Eigenvalues enclosures of skew symmetric/Hermitian matrices having bounded uncertainty

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Introduction

We have concentrated on binding the eigenvalues of skew symmetric/Hermitian interval matrices. It has been noticed that significantly less attention has been given to this kind of matrices. The benefits and drawbacks of handling these problems are discussed here.

Basic properties

The skew symmetric interval matrix $\boldsymbol{A}^{ss}$ is defined as

$$\boldsymbol{A}^{ss} = \{A \in \boldsymbol{A} | A = -A^T\}, \tag{1}$$

where $\boldsymbol{A}$ is a real interval matrix. We know that eigenvalues of a skew symmetric matrix are either zero or purely imaginary. Thus our focus is on the real eigenvalues of the interval matrix $i\boldsymbol{A}^{ss}$.

There is no way to compute exact eigenvalue bounds for the complex interval matrix. Therefore finding outer bounds of eigenvalues of the complex interval matrix $i\boldsymbol{A}^{ss}$ is a good option.

The exact interval eigenvalue $\boldsymbol{\Lambda}$ of $\boldsymbol{A}^{ss}$ is described as

$$\boldsymbol{\Lambda} = \{ib : Az = ibz, A \in \boldsymbol{A}^{ss}, z \neq 0\}. \tag{2}$$

Main results

One bound [4] of the eigenvalues of $i\boldsymbol{A}^{ss}$ can be found by Gershgorin bound as following,

$$-\Sigma_{k \neq j} |\boldsymbol{a}_{jk}| \leq \lambda \leq \Sigma_{k \neq j} |\boldsymbol{a}_{jk}|, \tag{3}$$

for $j = 1, \ldots, n$ and $\boldsymbol{A}^{ss} = (\boldsymbol{a}_{jk})$, where n is the order of the matrix.

Another bound [1] for complex interval matrices can be found by the following equation,

$$\underline{\lambda}_n \begin{pmatrix} \boldsymbol{O} & \boldsymbol{A}^{\boldsymbol{ss}T} \\ \boldsymbol{A}^{ss} & \boldsymbol{O} \end{pmatrix}^s \leq \lambda \leq \overline{\lambda}_1 \begin{pmatrix} \boldsymbol{O} & \boldsymbol{A}^{\boldsymbol{ss}T} \\ \boldsymbol{A}^{ss} & \boldsymbol{O} \end{pmatrix}^s. \tag{4}$$

Also, there are iterative approaches for the eigenvalue enclosure of complex interval matrices [2, 3].

Purely imaginary skew Hermitian interval matrices:

Let $i\boldsymbol{B}^s$ be a skew Hermitian interval matrix. Then $\boldsymbol{B}^s$ is a real symmetric matrix. We can bound the eigenvalues for symmetric interval matrices, and consequently, we will get eigenvalue bounds for $i\boldsymbol{B}^s$.

The eigenvalues of purely imaginary skew Hermitian interval matrices will be zero or purely imaginary. The main distinction with skew symmetric matrices is that eigenvalues do not occur in conjugate pairs.

Complex skew Hermitian interval matrices:

We can obtain enclosures for eigenvalue clusters of complex skew Hermitian interval matrices by different methods developed for complex interval matrices. However, we need to find the tighter enclosure of the eigenvalue clusters for the complex skew Hermitian interval matrices.

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