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Machine-Learning-Based Exploration of Bending Flexoelectricity in Novel 2D Van der Waals Bilayers

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Accurate examination of electricity generation stemming from higher-order deformation (flexoelectricity) in 2D layered materials is a highly challenging task to be investigated with either conventional computational or experimental tools. To address this challenge herein an innovative and computationally efficient approach on the basis of density functional theory (DFT) and machine-learning interatomic potentials (MLIPs) with incorporated long-range interactions to accurately investigate the flexoelectric energy conversion in 2D van der Waals (vdW) bilayers is proposed. In this approach, short-range interactions are accurately defined using the moment tensor potentials trained over computationally inexpensive DFT-based datasets. The long-range electrostatic (charge and dipole) and vdW interaction parameters are calibrated from DFT simulations. Elaborated comparison of mechanical and piezoelectric properties extracted from the herein proposed approach with available data confirms the accuracy of the devised computational strategy. It is shown that the bilayer transition metal dichalcogenides can show a flexoelectric coefficient 2–7 times larger than their monolayer counterparts. Noticeably, this enhancement reaches up to 20 times for Janus diamane and fluorinated boron-nitrogen derivatives of diamane bilayers. The presented results improve the understanding of the flexoelectric effect in vdW heterostructures and moreover the proposed MLIP-based methodology offers a robust tool to improve the design of novel energy harvesting devices.

1. Introduction

Flexoelectricity is the emerging electromagnetic coupling type that generates electricity under mechanical deformations and vice versa. Solid perovskite materials well studied to avail this electricity. However, recent research interest has turned to 2D materials due to their highly promising features, like achieving larger strain gradients, a wide choice of materials (also bio-compatible), and miniature and effective energy conversion rate. Applications involving flexoelectricity are not only limited to energy harvesting but also bone crack-healing, memory reading, memory writing, sensors, silicon Schottky diodes. On the other hand, a vast development of 2D van der Waals (vdW) materials is highly aspiring. Especially, the material property dependency with the number of layers, translational and rotational stack sequencing, and ability to stack different materials due to weak vdW forces shed light into new research directions and raise open questions. Deeply investigated the fundamental electronic, mechanical, and thermal properties of vdW layers. Review articles focus on the current state of research and the devices using the vdW layers. The inter-layer electronic interactions transfer the charges among the layers and help to generate polarization. Layered GdTe$_3$ material exhibit exceptional electron mobility than its bulk counterpart. The flexoelectricity coupled with photoelectric effect enhances the separation of electron–hole pairs in bent 2D semiconductors. The vertical piezoelectricity (electricity due to mechanical strain) for Janus multilayer transition metal dichalcogenides (TMDs) is higher than the conventional quartz crystal. Using two Janus TMD monolayers, a tribo-piezoelectric nano energy generator with high power densities is predicted. The symmetry breaking in twisted bilayer graphene induces flexoelectric polarization. In a double layer MoS$_2$ nanotubes, curvature induced voltage generation is estimated.

The experimental investigation on flexoelectric coefficients uses piezoresponse force microscopy (PFM) for monolayers of TMDs. Other studies also use PFM for thin (more than 2 nm) LaAlO$_3$ and MoS$_2$ to predict the flexoelectricity. In order to minimize the experimental burdens, expert investigation on fine-scale physics and faster exploration across different materials, numerical models are assuredly helpful. Theoretical studies on carbon nanoshells provide electronic rehybridization due to...
curvature,[26] which create a dipole moment. A recent density functional theory (DFT) study explores the flexoelectric property in many 2D materials by considering the changes in radial polarization,[27] and another one uses a potential difference across the nanotube cross-section.[28] Those studies differ in terms of the flexoelectric coefficients due to the involvement of different contributions for total polarization. To estimate flexoelectricity, the periodic ripple patterns are introduced in a 2D sheet.[29,30] Explored the flexoelectric dipole moment in nanocubes and nanotubes using DFT.[31,32] In regard of flexoelectricity in vdW layers, issues like verification and flexoelectric dipole moment in nanocones and nanotubes using developed potential parameters and then extracted the flexoelectricity with MD simulations alone. The present work focuses on handling these issues in molecular dynamics (MD) environment by deriving the accurate interatomic potentials from first-principle simulations.

In doing so, we consider three materials groups, TMDs, diamanes and BN-diamanes. We propose the BN-diamanes by replacing the carbons in the native diamane structure with alternate boron and nitrogen atoms. Monolayer diamane exhibited improved flexoelectric response compared to conventional TMD monolayer[33] thus, the boron–nitrogen bonds create extra asymmetry in BN-diamane and may help in flexoelectric effect. Figure S1, Supporting Information, shows the stacking configurations considered in the present work for these material groups. The dashed line indicates the atomic alignment in both the layers. To utilize the MD framework, the availability of accurate interatomic potential is essential. Recent machine learning based moment tensor potential (MTP) parameters provide high accuracy in predicting the mechanical and thermal properties for monolayer 2D materials. This study employs MTP for short-range interactions and derives the long-range interaction parameters from DFT simulations. To induce a natural bending response to the vdW layers the present work proposes compressing only the edges of 2D material and analyzing the resulting configuration. Such models are extremely impracticable through DFT simulations due to the limitation of periodic boundary condition. The proposed deformation process is highly and effectively possible with the derived inter-atomic potentials in MD environment. The performed simulations in the manuscript are divided into three parts: 1) Generating the inter-atomic potential parameters: First we prepare the data sets at high and low temperatures for the considered vdW bilayers and obtain MTP parameters through the machine-learning interatomatic potentials (MLIPs) scheme. Lennard-Jones (LJ) and charge-dipole (CD) potential parameters are obtained next by matching the DFT measured inter-layer energy and polarizability with the MD simulations. 2) Validating the potential parameters: The initial tests are assigned to validate the potential parameters by comparing the mechanical and piezoelectric properties with available data in the literature and through standalone DFT simulations. 3) Bending under compression: The compression simulations are performed for the bilayer systems with the developed potential parameters and then extracted the flexoelectric coefficients by processing the generated atomic data.

2. Results and Discussion

The atomic interactions in bilayer systems are divided into short-range and long-range contributions. Figure 1a illustrates the schematic of layer separation and consideration of neighbor atoms restricted to each layer separately. We perform ab initio molecular dynamics (AIMD) simulations over bilayer supercells with 1 nm² area at 50 and 600 K temperature, each for 1000 time steps. Out of these steps, half of the full trajectories were selected to create the training sets, which consist of positions r, energy e, forces f, and stress σ as input for passive fitting implemented in the machine learning inter-atomic potential package.[34] The methodology for efficient training of MTPs are explained in early works.[15–17] To derive the long-range vdW inter-layer interaction parameters, we modify the unitcell size in z-direction as 50 Å for the optimized bilayer unitcell. This helps to avoid the unnecessary interaction/contributions from the periodic replicas along the z-direction. We set this limit to be about five times larger than the maximum distance of separation between the two layers in a bilayer system (as seen in Figure S2, Supporting Information). Next, we vary the inter-layer distance h between the two monolayers in a bilayer unitcell and perform electronic self-consistent calculations taking into account the vdW dispersion correction using the DFT-D3 method of Grimme.[38,39] We record the total energy of the bilayer system as a function of inter-layer distance. The inter-layer energy from DFT simulations (ΦDFT) is the difference of energy from bilayer unitcell and twice the monolayer energy. We now perform classical single step MD simulation with no velocity involvement for a simulation cell area 4 nm² with in-plane (x and y) periodic boundary conditions. The generated MTP parameters and LJ parameters (ε and ψ) define the pair interaction potential. The inter-layer energy from MD (ϕ) simulations is defined as the energetic interaction across the atomic layers. Figure 1b indicates the neighbor atom selection. Atom belongs to the bottom layer has a set of neighbors from the top layer. The LJ parameters for each atom type in a bilayer unitcell are determined by establishing a close match between ϕ and ΦDFT. Figure 1b and Figure S2, Supporting Information, depicts the comparison between ϕ and ΦDFT. The atomic positions in optimized bilayer unitcell were updated with the minimum energetic distance of separation (hₚ). The CD model was added to the short-range MTP and LJ potentials to estimate the deformation-induced electrical polarization. According to CD model, each atom carries a charge q and dipole moment p. This model is dependent on the parameter R for each atom type, which is related to total polarizability. We adjust R to match the atomic polarizability estimated from DFT ϕDFT (using GAUSSIAN software[40]) to the calculated polarizability from CD model α (schematic shown in Figure 1c). The early works from authors cover the calculation of CD parameters and implementation of CD model in MD environment.[41,42]

The initial tests aim to benchmark the derived potential parameters. The in-plane elastic modulus (Eᵢ) under tensile stretching deformation estimated from the slope of stress σᵢᵧ to strain εᵢᵧ (upto 1%). Figure 2a shows the elastic modulus for bilayer systems with stacking order a, which are in excellent agreement with the DFT predicted elastic modulus EᵢDFT. The elastic modulus for bilayer SeMoS₂, HCF, and HBNF are 221, 917, and 699 N m⁻¹, respectively. The corresponding monolayer values were calculated as 109, 460, and 332 N m⁻¹. This fulfills the expected twice increment of elastic modulus due to the involvement of two monolayers. Note that, we avoided
using the thickness in the calculation of stress and polarization. Because of this, the reported Young’s modulus is double for bilayer as compared with the monolayer counterpart. The inter-layer vdW interaction energy $\phi$ estimated for the given configuration is in match with the DFT predictions $\phi_{\text{DFT}}$ (see Figure 2a and values belong to right-hand side vertical axis). We also observe that $\phi$ is closely related to the earlier DFT predictions ($\phi_{\text{REF}}$ as open markers in Figure 2a) for bilayer SMoS, SMoSe, and SeMoSe systems. This shows that the derived LJ parameters accurately capturing the inter-layer interactions. Diamane (TMD) bilayers have weak (strong) inter-layer energy. This trend is reversed in the case of elastic modulus. Figure 2b shows the high consistency exists between polarizabilities $\alpha$ and $\alpha_{\text{DFT}}$. The bilayer $\alpha$ for SMoS is 39.33 Å$^3$ which is 3.184 times higher than the monolayer $\alpha$. The associated CD parameter $R$ values increased by 1.27 times for the chalcogen (sulfur and selenium) atoms in bilayer SMoSe. In bilayer HCF, $\alpha$ rises only about 1.315 times that of monolayer HCF. The bilayer HBNF shows a rise of $\alpha$ nearly 1.9 times than the monolayer HBNF. The MD simulations were performed at a temperature of 1 K and DFT results acquired at ground state (0 K). Table S1, Supporting Information, indicates the derived LJ and CD parameters for bilayer materials with stacking order a. Tables S2–S4, Supporting Information, provide several physical quantities (lattice constant, inter-layer energy, inter-layer distance, polarizability, and elastic modulus) for the bilayer stacking orders along with DFT values. Note that for diamanes, the parameter derivation executed only for Janus structure HCF-a. Using that, simulations for all other stacked configurations (b, c, d, and e) performed. Also, used the same parameters for the HCH and FCF bilayer systems. For BNdiamanes, the potential parameters derived for Janus HBNF and non-Janus HBNH and FBNF bilayers. This is due to the missing bond information from Janus HBNF alone. For example, the parameters for HCF used to perform HCH or FCF simulations. Since the required interactions (C−H and C−F) are known from HCF itself. In case of HBNF, interactions N−H and B−F are missing when using it for HBNH and FBNF, respectively.

Figure 2c shows the piezoelectric coefficients under in-plane and out-of-plane uniaxial stretching deformations for bilayer materials with stacking orders from a to e. From the slope of the total polarization $P_y$ to the applied strain $\varepsilon_{yy}$, the piezoelectric coefficient $\varepsilon_{yyy}$ for SeMoS-d and SeMoS-e are noted
as 0.65 and 0.748 nC m$^{-1}$, which is nearly twice that of a monolayer SeMoS$^{[18,45]}$. Throughout the paper, the total polarization is the net dipole moment to the total area of the system. The 2D layer thickness is not utilized in defining the polarization. The stacking orders a–c are non-piezoelectric. The horizontal arrows in Figure 2d show \( P_y \) is opposite in both the layers for a, b, and c stacking orders. This cancels the total polarization. In stacking order d and e, direction of \( P_y \) is aligned in both layers and results \( e_y \) as non-zero. Similarly in non-Janus SMoS and SeMoSe bilayers, a to c stacking orders show \( e_y \) as zero. For d, e stacking orders \( e_y \) is twice that of monolayers. The \( e_y \) for bilayer diamanes and BN-diamanes are an order smaller than TMDs, similar to the earlier reports on monolayers.$^{[33]}$ The direction of polarization \( P_z \) is same as in TMDs (see Figure 2d,e).

In the out-of-plane direction, polarization \( P_z \) arises due to the Janus structure of each layer in SeMoS and HCF bilayers. Irrespective to the stacking order, \( P_z \) shows a linear change with strain \( \varepsilon_{yy} \) which leads non-zero \( e_{zy} \) piezoelectric coefficient. The sulfur atoms acquire higher polarization \( P_z \) than selenium atoms due to the high electron affinity of sulfur. For stacking orders, c and d noticed an enhanced \( P_z \), which makes \( e_{zy} \) as 1.72 times larger than SeMoS-a, SeMoS-b, and SeMoS-e. The \( e_{zy} \) value for SeMoS-a is 0.061 nC m$^{-1}$, and for multilayer SeMoS is 0.047 nC m$^{-1}$. This confirms that the derived interatomic potentials accurately predict the electromechanical properties of bilayers. Note that the present results are specifically for bilayers without assuming periodicity in out-of-plane direction, whereas DFT predictions use the periodicity assumption. The non-Janus diamanes (HCH and FCF) do not yield \( P_z \), and the \( e_{zy} \) is zero. HCF-a to HCF-e bilayers yield \( e_{zy} \) about two times higher than Janus TMDs. When compared with monolayer HCF, \( e_{zy} \) is four times higher in bilayer HCF. The collective support from all the carbon atom dipole moments makes the polarization \( P_z \) high (see Figure 2e). There is a cancellation
of the dipole moment due to the boron-to-nitrogen bond in the out-of-plane direction for HBNF bilayer. Figure 2f clearly indicates the oppositely directed $P_b$ for boron and nitrogen atoms for all stacking orders. Because of this, there is no possibility of $e_{yy}$. The single boron–nitride monolayer decorated with hydrogen and fluorine yield significant $e_{yy}$ and $e_{zy}$ values.\(^\text{[46]}\) The present HBNF consists of three boron–nitrogen bonds (similar to diamane), which cancels the generated polarization under stretching. Such a cancellation was slightly avoided in case of HBNH and FBNF bilayers. The resultant $e_{yy}$ is six times smaller than that in HCF bilayers. The observations for $e_{yy}$ and $e_{zy}$ under tensile deformation further support the accuracy of derived potential parameters and simulation settings.

The vertical tensile deformation helps to determine the elastic coefficients ($E_{zzzz}$ and $B_{yzzz}$) and the piezoelectric coefficient $e_{zzzz}$. We adopt the displacement control conditions to hold the top-most and bottom-most set of atoms. $E_{zzzz}$ and $B_{yzzz}$ for SmoS-a is 43.4 GPa (thickness as 9.35 Å) and from the previous report is 58 GPa.\(^\text{[47]}\) The observed difference (about 14 GPa) is mainly due to the removal of stress contributions from the holding atoms. Further, we perform only stretching simulations for three and four layered SmoS systems. Figure S3, Supporting Information, shows the out-of-plane elastic modulus variation with number of layers in SmoS. The resulting elastic coefficient $E_{zzzz}$ was noted as 51.2 and 49.3 GPa, respectively. For four-layered SmoS, $E_{zzzz}$ and $B_{yzzz}$ are highly coinciding with the experimental bulk sample\(^\text{[48]}\) as well as DFT reported values\(^\text{[49,50]}\) for multi-layered systems. The $e_{zzzz}$ coefficient is 0.07 nC m\(^{-1}\) for SmoS-a to SmoS-e (see Figure 2c). Whereas for multi-layer SmoS system, $e_{zzzz}$ is noted as 0.43 nC m\(^{-1}\)\(^\text{[48]}\). We extend our simulations for three and four layer SmoS system with stacking order a and found the $e_{zzzz}$ as 0.22 and 0.36 nC m\(^{-1}\), respectively. The observed values of $e_{zzzz}$ are also matching with the DFT predictions for multi-layers. This examines the results accuracy further. Other than bilayer HBNF ($e_{zzzz} = 0.05$ nC m\(^{-1}\)), the remaining diamane bilayers show zero $e_{zzzz}$ values.

We now focus on the compressive loading simulations. As shown in Figure 1d, the subjection of left and right edge atoms along y-direction to an equal and opposite displacements (0.1 Å) lead to a natural bending deformation. The bilayer system is initially compressed, and buckling occurs at the critical compression strain. Later, with the increase of compressive strain, the deformation follows the shape at the critical strain, and there is no observation of other higher-order deformations. Supporting videos visualize the deformation of the bilayer system. First, we calculate the averaged curvature for the compressed system using Equation (S4), Supporting Information. The bending energy ($E_b$) is estimated as the difference of total energy per unit area between initial and current atomic states using Equation (S6), Supporting Information. Figure S6a, Supporting Information, indicates the response of $E_b$ with square of curvature ($\kappa^2$). The bending stiffness $D_b$ computed using Equation (S8), Supporting Information, for bilayer SmoS-a is 33.12 eV, which is in the experimental prediction range of 14–55 eV for aligned bilayers.\(^\text{[50]}\) For bilayer SmoS-a, $D_b$ is about 21.41 eV. Due to the force imbalance through the different chalcogen atom bonding on either side of the molybdenum atom, the bending deformation is achieved easily in SmoS-a compared to SmoS-a. Similar observation made between FCF-a (142.73 eV) and HCF-a (114.57 eV).

Systems FBNF-a and FBNH-a show $D_b$ as 105.51 and 86.77 eV, respectively. The diamane and BNdiomane show enhanced $D_b$ compared to the TMDCs. The structural asymmetry in these layered systems helps to reduce the bending stiffness. Figure S4, Supporting Information, plots the bin averaged strain distribution along the compressive direction. The components $e_{xx}$, $e_{yy}$, and $e_{zy}$ are non-zero, and the remaining components are zero. From these non-zero strains, the total polarization $P_z$ including piezoelectric and flexoelectric effects expressed as

$$P_z = e_{yy} e_{yy} + e_{zz} e_{zz} + \mu_{zyy} \frac{\partial e_{xx}}{\partial y} + \mu_{yyz} \frac{\partial e_{yy}}{\partial y} + \mu_{zym} \frac{\partial e_{zy}}{\partial y}$$

where $e_{yy}$ and $\mu_{zyy}$ represent the piezoelectric and flexoelectric coefficients. $\alpha$, $\beta$, $\gamma$, and $\delta$ denote the Cartesian directions $x$, $y$, and $z$. The opposite variation of $e_{yy}$ and $e_{zy}$ (see Figure S4, Supporting Information) cancels the resultant piezoelectric part of polarizations $e_{yy} e_{yy}$ and $e_{zy} e_{zy}$. The associated average strain gradients for $e_{yy}$ and $e_{zy}$ are about 100 times smaller than the strain gradient $\frac{\partial e_{xx}}{\partial y}$. Also, the structural symmetry does not allow the coefficients $\mu_{yyz}$ and $\mu_{zym}$. Hence the only effective contribution is from $e_{yy}$ to the total polarization $P_z$. We calculate the average strain gradient $e_{yy}$ using Equation (S5), Supporting Information. Unlike earlier deformation prescribed reports\(^\text{[45,52]}\) the strain gradient and curvature are not equal. Note that, the curvature uses the averaged coordinates and not depend on the atomic strain. The current scheme is free from prescribing the deformation pattern and the bending arise as a natural process.

Figure 4a shows the linear variation of $P_z$ with $e_{yy}$ for SmoS, SmoS, HCF, and FBNF bilayers in stacking order a. The flexoelectric coefficient $\mu_{zyy}$ is 0.99 eC for SmoS-a, which is seven times higher than the monolayer SmoS\(^\text{[45]}\) (thickness of 6.5 Å is used to convert nC m\(^{-1}\) to eC units). A recent experimental study on thin SmoS film predicted the absolute value of flexoelectric coefficient as 0.23 nC m\(^{-1}\)\(^\text{[22,23]}\) which is also seven times larger than the monolayer SmoS\(^\text{[45]}\). DFT reports the flexoelectric coefficient value as 0.14, 0.04, and 0.032\(^\text{[53]}\) nC m\(^{-1}\). The difference in these values is attributed to the consideration of different contributions to polarization, deformation scheme, thickness definition, etc. The original experimental investigation on monolayer SmoS reports the measured effective piezoelectric coefficient $d_{zzzz}$ about 1 to 1.5 pm V\(^{-1}\)\(^\text{[22,23]}\) and then converted into effective flexoelectric response under certain assumptions. In the current simulations, we have the recorded data of stress $\sigma_{zz}$ during the compression process. Using that, we established a linear fitting across $P_z$ and $\sigma_{zz}$, which yields the coefficient $d_{zzzz}$ as 2.58 pm V\(^{-1}\) for SmoS-a. Figure S6b, Supporting Information, shows this variation for bilayer materials SmoS, SeMoS, HCF, and FBNF bilayers in stacking order a. For SmoS-e configuration, $\mu_{zyy}$ is 0.42 eC and $d_{zzzz}$ is noted as 1.69 pm V\(^{-1}\). Another recent experimental study\(^\text{[54]}\) reports $d_{zzzz}$ as 0.7–1.5 pm V\(^{-1}\) (thickness is 4–90 nm) under same stacking as SmoS-e. This represents the enhancement of $\mu_{zyy}$ over monolayer is strongly depending on the stacking order. Figure 4c shows $\mu_{zyy}$ for SmoS-a to SmoS-e. The flexoelectric response in SmoS-b to SmoS-d similar to SmoS-e. Identical observations were made from the stacking orders in SeMoSe bilayer system.
For SeMoS-a, $\mu_{zyzy}$ is 0.93 eC, which is only 1.8 times than monolayer SeMoS (thickness 6.9 Å used to convert 0.117 nC m$^{-1}$[45] to 0.504 eC) and for SeMoS-e, $\mu_{zyzy}$ is 1.98 eC. SeMoS-b, SeMoS-c, and SeMoS-d show higher $\mu_{zyzy}$ compared to SeMoS-a (see Figure 4c). We note the coefficient $d_{zzz}$ as 2.85 pm V$^{-1}$, and from DFT predictions for a multilayer SeMoS system is 2.58 pm V$^{-1}$[55] in stacking order e. An inverted trend is observed when compared to SMoS or SeMoSe bilayers. Figure 3a shows the polarization distribution along the compression direction for SeMoS-a and SeMoS-e. The polarization from atoms in the highlighted cells (A and C) is small in SeMoS-a and high in SeMoS-e. The bond angle variation at different strain gradients for cells A and B are plotted in Figure S7, Supporting Information. The bond angle for atoms in cell A is nearly constant and varies for cell B. Due to the no change in bond angle, the electron interactions ($\pi-\sigma$ or $\sigma-\sigma$) are not effective to induce a dipole moment. As a result, the polarization corresponding to cell A is minimal. A lower scale of rigidity is observed for bond angle in cell C of SeMoS-e (Figure S7, Supporting Information), which help to raise the polarization. Also unlike SMoS, there exist interaction between sulfur and selenium from the bottom and top layers. That interaction also plays a significant role with respect to the stacking order in defining the polarization.

Figure 4a,b plots the $P_z$ response for FCF-a and FCF-e. The generated $P_z$ is very small in these systems. There is no observation of linear variation of $P_z$ for all stacking orders of FCF system. To confirm this observation, we plot the distribution of $P_z$ over bins for each layer in Figure 3b. A cancellation of $P_z$ is understood. The polarization for cell A is zero, and for cell B is maximum. The edge cell C shows a polarization that opposing the polarization in cell B. A similar trend is observed on the other side of Figure 3b. The fluorine atoms connected to carbon atoms result in an equal and opposite dipole moment $p_z$ in cell A, which cancels the polarization $P_z$. In cell B, the bending induced change of bond length and bond angles creates an imbalance between carbon–fluorine bonds and results a $p_z$ of 0.0034 eCA. Cell C is near to the loading edge creates an opposite strain response to cell B, which reverses the direction of $p_z$. Overall, the strong charge localization of fluorine atoms and the weak symmetry breakage avoid the changes in polarization for FCF. Similar observations were made for HCH system. To confirm the above observations, we repeated our calculations with monolayer FCF and HCH systems using the current scheme and obtained $\mu_{zyzy}$ values that agree with earlier predictions.[33] In a monolayer, the complete cancellation of $P_z$ is avoided (as seen from bottom or top layer of Figure 3b). Whereas, in the bilayer system the response of $P_z$ in each layer is in contrast with each other, which makes the $P_z$ low and not linear.

The HBNF-a and HBNF-e configurations show very small $P_z$ and there is no observation of linear relationship with the
strain gradient (see Figure 4a,b). The atomic configurations in Figure 3c confirms the distribution of low $P_z$ in stacking orders a and e. This is mainly due to the cancellation of dipole moments associated with each unitcell. The vertical dipole moment $p_z$ is opposite for the bonds nitrogen–fluorine and boron–hydrogen. Out of the three central boron–nitrogen bonds, only the vertical bond maintain the dipole moment. The other two bonds align opposite to each other, and hence their dipole moments cancel with each other. The bending induced bond angle and bond length changes induce flexoelectric effect. However, the generated asymmetry in dipole moment is not overcoming the cancellation. As a total, the central bond is responsible for the total polarization in each layer. The $P_z$ for HBNF-e is 0.00014 eC Å$^{-1}$ at bending angle 11.52°, which is ten times smaller than SeMoS-e at same bending angle. Other stacking orders b to d also perform in the similar manner and there is no observation of linear response. Hence $μ_{zyzy}$ values are noted as zero in Figure 4c.

The HCF materials outperform in generating the polarization under bending deformation. The $μ_{zyzy}$ for HCF-a is doubled with respect to bilayer TMDs (see Figure 4c). Unlike HBNF and FCF cases, here the cancellation of dipole moments avoided. As fluorine atoms gains more charge due to electron negativity character, the resultant dipole moment directed toward it. The total $P_z$ for HCF-e is 0.0026 eC Å$^{-1}$, which is nearly twice that of SeMoS-e material at same bending angle 11.52°. This enhancement is similar in case of monolayer HCF[33] and monolayer SeMoS[45]. For the selected unitcell in HCF-e in Figure 3d and SeMoS-e in Figure 3a at nearly same location and same bending angles, the percentage of change in bond length (bond angle) is 0.16 (2.35) and 0.45 (0.97), respectively. The higher changes in atomic configuration lead to higher local electric fields and high dipole moments. The change in bond angle is about 1.4 times higher than that observed in monolayer HCF, which strongly support the enhanced flexoelectric coefficient through the bonding or valence electron interactions. Other stacking orders a to d also show similar enhancement with allowed changes due to the different stacking order.

The non-Janus FBNF interestingly showing high polarization values. In this case, the hydrogen atom in HBNF changed to fluorine atom. Because of the boron–fluorine bond, the dipole moment strengthens and reversed the direction toward fluorine atom. Figure 3e shows the unidirectional $p_z$ for all bins in stacking order a and e. The fluorine atom collects more charge from the boron atom compared to the collection by the nitrogen atom. The total dipole moment in the lower layer central cell for this bond is 0.44 eCÅ, and for other bonds is −0.18 eCÅ, which makes a non-zero contribution to the total polarization $P_z$. Figure 4a,b shows the linear variation for $P_z$ with strain gradient for FBNF-a and FBNF-e. The noted coefficient $μ_{zyzy}$ is similar to the HCF-e. The electron interaction strengths may differ in stacking orders b to d for FBNF causing a rise in $μ_{zyzy}$ compared to HCF-b to HCF-d (see Figure 4c). For the non-Janus HBNN, the $μ_{zyzy}$ is in the order of bilayer Janus TMDs. In
HBNH, the hydrogen–boron bond gains more dipole moment than the rest of the bonds, similar to the FBNF case. However, due to low electron negativity character for hydrogen, the induced dipole moment is weaker than boron–fluorine bond. As a result, $\mu_{xy}$ is reduced. An identical variation between stacking orders is observed for both FBNF and HBNH.

### 3. Conclusion

In summary, we propose an innovative and accurate computational approach on the basis of DFT and MLIPs with incorporated long-range vdW and electrostatic interactions to explore the flexoelectric response of 2D vdW bilayers. The short-range interactions are accurately defined using the MLIPs trained over computationally inexpensive DFT datasets and long-range interaction parameters are calibrated from DFT simulations. The in-plane and out-of-plane tensile loading conducted using the most popular platform for the standard MD simulations, enabled us to test the accuracy of the devised modeling strategy. The predicted mechanical properties by the proposed computationally efficient method show an excellent level of agreement with those by the plane-wave DFT. After ensuring the outstanding accuracy of our developed technique, we further explored the effects of stacking orders (a–e) in bilayer TMDs, diamanes, and BNdimanes. Our predicted piezoelectric coefficients for TMDs agree with the earlier experimental and full-DFT reports. The natural bending process is then simulated via the edge compression, from which, curvature, strain gradient, stress, and polarization are quantified. Bilayer TMDs are found to be about four times softer than diamanes. The effective piezoelectric coefficient predicted in this work matches excellently with earlier experiments. TMDs bending flexoelectric coefficient is predicted to be 2–7 times higher than that of the monolayer counterpart, depending on the stacking orders. Bilayer Janus diamane (HCF) outperforms TMDs, with the enhancement of 20 times. The non-Janus diamanes (FCF and HCH) cancel the change in polarization due to bending and the flexoelectric effect is diminished. The Janus bilayer HBNF system also cancels the polarization due to the dipole moment reversal across the boron–hydrogen and nitrogen–fluorine bonds. The non-Janus FBNF diamane performs similarly to HCF, whereas HBNH yields a low enhancement due to the lower electron interactions. The presented methodology of deriving potential parameters is easily extendable to handle grain boundaries, Moiré patterns, lattice-mismatched bilayer supercells, and to perform complex deformation conditions. Obtained flexoelectricity results can enhance the design of next-generation energy harvesters. The proposed MLIP-based methodology is moreover believed to offer a robust tool to simulate the complex flexoelectric response of novel vdW heterostructures and can be also extended for other cutting-edge applications.

### 4. Computational Methods

In DFT calculations, periodic boundary conditions were applied along all three Cartesian directions. The unitcell size in bilayer normal direction ($z$) was set to 50 Å, to avoid interactions with periodic replicas. Such unitcells undergo geometric optimization using conjugate-gradient method with a convergence criterion of $0.001 \text{ eV \ Å}^{-1}$ for Hellmann–Feynman forces with $15 \times 15 \times 1$ k-point mesh. The generalized gradient approximation and Perdew–Burke–Ernzerhof functional were considered with an energy cutoff of 500 eV for the plane waves and a convergence criterion of $10^{-5}$ eV for the electronic self-consistent-loop. The AIMD runs performed under Nosé thermostat with time step 1 fs using $2 \times 2 \times 1$ k-point grid.

We perform both in-plane and out-of-plane stretching deformations to study the mechanical and piezoelectric properties of bilayer atomic system size $8 \times 8 \text{ nm}^2$. Such simulations serve as a benchmark tests for the derived inter-atomic potential parameters. For in-plane stretching deformation, we supply same displacement of 0.0005 Å to the atoms in right edge and −0.0005 Å to the atoms in left edge. In out-of-plane stretching case, supplied similar displacement to the top and bottom most atom sets. The stretching simulations performed until the strain reaches to 0.01.

To achieve the bending deformation, we consider compressing the left and right edge atoms. Figure 1d shows the schematic setup for compression. Here, we supply displacement higher than the stretching case (0.1 Å). This helps to increase the atomic forces and lead to bending deformation. In tensile and compressive conditions of loading, we supply the displacement only for ten time steps and relax for 10 000 time steps. Out-of-these relaxation steps, the data is collected for every 1000 steps. Before applying the loading conditions, atomic systems are equilibrated using Nosé–Hoover thermostat under velocity Verlet integration scheme with time increment of 0.5 fs. During the relaxation, the edge atoms are held fixed. We use Large-scale Atomic/Molecular Massively Parallel Simulator (LAMMPS)\(^{[56]}\) to perform the MD simulations. Vienna Ab-initio Simulation Package (VASP)\(^{[57-59]}\) employed in creating training sets and estimating the inter-layer energy.

### Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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### Conflict of Interest

The authors declare no conflict of interest.

### Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.