0.1933	0.3408	0.0422	0.1910	0.3321	0.0402	0.1796	0.3178	0.0374	0.1639	0.2999	0.0313	0.1624	0.2723	0.0233	0.1508	0.2580
$t(\hat{\rho}^*)$	0.0278	0.1250	0.2313	0.0253	0.1100	0.2110	0.0211	0.0999	0.1908	0.0174	0.0891	0.1746	0.0141	0.0846	0.1620	0.0109	0.0675	0.1364
$Z(\hat{\rho})$				0.0385	0.1606	0.2948	0.0400	0.1700	0.3101	0.0391	0.1750	0.3207	0.0341	0.1730	0.3235	0.0279	0.1592	0.3180
$Z(t_{\hat{\rho}})$				0.0321	0.1343	0.2462	0.0336	0.1418	0.2554	0.0313	0.1416	0.2627	0.0292	0.1392	0.2626	0.0257	0.1266	0.2511
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.1620	0.4923	0.7055	0.1572	0.4777	0.6843	0.1458	0.4547	0.6669	0.1375	0.4358	0.6438	0.1234	0.4101	0.6178	0.1097	0.3979	0.5993
Q_ε	0.1544	0.4735	0.6788	0.1521	0.4606	0.6627	0.1397	0.4320	0.6424	0.1314	0.4172	0.6014	0.1201	0.3975	0.5861	0.0964	0.3733	0.5672
$t(\hat{\rho}^*)$	0.0942	0.3309	0.5313	0.0647	0.2629	0.4428	0.0498	0.2124	0.3717	0.0376	0.1766	0.3133	0.0289	0.1392	0.2640	0.0217	0.0957	0.1893
$Z(\hat{\rho})$				0.1310	0.4385	0.6540	0.1375	0.4562	0.6693	0.1351	0.4646	0.6780	0.1274	0.4591	0.6821	0.1074	0.4221	0.6701
$Z(t_{\hat{\rho}})$				0.1062	0.3577	0.5505	0.1088	0.3692	0.5620	0.1061	0.3725	0.5685	0.1015	0.3651	0.5701	0.0888	0.3355	0.5533
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.4299	0.8299	0.9352	0.3914	0.8048	0.9282	0.3803	0.7819	0.9158	0.3686	0.7692	0.8905	0.3435	0.7535	0.8735	0.2568	0.6823	0.8358
Q_ε	0.3899	0.7823	0.9131	0.3718	0.7615	0.9007	0.3758	0.7618	0.8923	0.3596	0.7359	0.8523	0.3365	0.7019	0.8185	0.2158	0.6585	0.7823
$t(\hat{\rho}^*)$	0.2535	0.6392	0.8190	0.1623	0.4806	0.6837	0.1070	0.3612	0.5557	0.0749	0.2767	0.4486	0.0490	0.2144	0.3699	0.0277	0.1387	0.2522
$Z(\hat{\rho})$				0.3518	0.7529	0.9024	0.3601	0.7624	0.9064	0.3532	0.7681	0.9110	0.3301	0.7653	0.9133	0.2760	0.7332	0.9096
$Z(t_{\hat{\rho}})$				0.2839	0.6547	0.8297	0.2867	0.6655	0.8355	0.2805	0.6691	0.8386	0.2637	0.6655	0.8403	0.2298	0.6288	0.8301

Table 4.3: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T=100$, $\theta=0$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0094	0.0498	0.0999	0.0093	0.0494	0.0984	0.0091	0.0486	0.0976	0.0088	0.0479	0.0961	0.0084	0.0455	0.0946	0.0075	0.0422	0.0922
Q_ε	0.0097	0.0499	0.0996	0.0095	0.0467	0.0970	0.0090	0.0469	0.0947	0.0086	0.0450	0.0933	0.0080	0.0432	0.0915	0.0069	0.0401	0.0899
$t(\hat{\rho}^*)$	0.0103	0.0485	0.1006	0.0095	0.0470	0.0977	0.0095	0.0458	0.1000	0.0113	0.0464	0.0966	0.0096	0.0470	0.0976	0.0081	0.0429	0.0930
$Z(\hat{\rho})$				0.0115	0.0540	0.1097	0.0122	0.0579	0.1133	0.0139	0.0597	0.1171	0.0138	0.0615	0.1195	0.0138	0.0645	0.1220
$Z(t_{\hat{\rho}})$				0.0107	0.0507	0.1030	0.0111	0.0534	0.1057	0.0121	0.0546	0.1088	0.0121	0.0557	0.1116	0.0116	0.0584	0.1136
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0242	0.1170	0.1980	0.0233	0.1120	0.1899	0.0225	0.1058	0.1842	0.0218	0.0999	0.1761	0.0204	0.0930	0.1705	0.0193	0.0853	0.1509
Q_ε	0.0226	0.1106	0.1931	0.0221	0.1095	0.1856	0.0213	0.1002	0.1787	0.0209	0.0976	0.1709	0.0195	0.0904	0.1623	0.0143	0.0818	0.1452
$t(\hat{\rho}^*)$	0.0173	0.0721	0.1456	0.0157	0.0720	0.1442	0.0145	0.0694	0.1422	0.0128	0.0689	0.1416	0.0121	0.0662	0.1362	0.0112	0.0608	0.1272
$Z(\hat{\rho})$				0.0201	0.0908	0.1733	0.0214	0.0952	0.1837	0.0215	0.0991	0.1882	0.0223	0.1025	0.1915	0.0220	0.1079	0.1987
$Z(t_{\hat{\rho}})$				0.0176	0.0773	0.1510	0.0182	0.0805	0.1563	0.0191	0.0826	0.1620	0.0198	0.0856	0.1639	0.0202	0.0887	0.1694
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0598	0.2470	0.4074	0.0588	0.2402	0.4013	0.0573	0.2353	0.3959	0.0572	0.2321	0.3858	0.0564	0.2296	0.3823	0.0534	0.2159	0.3629
Q_ε	0.0574	0.2433	0.3967	0.0567	0.2399	0.3924	0.0549	0.2319	0.3845	0.0525	0.2295	0.3793	0.0507	0.2224	0.3774	0.0488	0.2059	0.3493
$t(\hat{\rho}^*)$	0.0375	0.1547	0.2946	0.0341	0.1514	0.2755	0.0305	0.1389	0.2656	0.0291	0.1345	0.2512	0.0257	0.1273	0.2419	0.0235	0.1159	0.2188
$Z(\hat{\rho})$				0.0533	0.2162	0.3803	0.0560	0.2238	0.3909	0.0582	0.2303	0.3968	0.0585	0.2350	0.4034	0.0600	0.2455	0.4115
$Z(t_{\hat{\rho}})$				0.0409	0.1665	0.2993	0.0435	0.1732	0.3065	0.0444	0.1785	0.3136	0.0459	0.1828	0.3178	0.0454	0.1873	0.3260
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.2548	0.6560	0.8406	0.2395	0.6301	0.8183	0.2310	0.6258	0.8132	0.2289	0.6219	0.8044	0.2255	0.6135	0.7923	0.2019	0.5823	0.7723
Q_ε	0.2472	0.6359	0.8259	0.2222	0.6257	0.8085	0.2209	0.6197	0.7999	0.2195	0.6123	0.7883	0.2168	0.5952	0.7794	0.1624	0.5648	0.7549
$t(\hat{\rho}^*)$	0.1568	0.4665	0.6786	0.1337	0.4194	0.6216	0.1178	0.3772	0.5747	0.1015	0.3414	0.5329	0.0839	0.3037	0.4915	0.0678	0.2511	0.4209
$Z(\hat{\rho})$				0.2225	0.6068	0.7965	0.2314	0.6131	0.7991	0.2405	0.6210	0.8037	0.2443	0.6264	0.8083	0.2495	0.6338	0.8149
$Z(t_{\hat{\rho}})$				0.1673	0.4825	0.6801	0.1747	0.4922	0.6881	0.1778	0.5020	0.6966	0.1797	0.5056	0.7011	0.1810	0.5101	0.7076
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.8970	0.9964	0.9998	0.8794	0.9952	0.9994	0.8635	0.9937	0.9989	0.8523	0.9897	0.9942	0.8502	0.9822	0.9925	0.8421	0.9724	0.9909
Q_ε	0.8744	0.9911	0.9989	0.8624	0.9893	0.9983	0.8601	0.9874	0.9981	0.8509	0.9803	0.9923	0.8435	0.9772	0.9906	0.8206	0.9698	0.9834
$t(\hat{\rho}^*)$	0.7573	0.9686	0.9947	0.6212	0.9183	0.9781	0.4999	0.8496	0.9481	0.4017	0.7754	0.9060	0.3240	0.6917	0.8583	0.2130	0.5487	0.7453
$Z(\hat{\rho})$				0.8496	0.9904	0.9981	0.8518	0.9901	0.9981	0.8564	0.9889	0.9980	0.8609	0.9896	0.9980	0.8652	0.9903	0.9984
$Z(t_{\hat{\rho}})$				0.7609	0.9684	0.9938	0.7668	0.9678	0.9931	0.7705	0.9683	0.9918	0.7737	0.9690	0.9920	0.7782	0.9700	0.9921
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9993	1.0000	1.0000	0.9991	1.0000	1.0000	0.9988	1.0000	1.0000	0.9943	1.0000	1.0000	0.9933	1.0000	1.0000	0.9918	1.0000	1.0000
Q_ε	0.9986	1.0000	1.0000	0.9983	1.0000	1.0000	0.9976	1.0000	1.0000	0.9930	1.0000	1.0000	0.9911	1.0000	1.0000	0.9834	1.0000	1.0000
$t(\hat{\rho}^*)$	0.9928	0.9999	1.0000	0.9419	0.9974	0.9998	0.8460	0.9847	0.9973	0.7191	0.9512	0.9882	0.5904	0.8954	0.9674	0.3785	0.7409	0.8894
$Z(\hat{\rho})$				0.9979	1.0000	1.0000	0.9976	1.0000	1.0000	0.9976	1.0000	1.0000	0.9980	1.0000	1.0000	0.9980	1.0000	1.0000
$Z(t_{\hat{\rho}})$				0.9915	0.9999	0.9999	0.9918	0.9999	0.9999	0.9919	0.9998	1.0000	0.9923	0.9997	1.0000	0.9938	0.9998	1.0000

Table 4.4: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T=250$, $\theta=0$.

The Tables 4.5-4.8 report the rejection frequencies obtained when the data are simulated with a positive MA parameter. With $l = 0$ the tests Q_y, Q_ε and $t(\hat{\rho}^*)$ are conservative and reasonable power results for Q_y are obtained for $\rho \leq 0.95$ with $T = 100$ and $\rho \leq 0.975$ with $T = 250$ for both MA parameters.

The Q_ε and the DF-tests suffer more from substantial power losses and in general the results for no included lag truncation and no lags cannot be interpreted meaningfully with a positive MA parameter and $l = 0$.

Considering $l > 0$, all tests are conservative. The size distortions are larger for small T and high θ. The ADF-test is the only test which is close to the nominal size level for $T = 250$ and $\theta = 0.5$.

When the alternative is very close to the null hypothesis, that is $\rho = 0.99$, a non-stationary process is not distinguishable from a unit root at the small sample size $T = 100$. With more observations, $T = 250$, again the CUSUM test Q_y provides the best power results for both MA parameters and all l. The rejection frequencies of the ADF-test are higher than the rejection frequencies of Q_ε with $\theta = 0.5$ and, vice versa, with $\theta = 0.8$. When $\rho = 0.99$ the PP-tests suffer most from power losses. This property confirms the findings in the literature that a unit root process is not identifiable, when the autoregressive parameter is rather close to unity and an additional MA parameter is present in the data. Still, the test Q_y provides the best results and is more able to identify the alternative correctly, compared to the other tests.

In general for $\rho < 0.99$ the CUSUM test Q_y provides the highest power results in most cases and Q_ε is slightly smaller than Q_y with a high AR parameter. When the AR parameter decreases, the rejection frequencies of Q_ε are the highest with $l \in 2, 4$. The $Z(\hat{\rho})$ test is in most of the time between the CUSUM tests.

This effect increases with the large size and Q_ε provides better power results for $\rho \leq 0.95$ and all l. The ADF-test and the $Z(t_{\hat{\rho}})$ test suffer most from substantial power losses however, the power increases with decreasing ρ.

The main findings for the positive MA parameter are that all tests suffer from large size distortions and therefore are conservative. The CUSUM based tests provide the best power results and are the preferable tests.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0077	0.0378	0.0678	0.0069	0.0356	0.0721	0.0063	0.0362	0.0742	0.0048	0.0335	0.0722	0.0036	0.0301	0.0688	0.0020	0.0235	0.0613
Q_ε	0.0072	0.0332	0.0644	0.0024	0.0215	0.0541	0.0013	0.0189	0.0509	0.0001	0.0134	0.0427	0.0000	0.0064	0.0316	0.0000	0.0007	0.0120
$t(\hat{\rho}^*)$	0.0062	0.0257	0.0500	0.0068	0.0421	0.0801	0.0097	0.0477	0.0944	0.0074	0.0471	0.0966	0.0095	0.0474	0.0949	0.0083	0.0419	0.0899
$Z(\hat{\rho})$				0.0043	0.0269	0.0625	0.0041	0.0296	0.0685	0.0026	0.0261	0.0672	0.0010	0.0215	0.0616	0.0003	0.0110	0.0444
$Z(t_{\hat{\rho}})$				0.0057	0.0328	0.0697	0.0058	0.0336	0.0721	0.0053	0.0308	0.0703	0.0053	0.0286	0.0669	0.0056	0.0256	0.0590
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0067	0.0274	0.0466	0.0059	0.0410	0.0853	0.0060	0.0406	0.0878	0.0053	0.0380	0.0852	0.0047	0.0334	0.0803	0.0017	0.0261	0.0680
Q_ε	0.0062	0.0141	0.0358	0.0020	0.0257	0.0641	0.0006	0.0211	0.0647	0.0001	0.0136	0.0545	0.0000	0.0067	0.0393	0.0000	0.0003	0.0122
$t(\hat{\rho}^*)$	0.0050	0.0201	0.0417	0.0060	0.0399	0.0878	0.0087	0.0497	0.1017	0.0080	0.0470	0.1005	0.0083	0.0494	0.1023	0.0090	0.0441	0.0940
$Z(\hat{\rho})$				0.0019	0.0233	0.0583	0.0014	0.0249	0.0644	0.0004	0.0208	0.0612	0.0002	0.0157	0.0540	0.0003	0.0066	0.0355
$Z(t_{\hat{\rho}})$				0.0048	0.0293	0.0684	0.0047	0.0297	0.0725	0.0047	0.0278	0.0684	0.0048	0.0244	0.0631	0.0048	0.0231	0.0520
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0044	0.0386	0.0645	0.0106	0.0558	0.1174	0.0094	0.0559	0.1188	0.0076	0.0522	0.1148	0.0062	0.0472	0.1072	0.0034	0.0358	0.0916
Q_ε	0.0006	0.0265	0.0482	0.0040	0.0421	0.0974	0.0018	0.0356	0.0947	0.0003	0.0213	0.0747	0.0000	0.0091	0.0543	0.0000	0.0005	0.0182
$t(\hat{\rho}^*)$	0.0034	0.0187	0.0444	0.0087	0.0484	0.1015	0.0091	0.0558	0.1167	0.0100	0.0531	0.1174	0.0089	0.0551	0.1137	0.0084	0.0493	0.1056
$Z(\hat{\rho})$				0.0041	0.0379	0.0908	0.0034	0.0385	0.0988	0.0019	0.0305	0.0919	0.0008	0.0241	0.0814	0.0003	0.0116	0.0526
$Z(t_{\hat{\rho}})$				0.0056	0.0379	0.0869	0.0053	0.0381	0.0898	0.0045	0.0341	0.0815	0.0040	0.0290	0.0733	0.0032	0.0234	0.0571
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0085	0.0543	0.1212	0.0213	0.1048	0.2140	0.0198	0.1048	0.2133	0.0157	0.0934	0.2004	0.0132	0.0827	0.1867	0.0061	0.0616	0.1527
Q_ε	0.0042	0.0356	0.0816	0.0087	0.0789	0.1837	0.0040	0.0672	0.1733	0.0004	0.0427	0.1426	0.0000	0.0178	0.1024	0.0000	0.0018	0.0309
$t(\hat{\rho}^*)$	0.0021	0.0208	0.0544	0.0127	0.0726	0.1525	0.0177	0.0889	0.1788	0.0156	0.0882	0.1712	0.0149	0.0770	0.1571	0.0119	0.0657	0.1361
$Z(\hat{\rho})$				0.0085	0.0694	0.1656	0.0069	0.0699	0.1758	0.0035	0.0602	0.1622	0.0013	0.0448	0.1447	0.0005	0.0199	0.0890
$Z(t_{\hat{\rho}})$				0.0080	0.0546	0.1293	0.0063	0.0556	0.1328	0.0047	0.0459	0.1222	0.0038	0.0380	0.1073	0.0032	0.0261	0.0764
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0350	0.1697	0.3207	0.0690	0.2750	0.4644	0.0606	0.2563	0.4487	0.0485	0.2290	0.4192	0.0345	0.2020	0.3826	0.0159	0.1450	0.3069
Q_ε	0.0063	0.0714	0.1899	0.0388	0.2568	0.4722	0.0157	0.2072	0.4395	0.0016	0.1258	0.3538	0.0000	0.0518	0.2514	0.0000	0.0020	0.0611
$t(\hat{\rho}^*)$	0.0059	0.0478	0.1220	0.0357	0.1805	0.3345	0.0428	0.1919	0.3418	0.0378	0.1685	0.3009	0.0300	0.1363	0.2578	0.0205	0.0956	0.1885
$Z(\hat{\rho})$				0.0322	0.2282	0.4261	0.0248	0.2234	0.4355	0.0113	0.1871	0.4081	0.0049	0.1398	0.3584	0.0021	0.0548	0.2380
$Z(t_{\hat{\rho}})$				0.0208	0.1544	0.3203	0.0165	0.1455	0.3215	0.0100	0.1187	0.2896	0.0066	0.0869	0.2466	0.0048	0.0503	0.1534
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0879	0.3563	0.5667	0.1540	0.4881	0.6996	0.1311	0.4486	0.6617	0.1005	0.3960	0.6134	0.0737	0.3429	0.5604	0.0331	0.2429	0.4555
Q_ε	0.0235	0.2148	0.4523	0.1166	0.5196	0.7682	0.0447	0.4174	0.7019	0.0040	0.2556	0.5752	0.0002	0.1049	0.4076	0.0000	0.0034	0.0938
$t(\hat{\rho}^*)$	0.0132	0.1210	0.2764	0.0811	0.3359	0.5328	0.0802	0.3160	0.4994	0.0631	0.2537	0.4247	0.0401	0.1924	0.3414	0.0256	0.1249	0.2413
$Z(\hat{\rho})$				0.1020	0.4734	0.7241	0.0755	0.4540	0.7113	0.0356	0.3870	0.6709	0.0147	0.3006	0.6139	0.0059	0.1390	0.4526
$Z(t_{\hat{\rho}})$				0.0610	0.3352	0.5756	0.0438	0.3115	0.5590	0.0237	0.2541	0.5074	0.0153	0.1860	0.4425	0.0089	0.1005	0.2905

Table 4.5: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 100$, $\theta = 0.5$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0023	0.0183	0.0408	0.0070	0.0379	0.0779	0.0077	0.0413	0.0853	0.0078	0.0418	0.0853	0.0077	0.0425	0.0854	0.0074	0.0414	0.0847
Q_ε	0.0004	0.0031	0.0113	0.0033	0.0246	0.0582	0.0037	0.0279	0.0673	0.0030	0.0273	0.0683	0.0019	0.0249	0.0686	0.0008	0.0191	0.0578
$t(\hat{\rho}^*)$	0.0058	0.0237	0.0502	0.0079	0.0373	0.0786	0.0090	0.0434	0.0922	0.0093	0.0452	0.0976	0.0085	0.0456	0.0954	0.0082	0.0454	0.0947
$Z(\hat{\rho})$				0.0032	0.0230	0.0586	0.0042	0.0284	0.0687	0.0043	0.0303	0.0719	0.0035	0.0307	0.0721	0.0022	0.0273	0.0697
$Z(t_{\hat{\rho}})$				0.0064	0.0343	0.0729	0.0066	0.0382	0.0802	0.0068	0.0383	0.0828	0.0064	0.0377	0.0829	0.0057	0.0360	0.0790
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0042	0.0307	0.0678	0.0116	0.0629	0.1287	0.0132	0.0678	0.1388	0.0135	0.0681	0.1392	0.0123	0.0673	0.1388	0.0107	0.0641	0.1346
Q_ε	0.0002	0.0069	0.0230	0.0065	0.0453	0.1073	0.0071	0.0522	0.1226	0.0055	0.0511	0.1226	0.0032	0.0463	0.1182	0.0012	0.0356	0.1008
$t(\hat{\rho}^*)$	0.0040	0.0181	0.0437	0.0091	0.0498	0.1088	0.0140	0.0649	0.1330	0.0146	0.0693	0.1367	0.0136	0.0682	0.1310	0.0124	0.0609	0.1311
$Z(\hat{\rho})$				0.0059	0.0432	0.1000	0.0075	0.0520	0.1156	0.0068	0.0535	0.1202	0.0057	0.0522	0.1200	0.0037	0.0466	0.1152
$Z(t_{\hat{\rho}})$				0.0069	0.0416	0.0932	0.0079	0.0481	0.1053	0.0079	0.0488	0.1087	0.0073	0.0472	0.1073	0.0060	0.0419	0.1009
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0115	0.0787	0.1629	0.0314	0.1520	0.2868	0.0330	0.1603	0.2972	0.0315	0.1594	0.2960	0.0297	0.1547	0.2924	0.0257	0.1434	0.2762
Q_ε	0.0010	0.0226	0.0715	0.0211	0.1351	0.2730	0.0222	0.1504	0.2956	0.0165	0.1413	0.2901	0.0118	0.1276	0.2747	0.0031	0.0944	0.2384
$t(\hat{\rho}^*)$	0.0027	0.0237	0.0645	0.0207	0.1040	0.2074	0.0283	0.1315	0.2504	0.0292	0.1292	0.2476	0.0258	0.1235	0.2395	0.0237	0.1150	0.2232
$Z(\hat{\rho})$				0.0191	0.1195	0.2456	0.0224	0.1412	0.2783	0.0204	0.1440	0.2840	0.0182	0.1381	0.2785	0.0111	0.1224	0.2651
$Z(t_{\hat{\rho}})$				0.0141	0.0878	0.1833	0.0157	0.1015	0.2058	0.0146	0.1022	0.2129	0.0124	0.0983	0.2084	0.0087	0.0828	0.1942
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0594	0.2599	0.4544	0.1282	0.4191	0.6230	0.1306	0.4260	0.6321	0.1236	0.4156	0.6200	0.1146	0.3986	0.6038	0.0949	0.3629	0.5661
Q_ε	0.0119	0.1180	0.2896	0.1074	0.4447	0.6821	0.1113	0.4611	0.7020	0.0880	0.4340	0.6836	0.0625	0.3943	0.6492	0.0198	0.2959	0.5669
$t(\hat{\rho}^*)$	0.0079	0.0732	0.1848	0.0778	0.3058	0.5101	0.1031	0.3471	0.5449	0.0995	0.3272	0.5196	0.0877	0.3059	0.4884	0.0664	0.2467	0.4241
$Z(\hat{\rho})$				0.0923	0.4030	0.6371	0.1071	0.4404	0.6722	0.0994	0.4396	0.6719	0.0894	0.4225	0.6577	0.0546	0.3752	0.6227
$Z(t_{\hat{\rho}})$				0.0602	0.2839	0.4945	0.0691	0.3142	0.5279	0.0635	0.3090	0.5231	0.0523	0.2926	0.5093	0.0322	0.2483	0.4694
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.3504	0.7628	0.9157	0.5236	0.8829	0.9711	0.5052	0.8713	0.9666	0.4678	0.8487	0.9559	0.4275	0.8207	0.9425	0.3506	0.7559	0.9121
Q_ε	0.1945	0.7150	0.9202	0.6568	0.9626	0.9943	0.6149	0.9574	0.9921	0.5115	0.9345	0.9882	0.3813	0.8968	0.9805	0.1283	0.7602	0.9441
$t(\hat{\rho}^*)$	0.0956	0.4637	0.7387	0.4389	0.8311	0.9419	0.4422	0.8139	0.9315	0.3774	0.7519	0.8952	0.3060	0.6790	0.8455	0.2009	0.5281	0.7225
$Z(\hat{\rho})$				0.6020	0.9473	0.9907	0.6086	0.9484	0.9902	0.5715	0.9400	0.9875	0.5218	0.9268	0.9849	0.3840	0.8934	0.9789
$Z(t_{\hat{\rho}})$				0.4403	0.8662	0.9607	0.4473	0.8716	0.9624	0.4060	0.8560	0.9572	0.3511	0.8303	0.9462	0.2328	0.7633	0.9234
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.7218	0.9682	0.9968	0.8420	0.9903	0.9996	0.8128	0.9852	0.9989	0.7634	0.9765	0.9974	0.7110	0.9627	0.9948	0.6039	0.9234	0.9854
Q_ε	0.7207	0.9901	0.9997	0.9743	0.9999	1.0000	0.9510	0.9995	1.0000	0.8793	0.9982	1.0000	0.7467	0.9931	0.9996	0.2910	0.9425	0.9952
$t(\hat{\rho}^*)$	0.4663	0.9112	0.9841	0.8296	0.9862	0.9975	0.7763	0.9710	0.9943	0.6657	0.9321	0.9819	0.5524	0.8743	0.9564	0.3561	0.7274	0.8698
$Z(\hat{\rho})$				0.9604	0.9995	1.0000	0.9552	0.9994	1.0000	0.9338	0.9991	1.0000	0.9102	0.9987	1.0000	0.8240	0.9963	0.9998
$Z(t_{\hat{\rho}})$				0.8906	0.9965	0.9995	0.8792	0.9956	0.9995	0.8423	0.9922	0.9991	0.7927	0.9869	0.9984	0.6534	0.9766	0.9972

Table 4.6: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 250$, $\theta = 0.5$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
$\rho=1.00$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0019	0.0138	0.0334	0.0053	0.0341	0.0719	0.0049	0.0357	0.0752	0.0041	0.0341	0.0730	0.0037	0.0304	0.0688	0.0021	0.0234	0.0616
Q_ε	0.0001	0.0020	0.0085	0.0020	0.0200	0.0505	0.0007	0.0186	0.0507	0.0003	0.0125	0.0431	0.0000	0.0059	0.0325	0.0000	0.0006	0.0126
$t(\hat{\rho}^*)$	0.0073	0.0281	0.0481	0.0060	0.0323	0.0642	0.0074	0.0371	0.0747	0.0083	0.0386	0.0812	0.0086	0.0412	0.0819	0.0078	0.0398	0.0815
$Z(\hat{\rho})$				0.0032	0.0247	0.0572	0.0034	0.0267	0.0622	0.0017	0.0238	0.0595	0.0011	0.0196	0.0526	0.0005	0.0094	0.0367
$Z(t_{\hat{\rho}})$				0.0061	0.0319	0.0665	0.0060	0.0317	0.0682	0.0060	0.0299	0.0674	0.0054	0.0279	0.0627	0.0056	0.0269	0.0548
$\rho=0.99$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0023	0.0178	0.0416	0.0072	0.0414	0.0890	0.0071	0.0413	0.0928	0.0066	0.0390	0.0910	0.0048	0.0352	0.0863	0.0015	0.0274	0.0742
Q_ε	0.0001	0.0027	0.0105	0.0026	0.0257	0.0680	0.0019	0.0222	0.0674	0.0002	0.0156	0.0552	0.0001	0.0077	0.0418	0.0000	0.0007	0.0141
$t(\hat{\rho}^*)$	0.0055	0.0219	0.0446	0.0053	0.0359	0.0698	0.0069	0.0396	0.0806	0.0085	0.0438	0.0913	0.0088	0.0462	0.0943	0.0077	0.0420	0.0915
$Z(\hat{\rho})$				0.0022	0.0244	0.0620	0.0023	0.0251	0.0668	0.0011	0.0220	0.0614	0.0005	0.0169	0.0549	0.0003	0.0073	0.0352
$Z(t_{\hat{\rho}})$				0.0052	0.0328	0.0722	0.0047	0.0342	0.0745	0.0046	0.0312	0.0707	0.0044	0.0282	0.0659	0.0052	0.0244	0.0547
$\rho=0.975$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0022	0.0219	0.0566	0.0094	0.0561	0.1236	0.0092	0.0575	0.1242	0.0073	0.0546	0.1201	0.0052	0.0475	0.1140	0.0021	0.0341	0.0966
Q_ε	0.0003	0.0050	0.0149	0.0043	0.0383	0.0929	0.0020	0.0338	0.0948	0.0004	0.0195	0.0782	0.0001	0.0096	0.0557	0.0000	0.0006	0.0169
Q_c	0.0003	0.0050	0.0149	0.0043	0.0383	0.0929	0.0020	0.0338	0.0948	0.0004	0.0195	0.0782	0.0001	0.0096	0.0557	0.0000	0.0006	0.0169
$t(\hat{\rho}^*)$	0.0032	0.0161	0.0374	0.0057	0.0330	0.0810	0.0074	0.0478	0.1027	0.0095	0.0529	0.1065	0.0097	0.0520	0.1067	0.0078	0.0466	0.0965
$Z(\hat{\rho})$				0.0037	0.0332	0.0837	0.0029	0.0340	0.0917	0.0012	0.0250	0.0849	0.0003	0.0178	0.0707	0.0002	0.0069	0.0381
$Z(t_{\hat{\rho}})$				0.0051	0.0339	0.0812	0.0047	0.0342	0.0851	0.0040	0.0288	0.0760	0.0036	0.0250	0.0675	0.0033	0.0197	0.0514
$\rho=0.950$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0056	0.0444	0.1076	0.0185	0.1049	0.2017	0.0170	0.1055	0.2040	0.0141	0.0946	0.1963	0.0104	0.0829	0.1832	0.0049	0.0614	0.1496
Q_ε	0.0006	0.0086	0.0325	0.0077	0.0757	0.1742	0.0032	0.0652	0.1709	0.0003	0.0423	0.1413	0.0000	0.0200	0.0991	0.0000	0.0006	0.0299
$t(\hat{\rho}^*)$	0.0026	0.0164	0.0421	0.0080	0.0511	0.1130	0.0108	0.0625	0.1336	0.0129	0.0690	0.1454	0.0107	0.0687	0.1414	0.0109	0.0607	0.1247
$Z(\hat{\rho})$				0.0062	0.0651	0.1570	0.0045	0.0646	0.1658	0.0019	0.0518	0.1510	0.0006	0.0348	0.1298	0.0003	0.0130	0.0716
$Z(t_{\hat{\rho}})$				0.0059	0.0520	0.1246	0.0053	0.0509	0.1280	0.0039	0.0434	0.1143	0.0038	0.0322	0.0972	0.0032	0.0219	0.0655
$\rho=0.900$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0273	0.1396	0.2810	0.0646	0.2653	0.4470	0.0593	0.2516	0.4343	0.0479	0.2254	0.4058	0.0375	0.1959	0.3712	0.0180	0.1415	0.3004
Q_ε	0.0021	0.0432	0.1245	0.0366	0.2396	0.4546	0.0148	0.2017	0.4268	0.0019	0.1223	0.3465	0.0000	0.0520	0.2388	0.0000	0.0022	0.0637
$t(\hat{\rho}^*)$	0.0026	0.0309	0.0853	0.0158	0.1124	0.2338	0.0264	0.1332	0.2629	0.0246	0.1259	0.2438	0.0251	0.1167	0.2280	0.0167	0.0954	0.1812
$Z(\hat{\rho})$				0.0282	0.1961	0.4005	0.0183	0.1927	0.4097	0.0078	0.1528	0.3729	0.0023	0.1037	0.3188	0.0010	0.0350	0.1803
$Z(t_{\hat{\rho}})$				0.0170	0.1339	0.2848	0.0129	0.1248	0.2845	0.0076	0.0952	0.2503	0.0049	0.0641	0.2022	0.0037	0.0350	0.1163
$\rho=0.850$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0748	0.3150	0.5249	0.1577	0.4852	0.6964	0.1376	0.4489	0.6648	0.1048	0.4009	0.6178	0.0761	0.3515	0.5665	0.0342	0.2544	0.4624
Q_ε	0.0130	0.1398	0.3426	0.1179	0.4988	0.7545	0.0488	0.4141	0.6962	0.0051	0.2623	0.5758	0.0000	0.1153	0.4146	0.0000	0.0035	0.0957
$t(\hat{\rho}^*)$	0.0081	0.0790	0.1977	0.0416	0.2272	0.4124	0.0498	0.2316	0.4095	0.0461	0.2055	0.3602	0.0394	0.1728	0.3113	0.0240	0.1230	0.2354
$Z(\hat{\rho})$				0.0922	0.4353	0.6924	0.0645	0.4131	0.6808	0.0259	0.3385	0.6323	0.0092	0.2469	0.5650	0.0029	0.0857	0.3624
$Z(t_{\hat{\rho}})$				0.0504	0.3001	0.5380	0.0335	0.2720	0.5218	0.0189	0.2080	0.4635	0.0104	0.1372	0.3825	0.0065	0.0644	0.2173

Table 4.7: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T=100$, $\theta=0.8$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0016	0.0139	0.0323	0.0066	0.0339	0.0743	0.0073	0.0382	0.0811	0.0074	0.0395	0.0829	0.0072	0.0390	0.0822	0.0062	0.0367	0.0812
Q_ε	0.0001	0.0017	0.0070	0.0027	0.0226	0.0555	0.0033	0.0271	0.0661	0.0028	0.0284	0.0665	0.0017	0.0256	0.0640	0.0006	0.0190	0.0553
$t(\hat{\rho}^*)$	0.0067	0.0274	0.0517	0.0055	0.0331	0.0697	0.0070	0.0394	0.0834	0.0073	0.0410	0.0921	0.0069	0.0448	0.0937	0.0091	0.0472	0.0965
$Z(\hat{\rho})$				0.0040	0.0256	0.0613	0.0053	0.0331	0.0735	0.0053	0.0349	0.0764	0.0042	0.0342	0.0769	0.0026	0.0296	0.0721
$Z(t_{\hat{\rho}})$				0.0054	0.0345	0.0755	0.0059	0.0379	0.0818	0.0060	0.0382	0.0829	0.0057	0.0377	0.0827	0.0049	0.0346	0.0797
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0035	0.0253	0.0600	0.0119	0.0638	0.1323	0.0131	0.0709	0.1422	0.0131	0.0722	0.1434	0.0130	0.0713	0.1429	0.0116	0.0672	0.1393
Q_ε	0.0003	0.0035	0.0157	0.0059	0.0441	0.1041	0.0065	0.0514	0.1224	0.0057	0.0525	0.1243	0.0044	0.0498	0.1213	0.0013	0.0360	0.1049
$t(\hat{\rho}^*)$	0.0033	0.0152	0.0353	0.0061	0.0339	0.0778	0.0086	0.0449	0.1024	0.0108	0.0538	0.1161	0.0109	0.0608	0.1187	0.0120	0.0631	0.1216
$Z(\hat{\rho})$				0.0049	0.0413	0.0991	0.0061	0.0504	0.1152	0.0053	0.0532	0.1208	0.0053	0.0528	0.1220	0.0027	0.0441	0.1170
$Z(t_{\hat{\rho}})$				0.0059	0.0374	0.0902	0.0068	0.0442	0.1001	0.0065	0.0450	0.1051	0.0059	0.0437	0.1032	0.0048	0.0384	0.0975
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0101	0.0648	0.1460	0.0316	0.1500	0.2774	0.0338	0.1601	0.2943	0.0328	0.1592	0.2946	0.0314	0.1554	0.2897	0.0261	0.1443	0.2762
Q_ε	0.0010	0.0136	0.0479	0.0206	0.1289	0.2673	0.0226	0.1453	0.2984	0.0169	0.1417	0.2945	0.0119	0.1276	0.2830	0.0042	0.0948	0.2427
$t(\hat{\rho}^*)$	0.0020	0.0195	0.0518	0.0102	0.0676	0.1498	0.0168	0.0973	0.1944	0.0183	0.1058	0.2101	0.0211	0.1068	0.2137	0.0213	0.1044	0.2085
$Z(\hat{\rho})$				0.0168	0.1211	0.2394	0.0212	0.1414	0.2773	0.0193	0.1410	0.2828	0.0157	0.1359	0.2799	0.0098	0.1176	0.2617
$Z(t_{\hat{\rho}})$				0.0130	0.0888	0.1815	0.0154	0.1025	0.2061	0.0144	0.1021	0.2081	0.0123	0.0958	0.2042	0.0088	0.0818	0.1883
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0402	0.2118	0.3943	0.1128	0.3950	0.6154	0.1182	0.4075	0.6281	0.1134	0.3958	0.6176	0.1058	0.3819	0.6040	0.0883	0.3461	0.5640
Q_ε	0.0049	0.0728	0.2075	0.0962	0.4268	0.6640	0.0991	0.4537	0.6901	0.0826	0.4315	0.6753	0.0539	0.3909	0.6495	0.0165	0.2881	0.5742
$t(\hat{\rho}^*)$	0.0045	0.0427	0.1270	0.0356	0.1942	0.3793	0.0525	0.2470	0.4407	0.0622	0.2612	0.4513	0.0600	0.2561	0.4443	0.0560	0.2252	0.3949
$Z(\hat{\rho})$				0.0777	0.3806	0.6125	0.0895	0.4215	0.6537	0.0844	0.4204	0.6496	0.0683	0.4012	0.6427	0.0383	0.3451	0.6084
$Z(t_{\hat{\rho}})$				0.0480	0.2562	0.4680	0.0564	0.2897	0.5058	0.0482	0.2859	0.5066	0.0376	0.2655	0.4920	0.0214	0.2174	0.4447
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.3051	0.7119	0.8855	0.5123	0.8725	0.9692	0.4983	0.8630	0.9644	0.4628	0.8410	0.9543	0.4233	0.8141	0.9402	0.3534	0.7548	0.9067
Q_ε	0.1111	0.5788	0.8407	0.6204	0.9561	0.9951	0.5976	0.9503	0.9935	0.5038	0.9305	0.9886	0.3773	0.8914	0.9795	0.1293	0.7517	0.9403
$t(\hat{\rho}^*)$	0.0516	0.3420	0.6166	0.2806	0.6935	0.8806	0.3098	0.7120	0.8775	0.2906	0.6779	0.8434	0.2528	0.6260	0.8106	0.1913	0.5125	0.7115
$Z(\hat{\rho})$				0.5604	0.9368	0.9881	0.5744	0.9407	0.9900	0.5310	0.9289	0.9863	0.4705	0.9111	0.9820	0.3226	0.8650	0.9694
$Z(t_{\hat{\rho}})$				0.4004	0.8363	0.9530	0.4113	0.8523	0.9573	0.3670	0.8277	0.9498	0.3054	0.7945	0.9369	0.1724	0.7115	0.9029
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.6668	0.9523	0.9940	0.8362	0.9900	0.9990	0.8093	0.9838	0.9987	0.7641	0.9729	0.9974	0.7108	0.9587	0.9943	0.5968	0.9214	0.9848
Q_ε	0.5592	0.9679	0.9977	0.9668	0.9997	1.0000	0.9452	0.9992	1.0000	0.8766	0.9974	0.9999	0.7437	0.9916	0.9995	0.2859	0.9387	0.9946
$t(\hat{\rho}^*)$	0.3114	0.8282	0.9602	0.6721	0.9535	0.9898	0.6421	0.9342	0.9835	0.5676	0.8933	0.9707	0.4794	0.8400	0.9424	0.3229	0.6968	0.8609
$Z(\hat{\rho})$				0.9484	0.9993	1.0000	0.9412	0.9988	1.0000	0.9173	0.9975	1.0000	0.8772	0.9961	0.9997	0.7484	0.9922	0.9994
$Z(t_{\hat{\rho}})$				0.8670	0.9932	0.9994	0.8543	0.9907	0.9992	0.8048	0.9876	0.9978	0.7353	0.9819	0.9967	0.5403	0.9619	0.9938

Table 4.8: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 250$, $\theta = 0.8$.

The results with a negative MA parameter are reported in Tables 4.9-4.14. The interpretation of $l = 0$ is not useful as the size is about 20% for $\theta = -0.2$ and increases with increasing MA parameter.

First, considering the case where $\theta = -0.2$ in Table 4.9 and Table 4.10. The CUSUM test Q_y is oversized for $l = 2$ and $l = 4$ in both sample sizes, close to the nominal significance level for $l \in 4, 6$ and undersized for $l = 12$. The CUSUM test Q_ε suffers more from size distortions and is undersized for $l \geq 4$ with $T = 100$ and close to the nominal size level for $l = 6$ in $T = 250$. The size is decreasing for both CUSUM tests with increasing l for all negative MA parameters.

The ADF-test provides the best results in terms of size and is close to the nominal size for $l = 2$ with decreasing size as l increases. The size of the PP-tests is far over 10% in the small sample and close to 10% in the large sample with $\alpha = 5\%$. Therefore, these tests provide the best power results but are not meaningfully interpretable.

Comparing the CUSUM tests and the ADF-test in terms of power, the Q_y test provides the best results for $\rho \leq 0.95$ and $l \leq 4$ with $T = 100$. When ρ decreases the power of the Q_ε test rises very sharply when l is chosen to be small. In fact, the power is the highest with $l = 2$ or with $l = 4$ and $\rho \leq 0.9$. The reason for the better power properties of the Q_ε test for $l = 2$ is that it is oversized. This behaviour can be confirmed with the larger sample size. The best power results are provided from the CUSUM test Q_ε with few exceptions, for example $\rho = 0.9$ and $l \geq 8$.

Analysing the results with $\theta = -0.5$ intensifies these observations. The CUSUM test Q_y is oversized for all l and Q_ε provides rejection frequencies of 25% with $l = 2$ and 0.0022% for $l = 12$ with $T = 100$. For $T = 250$, Q_ε is close to the nominal size for $l = 12$. The ADF-test is close to the significance level with $l = 4$. Again, the PP-tests are largely oversized.

Reasonable power properties for $l \geq 6$ are only obtained from the ADF-test. When Q_ε is close to the nominal significance level, the test provides the best power results.

Increasing the MA parameter to $\theta = -0.8$ demonstrates that all tests are oversized. The ADF-test with $l = 12$ is close to the nominal size in the small sample but none of the tests provide reasonable results.

In general, all tests suffer from large size distortion for a large negative MA parameter. When the negative MA parameter is relatively small, for example $\theta = -0.2$, the CUSUM tests with $l \leq 4$ are the preferable tests. With a moderate MA parameter $\theta = -0.5$, the ADF-test provides the best size and power properties. When the MA parameter is large, the tests are not able to distinguish adequately between a non-stationary and a stationary process.

This simulation study shows that the test based on squared CUSUMs of residuals provide better results in terms of size and power as the common used tests for a unit root. Especially when no MA parameter is present in the data, the MA parameter is positive or negative and small. The CUSUM test Q_y provides the best properties followed by Q_ε. The size properties of the Q_ε test suggest to consider another estimator of the long-run variance.

	$l = 0$			$l = 2$			$l = 4$			$l = 6$			$l = 8$			$l = 12$		
$\rho = 1.00$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0303	0.1105	0.1862	0.0141	0.0688	0.1278	0.0104	0.0588	0.1118	0.0079	0.0493	0.1009	0.0058	0.0423	0.0909	0.0032	0.0322	0.0767
Q_ε	0.0615	0.1712	0.2672	0.0098	0.0663	0.1306	0.0018	0.0351	0.0907	0.0002	0.0177	0.0634	0.0001	0.0068	0.0417	0.0000	0.0008	0.0121
$t(\hat{\rho}^*)$	0.0440	0.1377	0.2160	0.0097	0.0522	0.0989	0.0087	0.0471	0.0959	0.0079	0.0436	0.0893	0.0066	0.0440	0.0862	0.0087	0.0414	0.0831
$Z(\hat{\rho})$				0.0464	0.1257	0.1989	0.0537	0.1355	0.2065	0.0601	0.1465	0.2187	0.0655	0.1580	0.2313	0.0720	0.1717	0.2528
$Z(t_{\hat{\rho}})$				0.0360	0.1085	0.1778	0.0396	0.1160	0.1840	0.0423	0.1231	0.1949	0.0469	0.1310	0.2026	0.0516	0.1400	0.2167
$\rho = 0.99$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0363	0.1281	0.2268	0.0185	0.0795	0.1525	0.0125	0.0647	0.1321	0.0090	0.0543	0.1191	0.0075	0.0465	0.1063	0.0041	0.0346	0.0881
Q_ε	0.0766	0.2145	0.3300	0.0130	0.0816	0.1660	0.0026	0.0460	0.1125	0.0003	0.0201	0.0799	0.0000	0.0088	0.0488	0.0000	0.0009	0.0126
$t(\hat{\rho}^*)$	0.0562	0.1639	0.2638	0.0121	0.0579	0.1147	0.0099	0.0517	0.1077	0.0079	0.0453	0.0956	0.0077	0.0454	0.0928	0.0070	0.0429	0.0884
$Z(\hat{\rho})$				0.0444	0.1311	0.2221	0.0512	0.1442	0.2346	0.0587	0.1587	0.2512	0.0645	0.1714	0.2667	0.0724	0.1897	0.2882
$Z(t_{\hat{\rho}})$				0.0443	0.1308	0.2097	0.0504	0.1385	0.2205	0.0543	0.1481	0.2343	0.0590	0.1574	0.2456	0.0636	0.1704	0.2616
$\rho = 0.975$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0544	0.1739	0.3018	0.0263	0.1117	0.1993	0.0180	0.0931	0.1741	0.0136	0.0791	0.1559	0.0105	0.0689	0.1421	0.0056	0.0509	0.1179
Q_ε	0.1157	0.2998	0.4374	0.0204	0.1191	0.2276	0.0032	0.0670	0.1586	0.0003	0.0325	0.1110	0.0000	0.0120	0.0719	0.0000	0.0007	0.0197
$t(\hat{\rho}^*)$	0.0734	0.2279	0.3441	0.0152	0.0723	0.1400	0.0125	0.0634	0.1273	0.0112	0.0573	0.1171	0.0116	0.0551	0.1104	0.0084	0.0509	0.1020
$Z(\hat{\rho})$				0.0618	0.1890	0.3047	0.0726	0.2048	0.3169	0.0832	0.2256	0.3402	0.0932	0.2446	0.3584	0.1058	0.2643	0.3890
$Z(t_{\hat{\rho}})$				0.0594	0.1779	0.2783	0.0655	0.1888	0.2902	0.0738	0.2037	0.3085	0.0794	0.2163	0.3221	0.0882	0.2342	0.3439
$\rho = 0.950$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0959	0.2944	0.4676	0.0457	0.1891	0.3299	0.0321	0.1549	0.2854	0.0234	0.1324	0.2515	0.0173	0.1144	0.2275	0.0080	0.0840	0.1830
Q_ε	0.2085	0.4862	0.6612	0.0372	0.2114	0.3914	0.0067	0.1190	0.2794	0.0002	0.0577	0.1963	0.0001	0.0218	0.1270	0.0000	0.0016	0.0287
$t(\hat{\rho}^*)$	0.1353	0.3596	0.5242	0.0224	0.1159	0.2197	0.0185	0.0984	0.1883	0.0176	0.0859	0.1696	0.0146	0.0770	0.1517	0.0126	0.0600	0.1252
$Z(\hat{\rho})$				0.1160	0.3281	0.4876	0.1342	0.3532	0.5077	0.1545	0.3839	0.5362	0.1714	0.4096	0.5616	0.1867	0.4390	0.5984
$Z(t_{\hat{\rho}})$				0.1058	0.2890	0.4348	0.1190	0.3084	0.4543	0.1332	0.3320	0.4794	0.1448	0.3513	0.4994	0.1589	0.3775	0.5280
$\rho = 0.900$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.2712	0.6237	0.8044	0.1498	0.4404	0.6483	0.1067	0.3690	0.5694	0.0750	0.3155	0.5097	0.0519	0.2708	0.4637	0.0261	0.1964	0.3779
Q_ε	0.5395	0.8553	0.9468	0.1447	0.5325	0.7588	0.0285	0.3334	0.6012	0.0014	0.1712	0.4577	0.0000	0.0589	0.3018	0.0000	0.0036	0.0651
$t(\hat{\rho}^*)$	0.3827	0.7238	0.8583	0.0759	0.2803	0.4667	0.0483	0.2208	0.3791	0.0373	0.1736	0.3153	0.0309	0.1412	0.2666	0.0197	0.0953	0.1895
$Z(\hat{\rho})$				0.3596	0.6959	0.8498	0.4022	0.7227	0.8662	0.4428	0.7577	0.8837	0.4744	0.7821	0.8977	0.4975	0.8110	0.9158
$Z(t_{\hat{\rho}})$				0.3231	0.6341	0.7867	0.3535	0.6629	0.8066	0.3853	0.6928	0.8268	0.4081	0.7172	0.8440	0.4302	0.7431	0.8656
$\rho = 0.850$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.5251	0.8711	0.9637	0.3187	0.7074	0.8736	0.2317	0.6046	0.8029	0.1713	0.5221	0.7395	0.1284	0.4534	0.6771	0.0645	0.3375	0.5629
Q_ε	0.8545	0.9824	0.9980	0.3570	0.8330	0.9504	0.0872	0.6156	0.8598	0.0057	0.3449	0.7115	0.0006	0.1217	0.4953	0.0000	0.0064	0.1020
$t(\hat{\rho}^*)$	0.7101	0.9406	0.9803	0.1812	0.5240	0.7238	0.1086	0.3813	0.5779	0.0750	0.2887	0.4723	0.0517	0.2160	0.3794	0.0282	0.1361	0.2537
$Z(\hat{\rho})$				0.6928	0.9352	0.9794	0.7372	0.9465	0.9831	0.7738	0.9578	0.9868	0.7986	0.9650	0.9899	0.8097	0.9707	0.9937
$Z(t_{\hat{\rho}})$				0.6461	0.9040	0.9615	0.6833	0.9181	0.9654	0.7160	0.9304	0.9727	0.7378	0.9397	0.9766	0.7505	0.9473	0.9822

Table 4.9: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 100$, $\theta = -0.2$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0265	0.1028	0.1796	0.0140	0.0637	0.1252	0.0108	0.0556	0.1112	0.0095	0.0501	0.1048	0.0080	0.0474	0.1000	0.0061	0.0433	0.0935
Q_ε	0.0597	0.1747	0.2653	0.0160	0.0734	0.1418	0.0080	0.0518	0.1107	0.0045	0.0407	0.0952	0.0025	0.0333	0.0828	0.0006	0.0221	0.0674
$t(\hat{\rho}^*)$	0.0430	0.1321	0.2201	0.0079	0.0437	0.0976	0.0096	0.0434	0.0914	0.0076	0.0442	0.0906	0.0073	0.0412	0.0916	0.0090	0.0419	0.0894
$Z(\hat{\rho})$				0.0316	0.0988	0.1655	0.0325	0.0994	0.1609	0.0351	0.1045	0.1676	0.0402	0.1118	0.1756	0.0479	0.1256	0.1929
$Z(t_{\hat{\rho}})$				0.0254	0.0874	0.1514	0.0251	0.0876	0.1479	0.0281	0.0927	0.1522	0.0304	0.0990	0.1586	0.0371	0.1084	0.1733
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.0532	0.1854	0.3069	0.0277	0.1216	0.2195	0.0230	0.1055	0.1967	0.0184	0.0954	0.1839	0.0157	0.0895	0.1735	0.0129	0.0812	0.1585
Q_ε	0.1237	0.3195	0.4552	0.0353	0.1525	0.2758	0.0205	0.1133	0.2222	0.0115	0.0866	0.1907	0.0065	0.0711	0.1673	0.0014	0.0439	0.1296
$t(\hat{\rho}^*)$	0.0884	0.2394	0.3647	0.0167	0.0767	0.1518	0.0149	0.0723	0.1424	0.0132	0.0695	0.1370	0.0131	0.0697	0.1360	0.0122	0.0647	0.1278
$Z(\hat{\rho})$				0.0562	0.1837	0.2956	0.0583	0.1798	0.2903	0.0652	0.1904	0.2978	0.0712	0.2032	0.3110	0.0868	0.2267	0.3357
$Z(t_{\hat{\rho}})$				0.0516	0.1553	0.2565	0.0526	0.1536	0.2524	0.0574	0.1645	0.2620	0.0634	0.1749	0.2719	0.0747	0.1938	0.2925
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.1295	0.3743	0.5606	0.0713	0.2582	0.4263	0.0595	0.2270	0.3867	0.0513	0.2095	0.3624	0.0456	0.1953	0.3441	0.0356	0.1733	0.3138
Q_ε	0.2993	0.6076	0.7691	0.0999	0.3455	0.5366	0.0545	0.2684	0.4560	0.0332	0.2195	0.4038	0.0192	0.1784	0.3586	0.0047	0.1150	0.2867
$t(\hat{\rho}^*)$	0.2063	0.4729	0.6357	0.0393	0.1680	0.3000	0.0349	0.1490	0.2749	0.0317	0.1388	0.2622	0.0288	0.1297	0.2476	0.0231	0.1144	0.2211
$Z(\hat{\rho})$				0.1468	0.3973	0.5657	0.1498	0.3926	0.5575	0.1678	0.4093	0.5706	0.1840	0.4308	0.5901	0.2186	0.4710	0.6279
$Z(t_{\hat{\rho}})$				0.1249	0.3290	0.4856	0.1284	0.3281	0.4824	0.1401	0.3438	0.4984	0.1545	0.3640	0.5136	0.1782	0.4005	0.5464
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.3744	0.7443	0.8963	0.2354	0.5939	0.7906	0.1950	0.5383	0.7399	0.1693	0.4999	0.7071	0.1501	0.4673	0.6772	0.1159	0.4140	0.6272
Q_ε	0.7242	0.9450	0.9877	0.3600	0.7729	0.9117	0.2259	0.6610	0.8537	0.1457	0.5775	0.8036	0.0885	0.4997	0.7533	0.0212	0.3507	0.6475
$t(\hat{\rho}^*)$	0.5746	0.8601	0.9461	0.1481	0.4452	0.6534	0.1196	0.3836	0.5844	0.1048	0.3471	0.5436	0.0915	0.3138	0.5035	0.0683	0.2571	0.4349
$Z(\hat{\rho})$				0.4698	0.8129	0.9228	0.4814	0.8066	0.9171	0.5121	0.8236	0.9227	0.5495	0.8402	0.9314	0.6089	0.8698	0.9480
$Z(t_{\hat{\rho}})$				0.4004	0.7291	0.8651	0.4156	0.7281	0.8595	0.4458	0.7467	0.8693	0.4756	0.7671	0.8809	0.5295	0.8031	0.9048
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.8785	0.9938	0.9996	0.7251	0.9654	0.9961	0.6490	0.9376	0.9888	0.5853	0.9099	0.9795	0.5285	0.8799	0.9698	0.4272	0.8190	0.9411
Q_ε	0.9977	1.0000	1.0000	0.9395	0.9986	0.9999	0.8291	0.9923	0.9997	0.6807	0.9785	0.9979	0.4943	0.9500	0.9935	0.1444	0.8251	0.9697
$t(\hat{\rho}^*)$	0.9895	0.9999	1.0000	0.6678	0.9362	0.9864	0.5308	0.8678	0.9549	0.4236	0.7887	0.9146	0.3374	0.7087	0.8655	0.2227	0.5594	0.7604
$Z(\hat{\rho})$				0.9759	0.9994	1.0000	0.9759	0.9993	1.0000	0.9807	0.9992	1.0000	0.9865	0.9996	1.0000	0.9927	0.9999	1.0000
$Z(t_{\hat{\rho}})$				0.9541	0.9973	0.9997	0.9572	0.9971	0.9997	0.9664	0.9974	0.9997	0.9737	0.9983	0.9999	0.9842	0.9990	1.0000
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9924	1.0000	1.0000	0.9550	0.9997	1.0000	0.9135	0.9969	1.0000	0.8633	0.9926	0.9998	0.8090	0.9829	0.9989	0.6990	0.9582	0.9940
Q_ε	1.0000	1.0000	1.0000	0.9997	1.0000	1.0000	0.9942	1.0000	1.0000	0.9615	0.9999	1.0000	0.8589	0.9987	1.0000	0.3310	0.9720	0.9988
$t(\hat{\rho}^*)$	0.9999	1.0000	1.0000	0.9620	0.9990	0.9998	0.8648	0.9879	0.9978	0.7436	0.9578	0.9896	0.6166	0.9066	0.9721	0.3896	0.7666	0.8997
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				0.9996	1.0000	1.0000	0.9997	1.0000	1.0000	0.9998	1.0000	1.0000	0.9998	1.0000	1.0000	0.9999	1.0000	1.0000

Table 4.10: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 250$, $\theta = -0.2$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
$\rho=1.00$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.1632	0.3674	0.5239	0.0543	0.1711	0.2733	0.0325	0.1186	0.2091	0.0246	0.0924	0.1729	0.0173	0.0768	0.1500	0.0097	0.0567	0.1183
Q_ε	0.4655	0.6642	0.7569	0.0712	0.2485	0.3809	0.0069	0.1053	0.2195	0.0004	0.0406	0.1324	0.0001	0.0120	0.0721	0.0000	0.0022	0.0174
$t(\hat{\rho}^*)$	0.3835	0.5864	0.6911	0.0272	0.0994	0.1750	0.0107	0.0551	0.1081	0.0089	0.0463	0.0928	0.0087	0.0439	0.0884	0.0071	0.0387	0.0818
$Z(\hat{\rho})$				0.3256	0.4874	0.5860	0.3629	0.5130	0.6012	0.4063	0.5522	0.6285	0.4466	0.5833	0.6562	0.5071	0.6327	0.7008
$Z(t_{\hat{\rho}})$				0.2881	0.4500	0.5444	0.3169	0.4738	0.5611	0.3521	0.5056	0.5936	0.3854	0.5338	0.6223	0.4371	0.5872	0.6610
$\rho=0.99$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.1813	0.4198	0.5819	0.0632	0.1916	0.3126	0.0367	0.1364	0.2341	0.0258	0.1094	0.1925	0.0188	0.0908	0.1682	0.0094	0.0669	0.1354
Q_ε	0.5289	0.7396	0.8329	0.0859	0.2852	0.4344	0.0096	0.1222	0.2534	0.0010	0.0464	0.1529	0.0002	0.0144	0.0899	0.0000	0.0020	0.0191
$t(\hat{\rho}^*)$	0.4371	0.6528	0.7569	0.0269	0.1133	0.1925	0.0109	0.0609	0.1171	0.0084	0.0486	0.1021	0.0080	0.0459	0.0922	0.0072	0.0417	0.0847
$Z(\hat{\rho})$				0.3329	0.5180	0.6251	0.3752	0.5456	0.6450	0.4268	0.5891	0.6836	0.4692	0.6284	0.7155	0.5350	0.6876	0.7619
$Z(t_{\hat{\rho}})$				0.3353	0.5049	0.6069	0.3684	0.5326	0.6241	0.4079	0.5661	0.6568	0.4407	0.6017	0.6881	0.5010	0.6540	0.7327
$\rho=0.975$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.2613	0.5456	0.7152	0.0905	0.2688	0.4195	0.0530	0.1904	0.3236	0.0366	0.1496	0.2677	0.0271	0.1229	0.2329	0.0146	0.0877	0.1828
Q_ε	0.6753	0.8601	0.9323	0.1329	0.4020	0.5735	0.0133	0.1808	0.3611	0.0004	0.0672	0.2206	0.0000	0.0187	0.1210	0.0000	0.0023	0.0230
$t(\hat{\rho}^*)$	0.5767	0.7857	0.8713	0.0442	0.1600	0.2677	0.0140	0.0778	0.1561	0.0110	0.0623	0.1270	0.0094	0.0521	0.1135	0.0091	0.0476	0.1018
$Z(\hat{\rho})$				0.4581	0.6679	0.7707	0.5119	0.6952	0.7880	0.5695	0.7381	0.8176	0.6157	0.7741	0.8439	0.6829	0.8215	0.8797
$Z(t_{\hat{\rho}})$				0.4572	0.6482	0.7471	0.4983	0.6759	0.7629	0.5437	0.7105	0.7926	0.5847	0.7435	0.8195	0.6482	0.7933	0.8577
$\rho=0.950$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.4213	0.7453	0.8892	0.1662	0.4298	0.6157	0.0989	0.3176	0.4912	0.0667	0.2562	0.4159	0.0488	0.2181	0.3691	0.0256	0.1618	0.2953
Q_ε	0.8727	0.9704	0.9896	0.2383	0.6129	0.7911	0.0275	0.3097	0.5565	0.0014	0.1276	0.3687	0.0004	0.0339	0.2166	0.0000	0.0040	0.0418
$t(\hat{\rho}^*)$	0.7854	0.9343	0.9701	0.0790	0.2640	0.4180	0.0274	0.1234	0.2370	0.0182	0.0970	0.1867	0.0169	0.0831	0.1695	0.0109	0.0628	0.1338
$Z(\hat{\rho})$				0.6761	0.8644	0.9296	0.7324	0.8858	0.9385	0.7857	0.9105	0.9525	0.8269	0.9304	0.9633	0.8785	0.9532	0.9763
$Z(t_{\hat{\rho}})$				0.6710	0.8482	0.9124	0.7143	0.8702	0.9222	0.7600	0.8949	0.9390	0.7965	0.9132	0.9509	0.8481	0.9394	0.9666
$\rho=0.900$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.7490	0.9588	0.9945	0.4008	0.7519	0.8978	0.2642	0.6137	0.7983	0.1898	0.5222	0.7193	0.1438	0.4486	0.6543	0.0797	0.3401	0.5500
Q_ε	0.9944	0.9998	1.0000	0.5705	0.9143	0.9813	0.1059	0.6474	0.8738	0.0061	0.3231	0.6896	0.0005	0.0933	0.4439	0.0000	0.0098	0.0876
$t(\hat{\rho}^*)$	0.9763	0.9985	0.9998	0.2326	0.5532	0.7256	0.0798	0.2914	0.4617	0.0453	0.1990	0.3508	0.0319	0.1539	0.2801	0.0188	0.1011	0.1995
$Z(\hat{\rho})$				0.9436	0.9937	0.9989	0.9626	0.9957	0.9989	0.9776	0.9976	0.9992	0.9861	0.9988	0.9995	0.9951	0.9994	0.9998
$Z(t_{\hat{\rho}})$				0.9390	0.9913	0.9973	0.9546	0.9940	0.9983	0.9699	0.9961	0.9989	0.9799	0.9975	0.9992	0.9901	0.9988	0.9995
$\rho=0.850$																		
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9440	0.9990	1.0000	0.6670	0.9374	0.9891	0.4929	0.8498	0.9533	0.3825	0.7699	0.9117	0.3011	0.6862	0.8633	0.1879	0.5553	0.7722
Q_ε	0.9999	1.0000	1.0000	0.8472	0.9949	0.9998	0.2450	0.8898	0.9840	0.0118	0.5527	0.8978	0.0011	0.1679	0.6636	0.0000	0.0146	0.1285
$t(\hat{\rho}^*)$	0.9996	1.0000	1.0000	0.4722	0.8116	0.9240	0.1774	0.5045	0.6961	0.0949	0.3423	0.5309	0.0610	0.2530	0.4241	0.0313	0.1516	0.2762
$Z(\hat{\rho})$				0.9984	1.0000	1.0000	0.9994	1.0000	1.0000	0.9998	1.0000	1.0000	0.9998	1.0000	1.0000	0.9999	1.0000	1.0000
$Z(t_{\hat{\rho}})$				0.9974	1.0000	1.0000	0.9988	1.0000	1.0000	0.9994	1.0000	1.0000	0.9998	1.0000	1.0000	0.9999	1.0000	1.0000

Table 4.11: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 100$, $\theta = -0.5$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.1501	0.3556	0.5102	0.0505	0.1602	0.2653	0.0300	0.1119	0.1993	0.0213	0.0894	0.1672	0.0175	0.0788	0.1498	0.0133	0.0647	0.1275
Q_ε	0.4881	0.6786	0.7693	0.1219	0.2891	0.4097	0.0452	0.1669	0.2732	0.0205	0.1076	0.2021	0.0089	0.0771	0.1594	0.0012	0.0406	0.1080
$t(\hat{\rho}^*)$	0.4133	0.6020	0.6986	0.0308	0.1018	0.1793	0.0119	0.0557	0.1097	0.0093	0.0477	0.0964	0.0096	0.0469	0.0909	0.0089	0.0449	0.0925
$Z(\hat{\rho})$				0.2556	0.4110	0.5125	0.2591	0.3999	0.4894	0.2841	0.4211	0.5045	0.3153	0.4494	0.5291	0.3705	0.5011	0.5756
$Z(t_{\hat{\rho}})$				0.2319	0.3745	0.4702	0.2365	0.3689	0.4558	0.2586	0.3889	0.4733	0.2835	0.4130	0.4982	0.3308	0.4600	0.5403
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.2526	0.5374	0.7112	0.0934	0.2638	0.4177	0.0600	0.1938	0.3244	0.0469	0.1609	0.2787	0.0367	0.1398	0.2505	0.0263	0.1166	0.2138
Q_ε	0.7182	0.8823	0.9427	0.2286	0.4838	0.6336	0.0921	0.3013	0.4562	0.0440	0.2074	0.3595	0.0203	0.1487	0.2900	0.0034	0.0795	0.2013
$t(\hat{\rho}^*)$	0.6275	0.8111	0.8872	0.0568	0.1727	0.2843	0.0215	0.0886	0.1711	0.0180	0.0739	0.1430	0.0162	0.0707	0.1344	0.0130	0.0642	0.1261
$Z(\hat{\rho})$				0.4015	0.6154	0.7245	0.4067	0.6029	0.7054	0.4507	0.6293	0.7249	0.4924	0.6615	0.7470	0.5624	0.7205	0.7943
$Z(t_{\hat{\rho}})$				0.3872	0.5801	0.6852	0.3952	0.5712	0.6665	0.4309	0.5988	0.6897	0.4680	0.6269	0.7170	0.5314	0.6853	0.7610
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.4965	0.8193	0.9396	0.2078	0.5112	0.7017	0.1361	0.3949	0.5855	0.1031	0.3312	0.5203	0.0856	0.2917	0.4725	0.0631	0.2451	0.4095
Q_ε	0.9501	0.9934	0.9989	0.4845	0.7951	0.9103	0.2272	0.5876	0.7706	0.1111	0.4392	0.6548	0.0530	0.3350	0.5649	0.0067	0.1916	0.4139
$t(\hat{\rho}^*)$	0.9052	0.9797	0.9930	0.1312	0.3571	0.5297	0.0457	0.1783	0.3225	0.0332	0.1396	0.2628	0.0301	0.1285	0.2402	0.0244	0.1116	0.2216
$Z(\hat{\rho})$				0.7174	0.9006	0.9529	0.7246	0.8927	0.9447	0.7646	0.9094	0.9539	0.8061	0.9241	0.9631	0.8667	0.9512	0.9769
$Z(t_{\hat{\rho}})$				0.6949	0.8706	0.9309	0.7034	0.8659	0.9233	0.7411	0.8867	0.9348	0.7787	0.9064	0.9469	0.8396	0.9337	0.9655
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.8634	0.9909	0.9995	0.5385	0.8721	0.9673	0.3952	0.7702	0.9163	0.3228	0.6955	0.8690	0.2693	0.6389	0.8272	0.2011	0.5535	0.7621
Q_ε	0.9997	1.0000	1.0000	0.8944	0.9932	0.9989	0.6322	0.9440	0.9892	0.3980	0.8552	0.9651	0.2220	0.7486	0.9255	0.0360	0.5215	0.8102
$t(\hat{\rho}^*)$	0.9987	0.9999	1.0000	0.4263	0.7566	0.8941	0.1723	0.4855	0.6885	0.1187	0.3889	0.5930	0.0977	0.3399	0.5378	0.0747	0.2676	0.4490
$Z(\hat{\rho})$				0.9818	0.9991	0.9997	0.9836	0.9989	0.9997	0.9895	0.9993	0.9997	0.9946	0.9995	0.9998	0.9978	0.9997	0.9999
$Z(t_{\hat{\rho}})$				0.9744	0.9971	0.9995	0.9770	0.9969	0.9993	0.9857	0.9983	0.9997	0.9907	0.9990	0.9997	0.9964	0.9997	0.9999
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9993	1.0000	1.0000	0.9527	0.9994	1.0000	0.8749	0.9937	0.9999	0.8031	0.9833	0.9984	0.7365	0.9681	0.9960	0.6181	0.9285	0.9870
Q_ε	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9918	1.0000	1.0000	0.9348	0.9996	1.0000	0.7678	0.9955	1.0000	0.2084	0.9433	0.9960
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.9433	0.9973	0.9998	0.6840	0.9390	0.9846	0.5014	0.8513	0.9461	0.3903	0.7582	0.9008	0.2499	0.6053	0.7889
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	1.0000	1.0000	1.0000	0.9982	1.0000	1.0000	0.9900	1.0000	1.0000	0.9714	0.9997	1.0000	0.9456	0.9989	1.0000	0.8782	0.9934	0.9996
Q_ε	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9990	1.0000	1.0000	0.9763	1.0000	1.0000	0.4236	0.9968	1.0000
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.9996	1.0000	1.0000	0.9554	0.9979	0.9996	0.8323	0.9801	0.9965	0.6905	0.9423	0.9846	0.4457	0.8033	0.9238
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.12: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 250$, $\theta = -0.5$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho = 1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.7931	0.9589	0.9905	0.4271	0.7161	0.8567	0.2852	0.5573	0.7211	0.2077	0.4603	0.6186	0.1625	0.3857	0.5496	0.1070	0.2958	0.4477
Q_ε	0.9927	0.9992	1.0000	0.6898	0.9015	0.9568	0.1649	0.6373	0.8102	0.0108	0.3027	0.5879	0.0018	0.0787	0.3438	0.0002	0.0134	0.0675
$t(\hat{\rho}^*)$	0.9843	0.9973	0.9993	0.4187	0.6443	0.7528	0.1119	0.2877	0.4119	0.0360	0.1421	0.2335	0.0204	0.0851	0.1558	0.0081	0.0449	0.0985
$Z(\hat{\rho})$				0.9676	0.9905	0.9958	0.9774	0.9923	0.9965	0.9850	0.9951	0.9981	0.9897	0.9970	0.9983	0.9944	0.9984	0.9993
$Z(t_{\hat{\rho}})$				0.9610	0.9867	0.9933	0.9686	0.9889	0.9945	0.9780	0.9922	0.9962	0.9840	0.9948	0.9974	0.9904	0.9975	0.9984
									$\rho = 0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.8525	0.9825	0.9976	0.4909	0.7860	0.9038	0.3349	0.6304	0.7865	0.2459	0.5232	0.6905	0.1876	0.4446	0.6189	0.1208	0.3444	0.5100
Q_ε	0.9991	0.9999	1.0000	0.7746	0.9475	0.9840	0.1899	0.7203	0.8768	0.0119	0.3482	0.6723	0.0018	0.0901	0.3868	0.0003	0.0137	0.0770
$t(\hat{\rho}^*)$	0.9960	0.9996	0.9999	0.4858	0.7219	0.8233	0.1267	0.3218	0.4680	0.0446	0.1565	0.2658	0.0219	0.0968	0.1747	0.0105	0.0493	0.1045
$Z(\hat{\rho})$				0.9861	0.9973	0.9992	0.9905	0.9984	0.9994	0.9946	0.9991	0.9998	0.9973	0.9996	0.9998	0.9991	0.9998	0.9999
$Z(t_{\hat{\rho}})$				0.9862	0.9967	0.9988	0.9905	0.9979	0.9992	0.9933	0.9987	0.9995	0.9959	0.9991	0.9997	0.9987	0.9997	0.9998
									$\rho = 0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9312	0.9965	0.9993	0.6114	0.8860	0.9645	0.4304	0.7475	0.8858	0.3259	0.6422	0.8038	0.2551	0.5588	0.7331	0.1734	0.4398	0.6242
Q_ε	0.9998	0.9999	1.0000	0.8787	0.9875	0.9983	0.2581	0.8358	0.9512	0.0182	0.4493	0.7904	0.0032	0.1206	0.4949	0.0007	0.0200	0.1001
$t(\hat{\rho}^*)$	0.9993	0.9999	0.9999	0.6032	0.8345	0.9107	0.1703	0.4203	0.5729	0.0552	0.2049	0.3324	0.0241	0.1186	0.2184	0.0096	0.0620	0.1268
$Z(\hat{\rho})$				0.9981	0.9995	0.9999	0.9985	0.9996	0.9999	0.9992	0.9998	0.9999	0.9995	0.9999	0.9999	0.9998	0.9999	0.9999
$Z(t_{\hat{\rho}})$				0.9979	0.9995	0.9997	0.9983	0.9996	0.9998	0.9991	0.9996	0.9999	0.9993	0.9998	0.9999	0.9996	0.9999	0.9999
									$\rho = 0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9885	0.9998	1.0000	0.7957	0.9723	0.9967	0.6199	0.8978	0.9727	0.4969	0.8195	0.9334	0.4031	0.7451	0.8890	0.2842	0.6259	0.8024
Q_ε	1.0000	1.0000	1.0000	0.9700	0.9996	0.9999	0.3979	0.9505	0.9946	0.0261	0.6132	0.9200	0.0050	0.1757	0.6575	0.0011	0.0279	0.1328
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.7912	0.9458	0.9803	0.2827	0.5896	0.7471	0.0943	0.3146	0.4793	0.0490	0.1921	0.3190	0.0186	0.0916	0.1769
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
									$\rho = 0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	1.0000	1.0000	1.0000	0.9811	0.9998	1.0000	0.9118	0.9961	0.9998	0.8136	0.9856	0.9989	0.7225	0.9655	0.9954	0.5743	0.9087	0.9804
Q_ε	1.0000	1.0000	1.0000	0.9997	1.0000	1.0000	0.6462	0.9988	1.0000	0.0495	0.8309	0.9936	0.0080	0.2740	0.8481	0.0012	0.0401	0.1862
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.9673	0.9984	0.9998	0.5546	0.8654	0.9459	0.2390	0.5711	0.7518	0.1102	0.3663	0.5509	0.0365	0.1694	0.3075
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
									$\rho = 0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	1.0000	1.0000	1.0000	0.9993	1.0000	1.0000	0.9898	1.0000	1.0000	0.9638	0.9994	1.0000	0.9246	0.9981	0.9999	0.8216	0.9881	0.9992
Q_ε	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7975	0.9999	1.0000	0.0753	0.9155	0.9991	0.0157	0.3471	0.9050	0.0030	0.0536	0.2166
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.9972	0.9999	1.0000	0.7943	0.9695	0.9924	0.4303	0.7907	0.9099	0.2081	0.5573	0.7455	0.0608	0.2416	0.4300
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.13: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 100$, $\theta = -0.8$.

	$l=0$			$l=2$			$l=4$			$l=6$			$l=8$			$l=12$		
									$\rho=1.00$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.7952	0.9622	0.9912	0.4333	0.7260	0.8575	0.2918	0.5622	0.7247	0.2172	0.4589	0.6205	0.1692	0.3874	0.5491	0.1187	0.2987	0.4435
Q_ε	0.9963	0.9996	1.0000	0.8464	0.9444	0.9752	0.6184	0.8224	0.8963	0.3857	0.6851	0.8028	0.2056	0.5444	0.7002	0.0226	0.3023	0.5116
$t(\hat{\rho}^*)$	0.9913	0.9984	0.9998	0.5529	0.7318	0.8123	0.1943	0.3708	0.4887	0.0692	0.1941	0.2978	0.0323	0.1124	0.1957	0.0148	0.0624	0.1234
$Z(\hat{\rho})$				0.9512	0.9833	0.9917	0.9525	0.9812	0.9893	0.9633	0.9846	0.9911	0.9708	0.9877	0.9935	0.9822	0.9932	0.9964
$Z(t_{\hat{\rho}})$				0.9495	0.9789	0.9891	0.9509	0.9777	0.9870	0.9601	0.9817	0.9894	0.9681	0.9858	0.9915	0.9786	0.9910	0.9953
									$\rho=0.99$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9454	0.9983	1.0000	0.6356	0.8987	0.9720	0.4581	0.7664	0.8973	0.3540	0.6635	0.8190	0.2873	0.5818	0.7526	0.2076	0.4700	0.6448
Q_ε	1.0000	1.0000	1.0000	0.9763	0.9983	0.9998	0.8348	0.9676	0.9898	0.6077	0.8920	0.9600	0.3581	0.7750	0.9017	0.0392	0.4938	0.7346
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.7731	0.9122	0.9564	0.3216	0.5657	0.6982	0.1257	0.3091	0.4479	0.0559	0.1880	0.3093	0.0215	0.0955	0.1849
$Z(\hat{\rho})$				0.9987	0.9999	1.0000	0.9990	0.9999	1.0000	0.9993	1.0000	1.0000	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				0.9988	0.9997	1.0000	0.9989	0.9997	0.9999	0.9994	0.9999	1.0000	0.9996	0.9999	1.0000	0.9999	1.0000	1.0000
									$\rho=0.975$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	0.9979	1.0000	1.0000	0.8905	0.9931	0.9991	0.7437	0.9566	0.9931	0.6192	0.9054	0.9759	0.5307	0.8518	0.9497	0.4075	0.7494	0.8943
Q_ε	1.0000	1.0000	1.0000	0.9994	1.0000	1.0000	0.9827	0.9993	0.9998	0.8847	0.9934	0.9987	0.6478	0.9636	0.9942	0.0965	0.7906	0.9483
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.9647	0.9948	0.9985	0.5931	0.8399	0.9239	0.2691	0.5588	0.7224	0.1335	0.3610	0.5388	0.0486	0.1973	0.3354
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
									$\rho=0.950$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	1.0000	1.0000	1.0000	0.9969	1.0000	1.0000	0.9744	0.9998	0.9999	0.9260	0.9981	0.9999	0.8679	0.9921	0.9997	0.7630	0.9736	0.9973
Q_ε	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9979	1.0000	1.0000	0.9461	0.9999	1.0000	0.2635	0.9843	0.9997
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	0.9998	1.0000	1.0000	0.9178	0.9908	0.9979	0.6236	0.8913	0.9593	0.3723	0.7183	0.8582	0.1567	0.4441	0.6396
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
									$\rho=0.900$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9990	1.0000	1.0000	0.9940	1.0000	1.0000
Q_ε	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.5719	1.0000	1.0000
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	1.0000	1.0000	0.9779	0.9996	1.0000	0.8636	0.9857	0.9970	0.5166	0.8519	0.9523
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
									$\rho=0.850$									
α	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
Q_y	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q_ε	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6842	1.0000	1.0000
$t(\hat{\rho}^*)$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	1.0000	1.0000	0.9871	0.9994	0.9999	0.7994	0.9735	0.9947
$Z(\hat{\rho})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$Z(t_{\hat{\rho}})$				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.14: Empirical Rejection Frequencies of the Tests Q_y, Q_ε, $t(\hat{\rho}^*)$, $Z(\hat{\rho})$ and $Z(t_{\hat{\rho}})$ with Different ρ and l, $T = 250$, $\theta = -0.8$.

4.6 Application to Nelson-Plosser Data

In the following section, the non-stationarity CUSUM tests are applied to the historical data analysed by Nelson and Plosser (1982). Many authors studied this data set in context of non-stationarity and stationarity tests (cf. Perron (1988), Kwiatkowski et al. (1992b), and DeJong et al. (1992a)). The data set includes U.S. annual data for 14 macroeconomic and financial time series starting in the year 1860 and ending in 1970. In Figure 4.1 the yearly data of the series are visualized with the respective measurement. They are of different length so that the number of observations differ from 62 to 111.

The main findings of the previous studies regarding this data set are the following. Nelson and Plosser (1982) find strong evidence for a unit root with the DF-test and Perron (1988) and DeJong et al. (1992a) support these findings. The hypothesis of a unit root is only rejected for the unemployment rate and the industrial production. Kwiatkowski et al. (1992b) analyse the series with the stationarity hypothesis and confirm that the unemployment rate is stationary, while they reject the stationarity hypothesis for the industrial production and therefore the results are not clear. In six time series (real per capita GNP, employment, unemployment rate, GNP deflator, wages and money) they can neither reject the unit root hypothesis nor the trend stationarity. The other series are clearly non-stationary.

Following Nelson and Plosser (1982), the natural logarithm of the time series is analysed. Calculating the logarithm of the data set is motivated by the fact that economic time series often exhibit a nonlinear trend. In proportion to the absolute value the variation of the time series increases in mean and variance. Hence, the transformed series exhibit a linear trend. The tests Q_y and Q_ε are applied to the Nelson-Plosser data to test the null hypothesis of a unit root against the alternative of level stationarity ($z_t = 1$) and the alternative that the process is trend stationary ($z_t = (1, t)'$). The lag truncation parameter $l \in \{0, 2, 4, 6, 8\}$ is used as the time series are short and the number of estimated autocovariances is chosen to be small. Despite that, the properties of the tests in the simulation study show the best results for low l. The results are reported in Table 4.6 and the findings in this study are the following.

The Q_y test rejects the unit root hypothesis in the constant case for the unemployment rate at 1% significance level for all l. This finding is supported by the Q_ε test which rejects the hypothesis for the unemployment rate at 1% for $l = 0$, 5% for $l \in \{2, 4\}$ and 10% for $l \in \{6, 8\}$. There is no evidence of level stationarity in the other economic time series. Note that the resulting test statistics are very low and many series would reject the explosive alternative on the left side of the distribution. This behaviour is not surprising and is caused by the obvious trend in the series and therefore the results of the tests without including a linear trend are not reasonable.

With the alternative hypothesis of trend stationarity the results mitigate. The non-stationarity hypothesis for the unemployment rate can be rejected with Q_y at 5% for $l \in \{0, 2\}$ and 10% for the rest and with Q_ε only at 10% for $l \in 0, 2$. Further rejections are not obtained and therefore, no evidence of trend stationarity can be found in 13 of the 14 time series. The results for the unemployment rate are reasonable and the test statistic is of lower magnitude for larger l which is supported by the simulation study. Furthermore, the Q_y test has more power than the Q_ε test and it can be assumed that the unemployment rate is stationary. The magnitudes of the test statistics for the other series are very low and far from rejecting the null hypothesis at the 10% level.

For the industrial production the previous findings of other researchers are not confirmed. Although the test statistics in the constant and linear trend case are the highest compared to the other 12 series, where the null hypothesis cannot be rejected, they are far from rejecting the unit root for the Q_y test. With $l = 0$ the result is $Q_\varepsilon = 34.88$ which is close to the 10% significance level of 34.929. In this time series the Q_ε has, compared to the Q_y test, relatively high values. If we assume that the industrial production is non-stationary or stationary but rather close the null hypothesis, high values of Q_ε in proportion to Q_y are observed with a negative MA parameter in the simulation study. The simulation shows further that all tests are largely oversized when a negative MA parameter is present in the data and therefore a negative MA parameter in the industrial production series could be the reason for the findings of the other authors, that the industrial production is stationary.

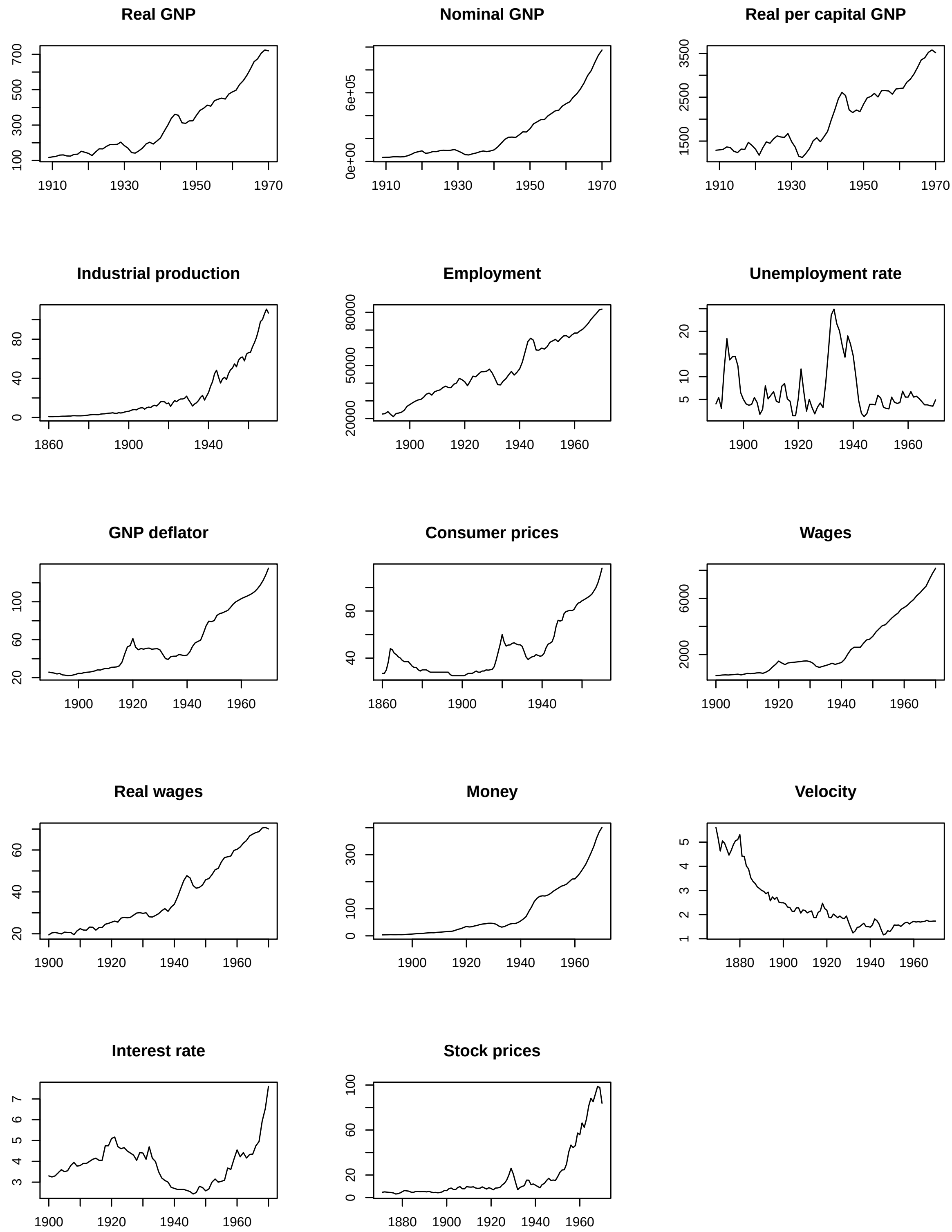


Figure 4.1: Nelson-Plosser Data Set of 14 Macroeconomic Time Series.

$z_t = 1$			Q_y					Q_ε		
Time series	$l=0$	$l=2$	$l=4$	$l=6$	$l=8$	$l=0$	$l=2$	$l=4$	$l=6$	$l=8$
Real GNP	54.01	94.60	107.98	116.52	129.88	0.95	1.67	1.91	2.06	2.30
Nominal GNP	47.52	93.32	114.31	133.20	156.24	0.79	1.54	1.89	2.19	2.57
Real per capital GNP	126.54	197.54	199.22	189.46	193.44	2.27	3.55	3.59	3.42	3.50
Industrial production	46.16	59.71	70.39	76.03	87.51	0.72	0.93	1.10	1.19	1.37
Employment	69.45	113.21	130.23	140.05	157.38	1.02	1.67	1.92	2.06	2.31
Unemployment rate	17051.12	15726.62	13602.65	10745.08	9802.52	40.08	37.01	32.10	25.52	23.42
GNP deflator	56.61	106.47	133.84	154.45	173.23	0.87	1.63	2.04	2.34	2.63
Consumer prices	107.03	206.48	240.70	262.35	278.81	1.98	3.78	4.39	4.78	5.07
Wages	38.81	78.18	100.94	120.12	141.80	0.62	1.24	1.59	1.89	2.23
Real wages	38.43	60.66	74.05	83.15	94.88	0.69	1.09	1.33	1.49	1.70
Money	18.82	46.49	67.90	86.21	103.26	0.31	0.75	1.10	1.39	1.67
Velocity	168.11	196.61	175.85	159.53	164.78	2.98	3.49	3.12	2.83	2.92
Interest rate	506.84	683.54	848.34	980.06	1071.98	5.93	7.32	8.74	9.95	10.87
Stock prices	249.56	316.71	298.92	258.27	259.25	3.24	4.12	3.91	3.40	3.44
$z_t = (1,t)'$			Q_y					Q_ε		
Time series	$l=0$	$l=2$	$l=4$	$l=6$	$l=8$	$l=0$	$l=2$	$l=4$	$l=6$	$l=8$
Real GNP	1515.76	2245.14	2065.74	1727.45	1570.63	14.68	21.75	20.04	16.85	15.41
Nominal GNP	739.77	1214.11	1190.77	1130.85	1174.50	8.59	14.10	13.83	13.14	13.65
Real per capital GNP	1879.99	2755.92	2545.82	2153.73	1985.35	15.26	22.39	20.72	17.63	16.34
Industrial production	4841.59	4699.98	4228.12	3154.73	2859.76	34.88	33.90	30.53	22.92	20.92
Employment	2783.33	3869.99	3681.58	3160.88	3020.80	17.41	24.23	23.07	19.85	18.99
Unemployment rate	15865.07	14634.75	12657.04	9995.21	9115.17	40.80	37.68	32.67	25.96	23.81
GNP deflator	1598.66	2736.25	3140.26	3318.61	3428.23	9.26	15.78	18.09	19.11	19.74
Consumer prices	314.42	590.80	670.83	715.51	744.66	5.11	9.55	10.82	11.53	11.99
Wages	1095.30	1844.61	1974.79	1974.76	2076.91	9.05	15.23	16.30	16.30	17.14
Real wages	1243.48	1535.58	1462.69	1213.31	1068.06	16.06	19.74	18.71	15.44	13.57
Money	1613.60	3273.42	3967.76	4181.50	4209.36	8.44	17.14	20.79	21.93	22.08
Velocity	766.90	859.93	721.48	604.24	587.85	12.97	14.53	12.19	10.19	9.91
Interest rate	487.88	656.92	814.67	940.64	1028.32	5.92	7.30	8.71	9.92	10.82
Stock prices	1379.37	1667.55	1469.40	1137.85	1043.44	16.45	19.86	17.53	13.64	12.61

Table 4.15: Q_y and Q_ε Tests for $z = 1$ and $z = (1,t)'$ Applied to Nelson-Plosser Data.

4.7 Conclusion

This paper presents two new tests for a unit root. The testing procedure differs from existing non-stationarity tests as it is not based on estimating the autoregressive parameter ρ or standardising existing unit root tests. The approach is based on squared CUSUMs of residuals and the test statistics are based on the sum of all possible partial sums of the series. Therefore, the behaviour of a unit root time series, where large positive and negative consecutive observations occur, is considered in the test statistics. The limiting distribution and consistency is derived and simulation results show that the tests perform well in finite samples and provide better size and power properties than the tests proposed by Said and Dickey (1984), Phillips (1987) and Phillips and Perron (1988). Especially when no MA parameter or a positive MA parameter is present in the simulated data, the CUSUM tests provide the best results. An interesting feature of the approach is the possibility to derive the limiting distribution under the alternative hypothesis. Therefore, it is possible to construct a statistic to test the stationarity under the null hypothesis with the correct standardisation against the alternative of a unit root with a rewritten test statistic. In addition, a test against a change in persistence can be constructed based on the ratio of the CUSUM tests in each sub-sample.

Chapter 5

Squared CUSUM based Tests for a Change in Persistence

Co-authored with Philipp Sibbertsen.

5.1 Introduction

Testing for changes in persistence is based on the requirement to divide a time series into stationary and non-stationary parts. Modelling a time series without regarding a change in persistence yield to incorrect inference and biased forecasts. The difference between $I(0)$ and $I(1)$ series has many practical and theoretical implications. Different models are considered for the correct specification, limiting distributions of hypotheses tests differ as the series are of different orders in probability and the optimal forecast differs.

Many tests have been developed to consider changes in persistence in time series. Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004) propose ratio tests based on partial sums of OLS residuals in the respective sub-sample and Leybourne et al. (2007b) suggest a CUSUM of squared residuals based test. Both test methods have some shortcomings.

The ratio based tests are based on sums of partial sum processes starting with the first observation for each partial sum. This results in convergence to Brownian bridges, but the first residuals are more weighted than the last residuals in this test statistic. The CUSUM of squares based tests consider squared residuals but ignore the cross-dependencies of consecutive residuals. These are expected to be large under the null hypothesis of a unit root, as the process consists sequences of large positive and negative consecutive observations.

Therefore, we construct two test statistics based on squared CUSUMs of OLS sub-sample residuals to test both, the null hypothesis of stationarity and the null hypothesis of non-stationarity. Our presented tests reflect the persistence of the process in the respective sub-sample as the sum of all squared partial sums is considered.

The rest of the paper is organised as follows. Section 5.2 introduces the persistence change model. Section 5.3 presents the ratio based and CUSUM of squares based test statistics. In Section 5.4 the new testing approach is presented, we derive the limiting distribution of the tests and show consistency. A Monte Carlo simulation study in Section 5.5 shows that our procedure works well and our tests are preferable with better size and power properties.

5.2 The Persistence Change Model

In the following study the persistence change model, following Kim (2000) among others, is considerd to analyse a stochastic process $\{y_t\}$ that exhibits a change in persistence. Assume the following data generating process (DGP)

$$y_t = x_t + \rho_t y_{t-1} + \varepsilon_t, \quad t = 1, \dots, T, \tag{5.1}$$

where x_t is either a constant ($x_t = \beta_0$), or a constant and a linear time trend ($x_t = \beta_0 + \beta_1 t$), ρ is the autoregressive parameter which determines the persistence of the process and $\{\varepsilon_t\}$ is the sequence of innovations.

Following Phillips (1987), Phillips and Perron (1988), the innovation process satisfies the following general conditions:

(a) $E(\varepsilon_t) = 0 \quad \forall t$;

(b) $\sup_t E|\varepsilon_t|^{\gamma+\epsilon} < \infty$ for some $\gamma > 2$ and $\epsilon > 0$;

(c) The long-run variance $\omega^2 = \sum_{j=0}^{\infty} E[\varepsilon_{j+1}\varepsilon_1']$ exists and $\omega^2 > 0$;

(d) $\{\varepsilon_t\}$ is strong mixing with mixing coefficients α_m, such that $\sum_{m=1}^{\infty} \alpha_m^{1-2/\gamma} < \infty$.

These conditions allow for a wide class of weakly dependent and heterogeneously distributed data and a detailed discussion is given by the authors.

Four different hypotheses are considered within model (5.1). The first is that the series y_t is $I(0)$ throughout with $\rho = 0$ for all t and is denoted by H_0. The second is with $\rho = 1$ for all t denoted by H_1 and is a series which is $I(1)$ throughout.

Two directions of persistence changes are possible. A change in persistence from $I(0)$ to $I(1)$ is the third hypothesis, denoted by H_{01} and implies $|\rho| < 1$ for $t = 1, \dots, [\tau^* T]$ and $\rho = 1$ for $t = [\tau^* T] + 1, \dots, T$. The last hypothesis is a change from $I(1)$ to $I(0)$ at time $[\tau^* T]$ and denoted by H_{10}. Here, $\rho = 1$ for $t = 1, \dots, [\tau^* T]$ and $|\rho| < 1$ for $t = [\tau^* T] + 1, \dots, T$. In these hypotheses the true change point is τ^* and $[\cdot]$ denotes the integer part of its argument.

5.3 Existing Tests for Structural Change

5.3.1 Ratio based Tests

Ratio based tests for the null hypothesis H_0 were independently developed by Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004). Define the partial sum processes

$$S_{0,t}(\tau) = \sum_{i=1}^{t} \hat{\varepsilon}_i, \qquad t = 1, \ldots, [\tau T], \tag{5.2}$$
$$S_{1,t}(\tau) = \sum_{i=\tau T+1}^{t} \tilde{\varepsilon}_i, \qquad t = [\tau T] + 1, \ldots, T,$$

where $\tau \in (0, 1)$, $\hat{\varepsilon}_t$ are the OLS residuals from the regression of y_t on x_t for $t = 1, \ldots, \tau T$ and $\tilde{\varepsilon}_t$ are the OLS residuals from the regression of y_t on x_t for $t = [\tau T] + 1, \ldots, T$. The test statistic against the alternative H_{01} is then based on the ratio of partial sums

$$K^f(\tau) = \frac{[(1-\tau)T]^{-2} \sum_{t=[\tau T]+1}^{T} S_{1,t}(\tau)^2}{[\tau T]^{-2} \sum_{t=1}^{[\tau T]} S_{0,t}(\tau)^2}, \tag{5.3}$$

which rejects for large values of the test statistic. When the true change point τ^* is unknown the authors suggest three appropriate functions based on the sequence of statistics $\{K^f(\tau), \tau \in \Lambda\}$, where Λ is a symmetric interval in $[0, 1]$ around 0.5. The maximum-statistic (cf. Andrews (1993)), the mean-statistic (cf. Hansen (2002)) and the mean-exponential-statistic (cf. Andrews and Ploberger (1994)) are given by:

$$K_1^f = \mathcal{H}_1(K^f(\cdot)) = \underset{\tau \in \Lambda}{max} K(\tau)^f, \tag{5.4}$$
$$K_2^f = \mathcal{H}_2(K^f(\cdot)) = \int_{\tau \in \Lambda} K(\tau)^f,$$
$$K_3^f = \mathcal{H}_3(K^f(\cdot)) = \log \left\{ \int_{\tau \in \Lambda} \exp(K(\tau)^f) \right\},$$

which all reject for large values. While tests based on Equation (5.3) are consistent against H_{01}, they are inconsistent against H_{10}. Hence, the authors suggest a test statistic based on the sequence of reciprocals of Equation (5.3) denoted by $K^r(\cdot) = K^f(\cdot)^{-1}$, which is consistent against H_{10}. Testing for a change in persistence in unknown direction is based on the maximum of sequences of test statistics

$$K_j = \max\{K_j^f, K_j^r\} \quad j = 1, 2, 3.$$

Tests based on K_j are consistent against H_{01} and H_{10} with rate $O_p(T^2)$ since K_j^f and K_j^r are of $O_p(1)$ under the wrong alternative.

5.3.2 CUSUM of Squares based Tests

The CUSUM of squared residuals based tests for a change in persistence are developed by Leybourne et al. (2007b). The test statistic for the null hypothesis H_1 against H_{01} is based on the standardised CUSUM of squared sub-sample OLS residuals

$$L^f(\tau) = \frac{[\tau T]^{-2} \sum_{t=1}^{[\tau T]} \hat{\varepsilon}_t^2}{\hat{\omega}^2(\tau)}, \tag{5.5}$$

where $\hat{\varepsilon}_t$ are the residuals in the denominator of Equation (5.3) and $\hat{\omega}^2$ is an estimator of the long-run variance,

$$\hat{\omega}^2(\tau) = \hat{\gamma}_0 + 2\sum_{s=1}^{m} w_{s,m}\hat{\gamma}_s, \quad \hat{\gamma}_s = [\tau T]^{-1} \sum_{t=1}^{[\tau T]} \Delta\hat{\varepsilon}_t \Delta\hat{\varepsilon}_{t-s}, \quad w_{s,m} = 1 - sl^{-1},$$

where m is the lag truncation parameter and $l = m+1$ the associated bandwidth. Testing against the alternative H_{10} is based on the analogue of Equation (5.5) in the reversed series $z_t \equiv y_{T-t+1}$

$$L^r(\tau) = \frac{(T - [\tau T])^{-2} \sum_{t=1}^{(T-[\tau T])} \tilde{\varepsilon}_t^2}{\tilde{\omega}^2(\tau)},$$

with

$$\tilde{\omega}^2(\tau) = \tilde{\gamma}_0 + 2\sum_{s=1}^{m} w_{s,m}\tilde{\gamma}_s, \quad \tilde{\gamma}_s = (T - [\tau T])^{-1} \sum_{t=1}^{T-[\tau T]} \Delta\tilde{\varepsilon}_t \Delta\tilde{\varepsilon}_{t-s}, \quad w_{s,m} = 1 - sl^{-1},$$

and $\tilde{\varepsilon}$ are the residuals from the regression of z_t on x_t.

Both test statistics reject for small values and the CUSUM of squared residuals ensure that the test statistics are of different magnitude for $\tau \leq \tau^*$ and $\tau > \tau^*$. The minimum of sequences of the respective test statistics converges in probability to zero under the correct alternative and is of $O_p(1)$ under the wrong alternative. Therefore, a two-tailed test based on the ratio of the minimum of sequential test statistics is constructed

$$L = \frac{\inf\limits_{\tau \in \Lambda} L^f(\tau)}{\inf\limits_{\tau \in \Lambda} L^r(\tau)} =: \frac{N}{D},$$

which rejects for large and small values. The test is consistent against H_{01} and H_{10} and conservative against H_0 with degenerating limiting distribution.

5.4 Squared CUSUM based Tests

In the following section we develop new tests for changes in persistence based on the squared CUSUMs of OLS sub-sample residuals. The presented ratio based tests for H_0 are constructed with the sum of partial sums $S_{0,t}(\tau)$ and $S_{1,t}(\tau)$, which converge to Brownian motions. The drawback of this approach is that the weights of the residuals differs within the test statistic. Starting with the partial sum processes for $t = 1$, the first residuals are higher weighted than the last residuals. The CUSUM tests developed for H_1 are constructed with equally weighted squared residuals but ignore the cross-dependencies of the residuals.

The persistence of a time series is reflected in the squared sum of consecutive observations and therefore we define new partial sum processes and construct test statistics with equally weighted squared CUSUMs of residuals. The cross-dependencies of the residuals are exploited in this testing approach.

5.4.1 Testing Stationarity against a Persistence Change

We define the partial sum processes

$$S_{0,t,j}(\tau) = \sum_{i=t}^{j} \hat{\varepsilon}_i, \qquad t = 1, \dots, [\tau T], \qquad t \leq j \leq [\tau T], \tag{5.6}$$

$$S_{1,t,j}(\tau) = \sum_{i=t}^{j} \tilde{\varepsilon}_i, \qquad t = [\tau T] + 1, \dots, T, \qquad t \leq j \leq T, \tag{5.7}$$

where the residuals $\hat{\varepsilon}_t$ and $\tilde{\varepsilon}_t$ are defined in Equation (5.3). The test statistic for the null hypothesis H_0 against H_{01} is then given by

$$\mathcal{Q}_0^f(\tau) = \frac{[(1-\tau)T]^{-3} \sum_{t=[\tau T]+1}^{T} \sum_{j=t}^{T} S_{1,t,j}(\tau)^2}{[\tau T]^{-3} \sum_{t=1}^{[\tau T]} \sum_{j=t}^{[\tau T]} S_{0,t,j}(\tau)^2} \tag{5.8}$$

and a test based on reciprocals of Equation (5.8) can be constructed to test against the alternative H_{10} and is denoted by $\mathcal{Q}_0^r(\cdot) = \mathcal{Q}_0^f(\cdot)^{-1}$. With unknown true change point τ^*, the same statistics from Equation (5.4) based on sequences of test statistics can be applied

$$\mathcal{Q}_0^{f,j} = \mathcal{H}_j(\mathcal{Q}_0^f(\cdot)), \quad \textit{and} \quad \mathcal{Q}_0^{r,j} = \mathcal{H}_j(\mathcal{Q}_0^r(\cdot)).$$

Both tests reject for large values and while they are inconsistent against the wrong alternative, the maximum of the forward and reversed statistic is a test for a change in persistence in unknown direction:

$$\mathcal{Q}_0^j = max(\mathcal{Q}_0^{f,j}, \mathcal{Q}_0^{r,j}) \quad j = 1, 2, 3.$$

The test $\mathcal{Q}_0^j$ rejects for large values and is consistent against H_{01} and H_{10} at rate $O_p(T^2)$. Note that the construction of the test is similar to the ratio based tests from Section 5.3.1 and the tests are independent of the long-run variance of $\{\varepsilon_t\}$, even though neither the numerator nor the denominator of any test statistic is scaled by a long-run variance estimator. Similar to Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004) the test has non-trivial power against constant H_1. In the following we use the maximum-statistic with the notation $Q_0^f = Q_0^{f,1}$, $Q_0^r = Q_0^{r,1}$ and $Q_0 = Q_0^1$. The results are transferable for the other statistics.

Theorem 5.4.1 provides the limiting distribution of the presented tests.

Theorem 5.4.1. Let $\{y_t\}$ be generated by model (5.1) under the null hypothesis H_0 and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let $W(s)$ denote a standard Brownian motion process on $[0, 1]$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($x_t = 1$) and the de-meaned and de-trended case ($x_t = (1, t)'$). Then, provided that as $T \to \infty$ it is true that

$$\mathcal{Q}_0^f(\tau) \Rightarrow \frac{(1-\tau)^{-3} \int_\tau^1 \int_a^1 V_\zeta(s-\tau)^2 ds da}{\tau^{-3} \int_0^\tau \int_a^\tau V_\zeta(s)^2 ds da} \equiv \mathcal{Q}_{0,\infty}^f(\tau), \quad \mathcal{H}(\mathcal{Q}_0^f) \Rightarrow \mathcal{H}(\mathcal{Q}_{0,\infty}^f),$$

$$\mathcal{Q}_0^r(\tau) \Rightarrow \frac{\tau^{-3} \int_0^\tau \int_a^\tau V_\zeta(s)^2 ds da}{(1-\tau)^{-3} \int_\tau^1 \int_a^1 V_\zeta(s-\tau)^2 ds da} \equiv \mathcal{Q}_{0,\infty}^r(\tau), \quad \mathcal{H}(\mathcal{Q}_0^r) \Rightarrow \mathcal{H}(\mathcal{Q}_{0,\infty}^r),$$

where the symbol $\Rightarrow$ signifies weak convergence of the associated probability measures and in the de-meaned case ($\zeta = 0$), $V_0(s)$ and $V_0(s-\tau)$ are Brownian bridges

$$V_0(s) = W(s) - s\tau^{-1}W(\tau),$$
$$V_0(s-\tau) = W(s) - W(\tau) - (s-\tau)(1-\tau)^{-1}[W(1) - W(\tau)]$$

and in the de-meaned and de-trended case ($\zeta = 1$), $V_1(s)$ and $V_1(s-\tau)$ are second level Brownian bridges

$$\begin{aligned} V_1(s) =& W(s) + (2s - 3s^2)\tau^{-1}W(\tau) + (-6s + 6s^2)\tau^{-1}\int_0^\tau W(r)dr, \\ V_1(s-\tau) =& W(s) - W(\tau) + (2(s-\tau) - 3(s-\tau)^2)(1-\tau)^{-1}[W(s) - W(\tau)] \\ &+ (-6(s-\tau) + 6(s-\tau)^2)(1-\tau)^{-1}\int_0^1 W(r)dr. \end{aligned}$$

Proof of Theorem 5.4.1. The proof follows by a functional central limit theorem and the continuous mapping theorem (CMT) from Chan and Wei (1988). Following Phillips and Perron (1988) it is well known that partial sum processes defined in (5.2) converge to a Brownian bridge under the assumptions (a)-(d)

$$T^{-2}\sum_{j=1}^{\tau T} S_{1,j}^2 \Rightarrow \omega^2 \int_0^{\tau T} V_\zeta(s)^2 ds.$$

Therefore, partial sum processes defined in (5.6) converge to a Brownian bridge and with the CMT

$$[\tau T]^{-3}\sum_{t=1}^{[\tau T]}\sum_{j=t}^{[\tau T]} S_{0,t,j}^2(\tau) \Rightarrow \tau^{-3}\int_0^\tau \int_a^\tau V_\zeta(s)^2 dsda,$$

$$[(1-\tau)T]^{-3}\sum_{t=[\tau T]+1}^{T}\sum_{j=t}^{T} S_{1,t,j}^2(\tau) \Rightarrow (1-\tau)^{-3}\int_\tau^1 \int_a^1 V_\zeta(s-\tau)^2 dsda.$$

The proof for the second sub-sample follows analogously. □

Theorem 5.4.2 provides the consistency of Q_0 under the alternative hypotheses H_{01}, H_{10}.

Theorem 5.4.2. Let τ^* denote the true change point. Let $\{y_t\}$ be generated by model (5.1) under the alternative hypothesis H_1 for $t = 1, \ldots, [\tau^* T]$ and be generated by model (5.1) under the null hypothesis H_0 for $t = [\tau^* T] + 1, \ldots, T$ and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let $W(s)$ denote a standard Brownian motion process on $[0, 1]$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($x_t = 1$) and the de-meaned and de-trended case ($x_t = (1, t)'$). Then for the sub-sample up to τ^* it is true that

$$[\tau^* T]^{-5}\sum_{t=1}^{[\tau^* T]}\sum_{j=t}^{[\tau^* T]} S_{0,t,j}^2 \Rightarrow \omega^2 {\tau^*}^{-5}\int_0^{\tau^*}\int_b^{\tau^*}\left(\int_b^a W_\zeta(s)ds\right)^2 dadb,$$

$$W_0(s) = W(s) - \int_0^{\tau^*} W(r)dr,$$
$$W_1(s) = W(s) + (6s-4)\int_0^{\tau^*} W(r)dr + (-12s+6)\int_0^{\tau^*} sW(r)dr.$$

Then it follows for $\mathcal{Q}_0^r(\tau^*)$

$$\frac{[\tau^* T]^{-3}\sum_{t=1}^{[\tau^* T]}\sum_{j=t}^{[\tau^* T]} S_{0,t,j}^2}{[(1-\tau^*)T]^{-3}\sum_{t=[\tau^* T]+1}^{T}\sum_{j=t}^{T} S_{1,t,j}^2} \Rightarrow O_p(T^2).$$

The forward statistic is the reciprocal of the reversed test statistic $\mathcal{Q}_0^f(\tau^*) = \mathcal{Q}_0^r(\tau^*)^{-1}$ and therefore $\mathcal{Q}_0^f(\tau^*)$ is of $O_p(T^2)$. Let $\{y_t\}$ be generated by H_{01} or H_{10} it is true that

$$\mathcal{Q}_0 \Rightarrow O_p(T^2).$$

Proof of Theorem 5.4.2. The limiting distribution of the sub-sample of the test statistic can be obtained with the assumptions in Theorem 5.4.2, $s \leq \tau^*$ and the well known results

$$T^{-1/2} y_{[sT]} = T^{-1/2}\sum_{i=1}^{[sT]} \varepsilon_i \Rightarrow \omega W(s),$$
$$T^{-1/2}\bar{y}(\tau) = \tau^{-1} T^{-3/2}\sum_{t=1}^{T} y_t \Rightarrow \omega \tau^{-1}\int_0^{\tau} W(s)ds.$$

It follows with $a \in [0,\tau^*]$, $b \in [0,\tau^*]$ and $\tau^* \geq a \geq b$

$$\begin{aligned} T^{-3/2} S_{bT,aT} &= T^{-3/2}\sum_{i=[bT]}^{aT} (y_i - \bar{y}(\tau^*))) \\ &= \left(T^{-1}\sum_{i=1}^{[aT]} y_i - ([aT]/T) T^{-1/2}\bar{y}(\tau^*)\right) - \left(T^{-1}\sum_{i=1}^{[bT]} y_i - (([bT])/T) T^{-1/2}\bar{y}(\tau^*)\right) \\ &\Rightarrow \omega\int_0^a W(s)ds - a\omega\int_0^{\tau^*} W(s)ds - \omega\int_0^b W(s)ds + b\omega\int_0^{\tau^*} W(s)ds \\ &= \omega\int_b^a W(s)ds - (a-b)\omega\int_0^{\tau^*} W(s)ds \\ &= \omega\int_b^a W_\varsigma(s)ds. \end{aligned}$$

Therefore

$$[\tau^* T]^{-5}\sum_{t=1}^{[\tau^* T]}\sum_{j=t}^{[\tau^* T]} S_{0,t,j}^2 \Rightarrow \omega^2\int_0^{\tau^*}\int_b^{\tau^*}\left(\int_b^a W_\varsigma(s)ds\right)^2 dadb.$$

□

5.4.2 Testing Non-Stationarity against a Persistence Change

The test statistic for the null hypothesis H_1 against H_{01} is based on squared CUSUMs of sub-sample OLS residuals and given by

$$\mathcal{Q}_1^f(\tau) = \frac{[\tau T]^{-5} \sum_{t=1}^{[\tau T]} \sum_{j=t}^{[\tau T]} \left(\sum_{i=t}^{j} \hat{\varepsilon}_i \right)^2}{\hat{\omega}^2}, \tag{5.9}$$

where $\hat{\varepsilon}_t$ are the residuals from the denominator of Equation (5.3) and the denominator $\hat{\omega}^2$ is an estimator of the long-run variance ω^2. In the following the estimator

$$\hat{\omega}^2(\tau) = \hat{\gamma}_0 + 2 \sum_{s=1}^{m} w_{s,m} \hat{\gamma}_s, \quad \hat{\gamma}_s = [\tau T]^{-1} \sum_{t=1}^{[\tau T]} \Delta\hat{\varepsilon}_t \Delta\hat{\varepsilon}_{t-s}, \quad w_{s,m} = 1 - sl^{-1},$$

is adopted with the lag truncation parameter m and the Bartlett window $w_{s,l}$.

For a test statistic against the alternative H_{10}, the analogue of Equation (5.9) for the reversed series is considered

$$\mathcal{Q}_1^r(\tau) = \frac{(T - [\tau T])^{-5} \sum_{t=1}^{(T-[\tau T])} \sum_{j=t}^{(T-[\tau T])} \left(\sum_{i=t}^{j} \tilde{\varepsilon}_i \right)^2}{\tilde{\omega}^2},$$

with

$$\tilde{\omega}^2(\tau) = \tilde{\gamma}_0 + 2 \sum_{s=1}^{m} w_{s,m} \tilde{\gamma}_s, \quad \tilde{\gamma}_s = [\tau T]^{-1} \sum_{t=1}^{[\tau T]} \Delta\tilde{\varepsilon}_t \Delta\tilde{\varepsilon}_{t-s}, \quad w_{s,m} = 1 - sl^{-1}.$$

Both tests reject for small values and are inconsistent against a change in the direction not covered under the alternative hypothesis. The test statistic against a change in persistence in unknown direction can be constructed with a ratio of the minimum of test statistic sequences

$$\mathcal{Q}_1 = \frac{\inf_{\tau \in \Lambda} \mathcal{Q}_1^f(\tau)}{\inf_{\tau \in \Lambda} \mathcal{Q}_1^r(\tau)} =: \frac{\mathcal{Q}_1^f}{\mathcal{Q}_1^r}.$$

The two-tailed test $\mathcal{Q}_1$ rejects for large and small values and is consistent against H_{01} and H_{10} at rate $O_p(T^2)$ and $O_p(1/T^2)$. These result is obtained due to the construction of the test. The tests Q_1^f and Q_1^r are consistent and of $O_p(1/T^2)$ under the alternative and of $O_p(1)$ under the null hypothesis. Note that the construction of the test is similar to the squared CUSUM based tests from Section 5.3.2 and the test $\mathcal{Q}_1$ can be calculated with $l = 0$ yielding OLS variance estimators. Similar to Leybourne et al. (2007b), the test is conservative against H_0 with degenerating limiting distribution.

Theorem 5.4.3 provides the limiting distribution of Q_1 under the null hypothesis H_1.

Theorem 5.4.3. Let $\{y_t\}$ be generated by model (5.1) under the null hypothesis H_1 and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let $W(s)$ denote a standard Brownian motion process on $[0,1]$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($x_t = 1$) and the de-meaned and de-trended case ($x_t = (1,t)'$). Then, provided that as $T \to \infty$ it is true that

$$\mathcal{Q}_1^f(\tau) \Rightarrow \omega^2 \int_0^\tau \int_b^\tau \left(\int_b^a W_\zeta^f(s,\tau)ds \right)^2 dadb \equiv \Psi_\zeta^f(\tau),$$
$$\mathcal{Q}_1^r(\tau) \Rightarrow \omega^2 \int_\tau^1 \int_b^1 \left(\int_b^a W_\zeta^r(s,\tau)ds \right)^2 dadb \equiv \Psi_\zeta^r(\tau),$$

and

$$\mathcal{Q}_1^f \Rightarrow \inf_{\tau\in\Lambda} \Psi_\zeta^f(\tau), \quad \mathcal{Q}_1^r \Rightarrow \inf_{\tau\in\Lambda} \Psi_\zeta^r(\tau), \quad \mathcal{Q}_1 \Rightarrow \frac{\inf_{\tau\in\Lambda} \Psi_\zeta^f(\tau)}{\inf_{\tau\in\Lambda} \Psi_\zeta^r(\tau)}$$

where in the de-meaned case ($\zeta = 0$)

$$W_0^f(s,\tau) = W(s) - \tau^{-1} \int_0^\tau W(r)dr,$$
$$W_0^r(s,\tau) = W(1-s) - (1-\tau)^{-1} \int_\tau^1 W(r)dr$$

and in the de-meaned and de-trended case ($\zeta = 1$)

$$\begin{aligned}
W_1^f(s,\tau) &= W(s) - B_0^f(\tau) - sB_1^f(\tau),\\
W_1^r(s,\tau) &= W(1-s) - W(1) - B_0^r(\tau) - sB_1^r(\tau),\\
B_0^f(\tau) &= 4\tau^{-1} \int_0^\tau W(r)dr - 6\tau^{-2} \int_0^\tau sW(r)dr,\\
B_1^f(\tau) &= 6\tau^{-2} \int_0^\tau W(r)dr + 12\tau^{-3} \int_0^\tau sW(r)dr,\\
B_0^r(\tau) &= 4(1-\tau)^{-1} \left[\int_\tau^1 W(r)dr - (1-\tau)W(1) \right]\\
&\quad - 6(1-\tau)^{-2} \left[\frac{(1-\tau)^2}{2} W(1) - \int_\tau^1 W(r)dr + \int_\tau^1 sW(r)dr \right],\\
B_1^r(\tau) &= 6(1-\tau)^{-2} \left[\int_\tau^1 W(r)dr - (1-\tau)W(1) \right]\\
&\quad + 12(1-\tau)^{-3} \left[\frac{(1-\tau)^2}{2} W(1) - \int_\tau^1 W(r)dr + \int_\tau^1 sW(r)dr \right].
\end{aligned}$$

Note that the processes $W_\zeta^f(s,\tau)$ and $W_\zeta^r(s,\tau)$ are the same as in Leybourne et al. (2007b).

Proof of Theorem 5.4.3. The limiting distribution is based on the results

$$T^{-1/2}y_{[sT]} = T^{-1/2}\sum_{i=1}^{[sT]}\varepsilon_i \Rightarrow \omega W(s),$$
$$T^{-1/2}\bar{y}(\tau) = \tau^{-1}T^{-3/2}\sum_{t=1}^{T}y_t \Rightarrow \omega\tau^{-1}\int_0^{\tau}W(s)ds.$$

The limiting distribution of the forward statistic $\mathcal{Q}_1^f$ can be derived with the partial sum process $S_{t,j}$ and $a \in [0,\tau]$, $b \in [0,\tau]$, $a \geq b$

$$\begin{aligned}
T^{-3/2}S_{bT,aT} &= T^{-3/2}\sum_{i=[bT]}^{aT}(y_i - \bar{y}(\tau)) \\
&= \left(T^{-1}\sum_{i=1}^{[aT]}y_i - ([aT]/T)T^{-1/2}\bar{y}(\tau)\right) - \left(T^{-1}\sum_{i=1}^{[bT]}y_i - (([bT])/T)T^{-1/2}\bar{y}(\tau)\right) \\
&\Rightarrow \omega\int_0^a W(s)ds - a\omega\int_0^{\tau}W(s)ds - \omega\int_0^b W(s)ds + b\omega\int_0^{\tau}W(s)ds \\
&= \omega\int_b^a W(s)ds - (a-b)\omega\int_0^{\tau}W(s)ds \\
&= \omega\int_b^a W_{\varsigma}(s)ds.
\end{aligned}$$

With the continuous mapping theorem it follows

$$\mathcal{Q}_1^f(\tau) \Rightarrow \omega^2\int_0^{\tau}\int_b^{\tau}\left(\int_b^a W_{\zeta}^f(s,\tau)ds\right)^2 dadb.$$

Similar arguments follow for the reversed statistic and it follows

$$\mathcal{Q}_1^r(\tau) \Rightarrow \omega^2\int_{\tau}^{1}\int_b^{1}\left(\int_b^a W_{\zeta}^r(s,\tau)ds\right)^2 dadb.$$

□

Theorem 5.4.4 provides the consistency of Q_1 under the alternative hypotheses H_{01}, H_{10}.

Theorem 5.4.4. Let τ^* denote the true change point. Let $\{y_t\}$ be generated by model (5.1) under the alternative hypothesis H_0 for $t = 1, \ldots, [\tau^* T]$ and be generated by model (5.1) under the null hypothesis H_1 for $t = [\tau^* T] + 1, \ldots, T$ and let assumptions (a)-(d) hold for $\{\varepsilon_t\}$. Let $W(s)$ denote a standard Brownian motion process on $[0, 1]$. Let the subscript $\zeta = 0, 1$ denote the de-meaned ($x_t = 1$) and the de-meaned and de-trended case ($x_t = (1, t)'$). Then for the sub-sample up to τ^* it is true that

$$[\tau T]^{-3} \sum_{t=1}^{[\tau T]} \sum_{j=t}^{[\tau T]} S_{0,t,j}^2 \Rightarrow \omega^2 \tau^{-3} \int_0^\tau \int_a^\tau V_\zeta(s)^2 ds da,$$

Then it follows for $\mathcal{Q}_1^f(\tau^*)$

$$\mathcal{Q}_1^f = [\tau^* T]^{-5} \sum_{t=1}^{[\tau^* T]} \sum_{j=t}^{[\tau^* T]} S_{0,t,j}^2 \Rightarrow O_p(1/T^2).$$

The same arguments follow for the reversed test statistic $\mathcal{Q}_1^r$. In the reversed time series it follows that

$$\mathcal{Q}_1^r \Rightarrow O_p(1/T^2).$$

Let $\{y_t\}$ be H_{01} it is true that $\mathcal{Q}_1^f \Rightarrow O_p(1/T^2)$, $\mathcal{Q}_1^r \Rightarrow O_p(1)$ and $\mathcal{Q}_1 \Rightarrow O_p(1/T^2)$.
Let $\{y_t\}$ be H_{10} it is true that $\mathcal{Q}_1^f \Rightarrow O_p(1)$, $\mathcal{Q}_1^r \Rightarrow O_p(1/T^2)$ and $\mathcal{Q}_1 \Rightarrow O_p(T^2)$.

Proof of Theorem 5.4.4. The proof of Theorem 5.4.4 follows from Theorem 5.4.1. □

5.5 Simulation Results

5.5.1 Critical Values of the Test Statistics

Table 5.1 and Table 5.2 report the critical values for the $\mathcal{Q}_0^f, \mathcal{Q}_0^r$ and $\mathcal{Q}_0$ tests for $z_t = 1$ and $z_t = (1, t)'$ and Table 5.3 reports the critical values for the $\mathcal{Q}_1^f, \mathcal{Q}_1^r$ and $\mathcal{Q}_1$ tests for $z_t = 1$ and $z_t = (1, t)'$. The finite sample critical values are obtained by Monte Carlo simulation methods for $T = 100, 250, 500, 1000$ with $100, 000$ replications.

The critical values in Table 5.1 and Table 5.2 are reported for the three considered functions in Equation (5.4), the maximum-statistic, mean-statistic and the mean-exponential-statistic. Critical values are obtained for different nominal significance levels α and the quartiles of the distribution are reported to analyse the behaviour of the limiting distribution of the test statistics. For the forward and the reserved test statistics the critical values are asymptotically the same, which is reasonable while the process under the null hypothesis is constant H_0. In general, the critical values converge with increasing sample size and small sample sizes approximate the limiting distribution reasonable well. The critical values of $\mathcal{Q}_0$ are higher as they are based on the maximum of $\mathcal{Q}_0^f$ and $\mathcal{Q}_0^r$. All test statistics reject for large values.

The critical values in Table 5.3 are reported for many different nominal significance levels α to use the one-sided tests $\mathcal{Q}_1^f, \mathcal{Q}_1^r$ and the two-sided test $\mathcal{Q}_1$ for $\alpha/2$. Again, the quartiles of the distribution are reported to analyse the behaviour of the limiting distribution. Note that the critical values for $\mathcal{Q}_1^f$ and $\mathcal{Q}_1^r$ are multiplied by 100 for a better overview, as they are very low. Notice, that the critical values for the forward and the reversed statistics are the same. Furthermore, the results are very similar for all sample sizes and converge with increasing T. The limiting distribution of $\mathcal{Q}_1$ is symmetric around 1 and the critical values converge relatively fast. The tests $\mathcal{Q}_1^f$ and $\mathcal{Q}_1^r$ reject for small values and the two-sided $\mathcal{Q}_1$ test for small and large values.

$z_t = 1$	$T\backslash\alpha$	0.01%	0.05%	0.10%	0.25%	0.50%	0.75%	0.90%	0.95%	0.99%
					Maximum-statistic					
$\mathcal{Q}_0^{f,1}$	100	0.6222	0.9476	1.1951	1.7926	2.7787	4.3164	6.4063	8.0601	12.1555
	250	0.6509	0.9932	1.2371	1.8162	2.7890	4.3555	6.4174	8.0809	12.4333
	500	0.6534	1.0171	1.2847	1.8771	2.8634	4.3838	6.4526	8.0987	12.7383
	1000	0.6899	1.0626	1.3212	1.9074	2.9031	4.4084	6.4700	8.1418	12.9043
$\mathcal{Q}_0^{r,1}$	100	0.6222	0.9476	1.1951	1.7926	2.7787	4.3164	6.4063	8.0601	12.1555
	250	0.6509	0.9932	1.2371	1.8162	2.7890	4.3555	6.4174	8.0809	12.4333
	500	0.6534	1.0171	1.2847	1.8771	2.8634	4.3838	6.4526	8.0987	12.7383
	1000	0.6899	1.0626	1.3212	1.9074	2.9031	4.4084	6.4700	8.1418	12.9043
$\mathcal{Q}_0^{1}$	100	1.8847	2.2355	2.5124	3.1212	4.1853	5.9016	8.4449	10.4472	16.1893
	250	1.8737	2.2621	2.5195	3.1210	4.1662	5.7668	7.9493	9.6945	14.4888
	500	1.9115	2.2988	2.5742	3.1902	4.2523	5.8709	8.0266	10.0203	14.6160
	1000	1.9365	2.3276	2.6002	3.2155	4.2415	5.8724	8.1261	9.8574	14.2861
					Mean-statistic					
$\mathcal{Q}_0^{f,2}$	100	0.2777	0.4202	0.5298	0.7711	1.1657	1.7882	2.6231	3.3043	5.0961
	250	0.2863	0.4227	0.5334	0.7715	1.1672	1.7819	2.5459	3.1636	4.8780
	500	0.2936	0.4292	0.5357	0.7784	1.1675	1.7547	2.5264	3.1388	4.7568
	1000	0.2976	0.4334	0.5422	0.7807	1.1698	1.7464	2.5263	3.1293	4.6997
$\mathcal{Q}_0^{r,2}$	100	0.2777	0.4202	0.5298	0.7711	1.1657	1.7882	2.6231	3.3043	5.0961
	250	0.2863	0.4227	0.5334	0.7715	1.1672	1.7819	2.5459	3.1636	4.8780
	500	0.2936	0.4292	0.5357	0.7784	1.1675	1.7547	2.5264	3.1388	4.7568
	1000	0.2976	0.4334	0.5422	0.7807	1.1698	1.7464	2.5263	3.1293	4.6997
$\mathcal{Q}_0^{2}$	100	1.0917	1.1749	1.2382	1.4202	1.7951	2.4333	3.3411	4.0859	5.9773
	250	1.0824	1.1579	1.2222	1.4051	1.7696	2.3665	3.1814	3.8492	5.6139
	500	1.0832	1.1544	1.2236	1.3966	1.7442	2.3467	3.1421	3.8474	5.7664
	1000	1.0845	1.1533	1.2194	1.3878	1.7351	2.3287	3.1704	3.8399	5.4349
					Mean-exponential-statistic					
$\mathcal{Q}_0^{f,3}$	100	0.2892	0.4532	0.5770	0.8756	1.4453	2.4291	4.1312	5.7475	10.6680
	250	0.2997	0.4624	0.5816	0.8760	1.4131	2.3677	3.8547	5.1764	9.1958
	500	0.3032	0.4637	0.5875	0.8787	1.4124	2.3305	3.7963	5.0776	8.8369
	1000	0.3153	0.4641	0.5908	0.8942	1.4095	2.3153	3.7777	5.0535	8.5226
$\mathcal{Q}_0^{r,3}$	100	0.2892	0.4532	0.5770	0.8756	1.4453	2.4291	4.1312	5.7475	10.6680
	250	0.2997	0.4624	0.5816	0.8760	1.4131	2.3677	3.8547	5.1764	9.1958
	500	0.3032	0.4637	0.5875	0.8787	1.4124	2.3305	3.7963	5.0776	8.8369
	1000	0.3153	0.4641	0.5908	0.8942	1.4095	2.3153	3.7777	5.0535	8.5226
$\mathcal{Q}_0^{3}$	100	1.1646	1.3010	1.4221	1.7346	2.3994	3.6718	5.7787	7.6363	12.8899
	250	1.1441	1.2800	1.3851	1.6835	2.2936	3.4492	5.1870	6.7298	11.0763
	500	1.1467	1.2710	1.3724	1.6665	2.2850	3.4121	5.0912	6.7897	10.9872
	1000	1.1515	1.2689	1.3738	1.6570	2.2464	3.3320	5.1005	6.6099	10.4484

Table 5.1: Critical Values of the $\mathcal{Q}_0^f$, $\mathcal{Q}_0^r$ and $\mathcal{Q}_0$ Tests for $z_t = 1$.

$z_t = (1,t)'$	$T\backslash\alpha$	0.01%	0.05%	0.10%	0.25%	0.50%	0.75%	0.90%	0.95%	0.99%
					Maximum-statistic					
$\mathcal{Q}_0^{f,1}$	100	0.7482	1.1290	1.3961	2.0417	3.0693	4.6071	6.7824	8.0452	12.0313
	250	0.8283	1.2259	1.4994	2.1073	3.1347	4.6576	6.8098	8.1684	12.3520
	500	0.8724	1.2789	1.5841	2.2106	3.2509	4.7990	6.8102	8.3945	12.6704
	1000	0.9153	1.3246	1.6258	2.2668	3.2699	4.8178	6.8241	8.5409	12.8448
$\mathcal{Q}_0^{r,1}$	100	0.7482	1.1290	1.3961	2.0417	3.0693	4.6071	6.7824	8.0452	12.0313
	250	0.8283	1.2259	1.4994	2.1073	3.1347	4.6576	6.8098	8.1684	12.3520
	500	0.8724	1.2789	1.5841	2.2106	3.2509	4.7990	6.8102	8.3945	12.6704
	1000	0.9153	1.3246	1.6258	2.2668	3.2699	4.8178	6.8241	8.5409	12.8448
$\mathcal{Q}_0^{1}$	100	2.0911	2.4844	2.7585	3.4319	4.5390	6.2129	8.5686	10.5788	15.8412
	250	2.1341	2.5401	2.8224	3.4633	4.4547	6.0559	8.2278	10.0118	14.8937
	500	2.2210	2.6192	2.9134	3.5601	4.5977	6.1772	8.3990	10.1672	14.4871
	1000	2.2518	2.6667	2.9516	3.5824	4.6065	6.2491	8.5523	10.2598	14.5720
					Mean-statistic					
$\mathcal{Q}_0^{f,2}$	100	0.3278	0.4776	0.5847	0.8187	1.1935	1.7247	2.3965	3.0518	4.4358
	250	0.3447	0.4858	0.5964	0.8232	1.1862	1.7056	2.3628	2.8940	4.2733
	500	0.3454	0.4893	0.5978	0.8266	1.1840	1.7049	2.3646	2.8823	4.2011
	1000	0.3471	0.4929	0.5989	0.8285	1.1813	1.6785	2.3572	2.8784	4.1935
$\mathcal{Q}_0^{r,2}$	100	0.3278	0.4776	0.5847	0.8187	1.1935	1.7247	2.3965	3.0518	4.4358
	250	0.3447	0.4858	0.5964	0.8232	1.1862	1.7056	2.3628	2.8940	4.2733
	500	0.3454	0.4893	0.5978	0.8266	1.1840	1.7049	2.3646	2.8823	4.2011
	1000	0.3471	0.4929	0.5989	0.8285	1.1813	1.6785	2.3572	2.8784	4.1935
$\mathcal{Q}_0^{2}$	100	1.1066	1.1825	1.2441	1.4197	1.7584	2.2958	3.0310	3.6123	5.1782
	250	1.0970	1.1694	1.2290	1.3961	1.7034	2.2006	2.9026	3.4487	4.9080
	500	1.0961	1.1665	1.2262	1.3871	1.6883	2.2034	2.8948	3.4683	4.7847
	1000	1.0972	1.1629	1.2160	1.3688	1.6666	2.1816	2.9082	3.4711	4.8237
					Mean-exponential-statistic					
$\mathcal{Q}_0^{f,3}$	100	0.3622	0.5241	0.6496	0.9634	1.5153	2.4403	4.0339	5.1047	9.8003
	250	0.3641	0.5304	0.6574	0.9574	1.4769	2.3654	3.8028	5.0967	8.9985
	500	0.3640	0.5342	0.6651	0.9540	1.4733	2.3613	3.7619	5.0936	8.9570
	1000	0.3675	0.5343	0.6703	0.9513	1.4513	2.3107	3.7388	5.0859	8.5719
$\mathcal{Q}_0^{r,3}$	100	0.3622	0.5241	0.6496	0.9634	1.5153	2.4403	4.0339	5.1047	9.8003
	250	0.3641	0.5304	0.6574	0.9574	1.4769	2.3654	3.8028	5.0967	8.9985
	500	0.3640	0.5342	0.6651	0.9540	1.4733	2.3613	3.7619	5.0936	8.9570
	1000	0.3675	0.5343	0.6703	0.9513	1.4513	2.3107	3.7388	5.0859	8.5719
$\mathcal{Q}_0^{3}$	100	1.2052	1.3478	1.4671	1.8039	2.4739	3.7104	5.7184	7.5579	12.5737
	250	1.1923	1.3216	1.4311	1.7304	2.3082	3.3966	5.1521	6.6541	11.2068
	500	1.1862	1.3191	1.4295	1.7031	2.2811	3.3309	5.0737	6.5352	10.6187
	1000	1.1857	1.3106	1.4141	1.6724	2.2320	3.2944	5.1114	6.5443	10.6045

Table 5.2: Critical Values of the $\mathcal{Q}_0^f, \mathcal{Q}_0^r$ and $\mathcal{Q}_0$ Tests for $z_t = (1,t)'$.

	$T\backslash\alpha$	0.005%	0.010%	0.025%	0.050%	0.100%	0.250%	0.500%	0.750%	0.900%	0.950%	0.975%	0.990%	0.995
$\mathcal{Q}_1^f$	100	0.0040	0.0050	0.0070	0.0097	0.0140	0.0266	0.0568	0.1225	0.2260	0.3134	0.4039	0.5325	0.6310
	250	0.0036	0.0047	0.0066	0.0091	0.0132	0.0256	0.0552	0.1206	0.2241	0.3146	0.4098	0.5413	0.6478
	500	0.0035	0.0044	0.0064	0.0088	0.0131	0.0253	0.0550	0.1200	0.2257	0.3170	0.4161	0.5477	0.6555
	1000	0.0034	0.0043	0.0063	0.0087	0.0130	0.0251	0.0549	0.1200	0.2268	0.3175	0.4221	0.5513	0.6597
$\mathcal{Q}_1^r$	100	0.0040	0.0050	0.0070	0.0097	0.0140	0.0266	0.0568	0.1225	0.2260	0.3134	0.4039	0.5325	0.6310
	250	0.0036	0.0047	0.0066	0.0091	0.0132	0.0256	0.0552	0.1206	0.2241	0.3146	0.4098	0.5413	0.6478
	500	0.0035	0.0044	0.0064	0.0088	0.0131	0.0253	0.0550	0.1200	0.2257	0.3170	0.4161	0.5477	0.6555
	1000	0.0034	0.0043	0.0063	0.0087	0.0130	0.0251	0.0549	0.1200	0.2268	0.3175	0.4221	0.5513	0.6597
$\mathcal{Q}_1$	100	0.0248	0.0335	0.0557	0.0889	0.1481	0.3632	0.9997	2.7179	6.6468	11.1930	17.4995	28.4952	38.6769
	250	0.0230	0.0331	0.0539	0.0842	0.1430	0.3575	0.9993	2.7883	6.9617	11.8097	18.5995	30.8489	43.4130
	500	0.0224	0.0317	0.0529	0.0807	0.1406	0.3513	0.9998	2.8188	7.1441	12.0742	18.9973	32.3165	45.9023
	1000	0.0220	0.0312	0.0525	0.0799	0.1397	0.3484	0.9995	2.8243	7.2531	12.1934	19.1235	33.2624	46.8374
	T							$z_t = (1,t)'$						
$\mathcal{Q}_1^f$	100	0.0020	0.0024	0.0032	0.0041	0.0054	0.0089	0.0154	0.0266	0.0433	0.0566	0.0706	0.0899	0.1032
	250	0.0017	0.0021	0.0029	0.0037	0.0050	0.0084	0.0150	0.0266	0.0435	0.0577	0.0727	0.0929	0.1070
	500	0.0017	0.0021	0.0028	0.0036	0.0049	0.0083	0.0150	0.0270	0.0444	0.0590	0.0737	0.0942	0.1104
	1000	0.0016	0.0020	0.0027	0.0035	0.0048	0.0082	0.0150	0.0271	0.0448	0.0602	0.0745	0.0948	0.1118
$\mathcal{Q}_1^r$	100	0.0020	0.0024	0.0032	0.0041	0.0054	0.0089	0.0154	0.0266	0.0433	0.0566	0.0706	0.0899	0.1032
	250	0.0017	0.0021	0.0029	0.0037	0.0050	0.0084	0.0150	0.0266	0.0435	0.0577	0.0727	0.0929	0.1070
	500	0.0017	0.0021	0.0028	0.0036	0.0049	0.0083	0.0150	0.0270	0.0444	0.0590	0.0737	0.0942	0.1104
	1000	0.0016	0.0020	0.0027	0.0035	0.0048	0.0082	0.0150	0.0271	0.0448	0.0602	0.0745	0.0948	0.1118
$\mathcal{Q}_1$	100	0.0580	0.0765	0.1140	0.1604	0.2392	0.4713	0.9991	2.1078	4.1219	6.1810	8.5365	12.5538	16.6078
	250	0.0504	0.0673	0.1021	0.1474	0.2227	0.4580	1.0040	2.1952	4.4523	6.9145	9.7990	14.7882	19.1500
	500	0.0497	0.0656	0.0998	0.1419	0.2186	0.4492	1.0038	2.2115	4.5083	6.9232	9.8367	14.8106	19.6444
	1000	0.0493	0.0644	0.0986	0.1397	0.2164	0.4457	1.0008	2.2239	4.5362	6.9357	9.9842	14.8337	19.9938

Table 5.3: Critical Values of the $\mathcal{Q}_1^f$, $\mathcal{Q}_1^r$ and $\mathcal{Q}_1$ Tests for $z_t = 1$ and $z_t = (1,t)'$.

5.5.2 Simulation Setup

In this Monte Carlo simulation study the properties of the new tests for a change in persistence are compared to the ratio based tests and the CUSUM of squared residuals tests. The behaviour of the tests $\mathcal{Q}_0$ and $\mathcal{Q}_1$ from Section 5.4 is analysed under the null hypotheses H_0, H_1 and under changes in persistence in both directions H_{01} and H_{10}. We investigate the size and power properties of the tests compared to the test presented in Section 5.3.

We are interested in detecting a change in persistence in unknown direction and therefore we compare K^f, K^r and K with $\mathcal{Q}_0^f, \mathcal{Q}_0^r$ and $\mathcal{Q}_0$ in the constant H_0 case. For the non-stationary H_1 hypothesis we analyse L^f, L^r, L and $\mathcal{Q}_1^f, \mathcal{Q}_1^r, \mathcal{Q}_1$. We choose the function $\mathcal{H}_1(\cdot)$ and calculate the maximum of sequences of test statistics.

The data are generated with $M = 10,000$ replications by the ARMA process

$$y_t = \rho_t y_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t, \quad t = -200, \dots, T,$$

where ε_t is independent and identically distributed $N(0,1)$. The autoregressive (AR) parameter is $\rho_t \in \{0, 0.1, 0.5, 0.7, 0.9, 1\}$, the moving-average (MA) parameter $\theta \in \{0, 0.5, -0.5\}$ and the lag truncation parameter is set to $m = [4(T/100)^{1/4}]$ for $\mathcal{Q}_1^f$, $\mathcal{Q}_1^r$, L^f and L^r and we use $m = 0$ for $\mathcal{Q}_1$ and L yielding OLS variance estimators. For all tests we use $\Lambda = [0.2, 0.8]$, which is typical in the literature. The series are simulated with a burn-in period of 200 observations to eliminate the initial effects.

The parameter ρ_t can change with t and is assumed to change at $\tau^* = 0.5$, while other choices of τ^* provide similar results.

The results are only reported for the tests in the de-meaned case, where y_t is regressed on intercept $(x_t = 1)$ as the differences to the de-meaned and de-trended $(x_t = (1, t)')$ case are small. Rejection frequencies are reported for the nominal significance level 5%.

5.5.3 Properties of the Tests under H_0 and H_1

In the following section we investigate the size properties of the tests. The degree of persistence is constant throughout the sample and therefore we set $\rho = \rho_t$ for all t. This leads to H_0, for $\rho < 1$ and H_1 for $\rho = 1$.

The tests with the null hypothesis H_0 have non-trivial power against H_1 and therefore size results can only be analysed for $\rho < 1$. The size for the one-sided non-stationarity tests can only be interpreted for $\rho = 1$. In contrast, the limiting distribution of the two-sided tests against H_1 degenerates for $\rho < 1$ and therefore the tests are conservative.

Table 5.4 and Table 5.5 show the rejections frequencies in the constant persistence case for $T = 100$ and $T = 250$.

First, we consider the stationarity tests. All tests meet the nominal significance level for $\rho = 0$ and $\theta = 0$. They are oversized with a positive MA parameter and undersized, when a negative MA parameter is present in the simulated series. The rejection frequencies are increasing with increasing ρ for all tests and it is true for both sample sizes, that the new presented tests $\mathcal{Q}_0^f, \mathcal{Q}_0^r$ and $\mathcal{Q}_0$ are closer to the nominal significance level than the ratio based tests K^f, K^r and K. Still, they are oversized with increasing ρ.

Considering the non-stationarity tests similar results occur. The one-sided tests L^f, L^r, $\mathcal{Q}_1^f$ and $\mathcal{Q}_1^r$ are close to the nominal significance level for $\rho = 1$ and $\theta = 0$ with higher rejection frequencies when ρ decreases. The rejection frequencies are lower for the positive MA parameter and the tests are clearly oversized with a negative MA parameter. The two-sided test statistics $\mathcal{Q}_1$ and L are at the nominal significance level for $\rho = 1$ and the rejection frequencies for $\rho < 1$ show the effect of the degenerating limiting distribution with rejection frequencies close to 0.

The main finding is that the new presented stationarity tests are better sized than existing ratio based tests, while the properties of the non-stationarity tests and the CUSUM of squares tests are similar.

		$\mathcal{Q}_0$			Kim			$\mathcal{Q}_1$			Leybourne		
ρ	θ	$\mathcal{Q}_0^f$	$\mathcal{Q}_0^r$	$\mathcal{Q}_0$	K^f	K^r	K	$\mathcal{Q}_1^f$	$\mathcal{Q}_1^r$	$\mathcal{Q}_1$	L^f	L^r	L
0.0	0.0	0.047	0.049	0.048	0.049	0.052	0.053	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.066	0.066	0.070	0.065	0.066	0.072	1.000	1.000	0.000	1.000	1.000	0.000
	-0.5	0.015	0.013	0.006	0.013	0.014	0.010	1.000	1.000	0.000	1.000	1.000	0.000
0.1	0.0	0.051	0.045	0.047	0.053	0.055	0.054	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.070	0.072	0.079	0.070	0.076	0.082	1.000	1.000	0.000	1.000	1.000	0.000
	-0.5	0.017	0.014	0.009	0.018	0.018	0.013	1.000	1.000	0.000	1.000	1.000	0.000
0.5	0.0	0.098	0.103	0.120	0.115	0.104	0.133	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.114	0.104	0.135	0.138	0.125	0.160	0.981	0.975	0.000	0.994	0.994	0.000
	-0.5	0.049	0.053	0.052	0.043	0.042	0.037	1.000	1.000	0.000	1.000	1.000	0.000
0.7	0.0	0.154	0.162	0.210	0.194	0.185	0.255	0.918	0.909	0.001	0.935	0.927	0.000
	0.5	0.162	0.169	0.224	0.204	0.201	0.273	0.741	0.729	0.001	0.761	0.753	0.000
	-0.5	0.105	0.106	0.125	0.120	0.107	0.137	0.973	0.981	0.000	1.000	1.000	0.000
0.9	0.0	0.331	0.334	0.479	0.413	0.413	0.572	0.195	0.187	0.010	0.212	0.208	0.001
	0.5	0.325	0.346	0.480	0.420	0.418	0.588	0.084	0.087	0.008	0.098	0.095	0.002
	-0.5	0.288	0.299	0.423	0.360	0.357	0.495	0.873	0.857	0.007	0.879	0.872	0.000
1.0	0.0	0.556	0.559	0.751	0.660	0.661	0.827	0.047	0.049	0.048	0.048	0.047	0.057
	0.5	0.572	0.560	0.755	0.671	0.656	0.835	0.027	0.025	0.043	0.022	0.021	0.059
	-0.5	0.541	0.536	0.726	0.624	0.624	0.796	0.358	0.339	0.042	0.391	0.375	0.011

Table 5.4: Empirical Rejection Frequencies of $\mathcal{Q}_0$, Kim, $\mathcal{Q}_1$, Leybourne with $\rho_t = \rho$ and T=100.

		$\mathcal{Q}_0$			Kim			$\mathcal{Q}_1$			Leybourne		
ρ	θ	$\mathcal{Q}_0^f$	$\mathcal{Q}_0^r$	$\mathcal{Q}_0$	K^f	K^r	K	$\mathcal{Q}_1^f$	$\mathcal{Q}_1^r$	$\mathcal{Q}_1$	L^f	L^r	L
0.0	0.0	0.051	0.051	0.050	0.055	0.053	0.054	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.059	0.055	0.062	0.062	0.058	0.068	1.000	1.000	0.000	1.000	1.000	0.000
	-0.5	0.027	0.022	0.018	0.024	0.020	0.014	1.000	1.000	0.000	1.000	1.000	0.000
0.1	0.0	0.056	0.049	0.053	0.058	0.052	0.054	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.055	0.062	0.064	0.070	0.067	0.076	1.000	1.000	0.000	1.000	1.000	0.000
	-0.5	0.030	0.021	0.020	0.025	0.019	0.016	1.000	1.000	0.000	1.000	1.000	0.000
0.5	0.0	0.075	0.065	0.079	0.086	0.081	0.093	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.070	0.066	0.078	0.087	0.085	0.104	1.000	1.000	0.000	1.000	1.000	0.000
	-0.5	0.048	0.044	0.047	0.057	0.045	0.051	1.000	1.000	0.000	1.000	1.000	0.000
0.7	0.0	0.094	0.101	0.114	0.117	0.120	0.153	1.000	1.000	0.000	1.000	1.000	0.000
	0.5	0.092	0.098	0.111	0.123	0.122	0.159	1.000	1.000	0.000	1.000	1.000	0.000
	-0.5	0.082	0.080	0.087	0.099	0.096	0.113	1.000	1.000	0.000	1.000	1.000	0.000
0.9	0.0	0.188	0.189	0.256	0.277	0.268	0.386	0.692	0.676	0.001	0.785	0.765	0.000
	0.5	0.197	0.198	0.274	0.278	0.278	0.395	0.508	0.493	0.001	0.500	0.485	0.000
	-0.5	0.185	0.176	0.243	0.268	0.248	0.362	1.000	1.000	0.002	1.000	1.000	0.000
1.0	0.0	0.553	0.561	0.745	0.691	0.696	0.858	0.049	0.048	0.048	0.050	0.049	0.049
	0.5	0.543	0.555	0.746	0.683	0.683	0.855	0.016	0.022	0.047	0.017	0.016	0.050
	-0.5	0.551	0.535	0.731	0.681	0.674	0.845	0.431	0.428	0.048	0.514	0.504	0.020

Table 5.5: Empirical Rejection Frequencies of $\mathcal{Q}_0$, Kim, $\mathcal{Q}_1$, Leybourne with $\rho_t = \rho$ and T=250.

5.5.4 Properties of the Tests under H_{01} and H_{10}

In the next step, the properties are studied under the alternative hypotheses. Consider first the case H_{01} when the persistence changes from stationarity to non-stationarity at $\tau^* = 0.5$. Then, the parameter ρ_t is given by $\rho_t \in \{0, 0.1, 0.5, 0.7, 0.9)\}$ in the first sub-sample up to $\tau^* T$ of the simulated data and $\rho_t = 1$ after $\tau^* T$ in the sub-sample after a change occurs. The rejection frequencies are reported in Table 5.6 and Table 5.7 with $T = 100$ and $T = 250$.

Regarding the stationarity tests $\mathcal{Q}_0^f$, $\mathcal{Q}_0^r$ and $\mathcal{Q}_0$ of the new testing approach, all versions provide higher rejection frequencies than the ratio based tests K^f, K^r and K. Still, the ratio based tests provide reasonable power results. The pattern is similar for both sample sizes with rejection frequencies close to 1. The results are not distorted by a positive or negative MA parameter, although the size properties are different with a MA component.

The new non-stationarity tests $\mathcal{Q}_1^f$, $\mathcal{Q}_1^r$ and $\mathcal{Q}_1$ provide much better results when compared to the CUSUM based tests L^f, L^r and L. Especially the rejection frequencies of the two-sided test $\mathcal{Q}_1$ to detect a change in persistence in unknown direction are higher than the power of the L test.

Table 5.8 and Table 5.9 report the rejection frequencies for the change in persistence in the other direction H_{10} with $\rho = 1$ in the first sub-sample and $\rho \in \{0, 0.1, 0.5, 0.7, 0.9)\}$ in the second sub-sample with $\tau^* = 0.5$. The results for the tests $\mathcal{Q}_0$, K, $\mathcal{Q}_1$ and L are almost the same as in the H_{01} case. This is not surprising, as the former tests are based on the maximum of forward and reversed test statistics, while the latter are based on the ratio. Furthermore, the results for the one-sided tests are exchanged, which is reasonable, because H_{10} is the same change in persistence as H_{01} in the reversed series.

The main findings for the simulation results including a change in persistence, are the better power properties provided by the new testing approach from the tests $\mathcal{Q}_0$ and $\mathcal{Q}_1$ in all cases. Therefore, the new method is preferable, when detecting a change in persistence in known or unknown direction is of interest.

		$\mathcal{Q}_0$			Kim			$\mathcal{Q}_1$			Leybourne		
ρ_t	θ	$\mathcal{Q}_0^f$	$\mathcal{Q}_0^r$	$\mathcal{Q}_0$	K^f	K^r	K	$\mathcal{Q}_1^f$	$\mathcal{Q}_1^r$	$\mathcal{Q}_1$	L^f	L^r	L
0.0	0.0	0.998	0.769	0.997	0.995	0.717	0.993	1.000	0.141	0.661	1.000	0.256	0.383
	0.5	0.998	0.773	0.998	0.996	0.717	0.994	1.000	0.104	0.506	1.000	0.234	0.324
	-0.5	0.995	0.751	0.992	0.992	0.710	0.988	1.000	0.328	0.751	1.000	0.494	0.109
0.1	0.0	0.998	0.763	0.997	0.995	0.716	0.994	1.000	0.139	0.594	1.000	0.250	0.334
	0.5	0.998	0.778	0.997	0.995	0.726	0.993	1.000	0.103	0.466	0.999	0.235	0.281
	-0.5	0.993	0.746	0.991	0.991	0.706	0.986	1.000	0.326	0.687	1.000	0.493	0.095
0.5	0.0	0.990	0.777	0.987	0.980	0.725	0.976	1.000	0.135	0.297	0.984	0.243	0.141
	0.5	0.986	0.775	0.985	0.979	0.718	0.979	1.000	0.095	0.293	0.945	0.221	0.169
	-0.5	0.978	0.753	0.972	0.970	0.705	0.964	1.000	0.312	0.333	1.000	0.495	0.044
0.7	0.0	0.965	0.775	0.965	0.947	0.717	0.949	0.982	0.128	0.159	0.862	0.254	0.079
	0.5	0.965	0.787	0.967	0.950	0.734	0.953	0.955	0.089	0.158	0.775	0.235	0.090
	-0.5	0.957	0.760	0.953	0.943	0.709	0.944	1.000	0.309	0.157	0.994	0.471	0.023
0.9	0.0	0.901	0.792	0.932	0.853	0.725	0.894	0.797	0.124	0.051	0.488	0.262	0.031
	0.5	0.901	0.801	0.930	0.861	0.722	0.899	0.758	0.086	0.051	0.437	0.222	0.039
	-0.5	0.880	0.783	0.909	0.845	0.727	0.885	0.952	0.305	0.045	0.792	0.469	0.009

Table 5.6: Empirical Rejection Frequencies of $\mathcal{Q}_0$, Kim, $\mathcal{Q}_1$, Leybourne with $\rho_t = 1$ for $\tau^* T > 0.5$ and T=100.

		$\mathcal{Q}_0$			Kim			$\mathcal{Q}_1$			Leybourne		
ρ	θ	$\mathcal{Q}_0^f$	$\mathcal{Q}_0^r$	$\mathcal{Q}_0$	K^f	K^r	K	$\mathcal{Q}_1^f$	$\mathcal{Q}_1^r$	$\mathcal{Q}_1$	L^f	L^r	L
0.0	0.0	1.000	0.685	1.000	1.000	0.598	1.000	1.000	0.158	0.977	1.000	0.197	0.839
	0.5	1.000	0.696	1.000	1.000	0.606	1.000	1.000	0.119	0.937	1.000	0.158	0.766
	-0.5	1.000	0.683	1.000	1.000	0.590	1.000	1.000	0.360	0.990	1.000	0.598	0.409
0.1	0.0	1.000	0.679	1.000	1.000	0.581	1.000	1.000	0.150	0.962	1.000	0.189	0.806
	0.5	1.000	0.693	1.000	1.000	0.601	1.000	1.000	0.115	0.923	1.000	0.155	0.734
	-0.5	1.000	0.684	1.000	1.000	0.598	1.000	1.000	0.356	0.981	1.000	0.593	0.375
0.5	0.0	1.000	0.676	0.999	0.998	0.584	0.996	1.000	0.146	0.809	1.000	0.188	0.550
	0.5	1.000	0.681	1.000	0.998	0.594	0.998	1.000	0.109	0.783	1.000	0.145	0.568
	-0.5	1.000	0.677	1.000	0.998	0.592	0.997	1.000	0.354	0.841	1.000	0.565	0.248
0.7	0.0	0.997	0.691	0.995	0.989	0.595	0.986	1.000	0.143	0.583	1.000	0.189	0.357
	0.5	0.996	0.692	0.995	0.990	0.607	0.987	1.000	0.105	0.576	0.999	0.160	0.397
	-0.5	0.994	0.685	0.992	0.987	0.590	0.985	1.000	0.349	0.582	1.000	0.566	0.160
0.9	0.0	0.946	0.712	0.951	0.909	0.606	0.918	0.869	0.137	0.167	0.755	0.189	0.109
	0.5	0.935	0.702	0.943	0.903	0.598	0.912	0.754	0.098	0.175	0.623	0.147	0.132
	-0.5	0.948	0.706	0.953	0.912	0.604	0.919	1.000	0.341	0.161	0.997	0.550	0.051

Table 5.7: Empirical Rejection Frequencies of $\mathcal{Q}_0$, Kim, $\mathcal{Q}_1$, Leybourne with $\rho_t = 1$ for $\tau^* T > 0.5$ and T=250.

		$\mathcal{Q}_0$			Kim			$\mathcal{Q}_1$			Leybourne		
ρ	θ	$\mathcal{Q}_0^f$	$\mathcal{Q}_0^r$	$\mathcal{Q}_0$	K^f	K^r	K	$\mathcal{Q}_1^f$	$\mathcal{Q}_1^r$	$\mathcal{Q}_1$	L^f	L^r	L
0.0	0.0	0.788	0.998	0.998	0.738	0.996	0.995	0.143	1.000	0.636	0.324	1.000	0.333
	0.5	0.783	0.998	0.997	0.727	0.996	0.995	0.107	1.000	0.489	0.288	1.000	0.283
	-0.5	0.775	0.996	0.994	0.725	0.994	0.991	0.326	1.000	0.752	0.569	0.999	0.086
0.1	0.0	0.794	0.998	0.997	0.742	0.995	0.995	0.137	1.000	0.563	0.319	0.999	0.293
	0.5	0.787	0.998	0.996	0.731	0.994	0.993	0.105	1.000	0.461	0.287	0.999	0.251
	-0.5	0.763	0.993	0.992	0.715	0.990	0.987	0.324	1.000	0.683	0.550	1.000	0.076
0.5	0.0	0.788	0.986	0.986	0.733	0.980	0.978	0.133	1.000	0.296	0.313	0.980	0.125
	0.5	0.790	0.985	0.982	0.737	0.980	0.978	0.098	1.000	0.262	0.297	0.956	0.124
	-0.5	0.765	0.977	0.972	0.723	0.967	0.964	0.311	1.000	0.356	0.535	1.000	0.037
0.7	0.0	0.795	0.963	0.962	0.737	0.948	0.949	0.124	0.934	0.136	0.317	0.866	0.059
	0.5	0.794	0.963	0.964	0.734	0.950	0.955	0.086	0.835	0.146	0.298	0.790	0.075
	-0.5	0.769	0.950	0.951	0.719	0.938	0.939	0.304	1.000	0.149	0.520	0.993	0.016
0.9	0.0	0.812	0.900	0.928	0.744	0.854	0.897	0.120	0.623	0.047	0.316	0.512	0.023
	0.5	0.825	0.902	0.934	0.757	0.860	0.903	0.082	0.536	0.047	0.286	0.472	0.028
	-0.5	0.810	0.881	0.918	0.751	0.834	0.883	0.299	0.837	0.047	0.528	0.795	0.006

Table 5.8: Empirical Rejection Frequencies of $\mathcal{Q}_0$, Kim, $\mathcal{Q}_1$, Leybourne with $\rho_t = 1$ for $\tau^* T \leq 0.5$ and T=100.

		$\mathcal{Q}_0$			Kim			$\mathcal{Q}_1$			Leybourne		
ρ	θ	$\mathcal{Q}_0^f$	$\mathcal{Q}_0^r$	$\mathcal{Q}_0$	K^f	K^r	K	$\mathcal{Q}_1^f$	$\mathcal{Q}_1^r$	$\mathcal{Q}_1$	L^f	L^r	L
0.0	0.0	0.723	1.000	1.000	0.634	1.000	1.000	0.161	1.000	0.968	0.257	1.000	0.808
	0.5	0.725	1.000	1.000	0.640	1.000	1.000	0.123	1.000	0.933	0.215	1.000	0.711
	-0.5	0.728	1.000	1.000	0.639	1.000	1.000	0.365	1.000	0.989	0.650	1.000	0.393
0.1	0.0	0.736	1.000	1.000	0.642	1.000	0.999	0.153	1.000	0.953	0.251	1.000	0.762
	0.5	0.728	1.000	1.000	0.637	1.000	1.000	0.119	1.000	0.916	0.205	1.000	0.673
	-0.5	0.723	1.000	1.000	0.625	1.000	1.000	0.357	1.000	0.982	0.645	1.000	0.381
0.5	0.0	0.721	0.999	0.999	0.629	0.997	0.996	0.142	1.000	0.782	0.259	1.000	0.475
	0.5	0.726	1.000	0.999	0.637	0.998	0.997	0.104	1.000	0.772	0.210	1.000	0.499
	-0.5	0.727	0.999	0.999	0.638	0.997	0.995	0.358	1.000	0.831	0.627	1.000	0.243
0.7	0.0	0.719	0.996	0.994	0.624	0.991	0.987	0.138	1.000	0.591	0.239	1.000	0.321
	0.5	0.737	0.996	0.994	0.639	0.992	0.990	0.100	1.000	0.568	0.218	0.999	0.336
	-0.5	0.728	0.995	0.995	0.640	0.990	0.988	0.343	1.000	0.578	0.623	1.000	0.142
0.9	0.0	0.750	0.944	0.951	0.646	0.905	0.913	0.132	0.819	0.175	0.245	0.786	0.090
	0.5	0.748	0.942	0.951	0.650	0.901	0.911	0.099	0.724	0.173	0.222	0.680	0.099
	-0.5	0.737	0.935	0.939	0.639	0.901	0.909	0.339	1.000	0.153	0.620	0.997	0.032

Table 5.9: Empirical Rejection Frequencies of $\mathcal{Q}_0$, Kim, $\mathcal{Q}_1$, Leybourne with $\rho_t = 1$ for $\tau^* T \leq 0.5$ and T=250.

5.6 Conclusion

This paper presents two new tests for a change in persistence based on squared CUSUMs of sub-sample OLS residuals. The shortcomings of the existing tests are that different weights of the residuals are considered in the partial sum processes in ratio based tests and cross-dependencies of the time series are not considered in CUSUM of squared residual based tests. The approach of our test statistic overcome these shortcomings and is based on the different behaviour of stationary and non-stationary processes.

The main feature of a $I(1)$ time series are sequences of large positive and negative consecutive observations, while these sequences are short for $I(0)$ series. Therefore, the persistence of a time series is reflected in the partial sums of different order and we calculate the squared sum of all partial sum processes. Similar to Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004) we construct a ratio based test for the H_0 hypothesis and related to Leybourne et al. (2007b) a two-tailed test statistic to test the H_1 hypothesis is developed. We derived the limiting distribution of the tests and consistency is shown. Simulation results suggest that our new procedure provide better results than the existing persistence change tests in finite samples. Therefore, in applications the new testing principle is recommended.

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