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INTRODUCTION TO THE SPECIAL ISSUE ON HYDRODYNAMIC MODEL EQUATIONS

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The increased interest in water wave theory over the last decade has been motivated, arguably, by two themes: first, noticeable progress in the investigation of the governing equations for water waves (well-posedness issues, as well as in-depth qualitative studies of regular wave patterns–see the discussion and the list of references in [1, 17], respectively in [9]), and secondly, by the derivation and study of various model equations that, although simpler, capture with accuracy the prominent features of the governing equations in a certain physical regime. The two themes are intertwined with one another.

This special issue aims to illustrate some important recent developments, offering also a description of the state-of-the-art in the subject. Well-established researchers in the field, as well as promising young researchers, have contributed to the issue. The papers are quite diverse in the subjects that are treated (all but one being research papers), as well as in the underlying analytical methods that are employed in them. The following is a synopsis of these contributions.

The papers by M. Euler & N. Euler and by V. S. Gerdjikov & G. G. Grahovski & R. I. Ivanov examine some equations of hydrodynamical relevance from the point of view of their integrals of motion, respectively their integrability properties. Motivated by recent progress in the understanding of the flow pattern beneath a Stokes wave (cf. [4, 11]), the papers by D. Ionescu-Kruse and by A.-V. Matioc describe the particle paths beneath waves of small amplitude in a flow of constant vorticity over a flat bed, respectively in an irotational deep-water wave. The papers by A. Geyer, by O. G. Mustafa and by R. Stuhlmeier are concerned with some aspects arising in the modelling of tsunamis. G. Tulzer extends the methods developed in [8, 6, 27] to prove the symmetry of small-amplitude steady periodic water waves with monotonic wave profiles for flows with stagnation points–see [13, 32] for the rigorous existence theory for this type of rotational water wave. The paper by V. M. Hur considers the Burgers equation with fractional dispersion as a model of deep-water waves and proves a blow-up result, the gradient blowup in finite time being suggestive of breaking waves (see [5, 7, 33] for a general discussion of the wave breaking phenomenon). T. Lyons also investigates the development of singularities for the dispersionless two-component Burgers equation, which is derived as a model for blood-flow in arteries with elastic walls. The paper by D. Henry & B.-V. Matioc explores an extension of recent regularity results for steady rotational waves in water of constant vorticity [9] to flows with stratification. The paper by A. Constantin describes the dispersion relation for certain water waves in flows with discontinuous vorticity (other known dispersion relations can be found in [5, 10, 12]). J. Escher, B. Kolev & M. Wunsch show that the modified Constantin-Lax-Majda equation [14] modeling vortex and quasi-geostrophic dynamics [28] can be recast as the equation for geodesic flow, and give a geometric approach towards its well-posedness. C.-H. Cho & M. Wunsch prove the existence of global weak solutions for a parameterized model for fluid convection and stretching in one space dimension. In the survey paper by R. S. Johnson the methods of asymptotic expansions, with a small parameter, are briefly outlined and then applied to three examples in the theory of water waves, thus convincingly demonstrating the effectiveness of this approach.

All these papers were first presented in two workshops, organized by us as part of the programme "Nonlinear Water Waves," in May and June 2011 at the "Erwin Schrödinger International Institute for Mathematical Physics" in Vienna, Austria. We hope that this issue will be an important source on recent developments in the field, as we are sure that the interest in the subject is going to grow, and that many exciting developments are still awaiting us.

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