A constraint programming approach for polytopic simulation of ordinary differential equations - a collision detection application

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Introduction

Starting from a set of possible initial points, the solution of an ODE can be represented by a *reachable tube* describing the evolution of the system from this initial set. *Abstract domains* can be used to enclose the tube: *boxes* (cartesian product of intervals), *zonotopes*, *ellipsoids*, and nonconvex sets such as *Taylor models* (see [2] for a review of these abstract domains). The more accurate an abstract domain is, *i.e.* the smallest the difference between the hull of the abstraction and the abstracted set is, the more accurate the enclosure of the reachable tube will be. Polytope enclosure is a promising approach as it is more precise than the interval or zonotope abstract domains, but suffers from the expensiveness of its geometrical computation. Considering a polytope as *an intersection of zonotopes* and therefore benefiting from affine arithmetic is a possible solution to overcome the limitations (e.g. zonotope bundles [3], i.e., a set of zonotopes is used and therefore the intersection is not computed; or the intersection is computed when necessary [1]).

Reachable tubes as abstract trees

Considering a tube, a disjunction of predicates, $\mathcal{T} = (t_1 \wedge e_1) \vee (t_2 \wedge e_2) \cdots \vee (t_n \wedge e_n)$ where each e_i represents the set of values of solution functions within time frame t_i, it is to be understood as the following property: *the solution is either in set e_1 during the time frame t_1, or in set e_2 during the time frame t_2, etc.* Considering initial values given as a polytope $\mathcal{P}$, we decompose $\mathcal{P}$ as the intersection of s zonotopes $\mathcal{Z}_i$. The reachable tube of the corresponding ODE is therefore described by the conjunction of s tubes $\mathcal{T}^1 \wedge \cdots \wedge \mathcal{T}^i \wedge \cdots \wedge \mathcal{T}^s$, each tube $\mathcal{T}^i$ being obtained by the zonotopic simulation with initial value taken in $\mathcal{Z}_i$. This conjunction of disjunctions can be efficiently solved with constraint programming and polytopes as abstract domains. For obstacle avoidance or collision detection, a predicate (or several ones) of the form "and not in" is added to the previous formula.

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