Efficient engineering analysis with imprecise probabilities

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Abstract

An efficient analysis of our engineering structures and systems is a key requirement for a most suitable design at the required level of reliability. This requirement, however, is challenging engineers to come up with innovative solutions that can cope with the increasing complexity of our structures and their behaviour and with the uncertainties involved. Imprecise probabilities have shown useful conceptual features to facilitate a modelling at a reasonable level of detail and capturing the remaining epistemic uncertainty in a set-valued manner. This approach allows for an optimal balance between model detailedness and imprecision of results to still derive useful decisions. However, it is also associated with some extensive numerical cost when applied in a crude way. This presentation will highlight selected solutions for efficient numerical analysis with imprecise probabilities, specifically for reliability analysis, to attack high-dimensional and nonlinear problems. After an introductory overview on conceptual pathways for solution one intrusive and three non-intrusive specific developments will be discussed. These solutions include operator norm theory to solve first passage problems by linear algebra, intervening variables to moderate nonlinearities for linearized approximate solutions, and the utilization

of high dimensional model representation of the failure probability for non-intrusive efficient sampling. Engineering examples are presented to demonstrate the capabilities of the approaches and concepts.

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