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Introduction

Over the period 12-23 October 2009, the program "Recent advances in integrable systems of hydrodynamic type" will be organized by us and will take place at the *Erwin Schrödinger International Institute for Mathematical Physics* (Vienna, Austria).

Water waves lie at the forefront of modern applied mathematics and theoretical physics. The study of water wave phenomena has been a rich source of deep mathematical theories for over 200 years and leads to a variety of issues, involving several domains of mathematics: partial differential equations, functional analysis, harmonic analysis, dynamical systems, geometry, topology [2]. On the other hand, the theory of completely integrable Hamiltonian systems is one of the most fascinating areas of pure and applied mathematics, bridging a large number of traditional disciplines of mathematics and physics [3]. In particular, it was very successful in providing insight into various aspects of water waves [18]. The program explored recent research advances in integrable systems of hydrodynamic type. Using simplifying assumptions[1], it is possible to derive from the governing equations for water waves simpler model equations of various degrees of sophistication amenable to a more detailed analysis [1]. Conclusions drawn on the basis of these simpler models sometimes reveal hidden phenomena. It is desirable that these models go beyond the linear level of first-order approximations to capture nonlinear effects. At the nonlinear level the mathematical complexity increases and structural properties of the model equation are mostly the key towards an in-depth study. Concentrating on integrable equations[2] among the various models for shallow water waves is motivated by the fact that one expects to be able to develop an inverse scattering/inverse spectral approach and consequently to solve the equations exactly, as initial-value problems, for large classes of initial data. For example, the integrable Korteweg-de Vries (KdV) equation provided the basic understanding of solitons in the context of water waves [18, 22]. Two recently derived nonlinear models for shallow water waves attracted a lot of attention: the Camassa-Holm (CH) equation [6] and the Degasperis-Procesi (DP) equation [17] (see [13, 22] for the derivation of these models as approximations to the governing equations for water waves). Both nonlinear equations share with the classical KdV model the properties that they have a bi-Hamiltonian structure and are formally integrable (that is, they have a Lax pair formulation). However, while all smooth solutions of the KdV equation exist for all times [27], both CH and DP admit global solutions as well as breaking waves: the solution remains bounded, but its slope becomes unbounded in finite time [9, 19, 25]. Another interesting aspect of these recent models is the presence of peaked solitary

[1]Such as small amplitude, long wave (or shallow water, in the sense of a small depth-to-wavelength ratio), and unidirectionality within certain regimes (e.g. for irrotational flows).

[2]Nonlinear partial differential equations with the structure of a completely integrable infinite-dimensional Hamiltonian system.

waves, called peakons [6, 15], as these wave forms replicate a feature characteristic of the traveling wave solutions to the governing equations for water waves of largest possible amplitude (see the discussion in [7, 10, 28]). As early as 1980, the need for shallow water models that exhibit soliton interaction, the existence of peaked waves, and allow for breaking waves was emphasized [29]. In the case of CH an inverse scattering/inverse spectral analysis was developed: its flow is equivalent to the flow of a (mostly) infinite set of parameters moving linearly at constant speed [11, 14]. An inverse scattering approach for the DP equation is not yet available. Related to this issue is the important open problem whether for CH and DP equations any permanent wave splits up into a finite number of solitons/peakons and a dispersive part vanishing asymptotically. Also intruiging is the behaviour of the solution after the occurence of wave breaking [4, 5]. In addition to these issues, the relevance of soliton theory to the modelling of tsunamis is of interest as contrasting viewpoints exist in the recent research literature [16, 26, 24, 8, 12].

The present volume is a theme issue of research papers related to the program. It reflects the various problems that the participants consider important in this context. Although most of the contributors to this volume also participated in the program, the list of authors represented here is broader.

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