PAPER • OPEN ACCESS

Irreducible Cartesian multipole decomposition of scattered light with explicit contribution of high order toroidal moments

IOP Conf. Series: Journal of Physics: Conf. Series **1461** (2020) 012054 doi:10.1088/1742-6596/1461/1/012054

Irreducible Cartesian multipole decomposition of scattered light with explicit contribution of high order toroidal moments

E Gurvitz[1,*], K Ladutenko[1,2], P Dergachev[3,4], A Evlyukhin[5,6], A Miroshnichenko[7], and A Shalin[1]

[1] ITMO University, 49 Kronverkskiy pr., St. Petersburg, Russia, 197101.
[2] Ioffe Institute, 26 Polytekhnicheskaya st.. St. Petersburg, Russia, 194021.
[3] National Research University Moscow Power Engineering Institute, 14 Krasnokazarmennaya st., Moscow, Russia, 111250.
[4] National University of Science and Technology «MISIS», 4 Leninsky pr., Moscow, Russia, 119049.
[5] Leibniz Universität Hannover, Institut für Quantenoptik, Welfengarten 1, D-30167 Hannover, Germany.
[6] Moscow Institute of Physics and Technology, 9 Institutsky Lane, Dolgoprudny, Russia, 141700.
[7] School of Engineering and Information Technology, University of New South Wales Canberra, 2600, Australia.

* egorgurvitz@gmail.com

Abstract. Multipole decomposition is a powerful tool for analysis of electromagnetic systems. This work considers high order irreducible Cartesian multipole moments in approximation of electric 32-pole and magnetic 16-pole. The explicit contributions to scattering of high order toroidal moments up to toroidal electric octupole and toroidal magnetic quadrupole are demonstrated for a dielectric high refractive index scatterer.

1. Introduction:

Multipoles are simplest configurations of charges and currents excited in a medium suitable for an electromagnetic analysis. Many years multipole concept remained unchanged before the work of Ya. Zeldovich, who first specified the features of a toroidal moment (he called it anapole moment) in connection with the problem of violation of parity symmetry in an atomic system with broken space-time mirror symmetry [1].

Recently, toroidal moments attract much interest in metamaterials science. Unlike naturally occurring media those artificial ones were for first time shown to produce electrical toroidal response [2,3]. It was demonstrated in microwave frequencies that metamaterials with the toroidal response produce strong circular dichroism [2] and possess extremely high Q-factor, due to destructive interference of dipolar and toroidal parts (this nonradiative state was called anapole state), etc.

In the optical frequency range, the investigation of high order multipoles and high order toroidal moments has been started since the magnetic responses have been found in high refractive index materials [4,5], such as Si, Ge. High order multipoles were applied for optical forces analysis [6], scattering light directivity control [7,8], studying of nonradiative states (e.g lasing and nonradiative mode transfer [9–12]), photovoltaics [13–15], cloaking [16,17], etc .

There are frequently used approaches for multipole decomposition, e.g., Cartesian [18–21], spherical [22], fractional [23], Fano-Feshbach modes [24], and others. The most popular one is Cartesian multipole decomposition. It allows to get explicit contribution of toroidal type multipole moments to scattering, and reveals their interference with basic ones. In this work we demonstrate multipole decomposition of a scattered electric field in far-field zone in limitation of 5th order Cartesian multipoles (electric 32 pole and magnetic 16 pole) with explicit contribution of high order toroidal multipoles up to the toroidal electric octupole and toroidal magnetic quadrupole.

2. Results

To obtain multipole decomposition of the scattered electric field in Lorentz gauge $\mathbf{E} = -\dot{\mathbf{A}} - \boldsymbol{\nabla}\Phi$ the scalar Φ and vector $\mathbf{A}$ retarded potentials are expanded to Taylor series :

$$\Phi(\mathbf{R}, t) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\mathbf{r}, t' + \Delta t)}{|\mathbf{R}-\mathbf{r}|} dv, \tag{1}$$

$$\mathbf{A}(\mathbf{R}, t) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{r}, t' + \Delta t)}{|\mathbf{R}-\mathbf{r}|} dv, \tag{2}$$

where ρ and $\mathbf{J}$ are charge and current densities in an infinitesimal volume dv, ε_0 and μ_0 are vacuum permittivity and permeability, $\mathbf{R}$ is a radius vector from the origin to a distant point and $\mathbf{r}$ is a radius vector from origin to a point with considered distribution of charges and currents; retarded time at dv is $(t' + \Delta t) = t - |\mathbf{R} - \mathbf{r}|/c$, where c is speed of light in vacuum and $t' = t - |\mathbf{R}|/c$ is the time retardation at the coordinates origin [19]. Symmetrisation and detracing [25] of primitive high order multipoles [21] allow to obtain tensors, which produce an independent contribution to the electric field in far-field. The symmetrized and traceless tensors match requirement for a Cartesian tensor to stay invariant under its translation and rotation in respect to SO3 group.

Using Einstein index notation the final equation for the scattered power in far-field zone is introduced in the irreducible representation of high order multipoles and explicit contributions of high order toroidal moments:

$$\begin{aligned} P_{scat} = {} & \frac{k^4\sqrt{\varepsilon_d}}{12\pi\varepsilon_0^2 c\mu_0}|p_i + \frac{ik}{c}T_i^{(e)} + \frac{ik^3}{c}T_i^{(2e)}|^2 + \frac{k^4\,\varepsilon_d\sqrt{\varepsilon_d}}{12\pi\varepsilon_0 c}|m_i + \frac{ik}{c}T_i^{(m)}|^2 + \\ & + \frac{k^6\,\varepsilon_d\sqrt{\varepsilon_d}}{160\pi\varepsilon_0^2 c\mu_0}|\bar{\bar{Q}}_{ij}^{(e)} + \frac{ik}{c}\bar{\bar{T}}_{ij}^{(Qe)}|^2 + \frac{k^6\,\varepsilon_d^2\sqrt{\varepsilon_d}}{160\pi\varepsilon_0 c}|\bar{\bar{Q}}_{ij}^{(m)} + \frac{ik}{c}\bar{\bar{T}}_{ij}^{(Qm)}|^2 + \\ & + \frac{k^8\,\varepsilon_d^2\sqrt{\varepsilon_d}}{3780\pi\varepsilon_0^2 c\mu_0}|\bar{\bar{O}}_{ijk}^{(e)} + \frac{ik}{c}\bar{\bar{T}}_{ijk}^{(Oe)}|^2 + \frac{k^8\,\varepsilon_d^3\sqrt{\varepsilon_d}}{3780\pi\varepsilon_0 c}|\bar{\bar{O}}_{ijk}^{(m)}|^2 + \\ & + \frac{k^{10}\varepsilon_d^3\sqrt{\varepsilon_d}}{145152\pi\varepsilon_0^2 c\mu_0}|\bar{\bar{S}}_{ijkl}|^2 + \frac{k^{10}\varepsilon_d^4\sqrt{\varepsilon_d}}{145152\pi\varepsilon_0 c}|\bar{\bar{Y}}_{ijkl}|^2 + \\ & + \frac{k^{12}\varepsilon_d^4\sqrt{\varepsilon_d}}{831600\pi\varepsilon_0^2 c\mu_0}|\bar{\bar{X}}_{ijkla}|^2. \end{aligned} \tag{3}$$

Here k is a vacuum wave number, ε_0 and μ_0 are vacuum permittivity and permeability, ε_d is relative permittivity, $\mathbf{n}$ is a unit vector codirected with $\mathbf{R}$, R is length of $\mathbf{R}$, $\mathbf{p}$ and $\mathbf{m}$ are basic electric and magnetic dipoles, $\bar{\bar{Q}}_{jk}^{(e)}$ and $\bar{\bar{Q}}_{jp}^{(e)}$are electric and magnetic quadrupoles (the double overline denotes symmetrized and detraced tensor), $\bar{\bar{O}}_{jkp}^{(e)}$ and $\bar{\bar{O}}_{jpl}^{(m)}$ are electric and magnetic octupoles, $\bar{\bar{S}}_{jkpl}$ and $\bar{\bar{Y}}_{jplt}$

are electric and magnetic 16 poles, and $\bar{\bar{X}}_{jkplt}$ is electric 32 pole. Toroidal moments in Equation 3 contain superscript, which refers to the corresponding multipole and shown in Table1.

Table 1. The high order toroidal moments

Order	Toroidal electric multipoles	Toroidal magnetic multipoles
1	$T_j^{(e)} = \frac{1}{10}\int(\mathbf{J}\cdot\mathbf{r})r_j - 2r^2 J_j dv$ $T_j^{(2e)} = \frac{1}{280}\int 3r^4 J_j - 2r^2(\mathbf{r}\cdot\mathbf{J})r_j dv$	-
2	$\bar{\bar{T}}_{jk}^{(Qe)} = \frac{1}{42}\int 4(\mathbf{r}\cdot\mathbf{J})r_j r_k + 2(\mathbf{J}\cdot\mathbf{r})r^2\delta_{jk} - -5r^2(r_j J_k + r_k J_j)dv$	$T_j^{(m)} = \frac{i\omega}{20}\int r^2(\mathbf{r}\times\mathbf{J})_j dv$
3	$\bar{\bar{T}}_{jkl}^{(Oe)} = \frac{1}{300}\int 35(\mathbf{r}\cdot\mathbf{J})r_j r_k r_l - 20r^2(J_l r_k r_j + +J_k r_l r_j + J_j r_l r_k) + (\delta_{lj}\delta_{kp} + \delta_{lk}\delta_{jp} + \delta_{lp}\delta_{jk})\cdot \cdot[(\mathbf{r}\cdot\mathbf{J})r^2 r_p + 4J_p r^4]dv$	$\bar{\bar{T}}_{jp}^{(Qm)} = \frac{i\omega}{42}\int r^2[r_j(\mathbf{r}\times\mathbf{J})_p + +(\mathbf{r}\times\mathbf{J})_j r_p]dv$

Taking into account a normalization of the scattered power on incident wave energy flux $I_0 = \frac{1}{2}\sqrt{\varepsilon_0\varepsilon_d}$ and geometrical cross-section of the scatterer we have obtained the multipole decomposition of the scattering efficiency for a spherical scatterer with refractive index n=4

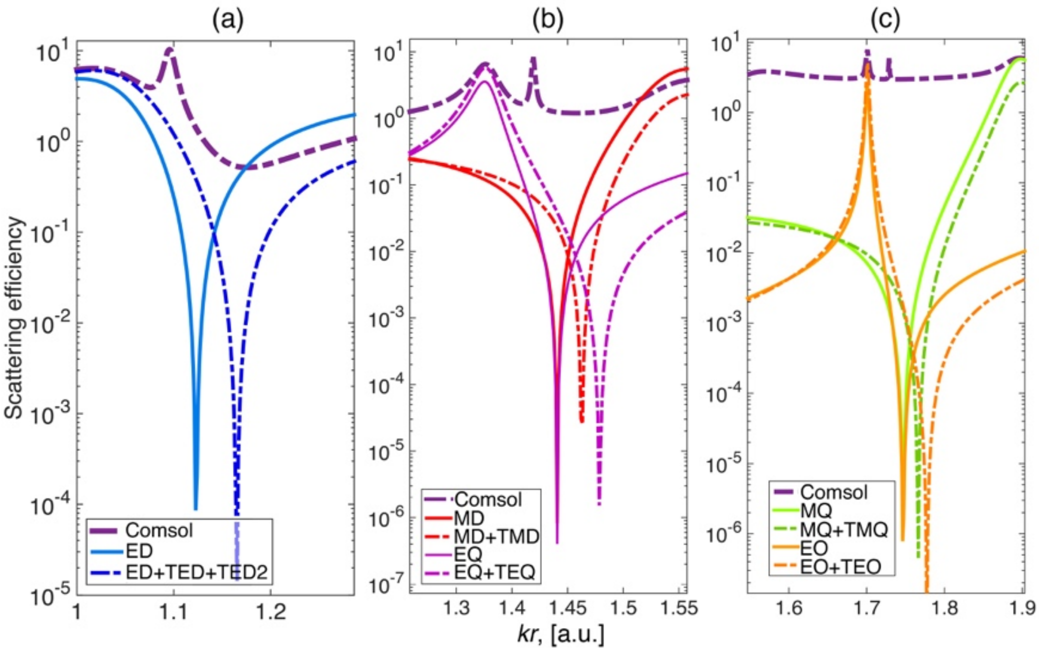

Figure1. Scattering efficiency with explicit contributions of toroidal moments of 1st order: TED and TED2 – toroidal electric dipoles (a), the 2nd order toroidal moments TMD - toroidal magnetic dipole and TEQ - toroidal electric quadrupole (b), and the 3rd order: TMQ toroidal magnetic quadrupole and TEO toroidal electric octupole (c).

Figure 1 shows that irreducible representation provides an additional information about configurations of charges and currents inside a scatterer and the integral contribution of basic and toroidal moments in the far field. Panels (a,b,c) demonstrate shifts of scattering efficiency minima, so-called anapole states, for basic moments having toroidal contributions. The scattering efficiency minima are explained by the destructive interference of basic and toroidal moments in far-field. At the same time the introduced toroidal moments reveal spectral points, where they have predominant contribution in the scattered field. Those spectral points are observed at the minima of basic multipole moments.

Conclusion:

Irreducible representation match the requirement for the tensors to be invariant regarding transformations on SO3 group. The primitive Cartesian multipoles were simplified to irreducible form using symmetrization and detracing processes. The combination of residual terms allows to reveal explicit contribution of toroidal moments to the scattered power. Toroidal moments have identical propagators as corresponding basic counterpart. Proposed multipole decomposition method allows to separate scattering contribution of the basic and toroidal multipoles in far-field zone.

Acknowledgement

The authors acknowledge the financial support provided by Russian Fund for Basic Research (Projects No. 18-02-00414, 18-52-00005); Government of Russian Federation, Grant 08-08.

References

[1] Zel'dovich Y B 1959 Parity nonconservation in the first order in the weak-inter- action constant in electron scattering and other effects *J. Exptl. Theor. Phys.* **36** 964 966

[2] Papasimakis N, Fedotov V A, Savinov V, Raybould T A and Zheludev N I 2016 Electromagnetic toroidal excitations in matter and free space *Nat. Mater.* **15** 263–71

[3] Fedotov V A, Rogacheva A V, Savinov V, Tsai D P and Zheludev N I 2013 Resonant Transparency and Non-Trivial Non-Radiating Excitations in Toroidal Metamaterials *Sci. Rep.* **3** 1–5

[4] Evlyukhin A B, Novikov S M, Zywietz U, Eriksen R L, Reinhardt C, Bozhevolnyi S I and Chichkov B N 2012 Demonstration of Magnetic Dipole Resonances of Dielectric Nanospheres in the Visible Region *Nano Lett.* **12** 3749–55

[5] Miroshnichenko A E, Evlyukhin A B, Yu Y F, Bakker R M, Chipouline A, Kuznetsov A I, Luk'yanchuk B, Chichkov B N and Kivshar Y S 2015 Nonradiating anapole modes in dielectric nanoparticles *Nat. Commun.* **6** 1–8

[6] Chen J, Ng J, Lin Z and Chan C T 2011 Optical pulling force *Nat. Photonics* **5** 531–4

[7] Terekhov P D, Baryshnikova K V., Shalin A S, Karabchevsky A and Evlyukhin A B 2017 Resonant forward scattering of light by high-refractive-index dielectric nanoparticles with toroidal dipole contribution *Opt. Lett.* **42** 1–4

[8] Terekhov P D, Baryshnikova K V., Artemyev Y A, Karabchevsky A, Shalin A S and Evlyukhin A B 2017 Multipolar response of nonspherical silicon nanoparticles in the visible and near-infrared spectral ranges *Phys. Rev. B* **96** 1–8

[9] Luk'yanchuk B, Paniagua-Domínguez R, Kuznetsov A I, Miroshnichenko A E and Kivshar Y S 2017 Hybrid anapole modes of high-index dielectric nanoparticles *Phys. Rev. A* **95** 1–8

[10] Wut P C, Liao C Y, Savinov V, Chung T L, Chen W T, Huang Y-W, Wu P R, Chen Y-H, Liu A-Q, Zheludev N I and Tsai D P 2018 Optical Anapole Metamaterial *ACS Nano* **12** 1920–7

[11] Totero Gongora J S, Miroshnichenko A E, Kivshar Y S and Fratalocchi A 2017 Anapole nanolasers for mode-locking and ultrafast pulse generation *Nat. Commun.* **8** 1–9

[12] Mazzone V, Sebastian J, Gongora T and Fratalocchi A 2017 Near-Field Coupling and Mode Competition in Multiple Anapole Systems *Appl. Sci.* **7** 1–9

[13] Shalin A S and Moiseev S G 2009 Optical Properties of Nanostructured Layers on the Surface

of an Underlying Medium *Opt. Spectrosc.* **106** 1004–13
[14] Simovski C R, Shalin A S, Voroshilov P M and Belov P A 2013 Photovoltaic absorption enhancement in thin-film solar cells by non-resonant beam collimation by submicron dielectric particles *J. Appl. Phys.* **114** 1–6
[15] Voroshilov P M, Simovski C R, Belov P A and Shalin A S 2015 Light-trapping and antireflective coatings for amorphous Si-based thin film solar cells *J. Appl. Phys.* **117** 1–7
[16] Terekhov P D, Babicheva V E, Baryshnikova K V, Shalin A S, Karabchevsky A and Evlyukhin A B 2019 Multipole analysis of dielectric metasurfaces composed of nonspherical nanoparticles and lattice invisibility effect *Phys. Rev. B* **99** 1–10
[17] Shalin A S, Ginzburg P, Orlov A A, Iorsh I, Belov P A, Kivshar Y S and Zayats A V. 2015 Scattering suppression from arbitrary objects in spatially dispersive layered metamaterials *Phys. Rev. B* **91** 1–7
[18] Jackson J D 1999 *Classical Electrodynamics*
[19] Raab R E and De Lange O L 2005 *Multipole theory in Electromagnetism* (Clarendon, Oxford)
[20] Markovich D, Baryshnikova K, Shalin A and Samusev A 2016 Enhancement of artificial magnetism via resonant bianisotropy *Sci. Rep.* **6** 1–8
[21] Gurvitz E A, Ladutenko K S, Dergachev P A, Evlyukhin A B, Miroshnichenko A E and Shalin A S 2018 The high-order toroidal moments and anapole states in all- dielectric photonics *Laser & Photonics Reviews*, **13**(5), 1800266.
[22] Bohren C F and Huffman D R 2008 *Absorption and Scattering of Light by Small Particles*
[23] Engheta N 1996 actional Calculus and Fracti ultipoles in Electromagnetism *IEEE Trans. Antennas Propag.* **44** 554–66
[24] Monticone F, Alù A, Decker M, Staude I, Rubinsztein-dunlop H, Forbes A, Berry M V, Chen H, Taylor A J, Yu N, Sebastian J, Gongora T, Favraud G and Fratalocchi A 2017 Fundamental and high-order anapoles in all- dielectric metamaterials via Fano – Feshbach modes competition *Nanotechnology* **28** 104001
[25] Applequist J 1989 Traceless cartesian tensor forms for spherical harmonic functions: New theorems and applications to electrostatics of dielectric media *J. Phys. A. Math. Gen.* **22** 4303–30