Essays on Asset Pricing Anomalies

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Abstract

Asset pricing anomalies refer to robust empirical patterns in asset prices and returns which are contradictory to theoretical asset pricing models. This compilation thesis comprises four empirical essays which study selected anomalies from a behavioral perspective and cover the following research topics:

The discounts on Closed-End Funds make a strong case for behavioral finance since the stylized fact that two identical cash flows have different prices is hard to reconcile with the law of one price, an important pillar of neoclassical finance. Research in psychology and decision theory suggests that investment decisions of retail investors are subject to a preference for skewness, tantamount to a higher demand for assets with lottery-like payoffs. The first essay introduces the difference between market prices and fundamental values of Closed-End Funds as a novel testing ground to quantify pricing effects of lottery demand. Skewness and economy-wide skewness preferences are a new piece towards solving the Closed-End Fund puzzle and explain variations in discounts above and beyond alternative propositions in the literature.

The positive trade-off between risk and return is a second pillar of neoclassical finance. Risk-averse investors accept higher risk only in expectation of higher returns. Empirically, however, the relationship between average returns and the two most widely adopted risk measures volatility and market beta is negative. The three remaining essays of this dissertation propose unaccounted factors as an explanation for the negative expected returns of risky stocks. Each of the three studies proposes novel approaches to quantify latent factors in the investors factor model without a conjecture about the source of unaccounted risk, thus facilitating an ex ante impartial evaluation of existing theories, e.g. short sale constraints or investor sentiment.

Although each study proposes a different technical identification strategy, thorough discrimination tests highlight that behaviorally motivated propositions attain the highest explanatory power which is not easily shared by alternative theories. Stocks with high idiosyncratic volatility and high beta are exposed to demand of sentimental investors, so-called noise traders. This unexpected buying pressure inflates prices and causes temporary spikes in volatility, followed by negative expected returns in the near future.

The results in this thesis are interesting for future research in empirical

finance and investors alike. Each essay pins down adverse effects due to skewness preferences and noise trader demand in aggregate asset prices and thus helps understanding determinants of asset prices and returns.

Keywords: Closed-End Fund puzzle, investor sentiment, risk-return trade-off

Zusammenfassung

Die empirische Kapitalmarktforschung liefert zahlreiche Widersprüche zu den Vorhersagen theoretischer Kapitalmarktmodelle, besser bekannt als Anomalien. Diese kumulative Dissertation thematisiert ausgewählte Kapitalmarktanomalien aus verhaltensökonomischer Perspektive und untersucht in vier empirischen Beiträgen die folgenden Themenbereiche:

Die Preisabschläge auf börsengehandelte Closed-End Funds gelten als Aushängeschild der Verhaltensökonomik, da zwei Preise für identische Kapitalströme nur schwer mit dem No-Arbitrage Prinzip der neoklassischen Theorie vereinbar sind. Erkenntnisse der Psychologie legen nahe, dass die Anlageentscheidung privater Investoren durch eine Schiefepräferenz geprägt ist und daher mit einer erhöhten Nachfrage nach Wertpapieren mit lotterieähnlichem Auszahlungsprofil einhergeht. Die Abschläge auf den Preis von Closed-End Funds relativ zum Nettoinventarwert liefern eine neue Möglichkeit, die Preiseffekte dieser Nachfrage zu quantifizieren. So zeigt der erste Beitrag dieser Dissertation, dass sowohl Schiefe als auch marktweite Schiefepräferenz einen statistisch und ökonomisch signifikanten Beitrag zur Lösung des Closed-End Fund Puzzles leisten.

Ein weiterer Baustein der neoklassischen Theorie ist der Zielkonflikt zwischen Risiko und Rendite. Aus der Annahme risikoaverser Investoren folgt, dass Investoren nur unter der Erwartung höherer Rendite bereit sind, höhere Risiken einzugehen. Empirisch hingegen hat der Zusammenhang zwischen erwarteten Renditen und den beiden wichtigsten Risikomaßen Volatilität und Marktbeta das entgegengesetzte Vorzeichen. Die drei anschließenden Beiträge zeigen auf, dass die negativen erwarteten Renditen riskanter Aktien auf unberücksichtigte Risikofaktoren zurückzuführen sind. Jede der drei Studien stellt eine neue Methodik vor, um die Sensitivität gegenüber latenten Risikofaktoren ohne die Unterstellung expliziter Faktorzeitreihen zu schätzen. Dies erlaubt es, existierende Erklärungen, beispielsweise Short-Sale Restriktionen oder Investorenstimmung, unvoreingenommen gegeneinander aufzuwiegen.

Trotz jeweils unterschiedlicher Methoden zur Identifikation latenter Faktoren liefern verhaltensökonomische Erklärungsansätze den größten Erklärungsgehalt. So sind Aktien mit hoher idiosynkratischer Volatilität und hohem Beta besonders sensitiv gegenüber der Marktstimmung privater Kleininvestoren, sogenannter Noise Trader. Die Nachfrage von Noise Tradern nach diesen Aktien führt kurzfristig zu Preiserhöhungen,

die mit temporär erhöhter Volatilität und negativen zukünftigen Renditen einhergehen.

Die Inhalte und Ergebnisse der vorgestellten Beiträge sind für Wissenschaftler aus dem Bereich Kapitalmarktforschung und Investoren gleichermaßen interessant. Die vorgeschlagenen Erklärungsansätze leisten einen Beitrag zum besseren Verständnis der Preisbildung auf Aktienmärkten und ermöglichen die Quantifizierung adverser Effekte durch Schiefepräferenz und Noise Trading auf aggregierter Ebene.

Schlagwörter: Closed-End Fund Puzzle, Investorenstimmung, Risiko-Ertrags-Verhältnis

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Chapter 1

Preface

1.1 Motivation

> *In the beginning, there was chaos. Practitioners thought that one only needed to be clever to earn high returns. Then came the CAPM. Every clever strategy to deliver high average returns ended up delivering high market betas as well. Then anomalies erupted, and there was chaos again.* Cochrane (2011)

Empirical patterns in security prices and returns qualify as anomalies if they contradict the predictions of theoretical asset pricing models or a prevalent paradigm, for example neoclassical finance with rational and efficient markets (Lee et al., 1990). "Asset prices should equal discounted expected cash flows" as Cochrane (2011) points out. Very often, however, they don't. From a neoclassical perspective, this equality rests on the two pillars of "beautiful markets" and "beautiful people" (De Bondt et al., 2008). In order to explain anomalies, the literature either studies the predictions of asset pricing models in less than beautiful markets by introducing frictions or relaxes the assumption of the methodologically beautiful homo oeconomicus. This thesis pursues the latter road and is built around two building blocks of behavioral finance, namely investor preferences and investor sentiment (De Bondt et al., 2008).

The analogy between prices and discounted expected cash flows is an ideal vantage point to organize the contributions of this thesis. Taken literally, the equality condition inheres the law of one price: Two assets with identical cash flows must sell for the same price (Ross, 2005). The discounts on Closed-End Funds are undoubtedly the most prominent violations of the law of one price. Chapter 2 exploits the Closed-End Fund Puzzle as a natural experiment to study connections between investor preferences and investor sentiment. Extreme investor sentiment reveals preferences for lottery-like returns measured from S&P 500 index options in a novel non-parametric approach. Periods of extremely high *and* low investor sentiment are tantamount to high aggregate probability weighting and come along with higher average Closed-End Fund discounts. This aggregate skewness preference interacts with the impact of lottery characteristics on fund level discounts. Lottery-like assets trade at higher prices due to skewness preferences, so the market prices of funds with more lottery-like stocks are lower than the sum of their holdings because diversification erodes desirable skewness. Conversely, if the fund itself is perceived as a lottery, fund share prices increase relatively to their net asset values and discounts are lower. The study is the first to relate skewness preferences to market-wide investor sentiment and quantify the impact of gambling demand on violations of the law of one price.

Usually, financial assets are claims on *future* cash flows which imply certain amounts of risk. Determining prices thus requires appropriate risk-adjusted discount rates. In terms of the analogy of Cochrane (2011), riskier assets should trade at lower prices today, equivalent to higher rates of return in the future. This positive risk-return relation is another important pillar of neoclassic finance. Empirically, the two most widely used risk measures volatility and market beta "point rather strongly in the wrong direction" (Baker et al., 2011). The negative relation between risk and returns is better known as the low-risk anomaly and comes in two flavors, depending on the risk measure at hand. In both cases, supposedly risky stocks earn lower average returns and exhibit negative alphas in a variety of factor models, including the CAPM.

However, as stressed out by Cochrane (2011): There is no alpha, only beta we do and beta we do not understand. The three remaining studies of this thesis embrace this premise and transform the negative alphas of high-risk stocks into betas. This transformation relies on the theoretical framework of MacKinlay (1995) and MacKinlay and Pastor (2000) who introduce the optimal orthogonal portfolio. Chapters 3, 4 and 5 each propose novel strategies to identify omitted factors represented by the optimal orthogonal portfolio. Thus, we can evaluate the role of potentially latent factors in the context of the low-risk anomaly without conjectures about explicit proxies for those factors. From an economic point of view, this technical advantage comes at the cost of an ambiguity with respect to the economic forces behind the low-risk effect. To this end, we perform thorough discriminating tests in Chapters 3 and 5 and use the optimal orthogonal portfolio as a tool to discriminate between existing theoretical explanations for the anomaly. Despite different technical approaches to the identification of latent factors in the CAPM and the Fama and French (1993) three factor model, the studies in Chapters 3 and 5 arrive at the conclusion that the low-risk anomaly is attributable to the demand of sentimental traders for high-volatility and high-beta stocks. Sentiment investors or noise traders bid up prices of risky stocks which goes along with temporary spikes in volatility and lower future returns. Investor sentiment outperforms alternative explanations based on market imperfections, most importantly leverage constraints.

Bringing order to the chaos is the current challenge in asset pricing. Naturally, this discourse is inherently chaotic, either until a new paradigm becomes accepted or until existing challenges are rationalized in the prevalent framework. Although this thesis will certainly not put an end to a 40 year-long debate, we introduce novel tools to distinguish between existing theories. For example, the non-parametric aggregate probability weighting measure in Chapter 2 might finally separate variations in risk aversion from investor sentiment, an important issue raised by Cochrane (2011) as well. The composite factor in Chapter 5 is systematic and behavioral at the same time, thus challenging old habits in the field, for example the seemingly clear distinction between characteristics and covariances to separate rational from behavioral theories (Kozak et al., 2018). In the end, the debate between rational and irrational explanations is more "philosophical than economic", as Tetlock (2007) points out. So all it takes is a new beauty standard.

1.2 Outline

Each of the chapters in this thesis provides an independent introduction to the respective research question as well as a conclusion. The remainder of this chapter summarizes the contribution of each paper:

Chapter 2: Lottery characteristics, time-varying skewness preference, and the Closed-End Fund Puzzle (joint work with Maik Dierkes) The Closed-End Fund Puzzle, i.e. the wedge between the fund's market prices and net asset values, is a long-standing challenge to the law of one price. The stock market effectively puts two prices on the same future cash flows, but typical market frictions such as agency costs and taxes neither account for the magnitude, nor the time dynamics of the discounts (Malkiel, 1977; Lee et al., 1991).

The study starts from the premise that retail investors prefer stocks with lottery-like payoffs. Recent literature vividly proves that this lottery demand leads to price patterns which differ from neoclassical predictions (e.g. Barberis and Huang, 2008; Boyer et al., 2010; Bali et al., 2011). Lottery-like assets trade at higher prices, while future expected returns are lower. We propose Closed-End Fund discounts as a novel measure to quantify violations of the law of one price due to lottery demand.

The study shows that the differential between lottery characteristics of the fund's share return and the assets in the fund's portfolio explains Closed-End Fund discounts. The more attractive gamble – either the fund share or the fund's constituent assets – ceteris paribus trades at a higher price. Since diversification erodes desirable skewness, average

Closed-End Funds trade at a discount. This result holds for a variety of well-accepted lottery characteristics in the literature, for example the lottery index of Kumar et al. (2016) or the five highest daily returns in the previous month proposed by Bali et al. (2017).

Furthermore, we introduce two measures for economy-wide skewness preferences. The first measure is based on S&P 500 index options, the second measure is derived from extreme investor sentiment. We relate both measures to aggregate discounts and times of high skewness preferences are tantamount to higher aggregate discounts. Interaction effects between skewness preferences and lottery characteristics indicate a particularly strong impact of lottery characteristics on fund-level discounts when skewness preferences are high. Skewness and skewness preferences explain Closed-End Fund discounts above and beyond alternative propositions, for example liquidity or manager ability as proposed by Cherkes et al. (2009) and Berk (2005).

Chapter 3: What is the latent factor behind the idiosyncratic volatility puzzle? (joint work with Arndt Claußen and Maik Dierkes) The underperformance of high-volatility stocks is another challenge to neoclassical theory. Theoretically, idiosyncratic risk either carries no risk premium at all in standard asset pricing theory, or a positive risk premium if investors are unable to diversify properly (Merton, 1987). Ang et al. (2006), however, document a negative empirical risk premium which coins the idiosyncratic volatility puzzle.

Our study shows that the idiosyncratic volatility puzzle originates from common risk in Fama and French (1993) three factor model residuals. We adapt the theoretical framework of MacKinlay (1995) and MacKinlay and Pastor (2000) to the nonlinear relationship between returns and idiosyncractic volatility to form an active portfolio which tracks this specific source of risk. Including this portfolio to the initial model explains the comovement of high idiosyncratic volatility portfolios and alleviates their negative average returns and factor model alphas. The proposed factor satisfies all criteria of a genuine risk factor according to the risk factor protocol of Pukthuanthong et al. (2019). Discriminating tests between competing theoretical explanations for the puzzle highlight noise trader risk as an economic mechanism behind the puzzle.

Our findings suggest that high-volatility stocks are attractive to sentimental investors who bid up prices and cause temporary volatility spikes, followed by negative subsequent returns. This is in line with many well-known characteristics of high-volatility stocks, for example a higher retail ownership (Brandt et al., 2010) or smaller size (Bali and Cakici, 2008). To the best of our knowledge, we are the first to relate the idiosyncratic volatility puzzle to systematic noise trader risk.

Chapter 4: Dissecting idiosyncratic volatility in the cross section of stock returns (joint work with Arndt Claußen and Maik Dierkes) This study generalizes the findings in the previous Chapter to the cross section of stock returns. The analysis in Chapter 3 relies on sorted portfolios as base assets which improves the parameter estimation in the Fama and French (1993) three factor model. However, Ang et al. (2018) point out that the construction of portfolios destroys valuable information and individual stocks are a better choice to evaluate asset pricing models.

We address this concern in Chapter 4 and derive an economically motivated regression-based procedure which facilitates the estimation of sensitivities to latent factors as long as the number of assets is considerably large. We decompose the full cross section of idiosyncratic volatility into two components: First, a stock's exposure to latent risk and second, *purely* idiosyncratic variation. Again, the optimal orthogonal portfolio of MacKinlay (1995) plays an important role in the derivation of the volatility decomposition.

Indeed, we find that the quantitatively small systematic component in supposedly idiosyncratic volatility explains a large fraction of the negative Fama and MacBeth (1973) risk premium on idiosyncratic volatility in the cross section of stock returns. At least 35% of this risk premium is attributable to the systematic component which is higher than any alternative explanation under consideration. The risk premium on the systematic component is historically stable and increasingly important, while the risk premium estimate on the purely asset-specific component is driven by a single peak in the early 1980s. The evidence in this study is consistent with the previous paper and supports a risk-based explanation for the idiosyncratic volatility puzzle. Existing explanations, for example aggregate variance or aggregate correlation as proposed by Chen and Petkova (2012), however, are unlikely to account for our findings.

Chapter 5: Betting against sentiment: Seemingly unrelated anomalies and the low-risk effect (joint work with Maik Dierkes) Idiosyncratic volatility is not the only risk measure which commands a negative risk premium in empirical studies. More broadly, the low-risk effect summarizes the historically negative alphas of high-beta and high-volatility stocks (Baker et al., 2011). Both findings contradict the predictions of the CAPM which implies a positive trade-off between returns and beta, while diversifiable risk such as volatility should yield no significant risk premium at all.

While the literature usually studies both phenomena separately (see e.g. Asness et al., 2019), we break with this habit and trace back the low-risk effect to unaccounted factors in the CAPM. Once more, the identification of unaccounted factors in the investor's factor model relies on the optimal orthogonal portfolio of MacKinlay (1995) and MacKinlay and Pastor (2000). We use decile portfolios of nine seemingly unrelated anomalies to construct a composite factor which identifies the optimal orthogonal portfolio empirically. While this difference to the two previous studies seems technical, the composite factor in this study does not only track the risk behind idiosyncratic volatility, but is expected to embody all relevant asset pricing information for the test assets under consideration. Since the composite risk factor is orthogonal to the market portfolio, we can extend the CAPM without affecting market beta estimates.

The extended CAPM explains not only the nine constituent anomalies, but also alleviates the low-risk effect. The negative alphas of high-beta and high-volatility decile portfolios become insignificant and returns increase in market beta as soon as we control for the unaccounted factors in the CAPM. This result extends to double sorted portfolios and cross-sectional Fama and MacBeth (1973) regressions.

Turning to economic explanations again highlights behavioral mechanisms as a likely source of the low-risk effect. High-risk stocks earn low returns because “betting against sentimental investors is costly and risky” (Baker and Wurgler, 2007). Conversely, we find little evidence for leverage constraints or disagreement, the propositions of Asness et al. (2019) and Hong and Sraer (2016).

Chapter 2

Lottery characteristics, time-varying skewness preference, and the Closed-End Fund puzzle

This Chapter refers to the working paper:

Dierkes, Maik and Sebastian Schroen (2019): 'Lottery Characteristics, Time-Varying Skewness Preference, and the Closed-End Fund Puzzle', Working Paper, Leibniz Universität Hannover.

Abstract

We test the impact of lottery features of Closed-End Fund shares and their equity portfolio holdings on Closed-End Fund premia. Proxies for lottery-characteristics include the previous month's maximum return, a lottery index based on idiosyncratic volatility, idiosyncratic skewness, and prices as well as a quantile-based skewness proxy. A one standard deviation increase of fund (asset) lottery features increases (decreases) monthly premia ceteri paribus up to 133 (151) basis points on average. The economic impact interacts with time-varying skewness preference which is related to extreme sentiment. Low skewness preference is tantamount to a desire for diversification and predicts Closed-End Fund IPOs.

Keywords: Closed-End Fund puzzle, lottery characteristics, skewness preference

JEL: G10, G12, G32.

2.1 Introduction

Closed-End Mutual Fund shares trade publicly in the stock market and provide a claim on a portfolio of traded securities. Effectively, the stock market puts two price tags on the same claim on future cash flows. At odds with traditional financial theory, especially the law of one price and the no-arbitrage principle, the two prices typically deviate systematically from each other. On average, seasoned Closed-End Funds trade at discounts (or negative premia) relative to the net asset value of their assets (NAV). Typical market frictions, such as taxes or agency costs, have a hard time explaining all characteristics of this price deviation (Malkiel, 1977; Lee et al., 1991). Any possible explanation must have enough differential impact on demand and supply of Closed-End Fund shares and fund holdings to disturb arbitrage forces.

In this paper, we show that (option-implied) skewness preferences and the differential in lottery characteristic of Closed-End Fund shares versus their assets are key determinants of the fund premium. Our results implicitly show that the liquid and competitive S&P 500 index option market and the seemingly remote corner of the Closed-End Fund market are linked by aggregate gambling preferences.

Closed-End Funds are ideal to study lottery demand's pricing impacts, because differences in market valuation and fundamental value, the NAV, are easily measurable. Put differently, our analyses give hints to what extent skewness preference can suppress the elimination of arbitrage opportunities. Our argument is as follows. All else equal, if assets in the fund portfolio provide more attractive gambles, the price for this asset increases because of lottery demand in the stock market (e.g. Kumar, 2009). Thereby, the fund's NAV increases. However, the Closed-End Fund share's return distribution does not necessarily inherit its single assets' lottery characteristic, because the fund's diversification across many stocks destroys the lottery characteristic on the portfolio level (Mitton and Vorkink, 2007). Therefore, the fund's stock price does not increase, at least not as much as the NAV does, and trades at an even lower premium relative to its NAV. Conversely, the premium increases if the fund's stock appears to provide better lottery characteristics, given fixed lottery characteristics of the holdings.

The recent literature vividly proves that skewness preferences lead to price patterns distinct from neoclassical theory's predictions. Theoretical models are based on, for example, Cumulative Prospect Theory's probability weighting (Barberis and Huang, 2008), optimal beliefs (Brunnermeier et al., 2007), or equilibrium underdiversification despite rational preferences for skewness (Mitton and Vorkink, 2007). Empirical support comes from, among others, Green and Hwang (2012) who analyze puzzling returns of IPO stocks (excluding Closed-End Funds), the cross section of stock returns (Boyer et al., 2010; Bali et al., 2011, 2017), or the option market (Boyer and Vorkink, 2014; Blau et al., 2016; Baele et al., 2019). Doran et al. (2012) find that gambling preference at the turn of a year impact option prices and stock returns. Kumar et al. (2011), Kumar et al. (2016), and Han and Kumar (2013) show that gambling preferences in the stock market vary geographically and drive prices in the cross section of returns. The relationship between lottery features and future Open-End Mutual Fund Flows reflects high investor demand in response to lottery-like fund returns in the past (Akbas and Genc, 2018; Goldie et al., 2017). Given that Open-End Mutual Fund shares - and similarly Exchange Traded Fund (ETFs) shares - are in perfectly elastic supply and investment companies issue or redeem shares in order to alleviate differences between share prices and net asset values (Malkiel, 1977), an analysis of Closed-End Funds yields more profound conclusions about the relationship between lottery characteristics and violations of the law of one price. Similar to Closed-End Funds, conglomerates trade at discounts relative to a matched portfolio of single-segments firms (Lang and Stulz, 1994; Berger and Ofek, 1995). Mitton and Vorkink (2010) relate this so-called diversification discount to the more skewed returns of single-segments firms. Schneider and Spalt (2016) explain the diversification discount with the tendency of CEOs to over-invest in highly skewed segments as a result of behavioral biases. Our results are consistent with Mitton and Vorkink (2010) and suggest that Closed-End Fund discounts are in part diversification discounts. Presumably, the measurement of discounts on Closed-End Funds is more precise because matching single-segment firms to conglomerates can introduce some noise (see Graham et al., 2002). Our results suggest that investment professionals, such as fund managers are also subject to the behavioral biases found by Schneider and Spalt (2016) among corporate managers. After all, the Closed-End Fund market is a particularly clean research environment - compared to Open-End Mutual Funds, ETFs, and conglomerates - for analyzing the limits of the law of one price due to skewness preferences.

Using five recently proposed proxies for lottery characteristics, our empirical baseline analyses of 51 US-equity Closed-End Funds for which we collected all holdings from 1997 to 2015 and 955 Closed-End Fund IPOs from 1986 to 2015 in the US are largely consistent with our predictions.

On the fund level, the crudest proxy is portfolio concentration measured as the sum of squared portfolio weights (SSPW, sometimes called Herfindahl index, see Goetzmann and Kumar, 2008). Less concentrated holdings are tantamount to better diversification, and diversification corrodes skewness and ultimately demand for the fund. We find that less concentration by one standard deviation leads to lower premia for Closed-End Funds by 115 basis points (bps) after controlling for other factors, including liquidity (Cherkes et al., 2009), managerial ability (Berk and Stanton, 2007), fund size and age, costs of arbitrage or leverage (Pontiff, 1996; Cherkes et al., 2009).

Bali et al. (2011) propose the maximum daily return of the previous month. This statistic is easily accessible and likely to catch investor attention because of its salience (Bordalo et al., 2013). More importantly, it is a good estimate for a stock's lottery feature in the subsequent month. Bali et al. (2017) use the average of the five maximum returns as a more robust estimate. Consistent with our hypothesis, higher average maximum returns on the fund level increase fund premia and higher average maximum returns of funds' assets decrease fund premia, all else equal. The latter effect of assets' maximum returns is particularly strong and reliable: a one standard deviation increase reduces the premium by 151 bps with absolute t-statistics exceeding four. However, increasing a fund's maximum return increases the premium by 133 bps, all else equal. Again, all reported figures are net of various controls. To compare with recently proposed liquidity-based explanations of Closed-End Fund premia (see Cherkes et al., 2009), increasing the assets' (fund's) illiquidity, measured according to Amihud (2002), by one standard deviation increases (decreases) the fund premium by 157 (182) bps.

Kumar et al. (2016) coin the lottery index (LIDX) measure, specifically designed to capture the lottery characteristic of a stock. It sorts stocks according to idiosyncratic skewness, idiosyncratic volatility, and price. Our adaption of LIDX on a monthly basis has a particularly strong and reliable impact, with absolute t-statistics around five. On the fund (assets) level, a one standard deviation increase changes the fund premium by 92 (-92) bps.

Hinkley (1975) proposes a quantile-based measure for skewness which is robust against outliers. Green and Hwang (2012) apply this measure to explain puzzling returns in IPOs with skewness preferences.[1] It is the normalized sum of the deviations of the 1st and 99th percentile from the median. Increasing this lottery characteristic estimated across the previous three months for funds (assets) changes the fund premium by 22 (-34) bps, all else equal, with both effects being significant at the 5% level.

Beyond our main focus on lottery demand effects, our estimates are in favor of a number of previously proposed explanations for Closed-

[1] Conrad et al. (2013) adapt this measure, compare it with option-implied skewness, and find return patterns consistent with skewness preference.

End Fund premia. Illiquidity of funds versus assets plays a cruicial role (Cherkes et al., 2009). Better manager ability leads to higher Closed-End Fund premia, as proposed by Berk and Stanton (2007) and Chay and Trzcinka (1999). Higher fund dividend yields increase premia (Cherkes et al., 2009; Pontiff, 1996). Still, lottery characteristics on the fund and particularly on the asset level, as measured by various proxies, have significant explanatory power above and beyond existing theories of Closed-End Funds. Robustness checks, including different definitions of lottery characteristics (e.g. estimation period) or weighting schemes of assets' lottery characteristics (value versus equal weighting) do not alter our conclusions.

There is evidence in the literature that skewness preference can vary over time, and very likely does so systematically. This variation in skewness preference leads to further testable predictions that are not shared by alternative interpretations of the Closed-End Fund market. Our skewness preference theory of Closed-End Fund premia predicts that the effects of lottery characteristics are stronger in times of high skewness preference. In addition, Closed-End Fund IPOs should be more likely in times of low skewness preference, when there is higher demand for diversified investments.

We employ two proxies for aggregate skewness preference. The first one is based on the psychologically motivated concept of probability weighting which is the driving pricing factor in lottery stocks in the Barberis and Huang (2008) model with Cumulative Prospect Theory agents. Polkovnichenko and Zhao (2013) estimate option-implied aggregate probability weighting and show that it varies over time. We use a new non-parametric methodology to estimate probability weighting from index options. This implicit link between the Closed-End Fund market and the index option market is new to the literature.

Our second skewness preference proxy is based on extreme sentiment. Specifically, we define a dummy that equals one in times of pessimistic or optimistic economic outlook, whereas it is zero during moderate sentiment measured by the University of Michigan Consumer Sentiment Index.[2] Recent literature lends support to the definition of our second proxy. For example, Kumar (2009) finds stronger gambling demand in the stock market during economic downturns. Green and Hwang (2012) analyze IPO returns in the short and long run and find a stronger impact of expected skewness in times of high investor sentiment, as measured by the Michigan Consumer Sentiment index. In the laboratory, Rottenstreich and Hsee (2001) find that probability weighting, and thus skewness preference, is stronger in affect-rich situations. Presumably, times of economic crises as well as economic exuberance can serve as affect-rich situations in our setting, which would be consistent with the mentioned findings in Kumar (2009) and Green and Hwang (2012). Bordalo et al. (2013) show that Salience Theory's probability distortion can generate variation in risk premia, and thus skewness preference. In a model with rational

[2]Kumar et al. (2016) note that measuring variation in skewness preference is notoriously difficult. They use the Catholic/Protestant ratio as a proxy of skewness preference and rely on retail investors trading local stock (see e.g. Coval and Moskowitz, 1999; Huberman, 2001; Ivkovic and Weisbenner, 2005). For our purposes, we are interested in a monthly measurement with sufficient time variation. Our sentiment-based dummy, therefore, appears as a more natural choice than survey data collected in 1990, 2000 and 2010 from the American Religion Data Archive (ARDA).

investors who like positive skewness, Mitton and Vorkink (2007) predict underdiversification, much in line with the empirical evidence above. Zhang (2014) elaborates on their model and shows that more gambling investors enter the market in economic upturns whereas the few investors remaining in the stock market during economic crises gamble more aggressively. In summary, there is more gambling or skewness preference in times of economic extremes, both in crises and in economic highs. Finally, both our skewness preference proxies are consistent since there is more extreme inverse-S shaped probability weighting implied in S&P500 index options when sentiment is extreme.

Our sentiment-based skewness preference proxy allows for a particularly easy interpretation because the dummy variable distinguishes between high and low skewness preference regimes. Consistent with our predictions, in high skewness preference regimes, a one standard deviation increase in the maximum return of assets leads to a 195 bps drop in the fund premium, exceeding the baseline effect mentioned above by 44 bps. In high skewness preference times, the LIDX impact on premia magnifies and becomes more reliable, too (both t-statistics are greater than five in absolute terms). The asset effect changes to -105 from -92 bps in the baseline scenario. For the Green and Hwang (2012) measure, the differential impact in high skewness preference times on fund premia is significant, but less strong (e.g. -42 instead of -34 bps for assets). Our option-implied skewness preference proxy delivers even more reliable results, thereby corroborating our conjectures. It is noteworthy that our preditions and results about the interaction between lottery characteristics and time-varying skewness preference are not easily shared by other theories about stylized facts in the Closed-End Funds market.

Two papers are close to ours. According to our hypothesis lower valuations of portfolios are driven by skewness preferences and a resulting desire to underdiversify on average. Hwang et al. (2017) construct similar valuation differences with heterogeneity in beliefs about single stocks such that portfolios of those stocks are less preferred on aggregate. For example, an increase of their belief-crossing measure by one standard deviation leads to a 49 basis point drop in Closed-End Fund premia on average. We consider both their and our explanation to be complementary, especially since our skewness preference proxies estimated from either the S&P500 index option market or aggregate sentiment are orthogonal to a belief-based explanation about single stocks. Liu's (2017) analysis of Closed-End Funds can be regarded as a special case of our analyses in the sense that he explains fund premia with disparity in maximum returns of a fund's stock and a fund's top 10 holdings in its portfolio. In contrast, our novel analyses reveal consistent results for up to four lottery characteristics based on overall holdings data and further find predictions about the interaction with time-varying skewness preference confirmed.

In addition, we look at Closed-End Fund IPOs to provide a broader perspective on the Closed-End Fund Puzzle. Recall that Closed-End Funds are better diversified investments and less-skewed returns than single stocks. Indeed, the average Hinkley (1975) measure is negative for funds and positive for funds' holdings. If the demand for Closed-End Funds is driven by time-varying skewness preference, then we should see more IPOs of Closed-End Funds in times of *low* skewness preference. Consistent with this hypothesis, Green and Hwang's (2012) conjecture that skewness preference is a key driver of IPO underpricing predicts no underpricing in the Closed-End Fund IPO market. Hanley et al. (1996) support this conjecture.

Given that IPOs need some organizational preparation (e.g. filings with the SEC) and can be called off, we propose that lagged skewness preference is the appropriate independent variable. Count regressions across Closed-End Fund categories, explaining the number of IPOs, are largely consistent with this prediction. The total number of IPOs, especially IPOs of US-Equity-focused funds, are significantly higher in times of low skewness preference as measured by both our proxies ($p < 0.05$). Taxable Bond Close-End Funds apparently address an investor clientele without much lottery demand. None of our skewness preference proxies is significant at conventional levels when analyzing Taxable Bond Closed-End Funds.

Lee et al. (1991) propose a sentiment story of Closed-End Funds. However, in linear regressions, Qiu and Welch (2004) find no significant relationship between the time series of discounts and survey-based sentiment indexes. We suggest a mechanism that can be consistent with both papers. Discounts depend on skewness preference which is higher during times of *extreme* sentiment. In other words, Closed-End Fund discounts do depend on sentiment, but in a u-shaped, i.e. non-linear way.

2.2 Data, variable definitions and descriptive statistics

2.2.1 Data

We use monthly Closed-End Fund prices and NAV to calculate the Closed-End Fund premium $Prem_{i,t}$ on a monthly basis as

$$Prem_{i,t} = \frac{P_{i,t} - NAV_{i,t}}{NAV_{i,t}}, \tag{2.1}$$

where $P_{i,t}$ is the price per share of Closed-End Fund i in month t and $NAV_{i,t}$ is the corresponding NAV. The core of the Closed-End Fund puzzle refers to the stylized fact that fund shares typically trade at a discount, i.e. $Prem_{i,t} < 0$. We obtain Closed-End Fund prices and NAV from several sources and perform cross validations. In general, we follow Cherkes et al. (2009) and use monthly fund prices from the Center for Research in Security Prices (CRSP).[3] For monthly NAV time series we cross-validate Datastream and Morningstar data and find an average difference of less than 0.1% with a standard deviation of 3.26%. Due to a different coverage of both sources, we maximize the number of observations by taking averages of both NAV sources. We find consistent results when using either of the individual sources.

[3]The Gabelli Global Deal Fund (GDL Fund) with ISIN US3615701048 is the only exception to this. Here, the CRSP adjusted price in the first 23 trading months of the fund is far off the prices from other sources, including the fund prospectus. For example, the first month-end adjusted price is $14.62, compared with an unadjusted price of $20.00 that equals the value in the shareholder report of March 2007. We use the unadjusted price on CRSP in this particular case, which is identical to the prices reported on Datastream and Morningstar.

We obtain quarterly holdings statements of in total 66 Closed-End Funds in the Morningstar category US Equity from Thomson One and Morningstar during the time from March 1997 to March 2015. Sample selection and sample period are restricted by the availability of holdings data. Detailed holdings are available from the first quarter in 1997 on Thomson One and early 2000 on Morningstar, independently of the fund's inception date. Thus, fund inception and initial filings can deviate significantly. The filings contain names and identification codes (Reuters Identification Code or ISIN) of the fund's holdings as well as corresponding portfolio weights in percent. In some cases, identification codes are missing and we search security names manually, such that only few holdings have to be excluded due to missing stock identification. We exclude funds with a filing history of less than 12 months or major breaks in the filing history, individual cases with inconsistent asset allocations, as well as funds of funds.[4] The final sample dataset comprises 51 US Equity Closed-End Funds due to the aforementioned restrictions in the data.[5] Portfolio weights are assumed to remain constant between filing dates. Although this might raise the concern of window dressing issues, it conveys the information set for Closed-End Fund investors. Unlike mutual fund managers, Closed-End Fund managers have no incentive to attract fund flows. Furthermore, we normalize portfolio weights to ensure

[4]We interpolate one-quarter gaps in the filing history, but exclude observations with longer gaps.

[5]A comparison with the Closed-End Fund literature puts this sample size into perspective. For example, Wu et al. (2016) analyze 83 US Equity funds during 1985 to 2010. The sample of Cherkes et al. (2009) comprises 65 funds in the category US Equity in the time from 1985 to 2004.

that the mimicking portfolios are as close as possible to the available information.

Fund characteristics are from Datastream and Morningstar. We obtain daily and monthly total return indices as well as market capitalizations and trading volumes from Datastream, and perform data screenings recommended by Ince and Porter (2006) in order to compute returns. Monthly fund dividend yields are from Datastream. Annual fund expense ratios and turnover ratios are from Morningstar. Furthermore, we use fund inception dates from Morningstar to analyze fund IPOs. The larger sample of Closed-End Fund IPOs is based on all funds with US domicile and legal structure Closed-End Fund Investment Company on Morningstar with non-missing Inception Dates, ISIN and Category. We restrict the analysis to the period from June 1986 to December 2015 due to availability of control variables (see below). The final sample comprises 955 Closed-End Fund IPOs.

Individual stock data are from Datastream. The Closed-End Fund holdings comprise 6214 individual stocks. Daily and monthly total returns, prices, market capitalizations and trading volumes are available for 6182 stocks. Daily and monthly stock returns are also screened according to Ince and Porter (2006).

Economic data are from common sources and are selected based on the literature. Constant maturity Treasury Rates with 20 years, 10 years, 1 year and 3 month times to maturity are from the Federal Reserve Bank of St. Louis database (FRED). The one-month Treasury Rate as well as daily and monthly Fama and French (1993) factors are from Kenneth

French's website. The monthly Pastor and Stambaugh (2003) liquidity factor return is from Lubos Pastor's website. The Chicago Board Options Exchange (CBOE) Volatility Index VIX and the CBOE S&P 100 Volatility Index VXO are from CBOE's website. We collect S&P 500 index option data from Optionmetrics and www.historicaloptiondata.com. The University of Michigan Index of Consumer Sentiment is from the University of Michigan website and Baker and Wurgler (2006) sentiment data are from Jeffrey Wurgler's website. Their sentiment data include the average Closed-End Fund discounts of equity funds from 1978 to 2015.

2.2.2 Variable definitions

We employ a variety of measures for expected skewness or lottery-like payoffs in Closed-End Fund returns.

Fund Diversification

Fund concentration is a simple measure to analyze the effect of diversification on Closed-End Fund discounts. Ex ante, this effect is rather ambiguous. Lee and Hong (2002) find diversification benefits from investing in Closed-End Country Funds, i.e. funds that invest in foreign markets. Although this argument is less applicable to US equity funds, the positive relationship between diversification and subsequent performance documented by Pollet and Wilson (2008) for US mutual funds possibly relates to Closed-End Funds.[6] Investors might perceive diversification as a sign of manager skill, and reward it with higher demand and thus a

[6] In contrast, Kacperczyk et al. (2005) find that a higher industry concentration of mutual funds is an indicator of higher performance and manager skill.

higher price. Conversely, the diversification discount on conglomerates supports an opposite effect of diversification. If investors prefer positively skewed returns, more diversified funds could trade at a discount in relation to the comparable portfolio of single assets, because diversification erodes desirable skewness (Mitton and Vorkink, 2010). To test the latter conjecture, we follow Blume and Friend (1975) and Goetzmann and Kumar (2008) and measure portfolio concentration of fund i in month t as the sum of squared portfolio weights (SSPW)

$$SSPW_{i,t} = \sum_{j=1}^{N} \left(w_{j,i} - w_{j,M}\right)^2 \approx \sum_{i=1}^{N} w_{j,i}^2, \tag{2.2}$$

where $w_{j,i}$ is the weight of stock j in fund i and $w_{j,M}$ is the weight of stock i in the market portfolio, which is assumed to be infinitely small. $SSPW_{i,t}$ is simply the sum of squared weights over all assets N in fund i. A higher level of $SSPW_{i,t}$ is associated with a lower diversification in the fund.[7] We expect that more diversified funds trade at higher discounts. By construction of the fund holdings, $SSPW$ fluctuates in a quarterly frequency. In unreported robustnes checks, we employ the natural logarithm of the number of stocks in the fund, motivated by Pollet and Wilson (2008) and Goetzmann and Kumar (2008), as an even simpler measure of diversification and find qualitatively identical results.

Lottery Demand: Max

[7] At first glance, the diversification measure $SSPW_{i,t}$ appears to be similar to the active share measure of Cremers and Petajisto (2009) who use absolute deviations of a stock's weight in the fund portfolio from the benchmark portfolio. However, Cremers and Petajisto (2009) use the current weight of the stock in the benchmark index which implies an active deviation from the benchmark rather than portfolio concentration.

Bali et al. (2011) present the highest daily return of asset i in month t as a predictor of lottery-like payoffs in the subsequent month. Although the authors technically separate lottery-like payoffs from skewness in statistical terms, Jacobs et al. (2016) show a mathematical connection between extremely high returns and the third moment of the return distribution as well as a strong predictive power for future skewness. Furthermore, this measure is highly intuitive and easily observable by investors. We follow Bali et al. (2017) and calculate Max as the average of the five highest daily returns of asset i in month $t-1$ in order to define a proxy for lottery qualities in month t. In unreported robustness checks, we also employ the originally proposed measure of Bali et al. (2011) in a strict sense as the (single) maximum daily return and draw similiar conclusions. The calculation of Max requires at least 15 daily returns.

LIDX

Most recently, Kumar et al. (2016) present a measure for the attractiveness of a stock as a gambling object, the lottery index $LIDX$. We compute a monthly adaption of the annual $LIDX$ measure. To construct $LIDX$, Kumar et al. (2016) sort stocks into vigintiles (20 bins) each year by price, idiosyncratic skewness and idiosyncratic volatility. Instead of annual sorts into vigintiles, we compute monthly rolling window estimates of idiosyncratic skewness and volatility over the preceding twelve months. The highest vigintile is assigned to stocks with the lowest price and the highest idiosyncratic volatility and skewness over the past twelve months. Then, the vigintile bins are added up to a score between 3 and 60. $LIDX$

of asset i in month t is calculated as

$$LIDX_{i,t} = \frac{Score - 3}{60 - 3}, \tag{2.3}$$

where the denominator ensures a lottery index value between zero and one. A higher value of $LIDX$ indicates a higher attractiveness for investors with a preference for lottery stocks. We refer to the monthly adaption of the measure as $LIDX_{i,t}$. We also employ the strict, i.e. annual, estimation as proposed by Kumar et al. (2016) in unreported robustness checks. Conclusions remain unchanged. We require at least 200 daily observations when estimating idiosyncratic skewness and volatility in the calculation of $LIDX$.

Skew

We furthermore adapt the Green and Hwang (2012) measure for expected skewness, which is defined as

$$Skew_{i,t}^{kM} = \frac{P_{99} + P_1 - 2 \cdot P_{50}}{P_{99} - P_1}, \tag{2.4}$$

where P_j is the jth percentile of the daily return distribution of asset i over the preceding k months. Our baseline regressions use $k = 3$ months, and unreported robustness checks confirm results for $k = 1$ or $k = 6$. A positive value of $Skew$ indicates more realizations to the right of the median, i.e. right skewness. The normalization in the denominator guarantees values in the interval of minus one and plus one and controls for the dispersion of the distribution. Green and Hwang (2012) argue that this measure captures lottery-like payoffs better than the third moment of the return distribution. Schneider and Spalt (2016) use it to measure skewness in segments of conglomerates. We adopt this quantile-based measure in the spirit of Hinkley (1975) in a time series context (see, e.g. Ghysels et al., 2011; White et al., 2008) rather than cross-sectionally (see, e.g. Green and Hwang, 2012; Conrad et al., 2013). We require at least 50 valid daily returns for $Skew_{i,t}^{3M}$.[8].

Control Variables

The choice of control variables is motivated by Cherkes et al. (2009), Chan et al. (2008) and Berk and Stanton (2007) as well as by the empirical work cited therein.

Fund Illiquidity ($FundIlliq$): Closed-End Funds are relatively liquid investment vehicles that grant investors access to illiquid underlying assets (Cherkes et al., 2009). Although this advantage is less applicable to US equity funds, illiquid funds are expected to trade at higher discounts. Similar to Chan et al. (2008), we measure fund illiquidity on a monthly basis with the adapted Amihud (2002) measure

$$FundIlliq_{i,t} = log\left(\frac{1}{D_t}\sum_{d=1}^{D_t}\frac{|r_{i,d}|}{Vol_{i,d}}\right). \tag{2.5}$$

$FundIlliq_{i,t}$ reflects the average ratio of absolute daily fund share returns $r_{i,d}$ over dollar trading volume $Vol_{i,d}$ on all days D_t during month t. We follow Amihud (2002) and employ the logarithmic transformation of the measure in order to reduce the skewness of the variable. We require 15 valid daily observations to calculate monthly fund illiquidity.

Asset Illiquidity ($AssetIlliq$): Since liquidity benefits strongly depend on the relation between liquidity of fund shares and their underlying

[8] In robustness checks, we require 15 valid returns for $Skew_{i,t}^{1M}$ and 100 valid returns for $Skew_{i,t}^{6M}$

assets, we further include the value-weighted average of stock-specific illiquidity measures. We calculate logarithmic stock illiquidity according to Equation (2.5) for all holdings of the fund and compute the value-weighted average for each fund i in month t. Similar to the fund illiquidity measure, taking natural logarithms reduces skewness. We expect higher fund premia when (value-weighted) average asset illiquidity is high.

Fund Size ($Size$): Lottery characteristics are most strongly pronounced among stocks with low market capitalizations (see, e.g. Bali et al., 2011). In order to separate the skewness effect from mere size effects, we include the market capitalization of the fund in million USD. Taking natural logarithms reduces the skewness of the variable.

Fund Age ($LogAge$): Seasoned funds trade at higher discounts (c.p) than newly issued funds. As common in the mutual fund literature, we include the natural logarithm of the fund age in years (see, e.g. Pollet and Wilson, 2008).

Fund Dividend Yield ($DivYld$): A higher payout to investors is expected to positively affect the fund premium, as shown by Cherkes et al. (2009) and Pontiff (1996). We include the fund dividend yield to control for this effect.

Fund Expense Ratio ($ExpRatio$): The effect of fund expenses is the exact opposite of dividends, i.e. payouts to investors are reduced. Thus, the fund expense ratio is expected to affect fund premia negatively (Cherkes et al., 2009) unless it signals superior fund manager ability (Berk and Stanton, 2007) in the recent compensation period.

Fund Turnover ($Turnover$): The role of fund turnover is ambiguous.

On the one hand, portfolio turnover increases transaction costs, thus lowering payouts to investors (Cherkes et al., 2009). Moreover, earlier research in the mutual fund literature finds a negative relationship between turnover and fund manager skill (Chevalier and Ellison, 1999). Both effects suggest a negative coefficient on turnover in the cross-section of fund premia. On the other hand, investors with a preference for lottery stocks are more likely to invest in funds with high turnover ratios (see Bailey et al. (2011) for evidence from mutual funds).

Managerial Ability ($T(\alpha)$): Chay and Trzcinka (1999) provide empirical evidence that manager ability can drive fund premia. Berk and Stanton (2007) develop a model that explains Closed-End Fund premia with the trade-off between managerial ability and fees. Whenever the value added by the fund manager exceeds the fees charged to investors, funds trade at a premium and vice versa. This effect, however, is non-linear. Investors reward manager ability with higher demand for the fund and thus a higher premium, but anticipate an increase in fund manager compensation in response to unusually good performance which in turn decreases premia. We closely follow Ramadorai (2012) who suggests to measure this relationship with the risk-adjusted performance measure

$$T(\alpha_{i,t}) = \frac{\alpha_{i,t}}{se(\alpha_{i,t})}, \tag{2.6}$$

where $\alpha_{i,t}$ is the constant of a Fama and French (1993) three-factor model estimated over months t to $t-11$. The author argues that normalizing α by its standard error $se(\alpha_{i,t})$ reduces the impact of outliers. Furthermore, the squared estimate $T(\alpha_{i,t})^2$ addresses the non-linearity of the relationship.

In line with Ramadorai (2012), we expect a positive sign on $T(\alpha_{i,t})$ and a negative sign on $T(\alpha_{i,t})^2$.[9] We estimate $T(\alpha_{i,t})$ from daily returns and require at least 200 valid observations.

Market Liquidity (Liq): Under a liquidity-based explanation, high aggregate market liquidity offsets potential liquidity benefits of Closed-End Funds and is thus negatively related to Closed-End Fund premia (Cherkes et al., 2009). As shown by Baker and Stein (2004), high market liquidity also indicates unusually high sentiment. Since we control for potential sentiment effects, we expect a negative sign of aggregate liquidity in the panel regressions. We include the Pastor and Stambaugh (2003) liquidity measure as a common proxy for market liquidity.

Cost of Leverage ($TERM$): Furthermore, Closed-End Funds are expected to provide liquidity benefits by using leverage. Cherkes et al. (2009) argue that this effect increases with the slope of the term-structure. We calculate two measures for the term spread which is defined as the slope of the Treasury yield curve due to limited data availability for certain time series on FRED. The first proxy for the term spread is the difference between the 20-year and the 3-month constant maturity Treasury rate (Cherkes et al., 2009). The second proxy is the difference between the 10-year and the 1-year Treasury rate. The first proxy is employed in the panel data analysis and we use the latter in the IPO regressions.

[9]Berk and van Binsbergen (2015) argue that neither net alpha, nor gross alpha accurately measure fund manager skills. The proposed value-added measure is generally less applicable to Closed-End Funds for which managers have no incentive to attract fund flows. We could not appropriately apply the methodology of Berk and van Binsbergen (2015) due to limited data. Nevertheless, alpha measures the risk-adjusted return earned by the fund which is sufficient for our purpose.

Investor Sentiment ($ConSent$): Lee et al. (1991) find various similarities between Closed-End Fund discounts and investor sentiment and argue that on aggregate, Closed-End Funds are an indicator for sentiment. Although several studies challenge this finding (see, e.g. Chen et al., 1993; Elton et al., 1998; Cherkes et al., 2009), a considerable body of literature presents empirical support (see, e.g. Baker and Wurgler, 2006; Anderson et al., 2013). We include the University of Michigan Consumer Sentiment Index as a direct proxy for the effect of sentiment on Closed-End Fund premia and expect a positive relationship with $Prem$.

VIX (VIX): The CBOE volatility index VIX is expected to be a negative predictor for investor sentiment, as argued by Cherkes et al. (2009) and Anderson et al. (2013). In addition, a low VIX indicates sufficient aggregate liquidity. Finally, the VIX might be related to probability weighting and thereby reflects skewness preference (Baele et al., 2019). Hence, a precise prediction of the impact of VIX on fund premia is unclear.

Skewness Preference ($SkewPref$): Probability weighting is largely considered to be a key driver behind lottery preferences (e.g. Tversky and Kahneman, 1992; Barberis and Huang, 2008; Snowberg and Wolfers, 2010). We use the degree of aggregate inverse-S shaped probability weighting as one skewness preference proxy. Polkovnichenko and Zhao (2013) show that aggregate probability weighting in an economy can be estimated with index options. A detailed description of how we estimate this skewness preference proxy $SkewPref_{option-implied}$ is given in Section 2.3.2.

Furthermore, we use *extreme* sentiment as a proxy for skewness preference. We define a dummy variable $SkewPref_{sentiment}$ equal to zero if the University of Michigan Sentiment Index $ConSent$ is in its middle time series tercile and equals one otherwise. It turns out that this definition is consistent with our skewness preference proxy based on option-implied probability weighting, i.e. aggregate probability weighting is more inverse-S shaped when sentiment is extreme (see below). Further support comes from Kumar (2009) and Green and Hwang (2012) who show that gambling activity is stronger in economic downturns and ups, respectively. Zhang (2014) builds on Mitton and Vorkink's (2007) model with heterogeneous skewness preference. She finds that more gamblers are lured to the equity market in good times while the few traders staying in the market during bad times gamble more aggressively. These papers motivate our tercile dummy definition of $SkewPref_{sentiment}$.

The advantage of the dummy definition is that it is available for a longer time series when analyzing Closed-End Fund IPOs while option-implied estimates start only in 1996. Also, this dummy is easily accessible for future research.

2.2.3 Descriptive statistics

Table 2.1 summarizes descriptive statistics and correlation coefficients of the all fund variables. If applicable, the prefix $Fund$ relates to the respective characteristic measured at the Closed-End Fund share level, whereas the prefix *Asset* depicts the value-weighted average of characteristics over all holdings of the corresponding fund.

Table 2.1: Summary statistics and correlations of lottery characteristics and control variables.

Panel A: Summary statistics

	$Prem$	$SSPW$	$FundMax$	$AssetMax$	$FundSkew^{3M}$	$AssetSkew^{3M}$	$FundLIDX$	$AssetLIDX$	$FundIlliq$	$AssetIlliq$	$Size$	$LogAge$	$DivYld$	$ExpRatio$	$Turnover$	$T(\alpha)$
Mean	-0.077	0.030	0.016	0.024	-0.029	0.043	0.255	0.470	2.793	0.532	5.635	2.147	0.078	0.012	0.664	0.118
Std. Dev.	0.089	0.043	0.013	0.012	0.198	0.073	0.106	0.058	1.650	0.873	1.203	1.111	0.049	0.005	1.641	0.618
Min.	-0.373	0.002	-0.009	0.006	-0.743	-0.225	0.018	0.272	0.255	0.012	1.340	0.000	0.000	0.002	0.000	-2.297
Max.	0.985	0.569	0.186	0.105	0.963	0.561	0.719	0.755	11.720	5.866	8.843	4.454	0.581	0.048	20.300	2.701
N	5,686	5,667	5,686	5,667	5,633	5,667	5,460	5,667	5,667	5,667	5,654	5,686	5,686	5,551	5,551	5,460

Panel B: Correlations

	$Prem$	$SSPW$	$FundMax$	$AssetMax$	$FundSkew^{3M}$	$AssetSkew^{3M}$	$FundLIDX$	$AssetLIDX$	$FundIlliq$	$AssetIlliq$	$Size$	$LogAge$	$DivYld$	$ExpRatio$	$Turnover$	$T(\alpha)$
$Prem$	1.000															
$SSPW$	-0.179	1.000														
$FundMax$	-0.061	0.029	1.000													
$AssetMax$	-0.140	-0.074	0.707	1.000												
$FundSkew^{3M}$	-0.029	0.052	0.240	0.185	1.000											
$AssetSkew^{3M}$	-0.087	0.081	0.163	0.306	0.242	1.000										
$FundLIDX$	-0.048	0.220	0.236	0.012	0.226	-0.017	1.000									
$AssetLIDX$	-0.018	-0.084	-0.106	-0.031	-0.151	0.111	0.114	1.000								
$FundIlliq$	-0.242	0.275	0.449	0.271	0.175	0.102	0.299	-0.165	1.000							
$AssetIlliq$	-0.021	-0.022	0.197	0.335	0.069	0.206	-0.040	-0.029	0.295	1.000						
$Size$	0.173	-0.159	-0.273	-0.098	-0.163	-0.066	-0.245	0.076	-0.738	-0.221	1.000					
$LogAge$	-0.151	0.109	-0.010	0.051	-0.051	-0.040	0.008	0.164	0.215	0.177	0.074	1.000				
$DivYld$	0.216	-0.244	0.177	0.192	0.024	0.049	-0.097	0.043	-0.157	-0.075	0.053	-0.393	1.000			
$ExpRatio$	-0.013	0.340	0.162	0.021	0.055	0.192	0.200	0.162	0.408	0.210	-0.494	-0.018	-0.085	1.000		
$Turnover$	0.126	-0.042	0.058	0.039	-0.002	0.060	0.075	0.086	-0.006	-0.049	-0.084	-0.211	0.179	0.134	1.000	
$T(\alpha)$	0.064	0.160	0.032	-0.083	-0.011	-0.030	0.326	-0.067	0.419	0.170	-0.221	0.319	-0.260	0.109	-0.083	1.000

Table 2.1 presents summary statistics of the sample Closed-End Fund premia *Prem*, our baseline lottery characteristics as well as control variables in the sample period from March 1997 to March 2015. If applicable, we present two distinct measures for each lottery characteristic and control variable. The prefix *Fund* indicates measures that are computed from Closed-End Fund shares and measures with the prefix *Asset* are the value-weighted average over all holdings of the corresponding fund. $SSPW$ is the sum of squared portfolio weights to measure portfolio concentration following Goetzmann and Kumar (2008). Max is the average of the highest five daily returns in month $t-1$ as proposed by Bali et al. (2017). $Skew$ is the adapted Green and Hwang (2012) quantile-based skewness measure, calculated over the preceding three months. $LIDX$ is an adaption of the Kumar et al. (2016) lottery index measure. $Illiq$ is the logarithm of the adapted Amihud (2002) illiquidity measure for the fund and its respective holdings. $Size$ is the natural logarithm of the market capitalization in million USD, $LogAge$ is the natural logarithm of the fund age in years and $DivYld$ is the fund's dividend yield. $ExpRatio$ is the expense ratio and $Turnover$ is the portfolio turnover. $T(\alpha)$ is the t-statistic of the fund alpha over the past twelve months.

Panel A presents descriptive statistics and reveals several interesting facts. First of all, considering the dependent variable, the negative average of *Prem* illustrates that the Closed-End Fund Puzzle, i.e. a negative average premium of seasoned funds, is given for the sample funds in the period from March 1997 to March 2015. The rather anomalous maximum *Prem* of 98.53% corresponds to the Herzfeld Carribean Baisin Fund in December 2006. A cross-validation of this value with the fund company's semi-annual report as of December 2006 confirms this value.[10] With regard to the baseline lottery measures, the average proxies for lottery characteristics are lower for funds relative to the holdings. This reflects the diversification effect of the funds in comparison to individual stocks. Average *FundSkew* is even negative, while average *AssetSkew* is positive. A closer look at the illiquidity measures for fund shares and holdings illustrates that the variables adequately measure the different aspects of liquidity. On average, US equity Closed-End Funds appear to be less liquid than the investment universe, which strongly distinguishes US equity funds from other categories. The illiquidity measure of fund holdings reflects the fund's respective focus. The minimum *AssetIlliq* of 0.0119 belongs to the Dow 30 Enhanced Premium & Income Fund – a fund investing in rather liquid blue chip stocks. Conversely, the maximum of 5.8658 which belongs to the Royce Micro Cap fund is well aligned

[10]The Herzfeld Carribean Baisin Fund with ISIN US42804T1060 trades with the ticker symbol CUBA and is invested in companies that supposedly benefit from a repeal of the Cuban trade embargo. Chairman Thomas J. Herzfeld relates the high premium in December 2006 to a structural improvement of the relationship between the USA and Cuba at the end of 2006: https://docs.wixstatic.com/ugd/7ba5b8_c31dd980e21d4c86b4d7668279c450fb.pdf.

with the fund's investment strategy, i.e. investing in small stocks with a market capitalization up to $1 billion. This observation also holds true for average *FundIlliq* and *AssetIlliq* of the two exemplary funds. For the Dow 30 Enhances Premium & Income Fund, illiquidity of holdings on average equals 0.026, while the fund exhibits an average illiquidity measure of 2.14. We observe the opposite relation for the Royce Micro Cap Fund, where average *AssetIlliq* of 3.87 exceeds average *FundIlliq* of 3.23. Panel B illustrates the correlation structure of the panel dataset. The negative correlation between Closed-End Fund premia *Prem* and the baseline measures for lottery characteristics does not account for the simultaneity of our research hypothesis and is not surprising due to the negative relationship between lottery characteristics and returns. Panel B further alleviates concerns of a high correlation between the skewness measures of funds and their respective holdings. Except for *Max*, the correlation between the respective *Fund* measure and the *Asset* measure is moderate, but always positive. Different measures for lottery characteristics unveil moderately positive correlation coefficients. Obviously, the baseline measures convey similarities, but capture distinct features of the respective return distribution. *FundIlliq* and *AssetIlliq* are only moderately correlated with a correlation coefficient of 0.2980. Similar to common stocks, fund liquidity strongly increases with the size of the fund, given a correlation of -0.7370. Larger funds also tend to have lower expense ratios and generate lower α.

2.3 Results

2.3.1 Baseline results on lottery characteristics and Closed-End Fund premia

To analyze the effect of lottery characteristics on the Closed-End Fund premium $Prem_{i,t}$ of fund i in month t, we estimate the following panel regression:

$$Prem_{i,t} = \alpha_i + \beta_1 \cdot FundLC_{i,t} + \beta_2 \cdot AssetLC_{i,t} + \gamma\lambda_{i,t} + \delta\chi_t + \epsilon_{i,t}. \qquad (2.7)$$

In Equation (2.7), $LC_{i,t}$ represents a placeholder for lottery characteristics, $\lambda_{i,t}$ represents fund-specific control variables, and χ_t represents systematic control variables. The prefix $Fund$ describes lottery characteristics of the Closed-End Fund shares and the prefix $Asset$ – not applicable for $SSPW$ – represents the lottery characteristics of the funds' underlying assets in terms of a value-weighted average.[11]

Specification (2.7) guarantees a simultaneous estimation of the two distinct dimensions in which we expect lottery-like characteristics to affect Closed-End Fund premia. Ex ante, this affects Closed-End Fund shares similarly to common stocks, and lottery characteristics are associated with more demand and a higher price. Consequently, funds that are attractive to lottery investors trade at higher premia and we expect $\beta_1 > 0$. Conversely, investors' lottery demand increases prices of common stocks with strong lottery characteristics, which directly manifests in higher NAV of funds holding the particular stock, therefore predicting $\beta_2 < 0$.

[11] In unreported robustness checks, we use an equal-weighting scheme without changes in our conclusions.

A pooled OLS regression of Equation (2.7) raises two major methodological concerns. First, residuals in Equation (2.7) are likely to be correlated in two dimensions, both serially and cross-sectionally. As a consequence, standard errors exhibit a downward bias, leading to an over-rejection of the null hypothesis (Petersen, 2009). Second, the dependent variable might be non-stationary.

We address the first concern parametrically, as suggested by Petersen (2009), by clustering standard errors in the more frequent dimension of the panel and including a dummy variable in the remaining dimension.[12] Given a panel dimension of $T = 220$ and $N = 51$, we include a fund dummy and estimate heteroscedasticity-consistent standard errors clustered by time (Rogers standard errors).[13] Panel unit-root tests alleviate the second concern regarding non-stationarity. We perform the Im et al. (2003) IPS test as well as the Pesaran (2007) CADF test, and both tests strongly reject the null hypothesis of a unit root.

Table 2.2 presents results of the panel regression in Equation (2.7) for the sample of US equity funds in the period from March 1997 to March 2015. The dependent variable is the Closed-End Fund premium of each fund i in month t, i.e. $Prem_{i,t}$. Columns (1)–(4) report regressions

[12]In specification tests we find that the number of clusters in the cross-sectional direction is insufficient for two-way clustering as proposed by Petersen (2009) and Thompson (2011) due to the strongly unbalanced panel. Unbalanced panels require more clusters than balanced datasets (Cameron and Miller, 2015). The parametric approach taken above yields similar standard errors given this particular dimension of the dataset (see Petersen, 2009, p. 460).

[13]The time effect, i.e. a correlation of residuals among funds at each point in time, is also expected to arise without systematic control variables for two reasons. First, the average stock in the sample is traded in about 3.2 funds such that lottery characteristics computed from holdings are correlated. Second, Closed-End Fund portfolios comprise a broad range of the US stock market and are thus subject to common systematic shocks.

Table 2.2: Lottery Characteristics and Closed-End Fund Premia.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$SSPW$	-0.067				0.267			
	(-3.38)				(7.91)			
$FundMax$		0.652				1.025		
		(1.78)				(3.19)		
$AssetMax$		-1.799				-1.258		
		(-5.51)				(-4.19)		
$FundLIDX$			0.056				0.087	
			(2.97)				(5.16)	
$AssetLIDX$			-0.187				-0.158	
			(-4.46)				(-4.83)	
$FundSkew^{3M}$				-0.010				0.011
				(-1.29)				(2.14)
$AssetSkew^{3M}$				-0.023				-0.046
				(-0.62)				(-2.28)
$FundIlliq$					-0.011	-0.010	-0.013	-0.010
					(-4.78)	(-4.45)	(-5.45)	(-4.63)
$AssetIlliq$					0.018	0.019	0.015	0.018
					(4.70)	(5.03)	(4.05)	(4.97)
$Size$					0.005	0.005	0.006	0.005
					(3.48)	(4.00)	(4.33)	(3.88)
$LogAge$					-0.040	-0.040	-0.032	-0.040
					(-14.27)	(-14.66)	(-10.50)	(-14.37)
$DivYld$					0.280	0.285	0.271	0.292
					(7.74)	(7.46)	(7.76)	(7.97)
$T(\alpha)$					0.113	0.108	0.107	0.109
					(24.70)	(24.31)	(24.19)	(24.39)
$T(\alpha)^2$					0.002	-0.001	-0.003	-0.003
					(0.63)	(-0.60)	(-1.21)	(-1.09)
$Turnover$					0.006	0.006	0.005	0.006
					(5.30)	(5.32)	(5.06)	(5.51)
$ExpRatio$					1.607	1.722	1.671	2.010
					(3.96)	(4.47)	(4.19)	(4.66)
VIX					-0.001	-0.001	-0.001	-0.001
					(-4.11)	(-2.59)	(-3.76)	(-4.01)
Liq					-0.046	-0.049	-0.034	-0.039
					(-1.83)	(-2.13)	(-1.38)	(-1.61)
$TERM$					0.188	0.161	0.216	0.200
					(1.24)	(1.13)	(1.46)	(1.31)
$ConSent$					0.000	0.001	0.001	0.001
					(2.84)	(3.12)	(3.75)	(2.96)
$\overline{R}^2$	0.370	0.403	0.388	0.379	0.589	0.597	0.591	0.588
N	5667	5617	5451	5572	5313	5289	5313	5289

Table 2.2 presents coefficient estimates of the panel regression in Equation (2.7). We include the following lottery characteristics: $SSPW$ is the sum of squared portfolio weights to measure portfolio concentration following Goetzmann and Kumar (2008). Max is the average of the highest five daily returns in month $t-1$ as proposed by Bali et al. (2017). $Skew$ is the adapted Green and Hwang (2012) quantile-based skewness measure, calculated over the preceding three months. $LIDX$ is a monthly adaption of the Kumar et al. (2016) lottery index measure. Fund-specific control variables are the following: $Illiq$ is the logarithm of the adapted Amihud (2002) illiquidity measure. $Size$ is the natural logarithm of the fund's market capitalization in million USD, $LogAge$ is the natural logarithm of the fund age in years, and $DivYld$ is the fund's dividend yield. $ExpRatio$ is the expense ratio and $Turnover$ is the portfolio turnover. $T(\alpha)$ is the t-statistic of the fund alpha over the past twelve months. Systematic control variables: $TERM$ is the term spread as the difference between 20-year and 3-month constant maturity Treasury rates, Liq is the Pastor and Stambaugh (2003) liquidity factor, $ConSent$ is the Michigan University Consumer Sentiment Index and VIX is the CBOE volatility index. The sample period is March 1997 to March 2015. Each regression contains a fund dummy variable to absorb the fund effect in the correlation of the residuals, and standard errors are clustered by month (Petersen, 2009). T-statistics in parentheses are calculated from heteroscedasticity-consistent clustered standard errors (Rogers standard errors).

without control variables, and in Columns (5)–(8) we extend the lottery characteristics by the baseline set of control variables. Starting with the most simplistic measure for lottery-like payoffs of fund shares, i.e. $SSPW$, results are mixed. In Column (1) we find a negative relation between $SSPW$ and $Prem_{i,t}$, suggesting higher premia for more diversified funds. This effect is highly statistically significant with a t-statistic of -3.38. The inclusion of control variables in Column (5) changes the sign of the $SSPW$ loading. This change suggests that the negative coefficient for $SSPW$ in Model (1) is likely to be driven by the omission of important fund characteristics. Apparently, size, age, managerial ability, or sophistication as might be indicated by the expense ratio can interfere with portfolio concentration $SSPW$. Including controls, the coefficient estimate for $SSPW$ matches the hypothesis regarding the relation between fund premia and lottery-like payoffs. The coefficient of 0.267 is positive and highly significant at any conventional level. Closed-End Funds with more concentrated portfolios, and thus a higher likelihood of lottery-like payoffs, trade at higher premia as soon as we include control variables.

Columns (2) and (6) repeat the analysis for the Bali et al. (2017) lottery demand measure Max. Both models, with and without control variables, support our theoretical hypotheses. While lottery characteristics in Closed-End Fund shares are granted with ceteri paribus higher fund premia, indicated by the positive coefficient on $FundMax$, the opposite holds true for lottery-like payoffs in the portfolio of the corresponding fund, measured by $AssetMax$. The latter effect is stronger, both statistically as well as economically. When including controls, a standard deviation increase of $FundMax$ is associated with an increase of the fund premium by 133 bps, compared with a decrease of 151 bps for $AssetMax$. Both $FundMax$ and $AssetMax$ attain t-statistics well beyond three in absolute terms.

$LIDX$ provides further support for our main hypothesis, independently of the exact specification. In Column (3), the regression coefficients are significant at any conventional level with the expected sign. Similar to Max, the $Asset$ characteristic performs better statistically and economically. This finding also extends to the full model in Column (7). We find significant coefficients, a positive coefficient of 0.087 for $FundLIDX$, and a negative coefficient of -0.158 for $AssetLIDX$. The respective changes in economic terms are 92 and -92 bps for a one standard deviation rise in $LIDX$. In both cases, t-statistics around ±5 indicate statistical significance well beyond conventional levels.

Finally, results for $Skew$ over the preceding three months are mixed to some extent, similar to $SSPW$. The specification without control variables in Column (4) yields insignificant results, while the full model in Column (8) provides further support to our hypotheses. The coefficients on $FundSkew$ and $AssetSkew$ are statistically significant at the 5% level, with t-statistics of 2.14 and -2.28, respectively. Again, the economic significance of $AssetSkew$ (-34 bps) exceeds $FundSkew$ (22 bps) in absolute terms. The results are somewhat weaker, but similar to Max.

A closer look at selected control variables also yields novel insights, especially with respect to the illiquidity measures. Although US Equity Closed-End Funds are less likely to provide liquidity benefits to investors, we find strong evidence for the importance of the relation between fund liquidity and the liquidity of assets in a fund's portfolio. More illiquid funds trade at lower premia. Conversely, funds grant liquidity benefits given rather illiquid stocks in their respective portfolio. Similar to lottery characteristics, $AssetIlliq$ is of stronger economic significance compared with $FundIlliq$. We thus, provide new empirical evidence in favor of Cherkes et al. (2009). Furthermore, there is rather weak evidence with respect to the non-linear relationship between managerial ability and Closed-End Fund discounts. While the coefficient on $T(\alpha)$ is positive and highly significant at any conventional level, evidence regarding the negative effect of $T(\alpha)^2$ is inconclusive. There is no statistical significance of the coefficient on $T(\alpha)^2$. The negative estimate for VIX supports an interpretation of the variable as a negative sentiment measure rather than as an indicator for systematic liquidity. This effect is significant in all specifications.

Our main conclusions are not altered when using different definitions of the lottery characteristics. In unreported robustness checks, we use the logarithm of the number of stocks in a fund's portfolio $NSTKS$ instead of $SSPW$ as a fund's diversification proxy. Max is calculated as the previous month's maximum daily return instead of the average of the top five returns. The quantile-based measure of skewness is calculated over just one month or six months instead of three months. Finally, the lottery index $LIDX$ is calculated on an annual instead of a monthly basis. These alternative variable definitions all yield significant results with predicted sign. Further, a change in the weighting scheme on the asset level, i.e.

equal instead of value-based weighting does not alter our conclusions.[14]

In summary, the majority of lottery characteristics is consistent with our initial hypotheses. Lottery-like funds trade at higher premia for given lottery characteristics in their underlying assets, while the opposite is true for lottery-like payoffs in the investment universe of US stocks. This effect is highly statistically significant, especially for Max and $LIDX$. This finding also holds after the inclusion of several control variables which, in turn, lend support to the models of Cherkes et al. (2009) and Berk and Stanton (2007).

2.3.2 Skewness preferences and Closed-End Funds

S&P 500 option prices and skewness preferences

In the previous Section, we showed that Closed-End Fund premia are driven to a significant degree by the lottery characteristics provided by fund stock returns relative to a fund's asset returns. A premise is that investors' preferences for idiosyncratic skewness or lottery demand drive stock prices. Theoretical models by, for example, Brunnermeier et al. (2007), Barberis and Huang (2008), and Mitton and Vorkink (2007) motivate this link. However, there is also evidence that skewness preference varies predictably over time. Kumar (2009) finds stronger gambling demand in the stock market during economic downturns. During the so-called Dotcom Bubble, with high investor sentiment, skewness characteristics had greater pricing impacts as shown by Green and Hwang

[14]These results are available upon request.

(2012), pointing to stronger skewness preferences. The Barberis and Huang (2008) model focuses on Cumulative Prospect Theory's probability weighting. The degree of probability weighting drives the magnitude of lottery stocks' overpricing. Polkovnichenko and Zhao (2013) infer aggregate probability weighting from index option prices, and find significant time variation.

Here, we nonparametrically estimate monthly probability weighting functions from S&P 500 index option prices. The curvature index of these weighting functions serves as a proxy for aggregate skewness preferences as motivated theoretically by Barberis and Huang (2008). We find that option-implied probability weighting is stronger in times of *extreme*, i.e. low and high sentiment. From laboratory experiments, we know that probability weighting is more extreme in affect-rich situations (Rottenstreich and Hsee, 2001). In our investment context, economic crises and periods of economic exuberance might be good proxies for affect-rich situations.[15]

Three testable predictions emerge. First, in times of strong skewness preference, the average Closed-End Fund premium is lower because only few investors want to hold diversified funds relative to individual stocks. Second, the impact of lottery characteristics found in the previous section is stronger during high skewness preference times. In Section 2.3.2, we

[15]There are more papers that motivate time varying skewness preferences with similar patterns. Bordalo et al.'s (2012) Salience Theory implies a different probability weighting scheme that can generate time-varying skewness preference (see Bordalo et al., 2013). Zhang (2014) elaborates on Mitton and Vorkink's (2007) equilibrium underdiversification and skewness preference model. She shows that, even though there are fewer gamblers in economic downturns, the remaining ones gamble more aggressively.

use regression analyses with interaction terms to test this prediction. Third, we should see fewer Closed-End Fund IPOs when there is high skewness preference because of low demand for diversified investments and fund managers do not want to start a fund with a high discount.

Our results further indicate that the seemingly remote corner of the Closed-End Fund Market is linked to the highly liquid S&P500 index option market via aggregate gambling preferences. Put differently, the same risk preferences that drive option prices in equilibrium are helpful in understanding the Closed-End Fund prices relative to their holdings. Further, this result highlights the complementary nature of our finding compared to Hwang et al. (2017) who focus on heterogeneity in beliefs as estimated from analysts' earnings forecasts for Closed-End Funds' holdings.

We now turn to the implications of probability weighting for option pricing and describe our nonparametric estimation procedure which closely follows Dierkes (2013). We assume a representative agent with monotonically increasing utility function u and monotonically increasing probability weighting function w. Both u and w are assumed to be smooth (twice continuously differentiable). Utility is derived from an index with S_t and S_T denoting values today and in the future, respectively. Following, among others, Jackwerth (2000), we normalize S_t to one and the representative agent derives utility over returns.

Let f_P and f_Q denote the density functions of the data-generating process and the risk-neutral measure with corresponding cumulative distribution functions F_P and F_Q, respectively. With probability weighting (non-linear w), the investor prices options as if the data-generating process' distribution function is $F_{\tilde{P}}(x) = 1 - w(1 - F_P(x))$. Thus, the corresponding density is given by

$$f_{\tilde{P}}(S_T) = f_P(S_T)) \cdot w'(1 - F_P(S_T)), \tag{2.8}$$

and the option-implied risk-neutral measure is calculated with a nonlinear w according to

$$f_Q(S_T) = f_P(S_T) \cdot w'\Big(1 - F_P(S_T)\Big) \cdot \beta \frac{u'(S_T)}{u'(S_t)}. \tag{2.9}$$

A formal derivation can be found in, e.g. Xia and Zhou (2014) or Polkovnichenko and Zhao (2013). Note that, given preferences by w and u, the pricing kernel $\frac{f_Q(S_T)}{f_P(S_T)} = w'\Big(1 - F_P(S_T)\Big) \cdot \beta \frac{u'(S_T)}{u'(S_t)}$ varies with the physical distribution F_P if w is not linear. Taking derivatives with respect to S_T and rearranging then yield

$$\frac{f_P'(S_T)}{f_P(S_T)} - \frac{f_Q'(S_T)}{f_Q(S_T)} = ARA_u(S_T) + \left(\frac{w''(1 - F_P(S_T))}{w'(1 - F_P(S_T))} f_P(S_T)\right), \tag{2.10}$$

where $ARA_u(S_T) = -\frac{u''(S_T)}{u'(S_T)}$ denotes the absolute risk aversion function across index levels S_T associated only with the agent's utility function u. The term $\frac{w''(1-F_P(S_T))}{w'(1-F_P(S_T))} f_P(S_T)$ on the right-hand side of (2.10) displays the probabilistic risk attitude. The denominator is always positive due to the strictly increasing weighting function.

A natural identification strategy that cleanly estimates w and u stems from the fact that the risk attitude associated with w varies with the physical distribution F_P whereas the risk attitude associated with u stays constant. We now set out to identify w and u nonparametrically. To the best of our knowledge we are the first to estimate w nonparametrically without parametric assumptions about u. Polkovnichenko and Zhao (2013) use a semiparametric approach and presuppose $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$ with $\alpha = 2$ or $\alpha = 0$ and aggregate risk aversion not explained by this utility function shows up in their weighting function estimate.

Consider the equilibrium condition (2.9). For convenience, we drop the time index T and convolve constants $u'(S_t)$ and β to a single normalization constant β. Obviously, for two different physical distributions P_1 and P_2, the term $w'(1 - F_{P_i}(S))$ changes with P_i whereas $u'(S)$ remains unaffected, $i = 1, 2$. This allows for the following identification strategy to estimate w nonparametrically. Rearrange Equation (2.9) for both distributions P_1 and P_2 to

$$\frac{f_{Q_1}(S)}{w'(1 - F_{P_1}(S))f_{P_1}(S) \cdot \beta_1} = u'(S),$$
$$\frac{f_{Q_2}(S)}{w'(1 - F_{P_2}(S))f_{P_2}(S) \cdot \beta_2} = u'(S).$$

Equating both left hand sides of the above equations with each other and rearranging yields:

$$w'(1 - F_{P_2}(S)) = \frac{f_{Q_2}(S)}{f_{Q_1}(S)} \frac{f_{P_1}(S)}{f_{P_2}(S)} \frac{\beta_1}{\beta_2} \cdot w'(1 - F_{P_1}(S)) \text{ for all states } S. \quad (2.11)$$

The only unknown in this equation is the function w because all other quantities are estimated from market outcomes. Under appropriate assumptions on F_{P_1} and F_{P_2}, Equation (2.11) constitutes a delay differential equation (DDE) of neutral type.[16] This is the case if, for example, $1 - F_{P_2}$ is

[16] DDEs are characterized by the fact that today's derivative of the unknown function depends on the function's behavior in the past. Neutral type means that today's derivative of the unknown function depends on its derivative in the past.

always above $1 - F_{P_1}$ and both actually coincide in one point. In this case, $w'(1 - F_{P_2}(S))$ is known at some "time point" $1 - F_{P_2}(S)$ because the "time point" $1 - F_{P_1}(S)$ lies in the past and therefore $w'(1 - F_{P_1}(S))$ is already known. How can we guarantee this assumption? If P_1 has more mass in the tails than P_2, then, for some value $\hat{S}$, it holds $F_{P_1}(S) \geq F_{P_2}(S)$ for all $S \leq \hat{S}$ and $F_{P_1}(S) \leq F_{P_2}(S)$ for all $S \geq \hat{S}$ and $F_{P_1}(\hat{S}) = F_{P_2}(\hat{S})$. We call this restriction the single crossing assumption. On the two intervals $[0, \hat{S}]$ and $[\hat{S}, \infty]$, the DDE can be solved for w.

We follow Dierkes (2013) and use different times to maturity to ensure the single crossing property. One advantage is that it allows for a time series of estimates of the probability weighting functions and, thus, implicitly idiosyncratic skewness preference.

DDEs require not only one initial value as initial condition (as do ordinary differential equations). Instead, a small *range* of values needs to be given. We get a reasonable initial condition from our environment as follows. Considering two distributions P_1 and P_2 with their risk neutral counterparts Q_1 and Q_2, respectively, and rearranging the decomposition of aggregate absolute risk aversion in Equation (2.10) yields the following two equations

$$\frac{\frac{f'_{P_1}(\hat{S})}{f_{P_1}(\hat{S})} - \frac{f'_{Q_1}(\hat{S})}{f_{Q_1}(\hat{S})} - ARA_u(\hat{S})}{f_{P_1}(\hat{S})} = \frac{w''(1 - F_{P_1}(\hat{S}))}{w'(1 - F_{P_1}(\hat{S}))},$$

$$\frac{\frac{f'_{P_2}(\hat{S})}{f_{P_2}(\hat{S})} - \frac{f'_{Q_2}(\hat{S})}{f_{Q_2}(\hat{S})} - ARA_u(\hat{S})}{f_{P_2}(\hat{S})} = \frac{w''(1 - F_{P_2}(\hat{S}))}{w'(1 - F_{P_2}(\hat{S}))}.$$

Given that $F_{P_1}(\hat{S}) = F_{P_2}(\hat{S})$ for the state $\hat{S}$, we equate the left hand sides of both equations with each other, rearrange, and have

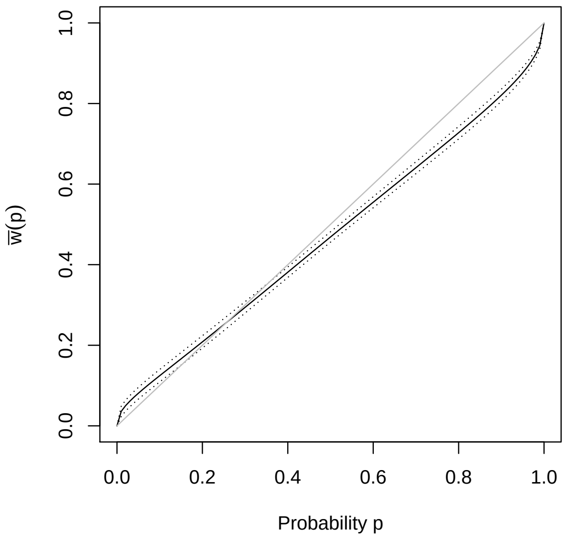
$$ARA_u(\hat{S}) = -\frac{u''(\hat{S})}{u'(\hat{S})} = \Big(\frac{ARA_{M_1}(\hat{S})}{f_{P_1}(\hat{S})} - \frac{ARA_{M_2}(\hat{S})}{f_{P_2}(\hat{S})}\Big)\Big/\Big(\frac{1}{f_{P_1}(\hat{S})} - \frac{1}{f_{P_2}(\hat{S})}\Big), \quad (2.12)$$

where $ARA_{M_i}(S) = \frac{f'_{P_i}(S)}{f_{P_i}(S)} - \frac{f'_{Q_i}(S)}{f_{Q_i}(S)}$, $i \in \{1,2\}$, is the market's aggregate risk aversion implied by asset prices. Importantly, Equation (2.12) can be solved without knowing w or u.

From Equation (2.12), we gain a rough estimate of u in the tiny neighborhood of $\hat{S}$. To come up with an initial condition for our delay differential equation, we make a parametric assumption about u for a tiny range around $\hat{S}$. Then, in the interval, say, $[\hat{S} - 0.001, \hat{S} + 0.001]$, we have an initial condition for w' in Equation (2.11) from Equation (2.9). Together with $w(0) = 0$ and $w(1) = 1$, we can identify w nonparametrically. We assume that $ARA_u(S) = ARA_u(\hat{S})$ for all $S \in [\hat{S} - 0.001, \hat{S} + 0.001]$, i.e. on this small interval absolute risk aversion associated with u is a constant and thus u is given by the exponential utility function on this interval. An unreported simulation study revealed that, even if the utility function u is not given by the exponential function, this parametric choice for u on a tiny interval around $\hat{S}$ does not distort our results for the probability weighting function w. There is virtually perfect identification.

Figure 2.1 depicts the average of 236 probability weighting functions estimated between February 1996 and September 2015. Additionally, ±2 times the pointwise standard error is plotted (dotted lines). The gray line indicates the identity line (45 degree line). The average probability weighting function clearly exhibits an inverse-S shape. This finding matches results by psychologists. The intersection with the identity line

Figure 2.1: Average nonparametric estimate of the probability weighting functions.



We estimate 236 probability weighting functions and utility functions without any parametric restrictions. Figure 2.1 depicts our estimate of the average probability weighting function $\overline{w}$ (solid line) and ±2 times the empirical pointwise standard error (dotted lines). The gray line corresponds to the identity function.

is roughly at probability $p = 0.26$. This value is remarkably close to those found in lab experiments (typically around 0.30).

To compare our nonparametric estimates to results by psychologists and behavioral economists, we fit the "linear in log odds" probability weighting function $w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$ to our average estimate $\overline{w}$. The linear regression of log odds for $p = 0.01, 0.02, \ldots, 0.98, 0.99$ is nearly perfect with adjusted $R^2 = 0.993$. We yield the curvature parameter $\gamma = 0.757$ and the logarithm of the elevation parameter δ is -0.171, i.e. $\delta = 0.843$. Standard errors are 0.007 and 0.011, respectively, again suggesting a strong deviation from EUT. These parameter values are slightly greater than what is reported by psychologists. Bleichrodt and Pinto (2000) review estimates from the literature and report values in $[0.44, 0.69]$ and $[0.65, 0.84]$ for γ and δ, respectively. That our values for γ and δ are closer to one indicates less pronounced probability weighting which is not surprising because our results are obtained from one of the most liquid and most competitive option markets in the world. So, ex ante we expect estimates to be closer to EUT.

Of particular interest to our analyses is the variation of the curvature of the probability weighting function over time. The upper panel of Figure 2.2 shows the times series of estimated γ. Recall that the curvature parameter γ can be interpreted as a measure of inverse skewness preference (Barberis and Huang, 2008). We thus, define our first skewness preference proxy as $SkewPref_{option-implied} = 1/\gamma$ in each month and standardize it for ease of interpretation (zero mean and unit standard deviation). To get a better sense of w's impact on aggregate risk aversion, the middle panel in Figure 2.2 depicts stacked bars that represent concave (black bar) and convex (white bar) parts of the fitted probability weighting function as displayed on the vertical axis. Recall that convex parts of w increase risk aversion and concave parts decrease it. For example, the first stacked bar from the left indicates that, in February 1996, the weighting function was concave for the probabilities in $[0, 0.51]$ and convex for probabilities in $[0.51, 1]$, i.e. probability weighting alone induces (local) risk proclivity for the 51% best wealth states according to the physical distribution and it induces risk aversion for the 49% worst wealth states.

The lower panel shows the same information as the upper one, but is less noisy because it depicts the curvature of the parametric probability

Figure 2.2: Curvature index of probability weighting functions over time.

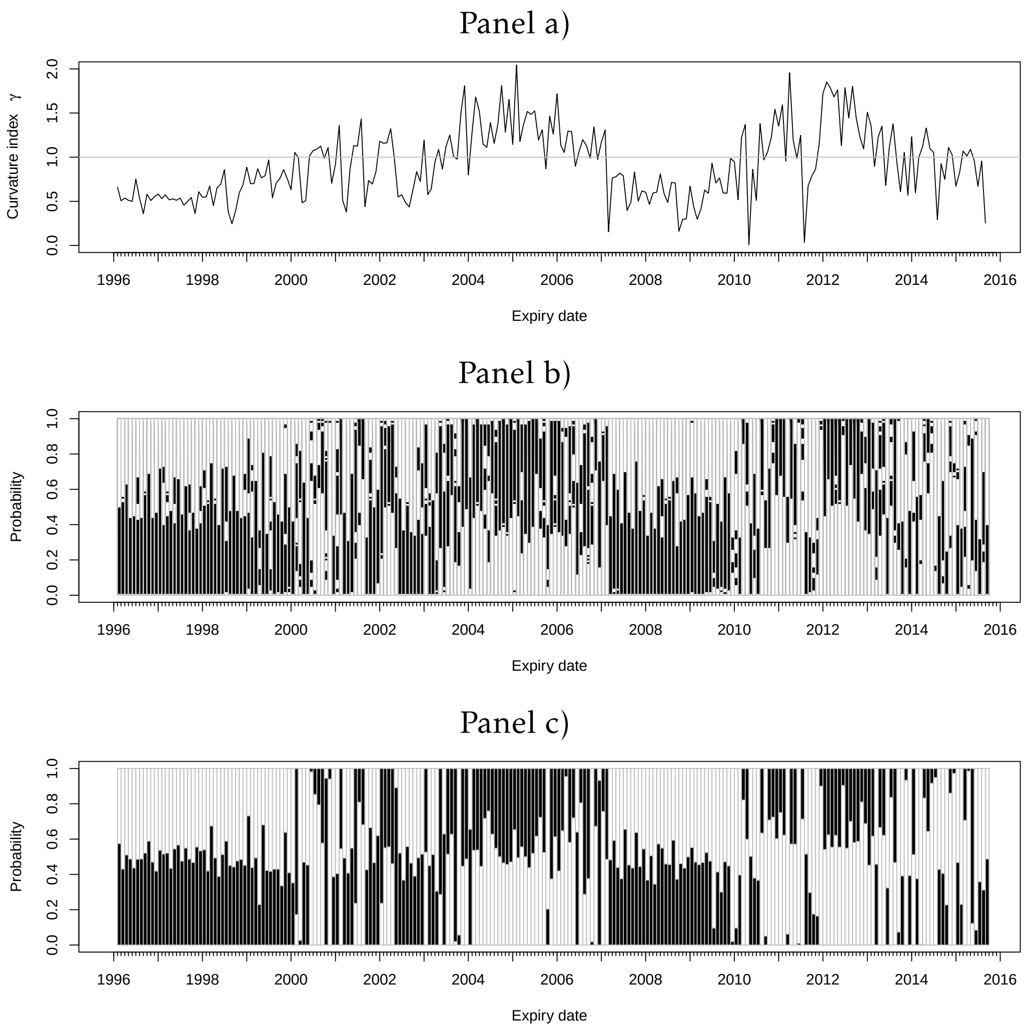


Figure 2.2 presents time series dynamics of the curvature index of probability weighting functions. We estimate 236 nonparametric probability weighting functions without any parametric restrictions on the utility function. Panel a) depicts the curvature parameter γ of the probability weighting function $w_{\gamma,\delta}(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$ when fitted to our nonparametric estimates of the probability weighting functions. Panel b) shows stacked bars that represent concave (black bar) and convex (white bar) parts of the probability weighting function. For example, the first stacked bar from the left indicates that in February 1996, w was concave for probabilities in $[0, 0.51]$ and convex for probabilities in $[0.51, 1]$. The lower Panel c) presents the same information as the middle one for the fitted parametric probability weighting function $w_{\gamma,\delta}$ instead of the nonparametric w.

weighting function $w_{\gamma,\delta}(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$ when fitted to w. The parametric fit of the "linear in log odds" probability weighting function $w_{\gamma,\delta}$ is in all months remarkably good. The adjusted R^2 of the linear regression of log odds for $p = 0.01, 0.02, \ldots, 0.98, 0.99$ never falls below 0.947.

We see three large chunks with rather stable general shape patterns. The probability weighting function was inverse-S shaped from February 1996 until January 2000 and from March 2007 until October 2009. From February 2004 to September 2005, w was S-shaped. With few exceptions, this period extends to February 2007. May 2002 until December 2002 is another, shorter period with inverse-S shape pattern. After 2010, S-shape dominates, with some months showing an inverse-S shape. For other time periods the general shape is less stable. However, it is surprising that we do not see entirely erratic behavior. For example, the curvature captures the beginnings of the financial crisis quite well. From February to March in 2007, the shape of the probability weighting function reverses from S-shaped to inverse-S shaped. In February 2007, HSBC had to announce one of the first major losses ($10.5 billion) from the US mortgage lending business. However, the information in the variation of the probability weighting function's curvature is much richer as we show below.

From lab experiments we know (see Rottenstreich and Hsee, 2001) that the probability weighting function is more inverse-S shaped when subjects are more emotional, i.e. hope and fear decrease the curvature index γ. We use the University of Michigan's Consumer Confidence Index to proxy extreme sentiment in the economy. Our prediction is an inverse-U shaped relation between our curvature index γ and the sentiment

Figure 2.3: The probability weighting functions' curvature and consumer sentiment.

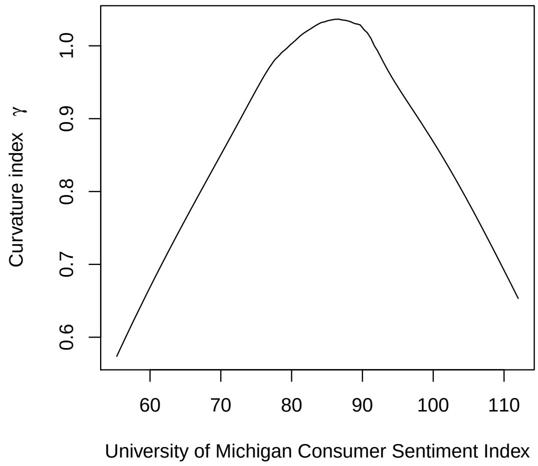


Figure 2.3 presents the relation between probability weighting and consumer sentiment. For 236 months, we estimate the representative agent's probability weighting function. For each weighting function, we estimate its curvature index. The curvature index γ is derived from fitting the "linear in log odds" probability weighting function $w_{\gamma,\delta}(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$ to our nonparametric estimates. We depict the fitted values of a locally polynomial regression of γ on the University of Michigan's Consumer Sentiment Index.

index.

Figure 2.3 shows a locally polynomial regression fit (see e.g. Cleveland, 1979) and confirms the inverse-U shaped relation. Skewness preference is stronger (lower γ values) during times of extreme sentiment, either good or bad. Hence, we define the skewness preference dummy $SkewPref_{sentiment}$ such that it equals one if the University of Michigan Sentiment Index is in the lowest or highest tercile; otherwise it equals zero.

Now, we provide first descriptive analyses between Closed-End Funds and our skewness preference proxies $SkewPref_{option-implied}$ and

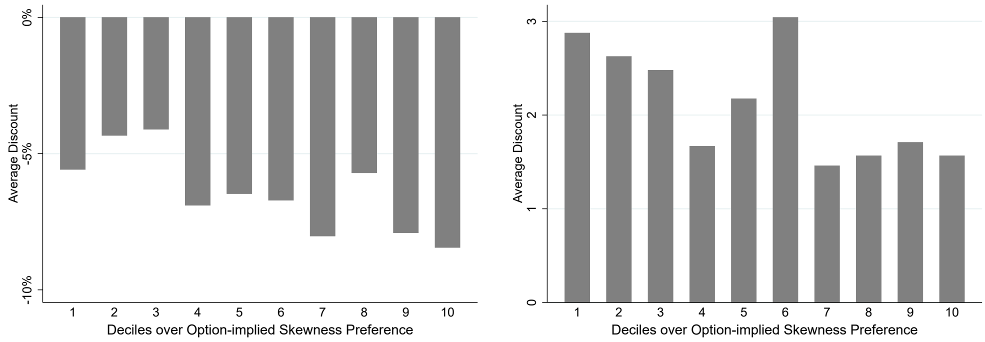
$SkewPref_{sentimewnt}$. The upper panels of Figure 2.4 depict the average premium (left panel) and the total number of Closed-End Fund IPOs (right panel) over deciles of option-implied skewness preference proxy $SkewPref_{option-implied}$. Consistent with our hypotheses, both the premium and the number of IPOs is lower when skewness preference is high. For example, the 286 bps premium difference between the first and tenth decile is significant at the 5% level (two sided t-test, $t = 2.16$). Similarly, the difference of 1.3 IPOs on average between the first and tenth decile is also significant at the 5% level (two-sided t-test, $t = 2.15$).

The middle panels presents the average Closed-End Fund premium (left panel) and average number of Closed-End Fund IPOs from two different sources over deciles of the University of Michigan Consumer Sentiment Index, respectively. Dark gray bars are computed from the full data of equity Closed-End Funds from 1978 to 2015. Note that we start in 1978 because monthly recording of $ConSent$ starts in this year. Light gray bars cover our sample of US Equity Closed-End Funds from 1997-2015. In line with our conjectures, both panels reveal a U-shaped pattern with lower premia and lower IPOs for extreme sentiment which we argued is tantamount to high skewness preference.

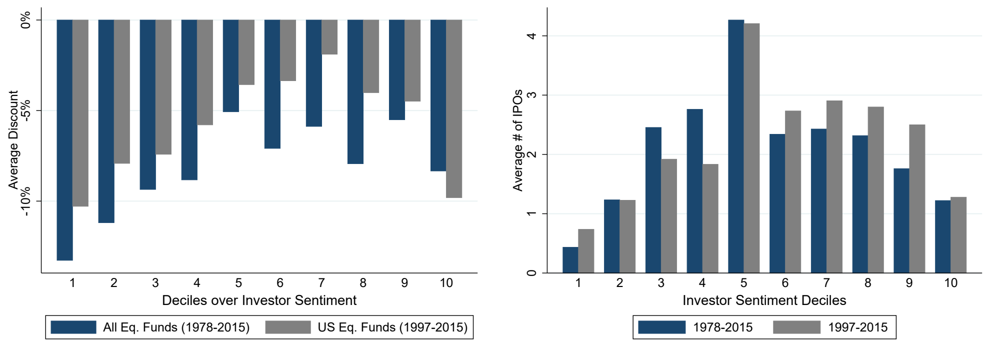
Our definition of the skewness preference dummy $SkewPref_{sentiment}$ further supports this insight. Similar to the middle panels, the bottom panels depict average premia (left panel) and the number of IPOs (right panel) for two samples (as above in dark or light gray), but now over $SkewPref_{sentiment}$ instead of Consumer sentiment. We find our hypotheses supported because there are lower premia and fewer IPOs when the

Figure 2.4: Average Premium and IPOs depending on Skewness Preference and Sentiment.

(a) Average premium (left panel) and number of IPOs (right panel) over deciles of $SkewPref_{Option-implied}$



(b) Average premium (left panel) and number of IPOs (right panel) over deciles of Consumer Sentiment $ConSent$



(c) Average premium (left panel) and number of IPOs (right panel) over $SkewPref_{Sentiment}$

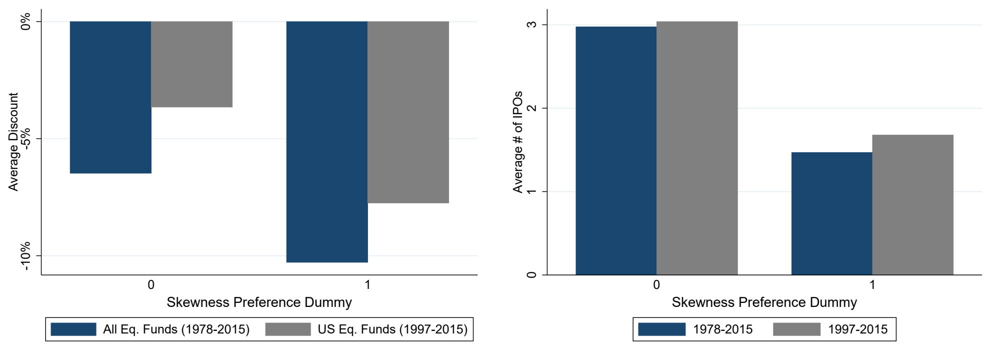


Figure 2.4 depicts the average monthly Closed-End Fund premia (left panels) and the average number of Closed-End Fund IPOs (right panels) over deciles of option-implied skewness preference $SkewPref_{Option-implied}$ (top panels), deciles of investor sentiment measured by the University of Michigan Consumer Sentiment Index $ConSent$ (middle panels), and the sentiment-based skewness preference dummy $SkewPref_{Sentiment}$ (bottom panels).

dummy variable equals one. The difference between premia of high versus low skewness preference regimes is around 400 bps for both samples with both t-statistics exceeding six. Similarly, the difference in IPOs across skewness preference regimes is in both samples around 1.4 on average and highly significant (both t-statistics exceed four).

Skewness preferences and premia

Now, we provide more rigorous statistical analyses. To begin with, simple time series regressions of the monthly average Closed-End Fund Premium $\overline{Prem_t}$, taken from Jeffrey Wurgler's website, on both our monthly skewness preference proxies yield

$$\overline{Prem_t} = \underset{(-3.09)}{-0.0026^{***}} SkewPref_{option-implied,t} + \delta\chi_t + \epsilon_t \tag{2.13}$$
$$N = 236, R^2 = 0.082,$$

$$\overline{Prem_t} = \underset{(-2.50)}{-0.0182^{**}} SkewPref_{sentiment,t} + \delta\chi_t + \epsilon_t \tag{2.14}$$
$$N = 354, R^2 = 0.106.$$

The control variables χ_t, motivated by Cherkes et al. (2009), are the S&P100 volatility index VXO[17], the term spread $TERM$ as the difference between 10-year and 1-year constant maturity Treasury Rates, the Pastor

[17] VXO has a longer time series starting in 1986 compared with the more common VIX which starts in 1990. Calculation methods are slightly different between both. Still, correlation between the two volatility indices is close to perfect, with a correlation coefficient of 0.9855.

and Stambaugh (2003) liquidity factor, and a constant.[18] While the option-implied skewness preference proxy $SkewPref_{option-implied}$ is available only since February 1996 (Regression (2.13)), data in Regression (2.14) is limited by the VXO starting in June 1986. As predicted, in periods of high skewness preference, the average Closed-End Fund premium is lower. If option-implied skewness preference increases by one standard deviation, all else equal, the premium is 26 bps lower (Newey/West t-statistic -3.09, $p < 0.01$). For the sentiment-based dummy $SkewPref_{sentiment}$, strong skewness preference decreases the average premium by 182 bps (Newey/West t-statistic -2.50, $p < 0.05$). The time series evidence favors our hypothesis and corroborates the descriptive analyses in the previous section.

We continue with cross-sectional analyses. The more specific prediction of our theory is that, in the cross section, there should be an interaction effect between skewness preference and lottery characteristics provided by Closed-End Funds and their assets. Given some lottery characteristic in fund stock returns and assets' returns, their impact should be stronger in high skewness preference times because then there is higher lottery demand in the stock market.

Table 2.3 shows results from a rerun of the baseline panel regressions in Table 2.2 while adding one of the skewness preference proxies, $SkewPref_{option-implied}$ (Columns (1)-(4)) and $SkewPref_{sentiment}$ (Columns (5)-(8)), and their interaction with the lottery characteristics on the fund

[18] Similar to the VXO, we apply the alternative measure for the term spread, i.e. the difference between 10-year and 1-year constant maturity Treasury rates, because of a better coverage on FRED. Furthermore we orthogonalized the VXO with respect to $SkewPref_{option-implied}$ because both variables are calculated from S&P500 index options.

and asset level. We are interested in cross-sectional effects and focus on lottery characteristics and their interaction with skewness preference as stated in Panel A. Recall that we standardized $SkewPref_{option-implied}$ to zero mean and unit standard deviation. Given a one standard deviation increase in $SkewPref_{option-implied}$, Column (1) in Table 2.3 suggests that, all else equal, a one standard deviation increase in portfolio concentration $SSPW$ leads to a significantly higher increase in a fund's premium than without stronger skewness preference because the interaction effect $SSPW \times SkewPref_{option-implied}$ is significant (t-stat > 4). However, this effect is economically small with less than 3 bps.[19] The interaction with the skewness preference dummy is not significant at conventional levels, as shown in Column (5). $SSPW$'s overall effect given high skewness preference is highly significant and economically about as large as in our baseline regressions. A one standard deviation increase in portfolio concentration $SSPW$ increases fund premia by more than 121 bps, on average (see Panel B).

As predicted, the interactions of $SkewPref$ with $AssetMax$, $AssetLIDX$, and $AssetSkew^{3M}$ are significantly negative at the 1% level. For the option-implied skewness preference proxy t-statistics range between -5.77 and -9.46. Given $SkewPref_{option-implied}$ is up by one standard deviation, the additional economic impact on top of the baseline effects are -47, -15, and -7 bps for a one standard deviation rise of $AssetMax$, $AssetLIDX$, and $AssetSkew^{3M}$, respectively. Similary, given the skewness preference

[19]SSPW has a standard deviation of 0.043. So, the additional effect as indicated by the interaction term is $0.006 \times 0.043 = 0.000258$ which corresponds to 2.58 bps.

Table 2.3: Closed-End Fund Premia depending on Lottery Characteristics and Skewness Preferences.

Panel A: Panel Regression with Interaction Effects

	Skewness Preference based on Index Options				Skewness Preference based on Sentiment			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$SSPW$	0.275				0.264			
	(8.22)				(3.92)			
$SSPW \times SkewPref$	0.006				0.025			
	(4.89)				(0.45)			
$FundMax$		1.014				1.360		
		(3.18)				(1.98)		
$FundMax \times SkewPref$		-0.183				-0.447		
		(-1.35)				(-0.65)		
$AssetMax$		-1.231				0.646		
		(-4.10)				(1.51)		
$AssetMax \times SkewPref$		-0.392				-2.129		
		(-5.77)				(-4.88)		
$FundLIDX$			0.084				0.065	
			(4.96)				(1.90)	
$FundLIDX \times SkewPref$			0.001				0.023	
			(0.32)				(0.66)	
$AssetLIDX$			-0.155				-0.064	
			(-4.67)				(-1.36)	
$AssetLIDX \times SkewPref$			-0.026				-0.145	
			(-9.46)				(-3.26)	
$FundSkew^{3M}$				0.011				0.008
				(2.14)				(1.11)
$FundSkew^{3M} \times SkewPref$				0.000				0.005
				(0.41)				(0.46)
$AssetSkew^{3M}$				-0.046				0.020
				(-2.33)				(0.76)
$AssetSkew^{3M} \times SkewPref$				-0.010				-0.083
				(-9.14)				(-2.62)
$\overline{R}^2$	0.588	0.595	0.589	0.586	0.594	0.611	0.596	0.592
N	5313	5289	5313	5289	5313	5289	5313	5289

Panel B: Tests for Joint Effects of Lottery Characteristics and Skewness Preferences

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta_1 + \beta_2$	0.282	0.831	0.084	0.011	0.289	0.913	0.088	0.012
	(8.508)	(2.419)	(5.217)	(2.272)	(8.603)	(3.002)	(5.161)	(1.765)
$\beta_3 + \beta_4$		-1.624	-0.181	-0.057		-1.483	-0.209	-0.063
		(-5.450)	(-5.431)	(-2.914)		(-4.877)	(-6.257)	(-2.493)

Panel A of Table 2.3 presents coefficient estimates of the panel regression

$$Prem_{i,t} = \alpha_i + \beta_1 \cdot FundLC_{i,t} + \beta_2 \cdot FundLC_{i,t} \times SkewPref_t + \beta_3 \cdot AssetLC_{i,t} + \beta_4 \cdot AssetLC_{i,t} \times SkewPref_t + \beta_5 SkewPref_t + \gamma \lambda_{i,t} + \delta \chi_t + \epsilon_{i,t},$$

where $Prem_{i,t}$ is the monthly premium of Closed-End Fund i in month t, LC is a placeholder for different lottery characteristics, $\lambda_{i,t}$ represents fund-specific control variables and χ_t represents systematic control variables. The prefix $Fund$ describes characteristics calculated on the Closed-End Fund share level. The prefix $Asset$ – if applicable – represents the value-weighted average of characteristics of all holdings of the corresponding funds. $SkewPref$ is a skewness preference dummy. In models (1)-(4), it is based on option-implied probability wieghting. In models (5)-(8), it is a dummy variable which equals one if Consumer Sentiment is in the lowest or highest tercile, otherwise it is zero. Lottery characteristics cover the following, well-accepted measures: $SSPW$ is the sum of squared portfolio weights to measure portfolio concentration following Goetzmann and Kumar (2008). Max is the average of the highest five daily returns in month $t-1$ as proposed by Bali et al. (2017). $Skew$ is the adapted Green and Hwang (2012) quantile-based skewness measure, calculated over the preceding three months. $LIDX$ is a monthly adaption of the Kumar et al. (2016) lottery index measure. The regressions include the following control variables: $Illiq$ as the logarithm of the adapted Amihud (2002) illiquidity measure. $Size$ as the natural logarithm of the fund's market capitalization in million USD, $LogAge$ as the natural logarithm of the fund age in years and $DivYld$ as the fund's dividend yield. $ExpRatio$ as the expense ratio and $Turnover$ as the portfolio turnover. $T(\alpha)$ as the t-statistic of the fund alpha over the past twelve months. $TERM$ as the term spread, i.e. the difference between 20-year and 3-month constant maturity Treasury rates, Liq as the Pastor and Stambaugh (2003) liquidity factor and VIX as the CBOE volatility index. The sample period is March 1997 to March 2015. Each regression contains a fund dummy variable to absorb the fund effect in the correlation of the residuals, and standard errors are clustered by month (Petersen, 2009). Panel B reports magnitude and statistical significance of the joint effect between lottery characteristics and skewness preferences for the model in the respective column. T-statistics in parentheses are calculated from heteroscedasticity consistent clustered standard errors (Rogers standard errors).

dummy $SkewPref_{sentiment}$ equals one, the corresponding additional economic effects due to high skewness preference – above and beyond the baseline impacts of $AssetMax$, $AssetLIDX$, and $AssetSkew^{3M}$ – are -255, -84, and -61 bps, respectively. t-statistics of these interaction effects range between -2.62 and -4.88. These results support our conjecture that higher skewness preference amplifies the impact of lottery characteristics in a fund's portfolio on the fund's premium. However, we do not find significant interactions with lottery characteristics of the fund's stock. One reason could be that, in times of high skewness preference, increases in lottery characteristics on the fund level, e.g. a rise in $FundMax$, are offset by the perceived detriments from diversification when investing in Closed-End Funds.

Panel B depicts the overall impact of lottery characteristics given high skewness preference by either $SkewPref_{option-implied}$ being up by one standard deviation (Columns (1)-(4)) or $SkewPref_{sentiment} = 1$ (Columns (5)-(8)). Given high skewness preference, a one standard deviation jump in Max, $LIDX$, or $Skew$ on the asset level leads to an average premium drop of at least 178, 105, or 42 bps, respectively.[20] On the fund level, given high skewness preference, a step-up of these lottery characteristics by one standard deviation boosts the fund premium on average by at least 108, 89, or 22 bps, with all figures being statistically significant.

Overall, the results are consistent with our conjecture that investors' preference for lottery characteristic in stock returns can drive a wedge be-

[20]We took the minimum overall impact on the asset level over both skewness preference proxies.

tween a Closed-End Fund's market valuation and its NAV. The differential is greater when investors' lottery appetite is greater, because diversified investments then appear less attractive.[21] Further, the change in fund premia (or discounts) is mainly driven by asset characteristics of the assets in a fund's portfolio.

Skewness preferences and Closed-End Fund IPOs

Finally, we analyze Closed-End Fund IPOs. Lee et al. (1991) state that Closed-End Funds start at a positive premium when initiated.[22] We propose that, above and beyond liquidity reasons, Closed-End Fund IPOs take place when investors look for diversified investments or, in our nomenclature, in times of *low* skewness preferences. Our conjecture precisely matches the empirical evidence of differential secondary market pricing after Closed-End Fund IPOs and Non-Closed-End Fund IPOs. Green and Hwang (2012) show that high skewness preference is a driver of underpricing in Non-Closed-End Fund IPOs. Our hypothesis of low skewness preference triggering Closed-End Fund IPOs implies that, in the Closed-End Fund Market, there is no significant underpricing of IPOs – which is exactly what is found by Hanley et al. (1996).

Following Cherkes et al. (2009), we distinguish four sectors: Closed-End Funds focussing on US Equity, International Equity, Municipal Bonds,

[21] In unreported robustness checks we address the concern that the state of the economy is a main driver behind this result. As motivated by Allen et al. (2012), we employ the Chicago Fed National Activity Index ($CFNAI$) as a measure for economic activity. We find that bad economic states (recessionary states), as indicated by $CFNAI < 0$ ($CFNAI < -0.7$) are not accountable for our results.

[22] Cherkes et al. (2009) argue that the premium should be sufficiently high to pay underwriting fees out of IPO proceeds.

and Taxable Bonds. We conjecture that our skewness preference proxies $SkewPref$ negatively predict the number of IPOs in US Equity. Since $SkewPref$ is estimated with US data, we expect a weaker impact on International Equity Closed-End Fund IPOs. $SkewPref$ might also negatively predict the number of IPOs of Closed-End Funds specializing in Municipal Bonds. Presumably, $SkewPref$ has the least explanatory power, if any, for IPOs of Taxable Bond Closed-End Funds because this investment category presumably attracts a different investor clientele who are not looking for lottery exposure in the first place.

Table 2.4 shows results from a negative binomial count regression. Columns (1)-(5) employ $SkewPref_{option-implied}$ whereas Columns (6)-(10) use $SkewPref_{sentiment}$ as skewness proxy. Since IPOs need some preparation time, e.g. for preparation of a prospectus or filing with the SEC, and can be called off, we want to explain the one-month-ahead number of Closed-End Fund IPOs. That is, all explanatory variables are lagged by one month. In addition to $SkewPref$, we include controls motivated by Cherkes et al. (2009), i.e. the CBOE S&P 100 Volatility Index VXO, the Pastor and Stambaugh (2003) liquidity index Liq, the one-month Treasury Bill $T-Bill$, and the term spread $TERM$ as the difference between 10-year and 1-year constant maturity Treasury Rates. The listed dispersion parameter α indicates whether a less elaborate Poisson count regression ($\alpha = 1$) would have been sufficient.

Consistent with our conjecture, $SkewPref$ negatively predicts the total number of Closed-End Fund IPOs (Models (1) and (6)) and IPOs of Closed-End Funds specializing in US Equity (Models (2) and (7)). The

Table 2.4: Count Data Regressions of Closed-End Fund IPOs in different categories

	Skewness Preference based on Index Options					Skewness Preference based on Sentiment				
	(1) Total	(2) US Equity	(3) Int. Equity	(4) Mun. Bonds	(5) Tax. Bonds	(6) Total	(7) US Equity	(8) Int. Equity	(9) Mun. Bonds	(10) Tax. Bonds
$SkewPref_{t-1}$	-4.816	-14.854	-0.382	-10.032	-3.951	-0.360	-0.832	-0.585	-0.359	-0.019
	(-2.491)	(-2.087)	(-0.272)	(-3.163)	(-1.576)	(-2.870)	(-2.515)	(-1.956)	(-1.467)	(-0.121)
Liq_{t-1}	-1.737	-7.919	-0.745	-1.375	-0.562	-1.123	-5.029	-1.232	-1.008	-0.192
	(-1.185)	(-1.895)	(-0.172)	(-0.618)	(-0.432)	(-0.787)	(-1.732)	(-0.429)	(-0.395)	(-0.169)
$TERM_{t-1}$	31.003	20.720	5.895	72.535	19.652	64.345	5.204	38.008	109.198	48.648
	(2.242)	(0.474)	(0.119)	(2.220)	(1.374)	(8.220)	(0.286)	(2.488)	(6.260)	(5.477)
$T-Bill_{t-1}$	193.384	189.917	314.830	591.526	-71.350	290.877	43.510	450.574	572.744	112.249
	(2.204)	(0.689)	(0.846)	(3.050)	(-0.849)	(8.484)	(0.442)	(5.196)	(7.836)	(2.157)
VXO_{t-1}	0.031	-0.085	-0.115	0.144	-0.012	-0.012	-0.061	-0.080	0.035	-0.026
	(2.234)	(-1.271)	(-2.243)	(5.434)	(-0.825)	(-1.346)	(-1.880)	(-3.055)	(2.384)	(-2.362)
(Intercept)	-1.646	-2.980	-0.714	-7.326	-0.883	-0.734	-0.747	-1.615	-4.163	-0.904
	(-3.218)	(-2.134)	(-0.879)	(-6.862)	(-1.391)	(-2.709)	(-1.087)	(-2.954)	(-6.527)	(-2.552)
log(α)	-0.085	-2.940	1.510	1.409	-88.812	-0.260	-0.613	0.729	1.041	-0.519
Pseudo-R^2	0.031	0.119	0.063	0.091	0.062	0.055	0.054	0.085	0.067	0.036
N	236	236	236	236	236	354	354	354	354	354
log likelihood	-378.992	-78.493	-83.434	-192.251	-222.672	-661.154	-141.033	-201.055	-416.410	-402.806

Table 2.4 presents coefficient estimates of negative binomial regressions

$$\#CategoryIPOs_t = \alpha + \beta_1 SkewPref_{t-1} + \delta \chi_{t-1} + \epsilon_t,$$

where $\#CategoryIPOs_t$ is the number of IPOs in the respective fund category, $SkewPref$ is our skewness preference proxy. For Models (1)-(5), it is equal to the option-implied skewness preference $SkewPref_{Option-implied}$. In Models (6)-(10), it is the sentiment-based dummy variable $SkewPref_{Sentiment}$ that takes a value of one if the University of Michigan Consumer Sentiment Index is in the highest or lowest tercile and zero otherwise. χ_t is a vector of control variables. Control variables cover the CBOE S&P 100 Volatility Index VXO, the Pastor and Stambaugh (2003) Liq, the one-month Treasury Bill $T-Bill$, and the term spread $TERM$ as the difference between 10-year and 1-year constant maturity Treasury Rates. Each column represents the respective fund category. In Models (1)-(5), the sample period is March 1996 to October 2015, in Models (6)-(10), the sample covers July 1986 to December 2015. T-statistics in parentheses.

evidence on International Equity Closed-End Fund IPOs is rather weak. Although loadings on both skewness preference proxies are negative, only $SkewPref_{sentiment}$ is significant at the 10% level. For Closed-End Funds specializing in Municipal Bond, both loadings on skewness preference are negative, but only $SkewPref_{option-implied}$ is significantly negative ($p < 0.01$). We find no statistical significance in the Taxable Bond categories which is, as argued above, less surprising.

Overall, the regression analyses confirm our impression from our descriptive outlines (see, e.g, right panels in Figure 2.4), net of controls suggested in the literature (Cherkes et al., 2009): Closed-End Fund IPOs are less likely during times of high lottery demand.

2.4 Conclusion

The existing literature presents strong evidence that investors' lottery demand in the stock market and lottery characteristics provided by stocks have pricing effects distinct from neoclassical equilibrium models. Still, quantifying the magnitude of mispricing is tedious because the fundamental value is not observable.

In the present study, we show that the wedge between a Closed-End Fund's market valuation and its NAV is related to the relative differential in lottery characteristics of a Closed-End Fund's stock return versus the lottery characteristics of assets in the fund's portfolio. Hence, the discount on Closed-End Funds can be used to quantify the mispricing in the stock market which originates from lottery preferences. The intuition is straight forward. On average, diversified funds trade at a discount, or negative premium, because there is lottery demand in the stock market on average and diversification removes lottery-features. This premium increases, all else equal, if the Closed-End Fund's stock return distribution provides more lottery-like features and vice versa. Conversely, if assets in the fund portfolio improve their lottery-characteristics then the premium decreases, all else equal, including the fund return distribution.

Our proxies for measuring the degree of lottery character include the sum of squared portfolio weights (Herfindahl index, Goetzmann and Kumar, 2008), the previous month's maximum returns (Bali et al., 2011, 2017), the lottery index developed by Kumar et al. (2016), and a quantile-based estimate of the return distribution's skewness (e.g. Green and Hwang, 2012). Across this wide range of proxies for lottery features, we find our conjecture confirmed. Results for maximum returns and the lottery index are particularly strong and reliable, especially for the assets in funds' portfolios with t-statistics exceeding four in absolute terms.

As a new contribution, we estimate option-implied skewness preference from S&P500 index options and show that pricing impacts of lottery characteristics interact with skewness preference. Further, we derive from the existing literature that extreme sentiment, i.e. low and high sentiment, also proxies for high skewness preference. In fact, both our skewness preference proxies are consistent with each other. The interaction effects between both our skewness preference proxies and lottery characteristics on Closed-End Fund premia are especially relevant for stocks in Closed-End Funds' portfolios.

Finally, time-varying skewness preference (negatively) predicts the number of Closed-End Fund IPOs for various Closed-End Fund sectors. One exception is the Taxable Bond sector, which is arguably less affected by lottery demand and presumably caters to a different investor clientele. In particular, aggregate gambling preferences manifest in index option prices as well as in the Closed-End Fund market.

Chapter 3

What is the latent factor behind the idiosyncratic volatility puzzle?

This Chapter refers to the working paper:

Claußen, Arndt, Maik Dierkes and Sebastian Schroen (2019): 'What is the latent factor behind the idiosyncratic volatility puzzle?', Working Paper, Leibniz Universität Hannover.

Abstract

The negative relation between idiosyncratic risk and subsequent returns, commonly known as the idiosyncratic volatility puzzle, is attributable to latent systematic risk. High idiosyncratic volatility stocks underperform in subsequent months because they have high exposures to this risk factor. The latent factor exhibits all characteristics of a genuine risk factor, but is unrelated to fundamental economic state variables and largely unexplained by well-accepted risk factors. Our evidence points to noise trader risk induced by sentiment as a promising solution to the puzzle and is consistent with many well known characteristics of high idiosyncratic volatility stocks.

Keywords: Idiosyncratic volatility, latent risk factor, mispricing

JEL: G10, G12, G32.

3.1 Introduction

The negative relation between idiosyncratic volatility ($IVol$) and subsequent returns, introduced by Ang, Hodrick, Xing and Zhang (2006, 2009) as the idiosyncratic volatility puzzle, presents a long-standing challenge to theoretical and empirical asset pricing. Theoretically, the negative relationship is hard to reconcile with standard asset pricing theory because idiosyncratic risk either carries no risk premium at all, or a positive risk premium if investors are unable to diversify properly (Merton, 1987). Empirically, the largest fraction of the puzzle remains unexplained (Hou and Loh, 2016) and the literature is tied between risk-based explanations (Chen and Petkova, 2012; Barinov, 2013) and mispricing (Stambaugh et al., 2015). A unified solution is yet to be found.

This paper shows that the $IVol$ puzzle originates from common risk in residuals of the investor's factor model. We propose an active portfolio which tracks this risk, explains the co-movement of high-$IVol$ stocks and fully alleviates the puzzle. $IVol$ stocks are exposed to a common risk factor – most likely attributable to noise trader risk – and earn low future returns due to this exposure. This exposure fully explains the $IVol$ puzzle in Fama and MacBeth (1973) regressions, the Hou and Loh (2016) decomposition and portfolio sorts.

We adapt the theoretical framework of MacKinlay (1995) and MacKinlay and Pastor (2000) to the empirical relationship between $IVol$ and stock returns to form an active portfolio based on factor model residuals. This active portfolio, referred to as OP, satisfies the orthogonality condition of the optimal orthogonal portfolio of MacKinlay (1995). Once we include it into the Fama and French (1993) three or Fama and French (2018) six factor model, the negative Fama and MacBeth (1973) coefficient on $IVol$ becomes insignificant and the mispricing of $IVol$ sorted portfolios reduces considerably. The extended factor model explains the alpha of 25 portfolios sorted by Size and $IVol$. Finally, we provide a full Hou and Loh (2016) decomposition of the Fama and MacBeth (1973) coefficient on $IVol$ in 200 $IVol$ portfolios. Almost 50% of the $IVol$ puzzle is attributable to the latent risk in the Fama and French (1993) three factor model. Taken together, the stock characteristics firm size, short-term reversal, illiquidity and mispricing account for the remainder of the puzzle.

A mimicking factor portfolio (FOP) which tracks the latent risk factor behind the $IVol$ puzzle accounts for the negative alpha of high-$IVol$ deciles in portfolio sorts. The trade-off between $IVol$ and alphas even turns positive if we combine FOP with the CAPM. FOP is significantly related to the covariance matrix of stock returns and earns a significant risk premium with an annualized Sharpe ratio below the upper bound of 0.6. Thus the latent factor behind the $IVol$ puzzle satisfies the conditions of a genuine risk factor according to the risk factor protocol of Pukthuanthong et al. (2019).

Discriminating tests between theoretical explanations based on arbitrage constraints and behavioral explanations for the $IVol$ puzzle motivated by Asness et al. (2019) favor behavioral explanations. FOP is significantly negatively related to the Baker and Wurgler (2006) Investor Sentiment Index and US equity mutual fund flows, while evidence with respect to leverage constraints is inconclusive. Focusing on behavioral explanations for the $IVol$ puzzle suggests that FOP traces back to systematic noise trader risk. FOP shares the explanatory power of investor sentiment with respect to market-wide stock price anomalies (Stambaugh et al., 2012; Jacobs, 2015). Anomalies earn higher alphas when arbitrageurs are exposed noise trader risk due to correlated trading of sentiment traders. Furthermore, in line with the theoretical model of De Long et al. (1990), noise trader risk as proxied by FOP is positively associated with temporary increases in aggregate volatility. Our findings are consistent with the theoretical model of Kozak et al. (2018) who show that time-varying sentiment imposes systematic state-variable risk. The positive contribution of noise trader risk to aggregate variance explains the findings of Chen and Petkova (2012) and Barinov (2013) who directly focus on volatility risk.

Our latent factor and its mimicking portfolio counterpart are more than just $IVol$ in disguise and we provide three robustness checks to illustrate this conclusion. First, the explanatory power of the latent factor in the Fama and MacBeth (1973) regressions is insensitive to the choice of assets which generate residuals. Including the weighted residuals of 25 portfolios sorted by Size and Book-to-Market or Size and Momentum explains the negative Fama and MacBeth (1973) coefficient on $IVol$ in 25 Size-$IVol$ portfolios just as well as OP formed on Size-$IVol$ residuals. Second, comparing the mimicking portfolio for the latent factor with two alternative factor candidates directly formed on $IVol$ reveals that its explanatory power in $IVol$ sorts is not mechanically driven by $IVol$. While the inclusion of FOP to the Fama and French (1993) model subsumes both alternative $IVol$ factor candidates in factor spanning tests (Barillas and Shanken, 2017, 2018), this does not hold in the opposite direction. Third, a discriminating test confirms that our adaption to the weight vector in the active portfolio of MacKinlay and Pastor (2000) pins down a systematic component in factor model residuals which is economically different from $IVol$. $IVol$ is a symptom of FOP which traces back to priced noise trader risk. In particular, FOP is not a statistical transformation of $IVol$ itself. Two additional robustness checks verify that our baseline results are robust to the choice of the residual generating factor model and a tradable version of the mimicking portfolio performs equally well in explaining the $IVol$ puzzle.

Our study contributes to two streams in the literature. First, the literature on the optimal orthogonal portfolio shows that mispricing due to a missing factor in asset pricing models induces a systematic component in the covariance matrix of factor model residuals (MacKinlay, 1995). MacKinlay and Pastor (2000) impose a strong form assumption on the residual covariance matrix to identify the exposure of the systematic component and improve portfolio selection. Under the same assumption, Asgharian (2011) presents a conditional version of the optimal orthogonal portfolio which allows an estimation of the exposure for a considerable number of portfolios, e.g. 48 industry portfolios. We relax the strong form assumption and adapt the active portfolio proposed by MacKinlay and Pastor (2000) to the nonlinear relationship between $IVol$ and alphas. Residual variance is positively related to squared alpha, not alpha and an active portfolio in the spirit of MacKinlay and Pastor (2000) who propose a weighting vector proportional to alpha is misspecified. Once we account for this empirical feature of the $IVol$-return relation, the active portfolio consistently captures $IVol$. Our approach facilitates the identification of risk-based deviations from the investor's factor model without conjecturing a characteristic which is potentially related to alphas.

Second, the negative relation between $IVol$ and expected returns has been explored extensively. The literature proposes a large number of mechanisms and potentially unaccounted factors, for example: Lottery demand (e.g. Bali et al., 2011; Han and Kumar, 2013; Boyer et al., 2010), liquidity risk (e.g. Bali and Cakici, 2008), credit risk (e.g. Duarte et al., 2014; Avramov et al., 2013), mispricing due to limits to arbitrage (e.g. Stambaugh et al., 2015; Stambaugh and Yuan, 2017) or variance risk (Ang et al., 2006; Chen and Petkova, 2012; Barinov, 2013).[1] Chen and Petkova (2012) describe a link between the optimal orthogonal portfolio and the $IVol$ puzzle and conjecture that average variance and average correlation are the two missing components of the Fama and French (1993) three factor model. We exploit this link and derive an active portfolio which precisely pins down the latent systematic component and is consistent with the theoretical framework of Chen and Petkova (2012). Our evidence is consistent with arbitrage risk due to noise trader risk (De Long et al., 1990; Stambaugh et al., 2015). This imposes genuine risk which is likely to explain the impact of sentiment on the risk-return tradeoff (Antoniou et al., 2016; Shen et al., 2017).

[1] For a detailed analysis of potential explanations we refer to Hou and Loh (2016).

3.2 Dissecting idiosyncratic volatility

3.2.1 The optimal orthogonal portfolio as a latent risk factor

We follow the framework of MacKinlay (1995) and MacKinlay and Pastor (2000) to examine the role of a latent systematic risk factor in the negative relationship between $IVol$ and subsequent stock returns. Let $r_{i,t}$ denote the excess return of asset $i \in \{1,...,N\}$ in period t and $\boldsymbol{\zeta_t} \in \mathbb{R}^K$ represent realizations of K observable risk factors. Assuming a linear relationship between asset returns and the risk factor returns, the return generating process is

$$r_{i,t} = \alpha_i + \boldsymbol{\beta_i}'\boldsymbol{\zeta_t} + \epsilon_{i,t}, \qquad (3.1)$$

$$\mathbb{E}(\boldsymbol{\epsilon_t}) = 0, \qquad \mathbb{E}(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}') = \Sigma \qquad \text{and} \qquad \text{cov}(\boldsymbol{\epsilon_t}, \boldsymbol{\zeta_t}) = 0,$$

where $\boldsymbol{\beta_i} \in \mathbb{R}^K$ are the sensitivities of asset i with respect to the K factors, $\epsilon_{i,t}$ is the error in each time period, and α_i denotes mispricing. An exact linear relation between the asset returns and the risk factor returns implies an intercept α_i of zero. An intercept which is significantly different from zero indicates mispricing.

In the presence of a missing factor, MacKinlay and Pastor (2000) show that the covariance matrix Σ contains information about the missing factor which drives α_i. This relationship can be developed using the optimal orthogonal portfolio defined as "*the unique portfolio given $\overline{N}$ assets that can be combined with the factor portfolios to form the tangency portfolio and is orthogonal to the factor portfolios*" (MacKinlay, 1995, p. 8). An advantage of the optimal orthogonal portfolio is that, by definition, it leaves the factor sensitivities $\boldsymbol{\beta_i}$ unaffected once the missing variable is included. We denote the return on the optimal orthogonal portfolio (OP) at time t with $r_{OP,t}$ which governs the asset return with sensitivity β_{OP} and its first two moments are $\mathbb{E}(r_{OP,t}) = \mu_{OP}$ and $\text{var}(r_{OP,t}) = \sigma^2_{OP}$. Per definition, it holds $\text{cov}(\boldsymbol{\zeta_t}, r_{OP,t}) = 0$. Replacing α_i in Equation (3.1) with the return of the optimal orthogonal portfolio yields

$$r_{i,t} = \beta_{OP,i} r_{OP,t} + \boldsymbol{\beta_i}'\boldsymbol{\zeta_t} + \nu_{i,t}, \tag{3.2}$$

$$\mathbb{E}(\boldsymbol{\nu_t}) = 0, \qquad \mathbb{E}(\boldsymbol{\nu_t}\boldsymbol{\nu_t}') = \Phi, \qquad \text{and} \qquad \text{cov}(\boldsymbol{\nu_t}, \boldsymbol{\zeta_t}) = \text{cov}(\boldsymbol{\nu_t}, r_{OP}) = 0.$$

MacKinlay and Pastor (2000) employ the assumption that the covariance matrix Φ is proportional to the identity matrix. We relax this *strong form* assumption and set Φ as a diagonal matrix with asset-specific error term variances $\sigma^2_{\nu,i} := \text{var}(\nu_{i,t})$. Equaling the expectation of Equation (3.1) and Equation (3.2) leads to

$$\alpha_i = \beta_{OP,i} E(r_{OP}) = \beta_{OP,i}\mu_{op}. \tag{3.3}$$

Given $\text{cov}(\boldsymbol{\nu_t}, r_{OP}) = 0$, we express the variance of the error term in Equation (3.1) in terms of two components

$$\sigma^2_{\epsilon,i} := \text{var}(\epsilon_{i,t}) = \beta^2_{OP,i}\sigma^2_{OP} + \sigma^2_{\nu,i}. \tag{3.4}$$

Equation (3.4) illustrates that $\sigma^2_{\epsilon,i}$, i.e. the idiosyncratic variance of the K factor model known to the investor, consists of two components. The first component $\beta^2_{OP,i}\sigma^2_{OP}$ reflects systematic deviations from the return generating process due to the latent factor r_{OP}. This component prevents the diversification of idiosyncratic risk to zero when forming a portfolio (MacKinlay, 1995). The second component, $\sigma^2_{\nu,i}$, is *truly* non-systematic.

3.2.2 An active portfolio to track IVol

Ang et al. (2006, 2009) employ the square root of idiosyncratic variance $\sigma^2_{\epsilon,i}$ in Equation (3.4) from the Fama and French (1993) three factor model (FF3) as a measure for idiosyncratic risk. Empirically, they find a negative relationship between $IVol$ and subsequent returns. This finding is known as the $IVol$ puzzle because theoretically, idiosyncratic risk carries no risk premium at all according to standard asset pricing theory, or a positive risk premium if investors cannot diversify properly (Merton, 1987).

The theoretical framework above proposes an explanation for this puzzle. Equation (3.4) indicates that the empirical measure for $IVol$ contains a systematic component which is attributable to a latent risk factor in the investor's factor model. In this case, the negative relation between $IVol$ and subsequent returns is not a puzzle, but the compensation for latent risk as pointed out by Chen and Petkova (2012).[2]

An empirical analysis of the framework above requires a model-based or empirical conjecture because OP is unobservable. To approximate OP empirically, MacKinlay and Pastor (2000) propose an active portfolio which assumes long positions in stocks with positive alphas and short positions in stocks with negative alphas. To guarantee that the active portfolio is zero-beta with respect to the factor portfolios of the initial model,

[2] Stambaugh et al. (2015) point out that the negative alphas of high $IVol$ stocks imply a negative risk premium for the latent risk factor.

the factor portfolios are included with corresponding weights. Including this active portfolio into the initial factor model in Equation (3.1) reduces the mispricing of the model.

Our main objective is the formation of an active portfolio which explains the latent risk behind the negative relation between $IVol$ and subsequent returns. If $IVol$ is a proxy for the sensitivity to a latent risk factor, an active portfolio in the spirit of MacKinlay and Pastor (2000) explains the $IVol$ puzzle if and only if $IVol$ and alphas are proportional. Combining Equation (3.3) and Equation (3.4) reveals that

$$\sigma_{\epsilon,i}^2 = \alpha_i^2 \frac{\sigma_{OP}^2}{\mu_{OP}^2} + \sigma_{\nu,i}^2. \tag{3.5}$$

Thus, idiosyncratic variance is linearly related to squared alpha, not alpha. This relationship, however, also holds for squared alpha and idiosyncratic volatility because we analyze a small range of values and the root function is approximately linear in this case.[3] We thus propose an active portfolio which addresses this finding and set the weight of asset i proportional to its squared alpha according to

$$w_i = \frac{\alpha_i^2}{\sum_{i=1}^N \alpha_i^2}. \tag{3.6}$$

Intuitively, this weighting scheme overweights mispriced assets, both overpriced and underpriced. As suggested from Equation (3.5), the weight is independent of the sign of alpha. Robustness checks in Section 3.7.3 illustrate that portfolio weights proportional to squared alpha correctly identify mispriced stocks instead of spuriously capturing $IVol$ itself.

[3] Results are almost identical if we choose weights proportional to absolute alphas.

The denominator guarantees that the portfolio weights sum up to one (MacKinlay and Pastor, 2000).

Instead of the zero-beta weights in the factor portfolios as proposed by MacKinlay and Pastor (2000), we directly form OP on factor model residuals. By construction, any cross-sectional linear combination of $\epsilon_{i,t}$ satisfies the orthogonality condition for OP. We multiply the weight in Equation (3.6) with the vector of residuals ϵ_t to approximate OP empirically as

$$OP_t := \mathbf{w}'\epsilon_t. \tag{3.7}$$

Our approach allows us to test the latent factor explanation for the $IVol$ puzzle without a conjecture about potential factors and implies three predictions. First, an inclusion of the active portfolio OP_t reduces the commonality in the residuals of the factor model in Equation (3.1) and thus the factor structure in the covariance matrix Σ. Second, the sensitivity of asset i to this active portfolio $\beta_{OP,i}$ explains the low alpha of high-$IVol$ stocks. Third, OP_t brings down the mispricing compared to the initial model.

3.3 Data and methodology

Our stock sample covers the CRSP common stock universe (share code 10 and 11) from July 1963 to December 2016. We obtain returns, market capitalizations, trading volumes and prices on a daily and monthly basis from CRSP. Returns are adjusted for delistings as motivated by Shumway (1997). We apply a five dollar price screen to exclude penny stocks. Chen et al. (2012) show that the $IVol$ puzzle is particularly robust under this price filter.

The empirical measure for $IVol$ is model-dependent. Despite the rapid growth in the number of factor models since the seminal paper of Ang et al. (2006), their proxy for $IVol$ as the standard deviation of residuals from a daily FF3 model regression is still the most adopted measure. For each asset i, we estimate monthly within-month time series regressions with at least 15 valid daily observations and refer to the standard deviation of these residuals as $IVol^{1M}_{FF3}$.[4]

We use the residuals of the FF3 model to compute OP_t. To compute the full sample counterpart of $\beta_{OP,i}$, we estimate the FF3 model in within-month regressions using daily data and compute alphas as well as residuals. Next, we use squared monthly alphas as weights to compute OP_t according to Equation (3.7). In the last step, we re-evaluate the FF3 model, but include OP_t as an additional factor to estimate $\beta_{OP,i}$ in an additional full sample regression according to Equation (3.2). By construction, this weighted average is orthogonal to the $FF3$ factors. In the rolling window analysis, we estimate the FF3 model over five years using daily data up to month t. Again we compute alphas and residuals and calculate OP_t according to Equation (3.7). We finally re-estimate the model with the additional factor to estimate $\beta_{OP,i}$ and repeat this step on a monthly basis.

Returns on the risk factors of the models proposed by Fama and French (1993), Carhart (1997) and Fama and French (2018) as well as several test

[4] In the robustness checks in Section 3.7.4 we adjust this model selection and also compute $IVol$ from the more recent Fama and French (2018) six factor model (FF6).

portfolios are from Kenneth French's website. We use 25 double sorted portfolios on Size and Operating Profit (5x5), Size and Beta (5x5), Size and Book-to-Market (5x5), as well as 32 three-way sorted portfolios on Size, Book-to-Market and Investment (2x4x4) as test assets. The lottery demand factor of Bali et al. (2017) is downloaded from Turan Bali's website. The Pastor and Stambaugh (2003) liquidity factor, the Stambaugh et al. (2015) mispricing measure and the four Stambaugh and Yuan (2017) mispricing factors are from Robert F. Stambaugh's website. Expected idiosyncratic skewness of Boyer et al. (2010) is from the website of Brian Boyer. The monthly Chicago Fed National Activity Index $CFNAI$ is from the Federal Reserve Bank of Chicago and the daily Aruoba et al. (2009) (ADS) business conditions index is from the Federal Reserve Bank of Philadelphia. Mutual fund flows of US equity mutual funds are from Morningstar, margin debt of NYSE customers in relation to NYSE market capitalization is from Datastream and the Ted Spread is from the Federal Reserve Bank of St. Louis. The Baker and Wurgler (2006) Investor Sentiment index is from the website of Jeffrey Wurgler and the Baker et al. (2016) economic policy uncertainty index is from policyuncertainty.com. The Chicago Board of Option Exchange (CBOE) S&P 100 Volatility Index VXO is from the CBOE's website. We follow Barinov (2018) and use this longer time series to approximate the VIX. Appendix 3.A.1 describes the control variables based on this data and their respective estimation. We gratefully acknowledge the provision of risk factors and economic data by fellow colleagues.

3.4 Absolving IVol of latent systematic risk

3.4.1 The IVol puzzle and preliminary evidence

Table 3.1 illustrates that the $IVol$ puzzle is present and not explained by the most prominent factor models in our sample period. Each month, we sort stocks into deciles conditional on $IVol_{FF3}^{1M}$, hold the portfolio for one month and record excess returns as well as alphas of different factor models. The right column presents the return and alpha of a portfolio which consists of a long position in the highest decile of $IVol_{FF3}^{1M}$ and a short position in the bottom decile of $IVol_{FF3}^{1M}$. The factor models cover a CAPM single factor model, the Fama and French (1993) model FF3, the Carhart (1997) four factor model (CAR), the Pastor and Stambaugh (2003) model (PS) which adds a liquidity factor to the FF3 model, as well as the the Fama and French (2015) five factor FF5 model. We furthermore present alphas from more recent factor models, i.e. the Stambaugh and Yuan (2017) four factor mispricing model M4, the Hou et al. (2015) q-factor model and the six factor model proposed by Fama and French (2018) (FF6) which adds the Momentum factor to the FF5 model.[5] Returns and alphas of the decile portfolios are presented in percent per month and we report equal-weighted sorts in Panel A as well as value-weighted sorts in Panel B. We report t-statistics based on Newey and West (1987) standard errors with six lags in parentheses.

Excess returns and alphas are statistically significant and the cor-

[5]Technically, Fama and French (2018) replace operating profitability with cash profitability, but this factor is not available online. We thus use operating profitability instead, but refer to this model as FF6.

Table 3.1: Revisiting the idiosyncratic volatility puzzle.

Panel A: Equal-weighted sorts

	Low $IVol_{FF3}^{1M}$	2	3	4	5	6	7	8	9	High $IVol_{FF3}^{1M}$	Diff
Excess Return	0.6576	0.8262	0.8500	0.9530	0.9424	0.9917	0.8169	0.7515	0.4834	-0.1769	-0.8345
	(4.17)	(4.55)	(4.25)	(4.44)	(4.14)	(4.03)	(3.08)	(2.62)	(1.57)	(-0.53)	(-3.38)
CAPM Alpha	0.3501	0.4104	0.3848	0.4429	0.3968	0.4073	0.1903	0.0841	-0.2309	-0.8892	-1.2394
	(3.47)	(4.17)	(3.72)	(4.08)	(3.57)	(3.38)	(1.46)	(0.61)	(-1.46)	(-4.70)	(-6.03)
FF3 Alpha	0.1733	0.2053	0.1787	0.2306	0.1923	0.2205	0.0228	-0.0431	-0.3299	-0.9588	-1.1320
	(2.24)	(3.24)	(2.85)	(4.13)	(3.76)	(4.35)	(0.48)	(-0.86)	(-5.45)	(-10.06)	(-7.88)
PS Alpha	0.1807	0.2111	0.1832	0.2254	0.1784	0.2232	0.0134	-0.0448	-0.3219	-0.9729	-1.1536
	(2.20)	(3.07)	(2.67)	(3.63)	(3.12)	(4.00)	(0.26)	(-0.85)	(-4.90)	(-9.42)	(-7.63)
C4F Alpha	0.1614	0.2169	0.1975	0.2655	0.2228	0.2539	0.0666	-0.0072	-0.2534	-0.8645	-1.0258
	(2.21)	(3.88)	(3.59)	(5.14)	(4.74)	(5.54)	(1.48)	(-0.15)	(-3.93)	(-8.50)	(-7.38)
FF5 Alpha	0.0516	0.0737	0.0550	0.1271	0.1089	0.1784	0.0244	0.0371	-0.1598	-0.6602	-0.7118
	(0.73)	(1.36)	(0.96)	(2.44)	(2.07)	(3.06)	(0.48)	(0.69)	(-2.39)	(-7.71)	(-6.64)
M4 Alpha	0.0132	0.0691	0.0581	0.1526	0.1430	0.2165	0.0641	0.0734	-0.0902	-0.6044	-0.6175
	(0.17)	(1.03)	(0.88)	(2.62)	(2.66)	(4.07)	(1.20)	(1.20)	(-1.13)	(-4.69)	(-3.92)
q-factor Alpha	0.0830	0.0848	0.0545	0.1366	0.1156	0.2004	0.0728	0.0942	-0.0422	-0.5418	-0.6248
	(0.84)	(0.91)	(0.58)	(1.61)	(1.44)	(2.32)	(1.05)	(1.22)	(-0.46)	(-4.54)	(-4.01)
FF6 Alpha	0.0561	0.0985	0.0849	0.1677	0.1435	0.2105	0.0608	0.0578	-0.1158	-0.6158	-0.6719
	(0.81)	(2.02)	(1.66)	(3.70)	(3.17)	(4.34)	(1.38)	(1.17)	(-1.87)	(-6.86)	(-6.11)

Panel B: Value-weighted sorts

	Low $IVol_{FF3}^{1M}$	2	3	4	5	6	7	8	9	High $IVol_{FF3}^{1M}$	Diff
Excess Return	0.5127	0.5404	0.5079	0.6117	0.6287	0.5956	0.6848	0.3000	0.2689	-0.3761	-0.8887
	(3.36)	(3.30)	(2.80)	(3.16)	(2.96)	(2.44)	(2.57)	(1.01)	(0.83)	(-1.07)	(-3.06)
CAPM Alpha	0.1522	0.1048	0.0164	0.0785	0.0508	-0.0345	0.0003	-0.4145	-0.5123	-1.1550	-1.3072
	(1.92)	(1.65)	(0.38)	(1.40)	(0.81)	(-0.42)	(0.00)	(-3.25)	(-3.20)	(-5.85)	(-5.17)
FF3 Alpha	0.0875	0.0591	-0.0120	0.0371	0.0268	-0.0395	0.0079	-0.3748	-0.4334	-1.1156	-1.2031
	(1.44)	(1.08)	(-0.29)	(0.66)	(0.44)	(-0.53)	(0.10)	(-3.87)	(-3.62)	(-6.89)	(-6.24)
PS Alpha	0.0948	0.0658	-0.0049	0.0412	0.0007	-0.0567	0.0129	-0.4171	-0.4633	-1.1755	-1.2703
	(1.41)	(1.08)	(-0.11)	(0.67)	(0.01)	(-0.71)	(0.15)	(-3.82)	(-3.49)	(-6.72)	(-6.08)
CAR Alpha	0.0445	0.0373	-0.0386	0.0308	0.0683	-0.0130	0.0469	-0.3498	-0.3510	-0.9621	-1.0065
	(0.73)	(0.64)	(-0.76)	(0.57)	(1.08)	(-0.17)	(0.60)	(-3.77)	(-3.02)	(-5.96)	(-5.25)
FF5 Alpha	-0.0374	-0.0337	-0.0907	-0.0169	0.0242	0.0470	0.1240	-0.1975	-0.1619	-0.6678	-0.6304
	(-0.65)	(-0.60)	(-1.83)	(-0.27)	(0.39)	(0.64)	(1.57)	(-2.25)	(-1.64)	(-4.85)	(-3.98)
M4 Alpha	-0.1107	-0.0666	-0.0975	0.0018	0.1055	0.1174	0.2089	-0.1133	-0.0458	-0.4972	-0.3864
	(-1.53)	(-1.03)	(-1.33)	(0.02)	(1.65)	(1.45)	(2.41)	(-1.17)	(-0.37)	(-2.76)	(-1.77)
q-factor Alpha	-0.0846	-0.0339	-0.1346	-0.0479	0.0109	0.0453	0.1964	-0.1620	-0.0657	-0.5686	-0.4840
	(-1.07)	(-0.50)	(-2.06)	(-0.67)	(0.14)	(0.50)	(2.30)	(-1.62)	(-0.50)	(-3.64)	(-2.51)
FF6 Alpha	-0.0584	-0.0412	-0.1043	-0.0165	0.0587	0.0578	0.1429	-0.1976	-0.1250	-0.5923	-0.5339
	(-0.95)	(-0.71)	(-1.83)	(-0.26)	(0.93)	(0.77)	(1.85)	(-2.24)	(-1.22)	(-4.21)	(-3.26)

Table 3.1 presents univariate portfolio sorts on Fama and French (1993) three factor model (FF3) idiosyncratic volatility $IVol_{FF3}^{1M}$ over the past month. The last column presents the difference in excess returns and respective alphas between stocks in the highest and the lowest $IVol_{FF3}^{1M}$ decile. Panel A presents sorts with equal weights, in Panel B we weight returns by market capitalization. We hold each portfolio for one month and record the monthly returns and different factor alphas. Factor alphas cover the following factor models: CAPM is a one factor model, FF3 is the Fama and French (1993) three factor model, PS is the FF3 model extended by the Pastor and Stambaugh (2003) liquidity factor, CAR is the Carhart (1997) four factor model and FF5 is the Fama and French (2015) five factor model. Furthermore, M4 is the Stambaugh and Yuan (2017) four factor mispricing model, q-factor is the four factor model of Hou et al. (2015) and and FF6 is the Fama and French (2018) six factor model. Returns and alphas are reported in % per month. Newey and West (1987) adjusted t-statistics with six lags in parentheses. The sample period for excess returns and alphas of the models CAPM, FF3, CAR, FF5, FF6 and M4 is July 1963 to December 2016. The sample period for the PS model is January 1968 to December 2015. The q-factor alpha covers January 1967 to December 2015.

responding t-statistics exceed minus three in Panel A. Regarding the value-weighted sorts in Panel B, we find two exceptions of this finding. The alphas of the M4 model and the q-factor model amount to -39 basis points (bps) and -48 bp per month with t-statistics of -1.77 and -2.51, respectively. In both models, however, high-$IVol_{FF3}^{1M}$ have negative alphas which are significant at any conventional level and the factor models leave highly significant alphas in equal-weighted sorts. The $IVol$ puzzle is well and alive, irrespectively of the factor model and the choice of decile portfolio weights. The negative alphas are mainly driven by the highest $IVol$ decile, i.e. the short leg of the difference portfolios.

The $IVol_{FF3}^{1M}$ decile portfolios are useful to illustrate the first prediction in Section 3.2.2. The latent factor in the investors model in Equation (3.1) prevents the diversification of idiosyncratic risk to zero. The covariance matrix Σ of the FF3 model residuals thus exhibits a factor structure. We address this prediction in Figure 3.1 which presents correlations of factor model residuals from the daily $IVol_{FF3}^{1M}$ decile portfolio returns. Each month, we estimate the FF3 model for daily decile portfolio returns and compute residuals on the portfolio level for which we plot the pairwise correlations between $IVol_{FF3}^{1M}$ deciles. The bars right from the main diagonal plot correlations of FF3 model residuals. Next, we use the FF3 model residuals, compute OP and add the latent factor to the initial model to calculate residuals for which we compute correlations. We plot correlations of the extended model left from the main diagonal.

In case of the FF3 model, residuals in high $IVol_{FF3}^{1M}$ deciles comove and the pairwise correlation between the highest two deciles amounts

Figure 3.1: Barplots of residual correlations.

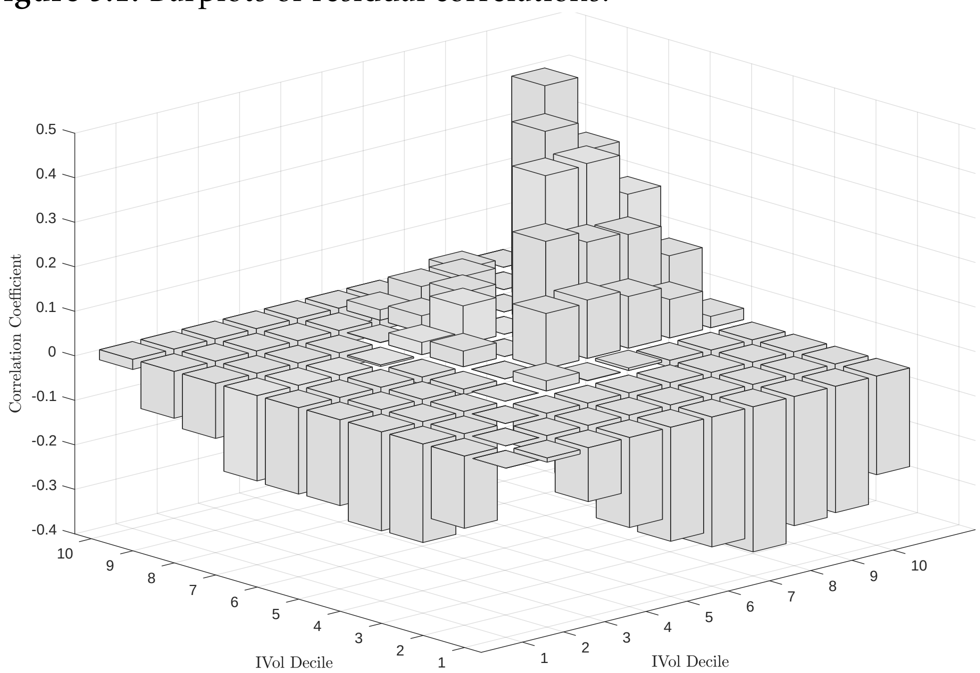


Figure 3.1 presents the correlation of factor model residuals before and after including OP into the Fama and French (1993) three factor model (FF3). The figure plots time series correlations of factor model residuals from 10 value-weighted portfolios sorted by idiosyncratic volatility $IVol_{FF3}^{1M}$ in ascending order from decile 1 to decile 10. We estimate the FF3 model as well as the extended model within each month on daily decile portfolio returns and compute residuals for which we estimate correlations. Right from the main diagonal, we plot correlations of the FF3 model residuals $\epsilon_{i,t}$ which are used to compute the standard measure $IVol_{FF3}^{1M}$. Left from the main diagonal, we extend the model by the latent factor OP, re-estimate the model and compute factor model residuals for which we plot the correlations. The sample period is August 1963 to December 2016.

to 0.43. Additionally, the correlation between high- and low-$IVol_{FF3}^{1M}$ deciles is negative. The highest and the lowest decile exhibit a correlation coefficient of -0.22. Extending the model by OP reduces the correlation in both examples. Now, the pairwise correlation between the highest two deciles is 0.02 and the negative correlation between the highest and the lowest decile reduces to -0.02. Both correlations are still statistically significant at the five percent level, but lower in comparison to the FF3

Figure 3.2: Scatter plot of residual standard deviations.

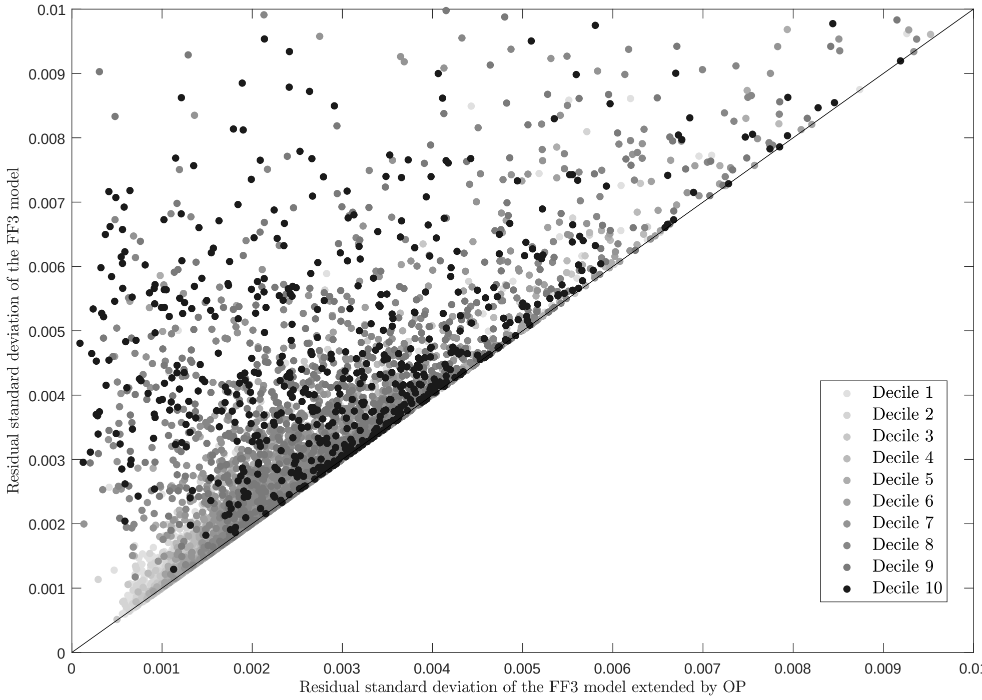


Figure 3.2 presents a scatter plot of residual standard deviations before and after including *OP* into the Fama and French (1993) three factor model FF3. Each month, we sort stocks in decile portfolios based on FF3 idiosyncratic volatility $IVol_{FF3}^{1M}$ and compute value-weighted returns on a daily basis. We estimate the FF3 model as well as the extended model including *OP* within each month on daily decile portfolio returns and compute residuals. Given these residuals, we estimate residual standard deviations of the FF3 model and the FF3 model extended by *OP*. The ordinate depicts the standard deviation of FF3 factor model residuals and the abscissa depicts the residual standard deviation of the extended model. The color of the markers indicates the decile portfolio. The sample period is August 1963 to December 2016.

model. The cluster in positive correlations between high $IVol_{FF3}^{1M}$ reduces considerably after including *OP* into the FF3 model.

Figure 3.2 extends the analysis to the standard deviations of the factor model residuals from both models, i.e. the FF3 model as well as the extended model including *OP*. The ordinate depicts the standard deviation of FF3 factor model residuals and the abscissa depicts the residual standard deviation of the extended model. The color of the markers indicates the decile portfolio.

The difference between the residual standard deviations depends on the $IVol$ decile. For low deciles, including OP into the FF3 model leaves the residual standard deviations unaffected. Both standard deviation measures are correlated with correlation coefficients well above 0.90 for the lowest five $IVol_{FF3}^{1M}$ deciles. From decile six to nine, the correlation monotonically decreases from 0.86 to 0.74 and then plunges down to 0.45 in the tenth decile. The additional explanatory power of OP is thus particularly large for high $IVol_{FF3}^{1M}$ stocks which are the prime cause of the $IVol$ puzzle.[6]

Both findings suggest that an omitted factor in the FF3 model is a likely explanation for the commonality in $IVol$ found by Herskovic et al. (2016). The FF3 model residuals and residual standard deviations of $IVol$ decile portfolios convey a factor structure which is largely attributable to the latent factor OP. The covariance matrix of the extended model is closer to diagonal, as predicted in Section 3.2.2.

3.4.2 Can a latent factor explain the IVol puzzle?

We move on to the second prediction in Section 3.2.2. If the $IVol$ puzzle is the result of a latent factor in the FF3 model, the inclusion of an asset's sensitivity to the latent factor β_{OP} explains the negative relation between $IVol$ and subsequent stock returns.

[6] In an untabulated analysis we evaluate the risk premium for the residual standard deviation of the extended model in Fama and MacBeth (1973) regressions for the $IVol_{FF3}^{1M}$ decile portfolios. The coefficient is insignificant, while $IVol_{FF3}^{1M}$ itself is highly significant.

The weight of asset i in the active portfolio in Equation (3.7) requires an estimate for the FF3 alpha which is subject to measurement error. To reduce the measurement error, we test risk-based $IVol$ explanations on the portfolio level. Each month, we sort stocks into five portfolios conditional on monthly market capitalizations (Size) and then further sort stocks by $IVol_{FF3}^{1M}$ into five portfolios. We hold the portfolios one month and calculate value-weighted excess returns. We follow Bali and Cakici (2008) and Chen and Petkova (2012) and use NYSE breakpoints for the market capitalizations.[7] Given the daily return on these 25 Size-$IVol_{FF3}^{1M}$ portfolios, we estimate the FF3 model on the portfolio level, use residuals to compute OP according to Equation (3.7) and include OP into the model to estimate β_{OP}. We estimate β_{OP} and all risk factor betas in 5-year rolling windows. Given the portfolio-level estimates for $IVol_{FF3}^{1M}$ and β_{OP} for each month t and portfolio p we perform the cross-sectional Fama and MacBeth (1973) regressions

$$r_{p,t+1} = \gamma_{0,t} + \gamma_{1,t} IVol_{p,t}^{1M} + \gamma_{2,t} \beta_{p,t}^{OP} + \mathbf{\Gamma}_t' \mathbf{B}_{p,t} + \epsilon_{p,t}, \tag{3.8}$$

where $r_{p,t+1}$ is the excess return of portfolio p one month ahead and $\mathbf{B}_{p,t}$ is a vector of risk factor betas.[8] We control for betas with respect to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyn-

[7] The findings are almost identical if we use the 25 Size-residual variance portfolios of Fama and French (2016) who suggest conditional NYSE breakpoints for residual variance as well.

[8] In this Section, we focus on risk-based explanations and thus include covariances rather than characteristics. We evaluate characteristics as alternative explanations in the Hou and Loh (2016) analysis in Section 3.4.3.

cratic volatility (market variance) as proposed by Herskovic et al. (2016), $FMax$ is the Bali et al. (2017) lottery demand factor and $FVIX$ is the Barinov (2018) aggregate variance factor.[9] Appendix 3.A.1 describes the estimation of factor betas. Table 3.2 presents the second stage coefficients

Table 3.2: Fama and MacBeth (1973) regressions on 25 Size-$IVol$ portfolios.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Intercept	0.3717	0.3837	0.5735	0.5602	0.1964	0.1943	0.2572	0.3972	0.3535	0.1694	0.1043	0.1995	0.1751	0.1097
	(4.70)	(4.97)	(3.01)	(3.81)	(2.90)	(2.77)	(4.42)	(4.91)	(4.00)	(2.14)	(1.68)	(2.53)	(2.16)	(1.40)
$IVol_{FF3}^{1M}$		-57.355		21.5872	-5.1652		-63.770	-115.05	-52.272	-39.164	-26.335	-34.411	-7.0241	-5.0191
		(-3.20)		(0.48)	(-0.30)		(-3.80)	(-4.53)	(-2.03)	(-2.16)	(-1.65)	(-1.34)	(-0.28)	(-0.28)
β_{OP}			-0.1966	-0.1905	-0.2755	-0.2606					-0.2889	-0.2781	-0.2507	-0.1491
			(-1.92)	(-2.48)	(-4.81)	(-4.79)					(-5.61)	(-5.21)	(-4.24)	(-2.11)
β_{Mkt}	0.1520	0.2613			0.3643	0.3573	0.3782	0.1697	0.2878	0.5642	0.4700	0.3598	0.3785	0.5597
	(0.85)	(1.49)			(2.04)	(1.99)	(2.07)	(1.03)	(1.71)	(2.49)	(2.58)	(2.10)	(2.20)	(2.47)
β_{SMB}	-0.0136	-0.0029			0.1189	0.1167	0.0539	-0.1817	0.0292	0.0913	0.1833	0.0132	0.1481	0.1714
	(-0.11)	(-0.02)			(0.87)	(0.86)	(0.42)	(-1.51)	(0.24)	(0.67)	(1.35)	(0.10)	(1.12)	(1.13)
β_{HML}	0.4269	0.3827			0.3087	0.3287	0.3333	0.1570	0.3233	0.3316	0.3442	0.2177	0.2813	0.2962
	(2.81)	(2.56)			(2.08)	(2.22)	(2.09)	(1.07)	(2.31)	(1.85)	(2.14)	(1.53)	(1.99)	(1.63)
β_{AV}							0.0463				0.0349			
							(0.63)				(0.45)			
β_{AC}							-2.4482				-1.8399			
							(-2.16)				(-1.87)			
β_{MV}								-0.0050				-0.0044		
								(-3.68)				(-3.42)		
β_{CIV}								-0.0166				-0.0373		
								(-0.80)				(-1.86)		
β_{FMax}									-0.1283				-0.1092	
									(-1.06)				(-0.91)	
β_{FVIX}										-0.1792				-0.1172
										(-2.57)				(-1.61)
$avg.\overline{R}^2$ in %	56.57	56.94	18.23	28.49	61.12	61.00	59.90	60.42	57.68	59.26	62.66	63.28	61.53	61.92

Table 3.2 presents average coefficients of Fama and MacBeth (1973) cross-sectional portfolio level regressions of excess returns in month $t+1$ on $IVol_{FF3}^{1M}$, the sensitivity β_{OP} to the latent factor OP and control variables in month t. $IVol_{FF3}^{1M}$ is the monthly idiosyncratic volatility of daily portfolio-level returns. Residuals are computed from the Fama and French (1993) three factor model (FF3). The base assets are 25 portfolios sorted on Size and monthly idiosyncratic volatility $IVol_{FF3}^{1M}$. Betas are calculated for the following risk factors: Mkt, SMB and HML are the FF3 factors. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016), $FMax$ is the Bali et al. (2017) lottery demand factor and $FVIX$ is the Barinov (2018) volatility risk factor. We estimate all betas in 5-year rolling window regressions. The sample period in all Columns is June 1968 to December 2016 except for Column (10) and (15) which start in May 1986. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in %. Average coefficients are multiplied by one hundred and the factor portfolios Mkt, SMB and HML are included among the test assets. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

and the average cross-sectional adjusted $\overline{R}^2$ in %. Average coefficients

[9] We use β_{FMax} instead of the maximum daily return (Max) because $IVol_{FF3}^{1M}$ and Max are almost perfectly correlated. Thus, it is impossible to distinguish between Max and $IVol_{FF3}^{1M}$ (Hou and Loh, 2016).

are multiplied by one hundred. We follow Lewellen et al. (2010) and include the factor portfolios Mkt, SMB and HML among the test assets. The t-statistics in parentheses calculated from Newey and West (1987) standard errors with six lags. The sample period is June 1968 to December 2016 except for Columns (10) and (14) where the sample period starts in May 1986 due to the inclusion of $FVIX$.

The coefficient on $IVol_{FF3}^{1M}$ is statistically significant in each specification without β_{OP} as revealed by Columns (2) and (7) to (10). Thus, neither the FF3 risk factors, nor the control variables explain the negative $IVol_{FF3}^{1M}$ coefficient in the 25 Size-$IVol_{FF3}^{1M}$ portfolios. As soon as we include β_{OP} in Columns (3) to (6) and (12) to (14), $IVol_{FF3}^{1M}$ becomes insignificant. Column (11) which controls for β_{AV} and β_{AC} is one exception, the $IVol_{FF3}^{1M}$ coefficient reduces considerably, but remains statistically significant at the edge of the 10% level. Conversely, the coefficient on β_{OP} is always significantly negative, with the least negative t-statistic of -1.92 in the univariate analysis in Column (3). In multivariate analyses, β_{OP} is statistically significant at least at the 5% level, but mostly at any conventional level with t-statistics beyond minus three.

Control variables are mostly statistically significant, but hardly affect the coefficients of $IVol_{p,t}^{1M}$ and β_{OP}, respectively. In Column (7) and (11), the beta with respect to average correlation β_{AC} is significantly negative. This result differs from the findings in Chen and Petkova (2012), but is in line with Hou and Loh (2016) and Barinov (2018) who present mixed evidence regarding β_{AV} and β_{AC}. The variance factor betas β_{MV}, β_{CIV} and β_{FVIX} are also significant in Columns (8), (10) and (12), but leave $IVol_{FF3}^{1M}$

statistically significant. β_{FVIX} becomes insignificant once we include β_{OP}. Regarding the FF3 factors, the inclusion of β_{OP} yields coefficient estimates closer to the risk factor premia of the factors Mkt and SMB. In the FF3 model in Column (1), β_{Mkt} is statistically insignificant, but once we include β_{OP} we obtain a coefficient of roughly 0.36 with a t-statistic of 1.99, although Mkt and OP are uncorrelated.

The inclusion of β_{OP} into the FF3 model reduces the Intercept from 37 bps in Column (1) to 19 bps in Column (6) with t-statistics of 4.70 and 2.77, respectively. β_{OP} reduces mispricing, but the Intercept remains statistically significant. Chen and Petkova (2012) show that full sample beta estimates are more precise and we re-evaluate the analysis in Table 3.3, but use full sample beta estimates instead of rolling windows. The Model number in Table 3.3 corresponds to the respective Column in Table 3.2.

Table 3.3: Fama and MacBeth (1973) regressions on 25 Size-$IVol$ portfolios with full sample betas.

Model	Intercept		$IVol^{1M}_{FF3}$		β_{OP}		β_{Mkt}		β_{SMB}		β_{HML}		$avg.\overline{R}^2$ in %
(1)	0.1572	(4.75)					0.3656	(2.01)	0.0741	(0.55)	0.8717	(4.95)	54.87
(2)	0.1610	(4.42)	-57.313	(-3.33)			0.4948	(2.79)	0.0828	(0.62)	0.8211	(5.07)	54.95
(5)	0.0378	(1.08)	-2.6421	(-0.17)	-0.3904	(-4.59)	0.5638	(3.02)	0.1859	(1.34)	0.5502	(3.84)	60.08
(6)	0.0418	(1.24)			-0.3835	(-4.69)	0.5547	(2.92)	0.1820	(1.31)	0.5596	(3.90)	60.24

Table 3.3 presents average coefficients of Fama and MacBeth (1973) cross-sectional portfolio level regressions of excess returns in month $t+1$ on $IVol^{1M}_{FF3}$, the sensitivity β_{OP} to the latent factor OP and the factors Mkt, SMB and HML of the Fama and French (1993) three factor model. We estimate betas over the full sample. The sample period is July 1963 to December 2016. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in %. Average coefficients are multiplied by one hundred and the factor portfolios Mkt, SMB and HML are included among the test assets. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

Model (1) illustrates that the FF3 model leaves a significant Intercept of about 16 bps with a t-statistic of 4.75. The coefficient on $IVol^{1M}_{FF3}$ in Model (2) amounts to -57.313 with a t-statistic of -3.33 if we control for full sample betas of the FF3 model. Including the full sample estimate for β_{OP} alleviates both of these observations. In Model (3), $IVol_{FF3}^{1M}$ becomes insignificant. The same holds true for the Intercept, both in Model (5) and (6). The extended model prices the the 25 Size-$IVol_{FF3}^{1M}$ more efficiently than the FF3 model. The risk premium on β_{Mkt} increases from 37 bps to 55 bps from Model (1) to Model (6) and the estimate for β_{HML} reduces from 87 bps to 56 bps. Both estimates are closer to the full sample risk premia of 51 bps and 37 bps for Mkt and HML. β_{SMB}, however, remains insignificant.

The portfolio regressions confirm that a latent factor in the FF3 model is a likely explanation for the negative relation between $IVol_{FF3}^{1M}$ and subsequent returns. In line with the predictions in Section 3.2.2, the sensitivity of a portfolio to the latent factor OP alleviates the negative $IVol_{FF3}^{1M}$ coefficient and the alphas of 25 Size-$IVol_{FF3}^{1M}$ portfolios. Conversely, we find little evidence for alternative explanations. Betas with respect to lottery demand or different measures of aggregate variance and volatility have no explanatory power with regard to the $IVol$ puzzle.

3.4.3 Hou and Loh (2016) analysis for alternative test assets

Finally, we apply the methodology of Hou and Loh (2016) to estimate the share of the negative risk premium on $IVol_{FF3}^{1M}$ in Fama and MacBeth (1973) regressions which is attributable to β_{OP}. The Hou and Loh (2016) approach provides an estimate for the fraction of the puzzle which can be explained by the explanatory variable. Furthermore, the approach provides statistical inference about the significance of this fraction as well as the unexplained fraction.

We follow Hou and Loh (2016) and apply the decomposition technique to 200 portfolios sorted by $IVol_{FF3}^{1M}$. Given the daily $IVol_{FF3}^{1M}$ portfolio returns, we estimate the FF3 model, compute OP from the residuals and include OP into the model to estimate β_{OP}. We estimate β_{OP} and all risk factor betas in 5-year rolling windows on the portfolio level. In contrast to the baseline analysis which only considers risk-based explanations for the $IVol$ puzzle, we also control for stock characteristics in the Hou and Loh (2016) analysis. Portfolio characteristics are the value-weighted average over all stocks in the respective portfolios. We include the following control variables: The Amihud (2002) illiquidity measure (Illiq), short-term reversal (LagRet), the logarithm of the monthly market capitalization (Size), idiosyncratic skewness of FF3 residuals (ISkew), the share of zero returns (ZeroRet), co-skewness (CoSkew), expected idiosyncratic skewness (EIS) and a mispricing score (MISP). We describe the calculation of stock characteristics in Appendix 3.A.2. Following Hou and Loh (2016), we compute $IVol_{FF3}^{1M}$ as the value-weighted portfolio average.

Table 3.4 presents results of the univariate Hou and Loh (2016) decomposition for β_{OP} and control variables. We report the $IVol_{FF3}^{1M}$ risk premium $\overline{\gamma}_t$ for each candidate regression separately and present the decomposed coefficients as well as the fraction of $\overline{\gamma}_t$ which is related to this candidate and the residual. All coefficients are multiplied with one hundred. The t-statistics indicate whether this fraction is statistically different from zero.

Table 3.4: Univariate Hou and Loh (2016) analysis for 200 $IVol$ portfolios.

	$r_{p,t+1} \sim \alpha_t + \gamma_t IVol_{p,t} + \epsilon_{p,t}$			Candidate			Residual		Sample Period
Candidate	$\overline{\gamma}_t$	t-stat	Coeff.	Fract. in %	t-stat	Coeff.	Fract. in %	t-stat	
β_{OP}	-33.470	(-3.85)	-0.2867	85.65	(12.41)	-0.0480	14.35	(2.08)	1968/06 – 2016/12
Illiq	-34.068	(-3.93)	-0.1702	49.96	(6.81)	-0.1705	50.04	(6.82)	1968/06 – 2016/12
LagRet	-33.470	(-3.85)	-0.0850	25.39	(4.34)	-0.2497	74.61	(12.76)	1968/06 – 2016/12
Size	-33.470	(-3.85)	-0.1707	50.99	(8.18)	-0.1640	49.01	(7.87)	1968/06 – 2016/12
ISkew	-33.570	(-3.87)	-0.0253	7.55	(2.46)	-0.3104	92.45	(30.18)	1968/06 – 2016/12
ZeroRet	-33.470	(-3.85)	-0.04	10.66	(3.92)	-0.2990	89.34	(32.90)	1968/06 – 2016/12
CoSkew	-33.570	(-3.87)	-0.0210	6.26	(1.20)	-0.3147	93.74	(18.04)	1968/06 – 2016/12
EIS	-34.489	(-3.65)	-0.0670	19.41	(3.12)	-0.2779	80.59	(12.95)	1969/07 – 2011/12
MISP	-34.753	(-3.97)	-0.1376	39.61	(6.59)	-0.2099	60.39	(10.06)	1968/06 – 2015/12
β_{AC}	-33.470	(-3.85)	-0.0260	7.77	(3.86)	-0.3087	92.23	(45.85)	1968/06 – 2016/12
β_{AV}	-33.470	(-3.85)	-0.0043	1.27	(0.56)	-0.3304	98.73	(43.51)	1968/06 – 2016/12
β_{MV}	-33.470	(-3.85)	-0.1100	32.87	(3.94)	-0.2247	67.13	(8.05)	1968/06 – 2016/12
β_{CIV}	-33.470	(-3.85)	-0.0695	20.77	(3.30)	-0.2652	79.23	(12.57)	1968/06 – 2016/12
β_{FMax}	-33.470	(-3.85)	-0.2070	61.84	(8.69)	-0.1277	38.16	(5.36)	1968/06 – 2016/12
β_{FVIX}	-26.575	(-2.52)	-0.1217	45.81	(3.58)	-0.1440	54.19	(4.23)	1986/05 – 2016/12

Table 3.4 presents results of the first and the final stage of the Hou and Loh (2016) decomposition methodology for β_{OP} and control variables. $IVol$ is the standard deviation of Fama and French (1993) factor model residuals. The table presents the average coefficient of the decomposed risk premium $\overline{\gamma}_t$ which is related to respective candidate variables and the residual, respectively. t-statistics in parentheses test the null hypothesis that the explained fraction is equal to zero. We include the following stock characteristics as control variables: Illiq is the Amihud (2002) illiquidity measure, LagRet is the return in month t, Size is the logarithm of the monthly market capitalization in 1,000 USD and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns, CoSkew is co-skewness as proposed by Harvey and Siddique (2000), EIS is expected idiosyncratic skewness as proposed by Boyer et al. (2010) and MISP is the Stambaugh et al. (2015) mispricing score. Betas are calculated to the following risk factors: AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016), FMax is the Bali et al. (2017) lottery demand factor and FVIX is the Barinov (2018) aggregate variance factor.

In the univariate analysis, β_{OP} explains 86.65% of the $IVol^{1M}_{FF3}$ coefficient $\overline{\gamma}_t$. This fraction is highly statistically significant with a t-statistic of 12.41. Conversely, the fraction of 14.35% which is attributable to the residual exhibits a t-statistic of 2.06. The common component in FF3 residuals of 200 $IVol$ portfolios accounts for the largest part of the puzzle. To put this into perspective, alternative explanations on average explain

27.15% of the $IVol$ puzzle.

The explanatory power of alternative explanations is comparable to Hou and Loh (2016). Illiquidity, Size and lottery demand – measured as β_{FMax} – attain the highest fractions of 49.96%, 50.99% and 61.84% (t-statistics of 6.81, 8.18 and 8.69). Conditional skewness (Co-Skew) of Harvey and Siddique (2000) and β_{AV} of Chen and Petkova (2012) explain insignificant fractions of 6.26% and 1.27%.

We evaluate the explanatory power of β_{OP} in the presence of alternative explanations in Table 3.5 which presents a multivariate decomposition of the $IVol_{FF3}^{1M}$ coefficient $\overline{\gamma}_t$. Table 3.5 presents the coefficient which is attributable to each candidate, the corresponding fraction in % and t-statistics in parentheses for the null hypothesis that this fraction is equal to zero.

We present kitchen sink models in Model 1 and Model 2. The only difference is the exclusion of β_{FVIX} in Model 1 due to the short sample period for which $FVIX$ is available. β_{OP} attains the highest individual explanatory power of 33.94% (t-statistic = 6.98), followed by Size with 24.2% (t-statistic = 6.49). Illiq and β_{FMax} each explain roughly 12.7%. For the remaining alternative explanations, the explanatory power reduces to 6% and less and the majority of candidates is insignificant. The unexplained fraction of 6.21% (t-statistic = 2.13) is statistically significant at the 5% level.

Model 2 is very similar and β_{FVIX} does not explain a significant fraction of $\overline{\gamma}_t$. The explained fraction of β_{OP} reduces to 25.94%, while the alternative explanations Illiq, LagRet and Size gain higher fractions. Com-

Table 3.5: Multivariate Hou and Loh (2016) analysis for 200 $IVol$ portfolios.

	Model 1			Model 2			Model 3		
Candidate	Coeff.	Fract. in %	t-stat	Coeff.	Fract. in %	t-stat	Coeff.	Fract. in %	t-stat
β_{OP}	-11.8087	33.94	(6.98)	-7.2455	25.94	(3.99)	-16.4427	46.69	(9.56)
Illiq	-4.4214	12.71	(4.51)	-5.363	19.2	(2.79)	-3.2729	9.29	(4.29)
LagRet	-2.002	5.75	(2.73)	-2.5271	9.05	(2.26)	-2.9968	8.51	(3.05)
Size	-8.4213	24.2	(6.49)	-7.9411	28.43	(3.91)	-7.7365	21.97	(5.89)
Iskew	0.5493	-1.58	(-1.12)	0.9876	-3.54	(-1.71)	-	-	-
ZeroRet	1.4772	-4.25	(-1.81)	1.5924	-5.7	(-1.66)	-	-	-
CoSkew	0.0456	-0.13	(-0.10)	-0.1632	0.58	(0.29)	-	-	-
EIS	-0.4964	1.43	(0.94)	-0.7018	2.51	(1.01)	-	-	-
MISP	-1.7437	5.01	(3.43)	-1.2463	4.46	(1.91)	-3.0013	8.52	(4.60)
β_{AC}	-0.3414	0.98	(1.16)	-0.368	1.32	(1.10)	-	-	-
β_{AV}	-0.4008	1.15	(1.83)	-0.2653	0.95	(0.84)	-	-	-
β_{MV}	-0.6278	1.8	(0.55)	-0.381	1.36	(0.24)	-	-	-
β_{CIV}	0.0034	-0.01	(-0.01)	0.9326	-3.34	(-1.13)	-	-	-
β_{FMax}	-4.4422	12.77	(1.92)	-3.0654	10.98	(0.62)	-	-	-
β_{FVIX}	-	-	-	0.2425	-0.87	(-0.13)	-	-	-
Residual	-2.162	6.21	(2.13)	-2.4146	8.65*	(1.72)	-1.7628	5.01	(1.08)
Sample Period	1969/07 – 2011/12			1986/05 – 2011/12			1968/06 – 2015/12		

Table 3.5 presents results of the final stage of the multivariate Hou and Loh (2016) decomposition methodology for β_{OP} and control variables. $IVol$ is the standard deviation of Fama and French (1993) factor model residuals. The table presents the average coefficient of the decomposed risk premium $\overline{\gamma}_t$ which is related to respective candidate variables and the residual, respectively. t-statistics in parentheses test the null hypothesis that the explained fraction is equal to zero. We include the following stock characteristics as control variables: Illiq is the Amihud (2002) illiquidity measure, LagRet is the return in month t, Size is the logarithm of the monthly market capitalization in 1,000 USD and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns, CoSkew is co-skewness as proposed by Harvey and Siddique (2000), EIS is expected idiosyncratic skewness as proposed by Boyer et al. (2010) and MISP is the Stambaugh et al. (2015) mispricing score. Betas are calculated to the following risk factors: AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016), FMax is the Bali et al. (2017) lottery demand factor and FVIX is the Barinov (2018) aggregate variance factor.

pared to Model 1, Size exhibits the largest fraction with 28.43%. In contrast, the explained fraction of 10.98% attributable to β_{FMax} is now insignificant. Since β_{FVIX} itself adds little to the $IVol$ puzzle, we consider Model 2 as a sample split.

Consequently, Model 3 only keeps candidates which have positive and significant explanatory power with respect to $\overline{\gamma}_t$ in both sample periods. The reduced Model 3 leaves an insignificant unexplained fraction of 5.01% and thus fully decomposes $\overline{\gamma}_t$. Again, the highest fraction is attributable to *OP* and β_{OP} alone explains 46.69% (t-statistic = 9.56). Among the characteristics, Size is the dominant candidate with an explained fraction of 21.97%, whereas the remainder is equally shared between illiquidity, short-term reversal and mispricing. The Hou and Loh (2016) analysis reveals that the common component in FF3 residuals explains almost 50% of the $IVol$ puzzle. Thus, $IVol$ is largely attributable to a common, yet unknown risk factor.

3.5 Mispricing versus risk

3.5.1 An economic tracking portfolio for the IVol puzzle

In this Section, we address the question whether the source of the $IVol$ puzzle, i.e. *OP*, represents mispricing or latent systematic risk. However, *OP* implicitly depends on the set of test assets and is not a tradable risk factor. To generalize the findings in Section 4.4 we follow Breeden et al. (1989) and form a mimicking portfolio which tracks *OP*. We estimate the regression

$$OP_t = a + \mathbf{b}\mathbf{X}_t + u_t, \qquad (3.9)$$

where OP_t are the weighted residuals of a FF3 regression for monthly excess returns of 25 Size-$IVol_{FF3}^{1M}$ portfolios. X_t is a matrix of the same 25 portfolios serving as base assets. The fitted values of the regression in

Equation (3.9) minus the constant constitute the mimicking factor for OP which we refer to as FOP. The correlation between OP and FOP amounts to 0.90. We follow Barinov (2018) and use full sample estimates. We provide robustness checks in Section 3.7.5 with an extending window estimation to eliminate the look-ahead bias.

To constitute a valid risk factor candidate for the $IVol$ puzzle, FOP should earn a significant risk premium and explain the underperformance of high $IVol$ stocks. First, we address the risk premium. The average monthly excess return of FOP is -57bp with a Newey and West (1987) t-statistic of -10.38 as shown in Column (1) of Table 3.6. This risk premium is also statistically significant after accounting for existing risk factors. In Columns (2) to (11) of Table 3.6 we present factor model alphas in % per month as well as the corresponding risk factor betas and the adjusted R^2 in %. We account for the following risk factors: Mkt, SMB, HML, RMW, CMA and MOM correspond to the factors of the FF6 model, Mkt_{M4}, SMB_{M4}, $MGMT$ and $PERF$ constitute the M4 model, Mkt_Q, ME, IA and ROE are the four factors of the q-factor model, $FVIX$ is the Barinov (2018) aggregate volatility factor and $FMax$ is the Bali et al. (2017) lottery demand factor. The sample period in Columns is August 1963 to December 2016 except Column (4), (8) and (10). The sample period in Column (4) is January 1968 to December 2015, January 1967 to December 2015 in Column (8) and May 1986 to December 2016 in Column (10). The t-statistics in parentheses are calculated from Newey and West (1987) standard errors with six lags.

The factor model alphas are significant at any conventional level and

Table 3.6: Factor model alphas of the mimicking factor *FOP*.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
α in %	-0.5720	-0.5722	-0.5060	-0.4924	-0.4668	-0.3774	-0.3384	-0.3218	-0.3594	-0.3425	-0.3867
	(-10.38)	(-10.04)	(-10.08)	(-9.09)	(-8.87)	(-8.72)	(-5.73)	(-5.47)	(-7.69)	(-3.81)	(-8.71)
Mkt		0.0003	-0.0388	-0.0382	-0.0471	-0.0757			-0.0788	0.4809	-0.1291
		(0.01)	(-2.30)	(-2.21)	(-3.18)	(-5.07)			(-5.38)	(2.54)	(-8.31)
SMB			0.0616	0.0474	0.0623	-0.0051			-0.0034		-0.0515
			(2.98)	(2.11)	(2.73)	(-0.27)			(-0.19)		(-2.68)
HML			-0.1669	-0.1691	-0.1826	-0.0960			-0.1094		-0.0554
			(-4.45)	(-4.18)	(-4.87)	(-4.67)			(-5.59)		(-2.69)
Liq				0.0024							
				(0.15)							
MOM					-0.0441				-0.0247		
					(-1.72)				(-1.57)		
CMA						-0.1510			-0.1408		
						(-4.41)			(-4.45)		
RMW						-0.2963			-0.2903		
						(-12.59)			(-13.42)		
Mkt_{M4}							-0.0899				
							(-5.28)				
SMB_{M4}							-0.0098				
							(-0.32)				
MGMT							-0.2250				
							(-6.49)				
PERF							-0.0754				
							(-3.44)				
Mkt_Q								-0.0592			
								(-3.55)			
ME								-0.0104			
								(-0.42)			
IA								-0.2413			
								(-5.29)			
ROE								-0.1989			
								(-5.34)			
FVIX										0.3403	
										(2.74)	
FMax											0.1979
											(10.86)
$\overline{R}^2$ in %	-	-0.16	15.69	15.38	17.73	40.02	20.50	25.20	40.59	8.98	41.09

Table 3.6 presents the alpha of the tracking portfolio for the latent factor *OP*, i.e. *FOP* for different factor models. Column (1) is the monthly average return of *FOP*, and Column (2) to (11) contain alphas of the following factor models in the respective order: A CAPM one factor model, the Fama and French (1993) three factor model FF3, the FF3 model extended by the Pastor and Stambaugh (2003) liquidity factor, the Carhart (1997) four factor model, the Fama and French (2015) five factor model, the Stambaugh and Yuan (2017) four factor mispricing model, the q-factor model of Hou et al. (2015), the Fama and French (2018) six factor model, the ICAPM model of Barinov (2018) and the FF3 model extended by the Bali et al. (2017) lottery demand factor. We report alphas in % per month and the adjusted $\overline{R}^2$ for each model in %. The sample period in Columns (1) – (3), (5) – (7), (9) and (11) is August 1963 to December 2016. The sample period in Column (4) is January 1968 to December 2015, January 1967 to December 2015 in Column (8) and May 1968 to December 2016 in Column (10). *t*-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

range from -57 bps for the CAPM in Column (2) to -32 bps per month in Column (8) which includes the Stambaugh and Yuan (2017) mispricing factors. At most, the factor models explain roughly 40% of the time series variation in FOP. The risk premium of FOP is significant after risk-adjustment.

Table 3.7 illustrates the correlation of FOP with the same set of risk factors as in Table 3.6. Although the mimicking factor does not satisfy the orthogonality condition of OP, the correlation between FOP, Mkt and SMB is close to zero. The correlation between FOP and HML amounts to -0.49. The highest correlation is found between FOP and the Robust-

Table 3.7: Correlation of FOP with existing risk factors.

	FOP	Mkt	SMB	HML	RMW	CMA	MOM	Mkt_{M4}	SMB_{M4}	MGMT	PERF	Mkt_Q	ME	IA	ROE	FVIX
Mkt	0.07															
SMB	0.16	0.19														
HML	-0.49	-0.23	-0.17													
RMW	-0.60	-0.38	-0.45	0.36												
CMA	-0.37	-0.39	-0.07	0.66	0.20											
MOM	-0.12	-0.18	0.03	-0.18	0.09	0.06										
Mkt_{M4}	0.07	1.00	0.19	-0.23	-0.38	-0.39	-0.18									
SMB_{M4}	0.05	0.17	0.94	-0.13	-0.36	-0.04	0.07	0.17								
MGMT	-0.45	-0.51	-0.34	0.69	0.45	0.76	0.07	-0.51	-0.29							
PERF	-0.19	-0.36	-0.10	-0.25	0.41	0.04	0.73	-0.36	-0.04	0.09						
Mkt_Q	0.07	1.00	0.20	-0.23	-0.39	-0.39	-0.19	1.00	0.18	-0.51	-0.37					
ME	0.12	0.21	0.97	-0.11	-0.44	-0.04	0.07	0.21	0.93	-0.31	-0.09	0.22				
IA	-0.39	-0.36	-0.18	0.68	0.29	0.91	0.02	-0.36	-0.15	0.77	-0.01	-0.37	-0.15			
ROE	-0.50	-0.33	-0.39	0.10	0.73	0.07	0.50	-0.33	-0.29	0.26	0.64	-0.35	-0.31	0.14		
FVIX	0.00	-0.98	-0.12	0.19	0.28	0.35	0.18	-0.98	-0.10	0.44	0.33	-0.97	-0.14	0.32	0.26	
FMax	0.55	0.60	0.51	-0.51	-0.75	-0.48	-0.11	0.60	0.44	-0.70	-0.30	0.61	0.48	-0.53	-0.58	-0.50

Table 3.7 presents pairwise correlations of the mimicking factor FOP and several risk factors. We include the following risk factors: Mkt , SMB, HML, RMW, CMA and MOM correspond to the factors of the Fama and French (2018) six factor model, Mkt_{M4}, SMB_{M4}. $MGMT$ and $PERF$ constitute the M4 mispricing factor model of Stambaugh and Yuan (2017), Mkt_Q, ME, IA and ROE are the four factors of the q-Factor model of Hou et al. (2015), $FVIX$ is the Barinov (2018) factor which tracks daily innovations in the VIX and $FMax$ is the Bali et al. (2017) lottery demand factor.

minus-Weak factor RMW of -0.60 followed by $FMax$ with a correlation of 0.55. The latter correlation is high by construction, because lottery demand Max and $IVol$ are mechanically correlated (see Hou and Loh, 2016, p. 172). Including $FMax$ into the analysis thus allows us to distinguish between $IVol$ and aggregate lottery demand. FOP is furthermore almost uncorrelated with $FVIX$, which is itself close to perfectly negatively correlated with Mkt. Considering the low correlation with FOP and collinearity concerns, we dismiss FVIX as an alternative risk-based candidate for the $IVol$ puzzle.[10] The correlations between FOP and the existing risk factors, however, raise no concerns that other models subsume FOP and we can add FOP to each of the models without collinearity issues.

Second, we include FOP to the factor models in Table 3.1 to evaluate the ability of FOP to explain the negative alphas of high $IVol_{FF3}^{1M}$ stocks. We present the results in Table 3.8 which is otherwise identical to the baseline sorts in Table 3.1. We report alphas in % per month and β_{FOP} of each decile. Estimates for β_{OP} which are significantly different from zero at the 10% level are printed in bold numbers.

The inclusion of FOP in the factor models consistently explains the negative alphas of the high $IVol_{FF3}^{1M}$ deciles as well as the difference portfolio between high and low $IVol_{FF3}^{1M}$ stocks. The extended models perform equally well in the equal-weighted sorts in Panel A and the value-weighted sorts in Panel B. Alphas of the difference portfolios are insignificant with three exceptions. In case of the extended CAPM, i.e. CAPM + FOP, alphas increase from low to high $IVol_{FF3}^{1M}$ deciles. In equal-weighted sorts, the alpha of the difference portfolio of 32 bps with a t-statistic of 1.75

[10] Although our mimicking regression shown in Appendix 3.A.1 is slightly different from the coefficients reported by Barinov (2018), we find that our fitted $FVIX$ factor perfectly matches the results in Table 1 in Barinov (2018). If we use his coefficients with our quintile VIX portfolios, we obtain an even higher correlation between $FVIX$ and Mkt. The correlation of -0.97 is most likely due to our reproduction of $FVIX$. However, we closely follow Barinov (2018).

Table 3.8: Revisiting the idiosyncratic volatility puzzle with FOP.

Panel A: Equal-weighted sorts

Model	Parameter	Low $IVol^{1M}_{FF3}$	2	3	4	5	6	7	8	9	High $IVol^{1M}_{FF3}$	Diff
CAPM + FOP	α	0.0199	0.0281	0.0451	0.1530	0.2128	0.3803	0.3622	0.5126	0.5309	0.3435	0.3236
		(0.20)	(0.31)	(0.46)	(1.53)	(2.00)	(3.03)	(2.50)	(3.15)	(2.93)	(1.77)	(1.75)
	β_{FOP}	**-0.5771**	**-0.6681**	**-0.5938**	**-0.5067**	**-0.3216**	-0.0472	**0.3005**	**0.7490**	**1.3314**	**2.1546**	**2.7317**
FF3 + FOP	α	-0.0866	-0.0988	-0.1161	-0.0481	-0.0267	0.0967	0.0322	0.1415	0.1251	-0.0852	0.0014
		(-1.05)	(-1.51)	(-1.76)	(-0.82)	(-0.46)	(1.57)	(0.59)	(2.31)	(1.70)	(-1.16)	(0.01)
	β_{FOP}	**-0.5135**	**-0.6011**	**-0.5825**	**-0.5507**	**-0.4328**	**-0.2447**	0.0186	**0.3650**	**0.8991**	**1.7263**	**2.2398**
PS + FOP	α	-0.0882	-0.1058	-0.1225	-0.0573	-0.0445	0.1007	0.0353	0.1431	0.1395	-0.1186	-0.0304
		(-1.00)	(-1.50)	(-1.70)	(-0.89)	(-0.69)	(1.49)	(0.59)	(2.22)	(1.77)	(-1.51)	(-0.28)
	β_{FOP}	**-0.5461**	**-0.6436**	**-0.6208**	**-0.5741**	**-0.4528**	**-0.2488**	0.0443	**0.3816**	**0.9370**	**1.7350**	**2.2811**
CAR + FOP	α	-0.0810	-0.0747	-0.0875	-0.0093	0.0058	0.1261	0.0617	0.1563	0.1534	-0.0670	0.0140
		(-1.02)	(-1.38)	(-1.61)	(-0.20)	(0.12)	(2.41)	(1.20)	(2.63)	(2.18)	(-0.88)	(0.13)
	β_{FOP}	**-0.5191**	**-0.6248**	**-0.6106**	**-0.5887**	**-0.4647**	**-0.2736**	-0.0104	**0.3504**	**0.8713**	**1.7083**	**2.2274**
FF5 + FOP	α	-0.0827	-0.0779	-0.0930	-0.0174	-0.0004	0.1237	0.0415	0.1381	0.1302	-0.0943	-0.0116
		(-1.06)	(-1.41)	(-1.63)	(-0.33)	(-0.01)	(2.26)	(0.74)	(2.37)	(1.87)	(-1.39)	(-0.13)
	β_{FOP}	**-0.3560**	**-0.4017**	**-0.3922**	**-0.3828**	**-0.2894**	**-0.1449**	0.0451	**0.2676**	**0.7682**	**1.4993**	**1.8552**
M4 + FOP	α	-0.1553	-0.1464	-0.1487	-0.0518	-0.0162	0.1263	0.0744	0.2104	0.2220	-0.0057	0.1496
		(-1.82)	(-2.19)	(-2.22)	(-0.94)	(-0.29)	(2.35)	(1.40)	(3.45)	(3.28)	(-0.07)	(1.38)
	β_{FOP}	**-0.4979**	**-0.6367**	**-0.6113**	**-0.6041**	**-0.4705**	**-0.2664**	0.0304	**0.4048**	**0.9226**	**1.7693**	**2.2671**
Q-Factor + FOP	α	-0.1274	-0.1597	-0.1760	-0.0877	-0.0630	0.0903	0.0518	0.1962	0.2173	-0.0330	0.0944
		(-1.24)	(-1.78)	(-1.91)	(-1.04)	(-0.74)	(0.95)	(0.68)	(2.27)	(2.15)	(-0.33)	(0.83)
	β_{FOP}	**-0.6537**	**-0.7599**	**-0.7162**	**-0.6970**	**-0.5550**	**-0.3421**	-0.0651	**0.3171**	**0.8063**	**1.5810**	**2.2347**
FF6 + FOP	α	-0.0743	-0.0531	-0.0644	0.0194	0.0306	0.1506	0.0690	0.1504	0.1536	-0.0808	-0.0065
		(-0.97)	(-1.08)	(-1.23)	(0.40)	(0.63)	(2.96)	(1.27)	(2.65)	(2.31)	(-1.18)	(-0.07)
	β_{FOP}	**-0.3627**	**-0.4217**	**-0.4152**	**-0.4125**	**-0.3143**	**-0.1665**	0.0229	**0.2576**	**0.7494**	**1.4885**	**1.8512**

Panel B: Value-weighted sorts

Model	Param.	Low $IVol^{1M}_{FF3}$	2	3	4	5	6	7	8	9	High $IVol^{1M}_{FF3}$	Diff
CAPM + FOP	α	-0.2079	-0.1266	-0.2001	-0.0684	0.0136	0.1249	0.3884	0.1715	0.3974	0.1761	0.3840
		(-2.72)	(-2.00)	(-3.17)	(-0.90)	(0.18)	(1.32)	(3.79)	(1.23)	(2.30)	(1.06)	(1.87)
	β_{FOP}	**-0.6294**	**-0.4044**	**-0.3783**	**-0.2568**	-0.0650	**0.2785**	**0.6783**	**1.0244**	**1.5900**	**2.3265**	**2.9559**
FF3 + FOP	α	-0.1355	-0.0715	-0.1646	-0.0723	-0.0265	0.0111	0.2274	-0.0639	0.1048	-0.1428	-0.0072
		(-2.01)	(-0.99)	(-2.81)	(-0.98)	(-0.37)	(0.13)	(2.64)	(-0.58)	(0.87)	(-0.90)	(-0.04)
	β_{FOP}	**-0.4408**	**-0.2581**	**-0.3017**	**-0.2162**	-0.1055	0.1000	**0.4339**	**0.6145**	**1.0635**	**1.9225**	**2.3633**
PS + FOP	α	-0.1322	-0.0754	-0.1722	-0.0791	-0.0594	-0.0098	0.2336	-0.1118	0.0724	-0.2012	-0.0690
		(-1.87)	(-0.96)	(-2.84)	(-1.03)	(-0.80)	(-0.11)	(2.46)	(-0.89)	(0.53)	(-1.16)	(-0.35)
	β_{FOP}	**-0.4610**	**-0.2868**	**-0.3397**	**-0.2444**	-0.1219	0.0954	**0.4482**	**0.6201**	**1.0879**	**1.9785**	**2.4395**
CAR + FOP	α	-0.1532	-0.0795	-0.1748	-0.0708	0.0047	0.0266	0.2425	-0.0632	0.1327	-0.0892	0.0640
		(-2.18)	(-1.10)	(-2.76)	(-0.97)	(0.07)	(0.30)	(2.85)	(-0.58)	(1.10)	(-0.57)	(0.35)
	β_{FOP}	**-0.4235**	**-0.2502**	**-0.2917**	**-0.2177**	**-0.1362**	0.0848	**0.4190**	**0.6139**	**1.0360**	**1.8698**	**2.2933**
FF5 + FOP	α	-0.1411	-0.0625	-0.1480	-0.0503	-0.0079	0.0369	0.2282	-0.0683	0.0989	-0.1424	-0.0014
		(-2.26)	(-0.91)	(-2.52)	(-0.73)	(-0.11)	(0.47)	(2.74)	(-0.66)	(0.85)	(-0.92)	(-0.01)
	β_{FOP}	**-0.2746**	**-0.0764**	**-0.1518**	-0.0883	-0.0849	-0.0267	**0.2759**	**0.3421**	**0.6910**	**1.3920**	**1.6666**
M4 + FOP	α	-0.2456	-0.1491	-0.2023	-0.0835	0.0388	0.1229	0.3361	0.0864	0.3110	0.1009	0.3465
		(-3.32)	(-2.13)	(-2.72)	(-0.97)	(0.56)	(1.43)	(3.84)	(0.87)	(2.64)	(0.68)	(2.01)
	β_{FOP}	**-0.3986**	**-0.2440**	**-0.3099**	**-0.2519**	**-0.1973**	0.0164	**0.3760**	**0.5902**	**1.0544**	**1.7674**	**2.1660**
Q-Factor + FOP	α	-0.2234	-0.1266	-0.2206	-0.1144	-0.0403	0.0556	0.3165	0.0238	0.2672	-0.0242	0.1992
		(-2.82)	(-1.54)	(-3.20)	(-1.44)	(-0.49)	(0.57)	(3.62)	(0.23)	(2.07)	(-0.15)	(1.08)
	β_{FOP}	**-0.4313**	**-0.2880**	**-0.2672**	**-0.2066**	**-0.1593**	0.0320	**0.3734**	**0.5773**	**1.0347**	**1.6918**	**2.1231**
FF6 + FOP	α	-0.1535	-0.0673	-0.1564	-0.0487	0.0201	0.0457	0.2390	-0.0732	0.1179	-0.1033	0.0503
		(-2.39)	(-1.00)	(-2.49)	(-0.71)	(0.29)	(0.58)	(2.87)	(-0.71)	(0.99)	(-0.67)	(0.30)
	β_{FOP}	**-0.2646**	**-0.0726**	**-0.1451**	-0.0896	**-0.1074**	-0.0337	**0.2672**	**0.3460**	**0.6757**	**1.3605**	**1.6251**

Table 3.8 revisits the univariate portfolio sorts in Table 3.1 with factor models extended by the mimicking factor FOP. The last column presents the difference in excess returns and respective alphas between stocks in the highest and the lowest $IVol^{1M}_{FF3}$ decile. Panel A presents sorts with equal weights, in Panel B we weight returns by market capitalization. We hold each portfolio for one month and record the monthly returns and different factor alphas. For each decile, we furthermore present the beta with respect to FOP, β_{FOP}. Bold values for β_{FOP} indicate estimates which are statistically significant at the 10% level. We present the same factor model setup as in Table 3.1. CAPM is a one factor model, FF3 is the Fama and French (1993) three factor model, PS is the FF3 model extended by the Pastor and Stambaugh (2003) liquidity factor, CAR is the Carhart (1997) four factor model and FF5 is the Fama and French (2015) five factor model. Furthermore, M4 is the Stambaugh and Yuan (2017) four factor mispricing model, q-factor is the four factor model of Hou et al. (2015) and FF6 is the Fama and French (2018) six factor model. Returns and alphas are reported in % per month. Newey and West (1987) adjusted t-statistics with six lags in parentheses. The sample period for excess returns and alphas of the models CAPM, FF3, CAR, FF5, FF6 and M4 is August 1963 to December 2016. The sample period for the PS model is January 1968 to December 2015. The q-factor alpha covers January 1967 to December 2015.

is significantly positive at the 10% level level. The same holds true for the extended CAPM in value-weighted sorts in Panel B. This alpha of 38 bps is even higher and exhibits a t-statistic of 1.87. Without the consideration of additional risk factor, the relationship between $IVol^{1M}_{FF3}$ and subsequent stock returns is positive, in line with Merton (1987). The alpha of the Stambaugh and Yuan (2017) mispricing model M4 is also significantly positive in Panel B. This finding challenges the interpretation of $IVol^{1M}_{FF3}$ as mispricing because high $IVol^{1M}_{FF3}$ stocks are not overpriced after adjusting for the latent risk factor FOP.

The portfolio estimates for β_{FOP} are in line with the theoretical considerations in Section 3.2.2. Stocks with high $IVol^{1M}_{FF3}$ have a high positive β_{FOP}. The different signs of β_{FOP} between high and low $IVol^{1M}_{FF3}$ stocks are also consistent with the theoretical predictions of Chen and Petkova (2012). However, when we consider equal-weighted sorts in Panel A, the relationship between $IVol^{1M}_{FF3}$ and β_{FOP} appears nonlinear for low $IVol^{1M}_{FF3}$ deciles. The mimicking factor FOP consistently explains the negative alpha of high-$IVol^{1M}_{FF3}$ stocks, earns a significant risk premium and is not subsumed by existing risk factors.

3.5.2 Relation to the covariance matrix of stock returns

The mimicking factor FOP tracks the latent factor behind the $IVol$ puzzle. Pukthuanthong et al. (2019) propose a two-stage protocol to separate genuine risk factors from firm characteristics and mispricing. First, a genuine risk factor is significantly related to the covariance matrix of the cross section of stock returns and second, it commands a significant risk premium with a reasonable Sharpe ratio. We address the first stage in this Section. The analysis closely follows Pukthuanthong et al. (2019).

To relate risk factor candidates to the covariance matrix of stock returns, Pukthuanthong et al. (2019) adopt the methodology of Connor and Korajczyk (1988) and compute $L = 10$ asymptotic PCs from the $T \times T$ matrix $\Omega_t = (\mathbf{1/T})\mathbf{RR}'$ where $\mathbf{R}$ is the matrix of sample stock returns. We also consider five subsamples: 1963 to 1976 and four decades thereafter up to 2016. We obtain ten PCs for five sub-periods. The factors under consideration are the six Fama and French (2018) risk factors Mkt, SMB, HML, RMW, CMA and MOM as well as the mimicking factor FOP.[11] We thus evaluate $K = 7$ risk factor candidates.

Given the two vectors $\mathbf{L}$ and $\mathbf{K}$ we compute canonical correlations between the two vectors. This allows us to test whether the group of factor candidates is correlated with the PC representation of the covariance matrix of stock returns. We obtain $\min(L, K) = 7$ orthogonal canonical variates, sorted from the highest to the smallest correlation between the PCs and the factor candidates. Table 3.9 summarizes the results for each of the five sub-periods.

We present canonical correlations between the ten PCs and the risk factor candidates as well as a z-statistic which tests the null hypothesis

[11] We also evaluate FOP in the context of other factors in untabulated robustness checks. We include FOP into the factor models of Stambaugh and Yuan (2017) and Hou et al. (2015) and also evaluate larger sets of factors. We include several combinations of non-redundant factors and also consider the Pastor and Stambaugh (2003) liquidity factor, short-term reversal as well as $FMax$. FOP passes the required threshold in contrast to existing factors. This discussion, however, is not subject of this paper. Our conclusions are robust to the factor choice in the canonical correlation.

Table 3.9: Risk factor protocol: Canonical correlations.

	1963 – 1976		1977 – 1986		1987– 1996		1997 – 2006		2007 – 2016	
Canonical Variate	Correlation	*z*-Statistic	Correlation	*z*-Statistic	Correlation	*z*-Statistic	Correlation	*z*-Statistic	Correlation	*z*-Statistic
1	0.9963	**27.9161**	0.9962	**23.9361**	0.9939	**22.7353**	0.9852	**20.3433**	0.9958	**23.6801**
2	0.8877	**15.6400**	0.8672	**12.7499**	0.8376	**11.8652**	0.9079	**14.2500**	0.6942	**8.7133**
3	0.8069	**12.8680**	0.6505	**7.9562**	0.7343	**9.4699**	0.7404	**9.5929**	0.6766	**8.4006**
4	0.6177	**8.6489**	0.6186	**7.4377**	0.5129	**5.8761**	0.6059	**7.2384**	0.4577	**5.1273**
5	0.4790	**6.3220**	0.3933	**4.2918**	0.4059	**4.4529**	0.4011	**4.3921**	0.3786	**4.1066**
6	0.3201	**3.9781**	0.3608	**3.8827**	0.3514	**3.7659**	0.3460	**3.6986**	0.3539	**3.7974**
7	0.1242	1.1923	0.0741	0.1982	0.2632	**2.6797**	0.1608	1.4096	0.0993	0.5810

Table 3.9 presents canonical correlations between the six Fama and French (2018) factors plus the mimicking factor FOP with ten principal components. The risk factors are Mkt, SMB, HML, CMA, RMW, MOM and FOP and the principal components are extracted according to the methodology of Connor and Korajczyk (1988) as proposed by Pukthuanthong et al. (2019). We report canonical correlations in descending order and a z-statistic to test the null hypothesis that the canonical correlation in the current row equals zero. Bold numbers indicate canonical correlations which are statistically significant (5% level).

that the canonical correlation in the given row equals zero. Bold figures indicate canonical correlations which are significant at the 5% level. The results are similar to Pukthuanthong et al. (2019). The first canonical variate exhibits a correlation close to one and the first six canonical correlations are statistically significant in each of the sub-periods. From 1987 to 1996, all canonical correlations are statistically significant.

The next step is the actual test whether the factor candidates are significantly related to the PC representation of the covariance matrix. Given the seven canonical variates, we regress the canonical variates on the full set of candidate factors in each sub-period. We thus estimate 35 individual regressions to evaluate whether the factor candidates are significantly related to the covariance matrix of stock returns. Panel A of Table 3.10 presents the average absolute t-statistic for each factor candidate as well as the average absolute t-statistic for canonical correlations which are statistically significant at the 5% level. Following Pukthuanthong et al. (2019), the latter is the relevant screening criterion. The risk factor pro-

Table 3.10: Risk factor protocol: Significance tests.

Panel A: Significance levels of the FF6 factors and FOP

	Risk factors						
	Mkt	SMB	HML	RMW	CMA	MOM	FOP
Mean |t-statistic|	**11.3742**	**7.1479**	**2.8551**	**2.0397**	1.7010	**2.9397**	**1.9619**
Mean |t-statistic| sign.	**12.8133**	**8.0284**	**3.1405**	**2.2404**	1.8388	**3.2671**	**2.1238**

Panel B: Number of \|t-statistics\| ≥ 1.96 out of 7

Subsample	Mkt	SMB	HML	RMW	CMA	MOM	FOP
1963 – 1976	3	5	4	3	4	6	3
1977 – 1986	3	3	2	5	0	4	4
1987 – 1996	3	4	3	3	4	4	3
1997 – 2006	4	3	4	4	5	4	3
2007 – 2016	2	4	4	3	3	3	0
Average #	**3**	**3.8**	**3.4**	**3.6**	**3.2**	**4.2**	**2.6**

Table 3.10 presents the second stage of the Pukthuanthong et al. (2019) risk factor protocol, i.e. a test whether factor candidates are significantly correlated with the cross-section of stock returns. Panel A presents the average absolute t-statistic of a multivariate regression of the canonical variates in Table 3.9 on the 7 factor candidates. We perform this regression for the five sub-periods and thus estimate 35 regressions. We further report the mean of significant absolute t-statistics, i.e. the mean for significant canonical correlations. Panel B presents the number of absolute t-statistics which exceed the critical value of 1.96 in each of the sub-periods. The last row reports the average over the sub-periods. Factors which pass the thresholds of Pukthuanthong et al. (2019), i.e. a mean absolute t-statistic ≥ 1.96 in Panel A and an average number of significant absolute t-statistics ≥ 2.5 in Panel B are presented in bold letters.

tocol requires that the mean of significant absolute t-statistics exceeds 1.96. Risk factor candidates which satisfy this criterion are printed in bold numbers. Panel B of Table 3.10 presents the second screening criterion. In each subsample, we estimate seven regressions which is the upper bound for the number of absolute t-statistics which exceed 1.96. We report the total number of |t-statistics| ≥ 1.96 as well as the average over all subsamples. Pukthuanthong et al. (2019) impose a threshold of 2.5 for genuine risk factors.[12]

[12] Pukthuanthong et al. (2019) originally estimate ten regressions in each subsample, such that the threshold of 2.5 is more rigorous in our analysis.

According to Table 3.10, only the conservative-minus-aggressive factor *CMA* falls short of the threshold in Panel A. This result is consistent with Pukthuanthong et al. (2019) who neglect *CMA* as a genuine risk factor. *FOP*, however, exhibits a mean absolute t-statistic higher than 1.96 in Panel A and an average number of 2.6 in Panel B. *FOP* passes the first stage of the risk factor protocol.

3.5.3 Is FOP priced?

In the second stage, the risk factor protocol requires that the risk factor under consideration earns a significant risk premium within reasonable bounds. Thus, we include β_{FOP} as an additional risk factor to the risk factor betas of the models FF3 and FF6. Following Lewellen et al. (2010), we use different test assets in Fama and MacBeth (1973) regressions.[13] The test assets are 25 portfolios sorted by Size and Operating Profitability (5x5) in Panel A, 25 portfolios sorted by Size and CAPM Beta (5x5) in Panel B and 32 portfolios sorted by Size, Book-to-Market and Investment (2x4x4) in Panel C. The test assets are also used in Fama and French (2015, 2018). In each analysis, we include the risk factors as dependent variables, as suggested by Lewellen et al. (2010) as well. We report full sample beta estimates in Column (1) to (6) and 5-year rolling window betas in Column (7) to (12). Other than that, Table 3.11 is identical to the Fama

[13]In untabulated robustness checks we perform Fama and MacBeth (1973) regressions in the cross-section of stock returns and find a significant risk premium on β_{FOP}. However, the nonlinearity becomes more severe in the cross-section of stock returns, also shown in Barinov (2018). Since high $IVol_{FF3}^{1M}$ stocks are very illiquid with a high fraction of zero returns, a precise estimation of β_{FOP} presents another challenge for future research.

and MacBeth (1973) regression in Section 3.4.2.

First, we consider the univariate analyses in Columns (1), (4), (7) and (10). The negative coefficient on β_{FOP} is significantly negative with t-statistics beyond minus three with two exceptions. In Column (4) of Panel A, the average coefficient on β_{FOP} of -0.27 exhibits a t-statistic of -2.09. Furthermore, β_{FOP} is insignificant in Column (10) . In both cases, however, t-statistics increase in absolute terms as soon as we control for the FF6 risk factors such that both coefficients are significant at conventional levels.

In multivariate analyses, β_{FOP} is always statistically significant with a negative risk premium. Although this premium depends on the set of test assets, it exhibits a similar magnitude around -45 bps. The inclusion of β_{FOP} alleviates alphas of the base assets in several cases of the full sample beta analysis. In Panel A, comparing Column (2) and (3), the negative Intercept of -0.1275 with a t-statistic of -4.98 becomes insignificant as soon as we include β_{FOP}. Similarly, in Panel C, the FF3 and the FF6 models both leave a significant negative Intercept in case of full sample betas. Including β_{FOP} into both of the models leads to insignificant Intercepts in Column (3) and (6). We conclude that β_{FOP} is priced with a significant risk premium in multiple test assets.

Finally, MacKinlay (1995) proposes an upper bound of 0.6 for the annual Sharpe ratio for the tangency portfolio. Since FOP is a zero-investment portfolio, we follow Pukthuanthong et al. (2019) and combine FOP with the market portfolio to test whether the resultant Sharpe ratio is greater than 0.6. The annualized Sharpe ratio of 0.0455 is well below this threshold. To rule out that the underlying characteristic itself violates

Table 3.11: Fama and MacBeth (1973) portfolio regressions with FOP.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Size/ Operating Profitability 5x5												
	Full Sample Betas						5-year rolling window betas					
Intercept	0.5507	-0.1275	-0.0371	0.5738	0.0717	0.0557	0.5867	-0.0555	-0.0168	0.5864	0.1521	0.1134
	(2.70)	(-4.98)	(-0.99)	(3.02)	(2.97)	(2.35)	(2.85)	(-1.27)	(-0.45)	(3.12)	(4.71)	(4.23)
β_{FOP}	-0.4475		-0.3885	-0.2721		-0.3103	-0.4476		-0.3384	-0.1136		-0.2094
	(-4.50)		(-3.91)	(-2.09)		(-3.04)	(-5.14)		(-4.63)	(-1.30)		(-2.93)
β_{Mkt}		0.5996	0.5132		0.4694	0.4543		0.5231	0.4813		0.3887	0.4208
		(2.95)	(2.48)		(2.31)	(2.23)		(2.71)	(2.45)		(1.92)	(2.08)
β_{SMB}		0.1821	0.1600		0.0964	0.1376		0.1733	0.2027		0.1487	0.1651
		(1.35)	(1.20)		(0.73)	(1.06)		(1.35)	(1.59)		(1.16)	(1.31)
β_{HML}		0.4493	0.3506		0.3613	0.3133		0.2792	0.3171		0.2634	0.2818
		(2.91)	(2.36)		(2.32)	(2.08)		(1.67)	(2.06)		(2.10)	(2.17)
β_{CMA}					0.2217	0.2657					0.1346	0.1839
					(2.13)	(2.51)					(1.61)	(2.18)
β_{RMW}					0.2551	0.2381					0.2088	0.2135
					(2.17)	(2.07)					(2.05)	(1.99)
β_{MOM}					0.5348	0.5745					0.3989	0.4502
					(2.80)	(3.03)					(2.16)	(2.50)
$avg.\overline{R}^2$ in %	6.57	52.41	61.23	1.82	67.26	70.53	4.13	55.23	59.35	0.79	67.00	68.97
Panel B: Size/ Beta 5x5												
	Full Sample Betas						5-year rolling window betas					
Intercept	0.5620	-0.1302	-0.0765	0.5564	-0.2379	-0.0860	0.5898	-0.0283	-0.0031	0.5713	0.0120	0.0580
	(2.73)	(-4.39)	(-2.03)	(3.06)	(-4.87)	(-2.32)	(2.89)	(-0.67)	(-0.08)	(3.14)	(0.35)	(1.80)
β_{FOP}	-0.3723		-0.3761	-0.5378		-0.3153	-0.2979		-0.2914	-0.2925		-0.2761
	(-3.54)		(-4.06)	(-4.85)		(-3.73)	(-3.18)		(-4.06)	(-3.36)		(-3.87)
β_{Mkt}		0.5771	0.4859		0.6525	0.5254		0.5068	0.4767		0.5169	0.4637
		(2.84)	(2.33)		(3.24)	(2.55)		(2.53)	(2.36)		(2.54)	(2.27)
β_{SMB}		0.1388	0.2302		0.2480	0.2366		0.1528	0.1736		0.1750	0.1780
		(1.07)	(1.80)		(1.89)	(1.80)		(1.19)	(1.35)		(1.37)	(1.39)
β_{HML}		0.8314	0.5562		0.6839	0.4651		0.4917	0.4355		0.3311	0.3011
		(4.59)	(3.77)		(3.53)	(2.84)		(2.90)	(2.95)		(2.26)	(2.16)
β_{CMA}					0.4988	0.4027					0.2668	0.2639
					(4.03)	(3.50)					(2.65)	(2.78)
β_{RMW}					0.4738	0.2603					0.1447	0.1191
					(2.88)	(2.02)					(1.46)	(1.12)
β_{MOM}					0.8905	0.7305					0.5589	0.5084
					(4.39)	(3.64)					(2.89)	(2.79)
$avg.\overline{R}^2$ in %	5.35	55.14	60.85	2.64	65.29	67.88	5.77	56.19	60.60	3.30	65.13	67.95
Panel C: Size/ Book-to-Market/ Investment 2x4x4												
	Full Sample Betas						5-year rolling window betas					
Intercept	0.5782	-0.0448	0.0120	0.5900	-0.1344	-0.0311	0.6161	0.0278	0.0456	0.5990	0.0644	0.0703
	(2.92)	(-2.11)	(0.48)	(3.27)	(-6.17)	(-1.29)	(3.09)	(1.05)	(1.68)	(3.30)	(2.58)	(2.79)
β_{FOP}	-0.4608		-0.4322	-0.5977		-0.3962	-0.3112		-0.2940	-0.2863		-0.2951
	(-4.99)		(-5.47)	(-5.15)		(-5.05)	(-6.10)		(-5.45)	(-5.14)		(-4.76)
β_{Mkt}		0.5728	0.4467		0.5858	0.4850		0.4793	0.4518		0.4653	0.4472
		(2.83)	(2.19)		(2.92)	(2.42)		(2.54)	(2.38)		(2.39)	(2.28)
β_{SMB}		0.1727	0.2421		0.2253	0.2350		0.1923	0.2015		0.1675	0.1819
		(1.35)	(1.90)		(1.75)	(1.83)		(1.52)	(1.59)		(1.32)	(1.44)
β_{HML}		0.3115	0.2778		0.3671	0.3083		0.2509	0.2587		0.2857	0.2910
		(2.06)	(1.87)		(2.39)	(2.06)		(1.83)	(1.88)		(2.11)	(2.14)
β_{CMA}					0.3703	0.3543					0.2491	0.2570
					(3.76)	(3.61)					(2.93)	(3.00)
β_{RMW}					0.4598	0.2767					0.0782	0.1415
					(3.36)	(2.32)					(0.85)	(1.44)
β_{MOM}					0.7777	0.6889					0.4651	0.4800
					(4.09)	(3.60)					(2.72)	(2.81)
$avg.\overline{R}^2$ in %	2.24	48.59	50.35	1.44	58.94	59.81	0.69	46.88	48.89	0.40	57.29	59.07

Table 3.11 presents average coefficients of Fama and MacBeth (1973) cross-sectional portfolio level regressions of excess returns in month $t+1$ the factor model betas of the Fama and French (1993) three factor model (FF3), the Fama and French (2018) six factor model (FF6) and both models extended by the mimicking factor *FOP* in month t. Columns (1) to (6) contain full sample betas and Columns (7) to (12) cover 5-year rolling window estimations with monthly data. We use the following base assets as dependent variables: 25 portfolios sorted by Size and operating profitability (5x5) in Panel A, 25 portfolios sorted by Size and beta (5x5) in Panel B and 32 portfolios sorted by Size, book-to-market and investment (2x4x4) in Panel C. The base assets are sorted according to the methodology in Fama and French (2015). Average coefficients are multiplied by one hundred and the factor portfolios of the respective model are included among the test assets. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in %. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses. The sample period is August 1963 to December 2016 in Columns (1) to (6) and June 1968 to December 2016 in Columns (7) to (12).

this threshold, we repeat the analysis and combine the $IVol_{FF3}^{1M}$ decile hedge portfolio in Table 3.1. This combination exhibits an annualized Sharpe ratio of 0.13, also well below the upper bound of 0.6.

The latent factor behind the $IVol$ puzzle satisfies all criteria of a genuine risk factor. The mimicking factor FOP is significantly related to the covariance matrix of stock returns and earns a negative risk premium within reasonable bounds. We conclude that the $IVol$ puzzle is the result of latent systematic risk and address potential sources of this risk in the following Section.

3.6 Testing economic drivers behind the IVol puzzle

3.6.1 Arbitrage constraints versus behavioral explanations

Having shown that FOP alleviates the $IVol$ puzzle and is attributable to systematic, yet unidentified risk, we can address potential sources of this risk. First, to facilitate the quest for the source of the IVol puzzle, we discriminate between arbitrage constraints (e.g. Frazzini and Pedersen, 2014; Asness et al., 2019) and behavioral explanations (e.g. Bali et al., 2011, 2017; Liu et al., 2018) as the two major theoretical propositions to the underperformance of high-risk stocks.

Following Asness et al. (2019), we run time series regressions of the mimicking factor FOP on proxies for arbitrage constraints and investor sentiment. The dependent variable is FOP in month t as well as one month ahead $t+1$. Proxies for arbitrage constraints cover the Ted Spread and Margin Debt of NYSE customers in relation to the NYSE market capitalization. A high Ted Spread indicates tight funding liquidity conditions while low Margin Debt is tantamount to tight leverage constraints (see Frazzini and Pedersen, 2014; Jacobs, 2015; Asness et al., 2019). Thus, the expected sign is negative for the Ted Spread and positive for Margin Debt. The two measures for investor sentiment are the Baker and Wurgler (2006) (BW) Investor Sentiment Index and Equity Fund Flows in Billion USD as the net flows into US equity mutual funds. Brown et al. (2002) and Da et al. (2015) provide evidence that flows to equity funds are an instrument for investor sentiment. We expect a negative sign for both sentiment proxies since the $IVol$ puzzle is stronger during periods of high investor sentiment (Stambaugh et al., 2015). We control for the economic state by including the CFNAI index.[14] All coefficients are multiplied with one hundred. The sample period is 1986 to 2016 in Columns (1) and (6), 1967 to 2016 in Columns (2) and (7), 1965 to 2015 in Columns (3) and (8) and 1993 to 2015 in Columns (4), (5), (9) and (10). The t-statistics in parentheses are calculated from Newey and West (1987) standard errors with six lags.

The results in Table 3.12 support behavioral explanations for the risk factor behind the $IVol$ puzzle. While the Ted Spread is overall insignificant, the coefficient for Margin Debt is mostly statistically significant, but switches signs. The sign on Margin Debt is significantly positive in the

[14] Excluding CFNAI yields identical results with respect to the variables of interest.

Table 3.12: Arbitrage constraints versus behavioral explanations.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			FOP_t					FOP_{t+1}		
Ted Spread	5.2124				24.2808	7.5907				42.4575
	(0.34)				(0.91)	(0.43)				(1.13)
Margin Debt		188.8351			72.9243		139.0789			-499.5209
		(2.49)			(0.33)		(1.87)			(-1.96)
BW Sentiment			-0.1300		-0.5592			-0.1194		-0.4946
			(-2.16)		(-2.70)			(-2.24)		(-2.48)
Equity Fund Flows				-0.0001	0.0122				-0.0160	-0.0251
				(-0.01)	(0.91)				(-1.72)	(-1.81)
CFNAI	-0.1235	-0.0524	-0.0717	-0.1673	-0.1143	-0.1893	-0.0200	-0.0368	-0.1870	-0.1469
	(-1.45)	(-1.07)	(-1.52)	(-2.20)	(-0.86)	(-1.79)	(-0.31)	(-0.56)	(-1.82)	(-1.03)
Intercept	-0.5311	-0.8766	-0.5632	-0.4378	-0.5780	-0.5531	-0.7922	-0.5640	-0.4423	0.5332
	(-4.75)	(-7.12)	(-10.25)	(-4.29)	(-1.11)	(-4.90)	(-6.53)	(-10.15)	(-4.25)	(0.92)
N	372	598	583	287	272	371	597	583	286	272

Table 3.12 presents time series regressions of the mimicking factor FOP on predictor variables for constraints to arbitrage and investor sentiment. We include the following variables: The Ted spread, Margin Debt of NYSE customers in relation to NYSE market capitalization, the Baker and Wurgler (2006) (BW) Investor Sentiment Index, Fund Flows as the net flow of equity minus bond mutual fund flows and the Chicago Fed National Activity Index CFNAI. All coefficients are multiplied with one hundred. The sample period is 1986 to 2016 in Columns (1) and (6), 1967 to 2016 in Columns (2) and (7), 1965 to 2015 in Columns (3) and (8) and 1993 to 2015 in Columns (4), (5), (9) and (10). t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

bivariate analyses in Columns (2) and (7) with t-statistics of 2.49 and 1.87, but becomes insignificant once we include alternative proxies in Column (5). In Column (10), Margin Debt is even significantly negative with a coefficient of -499.52 and a t-statistic of -1.96. By and large, the evidence with respect to arbitrage constraints is inconclusive at best.

Conversely, the coefficients on BW Sentiment are significantly negative in each Column with t-statistics of -2.16 and -2.24 in the bivariate analysis in Columns (3) and (8) as well as -2.70 and -2.48 in the multivariate analysis in Columns (5) and (10). Equity Fund Flows are unrelated to contemporary FOP_t, but negatively predict FOP_{t+1} bivariately and multivariately in Columns (9) and (10). Both findings are consistent with a behavioral explanation for the $IVol$ puzzle and in line with Asness et al.

(2019) who relate $IVol$ to behavioral factors as well. We thus focus on behavioral explanations for the $IVol$ puzzle.

3.6.2 Sentiment, noise traders and the IVol puzzle

The negative relationship between BW Investor Sentiment and FOP is consistent with a strand of literature which relates $IVol$ to noise trading (e.g. Foucault et al., 2011; Aabo et al., 2017). Black (1986) and De Long et al. (1990) provide a theoretical framework in which sentiment or noise traders trade on a noisy signal – for example sentiment – and expose arbitrageurs who encounter limits to arbitrage to systematic noise trader risk. As a consequence, changes in investor sentiment lead to greater mispricing and temporary volatility spikes (see Da et al., 2015; Brown, 1999). A formal test of this economic mechanism requires that FOP shares the predictive power of investor sentiment with respect to market-wide mispricing and is positively related to short-term volatility.

To test the first implication above, we make use of the explanatory power of BW Investor sentiment with respect to stock market anomalies. Stambaugh et al. (2012) and Jacobs (2015) show that several well-documented stock market anomalies are stronger during periods of high sentiment. If FOP represents noise trader risk, we expect that the results of Stambaugh et al. (2012) for BW Investor Sentiment translate to FOP, but with an opposite sign. Thus, anomalous returns are expected to be higher during periods of more negative FOP, tantamount to high levels of noise trader risk given the negative risk premium on FOP.

Following Stambaugh et al. (2012), we run the time series regression

$$r_{i,t} = \alpha_H d_{H,t} + \alpha_L d_{L,t} + \beta_{Mkt} Mkt + \beta_{SMB} SMB + \beta_{HML} HML + \epsilon_{i,t}, \quad (3.10)$$

where $d_{H,t}$ ($d_{L,t}$) is an indicator variables which is equal to one if BW Investor Sentiment or FOP are above (below) the sample median and zero otherwise.[15] We furthermore include the FF3 risk factors in order to compare our results to Stambaugh et al. (2012). Average benchmark-adjusted returns in the different states are the estimates on α_H and α_L. The left-hand side assets are spread portfolios of the anomalies considered in Stambaugh et al. (2012, 2015).[16]

Table 3.13 presents the coefficient estimates on the indicator variables $d_{H,t}$ and $d_{L,t}$, separately for BW Investors Sentiment (Sent) and FOP. We furthermore report the difference between high and low periods of Sent (FOP) and the corresponding t-statistic for the null hypothesis that this difference equals zero. The t-statistics in parentheses are computed from Newey and West (1987) standard errors with six lags. All coefficients are

[15] Stambaugh et al. (2012) use lagged sentiment to compute the indicator variables which yields almost identical results.

[16] Stambaugh et al. (2012, 2015) consider the following anomalies: Total Accruals as changes in working capital minus depreciation expense (Sloan, 1996), Asset Growth as the total growth rate of assets in the previous fiscal year (Cooper et al., 2008), Composite Stock Issue as the growth in market equity which is not attributable to stock returns (Daniel and Titman, 2006), Failure Probability as the expected default probability from a logit model (Campbell et al., 2008), Gross Profitability as the ratio of gross profits to assets (Novy-Marx, 2013), Investment-to-Asset as the annual change in gross property, plant and equipment plus inventories, scaled by lagged book value of assets (Titman et al., 2004), Momentum as the cumulative return over the previous eleven months (Jegadeesh and Titman, 1993), Net Operating Assets as the cumulative difference between operating income and free cash flow (Hirshleifer et al., 2004), the O-Score as a probability of bankruptcy from a static model with accounting data (Ohlson, 1980), Return on Assets as the ratio of quarterly earnings to last quarter's assets (Chen et al., 2011) and Net Stock Issue of Ritter (1991) as the growth rate of split-adjusted shares outstanding (see Stambaugh et al., 2015, pp. 1942–1944). For details regarding the measurement of the anomalies, we refer to Stambaugh et al. (2015).

Table 3.13: FOP, sentiment and unrelated anomalies.

Anomaly	BW Investor Sentiment High		BW Investor Sentiment Low		BW Investor Sentiment High–Low		FOP High		FOP Low		FOP High–Low	
Total Accruals	0.2166	(1.08)	0.7382	(4.20)	-0.5216	(-2.17)	0.7032	(3.40)	0.2235	(1.15)	0.4797	(1.73)
Asset Growth	0.3489	(2.08)	0.1253	(0.88)	0.2236	(1.03)	0.0036	(0.02)	0.4958	(2.83)	-0.4922	(-1.96)
Composite Equity Issue	0.5691	(3.39)	0.3480	(2.61)	0.2211	(1.04)	0.2615	(1.61)	0.6782	(4.60)	-0.4167	(-1.89)
Failure Probability	1.7865	(5.88)	0.5405	(2.08)	1.2461	(3.17)	0.4527	(1.56)	2.0631	(5.69)	-1.6104	(-3.22)
Gross Profitability	0.8285	(5.23)	0.5144	(3.01)	0.314	(1.39)	0.3990	(2.29)	0.9755	(5.39)	-0.5765	(-2.19)
Investment-to-Assets	0.4090	(2.35)	0.4610	(2.97)	-0.052	(-0.23)	0.5090	(3.16)	0.3537	(1.91)	0.1553	(0.63)
Momentum	1.6668	(5.06)	1.4781	(3.89)	0.1887	(0.37)	1.2402	(2.83)	1.9351	(5.32)	-0.695	(-1.08)
Net Operating Assets	0.7590	(4.27)	0.2364	(1.47)	0.5226	(2.28)	0.3784	(2.19)	0.6470	(3.53)	-0.2686	(-1.10)
O-Score	0.6265	(4.28)	0.2035	(1.29)	0.423	(2.00)	0.2247	(1.33)	0.6359	(4.39)	-0.4112	(-1.85)
Return on Assets	1.1642	(5.49)	0.5215	(2.16)	0.6427	(2.21)	0.1305	(0.53)	1.6573	(6.60)	-1.5268	(-4.23)
Net Stock Issue	0.7188	(4.67)	0.4952	(3.80)	0.2236	(1.12)	0.5022	(3.25)	0.7282	(5.11)	-0.226	(-1.05)

Table 3.13 presents average FF3 alphas of the 11 anomalies in Stambaugh et al. (2012, 2015) during periods of high and low investor sentiment (FOP) as well as the difference between high and low periods. High (low) periods are defined as months in which Baker and Wurgler (2006) Investor Sentiment or *FOP* are higher (lower) than the sample median. Periods of low *FOP* are tantamount to periods of high noise trader risk. Stambaugh et al. (2012, 2015) consider the following anomalies: Total Accruals as changes in working capital minus depreciation expense, Asset Growth as the total growth rate of assets in the previous fiscal year, Composite Stock Issue as the growth in market equity which is not attributable to stock returns, Failure Probability as the expected default probability from a logit model, Gross Profitability as the ratio of gross profits to assets, Investment-to-Asset as the annual change in gross property, plant and equipment plus inventories, scaled by lagged book value of assets, Momentum as the cumulative return over the previous eleven months, Net Operating Assets as the cumulative difference between operating income and free cash flow, the O-Score as a probability of bankruptcy from a static model with accounting data, Return on Assets as the ratio of quarterly earnings to last quarter's assets and Net Stock Issue as the growth rate of split-adjusted shares outstanding (see Stambaugh et al., 2015, pp. 1942–1944). The sample period is 1963 to 2016 except for the anomalies Accruals, Failure Probability and Return on Assets which start in 1966, 1973 and 1971, respectively. *t*-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

presented in % per month. The sample period is 1963 to 2016 except for the anomalies Accruals, Failure Probability and Return on Assets which start in 1966, 1973 and 1971, respectively. The results for the different BW Investor Sentiment regimes resemble the findings of Stambaugh et al. (2012). For five anomalies, the difference between benchmark-adjusted returns in the two sentiment regimes is statistically significant at the five percent level. Except for Total Accruals, anomaly spread returns are higher during periods of high sentiment, consistent with the hypothesis that overpricing is more likely to prevail when individual investors are optimistic.[17] However, the Total Accruals anomaly is exposed to sentiment as well with a difference of roughly -52 bps (t-statistic = -2.17).

Consistent with the noise trading hypothesis, the difference between high and low sentiment periods translate to *FOP*. Anomalies which are exposed to sentiment tend to have significant exposure to *FOP* and thus significantly different average returns in the two *FOP* regimes. This difference has the exact opposite sign compared to the BW Investor Sentiment regimes and is statistically significant for seven out of eleven anomalies. Comparing the magnitude of the difference further supports the hypothesis that *FOP* represents noise trader risk due to sentiment. For example, in case of Failure Probability which exhibits the highest difference between high and low sentiment regimes with 125 bps (t-statistic = 3.17), we find that *FOP* has the largest effect of all anomalies with -161 bps and a t-statistic of 3.22. Furthermore, the five anomalies Asset Growth, Composite Equity Issue, Failure Probability, O-Score and Return on Assets only prevail in the high noise trading regime, i.e. when *FOP* is below the sample median.[18]

The second implication of the proposed economic mechanism requires that noise trading is positively related to temporary volatility increases (De Long et al., 1990; Da et al., 2015). To illustrate that this effect is short-lived, we compute a daily counterpart of *FOP* and daily changes in

[17] The different sign is likely due to different data for the anomaly spreads and a more recent vintage of BW Investor Sentiment.

[18] Including *FOP* into the FF3 model alleviates the alphas on the anomalies Asset Growth, Failure Probability and O-Score. Since the latter two are also considered low-risk anomalies, this evidence further supports our findings.

Table 3.14: Noise trader risk and volatility.

	$GARCH_d$	$GARCH_{d+1}$	$GARCH_{d+2}$	$GARCH_{d+3}$	$GARCH_{d+4}$	$GARCH_{d+5}$	FOP_d^{neg}	FOP_{d+1}^{neg}	FOP_{d+2}^{neg}	FOP_{d+3}^{neg}	FOP_{d+4}^{neg}	FOP_{d+5}^{neg}
FOP^{neg}	0.0438	-0.0071	0.0033	-0.0037	-0.0045	-0.0009						
	(3.11)	(-0.41)	(0.37)	(-0.42)	(-0.64)	(-0.13)						
GARCH							0.1355	-0.0443	-0.0137	0.0257	0.0303	-0.0232
							(4.68)	(-1.34)	(-0.57)	(1.14)	(1.35)	(-1.07)
ADS	0.2873	0.1752	0.0619	-0.0080	-0.0085	0.0854	0.5648	0.5721	0.6000	0.5741	0.6340	0.7759
	(0.97)	(0.55)	(0.21)	(-0.03)	(-0.03)	(0.29)	(1.62)	(1.64)	(1.72)	(1.67)	(1.84)	(2.24)
EPU	0.0084	0.0027	-0.0014	-0.0041	-0.0040	0.0003	-0.0003	-0.0020	-0.0016	-0.0028	-0.0005	0.0043
	(2.44)	(0.96)	(-0.67)	(-1.90)	(-1.84)	(0.10)	(-0.11)	(-0.62)	(-0.48)	(-0.86)	(-0.15)	(1.25)

Table 3.14 presents results from time series regressions of changes in daily volatility (GARCH) on FOP and vice versa over different leads of the dependent variable. We estimate daily conditional variance from an asymmetric GARCH(1,1) model as proposed by Glosten et al. (1993). We control for the Aruoba et al. (2009) (ADS) business cycle index as well as a news-based policy uncertainty index (EPU) as proposed by Baker et al. (2016). The sample period is January 1985 to December 2016. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

volatility of the market portfolio from an asymmetric GARCH(1,1) model

$$r_d = a + b \cdot r_{d-1} + \epsilon_d, \tag{3.11}$$

$$\sigma_d^2 = \omega + \alpha_1 \epsilon_d^2 + \alpha_2 \sigma_{d-1}^2 + \beta \cdot I \cdot \epsilon_d^2, \tag{3.12}$$

where r_d is the daily return of the CRSP market portfolio and I is an indicator variable which is equal to one if $\epsilon_{d-1} > 0$ (see Glosten et al., 1993). We take the square root of σ_d^2 and include first differences. For the ease of interpretation we include the negative of FOP such that a higher level of FOP^{neg} is equivalent to higher levels of noise trader risk. Following Da et al. (2015), we control for daily economic conditions and policy uncertainty by including the Aruoba et al. (2009) business cycle index (ADS) and the Baker et al. (2016) policy uncertainty index (EPU). The t-statistics in parentheses are computed from Newey and West (1987) standard errors with six lags. All coefficients are multiplied by 10^4. The sample period is January 1985 to December 2016. The contemporaneous relationship between noise trader risk and aggregate volatility is significantly positive at conventional levels in both directions. High noise trader risk today is associated with high volatility in the market portfolio today and vice versa with t-statistics well beyond three. Consistent with noise trading, this effect is short-lived and thus insignificant over the following five trading days. While policy uncertainty is unrelated to FOP, the ADS business cycle index is weakly positively related to FOP^{neg}.

FOP resembles the explanatory power of BW Investor sentiment with respect to asset pricing anomalies, in line with the findings of Stambaugh et al. (2012) and Jacobs (2015). Anomalies earn higher average FF3 alphas when arbitrageurs are exposed to high levels of noise trading risk and market-wide mispricing is more likely to prevail. As suggested from the theoretical model of De Long et al. (1990), higher noise trader risk contributes to temporary increases in volatility. Both findings support the noise trading hypothesis as an explanation for the $IVol$ puzzle.

3.7 Robustness checks

3.7.1 OP and the choice of base assets

The framework of MacKinlay (1995) and MacKinlay and Pastor (2000) is agnostic with respect to the assets which constitute OP. Constructing OP from the residuals of 25 Size-$IVol_{FF3}^{1M}$ portfolios might be subject to the concern that the baseline results depend on this particular choice and thus, OP could be tautology.

We extend the set of portfolios which generate the residuals to form

Table 3.15: Fama and MacBeth (1973) regressions with OP from different portfolios.

Panel A: Explaining 25 Size-$IVol$ portfolios with OP from 25 Size-BM portfolios

Model	Intercept		$IVol^{1M}_{FF3}$		$\beta_{OP_{BM}}$		β_{Mkt}		β_{SMB}		β_{HML}		$avg.\overline{R}^2$ in %
(1)	0.1570	(4.26)	-55.791	(-3.20)			0.4827	(2.68)	0.0985	(0.72)	0.8113	(4.94)	55.38
(2)	0.6015	(4.40)	59.9537	(1.45)	-0.5842	(-3.00)							26.40
(3)	0.6852	(3.84)			-0.4661	(-1.80)							20.21
(4)	0.0130	(0.35)	-0.9493	(-0.06)	-0.7000	(-4.91)	0.5854	(3.10)	0.2190	(1.57)	0.6599	(4.23)	59.27
(5)	0.0099	(0.28)			-0.7022	(-4.89)	0.5736	(2.97)	0.2212	(1.57)	0.6841	(4.27)	59.45

Panel B: Explaining 25 Size-$IVol$ portfolios with OP from 25 Size-Mom portfolios

Model	Intercept		$IVol^{1M}_{FF3}$		$\beta_{OP_{Mom}}$		β_{Mkt}		β_{SMB}		β_{HML}		$avg.\overline{R}^2$ in %
(6)	0.1529	(4.00)	-50.802	(-3.04)			0.4752	(2.61)	0.0916	(0.68)	0.7514	(4.56)	55.89
(7)	0.6247	(4.70)	10.3992	(0.25)	-0.2874	(-2.29)							23.57
(8)	0.6305	(3.56)			-0.2866	(-1.63)							17.52
(9)	0.0825	(2.16)	-23.680	(-1.59)	-0.2547	(-2.56)	0.5245	(2.75)	0.1436	(1.04)	0.6715	(4.30)	60.13
(10)	0.0774	(2.06)			-0.2703	(-2.70)	0.4734	(2.43)	0.1429	(1.03)	0.6907	(4.30)	60.30

Table 3.15 presents average coefficients of Fama and MacBeth (1973) cross-sectional portfolio level regressions of excess returns in month $t+1$ on $IVol^{1M}_{FF3}$, the sensitivity $\beta_{OP_{PF}}$ to the latent factor OP and the factors Mkt, SMB and HML of the Fama and French (1993) three factor model. The subscript PF indicates the portfolios which generate residuals to from OP. We use three sets of double sorted portfolios: In Panel A, we use the residuals of 25 portfolios sorted by Size and Book-to-Market (Size-BM) to form OP, add OP to the $FF3$ factors and estimate the extended $FF3$ model to explain returns of 25 Size-$IVol$ portfolios. Panel B uses 25 portfolios sorted by Size and Momentum (Size-Mom) to form OP. We estimate betas over the full sample. The sample period is July 1964 to December 2016. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in %. Average coefficients are multiplied by one hundred and the factor portfolios Mkt, SMB and HML are included among the test assets. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

OP. We use the daily FF3 residuals of 25 portfolios sorted by Size and Book-to-Market (BM) as well Size and Momentum (Mom) to construct OP_{PF} according to Equation (3.7).[19] The subscript PF indicates the portfolios which generate the residuals for OP. Next, we add OP_{PF} to the FF3 factors, estimate the model for 25 Size-$IVol^{1M}_{FF3}$ portfolios and re-evaluate the analysis in Table 3.3 which relies on full sample beta estimates. Panel A (B) of Table 3.15 presents the results for 25 Size-BM (Size-Mom) portfolios and is otherwise identical to the baseline analysis.

[19]In untabulated robustness checks we also consider other sort variables and draw identical conclusions.

In Models (1) and (6), the FF3 model leaves a significant coefficient on $IVol^{1M}_{FF3}$ with a magnitude of -55.79 (t-statistic = -3.20) and -50.80 (t-statistic = -3.05), respectively. The inclusion of $\beta_{OP_{PF}}$ explains these negative coefficients. This holds true in the bivariate analysis in Models (2) and (7) as well as the multivariate analysis in Models (4) and (9) which controls for the FF3 risk factor betas. In general, Panel A is similar to the baseline analysis and the inclusion of $\beta_{OP_{BM}}$ does not only alleviate $IVol^{1M}_{FF3}$, but also fully explains the Intercept of the 25 Size-$IVol^{1M}_{FF3}$ portfolios. The reduction of the Intercept extends to Panel B, although the Intercept remains statistically significant in Models (9) and (10). The finding that the latent factor OP alleviates the $IVol$ puzzle is independent of the assets which constitute OP.

3.7.2 FOP versus FIVOL

To address the concern that OP and thus FOP are statistical transformations of $IVol$ itself, we directly form two factor candidates on $IVol$, referred to as $FIVOL$. For the first factor $FIVOL^{2x3}$ we follow the sorting procedure of Fama and French (1993). Each month, we sort stocks into two groups based on market capitalizations with NYSE breakpoints and then further sort stocks into three $IVol$ groups. The $IVol$ breakpoints are the 30th and the 70th percentile. The second factor $FIVOL^{10}$ is a value-weighted portfolio which is long in the highest $IVol$ decile and short in the lowest $IVol$ decile.

Table 3.16 presents correlations between FOP, the two $IVol$ factors

Table 3.16: FOP versus $FIVOL$: Correlations.

-	FOP	$FIVOL^{2x3}$	$FIVOL^{10}$	$IVol$ Dec 1	$IVol$ Dec 2	$IVol$ Dec 3	$IVol$ Dec 4	$IVol$ Dec 5	$IVol$ Dec 6	$IVol$ Dec 7	$IVol$ Dec 8	$IVol$ Dec 9
$FIVOL^{2x3}$	0.5362											
$FIVOL^{10}$	0.5249	0.9322										
$IVol$ Dec 1	-0.2149	0.2506	0.1532									
$IVol$ Dec 2	-0.1227	0.3902	0.3295	0.8929								
$IVol$ Dec 3	-0.1032	0.4605	0.4073	0.8657	0.9306							
$IVol$ Dec 4	-0.0641	0.5235	0.4654	0.8364	0.9057	0.9374						
$IVol$ Dec 5	-0.0141	0.6005	0.5372	0.7994	0.8852	0.9233	0.9333					
$IVol$ Dec 6	0.0599	0.6941	0.6269	0.7484	0.8387	0.8849	0.9021	0.9263				
$IVol$ Dec 7	0.1315	0.7602	0.6979	0.7155	0.8199	0.862	0.8814	0.9152	0.9323			
$IVol$ Dec 8	0.1829	0.8367	0.7478	0.6638	0.7699	0.8152	0.8438	0.8793	0.9186	0.9448		
$IVol$ Dec 9	0.2519	0.8674	0.794	0.6183	0.7321	0.7803	0.8087	0.8589	0.8998	0.9301	0.9439	
$IVol$ Dec 10	0.3465	0.8892	0.9038	0.5613	0.6626	0.7160	0.7519	0.7960	0.8491	0.8943	0.9138	0.9327

Table 3.16 presents correlations between FOP, the two $IVol$ factors $FIVOL^{2x3}$ and $FIVOL^{10}$ as well as value-weighted $IVol$ decile portfolios. The superscripts indicate the sorting methodology which is used to construct $IVol$ factors: $FIVOL^{2x3}$ is constructed according to the methodology of Fama and French (1993) as the intersection of two Size and three $IVol$ portfolios. $FIVOL^{10}$ is the high-minus low portfolio from the value-weighted $IVol$ decile sorts. The sample period is August 1963 to December 2016.

$FIVOL^{2x3}$ and $FIVOL^{10}$ as well as the value-weighted $IVol^{1M}_{FF3}$ decile portfolios which are the left-hand side assets in Table 3.8. The sample period is August 1963 to December 2016. The correlation between FOP and the two $IVol$ factors is moderate and amounts to 0.5362 for $FIVOL^{2x3}$ and 0.5249 for $FIVOL^{10}$, respectively. In contrast, $FIVOL^{2x3}$ and $FIVOL^{10}$ are close to perfectly correlated with a correlation coefficient of 0.9322. The correlation between FOP and the decile portfolios is moderate as well and increases monotonically from -0.2149 for the lowest to 0.3465 to the highest decile. Clearly, FOP is not an $IVol$ factor in disguise.

To further emphasize this conclusion, Table 3.17 presents spanning regressions in the spirit of Barillas and Shanken (2017, 2018) for FOP and the two $IVol$ factors $FIVOL^{2x3}$ and $FIVOL^{10}$. We explain each asset pricing factor with the two remaining factors extended by the FF3 risk factors. We present t-statistics from Newey and West (1987) adjusted standard errors with six lags in parentheses. The sample period is August 1963 to December 2016.

Table 3.17: *FOP* versus *FIVOL*: Spanning regressions.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	FOP				$FIVOL^{2x3}$					$FIVOL^{10}$				
Intercept in%	-0.5018	-0.3250	-0.4893	-0.3214	-0.5161	0.6928	0.1178	0.0630	-0.0553	-0.8887	0.8040	-0.0072	-0.2005	-0.2802
	(-10.70)	(-8.16)	(-10.29)	(-7.73)	(-2.57)	(2.74)	(1.05)	(0.85)	(-0.79)	(-3.06)	(2.49)	(-0.04)	(-1.92)	(-2.45)
FOP						2.1136	1.6045				2.9593	2.3633		
						(6.08)	(15.64)				(7.08)	(11.96)		
$FIVOL^{2x3}$	0.1360	0.2608											1.3334	1.3296
	(7.52)	(17.48)											(35.34)	(18.22)
$FIVOL^{10}$			0.0931	0.1534				0.6517	0.5310					
			(7.64)	(16.66)				(30.98)	(36.61)					
MktRF		-0.1460		-0.1135			0.4731		0.1525			0.5782		-0.0598
		(-11.91)		(-8.98)			(12.57)		(7.75)			(8.32)		(-1.75)
SMB		-0.1282		-0.1184			0.6289		0.1046			1.0279		0.2059
		(-6.71)		(-6.78)			(9.98)		(2.37)			(13.70)		(2.90)
HML		-0.0095		-0.0657			-0.3356		-0.2532			-0.2651		0.1427
		(-0.65)		(-3.93)			(-5.28)		(-6.27)			(-2.49)		(2.49)
$\overline{R}^2$ in %	28.64	50.89	27.44	46.17	-	28.64	80.55	86.88	90.17	-	27.44	73.92	86.88	87.97
N	641	641	641	641	641	641	641	641	641	641	641	641	641	641

Table 3.17 presents spanning regressions for FOP and the two $IVol$ factors $FIVOL^{2x3}$ and $FIVOL^{10}$. The superscripts indicate the sorting methodology which is used to construct $IVol$ factors: $FIVOL^{2x3}$ is constructed according to the methodology of Fama and French (1993) as the intersection of two Size and three $IVol$ portfolios. $FIVOL^{10}$ is the high-minus low portfolio from the $IVol$ decile sorts. t-statistics in parentheses are computed from Newey and West (1987) standard errors with six lags. The sample period is August 1963 to December 2016.

In Columns (1) to (4) we explain FOP with different combinations of the two $IVol$ factors and the FF3 factors. $FIVOL^{2x3}$ and $FIVOL^{10}$ explain FOP neither individually, nor jointly in combination with the FF3 risk factors. The Intercept remains statistically significant at conventional levels with t-statistics well beyond minus three.

Conversely, accounting for FOP individually alleviates the negative Intercept estimates of $FIVOL^{2x3}$ and $FIVOL^{10}$. In both cases, the negative Intercept changes its sign. In Columns (5) and (6), the negative risk premium on $FIVOL^{2x3}$ increases from -52 bps (t-statistic = -2.75) to 69 bps (t-statistic = 2.74). This finding extends to Columns (10) and (11), where the Intercept increases from -89 bps (t-statistic = -3.06) to 80 bps (t-statistic = 2.49). Similar to Table 3.8 in case of the extended CAPM, idiosyncratic risk earns a positive risk premium after controlling for FOP. For both $IVol$ factors, as shown in Columns (7) and (12), this positive risk premium is fully attributable to the FF3 risk factors. FOP is statistically and economically different from $IVol$.[20] Hence, FOP more likely proxies for the latent factor in the FF3 model which explains the $IVol$ puzzle.

3.7.3 FOP and the choice of weights

Section 3.2.2 proposes an adaption of the weights in the active portfolio of MacKinlay and Pastor (2000) from alpha to squared alpha. The positive relation between $IVol$ and squared alpha in Equation (3.5) might raise the concern that the weight vector in Equation (3.6) is tantamount to forming OP on $IVol$ itself.

To address this concern, we perform a litmus test by re-estimating FOP with an alternative weight vector which replaces squared alpha with $IVol$.[21] If the weight vector in Equation (3.6) resembles $IVol$, the resulting factor FOP_{IVol} performs just as well as FOP in Table 3.8. We present the results in Table 3.18 which is otherwise identical to the baseline analysis. For the sake of brevity, however, we do not present the sensitivity of each decile with respect to FOP_{IVol}. The $IVol$ puzzle remains well and alive in Table 3.18, similar to Table 3.1 which revisits the puzzle. Including FOP_{IVol} into the factor model setup leaves the negative return of the difference portfolio largely unaffected. The negative alphas of the

[20]In untabulated robustness checks we also regress FOP on the $IVol^{1M}_{FF3}$ decile returns components which are orthogonal to the FF3 risk factors. The cross correlation is lower after orthogonalization with the FF3 factors and we can perform spanning regressions without collinearity concerns. The orthogonalized $IVol^{1M}_{FF3}$ decile returns do not span FOP. We also repeat the analysis for orthogonalized versions of the $IVol$ factors $FIVOL^{2x3}$ and $FIVOL^{10}$ and find identical results. Put differently, the explanatory power of FOP is unrelated to the orthogonalization with the FF3 factors.

[21]The results are almost identical if we use idiosyncratic variance instead of $IVol$.

Table 3.18: Revisiting portfolio sorts with FOP formed on $IVol$.

Panel A: Equal-weighted sorts

Model	Low $IVol_{FF3}^{1M}$	2	3	4	5	6	7	8	9	High $IVol_{FF3}^{1M}$	Diff
CAPM + FOP_{IVol}	0.2949	0.3734	0.3653	0.4437	0.4196	0.4553	0.2814	0.2294	-0.0250	-0.6182	-0.9132
	(3.14)	(4.15)	(3.77)	(4.34)	(4.01)	(3.97)	(2.25)	(1.74)	(-0.17)	(-3.47)	(-5.09)
FF3 + FOP_{IVol}	0.1192	0.1673	0.1458	0.2046	0.1720	0.2048	0.0272	-0.0106	-0.2606	-0.8472	-0.9664
	(1.57)	(2.71)	(2.38)	(3.74)	(3.38)	(3.98)	(0.56)	(-0.21)	(-4.30)	(-9.03)	(-7.09)
PS + FOP_{IVol}	0.1271	0.1745	0.1539	0.1984	0.1629	0.2086	0.0236	-0.0073	-0.2424	-0.8540	-0.9811
	(1.61)	(2.68)	(2.35)	(3.34)	(2.95)	(3.83)	(0.46)	(-0.14)	(-3.85)	(-8.47)	(-6.93)
CAR + FOP_{IVol}	0.1233	0.1881	0.1716	0.2429	0.2048	0.2390	0.0652	0.0126	-0.2110	-0.7933	-0.9166
	(1.74)	(3.48)	(3.25)	(4.92)	(4.44)	(5.24)	(1.44)	(0.25)	(-3.34)	(-7.88)	(-6.93)
FF5 + FOP_{IVol}	0.0390	0.0721	0.0554	0.1292	0.1111	0.1778	0.0273	0.0437	-0.1449	-0.6411	-0.6800
	(0.55)	(1.35)	(0.98)	(2.52)	(2.15)	(3.10)	(0.54)	(0.82)	(-2.25)	(-7.57)	(-6.49)
M4 + FOP_{IVol}	0.0188	0.0730	0.0617	0.1564	0.1466	0.2207	0.0661	0.0726	-0.0940	-0.6127	-0.6315
	(0.25)	(1.14)	(0.97)	(2.81)	(2.83)	(4.36)	(1.24)	(1.18)	(-1.21)	(-4.97)	(-4.33)
Q-Factor + FOP_{IVol}	0.0688	0.0750	0.0461	0.1290	0.1094	0.1942	0.0707	0.0977	-0.0328	-0.5266	-0.5953
	(0.70)	(0.81)	(0.50)	(1.52)	(1.37)	(2.27)	(1.03)	(1.26)	(-0.35)	(-4.33)	(-3.82)
FF6 + FOP_{IVol}	0.0484	0.0965	0.0838	0.1673	0.1434	0.2088	0.0610	0.0609	-0.1087	-0.6063	-0.6547
	(0.71)	(1.99)	(1.65)	(3.68)	(3.16)	(4.32)	(1.38)	(1.23)	(-1.79)	(-6.79)	(-6.13)

Panel B: Value-weighted sorts

Model	Low $IVol_{FF3}^{1M}$	2	3	4	5	6	7	8	9	High $IVol_{FF3}^{1M}$	Diff
CAPM + FOP_{IVol}	0.5127	0.5404	0.5079	0.6117	0.6287	0.5956	0.6848	0.3000	0.2689	-0.3761	-0.8887
	(3.36)	(3.30)	(2.80)	(3.16)	(2.96)	(2.44)	(2.57)	(1.01)	(0.83)	(-1.07)	(-3.06)
FF3 + FOP_{IVol}	0.0224	0.0305	-0.0197	0.0720	0.0840	0.0693	0.1756	-0.1582	-0.1857	-0.7451	-0.7675
	(0.33)	(0.57)	(-0.49)	(1.25)	(1.44)	(0.89)	(2.13)	(-1.48)	(-1.40)	(-4.80)	(-4.07)
PS + FOP_{IVol}	0.0026	0.0167	-0.0270	0.0385	0.0501	0.0214	0.1162	-0.2211	-0.2423	-0.8545	-0.8571
	(0.04)	(0.30)	(-0.64)	(0.66)	(0.85)	(0.29)	(1.57)	(-2.67)	(-2.42)	(-5.81)	(-5.01)
CAR + FOP_{IVol}	0.0134	0.0108	-0.0229	0.0429	0.0176	-0.0020	0.1153	-0.2426	-0.2572	-0.9016	-0.9149
	(0.21)	(0.18)	(-0.51)	(0.69)	(0.29)	(-0.03)	(1.44)	(-2.67)	(-2.45)	(-5.76)	(-5.09)
FF5 + FOP_{IVol}	-0.0127	0.0087	-0.0468	0.0325	0.0809	0.0285	0.1216	-0.2407	-0.2207	-0.7884	-0.7757
	(-0.22)	(0.15)	(-0.91)	(0.57)	(1.32)	(0.37)	(1.69)	(-2.93)	(-2.20)	(-5.25)	(-4.45)
M4 + FOP_{IVol}	-0.0667	-0.0438	-0.0875	-0.0077	0.0383	0.0706	0.1661	-0.1396	-0.0966	-0.5878	-0.5211
	(-1.22)	(-0.77)	(-1.74)	(-0.12)	(0.63)	(0.97)	(2.21)	(-1.70)	(-1.05)	(-4.45)	(-3.56)
Q-Factor + FOP_{IVol}	-0.1018	-0.0628	-0.0968	0.0012	0.1041	0.1119	0.1970	-0.1311	-0.0669	-0.5242	-0.4224
	(-1.52)	(-0.99)	(-1.32)	(0.02)	(1.62)	(1.39)	(2.42)	(-1.47)	(-0.61)	(-3.28)	(-2.24)
FF6 + FOP_{IVol}	-0.1047	-0.0455	-0.1376	-0.0474	0.0142	0.0571	0.2193	-0.1278	-0.0247	-0.5181	-0.4134
	(-1.39)	(-0.66)	(-2.08)	(-0.65)	(0.19)	(0.62)	(2.65)	(-1.28)	(-0.20)	(-3.31)	(-2.18)
Q-Factor + FOP_{IVol}	-0.0749	-0.0468	-0.1018	-0.0110	0.0657	0.0714	0.1672	-0.1632	-0.0877	-0.5478	-0.4729
	(-1.28)	(-0.80)	(-1.78)	(-0.17)	(1.04)	(0.94)	(2.28)	(-1.97)	(-0.93)	(-4.06)	(-3.10)

Table 3.18 revisits the univariate portfolio sorts in Table 3.1 with factor models extended by the mimicking factor FOP_{IVol}. In contrast to the baseline analysis, we adapt the weighting vector of residuals and choose weights proportional to $IVol$ instead of α^2. Other than that, Table 3.18 is identical to Table 3.1.

highest $IVol_{FF3}^{1M}$ are statistically significant with t-statistics beyond minus three. FOP_{IVol} does not explain the $IVol$ puzzle.

3.7.4 Alternative residual generating factor model

This Section repeats the baseline analysis in Section 3.4.2, but varies the set of test assets as well as the residual generating factor model. Instead of 25 Size-$IVol_{FF3}^{1M}$ portfolios, we use 200 portfolios sorted by $IVol_{FF6}^{1M}$, i.e. idiosyncratic volatility of FF6 model residuals instead of the FF3 model. In contrast to the results in Table 3.2, we also consider the risk factor betas of the corresponding model as control variables and include RMW,

Table 3.19: Fama and MacBeth (1973) regressions on 200 $IVol$ portfolios with the Fama and French (2018) model.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.7262	0.5213	0.6061	0.3635	0.3400	0.3296	0.3155
	(4.36)	(2.47)	(3.58)	(4.45)	(4.10)	(4.10)	(3.86)
$IVol^{1M}_{FF6}$	-44.211		-10.832	-13.316		-5.9039	
	(-2.91)		(-1.07)	(-2.30)		(-1.02)	
β_{OP}		-0.5176	-0.4690			-0.2536	-0.2564
		(-3.29)	(-3.47)			(-2.83)	(-2.87)
β_{Mkt}				0.3718	0.3097	0.3195	0.2853
				(1.90)	(1.60)	(1.65)	(1.47)
β_{SMB}				-0.5285	-0.5577	-0.3139	-0.3231
				(-3.77)	(-3.94)	(-2.42)	(-2.48)
β_{HML}				0.0679	0.0994	0.0446	0.0559
				(0.61)	(0.89)	(0.39)	(0.50)
β_{RMW}				0.3698	0.3877	0.3316	0.3421
				(3.37)	(3.59)	(3.13)	(3.26)
β_{CMA}				-0.1199	-0.1090	-0.1521	-0.1461
				(-1.25)	(-1.11)	(-1.77)	(-1.68)
β_{MOM}				-0.2623	-0.2733	-0.0929	-0.0799
				(-1.44)	(-1.53)	(-0.53)	(-0.46)
$avg.\overline{R}^2 in\%$	4.96	6.86	8.51	13.83	13.61	14.55	14.36

Table 3.19 presents average coefficients of Fama and MacBeth (1973) cross-sectional portfolio level regressions of excess returns in month $t+1$ on $IVol^{1M}_{FF6}$, the sensitivity β_{OP} to the latent factor OP and control variables. $IVol^{1M}_{FF6}$ is the monthly idiosyncratic volatility of daily portfolio-level returns in month t. Residuals are computed from the Fama and French (2018) six factor model. The base assets are 200 portfolios sorted on monthly idiosyncratic volatility $IVol^{1M}_{FF6}$. Betas are calculated for the following risk factors: Mkt, SMB, HML, RMW, CMA and MOM are the factors of the Fama and French (2018) six factor model. We estimate all betas in 5-year rolling window regressions. The sample period in all columns is June 1968 to December 2016. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in %. Average coefficients are multiplied by one hundred and the factor portfolios are included among the test assets. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

CMA and MOM among the test assets. Other than that, the framework in Table 3.19 is identical to Table 3.2.

The $IVol$ puzzle persists in the Fama and MacBeth (1973) regressions if we use residuals from the recent model of Fama and French (2018). The negative coefficient on $IVol^{1M}_{FF6}$ in the univariate model in Column (1) amounts to -44.2 and is statistically significant at any conventional level. Once we control for the risk factor betas of the FF6 model, this coefficient reduces to -13.3, but remains significant at the 5% level with a t-statistic of -2.30. Including β_{OP} fully explains the negative $IVol_{FF6}^{1M}$, either in the bivariate analysis in Column (3) and in the full model in Columns (6) and (7). β_{OP} performs similar to the baseline analysis. The coefficient exhibits t-statistics around minus three and is thus highly statistically significant. Our main finding that the $IVol$ puzzle arises from a latent factor in the investor's factor model also extends to the residuals of the recent Fama and French (2018) six factor model.[22]

3.7.5 Extending window analysis

Section 3.5 relies on full sample beta estimates for the construction of the mimicking factor FOP. We re-evaluate the key findings in Section 3.5 with an extending window estimation (EW) of Equation (3.9) which eliminates the look-ahead bias. We refer to this mimicking factor as $FOP(EW)$.

First, we show that the inclusion of $FOP(EW)$ into the factor models under consideration alleviates the $IVol$ puzzle in the decile sorts of Table 3.1. Except for the different estimation of FOP, the analysis is identical. In contrast to the baseline sorts, the sample period starts in 1968. Table 3.20 presents the results.

$FOP(EW)$ performs equally well in equal-weighted sorts in Panel A and value-weighted sorts in Panel B. The negative alphas of the difference

[22] In untabulated robustness checks we repeat the analysis for residuals of the other factor models under consideration and draw identical conclusions.

Table 3.20: Robustness: revisiting the idiosyncratic volatility puzzle with FOP (EW).

Panel A: Equal-weighted sorts

Model	Parameter	Low $IVol_{FF3}^{1M}$	2	3	4	5	6	7	8	9	High $IVol_{FF3}^{1M}$	Diff
CAPM + FOP (EW)	α	-0.0561	-0.0396	-0.0274	0.0657	0.1109	0.2530	0.2109	0.3106	0.2844	0.0241	0.0803
		(-0.56)	(-0.47)	(-0.30)	(0.70)	(1.05)	(2.03)	(1.40)	(1.88)	(1.53)	(0.12)	(0.45)
	β_{FOP}	-0.7167	**-0.7938**	**-0.7272**	**-0.6653**	**-0.5044**	**-0.2722**	0.0364	**0.3996**	**0.9090**	**1.6112**	**2.3278**
FF3 + FOP (EW)	α	-0.0950	-0.0861	-0.0908	-0.0173	0.0075	0.1253	0.0572	0.1312	0.0824	-0.1969	-0.1019
		(-1.17)	(-1.39)	(-1.47)	(-0.32)	(0.14)	(2.07)	(1.00)	(2.08)	(1.08)	(-2.28)	(-0.88)
	β_{FOP}	**-0.5547**	**-0.6024**	**-0.5570**	**-0.5124**	**-0.3821**	**-0.1969**	0.0711	**0.3605**	**0.8523**	**1.5750**	**2.1296**
PS + FOP (EW)	α	-0.0902	-0.0835	-0.0872	-0.0165	-0.0057	0.1306	0.0493	0.1261	0.0897	-0.2362	-0.1459
		(-1.07)	(-1.27)	(-1.32)	(-0.28)	(-0.09)	(2.00)	(0.80)	(1.88)	(1.09)	(-2.64)	(-1.23)
	β_{FOP}	**-0.5749**	**-0.6252**	**-0.5739**	**-0.5133**	**-0.3907**	**-0.1966**	0.0761	**0.3626**	**0.8736**	**1.5636**	**2.1385**
CAR + FOP (EW)	α	-0.0895	-0.0619	-0.0624	0.0218	0.0399	0.1550	0.0877	0.1484	0.1168	-0.1681	-0.0786
		(-1.13)	(-1.17)	(-1.19)	(0.48)	(0.83)	(2.97)	(1.62)	(2.45)	(1.62)	(-1.98)	(-0.68)
	β_{FOP}	**-0.5591**	**-0.6215**	**-0.5794**	**-0.5433**	**-0.4077**	**-0.2204**	0.0470	**0.3469**	**0.8251**	**1.5522**	**2.1112**
FF5 + FOP (EW)	α	-0.1018	-0.0780	-0.0799	0.0029	0.0250	0.1464	0.0651	0.1343	0.0969	-0.1901	-0.0883
		(-1.33)	(-1.44)	(-1.43)	(0.06)	(0.47)	(2.61)	(1.12)	(2.23)	(1.34)	(-2.38)	(-0.88)
	β_{FOP}	**-0.4216**	**-0.4169**	**-0.3707**	**-0.3412**	**-0.2306**	-0.0878	0.1116	**0.2672**	**0.7054**	**1.2918**	**1.7134**
M4 + FOP (EW)	α	-0.1604	-0.1341	-0.1284	-0.0272	0.0084	0.1457	0.0894	0.1977	0.1868	-0.0913	0.0691
		(-1.95)	(-2.17)	(-2.08)	(-0.54)	(0.15)	(2.81)	(1.65)	(3.26)	(2.65)	(-1.04)	(0.64)
	β_{FOP}	**-0.5415**	**-0.6335**	**-0.5815**	**-0.5607**	**-0.4199**	**-0.2206**	0.0787	**0.3878**	**0.8638**	**1.6001**	**2.1416**
Q-Factor + FOP (EW)	α	-0.1299	-0.1456	-0.1525	-0.0600	-0.0362	0.1127	0.0678	0.1870	0.1878	-0.1079	0.0220
		(-1.35)	(-1.80)	(-1.81)	(-0.79)	(-0.45)	(1.22)	(0.89)	(2.13)	(1.79)	(-0.98)	(0.19)
	β_{FOP}	**-0.6757**	**-0.7313**	**-0.6568**	**-0.6239**	**-0.4817**	**-0.2784**	-0.0159	**0.2947**	**0.7298**	**1.3769**	**2.0526**
FF6 + FOP (EW)	α	-0.0927	-0.0521	-0.0504	0.0409	0.0566	0.1741	0.0937	0.1480	0.1243	-0.1700	-0.0773
		(-1.21)	(-1.07)	(-0.98)	(0.85)	(1.13)	(3.37)	(1.67)	(2.56)	(1.83)	(-2.21)	(-0.78)
	β_{FOP}	**-0.4271**	**-0.4324**	**-0.3883**	**-0.3639**	**-0.2495**	-0.1044	0.0945	**0.2590**	**0.6891**	**1.2798**	**1.7069**

Panel B: Value-weighted sorts

Model	Param.	Low $IVol_{FF3}^{1M}$	2	3	4	5	6	7	8	9	High $IVol_{FF3}^{1M}$	Diff
CAPM + FOP (EW)	α	-0.1658	-0.0825	-0.1662	-0.0556	0.0263	0.0896	0.3144	0.0238	0.1943	-0.1443	0.0215
		(-2.23)	(-1.27)	(-2.86)	(-0.77)	(0.36)	(1.00)	(3.05)	(0.17)	(1.12)	(-0.78)	(0.10)
	β_{FOP}	**-0.5610**	**-0.3304**	**-0.3220**	**-0.2365**	-0.0432	**0.2189**	**0.5541**	0.7733	**1.2465**	**1.4700**	**2.3439**
FF3 + FOP (EW)	α	-0.1227	-0.0502	-0.1446	-0.0533	0.0089	0.0328	0.2310	-0.1005	0.0358	-0.3196	-0.1969
		(-1.85)	(-0.67)	(-2.67)	(-0.76)	(0.12)	(0.38)	(2.61)	(-0.83)	(0.26)	(-1.72)	(-0.94)
	β_{FOP}	**-0.4347**	**-0.2261**	**-0.2742**	**-0.1869**	-0.0370	**0.1494**	0.4613	**0.5672**	**0.9700**	**1.6455**	**2.0802**
PS + FOP (EW)	α	-0.1198	-0.0477	-0.1390	-0.0452	-0.0161	0.0126	0.2265	-0.1584	-0.0034	-0.3899	-0.2701
		(-1.72)	(-0.58)	(-2.46)	(-0.60)	(-0.21)	(0.14)	(2.31)	(-1.16)	(-0.02)	(-1.98)	(-1.21)
	β_{FOP}	**-0.4553**	**-0.2409**	**-0.2846**	**-0.1834**	-0.0356	**0.1472**	**0.4531**	**0.5490**	**0.9759**	**1.6672**	**2.1225**
CAR + FOP (EW)	α	-0.1433	-0.0605	-0.1572	-0.0531	0.0405	0.0485	0.2479	-0.0966	0.0716	-0.2489	-0.1056
		(-2.07)	(-0.82)	(-2.66)	(-0.77)	(0.58)	(0.56)	(2.84)	(-0.81)	(0.52)	(-1.39)	(-0.52)
	β_{FOP}	**-0.4185**	**-0.2180**	**-0.2642**	**-0.1871**	-0.0620	**0.1370**	**0.4480**	**0.5642**	**0.9418**	**1.5896**	**2.0080**
FF5 + FOP (EW)	α	-0.1396	-0.0530	-0.1375	-0.0391	0.0268	0.0678	0.2425	-0.0869	0.0548	-0.2835	-0.1438
		(-2.25)	(-0.75)	(-2.43)	(-0.57)	(0.38)	(0.87)	(2.83)	(-0.78)	(0.43)	(-1.65)	(-0.78)
	β_{FOP}	**-0.2808**	-0.0530	**-0.1284**	-0.0608	0.0072	0.0570	**0.3255**	**0.3038**	**0.5956**	**1.0562**	**1.3370**
M4 + FOP (EW)	α	-0.2364	-0.1355	-0.1873	-0.0683	0.0669	0.1398	0.3376	0.0544	0.2551	-0.0232	0.2132
		(-3.22)	(-1.92)	(-2.54)	(-0.81)	(0.97)	(1.65)	(3.89)	(0.52)	(2.04)	(-0.14)	(1.13)
	β_{FOP}	**-0.3921**	**-0.2149**	**-0.2801**	**-0.2186**	-0.1204	0.0701	**0.4017**	**0.5230**	**0.9382**	**1.4782**	**1.8703**
Q-Factor + FOP (EW)	α	-0.2148	-0.1076	-0.2018	-0.0951	-0.0093	0.0767	0.3196	-0.0046	0.2173	-0.1413	0.0734
		(-2.77)	(-1.29)	(-3.06)	(-1.23)	(-0.11)	(0.77)	(3.63)	(-0.04)	(1.59)	(-0.81)	(0.37)
	β_{FOP}	**-0.4131**	**-0.2339**	**-0.2133**	**-0.1496**	-0.0642	0.0996	**0.3911**	**0.4996**	**0.8982**	**1.3560**	**1.7691**
FF6 + FOP (EW)	α	-0.1534	-0.0585	-0.1470	-0.0379	0.0552	0.0760	0.2539	-0.0909	0.0777	-0.2345	-0.0812
		(-2.39)	(-0.86)	(-2.42)	(-0.56)	(0.78)	(0.97)	(2.97)	(-0.82)	(0.60)	(-1.39)	(-0.45)
	β_{FOP}	**-0.2726**	-0.0497	**-0.1227**	-0.0615	-0.0098	0.0521	**0.3187**	**0.3062**	**0.5819**	**1.0269**	**1.2995**

Table 3.20 revisits the univariate portfolio sorts in Table 3.1 with factor models extended by the mimicking factor FOP from an extending window estimation (EW). Table 3.20 is otherwise identical to Table 3.8.

Table 3.21: Robustness risk factor protocol: Canonical correlations with FOP from extending window (EW).

Panel A: Significance levels of the FF6 factors and FOP

	Risk factors						
	Mkt	SMB	HML	RMW	CMA	MOM	FOP (EW)
Mean |t-statistic|	**5.2252**	**3.6871**	**1.9686**	1.4980	1.2667	1.8491	1.5314
Mean |t-statistic| sign.	**11.4564**	**6.3385**	**2.9331**	**2.9371**	**2.4629**	**3.1351**	**2.0614**

Panel B: Number of \|t-statistics\| $\geq$ 1.96 out of 7

Subsample	Mkt	SMB	HML	RMW	CMA	MOM	FOP (EW)
1968 – 1976	3	3	4	3	3	5	3
1977 – 1986	3	3	3	5	0	4	4
1987– 1996	2	3	3	2	4	3	3
1997 – 2006	4	3	4	4	3	5	3
2007 – 2016	2	4	4	3	3	3	0
Average #	**2.8**	**3.2**	**3.6**	**3.4**	**2.6**	**4**	**2.6**

Table 3.21 presents the second stage of the Pukthuanthong et al. (2019) risk factor protocol, i.e. a test whether factor candidates are significantly correlated with the cross-section of stock returns. In contrast to the baseline analysis, we estimate FOP from an extending window (EW) regression instead of a full sample estimation. Other than that, Table 3.21 is identical to Table 3.10.

portfolio between high and low $IVol_{FF3}^{1M}$ becomes insignificant for each of the factor model combinations. The only difference to the baseline analysis in Table 3.8 is that the positive alphas of the CAPM and the M4 model are now insignificant.

Second, we review the first stage of the risk factor protocol in Section 3.5.2 and focus on the actual test for the canonical correlations. $FOP(EW)$ also passes both thresholds of the risk factor protocol, i.e. a mean absolute t-statistic $\geq$ 1.96 in the second row of Panel A and an average number of absolute t-statistics $\geq$ 2.5 in Panel B.

Third, we address the question whether $FOP(EW)$ is also priced in the Fama and MacBeth (1973) regressions in Section 3.5.3. The analysis is identical to Table 3.11, except that we replace FOP with it's extending window equivalent $FOP(EW)$. The risk premium estimates on $\beta_{FOP(EW)}$

Table 3.22: Robustness: Fama and MacBeth (1973) portfolio regressions with FOP (EW).

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Panel A: Size/ Operating Profitability 5x5											
	Full Sample Betas						5-year rolling window betas					
Intercept	0.5459	-0.1408	-0.2061	0.5779	0.0688	0.0548	0.5692	-0.0497	-0.1488	0.5825	0.1520	0.1129
	(2.65)	(-5.23)	(-6.07)	(3.04)	(2.93)	(2.34)	(2.78)	(-1.18)	(-4.15)	(3.10)	(4.50)	(4.28)
$\beta_{FOP(EW)}$	-0.3382		-0.3911	-0.2395		-0.3096	-0.2115		-0.2861	-0.1382		-0.2260
	(-2.89)		(-3.30)	(-1.72)		(-2.94)	(-2.29)		(-3.31)	(-1.66)		(-2.97)
β_{Mkt}		0.6251	0.6531		0.4727	0.4545		0.5175	0.6008		0.3866	0.4206
		(3.09)	(3.24)		(2.34)	(2.24)		(2.69)	(3.10)		(1.91)	(2.09)
β_{SMB}		0.1582	0.1908		0.0909	0.1424		0.1654	0.2245		0.1404	0.1677
		(1.19)	(1.41)		(0.69)	(1.09)		(1.30)	(1.77)		(1.10)	(1.33)
β_{HML}		0.4568	0.4749		0.3576	0.3087		0.2683	0.3847		0.2615	0.2761
		(2.90)	(2.98)		(2.29)	(2.05)		(1.65)	(2.43)		(2.11)	(2.12)
β_{CMA}					0.2275	0.2676					0.1269	0.1881
					(2.20)	(2.53)					(1.51)	(2.22)
β_{RMW}					0.2555	0.2377					0.2115	0.2100
					(2.17)	(2.07)					(2.07)	(1.96)
β_{MOM}					0.5408	0.5756					0.4073	0.4509
					(2.84)	(3.03)					(2.17)	(2.50)
$avg.\overline{R}^2 in$ %	5.7039	52.2246	60.8671	2.1995	67.2982	70.4695	3.3687	55.2033	59.2435	0.9258	67.0087	68.9524
	Panel B: Size/ Beta 5x5											
	Full Sample Betas						5-year rolling window betas					
Intercept	0.5724	-0.1627	-0.2173	0.5556	-0.2313	-0.2115	0.5861	-0.0049	-0.0661	0.5711	0.0167	-0.0071
	(2.71)	(-5.58)	(-7.14)	(2.98)	(-4.69)	(-4.99)	(2.82)	(-0.11)	(-1.53)	(3.11)	(0.50)	(-0.23)
$\beta_{FOP(EW)}$	-0.2418		-0.3835	-0.2822		-0.2094	-0.0905		-0.2078	-0.1263		-0.1728
	(-1.33)		(-3.04)	(-1.73)		(-2.03)	(-0.81)		(-2.47)	(-1.24)		(-2.12)
β_{Mkt}		0.6117	0.6276		0.6473	0.6254		0.5211	0.5672		0.5125	0.5313
		(3.00)	(3.09)		(3.23)	(3.10)		(2.51)	(2.77)		(2.52)	(2.63)
β_{SMB}		0.1713	0.2317		0.2467	0.2552		0.1407	0.1817		0.1748	0.1812
		(1.29)	(1.77)		(1.88)	(1.95)		(1.10)	(1.43)		(1.37)	(1.42)
β_{HML}		0.7954	0.6803		0.6787	0.5960		0.4069	0.3640		0.3285	0.3577
		(4.30)	(4.16)		(3.48)	(3.45)		(2.68)	(2.33)		(2.24)	(2.47)
β_{CMA}					0.4946	0.4812					0.2644	0.2988
					(3.98)	(3.97)					(2.62)	(3.10)
β_{RMW}					0.4691	0.3838					0.1430	0.1474
					(2.85)	(2.81)					(1.45)	(1.35)
β_{MOM}					0.8843	0.8535					0.5552	0.5613
					(4.37)	(4.27)					(2.88)	(3.09)
$avg.\overline{R}^2 in$ %	7.0994	54.5413	59.3731	3.2643	65.2661	68.2396	5.5814	52.9725	57.2931	3.2896	65.0270	67.6436
	Panel C: Size/ Book-to-Market/ Investment 2x4x4											
	Full Sample Betas						5-year rolling window betas					
Intercept	0.6002	-0.0496	-0.1182	0.5978	-0.1324	-0.1384	0.6136	0.0250	-0.0296	0.5994	0.0658	0.0209
	(2.92)	(-2.21)	(-4.18)	(3.26)	(-5.84)	(-5.87)	(3.06)	(0.93)	(-1.04)	(3.29)	(2.72)	(1.00)
$\beta_{FOP(EW)}$	-0.1894		-0.3929	-0.1727		-0.2626	-0.1465		-0.2631	-0.1235		-0.2562
	(-1.40)		(-3.80)	(-1.13)		(-2.48)	(-2.07)		(-3.89)	(-1.95)		(-3.43)
β_{Mkt}		0.5787	0.5787		0.5819	0.5765		0.4820	0.5217		0.4633	0.4945
		(2.87)	(2.87)		(2.91)	(2.89)		(2.55)	(2.74)		(2.38)	(2.54)
β_{SMB}		0.1679	0.2425		0.2248	0.2440		0.1901	0.2125		0.1638	0.1862
		(1.32)	(1.90)		(1.75)	(1.90)		(1.51)	(1.67)		(1.30)	(1.47)
β_{HML}		0.3125	0.3029		0.3655	0.3477		0.2510	0.2671		0.2834	0.3025
		(2.06)	(2.01)		(2.37)	(2.29)		(1.83)	(1.92)		(2.10)	(2.19)
β_{CMA}					0.3702	0.3789					0.2472	0.2723
					(3.76)	(3.83)					(2.91)	(3.16)
β_{RMW}					0.4493	0.3974					0.0812	0.1574
					(3.30)	(3.14)					(0.85)	(1.56)
β_{MOM}					0.7794	0.7953					0.4590	0.5017
					(4.12)	(4.21)					(2.66)	(2.97)
$avg.\overline{R}^2 in$ %	2.7482	48.5051	50.0814	0.7369	58.9302	59.8995	0.6049	46.8371	48.4295	-0.1675	57.3278	58.7383

Table 3.22 presents average coefficients of Fama and MacBeth (1973) cross-sectional portfolio level regressions of excess returns in month $t+1$ on the factor model betas of the Fama and French (1993) three factor model (FF3), the Fama and French (2018) six factor model (FF6) and both models extended by the mimicking factor FOP in month t. In contrast to the baseline analysis, we estimate FOP from an extending window (EW) regression instead of a full sample estimation. Other than that, Table 3.22 is identical to Table 3.11.

are similar to the baseline analysis. However, $\beta_{FOP(EW)}$, performs slightly worse in univariate models, e.g. Columns (1), (7) and (10) of Panel B. If we consider the full set of risk factor betas, the estimate on $\beta_{FOP(EW)}$ is statistically significant with a similar magnitude to the baseline analysis.

$FOP(EW)$ explains the underperformance of high $IVol$ stocks, is significantly related to the covariance matrix of returns and earns a significant risk premium. The finding that FOP is a genuine risk factor is robust to the extending window analysis.

3.8 Conclusion

The $IVol$ puzzle originates from latent risk in the investor's factor model. We construct an active portfolio formed on factor model residuals to approximate this risk factor and include it in the residual generating factor models of Fama and French (1993) and Fama and French (2018). Empirically, the sensitivity to the latent factor OP as well as its mimicking portfolio FOP alleviate the $IVol$ puzzle in Fama and MacBeth (1973) regressions and portfolio sorts. The sensitivity to OP fully accounts for the unexplained fraction of the $IVol$ puzzle in the analysis of Hou and Loh (2016). High-$IVol$ portfolios perform poorly in subsequent months because they are exposed to noise trader risk. Our evidence is consistent with a risk-based explanation for the $IVol$ puzzle, but previously proposed risk factors are unlikely to account for our findings. Our results are robust to the choice of test assets and the residual generating factor model.

What is the latent factor behind the idiosyncratic volatility puzzle?

Our study explains several well-known characteristics of the $IVol$ puzzle. First, the $IVol$-return relation is nonlinear in alpha such that empirical factors which assume long positions in stocks with high alphas and short positions in stocks with low alphas only partly explain $IVol$. Second, this finding also explains the switching sign of the $IVol$-return relation between supposedly underpriced and overpriced stocks (Stambaugh et al., 2015). Alphas are proportional to the sensitivity to the latent factor, but the calculation of $IVol$ wipes out the sign of this relationship. Third, the $IVol$ effect is asymmetric because the distribution of β_{FOP} is not symmetric. Fourth, high $IVol$-stocks exhibit many attractive features for retail investors and are thus subject to the exposure to noise trader risk. Excluding these stocks weakens the puzzle (Bali and Cakici, 2008). This finding is consistent with Brandt et al. (2010) who show that increases in aggregate $IVol$ are related to speculative episodes. Fifth, noise trader risk contributes to aggregate volatility as pointed out by by Brown (1999) and Bollerslev et al. (2018) which explains the empirical support for volatility risk-based explanations for the $IVol$ puzzle (Chen and Petkova, 2012; Barinov, 2013).

3.A Appendix

3.A.1 Risk factor betas

In this Section, we describe the calculation of control variables used in the main paper. For variables which require daily data, we require 200 (15) valid daily observations within the annual (monthly) estimation periods. Variables with monthly estimation windows are computed with 60 monthly returns and require at least 24 valid observations.

Aggregate Lottery Demand (FMax): We estimate the beta of asset i with respect to aggregate lottery demand $FMax$ in month t from the regression

$$r_{i,t} = \alpha_i + \beta_{FMax,i} FMax_t + \epsilon_{i,t},$$

where $r_{i,t}$ is the excess return of asset i in month t and $FMax$ is the return on the Bali et al. (2017) lottery demand factor.

Market Variance (MV): Following Herskovic et al. (2016), innovations to market variance are monthly changes in the variance of value weighted market returns. We compute the variance beta of asset i in month t from the time series regression

$$r_{i,t} = \alpha_i + \beta_{MV,i,t} \Delta MV_t + \epsilon_{i,t},$$

where $r_{i,t}$ is the excess return of asset i in month t and ΔMV is the innovation in market variance MV.

Average Correlation and Average Variance (AC and AV): We follow Chen and Petkova (2012) in the construction of average correlation (AC)

and average variance (AV) and adapt the calculation of factor betas in line with Hou and Loh (2016). First, average variance in month t is

$$AV_t = \sum_{i=1}^{N_t} w_{i,t} \left[\sum_{d=1}^{D_t} r_{i,d}^2 + 2 \sum_{d=2}^{D_t} r_{i,d} r_{i,d-1} \right],$$

where $r_{i,d}$ is the excess return of stock i on date d in month t with D_t days per month, N_t is the number of stocks in that month and $w_{i,t}$ is the relative market capitalization of stock i in month t. The right-hand term adjusts for autocorrelation in daily returns. Second, average correlation is the value-weighted average of pairwise correlations of daily returns $r_{i,d}$ within each month t. We estimate the beta of asset i to innovations in AC and AV in the monthly time series regression

$$r_{i,t} = \alpha_i + \beta_{AC,i,t} \Delta AC_t + \beta_{AV,i,t} \Delta AV_t + \beta_{Mkt,i} Mkt_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + \epsilon_{i,t},$$

where $r_{i,t}$ is the excess return of asset i in month t and ΔAV and ΔAC are innovations in monthly average variance and average correlation, respectively. We follow Chen and Petkova (2012) and control for Mkt, SMB and HML in the beta estimation, but estimate a linear model instead of the VAR approach in the original paper.

Common Idiosyncratic Volatility (CIV): Herskovic et al. (2016) define common idiosyncratic volatility (CIV) as the equally-weighted average over all individual within-month idiosyncratic volatility measures of common stocks. Idiosyncratic volatility of asset i is the standard deviation of Fama and French (1993) three factor model residuals, estimated within month t using daily data. We follow Herskovic et al. (2016) and include MV when we estimate the following monthly time series regression

$$r_{i,t} = \alpha_i + \beta_{CIV,i,t}\Delta CIV_t + \beta_{MV,i,t}\Delta MV_t \epsilon_{i,t},$$

where $r_{i,t}$ is the excess return of asset i in month t and ΔCIV and ΔMV are innovations in monthly common idiosyncratic volatility and market variance, respectively.

Aggregate Volatility Risk Factor (FVIX): Barinov (2013, 2018) form a mimicking factor which tracks daily innovations in the daily CBOE S&P 100 Volatility Index (in the following referred to as VIX) from a regression of daily innovations in the VIX on quintile portfolios sorted by stock betas with respect to innovations in the VIX. We fit the regression given daily data and use the fitted values to form the mimicking factor as

$$\begin{aligned} FVIX = -0.157 \cdot VIX_{1,t} - 0.588 \cdot VIX_{2,t} - 0.365 \cdot VIX_{3,t} \\ -0.579 \cdot VIX_{4,t} + 0.164 \cdot VIX_{5,t} \end{aligned}$$

where $VIX_{1,t}$,..., $VIX_{5,t}$ are quintile portfolios sorted by within-month betas of individual stocks with respect to daily VIX innovations while controlling for market returns. The R^2 of this regression of 51.31% is almost identical to the value of 51.10% reported in Barinov (2018). We follow Barinov (2018) and use full sample estimates. In the second step, we estimate the suggested ICAPM specification of Barinov (2018) for each asset i as

$$r_{i,t} = \alpha_i + \beta_{FVIX,i,t} FVIX_t + \beta_{Mkt,i} Mkt_t + \epsilon_{i,t},$$

where $r_{i,t}$ is the excess return of asset i in month t.

3.A.2 Stock characteristics

Size: Size is the natural logarithm of a firm's market capitalization as provided by CRSP. Taking logs reduces skewness of market capitalization.

Illiquidity (Illiq): Illiquidity of stock i in month t is the adapted Amihud (2002) illiquidity measure given as

$$Illiq_{i,t} = \frac{1}{D_t} \sum_{d=1}^{D_t} \frac{|r_{i,d}|}{VOLD_{i,d}},$$

where $r_{i,d}$ is the return of stock i on day d, $VOLD_{i,d}$ is the trading volume in US Dollars and D_t is the number of days in month t.

Idiosyncratic Skewness (ISkew): Boyer et al. (2010) estimate historical idiosyncratic skewness of the returns of stock i in month t as

$$ISkew_{i,t} = \frac{1}{N(t)} \frac{\sum_{d \in S(t)} \epsilon_{i,d}^3}{IVol_{i,t}^3},$$

where $N(t)$ denotes the number of days d in the estimation period $S(t)$, and ϵ is the residual from the Fama and French (1993) three factor model estimated over the estimation period $S(t)$ using daily data. We estimate $ISkew_{i,t}$ over the past twelve months. We refer to the twelve month measure of idiosyncratic skewness relatively to the Fama and French (1993) three factor model as $ISkew$.

Short-Term Reversal (LagRet): Short-Term Reversal of stock i in month t is commonly the return of the stock in the previous month, i.e.

$$LagRet_{i,t} = r_{i,t-1}.$$

Momentum (Mom): Momentum of stock i in month t is the stock return during the 11 months prior to the most recent month t, i.e. months $t-11$ through $t-1$.

Co-Skewness (CoSkew): Co-skewness, adapted from Harvey and Siddique (2000), is the slope coefficient of a stock's excess return on the squared excess return of the market portfolio, measured over the past twelve months using daily data. This measure of co-skewness is not the baseline measure proposed by Harvey and Siddique (2000) and we follow, among others, Bali et al. (2017).

Mispricing (MISP): Stambaugh et al. (2015) construct the cross-sectional mispricing measure MISP as a composite rank based on eleven prominent anomalies. The highest rank is assigned to stocks with the lowest average returns based on each anomaly. The composite rank is the simple average over all eleven ranks. A higher value of MISP indicates a higher likelihood of a stock to be overpriced. For details regarding the anomalies, we refer to Stambaugh et al. (2015).

Expected idiosyncratic skewness (EIS): Boyer et al. (2010) present a model for the prediction of idiosyncratic skewness in the cross section of stock returns. The first step is a cross-sectional regression of contemporaneous idiosyncratic skewness on historical measures of skewness, volatility and a set of control variables

$$IS_{i,t} = \alpha_t + \beta_{1,t} IS_{i,t-T} + \beta_{2,t} IV_{i,t-T} + \gamma_t' X_{i,t-T},$$

where

$$IS_{i,t} = \frac{1}{N(t)} \frac{\sum_{d \in S(t)} \epsilon_{i,d}^3}{IV_{i,t}^3},$$

and

$$IV_{i,t} = \left(\frac{1}{N(t)} \sum_{d \in S(t)} \epsilon_{i,d}^2 \right)^{1/2}.$$

$N(t)$ denotes the number of days d in the estimation period $S(t)$ adjusted for degrees of freedom, and ϵ is the residual of a Fama and French (1993) three-factor model that we estimate for daily data in $S(t)$. $X_{i,t}$ represents a vector of firm-characteristic variables, namely momentum and turnover, as well as dummy variables for Size, industry, and NASDAQ stocks. In the next step, Boyer et al. (2010) use the fitted regression coefficients of the equation above to predict expected idiosyncratic skewness for the next T months

$$EIS = \hat{\alpha}_t + \hat{\beta}_{1,t} IS_{i,t} + \hat{\beta}_{2,t} IV_{i,t} + \hat{\gamma}_t' X_{i,t}.$$

In the baseline analysis, Boyer et al. (2010) estimate EIS with $T = 60$ months of data.[23]

[23]For details regarding the estimation of expected idiosyncratic skewness, we refer to the methodology appendix on the website of Brian Boyer: `http://boyer.byu.edu/Research/skew/skewmethodology.pdf`.

Chapter 4

Dissecting idiosyncratic volatility in the cross section of stock returns

This Chapter refers to the working paper:

Claußen, Arndt, Maik Dierkes and Sebastian Schroen (2019): 'Dissecting idiosyncratic volatility in the cross section of stock returns', Working Paper, Leibniz Universität Hannover.

Abstract

A simple, yet robust regression-based decomposition technique unveils that systematic noise trader risk accounts for the largest part of the negative relation between idiosyncratic risk and subsequent returns, commonly known as the idiosyncratic volatility puzzle. The systematic component in factor model residuals attributable to noise trader risk alone explains almost 50% of the puzzle. The pricing of the remaining *purely* idiosyncratic component of idiosyncratic risk is short-lived and historically unstable. Our results are robust to the choice of factor models as well as recently proposed rational and behavioral determinants of expected returns.

Keywords: Idiosyncratic volatility, latent risk factor, mispricing

JEL: G10, G12, G32.

4.1 Introduction

The underperformance of high-volatility stocks is widely considered to be one of the most compelling "*candidates for the greatest anomaly in finance*" (Baker et al., 2011, p. 40). Theoretically, idiosyncratic risk either carries no risk premium at all in standard asset pricing theory, or a positive risk premium if investors are unable to diversify properly (Merton, 1987). However, the seminal papers of Ang et al. (2006, 2009) find a strong negative relationship between idiosyncratic return volatility and future returns which marks the nucleus of the idiosyncratic volatility puzzle. Although the theoretical and empirical asset pricing literature[1] went to great lengths to explain the anomaly, "*all existing explanations still leave a sizeable portion of the puzzle unexplained*" (Hou and Loh, 2016, p. 191).

In this paper, we reduce this portion by providing further evidence for the results of Claußen et al. (2019) who show that the idiosyncratic volatility puzzle results from latent systematic risk in the Fama and French (1993) three factor model, most likely attributable to noise trader risk. We adapt the framework of Claußen et al. (2019) to the cross section of stock returns to derive an economically motivated regression-based procedure which treats this risk as a latent state variable and decompose idiosyncratic volatility into two components: First, a stock's exposure to latent but systematic risk – presumably induced by sentiment-driven noise trading – and second, *purely* idiosyncratic variation.

Approximately 93.5% of average monthly idiosyncratic volatility is

[1] For a summary of the long list of candidates and an extensive analysis of their explanatory power, we refer to Hou and Loh (2016).

indeed non-systematic. However, the quantitatively small latent factor volatility explains up to 55% of the puzzle. Accounting for alternative candidate variables, our latent factor volatility measure for noise trader risk attains an explained fraction of at least 35% which is by far the greatest share achieved by a single candidate in the multivariate analysis in Hou and Loh (2016) for the cross section of stock returns.

In full sample cross-sectional Fama and MacBeth (1973) regressions, the *purely* asset-specific component nevertheless earns a negative risk premium. A rolling window analysis, however, unveils that this premium is short lived and driven by a peak in the early 1980s. In contrast, the slope coefficient on the systematic component is steadily negative, highly significant and becomes economically more relevant in the recent past. Thus, latent systematic risk in supposedly idiosyncratic volatility reflects information above and beyond the *purely* asset-specific component, raw idiosyncratic volatility or alternative stock characteristics.

Our study generalizes the findings in Claußen et al. (2019) to the cross section of stock returns and contributes to the ongoing quest for the sources of the idiosyncratic volatility puzzle. More specifically, we provide further evidence in favor of a risk-based explanation for the idiosyncratic volatility puzzle, in line with the theoretical framework of Chen and Petkova (2012). The largest part of the negative premium on idiosyncratic volatility is attributable to a systematic component, most likely driven by sentiment-induced noise trading as shown by Claußen et al. (2019). Alternative risk-based explanations, especially aggregate variance risk as proposed by Chen and Petkova (2012) and Herskovic et al. (2016)

add little to this explanation. Our framework yields an explicit stock characteristic to further discriminate between risk-based explanations and mispricing, the main explanation in Stambaugh et al. (2015). The findings of Stambaugh et al. (2015) fully extend to the asset-specific component of idiosyncratic volatility, but the sign of the systematic component does not depend on mispricing. Therefore, mispricing is unlikely to be the sole explanation for the puzzle.

Furthermore, we contribute to the literature on the optimal orthogonal portfolio of MacKinlay (1995) and MacKinlay and Pastor (2000). By relaxing the strong from assumption in MacKinlay and Pastor (2000), we allow the residual variance to vary cross-sectionally and exploit this co-variation in a simple auxiliary regression on the cross-sectional average of factor model residuals. This facilitates an estimation of the exposure to latent factors in the investor's factor model without a proxy for the latent state variable as long as the number of assets is considerably large.

4.2 Dissecting idiosyncratic volatility in the cross section of stock returns

4.2.1 The optimal orthogonal portfolio

As motivated from Claußen et al. (2019), we follow the framework of MacKinlay (1995) and MacKinlay and Pastor (2000) to examine the role of a latent systematic risk factor in the negative relationship between idiosyncratic volatility and subsequent stock returns. Consequently, this section closely follows MacKinlay (1995) and MacKinlay and Pastor (2000).

Let $r_{i,t}$ denote the excess return of asset $i \in \{1,...,N\}$ in period t and $\boldsymbol{\zeta_t} \in \mathbb{R}^K$ represent realizations of K observable risk factors. Assuming a linear relationship between asset returns and the risk factor returns, the return generating process is

$$r_{i,t} = \alpha_i + \boldsymbol{\beta_i}'\boldsymbol{\zeta_t} + \epsilon_{i,t}, \tag{4.1}$$
$$\mathbb{E}(\boldsymbol{\epsilon_t}) = 0, \qquad \mathbb{E}(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}') = \Sigma \qquad \text{and} \qquad \text{cov}(\boldsymbol{\epsilon_t}, \boldsymbol{\zeta_t}) = 0,$$

where $\boldsymbol{\beta_i} \in \mathbb{R}^K$ are the sensitivities of asset i with respect to the K factors, $\epsilon_{i,t}$ is the error in each time period, and α_i denotes mispricing. An exact linear relation between the asset returns and the risk factor returns implies an intercept α_i of zero. An intercept which is significantly different from zero indicates mispricing.

In the presence of a missing factor, MacKinlay and Pastor (2000) show that the covariance matrix Σ contains information about the missing factor which drives α_i. This relationship can be developed using the optimal orthogonal portfolio defined as "*the unique portfolio given $\overline{N}$ assets that can be combined with the factor portfolios to form the tangency portfolio and is orthogonal to the factor portfolios*" (MacKinlay, 1995, p. 8). An advantage of the optimal orthogonal portfolio is that, by definition, it leaves the factor sensitivities $\boldsymbol{\beta_i}$ unaffected once the missing variable is included. We denote the return on the optimal orthogonal portfolio (op) at time t with $r_{op,t}$ which governs the asset return with sensitivity β_{op} and its first two moments are $\mathbb{E}\left(r_{op,t}\right) = \mu_{op}$ and $\text{var}\left(r_{op,t}\right) = \sigma_{op}^2$. Per definition, it holds $\text{cov}(\boldsymbol{\zeta_t}, r_{op,t}) = 0$. Replacing α_i in Equation (4.1) with the return of the optimal orthogonal portfolio yields

$$r_{i,t} = \beta_{op,i} r_{op,t} + \boldsymbol{\beta_i}'\boldsymbol{\zeta_t} + \nu_{i,t}, \tag{4.2}$$

$$\mathbb{E}(\boldsymbol{\nu_t}) = 0, \qquad \mathbb{E}(\boldsymbol{\nu_t}\boldsymbol{\nu_t}') = \Phi, \qquad \text{and} \qquad \operatorname{cov}(\boldsymbol{\nu_t}, \boldsymbol{\zeta_t}) = \operatorname{cov}\left(\boldsymbol{\nu_t}, r_{op}\right) = 0.$$

MacKinlay and Pastor (2000) employ the assumption that the covariance matrix Φ is proportional to the identity matrix. We relax this *strong form* assumption and set Φ as a diagonal matrix with asset specific error term variances $\sigma^2_{\nu,i} := \operatorname{var}(\nu_{i,t})$. Equaling the expectation of Equation (4.1) and Equation (4.2) leads to

$$\alpha_i = \beta_{op,i} E(r_{op}) = \beta_{op,i}\mu_{op}. \tag{4.3}$$

Further, comparison of Equation (4.1) and (4.2) results in

$$\epsilon_{i,t} = \beta_{op,i} r_{op,t} + \nu_{i,t} - \alpha_i. \tag{4.4}$$

Given $\operatorname{cov}\left(\boldsymbol{\nu_t}, r_{op}\right) = 0$, we express the variance of the error term in Equation (4.4) in terms of two components

$$\sigma^2_{\epsilon,i} := \operatorname{var}(\epsilon_{i,t}) = \beta^2_{op,i}\sigma^2_{op} + \sigma^2_{\nu,i}. \tag{4.5}$$

Equation (4.5) illustrates that $\sigma^2_{\epsilon,i}$, i.e. the idiosyncratic variance of the K factor model known to the investor, consists of two components. The first component on the right hand side of Equation (4.5), $\beta^2_{op,i}\sigma^2_{op}$, reflects systematic deviations from the return generating process due to the latent character of r_{op}. This component prevents the diversification of idiosyncratic risk to zero when forming a portfolio (MacKinlay, 1995). The second component, $\sigma^2_{\nu,i}$, is *truly* non-systematic.

4.2.2 A simple regression-based decomposition of idiosyncratic volatility

Instead of an economically motivated conjecture about the latent factor op and the subsequent estimation of $\beta_{op,i}$ for individual stocks, we implicitly derive $\beta_{op,i}$ without explicit knowledge about the unobserved state variable op.

First, we assume the linear model in Equation (4.1) to be known to the investor. The investor is thus able to estimate regression residuals $\epsilon_{i,t}$ based on this model. Consider an auxiliary regression of individual regression residuals $\epsilon_{i,t}$ on the cross-sectional average of all residuals $\overline{\epsilon}_t$, i.e.

$$\epsilon_{i,t} = \delta_{0,i} + \delta_{1,i} \cdot \overline{\epsilon}_t + \eta_{i,t}. \tag{4.6}$$

In contrast to MacKinlay and Pastor (2000) and Claußen et al. (2019) who suggest weights which are proportional to mispricing, we form the proxy for the active portfolio $\overline{\epsilon}_t$ as an equally-weighted average for two reasons. First, this reduces noise in the weighting vector which is supposedly larger for single stocks compared to portfolios used in Claußen et al. (2019). Second, we can now rewrite an analytical expression for the independent variable in Equation (4.6)

$$\overline{\epsilon}_t = \frac{1}{N}\sum_{\ell=1}^{N}\left(\beta_{op,\ell} r_{op,t} + \nu_{\ell,t} - \alpha_\ell\right) = \beta_{op,M} r_{op,t} + \frac{1}{N}\sum_{\ell=1}^{N} \nu_{\ell,t} - \alpha_M, \tag{4.7}$$

where $\beta_{op,M} = \frac{1}{N}\sum_{\ell=1}^{N} \beta_{op,\ell}$ and $\alpha_M = \frac{1}{N}\sum_{\ell=1}^{N} \alpha_\ell$. To express the regression coefficient analytically, we need the variance of $\overline{\epsilon}_t$, which is given by

$$\sigma_{\overline{\epsilon}}^2 := \operatorname{var}(\overline{\epsilon}_t) = \beta_{op,M}^2 \sigma_{op}^2 + \frac{1}{N^2} \sum_{\ell=1}^{N} \sigma_{\nu,\ell}^2, \tag{4.8}$$

because Φ is diagonal and $\operatorname{cov}\left(\boldsymbol{\nu}_t, r_{op}\right) = 0$. Recall that we allow Φ to have different entries on the diagonal which relaxes the strong form assumption in MacKinlay and Pastor (2000). This leads to a closed-form solution for the regression coefficient $\delta_{1,i}$:

$$\delta_{1,i} = \frac{\operatorname{cov}(\epsilon_{i,t}, \overline{\epsilon}_t)}{\sigma_{\overline{\epsilon}}^2} = \frac{\beta_{op,i}\beta_{op,M}\sigma_{op}^2 + \frac{\sigma_{\nu,i}^2}{N}}{\beta_{op,M}^2 \sigma_{op}^2 + \frac{1}{N^2}\sum\limits_{\ell=1}^{N} \sigma_{\nu,\ell}^2} \tag{4.9}$$

If the number of assets N is considerably large and $\sigma_{\epsilon,i}^2 < 1 \forall i$, the regression coefficient is proportional to the unknown factor sensitivity $\beta_{op,i}$, more precisely

$$\delta_{1,i} \approx \frac{\beta_{op,i}\beta_{op,M}\sigma_{op}^2}{\beta_{op,M}^2\sigma_{op}^2} = \frac{\beta_{op,i}}{\beta_{op,M}}. \tag{4.10}$$

Further, for large N, we can approximate Equation (4.8) with

$$\sigma_{\overline{\epsilon}}^2 \approx \beta_{op,M}^2 \sigma_{op}^2, \tag{4.11}$$

leading to

$$\beta_{op,i} \approx \frac{\delta_{i,1}\sigma_{\bar{\epsilon}}}{\sigma_{op}} \Rightarrow \hat{\beta}_{op,i} \approx \frac{\hat{\delta}_{i,1} \cdot \hat{\sigma}_{\bar{\epsilon}}}{\sigma_{op}}, \tag{4.12}$$

where the circumflex indicates estimates for the unknown true parameters. The yet unknown denominator σ_{op} cancels out in the final step. To estimate the second component of $\sigma_{\epsilon,i}^2$, i.e. $\sigma_{\nu,i}^2$, we insert the approximations in Equation (4.10) and (4.11) into the R-squared of the univariate auxiliary regression in Equation (4.6):

$$R_i^2 = \left(\frac{\text{cov}(\epsilon_{i,t}, \overline{\epsilon}_t)}{\sigma_{\epsilon,i}\sigma_{\overline{\epsilon}}}\right)^2 = \left(\frac{\delta_{1,i}\sigma_{\overline{\epsilon}}}{\sigma_{\epsilon,i}}\right)^2 \approx \left(\frac{\beta_{op,i}^2\beta_{op,M}^2\sigma_{op}^2}{\beta_{op,M}^2\sigma_{\epsilon,i}^2}\right) = \left(\frac{\beta_{op,i}^2\sigma_{op}^2}{\sigma_{\epsilon,i}^2}\right)$$

Given an estimate for $\hat{R}_i^2$, it follows

$$\sigma_{\nu,i} \approx \sqrt{(1-R_i^2)\sigma_{\epsilon,i}^2} \Rightarrow \hat{\sigma}_{\nu,i} \approx \sqrt{(1-\hat{R}_i^2)\hat{\sigma}_{\epsilon,i}^2}. \tag{4.13}$$

Finally, combining Equation(4.12) and (4.13) with Equation (4.5) we can pin down the two components of $\hat{\sigma}_{\epsilon,i}^2$:

$$\begin{aligned} \hat{\sigma}_{\epsilon,i}^2 = \hat{\beta}_{op,i}^2\sigma_{op}^2 + \hat{\sigma}_{\nu,i}^2 \approx \left(\frac{\hat{\delta}_{i,1}\hat{\sigma}_{\bar{\epsilon}}}{\sigma_{op}}\right)^2 \sigma_{op}^2 + \hat{\sigma}_{\nu,i}^2 \\ = \underbrace{(\hat{\delta}_{i,1}\hat{\sigma}_{\bar{\epsilon}})^2}_{\text{Exposure to latent factor variance}} + \underbrace{\hat{\sigma}_{\nu,i}^2}_{\text{Asset-specific variance}} \end{aligned} \tag{4.14}$$

We introduce the following nomenclature for the remainder of this paper: We define the square root of the exposure to the latent factor variance as latent factor volatility LFV_{FM} and refer to the asset-specific volatility as ASV_{FM}. The square-root of $\hat{\sigma}_{\epsilon,i}^2$, i.e. idiosyncratic volatility is referred to as $IVol_{FM}$, following the methodology of Ang et al. (2006, 2009). $\hat{\beta}_{op}^{FM}$ refers to the estimate for the sensitivity to the latent factor. FM indicates the corresponding factor model which is used to generate residuals.

4.2.3 Asset pricing implications of the volatility decomposition

Ang et al. (2006, 2009) employ the square root of idiosyncratic variance $\hat{\sigma}_{\epsilon,i}^2$ from the Fama and French (1993) three factor model as a measure for idiosyncratic risk. Empirically, they find a strong negative relationship between idiosyncratic volatility ($IVol$) and expected returns and alphas, also known as the idiosyncratic volatility puzzle. Based on the theoretical framework in Section 4.2.1, Claußen et al. (2019) identify latent but systematic noise trader risk as the most promising explanation for the puzzle. Their analysis relies on portfolios rather than individual stocks as base assets. Ang et al. (2018), however, argue that individual stocks permit more efficient factor pricing tests. To address this concern, we analyze the implications of the volatility decomposition in the cross section of stock returns. We organize the analysis in two testable hypotheses.

Ex ante, LFV is unlikely to be a large fraction of idiosyncratic risk because factor asset pricing models such as the Fama and French (1993) three factor model receive vast empirical support and capture a large amount of commonality in stock returns. Thus, we expect the latent factor component LFV to be only a small fraction of $IVol$. However, if latent systematic risk is an important determinant of the idiosyncratic volatility puzzle, we expect that this small fraction contributes a large share to the negative $IVol$ risk premium and hypothesize:

Hypothesis H1: *LFV is a small fraction of $IVol$, but carries a large fraction of the negative $IVol$ risk premium.*

Claußen et al. (2019) directly test the linear relationship in Equation (4.2) and show that high-$IVol$ portfolios exhibit negative alphas due to the exposure to noise trader risk. In line with the considerations of Stambaugh et al. (2015), the latent risk behind the $IVol$ puzzle commands a negative risk premium. Given this premise, high-$IVol$ stocks should have higher exposures $\beta_{op,i}$. Thus, the cross-sectional risk premium on $\beta_{op,i}$ should be negative. We expect that the linear relation in Equation (4.2) extends to the cross section as well and postulate:

Hypothesis H2: *Stocks with a high $\beta_{op,i}$ have low expected returns.*

4.3 Data, methodology and descriptive statistics

4.3.1 Data and methodology

Our stock sample covers the CRSP common stock universe (share code 10 and 11) from July 1963 to December 2016. We obtain returns, market capitalizations, trading volumes and prices on a daily and monthly basis from CRSP. We follow Hou and Loh (2016) and apply a one dollar price screen. Returns are adjusted for delistings as motivated by Shumway (1997).

Following Ang et al. (2006), we define $IVol$ in month t as the standard deviation of residuals from a daily Fama and French (1993) three factor model ($FF3$) regression. At least 15 valid daily observations are required. In robustness checks we adjust the estimation window as well as the factor model. This one-month ($1M$) baseline measure is referred to as $IVol_{FF3}^{1M}$. We perform an additional within-month regression on the daily residuals of each stock i in month t on the cross-sectional average of residuals according to Equation (4.6). We approximate the latent factor sensitivity $\beta_{op,i}^{FM}$, the volatility attributable to the latent systematic factor LFV_{FF3}^{1M}, as well as the asset specific component ASV_{FF3}^{1M} according to Equation (4.12) and Equation (4.14), respectively.

Returns on the risk factors of the models proposed by Fama and French (1993), Carhart (1997) and Fama and French (2015) are from Kenneth French's website. The Bali et al. (2017) lottery demand factor is downloaded from Turan Bali's website. The Pastor and Stambaugh (2003) liquidity factor and the Stambaugh, Yu and Yuan (2015) mispricing measure are from Robert F. Stambaugh's and Yu Yuan's website, respectively. Appendix 3.A describes the control variables based on this data and their respective estimation. We gratefully acknowledge the provision of risk factors and economic data by fellow colleagues.

4.3.2 Descriptive statistics

Table 4.1 provides summary statistics over the sample period from July 1963 to December 2016. We report means, standard deviations, the number of observations and quantile measures of our variables of interest, i.e. $IVol_{FF3}^{1M}$, LFV_{FF3}^{1M} and ASV_{FF3}^{1M}, as well as control variables. Descriptives of ASV_{FF3}^{1M} and $IVol_{FF3}^{1M}$ are very similar. $IVol_{FF3}^{1M}$ is largely idiosyncratic and the latent systematic factor on average accounts for 6.46% of its magnitude. The average monthly LFV_{FF3}^{1M} of 0.55% is consequently low compared to an average ASV_{FF3}^{1M} of 2.59% per month. This observation extends to extreme values of LFV_{FF3}^{1M}. Even in the 99th percentile, the share of LFV_{FF3}^{1M} accounts for 38.33% of $IVol_{FF3}^{1M}$. This empirical evidence is in line with the preliminary considerations that factor models capture a

Table 4.1: Summary statistics of the baseline analysis.

Variable	Mean	Std. Dev.	N	1st Pctl	Q1	Median	Q3	99th Pctl
$IVol_{FF3}^{1M}$	0.0267	0.0222	2,853,719	0.0023	0.0130	0.0209	0.0335	0.1063
LFV_{FF3}^{1M}	0.0055	0.0071	2,853,719	0.0000	0.0014	0.0034	0.0070	0.0324
ASV_{FF3}^{1M}	0.0259	0.0214	2,838,344	0.0035	0.0127	0.0203	0.0324	0.1029
LFV_{FF3}^{1M} (%)	0.0646	0.0840	2,838,344	0.0000	0.0071	0.0316	0.0895	0.3833
ASV_{FF3}^{1M} (%)	0.9354	0.0840	2,838,344	0.6168	0.9105	0.9684	0.9929	1.0000
β_{op}^{FF3}	0.0011	0.0090	2,853,719	-0.0210	-0.0027	0.0004	0.0042	0.0291
Mkt Cap (th.)	1,500,003	10,217,437	2,851,186	1,670	21,668	85,866	432,042	25,873,417
Return	0.0063	0.1596	2,845,668	-0.3713	-0.0682	-0.0038	0.0669	0.5083
Illiq	1.8997	10.3142	2,249,893	0.0001	0.0077	0.0839	0.6876	33.0751
CoSkew	-4.9360	26.5109	2,638,990	-89.6172	-13.1366	-2.6563	4.5527	66.7917
ISkew	0.6111	1.4613	2,638,990	-3.2040	0.0590	0.4434	0.9738	6.1830
Max	0.0419	0.0346	2,797,007	0.0064	0.0205	0.0325	0.0523	0.1682
MOM	0.1641	0.7086	2,616,597	-0.7505	-0.1791	0.0667	0.3452	2.5500
ZeroRet	0.2267	0.2358	2,853,208	0.0000	0.0476	0.1500	0.3333	0.9545
MISP	50.3301	13.4269	1,726,244	21.8800	40.8700	49.7600	59.2400	82.5800
β_{AV}	0.1237	7.1789	2,282,492	-21.3019	-1.7718	0.0895	2.0152	21.6210
β_{AC}	0.0067	0.3136	2,282,492	-0.8570	-0.1304	-0.0010	0.1332	0.9352
β_{Liq}	0.0112	0.8276	2,282,491	-2.1844	-0.3495	0.0103	0.3744	2.2194
β_{Max}	0.9821	0.9302	2,277,053	-0.5318	0.3342	0.8589	1.4836	3.5948
β_{CIV}	-0.5736	15.3343	2,282,492	-42.4499	-8.2273	-0.2146	7.2735	39.6381
β_{MV}	-252.2155	465.0155	2,282,492	-1,780.1260	-407.7942	-149.4328	-35.2598	810.9744
β_{Mkt}	0.8754	0.6381	2,638,990	-0.5150	0.4471	0.8482	1.2495	2.5890
β_{HML}	0.1141	0.9259	2,638,990	-2.5611	-0.3178	0.1318	0.5803	2.6054
β_{SMB}	0.7449	0.8192	2,638,990	-0.9922	0.2054	0.6585	1.1967	3.0979

Table 4.1 presents summary statistics of the baseline analysis. We report means, standard deviations, numbers of observations N and quantiles of: $IVol_{M}^{1M}$ is the one-month idiosyncratic volatility of Fama and French (1993) three factor model residuals. LVF_{FF3}^{1M} (ASV_{FF3}^{1M}) is the volatility attributable to the latent systematic factor (asset-specific factor) of $IVol_{FF3}^{1M}$ after its decomposition. We further report descriptive statistics for the share of each component in relation to $IVol_{FF3}^{1M}$ as well as the estimate for the sensitivity β_{op}^{FF3}. Mkt Cap is the monthly market capitalization in 1,000 USD and Return is the monthly return of the respective stock. Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000), ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals and Max is the average of the five highest daily return in the previous month (Bali et al., 2017). MOM is the cumulative return over the previous year. ZeroRet is the share of zero returns and MISP is the Stambaugh et al. (2015) mispricing measure. Betas are calculated to the following risk factors: AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Finally, Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. The sample period is July 1963 to December 2016.

large fraction of commonality in stock returns, leading to Hypothesis **H1**.

Table 4.2 presents time series averages of cross-sectional correlations. The correlation coefficient between $IVol_{FF3}^{1M}$ and ASV_{FF3}^{1M} is close to one. This mechanical relation is due to the decomposition in Equation (4.14), the large difference between the variances of the two $IVol_{FF3}^{1M}$ components by roughly factor ten and the positive skewness of the marginal distributions. The large difference in variances might also explain the results of Stambaugh et al. (2015) who find an almost perfect correlation between $IVol_{FF3}^{1M}$ and a model extended by a mispricing factor. The correlation between LFV_{FF3}^{1M} and $IVol_{FF3}^{1M}$ (ASV_{FF3}^{1M}) is lower and amounts to 0.64 (0.57). In general, the correlation between LFV_{FF3}^{1M} and other stock characteristics and risk factor betas is reduced compared to $IVol_{FF3}^{1M}$ (ASV_{FF3}^{1M}). For example, Max and $IVol_{FF3}^{1M}$ (ASV_{FF3}^{1M}) are highly correlated with a correlation of 0.87 (0.88), but the decomposed variance attributable to the latent factor conveys a smaller correlation of 0.57. This finding is very similar for the beta to $FMax$, the systematic lottery demand factor proposed in Bali et al. (2017), although the correlation is generally lower. LFV_{FF3}^{1M} is almost uncorrelated to the market beta and β_{SMB}. The correlation between LFV_{FF3}^{1M} and β_{HML} is also substantially reduced after the decomposition. This illustrates that the latent systematic factor behind LFV_{FF3}^{1M} is indeed not captured by the $FF3$ model or alternative candidate variables.

Table 4.2: Time-series average of cross-sectional correlations in the baseline analysis.

Variable	$IVol_{FF3}^{1M}$	LFV_{FF3}^{1M}	ASV_{FF3}^{1M}	β_{OP}^{FF3}	Mkt Cap	Return	Illiq	CoSkew	ISkew	Max	MOM	ZeroRet	MISP	β_{AV}	β_{AC}	β_{Liq}	β_{Max}	β_{CIV}	β_{MV}	β_{Mkt}	β_{HML}	β_{SMB}
LFV_{FF3}^{1M}	0.64	1.00																				
ASV_{FF3}^{1M}	1.00	0.57	1.00																			
β_{OP}^{FF3}	0.18	0.30	0.16	1.00																		
Mkt Cap	-0.13	-0.08	-0.13	-0.03																		
Return	0.12	0.08	0.12	0.01	0.02	1.00																
Illiq	0.38	0.23	0.38	0.07	-0.11	-0.04	1.00															
CoSkew	-0.09	-0.06	-0.09	-0.04	0.03	0.00	-0.03	1.00														
ISkew	0.13	0.08	0.13	0.03	-0.07	0.11	0.06	-0.04	1.00													
Max	0.90	0.57	0.90	0.16	-0.09	0.34	0.35	-0.08	0.17	1.00												
MOM	-0.09	-0.06	-0.09	-0.04	0.05	0.02	-0.11	-0.06	0.22	-0.09	1.00											
ZeroRet	-0.01	-0.01	0.01	0.03	-0.21	-0.06	0.19	-0.02	0.12	0.06	-0.13	1.00										
MISP	0.17	0.10	0.17	0.04	-0.05	-0.04	0.02	-0.02	-0.08	0.16	-0.25	0.02	1.00									
β_{AV}	0.01	0.01	0.01	0.00	0.00	0.03	0.00	0.01	0.02	0.02	0.03	-0.03	-0.03	1.00								
β_{AC}	0.05	0.03	0.05	0.01	-0.01	0.00	0.04	0.05	0.01	0.04	-0.01	-0.01	0.01	0.25	1.00							
β_{Liq}	-0.01	-0.01	-0.01	0.00	0.00	0.00	-0.01	-0.01	0.00	-0.01	0.01	0.01	-0.02	-0.10	-0.09	1.00						
β_{Max}	0.25	0.16	0.24	0.05	-0.03	-0.02	0.02	-0.05	0.04	0.23	-0.01	-0.11	0.12	0.08	0.11	-0.06	1.00					
β_{CIV}	-0.02	-0.02	-0.02	-0.01	0.01	-0.01	-0.03	0.03	-0.02	-0.03	-0.01	-0.08	0.01	-0.15	-0.17	-0.05	0.13	1.00				
β_{MV}	-0.11	-0.07	-0.11	-0.03	0.03	0.01	0.00	0.09	-0.03	-0.11	0.00	0.03	-0.06	-0.22	0.45	-0.06	-0.28	-0.02	1.00			
β_{Mkt}	0.11	0.07	0.10	-0.01	0.06	-0.01	-0.12	0.01	-0.04	0.11	0.04	-0.34	0.11	0.03	0.06	-0.02	0.39	0.09	-0.16	1.00		
β_{HML}	-0.02	-0.02	-0.02	0.00	-0.02	0.01	0.04	-0.03	0.01	-0.02	-0.02	0.04	-0.04	0.01	0.01	0.01	-0.13	-0.05	0.06	0.13	1.00	
β_{SMB}	0.25	0.16	0.24	0.04	-0.10	-0.01	0.04	-0.14	0.06	0.24	0.02	-0.05	0.13	0.03	0.05	-0.03	0.32	0.04	-0.16	0.41	0.06	1.00

Table 4.2 presents time series averages of cross-sectional correlations for the following variables: $IVol_{FF3}^{1M}$ is the one-month idiosyncratic volatility of Fama and French (1993) three factor model residuals. LVF_{FF3}^{1M} (ASV_{FF3}^{1M}) is the volatility attributable to the latent systematic factor (asset-specific factor) of $IVol_{FF3}^{1M}$ after its decomposition. β_{op}^{FF3} is the sensitivity of a stock to the latent systematic factor. Mkt Cap is the monthly market capitalization in 1,000 USD, Price is the share price at the end of month t and Return is the monthly return of the respective stock. Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000), ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals and Max is the average of the five highest daily return in the previous month (Bali et al., 2017). MOM is the cumulative return over the previous year. ZeroRet is the share of zero returns and MISP is the Stambaugh et al. (2015) mispricing measure. Betas are calculated to the following risk factors: AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Finally, Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. The sample period is July 1963 to December 2016.

Figure 4.1: Scatter plots of the $IVol$ components LFV and ASV.

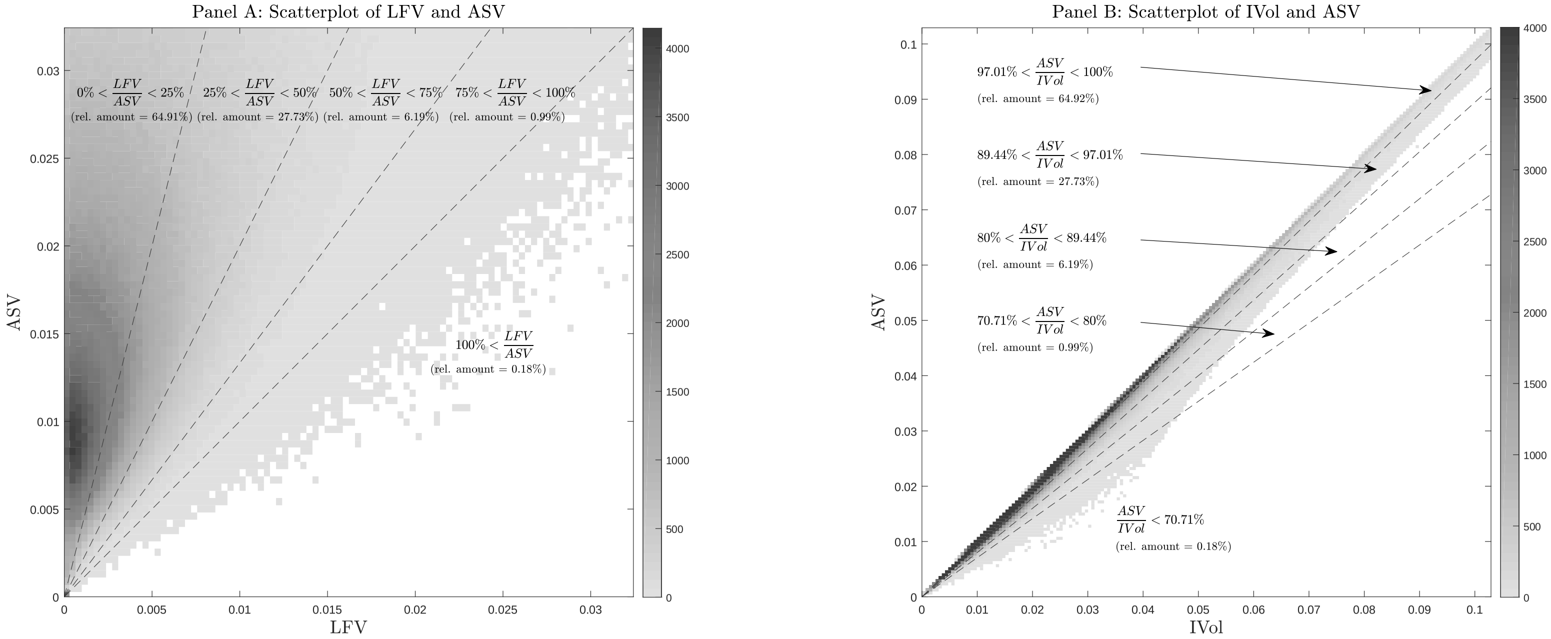


Figure 4.1 presents binned scatter plots of LFV, ASV and $IVol$. Panel A presents a binned scatter plot of the two components LFV and ASV as well as the relative amount of observations in different areas conditional on the ratio of LFV and ASV ($\frac{LFV}{ASV}$). Dashed lines separate observations in different $\frac{LFV}{ASV}$ regimes. Panel B presents a binned scatter plot of $IVol$ and its asset-specific component ASV. $\frac{ASV}{IVol}$ is the ratio of $IVol$ and ASV and dashed lines separate observations in different $\frac{ASV}{IVol}$ regimes. The color bar indicates the number of observations in each bin. The sample period is July 1963 to December 2016 and both figures consider 99% of the sample stocks to remove outliers from the illustration.

Figure 4.1 illustrates the effectiveness of our $IVol$ decomposition. Panel A presents binned scatter plots of the two $IVol_{FF3}^{1M}$ components LFV_{FF3}^{1M} and ASV_{FF3}^{1M} as well as the relative amount of stocks in different regions conditional on the ratio of both components, i.e. $\frac{LFV}{ASV}$. The dashed line separates observations in different $\frac{LFV}{ASV}$ regimes. The color bar indicates the number of observations in each bin. Both figures consider 99% of the sample stocks to remove outliers from the illustration.

In 65% of the observations, LFV_{FF3}^{1M} is less than a quarter of ASV_{FF3}^{1M}. This is consistent with our considerations which lead to hypothesis **H1** that the $FF3$ factor model captures a large fraction of common factors in stock returns and idiosyncratic volatility is indeed largely asset-specific. For the remaining 35%, the model does not appropriately account for the volatility of a latent risk factor. More specifically, one percentage point of ASV_{FF3}^{1M} is associated with at least 0.5 percentage points of latent factor volatility LFV_{FF3}^{1M} for roughly 7.5% of the stocks. Although average LFV_{FF3}^{1M} is small, Panel A unveils that a sizable fraction of the sample exhibits latent factor volatility in a magnitude comparable to ASV_{FF3}^{1M}. It turns out that the $\frac{LFV}{ASV}$-ratio amplifies the relation between $IVol$ and subsequent returns. The $IVol$ risk premium decreases monotonically from -18.13% in the first regime with $\frac{LFV}{ASV} < 25\%$ to -33.14% for stocks with $\frac{LFV}{ASV} > 75\%$. This is a first hint that LFV is a stronger driver of the negative idiosyncratic volatility premium than ASV.

Panel B presents binned scatter plots of $IVol_{FF3}^{1M}$ and its asset-specific component ASV_{FF3}^{1M} as well as the ratio between the two variables $\frac{ASV}{IVol}$. The dashed line separates the sample into different regions conditional on the $\frac{ASV}{IVol}$-ratio. We choose cutoff values to match the relative number of observations in Panel A.

If we consider the aforementioned 65% of observations, the fraction of latent factor volatility in LFV_{FF3}^{1M} is close to zero and ASV_{FF3}^{1M} accounts for the entire magnitude of $IVol_{FF3}^{1M}$. For observations with a sizable fraction of LFV_{FF3}^{1M} in relation to ASV_{FF3}^{1M}, the decomposition absolves $IVol_{FF3}^{1M}$ of the volatility attributable to the latent factor. LFV_{FF3}^{1M} accounts for at least 10% of $IVol_{FF3}^{1M}$ for the 7.5% of observations which exhibit 0.5 percentage points LFV_{FF3}^{1M} for each percentage point of ASV_{FF3}^{1M}. Despite the high correlation between ASV_{FF3}^{1M} and $IVol_{FF3}^{1M}$, the asset-specific component ASV_{FF3}^{1M} differs from $IVol_{FF3}^{1M}$ for observations which exhibit a sizable latent factor volatility LFV_{FF3}^{1M}.

4.4 Empirical analysis

4.4.1 Asset pricing implications of latent noise trader risk

First of all, Hypothesis **H1** implies an underperformance of stocks with high latent factor volatility LFV. Thus, we perform Fama and MacBeth (1973) cross-sectional regressions to test the negative relation between LFV_{FF3}^{1M} and subsequent excess returns. We control for $IVol_{FF3}^{1M}$ or its *purely* asset-specific component ASV_{FF3}^{1M} and other firm characteristics as well as risk factor betas.[2] Table 4.3 presents average coefficient estimates of

[2]We follow Hou and Loh (2016) and do not consider lottery demand Max due to the high mechanical correlation with $IVol_{FF3}^{1M}$ and ASV_{FF3}^{1M}, respectively. Instead, we

the monthly cross-sectional regressions. Newey and West (1987) adjusted t-statistics with six lags in parentheses test the null hypothesis that the average slope coefficient is zero. Average coefficients are multiplied with one hundred. The sample period is June 1964 to December 2016 in Columns (1) - (6) and June 1968 to December 2016 in Columns (7) - (12). We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis.

Column (1) in Table 4.3 confirms the results of Ang et al. (2006, 2009) in our sample period. The average coefficient on $IVol$ amounts to -20.882 and is statistically significant with a t-statistic of -4.02. Column (2) is in line with the mechanical relation between $IVol_{FF3}^{1M}$ and ASV_{FF3}^{1M}. We find an almost identical average coefficient on ASV_{FF3}^{1M} of -20.963 with a t-statistic of -3.95. Consequently, we include the asset-specific component ASV_{FF3}^{1M} instead of $IVol_{FF3}^{1M}$ in the further analysis.

Column (3) to (5) focus on the volatility attributable to the latent systematic factor LFV. In the univariate regression of Column (3), LFV_{FF3}^{1M} exhibits an average coefficient of -45.399 with a t-statistic of -4.76. Column (4) and (5) show that the coefficient on LFV_{FF3}^{1M} remains statistically significant once we control for ASV_{FF3}^{1M}, several stock characteristics and betas with respect to the three Fama and French (1993) factors. Although the magnitude of the coefficient decreases when we include ASV_{FF3}^{1M} as a control variable, this does not affect the statistical significance of LFV_{FF3}^{1M}

estimate the beta to aggregate lottery demand proposed in Bali et al. (2017) in order to capture the aggregate effect of lottery demand. In Section 4.5.4 we include Max and show that our results are robust to an inclusion of lottery demand as a stock characteristic as well.

Table 4.3: Fama and MacBeth (1973) regressions.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	1.2479	1.2316	0.9918	1.2393	3.7775	2.5669	2.3764	2.4475	2.3797	2.4454	2.3924	2.3235
	(6.47)	(6.39)	(4.32)	(6.47)	(7.50)	(4.47)	(4.25)	(4.24)	(4.19)	(4.23)	(4.50)	(4.48)
$IVol_{FF3}^{1M}$	-20.822											
	(-4.02)											
ASV_{FF3}^{1M}		-20.963		-16.892	-23.862							
		(-3.95)		(-3.31)	(-8.03)							
LFV_{FF3}^{1M}			-45.399	-19.735	-17.719	-40.557	-42.610	-42.486	-42.448	-42.178	-42.279	-41.433
			(-4.76)	(-4.97)	(-5.25)	(-8.78)	(-9.84)	(-9.76)	(-9.70)	(-9.68)	(-10.17)	(-10.11)
LagRet					-4.0599	-4.2864	-4.5263	-4.3785	-4.4060	-4.4263	-4.5983	-4.7115
					(-9.03)	(-9.61)	(-9.53)	(-9.27)	(-9.38)	(-9.26)	(-9.78)	(-9.62)
Mom					0.7157	0.7775	0.7311	0.7352	0.7231	0.7188	0.7043	0.7020
					(5.32)	(5.64)	(4.99)	(4.98)	(4.99)	(4.93)	(5.05)	(5.15)
Illiq					0.0324	0.0193	0.0115	0.0111	0.0107	0.0114	0.0109	0.0107
					(3.33)	(1.97)	(1.35)	(1.30)	(1.27)	(1.33)	(1.30)	(1.27)
ISkew					-0.0400	-0.0492	-0.0450	-0.0413	-0.0437	-0.0431	-0.0452	-0.0388
					(-2.37)	(-2.79)	(-2.55)	(-2.28)	(-2.47)	(-2.41)	(-2.67)	(-2.26)
CoSkew					-0.0085	-0.0078	-0.0053	-0.0080	-0.0074	-0.0087	-0.0059	-0.0049
					(-1.83)	(-1.69)	(-1.48)	(-1.68)	(-1.76)	(-1.83)	(-1.59)	(-1.59)
ZeroRet					-0.3280	-0.0098	-0.1595	-0.1607	-0.1731	-0.1670	-0.1444	-0.1561
					(-1.26)	(-0.04)	(-0.62)	(-0.61)	(-0.67)	(-0.64)	(-0.57)	(-0.64)
Size					-0.2054	-0.1336	-0.1139	-0.1178	-0.1135	-0.1179	-0.1151	-0.1103
					(-6.07)	(-3.40)	(-3.02)	(-3.04)	(-2.96)	(-3.03)	(-3.24)	(-3.22)
β_{Mkt}					-0.0318	-0.1330	-0.1656	-0.1464	-0.1505	-0.1324	-0.1901	-0.2000
					(-0.23)	(-0.89)	(-1.12)	(-0.94)	(-0.97)	(-0.84)	(-1.52)	(-1.63)
β_{HML}					0.1473	0.1835	0.1891	0.1833	0.1797	0.1769	0.1802	0.1773
					(2.21)	(2.64)	(2.72)	(2.55)	(2.54)	(2.50)	(3.09)	(3.10)
β_{SMB}					-0.1041	-0.1136	-0.1491	-0.1488	-0.1452	-0.1537	-0.1458	-0.1430
					(-2.10)	(-2.19)	(-3.04)	(-3.01)	(-2.96)	(-3.16)	(-3.19)	(-3.21)
β_{MV}							-0.0005					
							(-1.94)					
β_{AV}								-0.0071				-0.0055
								(-1.01)				(-0.82)
β_{AC}								0.0444				0.1124
								(0.56)				(1.28)
β_{CIV}									-0.0017			-0.0005
									(-0.61)			(-0.17)
β_{Liq}										0.0050		-0.0038
										(0.12)		(-0.09)
β_{FMax}											0.0476	0.0589
											(0.53)	(0.62)
$avg.\overline{R}^2$ in %	1.8708	1.8536	0.7687	1.9535	6.8888	6.5895	6.6191	6.6437	6.6187	6.6611	6.8409	7.1854
$avg.N$	3301	3301	3301	3301	3301	3301	3147	3147	3147	3147	3147	3147

Table 4.3 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ on $IVol_{FF3}^{1M}$ and its two components ASV_{FF3}^{1M} and LFV_{FF3}^{1M} in month t as well as several control variables. We include the following set of control variables: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period in Model (1) - (6) is June 1964 to December 2016 and the sample period in Model (7) - (12) is June 1968 to December 2016. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

with t-statistics around minus five.

To focus on the asset pricing implications of LFV_{FF3}^{1M}, we drop ASV_{FF3}^{1M} in Column (6) to Column (12) and include alternative explanations for the idiosyncratic volatility puzzle. The t-statistic of LFV_{FF3}^{1M} increases substantially from around minus five to roughly minus nine. The magnitude of the coefficient of -40.557 in Column (6) is similar to the univariate analysis in Column (3). Average coefficients on LFV_{FF3}^{1M} are largely unrelated to the inclusion of control variables. This supports the latent nature of the systematic risk factor. In Column (6), control variables are also significant with expected signs, except for the market beta and ZeroRet.

In Column (7) to (12) we include potential systematic risk factors which provide alternative explanations for the idiosyncratic volatility puzzle and thus might relate to LFV_{FF3}^{1M}. We start with aggregate market variance risk in Column (7), as motivated by Chen and Petkova (2012), and include a stock's beta to average monthly market variance. In line with the literature, this coefficient is significantly negative at the ten percent level, but its inclusion does not change the estimate for LFV_{FF3}^{1M}. The two components of aggregate variance risk, average variance and average cross-correlation proposed by Chen and Petkova (2012) in Column (8) are not significant. This result is similar to Hou and Loh (2016). The betas to the Herskovic et al. (2016) measure for common idiosyncratic volatility (CIV), Pastor and Stambaugh (2003) aggregate liquidity risk and the Bali et al. (2017) aggregate lottery demand factor are also insignificant in Columns (9) to (11). This also holds true for the full regression model in Column (12).[3] Either way, LFV_{FF3}^{1M} remains statistically significant at any conventional level with an average estimate of roughly -42. The difference between the estimates in Column (6) and the Columns (7) to (12) is caused by the shorter sample period starting in June 1968 instead of June 1964 due to the availability of control variables. In summary, the negative relationship between LFV_{FF3}^{1M} and subsequent returns is robust to the inclusion of well-accepted stock characteristics and risk factor betas. The results in Table 4.3 support hypothesis **H1**.

The asset-specific component ASV_{FF3}^{1M} remains significant when including LFV_{FF3}^{1M}. To illustrate that the full sample evidence draws an incomplete picture, we compute the Fama and MacBeth (1973) estimates in a rolling window analysis as motivated from Lewellen (2015). Figure 4.2 presents ten-year rolling window averages of the coefficient estimates in Column (4) in Table 4.3, i.e. an analysis of the two components without control variables. The figure plots the average coefficient estimate over the preceding ten years in bold lines as well as the upper and lower 95% confidence bound in dashed lines. The dates on the abscissa indicate the end of each ten-year window, so the depicted significance relates to the previous ten years. Panel A (B) shows the evolution of the coefficient on LFV_{FF3}^{1M} (ASV_{FF3}^{1M}) over time.

Before 1988, the upper 95% confidence bound for the average coefficient on LFV_{FF3}^{1M} is above zero, indicating that the slope coefficient is insignificant from 1963 to 1978. Thereafter, however, the upper bound is

[3] Instead of β_{MV}, we include the two components of aggregate market variance, i.e. β_{AC} and β_{AV}.

Figure 4.2: Rolling window Fama and MacBeth (1973) estimates.

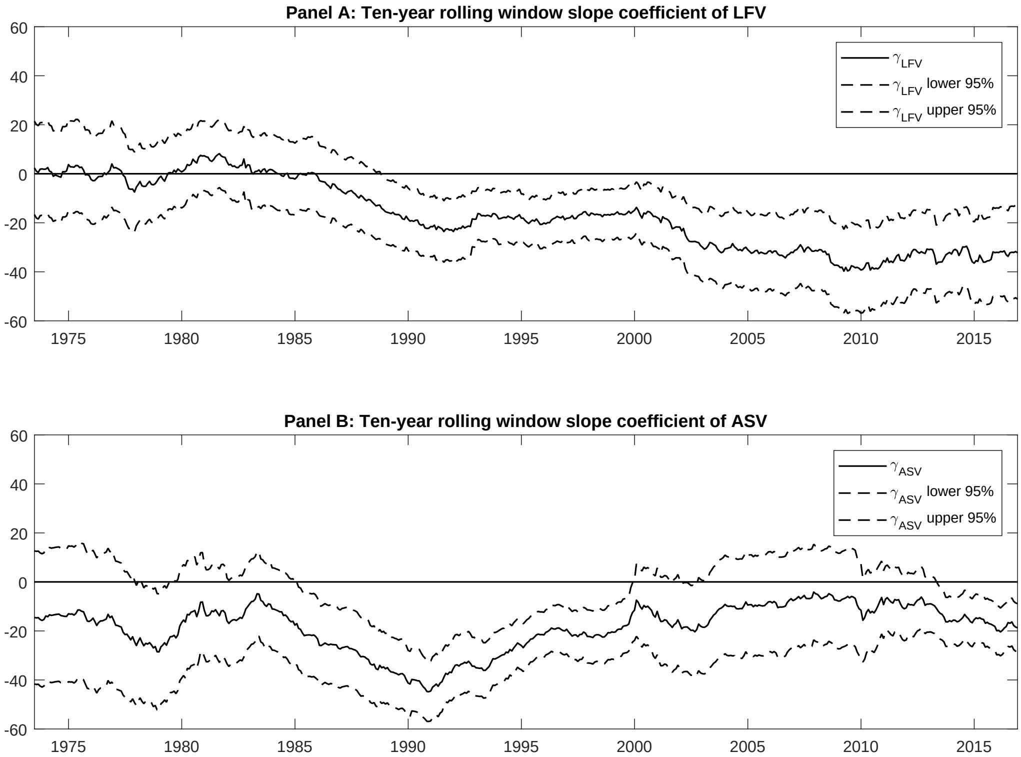


Figure 4.2 presents ten-year rolling window estimates of the Fama and MacBeth (1973) average coefficients in Column (4) of Table 4.3. Panel A (B) presents the ten-year rolling window average slope coefficient on LFV_{FF3}^{1M} (ASV_{FF3}^{1M}) in bold lines while controlling for both components of $IVol_{FF3}^{1M}$. Dashed lines represent 95% confidence bounds of the average slope coefficient. The dates on the abscissa indicate the end of each ten-year window. The sample period is 1963 to 2016.

well below zero and the rolling window coefficients indicate a significantly negative relation between LFV_{FF3}^{1M} and subsequent returns. Considering the more recent time period, the strength of this effect increases.

Similarly, the first part of the sample period shows mixed results for ASV_{FF3}^{1M} in Panel B. The upper bound of the 95% confidence bound falls below zero in 1985, tantamount to an insignificant slope coefficient before 1975. Recall that the dates on the abscissa indicate the end of the respective estimation window. Afterwards, the average slope coefficient on ASV_{FF3}^{1M} is significantly negative from 1975 to 1989 for approximately 14 years. The relationship is rather unstable with a stronger negative relation in recent years.

Considering the rolling window estimates of the Fama and MacBeth (1973) regressions, the full sample evidence on ASV_{FF3}^{1M} is intriguing and mostly driven by a negative peak in the 1980s. In contrast, LFV_{FF3}^{1M} is statistically significant for more than 38 consecutive years after October 1978. When controlling for this factor, the truly idiosyncratic component ASV_{FF3}^{1M} is rather short lived and concentrates in a small part of the full sample period. At the same time, the negative risk premium associated with LFV_{FF3}^{1M} becomes stronger in recent years. The time variation of the two slope coefficients differs substantially, indicating that LFV_{FF3}^{1M} and ASV_{FF3}^{1M} reflect different determinants of stock returns. From a historical perspective, the negative risk premium on LFV_{FF3}^{1M} seems more robust than the explanatory power of ASV_{FF3}^{1M}.

Both, the findings of the Fama and MacBeth (1973) regressions as well as the rolling window coefficient estimates support our hypothesis **H1**.

Stocks with high LFV_{FF3}^{1M} have low expected returns and alphas. Although the idiosyncratic component ASV_{FF3}^{1M} remains significant in the full sample analysis, the rolling window estimates unveil that this negative risk premium is driven by a short negative peak, while the slope coefficient LFV_{FF3}^{1M} is steadily negative after 1978 and becomes increasingly more negative in recent years. Furthermore, LFV_{FF3}^{1M} and ASV_{FF3}^{1M} are far from identical and perform differently in the cross section of stock returns.[4]

Hypothesis **H2** postulates low expected returns and alphas for stocks with a high sensitivity β_{op} to the latent factor. Therefore we replace LFV_{FF3}^{1M} with β_{OP}^{FF3} in the Fama and MacBeth (1973) cross-sectional regressions in Table 4.4. Other than that, the econometric framework is identical to Table 4.3. The sample period is June 1964 to December 2016 except for Column (5) with a sample period from June 1968 to December 2016.

We find consistently significant negative signs for the coefficient estimate on β_{OP}^{FF3} for each specification. In the univariate analysis in Column (1), β_{OP}^{FF3} attains an average coefficient of -13.04 with a t-statistic of -3.06. Including ASV_{FF3}^{1M} in Column and (3) reduces the coefficient estimates, but leaves the estimate β_{OP}^{FF3} statistically significant. The inclusion of control variables in Columns (4) to (10) has little effect on the average coefficient of β_{OP}^{FF3} in comparison to the univariate specification in Column (1). High β_{OP}^{FF3} earn low returns in the subsequent month and this effect is robust to the inclusion of control variables. The findings are in line with Hypothesis **H2** and highlight the importance of a systematic component

[4] We present additional empirical support in Section 4.5.1 and show that this results is robust to the orthogonalization of the two components LFV_{FF3}^{1M} and ASV_{FF3}^{1M}.

Table 4.4: Fama and MacBeth (1973) regressions with β_{op}.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.7787	1.2172	2.0457	1.8332	1.9059	1.8394	1.9054	1.8602	1.8030
	(3.02)	(6.30)	(3.41)	(3.15)	(3.17)	(3.11)	(3.17)	(3.37)	(3.36)
β_{op}^{FF3}	-13.040	-9.0947	-13.712	-13.873	-13.568	-13.703	-13.596	-13.916	-13.382
	(-3.06)	(-2.78)	(-4.34)	(-4.20)	(-4.09)	(-4.13)	(-4.12)	(-4.34)	(-4.27)
ASV_{FF3}^{1M}		-19.912							
		(-3.79)							
LagRet			-4.4132	-4.6663	-4.5236	-4.5512	-4.5727	-4.7468	-4.8508
			(-10.00)	(-9.98)	(-9.71)	(-9.83)	(-9.71)	(-10.26)	(-10.07)
MOM			0.8006	0.7517	0.7578	0.7442	0.7402	0.7245	0.7230
			(5.79)	(5.11)	(5.13)	(5.12)	(5.06)	(5.18)	(5.29)
Illiq			0.0135	0.0053	0.0050	0.0046	0.0053	0.0048	0.0048
			(1.38)	(0.63)	(0.59)	(0.54)	(0.62)	(0.57)	(0.56)
ISkew			-0.0548	-0.0513	-0.0474	-0.0500	-0.0493	-0.0515	-0.0449
			(-3.06)	(-2.87)	(-2.58)	(-2.78)	(-2.71)	(-2.99)	(-2.58)
CoSkew			-0.0083	-0.0060	-0.0086	-0.0080	-0.0094	-0.0066	-0.0056
			(-1.72)	(-1.55)	(-1.73)	(-1.80)	(-1.87)	(-1.64)	(-1.63)
ZeroRet			0.1763	0.0285	0.0264	0.0155	0.0197	0.0417	0.0300
			(0.73)	(0.11)	(0.10)	(0.06)	(0.08)	(0.17)	(0.13)
Size			-0.1035	-0.0826	-0.0866	-0.0823	-0.0867	-0.0844	-0.0804
			(-2.52)	(-2.10)	(-2.15)	(-2.07)	(-2.14)	(-2.29)	(-2.26)
β_{Mkt}			-0.1770	-0.2110	-0.1922	-0.1964	-0.1780	-0.2314	-0.2396
			(-1.17)	(-1.41)	(-1.21)	(-1.25)	(-1.12)	(-1.82)	(-1.93)
β_{HML}			0.1930	0.1983	0.1923	0.1891	0.1861	0.1879	0.1847
			(2.73)	(2.82)	(2.64)	(2.63)	(2.59)	(3.17)	(3.17)
β_{SMB}			-0.1169	-0.1535	-0.1534	-0.1495	-0.1580	-0.1495	-0.1466
			(-2.22)	(-3.11)	(-3.07)	(-3.01)	(-3.21)	(-3.25)	(-3.26)
β_{MV}				-0.0005					
				(-1.95)					
β_{AV}					-0.0071				-0.0053
					(-1.00)				(-0.79)
β_{AC}					0.0374				0.1086
					(0.47)				(1.23)
β_{CIV}						-0.0014			-0.0002
						(-0.49)			(-0.07)
β_{Liq}							0.0060		-0.0040
							(0.14)		(-0.10)
β_{FMax}								0.0395	0.0504
								(0.44)	(0.53)
$avg.\overline{R}^2 in$ %	0.3040	2.0903	6.9160	6.9919	7.0500	6.9931	7.0344	7.2164	7.6898
$avg.N$	3301	3301	3301	3147	3147	3147	3147	3147	3147

Table 4.4 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ on $IVol_{FF3}^{1M}$, its asset specific component ASV_{FF3}^{1M} and the sensitivity to the latent factor β_{op}^{FF3} as well as several control variables in month t. We include the following set of control variables: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period is June 1964 to December 2016 except for Column (5) with a sample period from June 1968 to December 2016. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

in supposedly idiosyncratic risk. Our evidence in favor of **H2** is consistent with Claußen et al. (2019) who relate the low average returns of β_{OP}^{FF3} to latent but systematic noise trader risk.

4.4.2 Dissecting the idiosyncratic volatility risk premium

Hypothesis **H1** further postulates that the negative risk premium on the latent factor volatility LFV_{FF3}^{1M} explains a large fraction of the risk premium on $IVol_{FF3}^{1M}$. Therefore we apply the methodology of Hou and Loh (2016) to estimate the share of the negative risk premium on $IVol_{FF3}^{1M}$ in the Fama and MacBeth (1973) regressions which is attributable to LFV_{FF3}^{1M}. Furthermore, the approach provides statistical inference about the significance of the fraction which is explained by the respective candidate variable as well as the unexplained fraction.

Table 4.5 presents results of the univariate decomposition methodology. We report the coefficient for the candidate variable as well as the fraction of the $IVol_{FF3}^{1M}$ risk premium γ_t which is related to this candidate. The t-statistics indicate whether this fraction is statistically different from zero. Motivated by the rolling window regressions in Figure 4.2, we present a sample split to illustrate the effect that the explanatory power of LFV_{FF3}^{1M} increases over time. The sample split in 1989 divides the total sample period from 1963 to 2016 equally.

Over the full sample in Column (1), LFV_{FF3}^{1M} explains 45.38% of the

Table 4.5: Quantization of the systematic risk in the idiosyncratic volatility puzzle.

		(1) Full Sample 1963 - 2016		(2) Sub period 1963 - 1989		(3) Sub period 1990 - 2016	
Description	Variable	Coeff	T	Coeff	T	Coeff	T
$r_{i,t+1} \sim \alpha_t + \gamma_t IVol_{i,t} + \epsilon_{i,t}$	Intercept	1.1687	(6.85)	1.0471	(4.24)	1.2885	(5.47)
	$IVol_{FF3}^{1M}$	-19.251	(-4.82)	-19.6043	(-3.00)	-18.904	(-4.07)
Decomposition of γ_t	LFV_{FF3}^{1M}	-8.7359		-6.9923		-10.4525	
	Fraction in %	45.38	(15.29)	35.67	(9.84)	55.29	(12.09)
	Residual	-10.5155		-12.6117		-8.4517	
	Fraction in %	54.62	(15.01)	64.33	(17.76)	44.71	(9.78)
	# Stocks/ Month	4388		3845		4962	

Table 4.5 presents results of the first and the final stage of the Hou and Loh (2016) decomposition methodology over three different sample periods. $IVol_{FF3}^{1M}$ is the idiosyncratic volatility as the standard deviation of Fama and French (1993) factor model residuals and LFV_{FF3}^{1M} is the variance of these residuals which is attributable to a latent systematic risk factor. The table presents the average coefficient of the decomposed risk premium γ_t which is related to LFV_{FF3}^{1M} and the residual as well as the fraction of γ_t explained by either LFV_{FF3}^{1M} or the residual. Average coefficients are multiplied by one hundred. Independent variables are winsorized at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics are presented in parentheses. The sample period as well as the average number of stocks per month are presented for each respective model.

average Fama and MacBeth (1973) coefficient γ_t on $IVol_{FF3}^{1M}$.[5] This fraction is statistically significant with a t-statistic of 15.29. In comparison with the results of Hou and Loh (2016) who consider a larger set of potential candidate variables for the idiosyncratic volatility puzzle, this is by far the largest fraction attained by a single candidate.[6] In the first half of the sample, as shown in Column (2), this fraction is smaller with 35.67% which is still statistically significant. In contrast, LFV_{FF3}^{1M} explains more than 55% of the average γ_t in the second half of the sample period from

[5]This estimate for γ_t slightly differs from the univariate average Fama and MacBeth (1973) coefficient in Table 4.3 because the latter estimate does not consider stocks for which control variables are missing.

[6]One exception of this observation is the highest daily return Max which attains an explanatory fraction of 112% due to almost perfect correlation with $IVol_{FF3}^{1M}$. In the light of this strong result, Hou and Loh (2016) argue that Max is an alternative measure rather than a candidate variable. To capture the aggregate effect of lottery demand, we include the beta to the lottery demand factor FMax as proposed by Bali et al. (2017).

1990 to 2016 in Column (3). This is in line with the rolling window estimates in Figure 4.2 which confirms that the negative risk premium on LFV_{FF3}^{1M} is stronger in the recent half of the sample.

Table 4.6 extends the analysis to a multivariate setting and we include the same set of control variables as in the Fama and MacBeth (1973) regressions.[7] In Column (1), we restrict the choice of candidate variables to the candidates comparable to Hou and Loh (2016). Still, LFV_{FF3}^{1M} explains more than 40% of the puzzle, followed by LagRet with an explained fraction of roughly ten percent. To put this into perspective, the fraction explained by LFV_{FF3}^{1M} alone is almost as high as the total explained fraction in the best performing model of Hou and Loh (2016) which amounts to roughly 55% and includes as many as nine candidates. The inclusion of Momentum and the Fama and French (1993) risk factors reduces this share to approximately 37% which is, again, by far the largest fraction of a single candidate. Momentum attains a fraction of 8.19% which is statistically significant at the one percent level. Column (3) includes the risk factor betas to average variance (AV), average correlation (AC), common idiosyncratic volatility (CIV) and aggregate lottery demand (FMax). Although the results are similar to the Fama and MacBeth (1973) regressions above, the beta to aggregate lottery demand performs better in the decomposition methodology. Here, β_{FMax} explains almost nine percent of the average γ_t. The fraction explained by LFV_{FF3}^{1M} still amounts to 34.91%

[7] We make two exceptions and exclude Size and β_{MV}. The exclusion of Size is motivated by the choice of Hou and Loh (2016). An inclusion of Size in Column (2) in unreported robustness checks leads to an explained fraction of approximately -10% which makes Size an unlikely candidate for the idiosyncratic volatility puzzle. Instead of β_{MV} we include its two components β_{AC} and β_{AV}.

Table 4.6: Multivariate Hou and Loh (2016) decomposition.

		(1)		(2)		(3)		(4)	
Description	Variable	Coeff	T	Coeff	T	Coeff	T	Coeff	T
$r_{i,t+1} \sim \alpha_t + \gamma_t IVol_{i,t} + \epsilon_{i,t}$	Intercept	1.2525	(6.89)	1.2478	(6.86)	1.2426	(6.51)	1.2426	(6.51)
	$IVol_{FF3}^{1M}$	-21.147	(-4.78)	-20.820	(-4.70)	-24.445	(-5.51)	-24.445	(-5.51)
Decomposition of γ_t	LFV_{FF3}^{1M}	-8.5174		-7.6306		-8.5330			
		40.28	(11.87)	36.65	(10.72)	34.91	(12.50)		
	LagRet	-2.0904		-2.3014		-2.3075		-2.9867	
		9.89	(3.39)	11.05	(3.70)	9.44	(3.88)	12.22	(3.81)
	CoSkew	0.0706		-0.1076		-0.2030		-0.2714	
		-0.33	(-0.29)	0.52	(0.92)	0.83	(1.88)	1.11	(1.72)
	ISkew	0.1557		0.2202		0.0059		-0.0063	
		-0.74	(-0.67)	-1.06	(-1.09)	-0.02	(-0.04)	0.03	(0.03)
	ZeroRet	0.4624		0.4056		0.2670		0.3474	
		-2.19	(-1.67)	-1.95	(-1.93)	-1.09	(-1.66)	-1.42	(-1.66)
	Illiq	0.1819		0.3661		-0.5092		-0.7760	
		-0.86	(-0.22)	-1.76	(-0.44)	2.08	(0.74)	3.17	(0.88)
	Mom			-1.7054		-1.6772		-2.3543	
				8.19	(3.89)	6.86	(4.06)	9.63	(4.30)
	β_{Mkt}			-0.7042		-0.2496		-0.3749	
				3.38	(1.32)	1.02	(0.53)	1.53	(0.63)
	β_{SMB}			-0.3583		-0.9639		-1.3336	
				1.72	(0.48)	3.94	(1.79)	5.46	(1.91)
	β_{HML}			-0.7500		-0.2189		-0.4863	
				3.6	(2.38)	0.9	(0.90)	1.99	(1.54)
	β_{AV}					0.0032		-0.0485	
						-0.01	(-0.04)	0.2	(0.40)
	β_{AC}					-0.0463		-0.0672	
						0.19	(0.89)	0.27	(1.05)
	β_{CIV}					0.3462		0.3889	
						-1.42	(-2.44)	-1.59	(-1.99)
	β_{Liq}					-0.0919		-0.2214	
						0.38	(1.08)	0.91	(2.11)
	β_{FMax}					-2.1055		-2.7461	
						8.61	(3.40)	11.23	(3.11)
	Residual	-11.4099		-8.2547		-8.1611		-13.5085	
		53.95	(13.97)	39.65	(8.54)	33.39	(9.21)	55.26	(9.28)
	# Stocks/ Month	3322		3302		3147		3147	
	Sample Period	1964/06 - 2016/12		1964/06 - 2016/12		1968/06 - 2016/12		1968/06 - 2016/12	

Table 4.6 presents results of the first and the final stage of the Hou and Loh (2016) decomposition methodology in an multivariate analysis. $IVol_{FF3}^{1M}$ is the idiosyncratic volatility as the standard deviation of Fama and French (1993) factor model residuals and LFV_{FF3}^{1M} is the variance of these residuals which is attributable to a latent systematic risk factor. The table presents the average coefficient of the decomposed risk premium γ_t which is related to LFV_{FF3}^{1M} or several control variables and the residual as well as the fraction of γ_t explained by either LFV_{FF3}^{1M}, the control variables or the residual. Control variables cover the following stock characteristics and risk factor betas: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. Average coefficients are multiplied by one hundred. Independent variables are winsorized at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics are presented in parentheses. The sample period as well as the average number of stocks per month are presented for each respective model.

with a t-statistic of 12.50.

To emphasize the relevance of this fraction, we exclude LFV_{FF3}^{1M} in Column (4). The joint explanatory power of the control variables amounts to 45.74% which marginally exceeds the univariate fraction of LFV_{FF3}^{1M} in Column (1) of Table 4.5. Individually, the control variables LagRet, Momentum and β_{FMax} attain higher fractions in comparison to Column (3). This suggests that LFV_{FF3}^{1M} captures effects of these variables to some extent, but the largest part of the 35.91% fraction explained by LFV_{FF3}^{1M} is not related to control variables. The inclusion of LFV_{FF3}^{1M} reduces the unexplained fraction of the idiosyncratic volatility puzzle above and beyond alternative candidates.

From the results in Table 4.5 and Table 4.6, we conclude that the qualitatively small latent factor volatility LFV_{FF3}^{1M} explains a sizable fraction of the negative $IVol$ risk premium. This fraction is substantially higher than any individual alternative candidate presented in Hou and Loh (2016). LFV_{FF3}^{1M} performs almost equally well in multivariate settings. This lies in contrast to other alternative candidates, which lose explanatory power in multivariate decompositions and supports our conjecture that the idiosyncratic volatility puzzle is driven by unaccounted risk in the Fama and French (1993) three factor model. These findings support hypothesis **H1** and the importance of a latent systematic risk factor in the cross section of stock returns.

4.5 Robustness checks

4.5.1 Orthogonal components of idiosyncratic volatility

The correlation between ASV_{FF3}^{1M} and LFV_{FF3}^{1M} might raise the concern that the results in Section 4.4.1 are driven by the information in LFV_{FF3}^{1M} which is related to ASV_{FF3}^{1M}. We therefore orthogonalize the two components LFV_{FF3}^{1M} and ASV_{FF3}^{1M} and refer to the orthogonal component of LFV_{FF3}^{1M} as $^{\perp}LFV_{FF3}^{1M}$.

We review the cross-sectional Fama and MacBeth (1973) regressions in Section 4.4.1 with $^{\perp}LFV_{FF3}^{1M}$ instead of LFV_{FF3}^{1M} in Table 4.7. Independently of the exact specification, $^{\perp}LFV_{FF3}^{1M}$ is highly significant with t-statistics beyond four in absolute terms. The results in Section 4.4.1 are therefore robust to the orthogonalization. This analysis further supports Hypothesis **H1** and highlights the difference between LFV_{FF3}^{1M} and ASV_{FF3}^{1M}.

4.5.2 Alternative factor models and estimation windows

The Fama and French (1993) three factor model benchmark and the one-month estimation window is in line with Ang et al. (2006) and the literature, but this choice might not capture systematic components in stock returns sufficiently and thus drive the results.

We perform two robustness checks to address this concern. First, we estimate $IVol_{FM}^{1M}$ and its two components LFV_{FM}^{1M} and ASV_{FM}^{1M} from the Carhart (1997) four factor model (CAR) and the Fama and French (2015) five factor model ($FF5$). Second, we estimate the three models,

Table 4.7: Fama and MacBeth (1973) regressions with ${}^{\perp}LFV_{FF3}^{1M}$.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	1.2479	1.2316	0.7647	1.2294	3.7695	1.9642	1.7580	1.8300	1.7615	1.8292	1.7896	1.7378
	(6.47)	(6.39)	(2.93)	(6.39)	(7.48)	(3.22)	(2.97)	(3.00)	(2.93)	(2.99)	(3.20)	(3.19)
$IVol_{FF3}^{1M}$	-20.822											
	(-4.02)											
ASV_{FF3}^{1M}		-20.963		-20.904	-27.519							
		(-3.95)		(-3.92)	(-8.95)							
${}^{\perp}LFV_{FF3}^{1M}$			-18.693	-19.735	-17.719	-14.030	-15.799	-15.868	-15.673	-15.480	-15.616	-15.104
			(-4.74)	(-4.97)	(-5.25)	(-4.34)	(-4.82)	(-4.86)	(-4.76)	(-4.76)	(-4.78)	(-4.68)
LagRet					-4.0599	-4.4078	-4.6521	-4.5006	-4.5320	-4.5514	-4.7283	-4.8346
					(-9.03)	(-9.92)	(-9.87)	(-9.58)	(-9.71)	(-9.58)	(-10.14)	(-9.96)
MOM					0.7157	0.8045	0.7558	0.7608	0.7484	0.7440	0.7284	0.7264
					(5.32)	(5.80)	(5.12)	(5.13)	(5.14)	(5.07)	(5.19)	(5.29)
Illiq					0.0324	0.0137	0.0055	0.0053	0.0049	0.0055	0.0050	0.0049
					(3.33)	(1.38)	(0.64)	(0.61)	(0.57)	(0.64)	(0.59)	(0.58)
ISkew					-0.0400	-0.0559	-0.0522	-0.0483	-0.0508	-0.0502	-0.0521	-0.0455
					(-2.37)	(-3.10)	(-2.91)	(-2.62)	(-2.81)	(-2.74)	(-3.03)	(-2.61)
CoSkew					-0.0085	-0.0074	-0.0050	-0.0076	-0.0070	-0.0084	-0.0056	-0.0046
					(-1.83)	(-1.57)	(-1.36)	(-1.57)	(-1.64)	(-1.72)	(-1.47)	(-1.44)
ZeroRet					-0.3280	0.1733	0.0216	0.0188	0.0091	0.0126	0.0352	0.0229
					(-1.26)	(0.72)	(0.09)	(0.07)	(0.04)	(0.05)	(0.14)	(0.10)
Size					-0.2054	-0.0980	-0.0776	-0.0816	-0.0772	-0.0817	-0.0798	-0.0761
					(-6.07)	(-2.35)	(-1.94)	(-1.99)	(-1.90)	(-1.98)	(-2.13)	(-2.11)
β_{Mkt}					-0.0318	-0.1821	-0.2150	-0.1973	-0.2011	-0.1828	-0.2332	-0.2415
					(-0.23)	(-1.19)	(-1.42)	(-1.23)	(-1.26)	(-1.14)	(-1.82)	(-1.93)
β_{SMB}					0.1473	0.1982	0.2038	0.1984	0.1951	0.1920	0.1925	0.1891
					(2.21)	(2.79)	(2.88)	(2.70)	(2.70)	(2.66)	(3.24)	(3.24)
β_{HML}					-0.1041	-0.1129	-0.1487	-0.1483	-0.1446	-0.1536	-0.1438	-0.1413
					(-2.10)	(-2.12)	(-2.97)	(-2.94)	(-2.88)	(-3.09)	(-3.08)	(-3.11)
β_{MV}							-0.0005					
							(-1.88)					
β_{AV}								-0.0069				-0.0048
								(-0.97)				(-0.72)
β_{AC}								0.0383				0.1120
								(0.48)				(1.25)
β_{CIV}									-0.0014			-0.0001
									(-0.51)			(-0.05)
β_{Liq}										0.0050		-0.0037
										(0.12)		(-0.09)
β_{FMax}											0.0329	0.0436
											(0.36)	(0.45)
$avg.\overline{R}^2$ in %	1.9047	1.8876	0.1276	2.0213	7.2735	6.8394	6.9172	6.9753	6.9162	6.9586	7.1464	7.6245
$avg.N$	3301	3301	3301	3301	3301	3301	3147	3147	3147	3147	3147	3147

Table 4.7 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ on $IVol_{FF3}^{1M}$ and its two orthogonalized components ASV_{FF3}^{1M} and ${}^{\perp}LFV_{FF3}^{1M}$ in month t as well as several control variables. We include the following set of control variables: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period in Model (1) - (6) is June 1964 to December 2016 and the sample period in Model (7) - (12) is June 1968 to December 2016. We report the average cross-sectional r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

i.e. $FF3$, CAR and $FF5$ with a longer estimation window instead of within-month regressions. We estimate the corresponding factor model in twelve-month rolling window regressions with daily data, calculate residuals and perform the decomposition methodology on the longer daily time series of residuals. At least 200 valid daily observations are required for the twelve-month window. The target variables are referred to as $IVol_{FM}^{12M}$, LFV_{FM}^{12M} and ASV_{FM}^{12M}, where FM denotes the corresponding factor model $FF3$, CAR or $FF5$.

Table 4.8 (Table 4.9) present results for the alternative factor models CAR ($FF5$). In general, the results are qualitatively identical to the baseline analysis in Table 4.3. LFV_{FM}^{1M} performs equally well in the CAR model and the $FF5$ model, independently of the exact specification. The t-statistics exceed values of minus four in any case. We conclude that our main findings are independent of the underlying factor model which generates residuals.

Table 4.10 presents results from an extended estimation window, i.e. twelve-month rolling window regressions to generate estimates for LFV_{FM}^{12M} and ASV_{FM}^{12M}. We furthermore present different factor models, the baseline model $FF3$, as well as the Carhart (1997) CAR and the Fama and French (2015) model $FF5$. We omit the univariate results since idiosyncratic volatility is not significant in univariate regressions for the longer estimation window. In Column (1), (4) and (7) when we include both components of $IVol_{FM}^{12M}$, the performance of LFV_{FM}^{12M} is lower compared to the one-month regressions. Still, the average LFV_{FM}^{12M} coefficient is highly significant, especially with regard to the CAR model. Neverthe-

Table 4.8: Fama and MacBeth (1973) regressions: Carhart (1997) model residuals.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	1.2341	1.2195	0.9733	1.2253	3.7489	2.5447	2.3331	2.4030	2.3357	2.4043	2.3525	2.2839
	(6.41)	(6.33)	(4.25)	(6.40)	(7.46)	(4.44)	(4.18)	(4.17)	(4.12)	(4.17)	(4.44)	(4.42)
$IVol_{CAR}^{1M}$	-20.980											
	(-3.95)											
ASV_{CAR}^{1M}		-21.196		-17.789	-24.361							
		(-3.87)		(-3.38)	(-8.01)							
LFV_{CAR}^{1M}			-42.661	-16.246	-16.059	-38.881	-39.509	-39.361	-39.349	-39.277	-39.176	-38.462
			(-4.49)	(-4.45)	(-4.74)	(-8.27)	(-8.58)	(-8.52)	(-8.48)	(-8.49)	(-8.83)	(-8.81)
LagRet					-4.0696	-4.2973	-4.5500	-4.4028	-4.4305	-4.4483	-4.6204	-4.7327
					(-9.04)	(-9.65)	(-9.61)	(-9.35)	(-9.46)	(-9.34)	(-9.85)	(-9.69)
MOM					0.7196	0.7816	0.7343	0.7384	0.7264	0.7217	0.7074	0.7042
					(5.34)	(5.67)	(5.01)	(5.01)	(5.02)	(4.96)	(5.08)	(5.16)
Illiq					0.0318	0.0191	0.0111	0.0107	0.0103	0.0110	0.0105	0.0104
					(3.28)	(1.95)	(1.30)	(1.26)	(1.23)	(1.29)	(1.25)	(1.23)
ISkew					-0.0414	-0.0505	-0.0462	-0.0425	-0.0449	-0.0443	-0.0464	-0.0399
					(-2.46)	(-2.87)	(-2.63)	(-2.36)	(-2.55)	(-2.49)	(-2.75)	(-2.34)
CoSkew					-0.0082	-0.0076	-0.0051	-0.0078	-0.0072	-0.0086	-0.0057	-0.0048
					(-1.82)	(-1.68)	(-1.46)	(-1.67)	(-1.75)	(-1.83)	(-1.56)	(-1.57)
ZeroRet					-0.3212	0.0003	-0.1426	-0.1435	-0.1555	-0.1509	-0.1283	-0.1402
					(-1.24)	(0.00)	(-0.56)	(-0.55)	(-0.60)	(-0.58)	(-0.51)	(-0.58)
Size					-0.2038	-0.1324	-0.1114	-0.1152	-0.1109	-0.1155	-0.1128	-0.1080
					(-6.03)	(-3.37)	(-2.96)	(-2.97)	(-2.90)	(-2.97)	(-3.18)	(-3.16)
β_{Mkt}					-0.0422	-0.1406	-0.1762	-0.1573	-0.1613	-0.1426	-0.1993	-0.2088
					(-0.30)	(-0.94)	(-1.19)	(-1.00)	(-1.04)	(-0.91)	(-1.59)	(-1.70)
β_{HML}					0.1513	0.1871	0.1933	0.1876	0.1837	0.1809	0.1835	0.1803
					(2.27)	(2.68)	(2.77)	(2.60)	(2.59)	(2.55)	(3.13)	(3.14)
β_{SMB}					-0.1042	-0.1141	-0.1492	-0.1489	-0.1453	-0.1540	-0.1459	-0.1432
					(-2.10)	(-2.19)	(-3.04)	(-3.00)	(-2.95)	(-3.16)	(-3.18)	(-3.20)
β_{MV}							-0.0005					
							(-1.92)					
β_{AV}								-0.0070				-0.0054
								(-1.00)				(-0.81)
β_{AC}								0.0400				0.1095
								(0.51)				(1.24)
β_{CIV}									-0.0016			-0.0004
									(-0.58)			(-0.14)
β_{Liq}										0.0038		-0.0055
										(0.09)		(-0.14)
β_{FMax}											0.0449	0.0563
											(0.50)	(0.59)
$avg.\overline{R}^2$ in %	1.8494	1.8349	0.7378	1.9128	6.8747	6.5831	6.6161	6.6406	6.6154	6.6584	6.8385	7.1832
$avg.N$	3301	3301	3301	3301	3301	3301	3147	3147	3147	3147	3147	3147

Table 4.8 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ on $IVol_{CAR}^{1M}$ and its two components ASV_{CAR}^{1M} and LFV_{CAR}^{1M} in month t as well as several control variables. In contrast to the baseline results, residuals are calculated from a Carhart (1997) four factor model (CAR). We include the following set of control variables: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period in Model (1) - (6) is June 1964 to December 2016 and the sample period in Model (7) - (12) is June 1968 to December 2016. We report the average cross-sectional r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

Table 4.9: Fama and MacBeth (1973) regressions: Fama and French (2015) model residuals.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	1.2346	1.2173	0.9865	1.2243	3.7477	2.5695	2.3565	2.4255	2.3579	2.4257	2.3713	2.3039
	(6.39)	(6.31)	(4.29)	(6.38)	(7.44)	(4.47)	(4.22)	(4.21)	(4.16)	(4.20)	(4.48)	(4.47)
$IVol_{FF5}^{1M}$	-21.694											
	(-4.00)											
ASV_{FF5}^{1M}		-21.816		-17.382	-24.350							
		(-3.90)		(-3.23)	(-7.81)							
LFV_{FF5}^{1M}			-46.366	-21.039	-19.806	-41.834	-41.791	-41.667	-41.673	-41.430	-41.508	-40.719
			(-4.94)	(-5.94)	(-6.59)	(-9.48)	(-9.36)	(-9.26)	(-9.25)	(-9.23)	(-9.59)	(-9.50)
LagRet					-4.0735	-4.2845	-4.5365	-4.3881	-4.4159	-4.4366	-4.6066	-4.7196
					(-9.04)	(-9.59)	(-9.56)	(-9.29)	(-9.40)	(-9.29)	(-9.80)	(-9.63)
MOM					0.7179	0.7777	0.7306	0.7347	0.7223	0.7183	0.7041	0.7019
					(5.34)	(5.66)	(5.00)	(5.00)	(5.00)	(4.94)	(5.06)	(5.16)
Illiq					0.0316	0.0190	0.0110	0.0107	0.0103	0.0110	0.0105	0.0103
					(3.26)	(1.95)	(1.31)	(1.26)	(1.23)	(1.29)	(1.26)	(1.23)
ISkew					-0.0407	-0.0494	-0.0451	-0.0414	-0.0438	-0.0432	-0.0453	-0.0389
					(-2.40)	(-2.78)	(-2.55)	(-2.28)	(-2.47)	(-2.41)	(-2.67)	(-2.26)
CoSkew					-0.0084	-0.0077	-0.0052	-0.0079	-0.0072	-0.0086	-0.0058	-0.0048
					(-1.83)	(-1.69)	(-1.46)	(-1.67)	(-1.75)	(-1.82)	(-1.57)	(-1.56)
ZeroRet					-0.3195	-0.0029	-0.1421	-0.1434	-0.1559	-0.1508	-0.1283	-0.1404
					(-1.23)	(-0.01)	(-0.55)	(-0.54)	(-0.60)	(-0.58)	(-0.51)	(-0.58)
Size					-0.2037	-0.1339	-0.1129	-0.1167	-0.1123	-0.1169	-0.1140	-0.1093
					(-6.01)	(-3.40)	(-2.99)	(-3.01)	(-2.93)	(-3.00)	(-3.22)	(-3.20)
β_{Mkt}					-0.0425	-0.1369	-0.1707	-0.1522	-0.1563	-0.1378	-0.1955	-0.2051
					(-0.30)	(-0.92)	(-1.15)	(-0.97)	(-1.01)	(-0.88)	(-1.56)	(-1.68)
β_{HML}					0.1492	0.1845	0.1910	0.1853	0.1817	0.1789	0.1818	0.1788
					(2.23)	(2.65)	(2.75)	(2.57)	(2.56)	(2.52)	(3.12)	(3.12)
β_{SMB}					-0.1051	-0.1133	-0.1497	-0.1494	-0.1458	-0.1547	-0.1462	-0.1438
					(-2.12)	(-2.17)	(-3.05)	(-3.01)	(-2.96)	(-3.18)	(-3.19)	(-3.22)
β_{MV}							-0.0005					
							(-1.91)					
β_{AV}								-0.0073				-0.0056
								(-1.04)				(-0.84)
β_{AC}								0.0459				0.1140
								(0.58)				(1.29)
β_{CIV}									-0.0017			-0.0005
									(-0.60)			(-0.17)
β_{Liq}										0.0050		-0.0039
										(0.12)		(-0.09)
β_{FMax}											0.0464	0.0581
											(0.52)	(0.61)
$avg.\overline{R}^2$ in %	1.8337	1.8183	0.7349	1.9034	6.8732	6.5812	6.6150	6.6399	6.6147	6.6571	6.8387	7.1842
$avg.N$	3301	3301	3301	3301	3301	3301	3147	3147	3147	3147	3147	3147

Table 4.9 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ on $IVol_{FF5}^{1M}$ and its two components ASV_{FF5}^{1M} and LFV_{FF5}^{1M} in month t as well as several control variables. In contrast to the baseline results, residuals are calculated from a Fama and French (2015) five factor model ($FF5$). We include the following set of control variables: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period in Model (1) - (6) is June 1964 to December 2016 and the sample period in Model (7) - (12) is June 1968 to December 2016. We report the average cross-sectional r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

Table 4.10: Fama and MacBeth (1973) regressions: Twelve-month windows.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	3.7262	2.3107	2.1252	3.7438	2.3850	2.1927	3.7298	2.3094	2.1203
	(8.06)	(4.13)	(4.02)	(8.12)	(4.27)	(4.17)	(8.06)	(4.11)	(4.00)
LFV_{FF3}^{12M}	-24.733	-71.322	-74.342						
	(-2.14)	(-4.45)	(-4.97)						
ASV_{FF3}^{12M}	-27.379								
	(-5.00)								
LFV_{CAR}^{12M}				-34.661	-81.000	-84.863			
				(-3.03)	(-5.13)	(-5.68)			
ASV_{CAR}^{12M}				-26.549					
				(-4.78)					
LFV_{FF5}^{12M}							-25.158	-72.226	-74.503
							(-2.24)	(-4.60)	(-5.00)
ASV_{FF5}^{12M}							-27.481		
							(-4.94)		
LagRet	-4.4491	-4.4546	-4.8658	-4.4464	-4.4451	-4.8612	-4.4433	-4.4479	-4.8668
	(-8.61)	(-8.64)	(-8.41)	(-8.60)	(-8.62)	(-8.39)	(-8.60)	(-8.61)	(-8.41)
MOM	0.8342	0.8417	0.7541	0.8369	0.8451	0.7555	0.8346	0.8435	0.7556
	(5.40)	(5.39)	(4.77)	(5.41)	(5.39)	(4.77)	(5.40)	(5.39)	(4.78)
Illiq	0.0260	0.0137	0.0026	0.0260	0.0141	0.0032	0.0262	0.0138	0.0026
	(2.36)	(1.20)	(0.32)	(2.34)	(1.23)	(0.38)	(2.37)	(1.20)	(0.31)
ISkew	-0.0092	-0.0467	-0.0346	-0.0093	-0.0451	-0.0329	-0.0095	-0.0470	-0.0350
	(-0.55)	(-2.87)	(-2.12)	(-0.55)	(-2.74)	(-2.00)	(-0.56)	(-2.86)	(-2.14)
CoSkew	-0.0083	-0.0073	-0.0045	-0.0089	-0.0080	-0.0050	-0.0084	-0.0073	-0.0045
	(-1.81)	(-1.61)	(-1.44)	(-1.85)	(-1.69)	(-1.57)	(-1.82)	(-1.61)	(-1.44)
ZeroRet	0.1816	0.1472	-0.0161	0.1829	0.1496	-0.0058	0.1839	0.1492	-0.0171
	(0.78)	(0.61)	(-0.07)	(0.78)	(0.61)	(-0.02)	(0.79)	(0.61)	(-0.07)
Size	-0.2051	-0.1202	-0.1011	-0.2066	-0.1251	-0.1052	-0.2055	-0.1201	-0.1007
	(-6.75)	(-3.12)	(-2.89)	(-6.82)	(-3.23)	(-3.02)	(-6.76)	(-3.09)	(-2.87)
β_{Mkt}	0.0220	-0.1311	-0.1944	0.0234	-0.1220	-0.1895	0.0213	-0.1296	-0.1959
	(0.20)	(-0.97)	(-1.76)	(0.21)	(-0.88)	(-1.71)	(0.19)	(-0.94)	(-1.76)
β_{HML}	0.1149	0.1809	0.1708	0.1110	0.1740	0.1628	0.1169	0.1841	0.1740
	(1.73)	(2.52)	(2.86)	(1.65)	(2.40)	(2.74)	(1.75)	(2.54)	(2.91)
β_{SMB}	-0.0913	-0.1065	-0.1404	-0.0952	-0.1106	-0.1422	-0.0932	-0.1090	-0.1408
	(-1.91)	(-1.98)	(-2.87)	(-2.02)	(-2.09)	(-2.98)	(-1.94)	(-2.02)	(-2.88)
β_{AV}			-0.0035			-0.0036			-0.0034
			(-0.54)			(-0.56)			(-0.53)
β_{CIV}			-0.0002			-0.0002			-0.0002
			(-0.08)			(-0.07)			(-0.08)
β_{AC}			0.0489			0.0516			0.0467
			(0.51)			(0.52)			(0.48)
β_{Liq}			-0.0086			-0.0076			-0.0087
			(-0.20)			(-0.18)			(-0.21)
β_{FMax}			0.0425			0.0461			0.0401
			(0.43)			(0.46)			(0.40)
$avg.\overline{R}^2$ in %	7.1644	6.7140	7.3034	7.1341	6.6813	7.2759	7.1536	6.6966	7.2889
$avg.N$	3268	3268	3112	3268	3268	3112	3268	3268	3112

Table 4.10 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ of the two components of twelve-month $FF3$ idiosyncratic volatility ASV_{FM}^{12M} and LFV_{FM}^{12M} in month t as well as several control variables. In contrast to the baseline results, residuals are calculated from twelve-month rolling window regressions with different factor models. CAR indicates residuals from a Carhart (1997) four factor model and $FF5$ indicates the Fama and French (2015) five factor model. We include the following set of control variables: LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period in Model (1) - (6) is June 1964 to December 2016 and the sample period in Model (7) - (9) is June 1968 to December 2016. We report the average cross-sectional r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with twelve lags in parentheses.

less, LFV_{FM}^{12M} is highly significant when we control for the baseline set of control variables. The t-statistics for LFV_{FM}^{12M} vary around minus five, indicating statistical significance at any conventional level. Our results are robust and neither an extension of risk factors nor a longer estimation window changes our main conclusions.

4.5.3 Mispricing

Stambaugh et al. (2015) argue that relative mispricing explains the idiosyncratic volatility puzzle. The sign of the relationship between idiosyncratic volatility and subsequent returns depends on mispricing. While underpriced stocks exhibit a positive relationship between idiosyncratic volatility and subsequent returns, the negative relation shown in Section 4.4 only holds among overpriced stocks (Stambaugh et al., 2015, p. 1916). Arbitrage asymmetries imply that the effect of overpriced stocks is stronger and thus the negative sign extends to the whole cross section of stocks. If mispricing also explains the negative sign on the systematic component LFV_{FF3}^{1M}, we would expect that this sign also depends on mispricing.

Following the approach of Stambaugh et al. (2015), Table 4.11 presents bivariate double sorts on mispricing (Misp) and the two components of $FF3$ idiosyncratic volatility.[8] Each month, we sort stocks into quintiles based on mispricing (Misp). In each Misp quintile, we then further sort

[8]For details regarding the estimation of Misp, we refer to Appendix 3.A.2 or Stambaugh et al. (2015). Due to the fact that Stambaugh et al. (2015) provide Misp with a price filter of five USD, the analysis in this Section imposes a higher price filter compared to the baseline analysis.

Table 4.11: Conditional double sorts on mispricing and the two components of $IVol$.

Panel A: Mispricing and the idiosyncratic component ASV

	Quintiles of ASV_{FF3}^{1M}: Excess Returns						High minus low ASV_{FF3}^{1M} factor alphas				
Quintiles of $Misp$	Low ASV_{FF3}^{1M}	2	3	4	High ASV_{FF3}^{1M}	Diff	CAPM Alpha	FF3 Alpha	CAR Alpha	PS Alpha	FF5 Alpha
Low Misp	0.4567	0.7302	1.0013	0.9660	1.0154	0.5587	0.3412	0.3298	0.3127	0.3589	0.5962
	(2.73)	(4.05)	(5.07)	(4.13)	(3.61)	(2.58)	(1.72)	(1.93)	(1.80)	(1.99)	(3.61)
2	0.5136	0.6260	0.6456	0.6410	0.5781	0.0645	-0.1881	-0.1730	-0.2170	-0.1973	0.1056
	(2.97)	(3.15)	(2.83)	(2.36)	(1.91)	(0.29)	(-0.91)	(-0.97)	(-1.09)	(-1.09)	(0.58)
3	0.4927	0.4393	0.5834	0.5609	0.5186	0.0259	-0.2932	-0.2050	-0.2510	-0.2208	0.0927
	(2.70)	(2.05)	(2.50)	(2.04)	(1.60)	(0.11)	(-1.36)	(-1.08)	(-1.40)	(-1.13)	(0.51)
4	0.3799	0.3369	0.5064	0.2754	-0.0460	-0.4259	-0.7674	-0.6767	-0.5521	-0.7229	-0.3361
	(2.05)	(1.49)	(1.79)	(0.88)	(-0.13)	(-1.56)	(-3.12)	(-3.39)	(-2.80)	(-3.46)	(-1.76)
High Misp	0.2140	0.0099	-0.0071	-0.4943	-1.0663	-1.2803	-1.6355	-1.5445	-1.3893	-1.5944	-1.1476
	(0.93)	(0.03)	(-0.02)	(-1.29)	(-2.53)	(-4.14)	(-5.95)	(-7.10)	(-6.23)	(-6.78)	(-5.42)

Panel B: Mispricing and the systematic component LFV

	Quintiles of LFV_{FF3}^{1M}: Excess Returns						High minus low LFV_{FF3}^{1M} factor alphas				
Quintiles of $Misp$	Low LFV_{FF3}^{1M}	2	3	4	High LFV_{FF3}^{1M}	Diff	CAPM Alpha	FF3 Alpha	CAR Alpha	PS Alpha	FF5 Alpha
Low Misp	0.6761	0.5804	0.7568	0.6918	0.9365	0.2604	0.1409	0.1328	0.0757	0.0902	0.2301
	(3.78)	(3.14)	(4.48)	(3.67)	(4.08)	(1.66)	(0.93)	(0.93)	(0.51)	(0.60)	(1.58)
2	0.5994	0.5502	0.5820	0.5485	0.6107	0.0114	-0.1072	-0.0934	-0.1637	-0.1177	0.0151
	(3.21)	(2.79)	(2.92)	(2.49)	(2.35)	(0.07)	(-0.69)	(-0.65)	(-1.10)	(-0.79)	(0.10)
3	0.4320	0.5470	0.4889	0.5820	0.4699	0.0379	-0.1204	-0.0665	-0.0651	-0.1141	0.1176
	(1.99)	(2.77)	(2.26)	(2.61)	(1.73)	(0.25)	(-0.79)	(-0.44)	(-0.44)	(-0.72)	(0.76)
4	0.5200	0.3926	0.4279	0.3020	0.1512	-0.3688	-0.5447	-0.4788	-0.3809	-0.4999	-0.3507
	(2.46)	(1.85)	(1.93)	(1.19)	(0.49)	(-1.98)	(-3.20)	(-3.14)	(-2.55)	(-3.05)	(-2.36)
High Misp	0.0867	0.0178	0.0260	-0.0877	-0.5981	-0.6848	-0.8952	-0.7992	-0.7508	-0.8029	-0.5546
	(0.33)	(0.07)	(0.09)	(-0.28)	(-1.54)	(-2.97)	(-4.19)	(-3.75)	(-3.44)	(-3.62)	(-2.83)

Table 4.11 presents dependent bivariate portfolio sorts (value-weighted) conditional on mispricing. In Panel A (B), we first sort all stocks into quintiles based on the Stambaugh et al. (2015) mispricing measure and then in each quintile we further sort stocks into quintiles based on the asset specific (systematic) component of monthly $FF3$ idiosyncratic volatility, i.e. ASV_{FF3}^{1M} (LFV_{FF3}^{1M}). In Panel A (B), we further present excess returns of each double sorted portfolios as well as the difference between the portfolio in the highest and the lowest quintile of ASV_{FF3}^{1M} (LFV_{FF3}^{1M}) conditional on $Misp$. All portfolio returns are weighted by market capitalization. We hold each portfolio for one month and record the monthly returns and different factor alphas. Factor alphas cover the following factor models: CAPM is a simple one factor model, $FF3$ is the Fama and French (1993) three factor model, PS is the $FF3$ model extended by the Pastor and Stambaugh (2003) liquidity factor, CAR is the Carhart (1997) four factor model and $FF5$ is the Fama and French (2015) five factor model. Returns and alphas are reported in % per month. Newey and West (1987) adjusted standard t-statistics with six lags in parentheses. The sample period for excess returns and alphas of the models $CAPM$, $FF3$, CAR, $FF5$ and is July 1965 to December 2015. The PS model covers January 1968 to December 2015.

stocks in quintiles based on ASV_{FF3}^{1M} in Panel A or LFV_{FF3}^{1M} in Panel B, hold the respective portfolios for one month and record monthly returns. We calculate the difference in excess returns between stocks in the highest and the lowest quintile of LFV_{FF3}^{1M} within each ASV_{FF3}^{1M} quintile. We also present alphas of the following factor models: The CAPM single factor model, the Fama and French (1993) three factor model ($FF3$), the Carhart (1997) four factor model (CAR), the Pastor and Stambaugh (2003) model (PS) which adds a liquidity factor to the $FF3$ model, as well as the Fama and French (2015) five factor ($FF5$) model. Monthly excess returns and alphas are reported in % per month and are weighted by market capitalization. We present Newey and West (1987) adjusted standard t-statistics with six lags in parentheses.

Panel A supports the results of Stambaugh et al. (2015). The relationship between the asset-specific component of idiosyncratic volatility ASV_{FF3}^{1M} and expected returns is positive and statistically significant among underpriced stocks in the first quintile of Misp. High ASV_{FF3}^{1M} outperform low ASV_{FF3}^{1M} stocks by 0.5587% per month with a t-statistic of 2.58. This outperformance also withstands the various risk adjustments. Alphas are significantly positive and vary from 0.3127% for the Carhart (1997) four factor model to almost 0.60% for the Fama and French (2015) five factor model. The latter is highly statistically significant at any conventional level. While the spread between high and low ASV_{FF3}^{1M} is almost flat for stocks in the second and third quintile of Misp, overpriced stocks in the two highest Misp quintiles show a strong negative pattern when sorting on ASV_{FF3}^{1M}. This supports both of the findings in Stambaugh et al. (2015). The asset-specific part of idiosyncratic volatility is related to relative mispricing and more pronounced among overpriced stocks, which accounts for the overall negative relation between idiosyncratic volatility and subsequent returns.

This finding does not fully translate to Panel B, i.e. the sorts on LFV_{FF3}^{1M} in each Misp quintile. Here, the positive spread between high and low LFV_{FF3}^{1M} among underpriced stocks is not significant after risk-adjustment. Although the negative spread between high and low LFV_{FF3}^{1M} stocks is highly significant among overpriced stocks, the sign in the risk-return trade-off does not change.

Consequently, mispricing is unlikely to explain the full extent of our results. Nevertheless, the results in Panel A unveil that mispricing is an important aspect in explaining the negative sign on the asset-specific component of idiosyncratic volatility.

4.5.4 Lottery demand

In the baseline analysis of Section 4.4, we follow Hou and Loh (2016) and exclude the Bali et al. (2017) lottery demand measure Max due to the high correlation with idiosyncratic volatility $IVOL_{FF3}^{1M}$. Given a lower cross-sectional correlation between Max and LFV_{FF3}^{1M}, Table 4.12 presents a reevaluation of the negative risk premium of the latent factor volatility from different factor models and time horizons. In addition to the baseline measure LFV_{FF3}^{1M} in Column (1) and (2), we report the estimate for the alternative factor models (Columns (3) – (6)) and estimation windows

(Columns (7) – (12)) presented in Section 4.5.2.

The negative coefficient on Max, as presented by Bali et al. (2011, 2017) is highly significant at any conventional level. Although this is also true for LFV_{FM} independently of the estimation window or factor model, t-statistics and coefficient estimates are lower compared to the baseline analysis in Table 4.3. Nevertheless, the consideration of lottery demand does not explain or alleviate the baseline results.

4.6 Conclusion

We derive a regression-based procedure to decompose residual standard deviations into two components: A stock's exposure to latent but systematic risk and a purely idiosyncratic component. Our theoretical framework is consistent with Claußen et al. (2019) who relate the systematic risk in supposedly idiosyncratic volatility of the Fama and French (1993) three factor model to sentiment-induced noise trader risk. Theoretically and empirically, the volatility attributable to noise trader risk is a promising candidate to explain the idiosyncratic volatility puzzle in the cross section of stock returns. We find a strong and robust negative relationship between the latent factor volatility and subsequent returns. The systematic component accounts for up to 55% of the idiosyncratic volatility puzzle. This fraction is largely unrelated to alternative candidate variables and factor models, although alternative candidates convey explanatory power as well. Alternative candidates perform worse when combined with each other, while our approach pins down a systematic component

Table 4.12: Fama and MacBeth (1973) regressions controlling for Max.

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	3.7992	3.4585	3.8042	3.4593	3.8098	3.4665	3.6431	3.5335	3.7014	3.5817	3.6449	3.5320
	(7.86)	(7.65)	(7.86)	(7.65)	(7.90)	(7.69)	(7.56)	(7.60)	(7.71)	(7.76)	(7.56)	(7.61)
LFV_{FF3}^{1M}	-13.123	-14.535										
	(-3.88)	(-4.67)										
LFV_{CAR}^{1M}			-10.703	-10.638								
			(-3.24)	(-3.33)								
LFV_{FF5}^{1M}					-14.851	-13.648						
					(-4.90)	(-4.38)						
LFV_{FF3}^{12M}							-38.285	-37.189				
							(-2.72)	(-2.86)				
LFV_{CAR}^{12M}									-48.982	-47.518		
									(-3.55)	(-3.65)		
LFV_{FF5}^{12M}											-38.898	-37.360
											(-2.84)	(-2.89)
Max	-18.622	-18.095	-18.994	-18.666	-18.455	-18.330	-18.737	-18.633	-18.552	-18.472	-18.768	-18.666
	(-7.51)	(-7.79)	(-7.75)	(-8.21)	(-7.43)	(-7.97)	(-7.96)	(-8.47)	(-7.79)	(-8.35)	(-7.87)	(-8.42)
LagRet	-3.0982	-3.5063	-3.0832	-3.4898	-3.1146	-3.4958	-3.2272	-3.5444	-3.2348	-3.5528	-3.2226	-3.5417
	(-6.51)	(-6.68)	(-6.51)	(-6.68)	(-6.55)	(-6.66)	(-5.57)	(-5.80)	(-5.58)	(-5.81)	(-5.57)	(-5.80)
MOM	0.7103	0.6405	0.7122	0.6414	0.7097	0.6397	0.6885	0.6690	0.6902	0.6708	0.6899	0.6698
	(5.21)	(4.79)	(5.22)	(4.80)	(5.21)	(4.80)	(4.15)	(4.35)	(4.14)	(4.36)	(4.15)	(4.35)
Illiq	0.0357	0.0270	0.0356	0.0269	0.0354	0.0268	0.0218	0.0213	0.0223	0.0217	0.0219	0.0214
	(3.69)	(3.22)	(3.70)	(3.24)	(3.69)	(3.22)	(2.46)	(2.49)	(2.48)	(2.52)	(2.45)	(2.48)
ISkew	-0.0363	-0.0271	-0.0368	-0.0273	-0.0358	-0.0264	-0.0285	-0.0221	-0.0268	-0.0207	-0.0286	-0.0222
	(-2.16)	(-1.63)	(-2.19)	(-1.65)	(-2.13)	(-1.59)	(-1.76)	(-1.37)	(-1.64)	(-1.28)	(-1.76)	(-1.38)
CoSkew	-0.0081	-0.0051	-0.0079	-0.0049	-0.0081	-0.0050	-0.0077	-0.0046	-0.0085	-0.0051	-0.0079	-0.0047
	(-1.76)	(-1.68)	(-1.75)	(-1.65)	(-1.76)	(-1.67)	(-1.63)	(-1.53)	(-1.70)	(-1.64)	(-1.65)	(-1.55)
ZeroRet	-0.3359	-0.4634	-0.3409	-0.4679	-0.3393	-0.4617	-0.4450	-0.4710	-0.4245	-0.4541	-0.4423	-0.4705
	(-1.29)	(-1.79)	(-1.31)	(-1.81)	(-1.30)	(-1.78)	(-1.61)	(-1.81)	(-1.51)	(-1.73)	(-1.59)	(-1.80)
Size	-0.2062	-0.1768	-0.2066	-0.1769	-0.2070	-0.1774	-0.1891	-0.1826	-0.1930	-0.1856	-0.1893	-0.1824
	(-6.34)	(-6.06)	(-6.33)	(-6.05)	(-6.36)	(-6.08)	(-5.97)	(-6.10)	(-6.09)	(-6.24)	(-5.96)	(-6.11)
β_{Mkt}	0.0635	-0.0153	0.0630	-0.0161	0.0613	-0.0181	0.0644	-0.0001	0.0751	0.0044	0.0669	-0.0001
	(0.49)	(-0.14)	(0.48)	(-0.15)	(0.47)	(-0.17)	(0.51)	(-0.00)	(0.58)	(0.05)	(0.52)	(-0.00)
β_{SMB}	0.1322	0.1346	0.1347	0.1369	0.1328	0.1352	0.1278	0.1227	0.1203	0.1158	0.1298	0.1250
	(2.08)	(2.54)	(2.11)	(2.58)	(2.08)	(2.54)	(1.91)	(2.27)	(1.80)	(2.16)	(1.93)	(2.31)
β_{HML}	-0.0941	-0.1341	-0.0951	-0.1343	-0.0949	-0.1346	-0.1354	-0.1349	-0.1399	-0.1375	-0.1372	-0.1353
	(-1.93)	(-3.14)	(-1.95)	(-3.13)	(-1.95)	(-3.15)	(-2.73)	(-2.89)	(-2.87)	(-3.02)	(-2.76)	(-2.90)
β_{AV}		-0.0068		-0.0068		-0.0069		-0.0057		-0.0058		-0.0057
		(-1.03)		(-1.04)		(-1.05)		(-0.91)		(-0.92)		(-0.90)
β_{AC}		0.1124		0.1093		0.1141		-0.0013		-0.0013		-0.0013
		(1.35)		(1.31)		(1.37)		(-0.47)		(-0.47)		(-0.47)
β_{CIV}		-0.0015		-0.0014		-0.0015		0.0609		0.0612		0.0592
		(-0.54)		(-0.52)		(-0.54)		(0.65)		(0.64)		(0.62)
β_{Liq}		-0.0024		-0.0038		-0.0023		-0.0081		-0.0065		-0.0083
		(-0.06)		(-0.10)		(-0.06)		(-0.20)		(-0.16)		(-0.20)
β_{FMax}		0.0869		0.0864		0.0874		0.0752		0.0791		0.0738
		(0.96)		(0.96)		(0.97)		(0.79)		(0.82)		(0.77)
$avg.\overline{R}^2$ in %	7.3200	8.0424	7.3130	8.0353	7.3167	8.0426	7.3896	8.1673	7.3570	8.1429	7.3734	8.1575
$avg.N$	3301	3147	3301	3147	3301	3147	3112	3112	3112	3112	3112	3112

Table 4.12 presents average coefficients of Fama and MacBeth (1973) cross-sectional regressions of excess returns in month $t+1$ on the systematic component LFV_{FM} of $FF3$ idiosyncratic volatility. The superscript indicates the time-horizon of the estimation and the subscript refers to the factor model which generates residuals. $FF3$ refers to the (Fama and French, 1993) three factor model, CAR indicates residuals from a Carhart (1997) four factor model and $FF5$ indicates the Fama and French (2015) five factor model. We include the following set of control variables: Max is the average of the five highest returns in month t (Bali et al., 2017). LagRet is the return in month t, Mom is the cumulative return over the previous year, Illiq is the Amihud (2002) illiquidity measure, CoSkew is co-skewness as proposed by Harvey and Siddique (2000) and ISkew is idiosyncratic skewness of Fama and French (1993) three factor model residuals. ZeroRet is the share of zero returns and Size is the logarithm of the monthly market capitalization in 1,000 USD. Betas are calculated to the following risk factors: Mkt, SMB and HML are the factors of the Fama and French (1993) three factor model. AV (AC) is monthly average variance (correlation) of Chen and Petkova (2012), CIV (MV) is common idiosyncratic volatility (market variance) as proposed by Herskovic et al. (2016). Liq is the Pastor and Stambaugh (2003) liquidity factor and FMax is the Bali et al. (2017) lottery demand factor. The sample period in Model (1), (3) and (6) is June 1964 to December 2016 and the sample period in the remaining Models is June 1968 to December 2016. We report the average cross-sectional r-squared $avg.\overline{R}^2$ in % as well as the average number of observations N. Average coefficients are multiplied by one hundred. We winsorize all explanatory variables at the 0.5% level (0.25% in each tail) on a monthly basis. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

in idiosyncratic risk which is unrelated to existing models and most likely attributable to noise trader risk, as highlighted in the previous work of Claußen et al. (2019).

Chapter 5

Betting against sentiment: Seemingly unrelated anomalies and the low-risk effect

This Chapter refers to the working paper:

Dierkes, Maik and Sebastian Schroen (2019): ‘Betting against sentiment: Seemingly unrelated anomalies and the low-risk effect’, Working Paper, Leibniz Universität Hannover.

Abstract

The negative relation between CAPM alphas and the two most widely adopted risk measures – beta and volatility – is attributable to unaccounted factors in the CAPM. We use seemingly unrelated anomaly portfolios to construct a composite factor in the spirit of the optimal orthogonal portfolio. Controlling for the exposure to this factor re-establishes a significantly positive relation between beta and average returns and explains the negative alphas of high-beta and high-volatility stocks. A thorough evaluation of existing explanations for the low-risk effect suggests a unified behavioral explanation: Risky stocks earn lower returns because betting against investor sentiment is risky and costly.

Keywords: Low-risk effect, CAPM, optimal orthogonal portfolio

JEL: G10, G12, G40.

5.1 Introduction

Empirical asset pricing provides rich and robust evidence that the relationship between average returns and the two most widely adopted risk measures in finance – market beta and volatility – points in the wrong direction (Baker et al., 2011). This so-called low-risk effect[1] presents a standing challenge to the capital asset pricing model (CAPM) which predicts a positive trade-off between market beta and returns, whereas diversifiable risk such as volatility should yield no significant risk premium at all.[2] While early evidence goes back more than 40 years to Black (1972), the seminal papers of Ang et al. (2006) and Frazzini and Pedersen (2014) refueled the debate about the underlying mechanisms of the low-risk effect. Asness et al. (2019) boil down this debate to the two most promising explanations: Leverage constraints and thus systematic risk versus idiosyncratic risk due to behavioral biases for lottery-like returns.

Our paper seeks to resolve this debate and shows that the low-risk effect is both, behaviorally driven *and* attributable to a common systematic factor. To illustrate this, we break up with the old habit of separating between the two risk measures beta and variance by tracing back both phenomena to unaccounted factors in the CAPM. Our empirical proxy for the optimal orthogonal portfolio of MacKinlay and Pastor (2000) – referred to as *FOP* – explains the negative CAPM alphas of high-beta

[1] We follow Asness et al. (2019) and use the term "low-risk effect" to summarize the negative alphas of high-beta and high-volatility stocks. The literature mostly considers both phenomena separately.

[2] If investors are unable to diversify properly, Merton (1987) predicts a positive risk premium for volatility-risk. The negative relationship, however, remains a puzzle.

and high-variance stocks, both in time series regressions as well as cross-sectional. Furthermore, controlling for the exposure to FOP reestablishes a significantly positive relation between beta and average returns. After having shown that FOP explains the flat and sometimes even negative slope of the empirical security market line (SML), we use FOP to challenge theoretical propositions for the low-risk anomaly. Baker and Wurgler (2006) Investor Sentiment is the most promising state variable behind FOP and our results suggest that high-risk stocks earn low returns because "betting against sentimental investors is costly and risky" (Baker and Wurgler, 2007).

We use the optimal orthogonal portfolio of MacKinlay (1995) to construct the composite factor FOP from seemingly unrelated anomalies. FOP captures unaccounted factors in the CAPM, explains the anomalies in Fama and French (1993, 2015, 2016) with the exception of momentum and spans the risk factor models of Fama and French (2018) as well as Stambaugh and Yuan (2017). By construction, FOP is uncorrelated to the market portfolio which allows us to extend the CAPM by FOP while leaving market beta (β_{Mkt}) estimates unchanged. The extended CAPM predicts that high-beta and high-volatility stocks exhibit negative exposures to FOP which alleviates their negative CAPM alphas.

Once we extend the CAPM by FOP, the negative CAPM alphas of high-beta and high-variance (Var) stocks in univariate portfolio sorts become insignificant. Returns of β_{Mkt} decile portfolios increase in β_{Mkt} after controlling for the exposure to FOP. This result extends to 25 $Size$-β_{Mkt} and 25 $Size$-Var portfolios, albeit to a lesser extent. Accounting for the exposure to *FOP* reestablishes a positive trade-off between β_{Mkt} and average returns in Fama and MacBeth (1973) regressions and explains the cross-sectional pricing errors of the CAPM.

Turning to the economic explanations for the low-risk effect, we reevaluate the theoretical propositions of Frazzini and Pedersen (2014), Antoniou et al. (2016), and Hong and Sraer (2016), i.e. leverage constraints, investor sentiment, and disagreement. All three explanations share the common prediction that the slope of the SML depends on the respective economic state variables and is flatter in times of high leverage constraints, sentiment or disagreement. Since *FOP* and the exposure of β_{Mkt} deciles to *FOP* fully capture this time-variation in the slope of the SML, *FOP* facilitates an impartial horse race to discriminate between the three competing explanations. Any potential candidate for the low-risk effect should not only affect the slope of the SML, but also explain the time series dynamics of *FOP*. Baker and Wurgler (2006) Investor Sentiment is the only state variable which consistently satisfies both criteria and turns out to be the most promising candidate to explain the low-risk anomaly.

Our study contributes to three strands in the literature. First and most importantly, we shed further light on the mechanisms behind the low-risk effect. We pick up where Asness et al. (2019) left off the debate and focus on the controversy between risk-based and behavioral explanations. Since *FOP* is a priori unrelated to the low-risk anomaly, our perspective on the explanation starts purely agnostic. In line with Asness et al. (2019), our results suggest that the low-risk effect is indeed systematic. The exposure to our composite factor *FOP* explains the underperformance of both high-beta and high-variance portfolios. *FOP* serves as a powerful tool to discriminate between so far observationally equivalent predictions of leverage constraints, disagreement and sentiment and our results make a strong case for a common sentiment-based and thus behavioral explanation.

In the current literature, papers explaining the low-risk effect with price pressure from demand for lottery-like stocks (e.g. Bali et al., 2017) focus on idiosyncratic risk. To this end, Liu et al. (2018) argue that volatility is the driver behind the anomaly and beta is guilty by correlation. Although high-volatility stocks tend to have high market betas, returns significantly increase in market beta after controlling for *FOP*, but not in variance. Risky stocks are likely to be exposed to common but unaccounted factors attributable to investor sentiment which goes beyond and above correlation.

Second, there is closely related and growing evidence that investor sentiment affects the aggregate risk-return trade-off (Yu and Yuan, 2011; Antoniou et al., 2016; Shen et al., 2017). Antoniou et al. (2016) and Shen et al. (2017) both investigate the spreads of market beta sorted portfolios and find that the slope of the SML decreases in sentiment and even becomes negative during periods of high sentiment. Our evidence offers a new perspective on this finding. The tilted SML is attributable to the negative exposure of risky stocks to the unaccounted factor *FOP* and the average return on *FOP* is higher during periods of high investor sentiment. The return spread of high-beta and high-variance stocks attributable to *FOP* exceeds their expected return from the CAPM during waves of high sentiment and the slope SML appears to be negative.

Third, empirical asset pricing recently went from a zoo of factors (Cochrane, 2011; Harvey et al., 2016) to a variety of factor models with substantial common ground (Hou et al., 2019). Clearly, the fact that *FOP* explains its constituting anomalies better than alternative factor models has little implication beyond the law of one price (Kozak et al., 2018). However, the explanatory power of *FOP* with respect to seemingly unrelated anomalies indicates that many characteristics align with the exposure to a few common factors as pointed out by Kelly et al. (2019). In this spirit, *FOP* fully captures the CAPM alphas of portfolios sorted by Investment, although that particular anomaly attains a zero weight in the construction of *FOP*. As Asness et al. (2019) argue, existing factors are correlated with one another or the market portfolio – for example the Frazzini and Pedersen (2014) betting-against-beta (*BAB*) and the Bali et al. (2017) lottery demand factor (*FMAX*) – which impedes discriminating tests between the factors. *FOP* on the other hand captures only mispricing above and beyond market risk and leaves existing market beta estimates unchanged. Thus, the theoretically motivated factor *FOP* might help separating important from redundant factors without suffering from guilt by association (Liu et al., 2018).

5.2 The optimal orthogonal portfolio, seemingly unrelated anomalies and the low-risk effect

5.2.1 Introducing the optimal orthogonal portfolio

Our explanation for the low-risk effect relies on unaccounted factors in the CAPM. We treat this factor as a latent variable and propose an empirical approach to the theoretical framework of MacKinlay (1995) and MacKinlay and Pastor (2000) who show that mispricing due to latent factors is embodied in the covariance matrix of factor model residuals.

Starting from the CAPM, the excess return $r_{i,t}$ of asset $i \in \{1, \ldots, N\}$ in period t is

$$r_{i,t} = \alpha_i + \beta_{Mkt,i} r_{Mkt,t} + \epsilon_{i,t}, \tag{5.1}$$

$$\mathbb{E}(\boldsymbol{\epsilon_t}) = 0, \qquad \mathbb{E}\left(\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}'\right) = \Sigma \qquad \text{and} \qquad \mathrm{Cov}\left(\boldsymbol{\epsilon_t}, r_{Mkt,t}\right) = 0,$$

where $\beta_{Mkt,i}$ is the market beta of asset i, $\epsilon_{i,t}$ is the error in each time period, and α_i denotes mispricing. As long as an exact factor which proxies for additional state variable risk is missing in Equation (5.1), all deviations from the return generating process are embodied in a nonzero intercept α_i. In this case, MacKinlay and Pastor (2000) show that the covariance matrix Σ contains information about the missing factor which drives α_i. This relationship can be developed using the optimal orthogonal portfolio (OP).[3] OP is optimal and orthogonal such that the inclusion

[3]MacKinlay (1995) defines the optimal orthogonal portfolio as *"the unique portfolio*

of OP to the factor model in Equation (5.1) alleviates the mispricing α_i while preserving the coefficient estimate $\beta_{Mkt,i}$.

We denote the return on OP at time t with $r_{OP,t}$ which governs the asset return with sensitivity β_{OP} and its first two moments are $\mathbb{E}(r_{OP,t}) = \mu_{OP}$ and $\text{var}(r_{OP,t}) = \sigma^2_{OP}$. Per definition, it holds $\text{Cov}(r_{Mkt,t}, r_{OP,t}) = 0$. Replacing α_i in Equation (5.1) with the return of the optimal orthogonal portfolio yields

$$r_{i,t} = \beta_{OP,i} r_{OP,t} + \beta_{Mkt,i} r_{Mkt,t} + \nu_{i,t}, \tag{5.2}$$

$$\mathbb{E}(\boldsymbol{\nu_t}) = 0, \qquad \mathbb{E}(\boldsymbol{\nu_t}\boldsymbol{\nu_t}') = \Phi, \qquad \text{and} \qquad \text{Cov}(\boldsymbol{\nu_t}, \boldsymbol{\zeta_t}) = \text{Cov}(\boldsymbol{\nu_t}, r_{OOP}) = 0.$$

Taking the unconditional expectations of Equations (5.1) and (5.2) leads to

$$\alpha_i = \beta_{OP,i} E(r_{OP}) = \beta_{OP,i} \mu_{OP}. \tag{5.3}$$

It follows that the variance of the residual in Equation (5.1) is positively linked to the mispricing vector α according to

$$\Sigma = \beta_{OP}\beta'_{OP}\sigma^2_{OP} + \Phi = \alpha\alpha' \frac{1}{s^2_{OP}} + \Phi, \tag{5.4}$$

where s_{OP} is the Sharpe ratio of OP. In absence of this link, near-arbitrage opportunities arise (MacKinlay and Pastor, 2000, p. 886).

MacKinlay and Pastor (2000) propose an active portfolio which alleviates mispricing by using the strong-form link between Σ and the mispricing vector α. MacKinlay and Pastor (2000) show that the weight

given $\overline{N}$ assets that can be combined with the factor portfolios to form the tangency portfolio and is orthogonal to the factor portfolios" (MacKinlay, 1995, p. 8).

vector $\mathbf{w}$ of $N+1$ assets in the active portfolio is

$$\mathbf{w} = c \begin{bmatrix} \alpha \\ -\beta'\alpha \end{bmatrix}, \tag{5.5}$$

where c is a normalizing constant such that portfolio weights add up to one. Thus, the weights of the N assets in the active portfolio are proportional to the mispricing vector α (see MacKinlay and Pastor, 2000, p. 891). The weight $-\beta'\alpha$ in the $(N+1)^{th}$ asset, i.e. the factor portfolio, guarantees that the active portfolio is orthogonal to the market factor.

5.2.2 Tracking down the optimal orthogonal portfolio empirically

To construct an empirical counterpart to the optimal orthogonal portfolio, we directly form the active portfolio from the residuals in Equation (5.1). Following MacKinlay (1995), we employ subsets $S \subset \{1, \ldots, N\}$ of the N assets. The sample representation of the optimal orthogonal portfolio for a given subset S is

$$OP_{t,S} = \sum_{s \in S} \alpha_s \epsilon_{s,t} \tag{5.6}$$

In Equation (5.6), α_s is the weight of asset s and S is the subset of assets we employ in the estimation of Equation (5.1). By construction, this linear combination is orthogonal to the traded factor. To address the fact that OP_S is formed on residuals and thus not tradable, we form a mimicking portfolio from the sample assets. We estimate the regression

$$OP_{S,t} = a + \mathbf{b}\mathbf{X}_{\mathbf{S},\mathbf{t}} + u_{S,t} \tag{5.7}$$

and the fitted values of this regression minus the constant constitute the mimicking factor for OP_S, referred to as FOP_S. The base assets' returns $\mathbf{X}_{S,t}$ are the returns of the same subset of assets S. FOP_S contains information about the mispricing of the subset S with respect to the CAPM which is unrelated to the market risk of the asset because FOP_S and r_{Mkt} are uncorrelated. We estimate Equation (5.7) over the full sample period to reduce the measurement error. FOP_S thus represents an *ex-post* estimate for the optimal orthogonal portfolio for the subset S.

To further reduce dimensionality, we form an aggregate mimicking factor for the optimal orthogonal portfolio FOP from the sample representations FOP_S.[4] FOP is the linear combination of the sample FOP_S which maximizes the Sharpe ratio s. In particular, for any given subset S of assets, it holds that $s^2_{FOP_S} \leq s^2_{FOP}$ (MacKinlay, 1995).

A formal analysis of the theoretical framework above requires sample assets to construct FOP empirically. These sample assets should be informative about CAPM deviations and allow a precise estimation of the weight vector in Equation (5.5). Anomaly portfolios satisfy these requirements. Anomalies typically refer to patterns in stock returns which are not explained by the CAPM (Fama and French, 1996) and are thus particularly informative with respect to CAPM violations. Furthermore, the portfolios are homogeneous in the characteristics behind the CAPM deviation which reduces the measurement error of α.

[4] In untabulated robustness checks we estimate the residuals for all anomaly portfolios except beta and compute FOP according to $OP_t = w'(\alpha + \epsilon_t)$ directly from all anomaly portfolios. Both representations of FOP are highly correlated. The weights in this approach, however, are more unbalanced and less tractable.

We use decile portfolios of the following nine anomalies as base assets to construct OP: Accruals ($Accr$), book-to-market (BM), investment (Inv), long-term reversal ($LRev$), momentum (Mom), net share issues ($NetIss$), operating profitability ($Prof$), short-term reversal ($ShRev$) and size ($Size$). See Appendix 5.A.1 and Fama and French (1993, 2015, 2016) for further details. We treat each set of anomaly decile portfolios as a subset, form nine FOP_S according to Equations (5.6) and (5.7) and then estimate the linear combination with the highest Sharpe ratio referred to as FOP.

5.2.3 Seemingly unrelated anomalies and the low-risk effect

Now that we are equipped with an empirical measure for unaccounted factors in the CAPM, we can assemble the pieces in the novel context of the low-risk anomaly. In the first part of the paper, we take the source of the unaccounted factors as exogenous and focus on the asset pricing implications of the two factor model in Equation (5.2). We facilitate this analysis with two testable predictions.

First, as an empirical counterpart of the optimal orthogonal portfolio, FOP is expected to embody all relevant asset pricing information for a given set of test assets (Asgharian, 2011). FOP should therefore not only explain the nine constituent anomalies, but also span multi factor models which rely on related anomalies, e.g. the factor models in Fama and French (2015, 2018) and Stambaugh and Yuan (2017).

Second, turning to the low-risk effect, Equations (5.3) and (5.4) point out how unaccounted factors in the CAPM might explain the underperformance of risky stocks. For the negative CAPM alphas of high-beta stocks, the prediction of the two factor model is straightforward. Since the expected return of FOP is positive by construction, Equation (5.3) implies negative β_{FOP} for high-beta stocks. To alleviate the low-beta anomaly, this exposure should account for the negative alphas of high-beta stocks and furthermore re-establish a positive trade-off between β_{Mkt} and average returns.

Implications for volatility as a risk measure are less obvious. As shown in Equation (5.4), β_{FOP} also contributes to variance, i.e. idiosyncratic risk. Thus, the negative CAPM alphas of high-volatility stocks might compensate for unaccounted factors in the initial model (see Chen and Petkova, 2012). This additional systematic component prevents the diversification of idiosyncratic risk to zero when forming a portfolio (MacKinlay, 1995). We focus on return variance rather than the more common residual variance to measure idiosyncratic risk because the latter measure is model dependent and usually measured from multifactor models. Robustness checks in Section 5.6, however, illustrate that our results are robust to this choice.

5.3 Data

We obtain value-weighted monthly returns of eleven decile portfolios for the following anomalies: Accruals ($Accr$), market beta (β_{Mkt}), book-to-market (BM), investments (Inv), long-term reversal ($LRev$), momentum

(Mom), net share issues ($NetIss$), operating profitability ($Prof$), short-term reversal ($ShRev$), size ($Size$) and return variance (Var). We provide a detailed description with respect to portfolio formation in Appendix 5.A.1 and refer to Fama and French (2015, 2016) for further information. Furthermore, we obtain 25 portfolios sorted by size and market beta ($Size$-β_{Mkt}, 5x5) and size and return variance ($Size$-Var, 5x5).

The aggregate market return is proxied by the market factor Mkt which is the value-weighted excess return of all stocks in the CRSP universe and we use the risk factors of the models of Fama and French (1993, 2015, 2018). SMB and HML are the Small-minus-Big and the High-minus-Low factors of the Fama and French (1993) three factor model ($FF3$). Fama and French (2015) extend this model by the profitability factor RMW (Robust-minus-Weak) and the investment factor CMA (Conservative-minus-Aggressive) to the five factor model $FF5$. Most recently, Fama and French (2018) add the Carhart (1997) momentum factor UMD (Up-minus-Down) to constitute the six factor model $FF6$.[5] All of the above data is from Kenneth French's website. We furthermore obtain decile portfolios sorted by and idiosyncratic volatility ($IVol$) from the website of Robert Novy-Marx.

The risk factors of the Stambaugh and Yuan (2017) mispricing model $M4$ are from the website of Robert F. Stambaugh. The $M4$ model comprises Mkt, a size factor SMB_{M4} as well as the mispricing factors $PERF$

[5]Technically, the six factor model $FF6$ replaces the operating profitability factor RMW with a cash profitability factor RMW_C. However, this version of the factor is not publicly available and we use the initial definition of RMW instead, but refer to the model as $FF6$.

and $MGMT$ which are formed on anomaly portfolios.

Other economic data is from common sources: The University of Michigan Index of Consumer Sentiment is from the University of Michigan website and Baker and Wurgler (2006) (BW) sentiment data are from Jeffrey Wurgler's website.[6] The Chicago Fed National Activity Index (CFNAI) is from the website of the Federal Reserve Chicago and the Ted Spread is from the Federal Reserve Bank of St. Louis. Margin debt of NYSE customers in relation to NYSE market capitalization is from Datastream. Disagreement as the standard deviation of analysts long-term EPS growth forecasts (time series item LTSD) is also from Datastream. In constructing aggregate disagreement, we follow Hong and Sraer (2016) and weight the standard deviation of individual stocks by the pre-ranking market beta. Betas are estimated over the previous five years with monthly return data from Datastream as well. We thank our fellow colleagues for the provision of the research data.

5.4 Explaining the low-risk effect

5.4.1 Seemingly unrelated anomalies and the optimal orthogonal portfolio

The first prediction states that FOP embodies all relevant asset pricing information for the given set of test assets. To illustrate that this prediction does not hold for the initial factor model, we start with the CAPM. Panel A

[6] We use the orthogonalized BW Investor Sentiment Index.

Table 5.1: CAPM deviations and the two factor model

	β_{Mkt}	Var	$Accr$	BM	Inv	$LRev$	Mom	$NetIss$	$Prof$	$ShRev$	$Size$
Panel A: GRS test for the null hypothesis that all ten CAPM anomaly alphas are zero											
GRS statistic	1.9754	3.9353	3.4662	2.1825	4.3445	2.0026	5.5176	4.2293	2.4017	2.0110	2.4015
p–value	0.0336	<0.001	0.0002	0.0173	<0.001	0.0308	<0.001	<0.001	0.0084	0.0300	0.0084
Panel B: Average returns of anomaly FOP_S											
Excess return in %	0.1561	0.4662	0.1709	0.2070	0.2048	0.1550	0.5700	0.2533	0.1928	0.1733	0.1085
	(2.42)	(4.40)	(4.81)	(2.65)	(4.28)	(2.72)	(4.92)	(5.16)	(3.45)	(3.77)	(1.17)
SD in %	1.4827	2.6269	0.8533	1.6983	1.0456	1.3091	2.8731	1.0740	1.2822	1.1986	2.1597
Sharpe ratio (ann)	0.3646	0.6148	0.6938	0.4222	0.6785	0.4103	0.6873	0.8170	0.5208	0.5008	0.1740
N	642	642	642	642	642	642	642	642	642	642	642
Panel C: GRS test for the null hypothesis that all ten anomaly alphas from selected factor models are zero											
$Mkt + FOP$											
GRS statistic	1.2663	1.2221	0.8946	1.2106	1.1563	0.8291	2.3730	1.3152	0.7748	0.3266	1.0411
p–value	0.2459	0.2731	0.5379	0.2806	0.3177	0.6007	0.0092	0.2182	0.6533	0.9741	0.4068
$FF6$ model											
GRS statistic	2.0348	2.2082	4.0555	1.1183	2.5343	0.9630	3.3039	3.0621	1.6837	1.1926	1.6781
p–value	0.0278	0.0159	<0.001	0.3456	0.0053	0.4747	0.0003	0.0008	0.0808	0.2926	0.0821
$M4$ model											
GRS statistic	1.2947	1.9837	2.8288	0.7098	1.2188	0.9803	2.3263	2.0121	1.3970	1.3827	1.9476
p–value	0.2295	0.0327	0.0019	0.7157	0.2753	0.4592	0.0108	0.0299	0.1774	0.1840	0.0366

Table 5.1 compares the CAPM with the two factor model. The two factor model extends the CAPM by the mimicking portfolio for the optimal orthogonal portfolio FOP. Panel A presents Gibbons et al. (1989) (GRS) test statistics and the corresponding p-values for the null hypothesis that all CAPM alphas of the decile anomalies are zero. Panel B presents monthly excess returns (in %), t-statistics for the null hypothesis that excess returns are zero, monthly standard deviations in % as well as annualized Sharpe ratios for FOP_S from the anomaly portfolios. t-statistics in parentheses are computed from Newey and West (1987) with six lags. Panel C repeats the GRS test for selected multifactor models. The two factor model $Mkt + FOP$, the Fama and French (2018) six factor model $FF6$ and the Stambaugh and Yuan (2017) four factor model $M4$. The following anomalies are covered: Market beta (β_{Mkt}), return variance (Var), accruals ($Accr$), book-to-market (BM), investments (Inv), long-term reversal ($LRev$), momentum (Mom), net share issues ($NetIss$), operating profitability ($Prof$), short-term reversal ($ShRev$) and size ($Size$). The sample period is July 1963 to December 2016.

of Table 5.1 presents results of the Gibbons, Ross and Shanken (1989) GRS test for the null hypothesis that the CAPM alphas of the decile anomaly portfolios are jointly equal to zero. We present the test statistic as well as a p-value for each of the eleven anomalies. The sample period is July 1963 to December 2016.

The GRS test rejects the null hypothesis for each and every anomaly at conventional significance levels. This also holds true for β_{Mkt} and Var

portfolios, indicating the existence of the low-risk effect in our sample. For β_{Mkt}, the GRS test rejects the null hypothesis at the 5% level with a p-value of 0.0336. The GRS test statistic for Var portfolios is more than twice as high, tantamount to a rejection of the null hypothesis at any conventional level. Consequently, the CAPM fails to price all anomaly portfolios and we compute FOP_S for the full set of anomalies according to the procedure in Section 5.2.2.

Panel B presents descriptive statistics for each FOP_S. We provide average monthly excess returns in %, a t-statistic for the null that this excess return equals zero as well as monthly standard deviations and an annualized Sharpe ratio. Unless stated otherwise, t-statistics throughout this paper are computed from Newey and West (1987) standard errors with six lags.

With the exception of $Size$ with an average return of roughly 11 basis points (bps) and a t-statistic of 1.17, the average returns are statistically significant. These anomaly returns provide significant information after accounting for market risk. Significant average returns vary from 16 bps for $FOP_{\beta_{Mkt}}$ to 57 bps for FOP_{Mom} with t-statistics of 2.42 and 4.92, respectively. FOP_{Var} attains an average return of 47 bps with a t-statistic of 4.40. We use the full set of sample FOP_S except β_{Mkt} and Var to form a single factor representation as the linear combination which maximizes the Sharpe ratio. Since FOP_S are zero investment portfolios, we form FOP with long-only portfolio weights. The exclusion of β_{Mkt} and Var guarantees that FOP is a priori unrelated to the low-risk effect. Figure 5.1 presents the weights in FOP which maximize its Sharpe ratio.

Figure 5.1: Pie chart of anomaly weights in FOP.

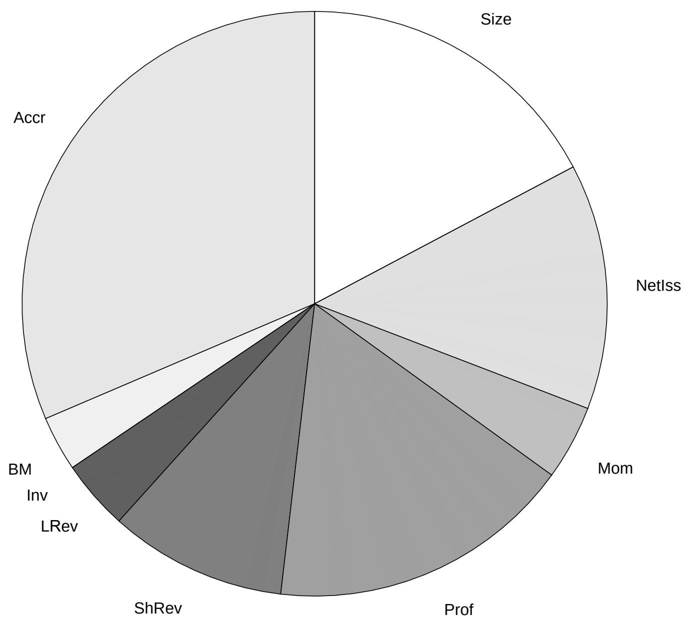


Figure 5.1 presents the weights of the anomaly FOP_S in the maximum Sharpe ratio combination FOP. The anomalies are: Accruals ($Accr$), book-to-market (BM), investments (Inv), long-term reversal ($LRev$), momentum (Mom), net share issues ($NetIss$), operating profitability ($Prof$), short-term reversal ($ShRev$) and size ($Size$). The sample period is July 1963 to December 2016.

$Accr$ attains the largest fraction in FOP with a weight of roughly 32%, followed by $Size$ with 18%, $Prof$ with 17%, and $NetIss$ with 14%, respectively. Other than that, weights in FOP are rather balanced. Despite their high individual Sharpe ratios, Mom attains a comparatively low weight of 4% whereas Inv even drops out completely. The average monthly excess return of FOP is 19 bps with a Newey and West (1987) t-statistic of 9.51. FOP exhibits an annualized Sharpe ratio of 1.5648 which is, by construction, higher than each of its subsample counterparts in Panel B of Table 5.1.

In Panel C of Table 5.1 we present the ability of FOP to explain its constituent anomaly portfolios as well as the two low-risk anomaly portfolios. Again, we present GRS test statistics and p-values for the null that all ten anomaly alphas are zero. For β_{Mkt} and Var portfolios, the null hypothesis cannot be rejected with p-values of 0.2107 and 0.2731, respectively. This finding extends to the other anomalies with the exception of Mom where the GRS test indicates that the alphas of the two factor model are jointly different from zero. Interestingly, the two factor model also performs well in explaining Inv, although FOP_{Inv} attains a zero weight in FOP.

To put the negative result in case of Mom into perspective, we report GRS test statistics and p-values for the factor models $FF6$ and $M4$ as well since both models explicitly account for Mom. In both cases, the null hypothesis is rejected and the two factor model performs similar to $M4$ and better than $FF6$. Although this is not a fair comparison since FOP is constructed from an ex post perspective, this result illustrates that FOP performs similarly well in terms of dimensionality reduction.

To further emphasize the latter finding, Table 5.2 presents spanning regressions which are less sensitive to the choice of test assets (see Barillas and Shanken, 2017; Hou et al., 2019).[7] In Panel A of Table 5.2, we regress risk factors of the factor models $FF6$ and $M4$ on FOP to evaluate the factor's alphas. We follow Stambaugh and Yuan (2017) and focus on unique factors, i.e. do not include Mkt, to analyze whether FOP subsumes

[7] The objective of the spanning regressions is not a model comparison, but the indication to what extent *FOP* explains existing asset pricing factors to reduce dimensionality. For an extensive comparison of factor models we refer to Fama and French (2018), Ahmed et al. (2019) and Hou et al. (2019).

Table 5.2: Spanning regressions and GRS test

Panel A: Spanning regressions

		FF6 Factors					M4 Factors		
	Mkt	SMB	HML	RMW	CMA	UMD	SMB_{M4}	$PERF$	$MGMT$
FOP	0.0471	0.9457	2.0662	1.6447	1.7726	3.4792	1.2393	2.7652	2.5844
	(0.06)	(2.50)	(4.25)	(3.87)	(5.16)	(6.50)	(3.41)	(4.13)	(5.00)
α in %	0.5012	0.0834	-0.0245	-0.0744	-0.0310	-0.0048	0.2108	0.1468	0.0855
	(2.19)	(0.63)	(-0.17)	(-0.74)	(-0.31)	(-0.02)	(1.73)	(0.74)	(0.65)
N	642	642	642	642	642	642	642	642	642

Panel B: GRS test for joint alphas of unique factors

	$\alpha_{SMB} = \alpha_{HML} = 0$	$\alpha_{SMB} = \alpha_{HML} = \alpha_{RMW} = \alpha_{CMA} = \alpha_{UMD} = 0$	$\alpha_{SMB_{M4}} = \alpha_{PERF} = \alpha_{MGMT} = 0$
GRS	0.2297	0.2416	2.5367
p-value	0.7948	0.9440	0.0558

Table 5.2 presents spanning regressions and Gibbons et al. (1989) (GRS) test results for different asset pricing factors. Panel A presents spanning regressions for the market factor Mkt and the asset pricing factors of the Fama and French (2018) six factor model $FF6$ and the Stambaugh and Yuan (2017) four factor model $M4$. Mkt enters both of the factor models $FF6$ and $M4$. t-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. In Panel B, we perform the GRS test for the null hypothesis that the alphas of unique asset pricing factors are jointly zero. We present the GRS test statistic and the corresponding p-value. The sample period is July 1963 to December 2016.

the asset pricing qualities of multifactor models. We report coefficient estimates as well as t-statistics from Newey and West (1987) standard errors in parentheses. The first Column with Mkt as the dependent variable indicates that FOP and Mkt are unrelated in statistical terms. The coefficient on FOP of 0.04714 is insignificant (t-statistic = 0.06). The correlation between FOP and Mkt amounts to 0.0045 and is insignificant as well, in line with the orthogonality condition of the optimal orthogonal portfolio. We find quite the opposite for the $FF6$ factors SMB, HML, RMW, CMA and UMD. All factor alphas are statistically insignificant. The GRS test for the joint alphas of the factor models $FF3$ and $FF6$ in Panel B does not reject the null hypothesis that all alphas are jointly zero. The p-values are 0.7948 and 0.9440, respectively. FOP consistently spans the $FF6$ risk factors and represents a reasonable univariate representation of the mutlifactor models in Fama and French (1993) and Fama and French (2018). This finding extends to the $M4$ model, however, to a lesser extent. FOP spans the mispricing factors $PERF$ and $MGMT$, but not the $M4$ counterpart of the SMB factor. The difference between SMB and SMB_{M4} is due to different breakpoints in the portfolio formation. Nevertheless, the GRS test in Panel B does not reject the null hypothesis at the 5% level. We conclude that FOP does not only explain its constituent anomaly portfolios, but also subsumes the largest part of the information in the multifactor models $FF6$ and $M4$. FOP is a reasonable single factor representation of the multifactor models and embodies all important information for the set of test assets.

5.4.2 Explaining the low-risk effect in time series regressions

Having shown that FOP satisfies the theoretical properties of the optimal orthogonal portfolio, we can turn to the performance of FOP in the context of the low-risk anomaly. The second prediction postulates that the inclusion of FOP into the CAPM alleviates the negative alphas of high-beta and high-volatility stocks.

Table 5.3 revisits the single sorted β_{Mkt} and Var portfolios in further detail and presents unadjusted monthly excess returns, alphas of several risk factor combinations, as well as the exposure of each decile with respect to the two factors Mkt and FOP. Returns and alphas are presented in % per month with Newey and West (1987) adjusted t-statistics in

Table 5.3: Explaining the returns of decile portfolios.

	Low β_{Mkt}	2	3	4	5	6	7	8	9	High β_{Mkt}	Diff
					Panel A: β_{Mkt} decile portfolios						
Unadjusted	0.5382	0.5021	0.5635	0.6475	0.5300	0.6146	0.5034	0.6540	0.6193	0.6118	0.0736
	(3.89)	(3.12)	(3.30)	(3.55)	(2.74)	(3.04)	(2.14)	(2.57)	(2.18)	(1.81)	(0.26)
Mkt	0.2243	0.1268	0.1383	0.1508	0.0102	0.0617	-0.0882	0.0022	-0.0909	-0.2106	-0.4349
	(2.57)	(1.56)	(1.96)	(1.98)	(0.19)	(0.98)	(-1.20)	(0.02)	(-0.80)	(-1.38)	(-2.06)
FOP	0.2933	0.2562	0.3311	0.5411	0.4018	0.5875	0.4569	0.7213	0.7872	0.9551	0.6618
	(1.82)	(1.42)	(1.64)	(2.38)	(1.70)	(2.25)	(1.64)	(2.33)	(2.31)	(2.30)	(1.98)
$FOP+Mkt$	-0.0147	-0.1121	-0.0863	0.0533	-0.1087	0.0445	-0.1242	0.0811	0.0893	0.1469	0.1617
	(-0.15)	(-1.29)	(-1.20)	(0.70)	(-1.69)	(0.61)	(-1.61)	(0.81)	(0.79)	(0.97)	(0.76)
β_{Mkt}	0.6146	0.7350	0.8327	0.9733	1.0185	1.0834	1.1593	1.2775	1.3923	1.6125	0.9979
	(20.82)	(22.26)	(33.87)	(47.33)	(47.75)	(48.18)	(39.13)	(41.43)	(39.38)	(38.39)	(16.15)
β_{FOP}	1.2449	1.2442	1.1698	0.5076	0.6191	0.0897	0.1874	-0.4106	-0.9388	-1.8619	-3.1068
	(4.15)	(3.55)	(3.90)	(1.42)	(3.40)	(0.36)	(0.97)	(-1.24)	(-2.98)	(-4.36)	(-4.99)
					Panel B: Var decile portfolios						
	Low Var	2	3	4	5	6	7	8	9	High Var	Diff
Unadjusted	0.4543	0.5817	0.5917	0.5842	0.6084	0.7089	0.7424	0.7549	0.5732	-0.0169	-0.4712
	(3.24)	(3.55)	(3.51)	(3.06)	(3.03)	(3.11)	(2.97)	(2.69)	(1.79)	(-0.04)	(-1.39)
Mkt	0.1577	0.1827	0.1432	0.0809	0.0723	0.1089	0.0965	0.0401	-0.2106	-0.8682	-1.0259
	(1.83)	(3.05)	(2.25)	(1.10)	(1.04)	(1.35)	(1.02)	(0.39)	(-1.58)	(-4.33)	(-3.89)
FOP	0.1961	0.3436	0.4009	0.4301	0.5004	0.6791	0.7625	0.8850	0.8283	0.7028	0.5067
	(1.26)	(1.81)	(1.99)	(1.90)	(2.06)	(2.46)	(2.43)	(2.58)	(2.08)	(1.43)	(1.22)
$OP+Mkt$	-0.0949	-0.0481	-0.0395	-0.0642	-0.0260	0.0898	0.1280	0.1827	0.0581	-0.1342	-0.0393
	(-1.11)	(-0.62)	(-0.52)	(-0.83)	(-0.31)	(1.11)	(1.23)	(1.76)	(0.43)	(-0.64)	(-0.15)
β_{Mkt}	0.5807	0.7816	0.8787	0.9861	1.0504	1.1759	1.2659	1.4011	1.5368	1.6701	1.0894
	(21.33)	(32.98)	(43.23)	(34.93)	(40.56)	(45.71)	(46.19)	(44.37)	(41.81)	(26.95)	(14.03)
β_{FOP}	1.3158	1.2019	0.9510	0.7554	0.5122	0.0996	-0.1644	-0.7427	-1.3994	-3.8226	-5.1384
	(5.04)	(3.85)	(3.06)	(2.11)	(1.05)	(0.32)	(-0.46)	(-2.28)	(-3.61)	(-5.33)	(-5.80)

Panel A (B) of Table 5.3 presents returns and factor model alphas of decile portfolios sorted by market beta β_{Mkt} (return variance Var) as well as a difference portfolio which is long in the highest and short in the lowest decile. Alphas and excess returns are multiplied with one hundred and the t-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. Mkt is the market factor and FOP is the mimicking factor for the optimal orthogonal portfolio. The sample period is July 1963 to December 2016.

parentheses.

Panel A presents β_{Mkt} decile portfolios. The unadjusted excess returns and the CAPM alphas confirm the beta anomaly. The relation between β_{Mkt} and average excess returns is flat with an insignificant difference between high and low β_{Mkt} stocks of roughly 7 bps. Controlling for Mkt leaves an alpha of approximately -44 bps with a t-statistic of -2.06. In contrast to the predictions of the CAPM, high-β_{Mkt} stocks significantly underperform low-β_{Mkt} stocks.

Extending the CAPM by FOP alleviates the anomaly. Once we control for the portfolio's exposure with respect to FOP, alphas increase in β_{Mkt}

and the difference portfolio exhibits an alpha of approximately 66 bps with a t-statistic of 1.98. Adding FOP to the market factor Mkt fully wipes out this unexplained return and the alpha of the difference portfolio of 16 bps is insignificant. In line with our second prediction, high-β_{Mkt} stocks have negative exposures to FOP. β_{FOP} decreases in the β_{Mkt} deciles, but this effect is asymmetric and nonlinear. Stocks in the high β_{Mkt} decile have a significantly negative β_{FOP} of -1.86 (t-statistic = -4.36), while low-β_{Mkt} exhibit portfolios a β_{FOP} of 1.24 (t-statistic = 4.15).

Panel B of Table 5.3 repeats the analysis for Var decile portfolios and is otherwise identical to Panel A. The underperformance of high Var deciles is stronger compared to β_{Mkt}. Unadjusted returns decrease from low to high Var, but the return of the difference portfolio is insignificant. This lies in stark contrast to the CAPM regressions. Here, high-Var stocks earn significantly negative alphas of -87 bps (t-statistic = -4.33). The negative alpha of the difference portfolio of -103 bps is highly significant with a t-statistic of -3.89, even when considering the standards of Harvey et al. (2016).

Including FOP alone reveals an interesting pattern. The average alphas slightly increase in the Var deciles, but the difference of roughly 51 bps is now insignificant (t-statistic = 1.22). Although the Var decile portfolios and the beta sorted portfolios have almost identical β_{Mkt}, the increasing return pattern of the beta portfolios when controlling for FOP does not extend to the Var deciles. Again, combining Mkt and FOP wipes out unexplained returns in the individual decile portfolios and reduces alpha of the difference portfolio to -4 bps (t-statistic = -0.15). High-Var

deciles also exhibit highly negative exposures to FOP. While the positive exposures in the lowest deciles of 1.3158 and 1.2019 are similar to the β_{Mkt} sorted portfolios, the negative β_{FOP} exposure in the highest decile is more than twice as large as for the top β_{Mkt} decile. Furthermore, the nonlinearity appears even stronger.

We extend the set of test assets to double-sorted portfolios. Table 5.4 presents time series regressions for 25 portfolios sorted by $Size$ and β_{Mkt}. Panel A (B) reports unadjusted excess returns (CAPM alphas) with corresponding t-statistics in parentheses which confirm the results of the univariate decile portfolios.

In Panel A, the relationship between β_{Mkt} and excess returns is flat in each of the $Size$ quintiles, whereas CAPM alphas in Panel B decrease from low to high β_{Mkt} quintiles. The beta anomaly persists in double sorted portfolios. The GRS test rejects the null hypothesis for the CAPM at conventional levels with a test statistic of 2.14 (p-value = 0.0011).

Panel C presents results for the two factor model which includes Mkt and FOP. We present alphas as well coefficient estimates for β_{Mkt} and β_{FOP}. The GRS test statistic for the two factor model amounts to 1.26 with a p-value of 0.1795, thus not rejecting the null hypothesis that all alphas are jointly zero. None of the alphas in Panel C is statistically significant, in contrast to the CAPM estimates in Panel B. Again, β_{FOP} decreases in β_{Mkt} quintiles and stocks in the highest β_{Mkt} quintiles have negative β_{FOP} except for the Small quintile. These quintiles largely correspond to the stocks which exhibit negative CAPM alphas in Panel B. β_{FOP} estimates furthermore monotonically decrease from Small to Big quintiles. This

Table 5.4: Explaining the returns of 25 $Size$-β_{Mkt} portfolios.

$\beta_{Mkt} \rightarrow$	Low	2	3	4	High	Low	2	3	4	High
			Coefficients					t-statistics		
					Panel A: Unadjusted excess returns					
Small	0.7442	0.8991	0.9154	0.9748	0.7733	(4.29)	(4.48)	(3.92)	(3.81)	(2.39)
2	0.7129	0.8723	0.9570	0.8928	0.6932	(4.20)	(4.64)	(4.45)	(3.66)	(2.22)
3	0.6916	0.8611	0.8480	0.7965	0.7277	(4.57)	(4.74)	(4.12)	(3.38)	(2.42)
4	0.6643	0.7687	0.7316	0.5935	0.7433	(4.37)	(4.24)	(3.61)	(2.59)	(2.49)
Big	0.4889	0.5235	0.4916	0.4956	0.4188	(3.44)	(3.14)	(2.57)	(2.21)	(1.49)
					Panel B: CAPM α					
Small	0.3588	0.4390	0.3822	0.3598	0.0188	(3.16)	(3.51)	(2.61)	(2.46)	(0.10)
2	0.3180	0.4109	0.4122	0.2736	-0.0995	(3.07)	(4.01)	(3.82)	(2.25)	(-0.64)
3	0.3261	0.3859	0.3082	0.1816	-0.0566	(3.79)	(4.72)	(3.36)	(1.71)	(-0.41)
4	0.2968	0.2819	0.1805	-0.0282	-0.0410	(3.45)	(3.91)	(2.44)	(-0.33)	(-0.31)
Big	0.1495	0.0707	-0.0382	-0.1197	-0.3069	(1.81)	(1.14)	(-0.62)	(-1.53)	(-2.30)
					Panel C: Two Factor Model Coefficients					
					α					
Small	0.0315	0.0637	-0.0049	0.0723	-0.0537	(0.26)	(0.48)	(-0.03)	(0.46)	(-0.25)
2	-0.0354	0.0483	0.0840	-0.0115	-0.1032	(-0.33)	(0.45)	(0.74)	(-0.09)	(-0.61)
3	0.0400	0.1176	0.0570	-0.0087	0.0708	(0.44)	(1.37)	(0.58)	(-0.08)	(0.47)
4	0.0239	0.0865	-0.0157	-0.1223	0.1668	(0.26)	(1.13)	(-0.20)	(-1.30)	(1.17)
Big	-0.0741	-0.0684	-0.0596	-0.0393	0.1283	(-0.84)	(-1.02)	(-0.88)	(-0.46)	(0.91)
					β_{Mkt}					
Small	0.7546	0.9009	1.0441	1.2046	1.4786	(30.59)	(33.19)	(32.62)	(37.22)	(33.67)
2	0.7732	0.9034	1.0671	1.2129	1.5534	(34.78)	(41.25)	(45.62)	(45.22)	(44.67)
3	0.7157	0.9306	1.0574	1.2046	1.5375	(38.68)	(52.91)	(52.83)	(51.04)	(50.09)
4	0.7196	0.9537	1.0795	1.2183	1.5376	(38.68)	(60.65)	(66.65)	(63.29)	(52.70)
Big	0.6646	0.8872	1.0382	1.2061	1.4233	(36.94)	(64.63)	(74.86)	(68.85)	(49.45)
					β_{FOP}					
Small	1.7044	1.9544	2.0158	1.4974	0.3776	(6.65)	(6.93)	(6.06)	(4.45)	(0.83)
2	1.8406	1.8884	1.7090	1.4846	0.0198	(7.96)	(8.29)	(7.03)	(5.32)	(0.05)
3	1.4898	1.3974	1.3083	0.9910	-0.6635	(7.75)	(7.64)	(6.29)	(4.04)	(-2.08)
4	1.4212	1.0172	1.0217	0.4902	-1.0819	(7.35)	(6.22)	(6.07)	(2.45)	(-3.57)
Big	1.1647	0.7241	0.1113	-0.4187	-2.2667	(6.23)	(5.07)	(0.77)	(-2.30)	(-7.58)

Table 5.4 presents time series regressions of 25 $Size$-β_{Mkt} portfolios on different factor models. Intercepts are multiplied with one hundred to facilitate interpretation. The corresponding t-statistics are presented in parentheses. Panel A (B) presents unadjusted excess returns (CAPM alphas). Panel C presents results for the two factor model which extends the CAPM by the mimicking factor for the optimal orthogonal portfolio FOP. We present the model Intercept as well as coefficient estimates for the two factors, i.e. β_{Mkt} and β_{FOP}. The sample period is July 1963 to December 2016.

pattern, however, perfectly matches the patterns in average returns in Panel A.

Table 5.5 repeats the analysis for $Size$-Var portfolios and is otherwise identical to Table 5.4. In Panel A and B, unadjusted excess returns and CAPM alphas decrease from low to high Var quintiles. The highest Var quintiles exhibit significantly negative CAPM alphas over all $Size$ quintiles. The strength of this relationship decreases from Small to Big quintiles. Consequently, the GRS test rejects the null in case of the CAPM with a test statistic of 6.67 at conventional levels (p-value < 0.001).

The extended CAPM again reduces the mispricing considerably and largely accounts for the negative alphas of the highest Var quintiles. The smallest quintile – referred to as the lethal combination (Fama and French, 2016) – is the only exception and alphas still significantly decrease from low to high Var quintiles. Similar to Table 5.4, portfolios with negative CAPM alphas exhibit negative β_{FOP}. Although FOP improves the asset pricing abilities of the CAPM, the GRS test still rejects the null hypothesis with a test statistic of 4.71 (p-value < 0.001).[8] The results of Table 5.4 extend to 25 $Size$-Var portfolios, albeit to a lesser extent.

The time series regressions make a strong case for the second prediction. Adding FOP to the CAPM explains the negative alphas of high-β_{Mkt} and high-Var stocks in decile portfolios. As predicted, high-risk portfolios exhibit highly negative exposures with respect to FOP. Both findings extend to double sorted portfolios in case of 25 $Size$-β_{Mkt}, but the negative

[8] To put this into perspective, the $FF6$ model attains a GRS test statistic of 4.80 (p-value < 0.001) and the negative Var spread in the second $Size$ quintile remains significant as well.

Table 5.5: Explaining the returns of 25 *Size*-*Var* portfolios.

$Var \rightarrow$	Low	2	3	4	High	Low	2	3	4	High
			Coefficients					*t*-statistics		
			Panel A: Unadjusted excess returns							
Small	1.0240	1.1668	1.0751	0.7734	-0.1949	(6.39)	(5.24)	(4.20)	(2.60)	(-0.54)
2	0.9126	1.0350	1.0406	0.9065	0.2719	(5.75)	(5.01)	(4.48)	(3.39)	(0.79)
3	0.7557	0.8384	0.9512	0.8738	0.4225	(5.22)	(4.47)	(4.50)	(3.59)	(1.33)
4	0.6866	0.7439	0.7663	0.7563	0.4771	(4.71)	(4.24)	(3.82)	(3.33)	(1.59)
Big	0.4317	0.5405	0.5261	0.4634	0.4743	(3.22)	(3.45)	(3.00)	(2.32)	(1.80)
			Panel B: CAPM α							
Small	0.6630	0.6350	0.4560	0.0609	-0.9940	(6.45)	(4.91)	(3.16)	(0.36)	(-4.10)
2	0.5329	0.5228	0.4517	0.2161	-0.5853	(5.80)	(4.76)	(3.88)	(1.70)	(-3.20)
3	0.4042	0.3539	0.4004	0.2336	-0.3954	(4.98)	(3.99)	(4.14)	(2.18)	(-2.61)
4	0.3433	0.2885	0.2303	0.1409	-0.3174	(3.96)	(3.54)	(2.83)	(1.65)	(-2.46)
Big	0.1119	0.1302	0.0478	-0.0917	-0.2339	(1.43)	(1.86)	(0.76)	(-1.47)	(-2.25)
			Panel C: Two Factor Model Coefficients							
					α					
Small	0.3136	0.2652	0.2035	0.0215	-0.7120	(2.91)	(1.93)	(1.30)	(0.11)	(-2.69)
2	0.1126	0.1002	0.1043	0.0185	-0.2922	(1.22)	(0.88)	(0.85)	(0.13)	(-1.47)
3	0.0093	0.0499	0.0807	-0.0029	-0.0521	(0.12)	(0.54)	(0.80)	(-0.03)	(-0.32)
4	-0.0205	0.0275	-0.0098	0.0549	0.1122	(-0.23)	(0.32)	(-0.11)	(0.59)	(0.83)
Big	-0.0764	-0.0814	-0.1021	-0.0635	0.1914	(-0.91)	(-1.10)	(-1.51)	(-0.93)	(1.80)
					β_{Mkt}					
Small	0.7067	1.0414	1.2128	1.3962	1.5668	(32.02)	(37.00)	(37.80)	(36.30)	(28.92)
2	0.7433	1.0029	1.1533	1.3526	1.6806	(39.31)	(43.13)	(45.71)	(47.79)	(41.20)
3	0.6880	0.9488	1.0788	1.2541	1.6037	(41.67)	(49.90)	(51.85)	(53.07)	(48.11)
4	0.6719	0.8919	1.0498	1.2059	1.5581	(37.08)	(50.65)	(59.35)	(62.86)	(56.20)
Big	0.6264	0.8035	0.9372	1.0880	1.3891	(36.40)	(52.93)	(67.49)	(77.48)	(63.58)
					β_{FOP}					
Small	1.8201	1.9257	1.3148	0.2050	-1.4689	(7.93)	(6.58)	(3.94)	(0.51)	(-2.61)
2	2.1890	2.2012	1.8095	1.0292	-1.5265	(11.14)	(9.11)	(6.90)	(3.50)	(-3.60)
3	2.0567	1.5831	1.6650	1.2320	-1.7875	(11.98)	(8.01)	(7.70)	(5.02)	(-5.16)
4	1.8948	1.3593	1.2506	0.4477	-2.2373	(10.06)	(7.43)	(6.80)	(2.25)	(-7.76)
Big	0.9807	1.1021	0.7804	-0.1469	-2.2152	(5.48)	(6.98)	(5.41)	(-1.01)	(-9.75)

Table 5.5 presents time series regressions of 25 *Size*-*Var* portfolios on different factor models. Intercepts are multiplied with one hundred to facilitate interpretation. The corresponding *t*-statistics are presented in parentheses. Panel A (B) presents unadjusted excess returns (CAPM alphas). Panel C presents results for the two factor model which extends the CAPM by the mimicking factor for the optimal orthogonal portfolio *FOP*. We present the model Intercept as well as coefficient estimates for the two factors, i.e. β_{Mkt} and β_{FOP}. The sample period is July 1963 to December 2016.

alphas of small high-Var portfolios remain statistically significant. The low-risk effect is likely to arise from unaccounted factors in the CAPM.

5.4.3 Cross-sectional evidence

We use the time series estimates in Tables 5.4 and 5.5 to evaluate the asset pricing performance of the two factor model cross-sectionally as well. We perform cross-sectional Fama and MacBeth (1973) regressions with the full sample estimates according to

$$r_{p,t} = \gamma_{0,t} + \gamma_{1,t}\beta_{Mkt,p} + \gamma_{2,t}\beta_{OP,p} + \epsilon_{p,t}, \tag{5.8}$$

where $r_{p,t}$ is the excess return of portfolio p in month t. Table 5.6 presents the second stage coefficients. We report the average cross-sectional adjusted r-squared $avg.\overline{R}^2$ in %. Average coefficients are multiplied by one hundred. We follow Lewellen et al. (2010) and include the factor portfolios Mkt and FOP among the test assets. t-statistics calculated from Newey and West (1987) standard errors are shown in parentheses.

Panel A of Table 5.6 presents the results for 25 $Size$-β_{Mkt} portfolios. Comparing the CAPM in Model (1) and the two factor Model (2) reveals two major differences. First, the risk premium for β_{Mkt} increases from roughly 20 bps to 52 bps after the inclusion of β_{FOP}, with t-statistics of 0.88 and 2.12, respectively. This estimate is close to the full sample risk premium of Mkt of 51 bps. Second, the pricing error of the CAPM of 50 bps with a t-statistic of 4.19 becomes insignificant once we include β_{FOP}. Since we include excess returns on the left-hand side, a nonzero Intercept

Table 5.6: Fama and MacBeth (1973) regressions for double sorted portfolios.

Panel A: 25 $Size$-β_{Mkt} portfolios								
Model	Intercept		β_{Mkt}		β_{FOP}		avg. $\overline{R}^2$ in %	N
(1)	0.4976	(4.19)	0.1976	(0.88)			31.01	27
(2)	0.0119	(0.09)	0.5181	(2.12)	0.1856	(3.34)	43.88	27
Panel B: 25 $Size$-Var portfolios								
Model	Intercept		β_{Mkt}		β_{FOP}		avg. $\overline{R}^2$ in %	N
(3)	0.7905	(5.57)	-0.1020	(-0.42)			34.61	27
(4)	-0.0281	(-0.23)	0.5097	(2.05)	0.2478	(5.30)	44.65	27

Panel A (B) of Table 5.6 presents second stage Fama and MacBeth (1973) estimates for 25 $Size$-β_{Mkt} (25 $Size$-Var) portfolios. All coefficients are multiplied with one hundred and the t-statistics in parentheses are computed from Newey and West (1987) standard errors with six lags. β_{Mkt} (β_{FOP}) is the beta with respect to the market portfolio (mimicking factor FOP). We follow Lewellen et al. (2010) and include the factor portfolios Mkt and FOP among the test assets. The sample period is July 1963 to December 2016.

indicates mispricing. Model (2) prices the test assets more efficiently and explains a larger fraction of cross-sectional variation than the CAPM.

This result also extends to Panel B in which 25 $Size$-Var portfolios serve as base assets. Again, the risk premium for β_{Mkt} increases from -10 bps to 51 bps due to the consideration of β_{FOP}. The latter estimate is statistically significant at the five percent level with a t-statistic of 2.05 and again close to the full sample market risk premium. The pricing error of the two factor model becomes insignificant and reduces from 79 bps (t-statistic = 5.57) to -3 bps (t-statistic = -0.23). The average coefficient for β_{FOP} of 25 bps is also close to the full sample risk premium of FOP.

To further emphasize the difference between the CAPM and the two factor model, Figure 5.2 plots average realized returns against the expected returns from the respective models. Panel A (B) plots the expected

Figure 5.2: Expected returns of the CAPM versus the two factor model.

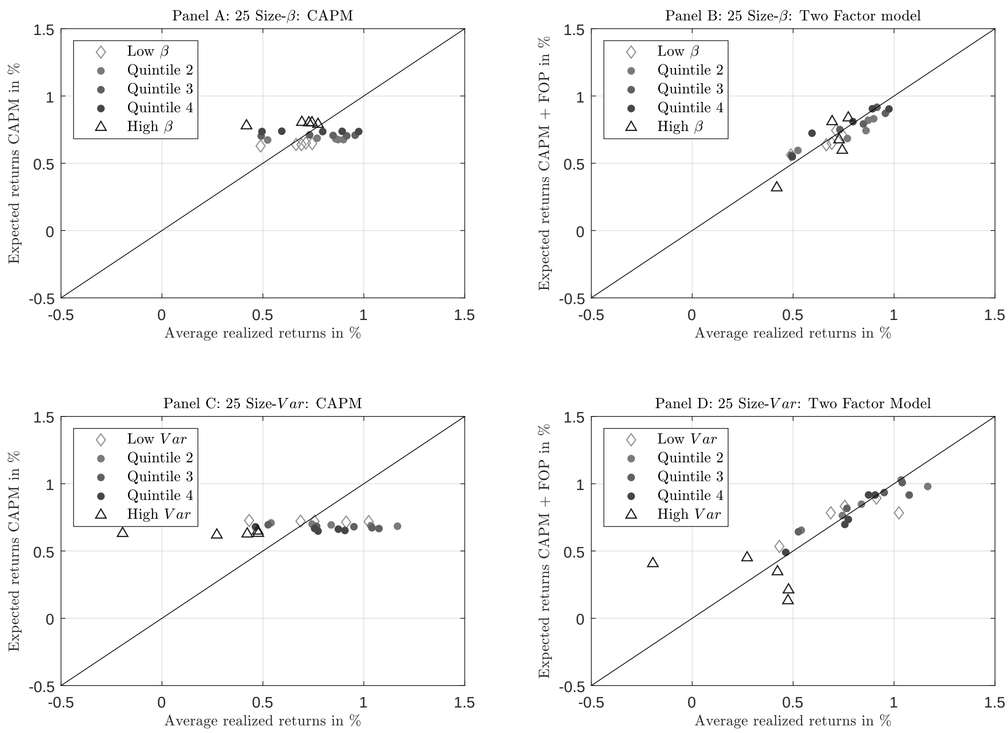


Figure 5.2 plots expected versus realized returns of the CAPM (Panels A and C) and the CAPM extended by the mimicking factor for the optimal orthogonal portfolio FOP (Panel B and D). The test assets are 25 portfolios sorted by $Size$ and β_{Mkt} ($Size$ and Var) in Panel A and B (C and D). The 45-degree line indicates a perfect relationship between realized and expected returns. The sample period is July 1963 to December 2016.

returns from the CAPM model (CAPM extended by FOP) in % per month against the average realized returns of the 25 $Size$-β_{Mkt} portfolios. Panels C and D repeat the same analysis for 25 $Size$-Var portfolios. The solid 45-degree line corresponds to a perfect relationship between expected returns and average realized returns.

For both sets of test assets, the relation between realized and expected returns is flat in case of the CAPM shown in Panel A and C. In Panel A, low-β_{Mkt} portfolios earn higher realized return than expected from the model, whereas the opposite is true for high-β_{Mkt} stocks. This result extends to Panel C where high-Var portfolios are plotted well above the 45-degree line, indicating that expected returns from the CAPM are too low compared to realized returns. In both cases, the model leaves significant pricing errors.

Including β_{FOP} into the model improves both of the problems in the CAPM. Expected returns of 25 $Size$-β_{Mkt} and 25 $Size$-Var portfolios are now closer to the 45-degree line. As expected from the time series regressions, the two factor model performs better in case of $Size$-β_{Mkt} portfolios since high-Var portfolios do not line up well with the 45-degree line in Panel D. The improvements over the CAPM, however, are easily visible.

Extending the CAPM with the mimicking factor for the optimal orthogonal portfolio FOP explains the underperformance of high-β_{Mkt} and high-Var portfolios. The negative exposure of risky stocks with respect to FOP explains their negative CAPM alphas. Furthermore, the positive trade-off between β_{Mkt} in decile portfolio sorts extends to cross-sectional regressions.

5.5 Testing economic theories

5.5.1 FOP and the slope of the security market line

The two factor model solves the issues of the CAPM in pricing risky portfolios, but remains agnostic with respect to the economic mechanisms behind FOP. The most prominent economic explanations – no matter whether they are based on leverage constraints, investor sentiment or disagreement – share the common prediction that the slope of the security market line (SML) depends on the respective state variables. During periods of high leverage constraints, investor sentiment or disagreements, the SML takes on a flatter slope because high-β_{Mkt} stocks tend to be overpriced and earn lower future returns (see, e.g. Frazzini and Pedersen, 2014; Hong and Sraer, 2016; Antoniou et al., 2016; Jylhä, 2018).

If FOP is related to existing theoretical explanations, we expect the same prediction for the SML. Since the CAPM suffers from the omission of the latent factor FOP in the first place, the exposure to β_{FOP} should fully account for variations in the slope of the SML. This powerful additional prediction is possible because the inclusion of FOP to the CAPM leaves estimates for β_{Mkt} unchanged. This Section focuses on β_{Mkt} sorted portfolios and leaves Var portfolios for a litmus test of the most promising explanation in Section 5.5.2.

Figure 5.3 presents sample splits at the median of FOP for β_{Mkt} decile portfolios. We plot the monthly realized returns and expected returns from the CAPM against post formation β_{Mkt}. The dashed line is the theoretical SML as expected from the CAPM, the solid line is a least squares fit of the relationship between realized returns and post formation β_{Mkt}. Marker colors indicate high and low deciles. For each sample split, we plot the return spread of the decile portfolios which is attributable to FOP, i.e. β_{FOP} times the average return of FOP in the respective subsample period.

Figure 5.3: *FOP* and the slope of the SML.

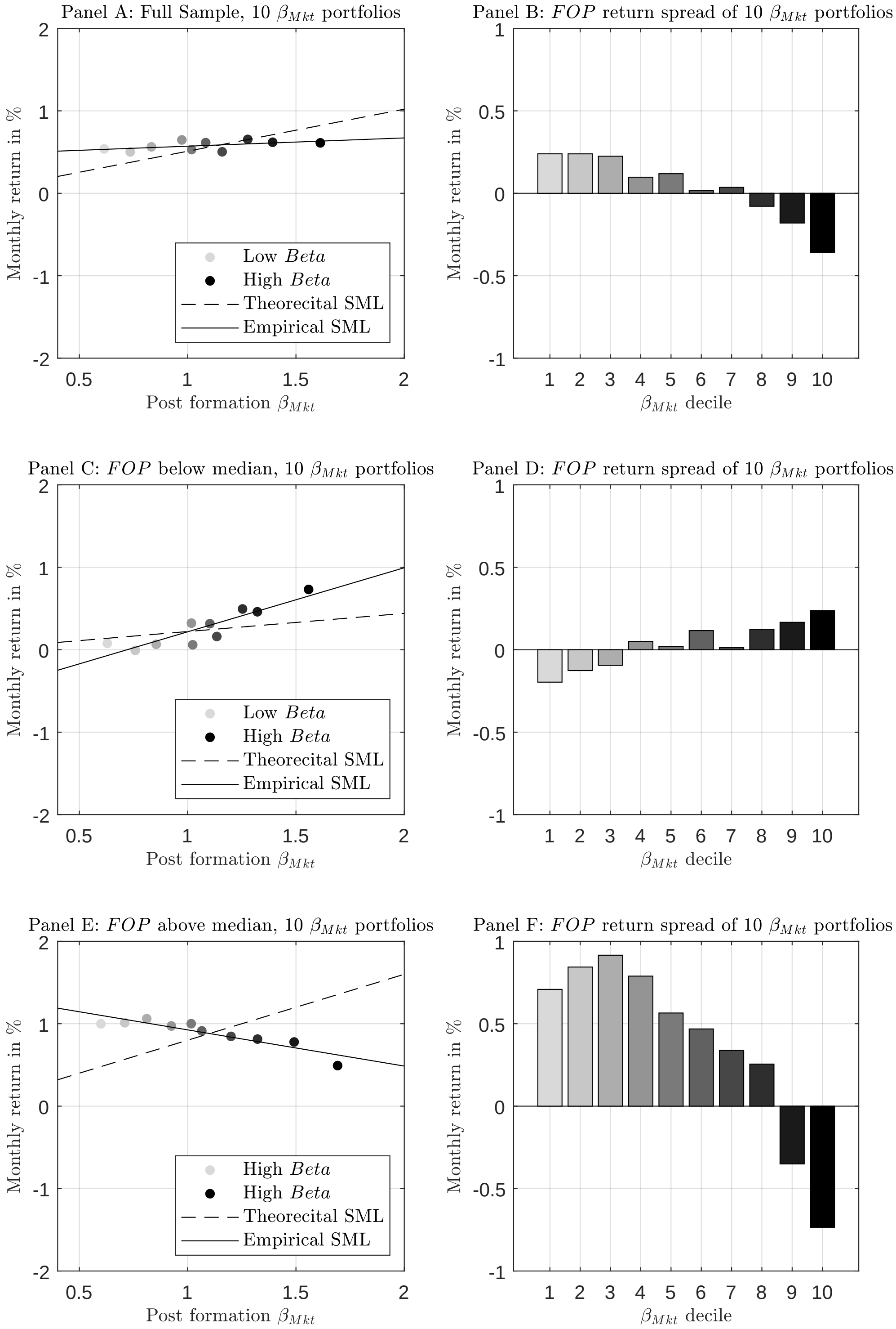


Figure 5.3 plots the empirical (solid) versus the theoretical (dashed) slope of the Security Market Line (SML) for decile β_{Mkt} portfolios. Panel A and B consider the full sample, in Panel C and D (E and F) we present the slopes during months in which the mimicking factor for the optimal orthogonal portfolio *FOP* is lower (higher) than the sample median. The sample period is July 1963 to December 2016.

Panel A presents the full sample period from July 1963 to December 2016. In line with results in the previous literature, the empirical SML is flat. The difference between the theoretical and the empirical SML almost perfectly lines up with the *FOP* return spread in Panel B. For example, the lowest β_{Mkt} decile earns an average return of 54 bps, while the expected return in the CAPM amounts to 31 bps. The β_{FOP} exposure times the average return on *FOP* is 24 bps and thus matches this difference. Overall, the correlation between the difference of the two SML lines and the *FOP* spread is 0.90. This finding, however, is no surprise considering the good performance of the two factor model in the previous Section.

Panel C presents the same estimates for the subperiod in which *FOP* is below the historical median and reveals the expected pattern. Now the empirical SML is steeper than its theoretical counterpart, in line with periods of low leverage constraints, disagreement, or sentiment as presented in Jylhä (2018), Hong and Sraer (2016), and Antoniou et al. (2016). Now that the realized returns exceed their expectations from the CAPM, the *FOP* return spread in Panel D lines up positively from low to high β_{Mkt} deciles. This switch is due to a negative average *FOP* of -11 bps in this subsample whereas the β_{FOP} exposures of the β_{Mkt} decile portfolios hardly change and still decrease monotonically from low to high deciles.

Panel E plots the most interesting case, subperiods with *FOP* above the sample median. If *FOP* is consistent with the theoretical explanations above, the negative slope of the SML should be fully attributable to the β_{FOP} exposure. The slope of the empirical SML now turns negative, in line with previous studies. Again, the pricing error of the theoretical SML almost perfectly lines up with the *FOP* return spread. Interestingly, the spread increases from the first to the third decile and then decreases monotonically. The overall spread is stronger compared to Panel B which might reflect an arbitrage asymmetry documented by Stambaugh et al. (2015). Now the correlation between the *FOP* return spread and the differences between empirical and theoretical SML is even higher and amounts to 0.96.

Overall, the sample splits reveal familiar patterns with respect to the slope of the SML. The finding that this pattern is fully attributable to the exposure to *FOP*, however, provides another powerful implication to test theoretical propositions for the tilted SML. In order to constitute a consistent explanation for the low-risk effect, any potential state variable should induce a higher average return on *FOP* and significantly affect the sign of the β_{Mkt} decile return spread in the same direction as *FOP*. The factor *FOP* thus facilitates a horse race to discriminate between the otherwise observationally equivalent predictions of leverage constraints, disagreement, and sentiment.

5.5.2 Leverage constraints versus behavioral explanations

Now that we established *FOP* as the new contender in the context of the low-risk effect, we can evaluate existing theoretical propositions. Following Asness et al. (2019), we focus on leverage constraints and promising behavioral alternatives. Specifically, our analysis considers leverage constraints, investor sentiment and disagreements as proposed by Frazzini and Pedersen (2014), Antoniou et al. (2016) and Hong and Sraer (2016).[9] We include the Ted spread and Margin Debt of NYSE customers as two proxies for leverage constraints (Frazzini and Pedersen, 2014; Asness et al., 2019). Since the latter exhibits a time trend and is therefore nonstationary, we remove the trend in a linear regression. To facilitate the interpretation, we furthermore multiply Margin Debt by minus one such that a higher value of Margin Debt in our analysis is tantamount to higher leverage constraints.[10] Our two proxies for sentiment are the BW Investor Sentiment Index and the University of Michigan Consumer Confidence Index. As in Hong and Sraer (2016), Disagreement is the beta-weighted average of the standard deviation from analyst forecasts for the long-term EPS growth rate.

Table 5.7 presents time series regressions of *FOP* on proxies for leverage constraints and investor sentiment. We include explanatory variables in terms of levels in Columns (1) to (6) and first differences in Columns (7) to (12). The sample period is 1986 to 2016 in Columns (1) and (7), 1967 to 2016 in Columns (2) and (8), 1965 to 2015 in Columns (4) and (10) and 1978 to 2016 in Columns (5) and (11). The kitchen sink models

[9]In unreported robustness checks, we account for the following alternatives, but find no significant evidence: The CBOE VIX (Ang et al., 2006; Barinov, 2018), average variance (Chen and Petkova, 2012), the CFNAI, Economic Policy Uncertainty of (Baker et al., 2016), inflation (Cohen et al., 2005), the term spread, the earnings price ratio and the default yield spread (all as defined in Welch and Goyal, 2008).

[10]We refer to Asness et al. (2019) for the discussion regarding the interpretation of Margin Debt as a measure of leverage constraints. Unreported robustness checks reveal that the detrended time series exhibits an even better predictive power for the Frazzini and Pedersen (2014) betting-against-beta factor *BAB*, a key result in Asness et al. (2019).

Table 5.7: Leverage constraints versus behavioral explanations.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Levels						First differences					
Ted	-6.3626					-4.9871	-10.4614					-5.0251
	(-1.16)					(-0.99)	(-0.73)					(-0.33)
Margin Debt		30.9768				-82.9420		573.3206				458.8819
		(0.47)				(-1.25)		(2.16)				(1.51)
BW Sentiment			0.0615			0.2103			0.3648			0.4365
			(2.01)			(4.16)			(2.37)			(1.61)
Consumer Confidence				0.0043		-0.0002				0.0021		0.0031
				(1.83)		(-0.10)				(0.46)		(0.54)
Disagreement					0.1662	0.1223					0.2198	0.2434
					(4.01)	(3.57)					(1.38)	(1.49)
Intercept	0.2349	0.1924	0.1934	-0.1735	-0.2043	-0.1016	0.1968	0.1922	0.1914	0.1918	0.1994	0.1979
	(4.61)	(9.50)	(9.36)	(-0.89)	(-2.23)	(-0.49)	(6.09)	(9.57)	(9.01)	(7.32)	(7.02)	(5.92)

Table 5.7 presents time series regressions of *FOP* on predictor variables for constraints to arbitrage and investor sentiment. We include the following variables: The Ted spread, Margin Debt of NYSE customers in relation to NYSE market capitalization, the Baker and Wurgler (2006) (BW) Investor Sentiment Index, the University of Michigan Consumer Confidence Index and aggregate Disagreement. All coefficients are multiplied with one hundred. We include explanatory variables in terms of levels in Columns (1) to (6) and first differences in Columns (7) to (12). The sample period is 1986 to 2016 in Columns (1) and (8), 1967 to 2016 in Columns (2) and (9), 1965 to 2015 in Columns (4) and (11), 1978 to 2016 in Columns (5) and (12) and 1982 to 2016 in Columns (6) and (3). The kitchen sink models in Columns (6) and (12) reduce the sample period to 1986 to 2015. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

in Columns (6) and (12) reduce the sample period to 1986 to 2015. All coefficients are multiplied with one hundred with t-statistics from Newey and West (1987) standard errors in parentheses.

Results in levels, i.e. Columns (1), (2) and (6) provide little support to leverage-based explanations. The Ted spread and Margin Debt exhibit insignificant coefficients, both in the univariate models in Columns (1) and (2) as well as the kitchen sink regression in Column (6). Both sentiment measures are significantly positive with coefficients of 0.0614 (t-statistic = 2.01) for BW Sentiment and 0.0043 (t-statistic = 1.83). Only the latter, however, survives when we control for all predictive variables in Column (6) with a highly significant coefficient of 0.2103 (t-statistic = 4.16). This also holds true for Disagreement which is statistically significant in

Columns (5) and (6).

In terms of first differences, results are mixed at best. Margin Debt and BW Sentiment are statistically significant at the 5% level in Columns (8) and (9) with coefficients of 573.32 and 0.3648 (t-statistics = 2.16 and 2.37, respectively). Both predictors, however, lose explanatory power in the multivariate regression in Column (14) and become insignificant. The time series regressions highlight behavioral explanations, most importantly BW Sentiment and Disagreement. Conversely, we find little support for leverage constraints.

As stated above, the two factor model yields a second, even stronger prediction to identify economic state variables behind variations in the slope of the SML. In order to account for the effects in Figure 5.3, a regime switch from low to high states in the economic variable should induce a significantly positive change in FOP and negatively affect the sign of the return difference between high and low β_{Mkt} deciles. To test this prediction formally, we follow Stambaugh et al. (2012) and run the time series regression

$$r_t = \alpha_H d_{H,t} + \alpha_L d_{L,t} + \epsilon_{i,t}, \tag{5.9}$$

where $d_{H,t}$ ($d_{L,t}$) is an indicator variables which is equal to one if the respective predictor variable in the previous month is above (below) the sample median and zero otherwise. r_t is either the return on FOP or the difference between the highest and the lowest β_{Mkt} decile portfolio. In the latter regressions we include Mkt to effectively measure the CAPM alpha of the β_{Mkt} decile spread.[11]

[11] Including both, Mkt and FOP yields qualitatively identical results.

Table 5.8: Testing economic theories.

	Panel A: *FOP*						Panel B: β_{Mkt} decile return spread					
	High		Low		Diff		High		Low		Diff	
Ted	0.1930	(4.18)	0.2007	(4.87)	-0.0078	(0.13)	-0.4978	(-1.18)	-0.6667	(-1.94)	0.1688	(0.31)
Margin Debt	0.1872	(6.86)	0.1974	(6.98)	-0.0102	(0.27)	-0.3479	(-1.36)	-0.5239	(-1.60)	0.1761	(0.43)
BW Sentiment	0.2408	(7.1)	0.1459	(6.19)	0.0949	(2.32)	-1.1652	(-4.96)	0.3347	(1.03)	-1.4999	(-3.82)
Consumer Confidence	0.2333	(5.97)	0.1509	(5.19)	0.0824	(1.77)	-1.1469	(-3.45)	-0.1289	(-0.43)	-1.0181	(-2.25)
Disagreement	0.2567	(5.05)	0.1417	(6.92)	0.1151	(2.14)	-0.5634	(-1.48)	-0.9395	(-3.27)	0.3761	(0.83)

Table 5.8 presents time series regressions with different indicator variables based on the median split of constraints to arbitrage and investor sentiment. The dependent variable is FOP in Panel A and the decile return spread of β_{Mkt} decile portfolios in Panel B. Regressions in Panel B include Mkt as an explanatory variable. We include the following variables: The Ted spread, Margin Debt of NYSE customers in relation to NYSE market capitalization, the Baker and Wurgler (2006) (BW) Investor Sentiment Index, the University of Michigan Consumer Confidence Index and aggregate Disagreement. All coefficients are multiplied with one hundred. The sample period is 1986 to 2016 for the Ted Spread, 1967 to 2016 for Margin Debt, 1965 to 2015 for BW Sentiment, 1978 to 2016 for Consumer Confidence and 1981 to 2016 for aggregate Disagreement. t-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

Table 5.8 presents the coefficient estimates as well as their respective difference $\alpha_H - \alpha_L$ which indicates whether the difference of the dependent variable in the two states is significantly different from zero. The dependent variables are the returns on FOP in Panel A and the return spread between the highest and the lowest β_{Mkt} decile in Panel B. Coefficients are multiplied with one hundred and t-statistics in parentheses are computed from Newey and West (1987) robust standard errors.

Panel A, again, accentuates behaviorally motivated predictive variables over leverage constraint-based explanations. The difference in FOP between high and low leverage constraint regimes is insignificant both in case of the Ted spread as well as Margin Debt. Hence, leverage constraints are unlikely to explain variation in the state variable proxied by FOP. BW Sentiment, Consumer Confidence and Disagreement all induce significantly higher average returns on FOP. For example, when previous months BW Sentiment is high, FOP is also higher on average and a high-FOP state – tantamount to a negative slope of the SML – is more likely.

The second condition refers to the sign of the β_{Mkt} decile spread during periods of high leverage constraints, sentiment or disagreement. Panel B presents the coefficient estimates as well as their respective difference while controlling for Mkt. Once more, behavioral explanations attain more promising results. When sentiment is high – either measured by BW Sentiment or Consumer Confidence – the decile return spread on β_{Mkt} is significantly negative and insignificant otherwise. The difference of -150 bps and -102 bps is statistically significant at least at the 5% level with t-statistics of -3.82 and -2.25, respectively. Disagreement is not in line with the second prediction. Here, the β_{Mkt} decile spread is significantly negative when disagreement is low, opposite of what the model in Hong and Sraer (2016) predicts. Again, predictors for leverage constraints are insignificant.

In summary, investor sentiment satisfies the predictions from Section 5.5.1 best and is a likely source to explain both parts of the low-risk anomaly. To further emphasize this finding, we test the pricing performance of FOP for Var decile portfolios during periods of high or low BW Investor Sentiment. Table 5.9 presents the results. Sentiment is referred to as high (low) when BW Investor Sentiment in the previous month is above (below) the sample median. Other than that, Table 5.9 matches Table 5.3.

Similar to β_{Mkt} deciles, the negative difference between high and low

Table 5.9: Explaining the returns of Var decile portfolios conditional on sentiment.

	Low Var	2	3	4	5	6	7	8	9	High Var	Diff
					Low Sentiment						
Mkt	-0.1881	0.0026	-0.0105	-0.0369	0.0736	0.2497	0.2312	0.2263	0.0784	-0.2244	-0.0363
	(-1.54)	(0.03)	(-0.15)	(-0.42)	(0.85)	(2.08)	(1.59)	(1.32)	(0.37)	(-0.74)	(-0.09)
$Mkt + FOP$	-0.3006	-0.0746	-0.0448	-0.0635	0.1558	0.3164	0.3084	0.3136	0.1850	0.1053	0.4059
	(-2.36)	(-0.80)	(-0.54)	(-0.67)	(1.80)	(2.56)	(1.97)	(1.90)	(0.86)	(0.30)	(0.92)
					High Sentiment						
Mkt	0.4883	0.3397	0.2816	0.1951	0.0714	-0.0222	-0.0100	-0.1133	-0.5393	-1.5192	-2.0075
	(3.92)	(4.12)	(2.72)	(1.84)	(0.62)	(-0.20)	(-0.08)	(-0.95)	(-3.26)	(-6.15)	(-6.24)
$Mkt + FOP$	0.0961	-0.0392	-0.0618	-0.0592	-0.2455	-0.1488	-0.0154	0.0920	-0.0416	-0.2668	-0.3630
	(0.72)	(-0.31)	(-0.46)	(-0.47)	(-1.73)	(-1.42)	(-0.10)	(0.67)	(-0.23)	(-1.12)	(-1.13)

Panel A (B) of Table 5.9 presents returns and factor model alphas of decile portfolios sorted by market beta β_{Mkt} (return variance Var) as well as a difference portfolio which is long in the highest and short in the lowest decile. Alphas and excess returns are multiplied with one hundred and the t-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. Mkt is the market factor and FOP is the mimicking factor for the optimal orthogonal portfolio. The sample period is July 1963 to December 2016.

Var deciles concentrates in high sentiment periods. When sentiment is low, both the CAPM and the two factor model leave no significant unexplained return in the difference portfolio. During high sentiment periods, the difference between high and low Var deciles increases to -201 bps (t-statistic = -6.24) which is significant at any conventional level. Including FOP fully wipes out this unexplained return which is also reflected in the GRS test statistic of 1.2727 (p-value = 0.2446).

5.6 Robustness checks

The literature on idiosyncratic risk usually measures idiosyncratic risk in terms of idiosyncratic volatility ($IVol$) as the standard deviation of Fama and French (1993) three factor model residuals. Since this measure is model-dependent, we focus on return variance in the baseline analysis. However, to illustrate that our findings are robust to this choice, we extend

Table 5.10: Explaining idiosyncratic volatility ($IVol$) portfolios.

	Low $IVol$	2	3	4	5	6	7	8	9	High $IVol$	Diff
CAPM	0.1819	0.1161	0.1212	0.0508	0.0825	0.1505	0.0258	0.0487	-0.1330	-0.8920	-1.0739
	(2.45)	(1.99)	(2.20)	(0.69)	(0.94)	(1.88)	(0.29)	(0.47)	(-0.92)	(-4.25)	(-4.03)
CAPM + FOP	-0.0295	-0.0519	-0.0470	-0.1115	0.0101	0.1355	0.1332	0.2015	0.2877	-0.2679	-0.2384
	(-0.39)	(-0.73)	(-0.59)	(-1.53)	(0.11)	(1.61)	(1.49)	(1.83)	(1.84)	(-1.18)	(-0.86)

Table 5.10 presents returns and factor model alphas of decile portfolios sorted by and idiosyncratic volatility ($IVol$) as well as difference portfolios which are long in the highest and short in the lowest deciles. Alphas are multiplied with one hundred and the t-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. Mkt is the market factor and FOP is the mimicking factor for the optimal orthogonal portfolio. The sample period is August to December 2013 in Panel A (B).

the set of test portfolios by decile portfolios sorted by $IVol$. We present results in Table 5.10 which is otherwise identical to the baseline analysis in Table 5.3. The sample period is August 1963 to December 2013. The alternative risk measure $IVol$ hardly affects the baseline findings. The difference portfolio earns a highly significant negative CAPM alpha of -107 bps with a t-statistic of 4.03. Extending the CAPM fully explains this anomalous return. The alpha of the difference portfolio reduces to roughly -24 bps with a t-statistic of -0.86. Thus, our baseline findings fully extend to $IVol$ sorted portfolios.

5.7 Concluding remarks

We use seemingly unrelated anomaly portfolios to construct the mimicking factor FOP which approximates the optimal orthogonal portfolio of MacKinlay and Pastor (2000) and test the asset pricing implications of the extended CAPM. The exposure to FOP explains the negative alphas of high-beta and high-variance stocks and reestablishes a positive trade-off between beta and returns. Our extended CAPM is theoretically motivated, computationally tractable, and allows a multidimensional approach to the identification of characteristics which provide independent information about average returns (Cochrane, 2011). For example, our composite factor explains average return differentials caused by investment, although the characteristic itself attains a zero weight in its construction. Our evidence promotes sentiment as an explanation for the low-risk effect and is not supported by alternative predictors, e.g. leverage constraints.

5.A Appendix

5.A.1 Anomalies

Following Fama and French (1993, 2015, 2016), we consider the following setup of anomalies. All portfolios are formed on NYSE breakpoints at the end of June each year. Double-sorted portfolios are independent sorts with NYSE breakpoints as well.

Accruals (Accr): Sloan (1996) shows that companies with high accruals earn lower future returns. Accruals are the change in operating working capital per split-adjusted share divided by the book equity per share (Fama and French, 2016, p. 74).

Book-to-Market (BM): Fama and French (1993) show that average returns are related to the book-to-market ratio which is defined as the ratio of book equity to market equity.

Investments (Inv): Investments is the growth of total assets from the fiscal year $t-2$ to $t-1$ (Fama and French, 2015, p. 4).

Long-term Reversal (LRev): Long-term reversal is the prior return over the prior 13 to 60 months.

Momentum (Mom): Momemtum, as documented by Jegadeesh and Titman (1993), is the cumulative return over the prior 2 to 12 months (Fama and French, 2016, p. 75).

Net Share Issues (NetIss): Returns following share issues are lower, as documented by Loughran and Ritter (1995). We use decile portfolios formed on NetIss, defined as the change in the natural log of split-adjusted shares outstanding from fiscal year-end in $t-2$ to $t-1$ (Fama and French, 2016, p. 74).

Operating Profitability (Prof): Novy-Marx (2013) shows that profitable firms earn higher returns. Operating profitability is annual revenues minus cost of goods sold, interest expense, selling, general and administrative expenses divided by book equity (Fama and French, 2015, p. 4).

Short-term Reversal (ShRev): Short-term Reversal is the return in the previous month.

Size (Size): Size is the market equity at the end of June.

Return Variance (Var): Ang et al. (2006) show that highly volatile stocks earn lower future returns. We consider portfolios on the variance of daily returns over the previous 60 days with a minimum of 20 days (Fama and French, 2016, p. 74).

Market Beta (β_{Mkt}): Market Beta is estimated over the previous 5 years of monthly returns with a minimum of 24 observations (Fama and French, 2016, p. 74).

Additionally, we obtain decile portfolios sorted by idiosyncratic volatility ($IVol$) as considered in Novy-Marx and Velikov (2016). Returns are value-weighted and rebalanced on a monthly basis using NYSE breakpoints:

Idiosyncratic Volatility (IVol): Idiosyncratic volatility is the standard deviation of Fama and French (1993) three factor model residuals over the previous three months (Novy-Marx and Velikov, 2016, p. 117). High $IVol$ stocks earn low returns and negative alphas (Ang et al., 2006).

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