A new type of intervals for solving problems involving partially defined functions

Pierre Filiol, Théotime Bollengier, Luc Jaulin,
Jean-Christophe Le Lann

Lab-STICC, ENSTA Bretagne
Brest, France

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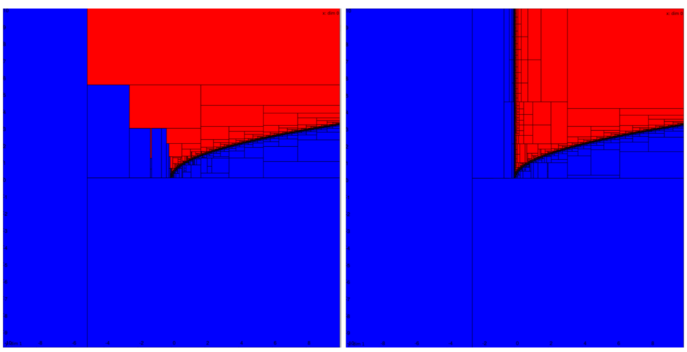
Introduction

If we want to characterize an inner and an outer approximation of

$$\mathbb{S} = \{(x, y) \,|\, y - \sqrt{2x - x} \geq 0\}$$

a classical set inversion algorithm [1], yields the left figure, where as we would like to obtain the right figure



The outer contractor works well, but the inner contractor is over-contracting. Note that, the multi-occurence of x in the expression $\sqrt{2x - x}$, allows the inner contractor to show its weakness. This type of problems occurs several times in our real applications when dealing with functions such as log, $\sqrt{\cdot}$ that are not defined everywhere. We want to identify the reasons of the problem and find a way to fix it.

New type of interval

Consider the extended set of reals $\mathring{\mathbb{R}} = \mathbb{R} \cup \iota$ where ι stands for *Not A Number* [2]. Operations on real numbers can be extended to $\mathring{\mathbb{R}}$ as follows:

$$\begin{array}{ll} f(x) = \iota & \text{if } x \notin \mathrm{dom}(f) \\ f(\iota) = \iota & \\ \iota \diamond x = \iota & \end{array}$$

where $f : \mathbb{R} \to \mathbb{R}$, $x \in \mathbb{R}$ and $\diamond$ is a binary operator. The set $\mathring{\mathbb{R}}$ can be equipped with a partial order relation derived from rules:

$$\begin{array}{l} \iota \leq \iota \\ a \in \mathbb{R}, b \in \mathbb{R} \quad \text{then} \quad a \leq_{\mathring{\mathbb{R}}} b \text{ iff } a \leq_{\mathbb{R}} b \end{array}$$

and intervals can be derived from these relations. Examples of intervals of $\mathring{\mathbb{R}}$ are $[2, 5], [2, 5] \cup \{\iota\}, \{\iota\}, \emptyset$. In the extended paper, we show that this new type of intervals allows us to solve inequalities where functions are not defined everywhere.

References

[1] L. Jaulin, M. Kieffer, O. Didrit, E. Walter, *Applied Interval Analysis*, Springer-Verlag, 2022.

[2] Institute of Electrical and Electronics Engineers A.N.S.I. *A standard for binary floating-point arithmetic.* ANSI/IEEE Std. 754-1985, New York, 1985