Stochastic Forecasting in Demography and Social Insurance

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Widmung

Dieses Werk entstand als kumulative Dissertationsschrift während meiner Tätigkeit als wissenschaftlicher Mitarbeiter am Institut für Versicherungsbetriebslehre der Gottfried Wilhelm Leibniz Universität Hannover.

Ich bedanke mich an dieser Stelle herzlich für die großartige Betreuung während dieser Zeit durch meinen Doktorvater Prof. Dr. J.-Matthias Graf von der Schulenburg und meine Forschungsleiterin Dr. Ute Lohse. Beide haben mich im Rahmen zahlreicher Diskussionen konsequent dazu angeregt, eigene Vorgehensweisen zu überdenken und über Grenzen hinauszudenken. Die Zeit am Institut war sowohl sehr lehrreich und erfolgreich, als auch von viel Freude erfüllt. Ich werde diese Periode immer in positivster Erinnerung behalten.

Weiterhin danke ich meiner Familie, allen voran meinen Eltern Rita und Giovanni, die auf ihre jeweils individuelle Art als wunderbare Vorbilder dienten und Bildung immer in den Vordergrund stellten. Zudem danke ich nicht zuletzt meiner Frau Nevin, die meinen Weg immer unterstützte und mir moralische Stärke verlieh.

Patrizio Vanella

Zusammenfassung

Die Prognose demografischer Prozesse ist in vielen Bereichen von großer Bedeutung, um sich auf diese Entwicklungen im Rahmen von Planungsprozessen frühzeitig einstellen zu können. In den meisten Fällen werden dafür anstelle echter Prognosen deterministische Projektionen herangezogen. Diese unterliegen einigen Limitationen, welche im Verlaufe dieser Dissertation diskutiert werden.

Die Arbeit stellt ein Rahmenwerk für die stochastische Prognose demografischer Prozesse und darauf aufbauender Probleme, wie die Versicherungsnachfrage, dar. Der Fokus der Arbeit liegt dabei auf der demografischen Entwicklung in Deutschland und ihrer Konsequenzen für die gesetzliche Rentenversicherung und die Pflegeversicherung. Der Beitrag besteht aus sieben Kapiteln, die für sich genommen wissenschaftliche Papiere darstellen, die individuell entweder bereits in wissenschaftlichen Fachzeitschriften oder –büchern publiziert wurden oder sich derzeit im Begutachtungsverfahren befinden. Nichtsdestotrotz ist die Zusammenstellung und Anreihung der einzelnen Kapitel dergestalt, dass diese schlüssig verbunden und logisch aufeinander aufbauend sind.

Die Zielgruppen dieser Arbeit sind sowohl Bevölkerungswissenschaftler und Forscher in angrenzenden Disziplinen, als auch Abteilungen in Statistikämtern oder Lebensversicherungsunternehmen, die Prognosen für die Bevölkerung oder Subpopulationen erstellen. Insbesondere sollen den Lesern die beiden Primärmethoden Hauptkomponentenanalyse und Zeitreihenanalyse nähergebracht werden, die außerhalb der akademischen Forschung selten praktische Anwendung finden, insbesondere für Prognosen. Das mag darauf zurückzuführen sein, dass Modellierer die Methoden entweder überhaupt nicht kennen oder ihnen die praktische Erfahrung in der Anwendung dieser auf reale Daten fehlt.

Die Dissertation adressiert diese Probleme, da das besondere Augenmerk auf einer verständlichen und nachvollziehbaren Illustration der methodischen Anwendung und der Betonung der Anwendungsmöglichkeiten liegt, ohne dass die Arbeit zu technisch wird. Dennoch dürften die konkreten Prognosen auch für diejenigen von Interesse sein, die an der zukünftigen Entwicklung der Bevölkerungsstruktur und der daraus entstehenden Herausforderungen für das Sozialversicherungssystem interessiert sind. Auch Prognostiker aus anderen Disziplinen könnten an der Dissertation Gefallen finden, da viele Probleme in der Prognostik realer Prozesse genereller Natur sind und nicht nur in der Demografie auftreten.

Schlagworte: Demografie, Sozialversicherung, Prognostik, Stochastik, Multivariate Methoden, Zeitreihenanalyse

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1 Background

The development of the structure and size of the population is of great importance in many areas of life. On a global scale, population is increasing due to high birth numbers, which are prevalent in the less developed parts of the world. The level of period fertility is well covered by the total fertility rate (TFR). The TFR is the sum of all age-specific fertility rates (ASFRs) over a year and is therefore a synthetic measure representing the average number of children a cohort of women would conceive if the ASFRs were to remain constant in the long term (Bongaarts and Feeney 1998: 271 – 272). Whereas replacement-level fertility[1] is approximately 2.1 births per woman (Bongaarts and Feeney 1998: 285), the TFR in Africa is approximately 4.7 for the period 2010–2015 (United Nations 2015: 3). The estimated total world population has more than doubled since 1970 (United Nations 2017: 46), which has had a massive impact on the environment, since the human population is spatially expanding for the purposes of living and food production, thus reducing natural parts of the landscape (Food and Agriculture Organization of the United Nations 2016). Along with a higher degree of industrialization, waste has also increased, which has led to a large disposal of waste in woods and oceans. According to recent studies, the total weight of garbage in the oceans by the mid-21st century is expected to exceed the weight of the fish (World Economic Forum et al. 2016: 17).

From a European perspective, the latter developments greatly impact the Mediterranean countries as well. Apart from these, the demographic trends affecting Europe are of a different nature. The more developed countries in Europe are experiencing fertility levels that are significantly lower than the replacement level (United Nations 2015: 3). In addition to a low fertility level, the more developed countries have been experiencing significant decreases in mortality for decades. Figure 1 illustrates the development of life expectancy in Europe and Central Asia since 1960, according to the data from the World Bank.

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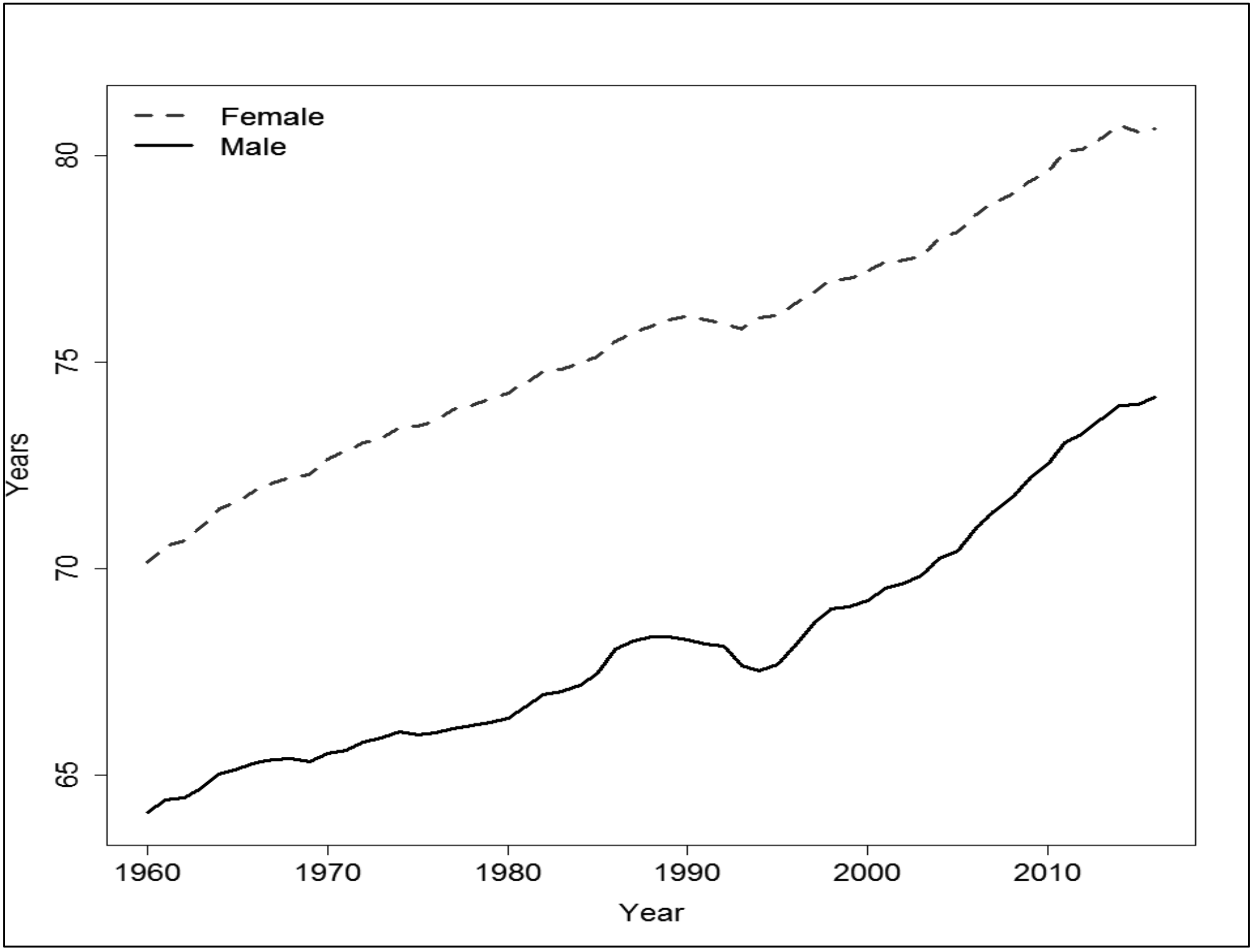
[1] The fertility level needed to keep the population, ceteris paribus, at a stable level.

Figure 1. Life Expectancy at Birth for Europe and Central Asia by Sex 1960-2016



Sources: The World Bank DataBank 2018; own design

These developments pose great challenges to the respective societies. Low fertility and mortality result in aging and, ceteris paribus (c.p.), in the decline of the population (Fuchs et al. 2018: 44 – 49). The preferences and needs of elderly people may differ greatly from those of the younger population. This may lead to large changes in the power of certain political parties, because elderly people vote for the parties that best represent their individual positions. Therefore, the population structure may have an enormous impact on a country's politics (Bujard 2015: 149 – 150). The structure and size of the population also have a large impact on the housing market, since elderly people often prefer very different places to live in comparison to the younger population (Meng et al. 2017). Moreover, the labor market is immensely affected by the structure of the population. Demographic change leads to a decrease in the working-age population (Fuchs et al. 2018: 51 – 53), whereas the number of pensioners increases (European Union 2018: 3 – 8).

Population aging affects individual risk as well, impacting both social and private insurance. For instance, the risk of being involved in traffic accidents increases after the age of 45 (Stamatiadis and Deacon 1994: 447 – 458). Naturally, the biometric risks of mortality (Thatcher et al. 1998: 74 – 104), morbidity (Salive 2013: 76 – 77) and frailty (Fuino and Wagner 2018: 56 – 57) increase at higher ages. These trends impact the risk management of both private life insurance (Gatzert and Wesker 2014: 57 – 60) and social insurance. Bismarck-type social security systems are especially greatly affected by the aging of the population, since in these systems, the working-age population transfers social security payments to finance the contemporaneous population in need (Graf von der Schulenburg and Lohse 2014: 427 – 430)[2]. In particular, statutory pension systems and long-term care insurance are greatly affected by an aging population. In developed countries that use the Bismarckian system, given the demographic developments explained earlier, financial pressure is expected to increase even further in the future due to demographic developments (see, e.g., Wilke 2009; Bowles 2015).

The difference between births and deaths constitutes the natural population growth, which is becoming negative for the type of countries described above, leading to a c.p. shrinkage of the population. That development might be counterbalanced by a positive net migration of working-age people. International migration is frequently the target of controversial discussions. The effects of international migration on social security must be investigated very cautiously and from a long-term perspective, since the effects of migration on the labor market may take time (see, e.g., Brücker 2018 on this). Moreover, increased cross-border migration often brings societal concerns about job security (Schmieder 2016: 1) and especially raises questions about increasing security risks regarding crime (Walburg 2018) and terrorism (Reimann 2016), if the immigrants originate from less developed and less egalitarian countries.

Forecasting demographic processes is therefore of high importance for many areas. Naturally, we are interested in the future outlook so that we can react to expected developments early. Companies need to adjust their production to the changing demands of the costumers (McNair et al. 2012: 32 – 43), and the demand for housing has to be planned in the long-term for possible new building projects (Mulder 2006: 403; Henger et al. 2017). The labor market has to adjust to the future structure of the labor force (Fuchs et al. 2018: 48 – 54). Politicians need to be

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[2] In fact, the total amount of social security payments is regularly subsidized by additional tax-funded statutory payments and contributions by employers.

informed about the future population structure for the purposes of adjusting social or demographic policy (Lee and Tuljapurkar 1998: 393 – 394; Deschermeier 2015: 98; Bundesministerium des Innern 2017). Deterministic projections, rather than real forecasting, are mostly conducted (see, e.g., Pötzsch and Rößger 2015: 13 – 53). These are valuable as well, for instance, for estimating the impact of certain policies under specified assumptions or for measuring the sensitivity of certain variables to changes in other factors (see, e.g., Wilke 2009 on the sensitivity of the German statutory pension system and Bowles 2015 for similar studies on the statutory long-term care insurance in Germany). Deterministic modeling gives a framework that is easy to implement and understand even for nonexperts in statistical modeling (Deschermeier 2015: 98 – 100).

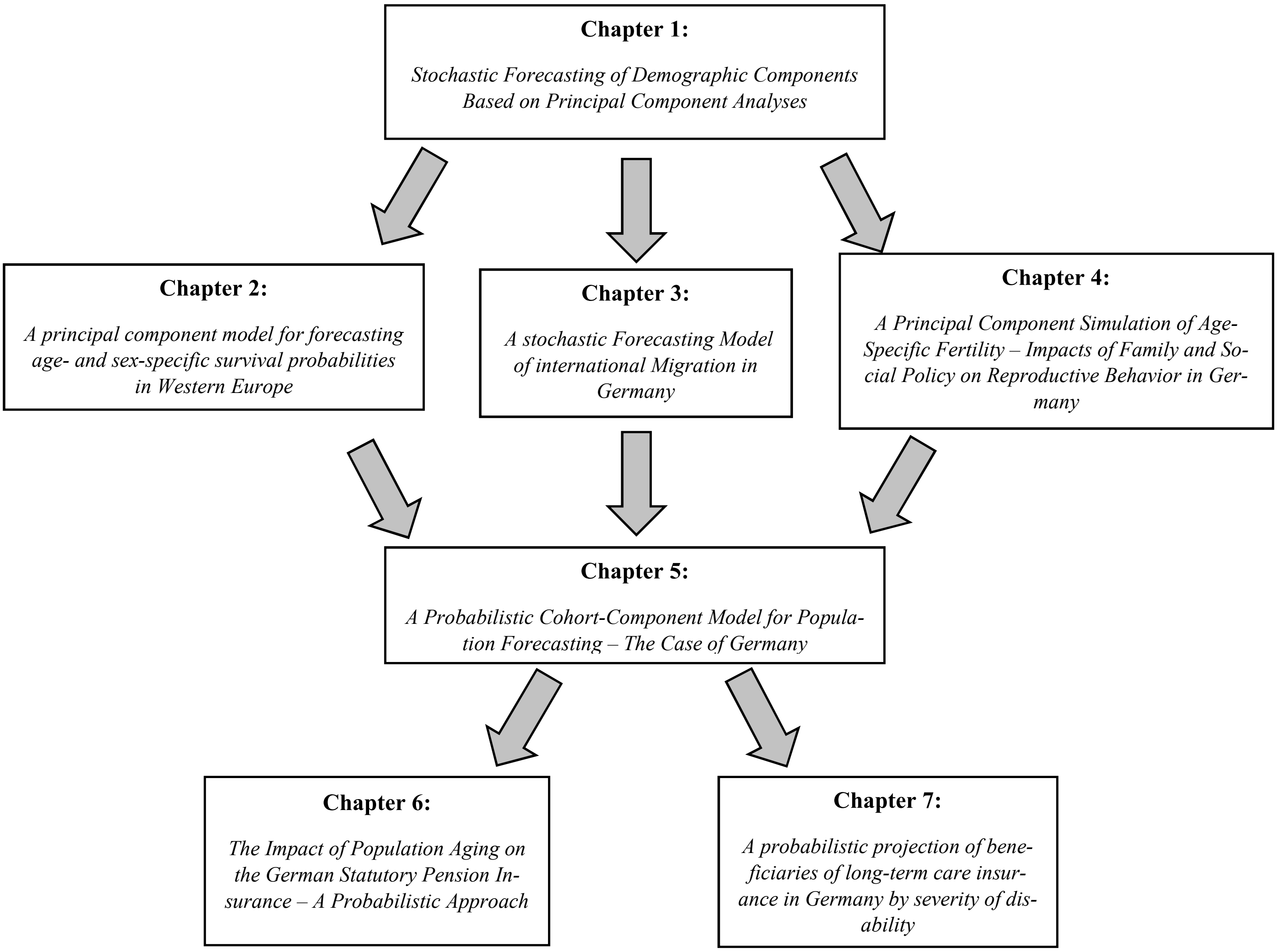
Conversely, one might argue against the use of deterministic projections, since they suffer from some limitations. First, deterministic projections impose rather hard assumptions about future development. From a statistical point of view, the respective trajectories are highly improbable (Keilman et al. 2002: 410). Second, the analysis is limited to a rather small number of scenarios (see, e.g., Pötzsch and Rößger 2015: 13 – 53) that do not sufficiently consider the future uncertainty. Third, the specified scenarios are seldom quantified with the likelihood of taking place. Fourth, the projections are not necessarily rooted in statistical data but rather in the judgment of a limited number of experts. These judgments have a tendency to be rather subjective and therefore have some bias because of the personal opinions of the persons interviewed. Even good experts tend to perform worse on judgment-based forecasting than forecasts that are conducted on solid statistical ground. Moreover, even when they have good ideas about realistic future developments, experts experience difficulties in transforming their subjective assessments into probabilities (Lee 1998: 156 – 170). While there exist statistical methods that include qualitative judgment in forecasting along with purely quantitative modeling (e.g., Lutz et al. 1998), this contribution is restricted to quantitative methods, following Lee’s argumentation.

2 Structure of the Dissertation

This dissertation deals with stochastic forecasting, with special emphasis on issues in demography and social insurance. The main focus is on Germany, but the methods are applicable to other countries or alternative regional units, such as federal states or cities.

This contribution consists of seven chapters that are individual papers either already published in scientific journals or books or currently under consideration for publication in scientific journals. Nevertheless, the individual parts are concisely connected, such that they follow a logical structure. The dissertation starts with a methodological paper called *Stochastic Forecasting of Demographic Components Based on Principal Component Analyses*, which presents the main features of principal component analysis (PCA) and time series analysis (TSA) in an applied way. The main focus is on applying the presented statistical methods to demographic forecasting, specifically the forecasting of age-specific survival rates; nevertheless, the methods are applicable to a wide range of topics as well. The paper introduces the abovementioned methods mathematically while focusing on a clear and an applied description. The goal is to help non-mathematicians understand the methods and explain how to use them practically and to present the advantages of the methods to those conducting statistical analysis in a nonscientific environment, such as statistical offices or political consulting. The paper lays the foundation for the rest of the dissertation, since PCA and TSA are used in nearly all of the remaining chapters. Chapter 2: *A principal component model for forecasting age- and sex-specific survival probabilities in Western Europe* applies the methods introduced in Chapter 1 to the joint forecasting of age- and sex-specific survival rates (ASSSRs) in 18 European countries until 2070. The ASSSRs are cumulated to probabilistic forecasts of the median life span as a summary statistic, giving an outlook on the immense developments in longevity that Western Europe will face for the next five decades. The paper clarifies the advantages of PCA by showing how a very complex forecasting problem with a magnitude of correlated variables can be effectively reduced to a small number of dimensions without losing much information. Moreover, the contribution shows that mortality trends in Europe are highly correlated. These common trends can be covered well by the PCA. Chapter 3: *A stochastic Forecasting Model of international Migration in Germany* discusses the problems in migration forecasting and the possible solutions to deal with these, as migrational processes are caused by many external political, economic, societal and environmental factors that are themselves extremely difficult to predict. Moreover, the data on migration are of lower quality and less reliable, which is pointed out in this paper. Germany is a very interesting example in migration forecasting, as it is one of the world's more important target countries for international migration due to its size and economic and political stability. The paper proposes a PCA-based approach for forecasting age-, sex- and nationality-specific net migration (ASNSNM) into Germany, which indirectly includes the impact of crises and economic trends on the migration flows through the modeling procedure. The model is applied to forecast net migration into Germany until 2040. The forecast illustrates the great uncertainty associated with future migration developments, showing the advantage of probabilistic methods in comparison to deterministic ones, which cannot sufficiently cover these risks. Chapter 4: *A Principal Component Simulation of Age-Specific Fertility – Impacts of Family and Social Policy on Reproductive Behavior in Germany* deals with the complexity of fertility forecasting. Fertility forecasts are commonly done using very restrictive models with strong underlying assumptions. Whereas discussions about the impact of a pro-natalistic family policy on a population's reproductive behavior often take place and are controversial in low-fertility countries, the effects are very rarely included in fertility forecasting. The study appears to confirm the results of past studies for some family policy measures. The impact of pension policy on fertility is discussed as well. Embedded into the PCA framework, conditional forecasts of the fertility level, represented by the TFR, are estimated under different assumptions regarding future policy. The model therefore allows a stochastic sensitivity analysis of the impact of policy innovations on age-specific fertility, quantifying the actual change in entrance probability of a variety of future scenarios. The results of the forecasts presented in Chapters 2-4 are then merged in Chapter 5: *A Probabilistic Cohort-Component Model for Population Forecasting – The Case of Germany*, where the probabilistic forecasts of the ASSSRs, the ASNSNM numbers and the ASFRs are combined into a fully probabilistic cohort-component forecast of the age- and sex-specific population in Germany until 2045, stochastically visualizing the future make-up of the population in Germany. The paper reveals the great uncertainty regarding the future population size and structure of the younger population, underlining once more the limitations of the less flexible deterministic projection approaches. Moreover, a further increase in the elderly population is confirmed, which is an important result for the labor market as well as social and life insurance, among other fields. In contrast to many past studies, we see that a decrease in the population size over the next two decades is rather improbable. This population forecast is then used as the basis for two applications in social insurance. Chapter 6: *The Impact of Population Aging on the German Statutory Pension Insurance – A Probabilistic Approach* quantifies age- and sex-specific risks of claims of old-age and disability pensions for persons aged 60 and older. Proposing a PCA-TSA model for these risks, the estimation of future pension risks is done probabilistically and is combined with the population forecast presented in Chapter 5, resulting in stochastic forecasts of old-age and disability pensioner numbers through 2040, while disaggregating by sex, age and pension type. The model takes the different legal retirement ages and their change over time into account as well. The results reveal large increases in pensioner numbers by 2040, implying the need for further pension reforms, as the already introduced increases in legal retirement ages obviously do not suffice to stabilize the balance between the working-age population and the pensioners. The paper offers the opportunity to conduct further studies on the topic, especially financial analyses. Chapter 7: *A probabilistic projection of beneficiaries of long-term care insurance in Germany by severity of disability* proposes a stochastic framework to estimate the future impact of the population trends on the prevalence of frailty and the future demand for services from statutory long-term care insurance (GPV) via a theory-based Monte Carlo simulation approach. As there exist no valid time series data on long-term care in Germany, the paper discusses the theoretical link between trends in mortality and morbidity. Based on this connection and the estimated mortality trends forecast in Chapter 2, the future development of frailty is simulated. The study reveals great increases in frailty risks and gives a stochastic basis for estimating the future demand for care nurses and the financial challenge the GPV will be facing in the future. The results show that the development of the population is of the utmost importance for the future numbers of cared-for persons, whereas the actual change in age-specific care risks does not affect the GPV to a large extent. The results imply a massive future demand for care nurses and financial compensation by the GPV. The structure of the dissertation is illustrated in Figure 2.

Figure 2. *Structure of the Dissertation*



3 Intended Goals and Audience

The present contribution addresses both scientists doing their research in population studies or related areas and units in statistical offices or life insurance companies doing forecasts of a certain population or subpopulation. The contribution may be understood as a guide on methods in applied population forecasting and is therefore of a more general nature. In particular, the two “leading methods” of PCA and TSA are, outside of academic science, rarely used in practical applications, especially for forecasting, either because they are not known at all or because the modelers lack practical training in applying them to real data. The dissertation addresses both problems, since the special emphasis is on presenting an understandable and a comprehensible illustration of the methods to stimulate their use and give suggestions on how to use them, without becoming too technical. Nevertheless, concrete forecasts may be of interest to those people interested in the future course of the population structure and the resulting challenges for the German social insurance system. Forecasters from different disciplines may find the dissertation interesting reading material as well. The methods are not restricted to forecasting but are useful for explorative studies as well. Indeed, they were originally developed for these very purposes, but forecasting appears to be the most sophisticated statistical problem, as may become clear over the course of this contribution. Successful application of the methods for forecasting requires the ability to apply the methods in cross-sectional analysis.

In addition to presenting the application of the statistical methods and discussing the concrete problems that demographers are confronted with in practical modeling, this work addresses a rarely discussed problem in detail: the uncertainty of the future. Whereas deterministic models basically ignore this problem, the preceding stochastic approaches tend to underestimate the stochasticity of the future course of a certain variable. Stochasticity in the case of population and social security forecasting is a greatly emphasized point, which will be discussed in detail over the course of the dissertation, starting with Chapter 2. Therefore, the contribution motivates a deeper consideration of risk in predictions of the future and discusses the risks further, making these studies and their limitations even more transparent.

4 Discussion and Outlook

The "Demography-Social Insurance Nexus" is an extremely complex system that is greatly influenced not only by pure population trends but also by political decision making, risks in global politics and society, global and national economic impacts and environmental trends. The mentioned system is obviously highly probabilistic, which once again stresses the importance of using stochastic models if we are to be able to make predictions about the future. As the inherent complexity of this system is extremely hard to forecast, it is impossible to develop a perfect forecasting model for all these processes. As the abovementioned external factors that influence the population, labor market and social insurance are very hard to forecast (and which would exceed the scope of this dissertation), some of the variables have been modeled only implicitly in the models within the PCA. Other variables, such as political measures, have been directly included in econometric forecast models that have a high potential for misspecification if the underlying data are scarce, which is the fundamental problem of time series data, especially in demography. The stochasticity once more alleviates this problem, as the base assumptions of the models are relativized to some extent.

The dissertation relies on frequentist methods from statistics. These have a major limitation, as they exclusively base their results on the underlying data. This might be advantageous if good data are available, as the resulting estimates are derived in a very objective manner. Many modern applications in demography are conducted based on Bayesian statistics, which is able to incorporate additional knowledge or expert assessment in the analysis. This might be vital in cases where data are scarce or even nonexistent. Therefore, Bayesian approaches could represent possible extensions or alternatives to the methods presented in this dissertation. As Bayesian statistics tends to be relatively subjective and actually needs a very good understanding of the research subject by the modeler, these approaches can be quite harmful if the prior information plugged into the models is not of a good enough quality. Therefore, the approach chosen in this contribution might be safer and more conservative. Bayesian approaches might very well present reasonable extensions to the models developed in this contribution, especially in cases of long-term care forecasting, international migration and fertility, where the data availability or quality is limited.

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6 Chapters of the Dissertation

Chapter 1. Vanella, P. 2018. "Stochastic Forecasting of Demographic Components Based on Principal Component Analyses." *Athens Journal of Sciences* 5(3): 223 – 246.

Chapter 2. Vanella, P. 2017. "A principal component model for forecasting age- and sex-specific survival probabilities in Western Europe." *Zeitschrift für die gesamte Versicherungswissenschaft (German Journal of Risk and Insurance)* 106(5): 539 – 554.

Chapter 3. Vanella, P.; Deschermeier, P. 2018. **"**A stochastic Forecasting Model of international Migration in Germany." *Familie – Bildung – Migration. Familienforschung im Spannungsfeld zwischen Wissenschaft, Politik und Praxis. Tagungsband zum 5. Europäischen Fachkongress Familienforschung*, edited by Kapella, O.; Schneider, N.F.; Rost, H. (pp. 261-280). Opladen, Berlin, Toronto: Verlag Barbara Budrich.

Chapter 4. Vanella, P.; Deschermeier, P. 2019. "A Principal Component Simulation of Age-Specific Fertility – Impacts of Family and Social Policy on Reproductive Behavior in Germany.“ *Population Review* 58(1): 78 – 109.

Chapter 5. Vanella, P.; Deschermeier, P. 2020. "A Probabilistic Cohort-Component Model for Population Forecasting – The Case of Germany." *Journal of Population Ageing, online first:* doi: 10.1007/s12062-019-09258-2.

Chapter 6. Vanella, P.; Rodriguez Gonzalez, M.A.; Wilke, C.B. "The Impact of Population Aging on the German Statutory Pension Insurance – A Probabilistic Approach." *Journal of Labour Market Research [Submitted].*

Chapter 7. Vanella, P.; Heß, M.; Wilke, C.B. 2020. "A probabilistic projection of beneficiaries of long-term care insurance by severity of disability." *Quality & Quantity. International Journal of Methodology, online first:* doi: 10.1007/s11135-020-00968-w.

Chapter 1

Stochastic Forecasting of Demographic Components Based on Principal Component Analyses

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Stochastic Forecasting of Demographic Components Based on Principal Component Analyses

By Patrizio Vanella[*]

Adequate forecasts of future population developments that are based on cohort-component methods demand an age- and sex-specific analysis; otherwise, the structure of the future population cannot be specified correctly. Age-specific demographic measures are both highly correlated and highly dimensional. Thus, a methodology that not only considers the correlations between the random variables but also reduces the effective dimensionality of the forecasting problem is needed: principal component analysis serves both purposes simultaneously. This study presents principal component analysis, from a mathematical-statistical perspective, to users from the field of population studies. Furthermore, important aspects of time series analysis, which are vital for an accurate stochastic forecast, are explained. The application is illustrated via the simultaneous projection of selected age- and sex-specific survival rates with projection intervals for Germany, Italy, Austria, and Switzerland.

Keywords: *Forecasting, Multivariate Methods, Principal Component Analysis, Quantitative Population Studies, Time Series Analysis.*

Introduction and Motivation

Official population projections are commonly conducted on the basis of deterministic cohort-component models (Alho, 1990; Pötzsch and Rößger, 2015). Stochastic forecasts are favorable compared to deterministic approaches (Keilman and Pham, 2000) since, in addition to the most probable scenario, they identify and quantify with respective probabilities infinitely many possible scenarios. Stochastic models may also be based on the components of fertility, migration and mortality. Autocorrelation and cross-correlation must be considered in future forecasts because there are correlations between the different age groups and genders as well as between observations at different points in the time series. Therefore, this paper first presents a short introduction to principal component analysis (PCA), where the focus is on explaining its use and a short illustration of its functionality.

Moreover, important concepts of time series analysis (TSA) are considered, with the explanation restricted to the aspects that are required for practical applications, with no claim on completeness in mind. This contribution therefore may be understood as a guide for statistical offices or demographic research institutes; the concrete projections serve only illustrative purposes and should not be mistaken as actual forecasts for future development. The methods are

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not restricted to population studies; forecasters from various disciplines might find this contribution interesting reading material. The explanation of the method is the focal point of the paper, especially the implementation of the illustrated statistical concepts of modeling and forecasting the components of demographic developments. The method is demonstrated by simulating selected age- and sex-specific survival rates (ASSSRs) for Germany, Italy, Austria, and Switzerland, but the model could be applied to other countries or regions as well. Application to other problems in demography or in other fields is not shown but may be done in principle the same way it is presented in this contribution.

Introduction to Principal Component Analysis

Detailed population forecasts by sex and age show a high degree of dimensionality, since for two genders up to 116 age groups should be investigated (see Vanella, 2017). Moreover, these quantities are highly correlated with each other. Forecasters have to address these two problems with appropriate methods. PCA is recommended because it simultaneously addresses both problems. PCA was originally developed by Pearson (1901) geometrically and involves applying an orthogonal transformation to the original variables into the same number of new, uncorrelated variables, which are labeled principal components (PCs). The method is especially well suited for situations in which no causal relationship between the variables is quantified, as is the case for regression analyses. Therefore, PCA is especially appropriate for forecasting age- and sex-specific measures in a demographic context. Each PC is a linear combination of N original variables. Let $S_{i,t}$ be the ith ASSSR in period t. Then, the jth PC $P_{j,t}$ in the same period is calculated by the following (Chatfield and Collins, 1980):

$$P_{j,t} = \sum_{i=1}^{N} e_{i,j,t} S_{i,t} =: \overrightarrow{e_{j,t}^{T}} \overrightarrow{S_t}, \tag{1}$$

where $e_{i,j,t}$ can be interpreted as a correlation coefficient between the ith ASSSR and the jth PC in period t.

Within the PCA framework, the PCs are deduced in decreasing order of the magnitude of the total variance they explain. This means the first PC explains the largest share of the variance in the original variables. Through the transformation, a complex system with many variables can effectively be reduced to few dimensions since the first few PCs explain the majority of the variance.

The first principal component (PC 1) is chosen to explain as much of the variance as possible. Statistically, this means that the coefficients (or *loadings*, as they are also called in this context) of the first linear combination are adjusted to maximize the amount of covariance in the original variables that is explained. The calculation is now illustrated with ASSSRs. For simplicity, the loadings are assumed to be invariant through time;[1] thus the index t is omitted. Given

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[1] Hyndman and Ullah (2007) have proposed a different approach with variable loadings.

the covariance matrix of $\boldsymbol{S}$ (the matrix of all ASSSRs), labeled $\boldsymbol{\Sigma}$, the variance of PC 1 is given by equation (2):

$$Var[P_1] = Var\left[\overrightarrow{e_1^T}\boldsymbol{S}\right] = \overrightarrow{e_1^T}\boldsymbol{\Sigma}\overrightarrow{e_1}. \tag{2}$$

The vector $\overrightarrow{e_1}$ can be chosen arbitrarily. To reach a unique solution of the maximization problem, a restriction for the elements of $\overrightarrow{e_1}$ (also called the *eigenvector*) must be stated. Normalizing $\overrightarrow{e_1}$ to a length of one ensures an orthogonal transformation. A vector has length one if its scalar product with itself is one (Handl, 2010):

$$\overrightarrow{e_1^T}\overrightarrow{e_1} = 1. \tag{3}$$

Due to the method of Lagrange multipliers, the stationary points[2] of a function $f(\vec{x})$ under condition $g(\vec{x}) = c$ can be identified through the identification of the stationary points of the affiliated Lagrange function $\mathcal{L}(\vec{x},\lambda)$. The Lagrangiana is defined as follows:

$$\mathcal{L}(\vec{x},\lambda) = f(\vec{x}) - \lambda[g(\vec{x}) - c] \tag{4}$$

Therefore, the maximization problem for the variance can be solved by finding the stationary point of the following Lagrange function:

$$\mathcal{L}(\vec{x},\lambda) = \overrightarrow{e_1^T}\boldsymbol{\Sigma}\overrightarrow{e_1} - \lambda[\overrightarrow{e_1^T}\overrightarrow{e_1} - 1]. \tag{5}$$

Accordingly, the stationary point is determined as follows:

$$\begin{gathered} \frac{\partial\mathcal{L}(\overrightarrow{e_1},\lambda)}{\partial\overrightarrow{e_1}} = 2\boldsymbol{\Sigma}\overrightarrow{e_1} - 2\lambda\overrightarrow{e_1} = 2(\boldsymbol{\Sigma} - \lambda\boldsymbol{I})\overrightarrow{e_1} = \vec{0} \wedge \\ 1 - \overrightarrow{e_1^T}\overrightarrow{e_1} = 0. \end{gathered} \tag{6}$$

Here, $\boldsymbol{I}$ is an identity matrix, which for $\overrightarrow{e_1}$, consisting of p elements with dimensions $p \times p$, becomes the following:

$$\boldsymbol{I} := \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix}. \tag{7}$$

The first equation in (6) indicates that the matrix on the left-hand side of the equation must be singular. Since $\overrightarrow{e_1}$ must not be a null vector, guaranteeing a nontrivial solution, it follows that the determinant of the matrix $\boldsymbol{\Sigma} - \lambda\boldsymbol{I}$ has to equal zero (Chatfield and Collins, 1980):

$$|\boldsymbol{\Sigma} - \lambda\boldsymbol{I}| = 0. \tag{8}$$

This is illustrated with a practical example. From age- and sex-specific data on deaths and the end-of-year population, provided by the federal statistical offices of Germany, Italy, and Austria and complemented by downloads from the Human Mortality Database and the Eurostat Database, the ASSSRs for Germany, Italy, Austria, and Switzerland are calculated for the years 1952-2016 (Destatis, 2005, 2015a, 2015b, 2015c, 2016, 2017a, 2017b, 2018a, 2018b; Eurostat, 2018; Human Mortality Database, 2018a, 2018b, 2018c, 2018d; Istat, 2018a, 2018b, 2018c,

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[2] Those might be either local minima, maxima or saddle points.

2018d, 2018e, 2018f; STATcube, 2018). For illustration, PCA is performed on the covariance matrix of the logit transformed ASSSRs of 25-year-old (cohort based) males for the four mentioned countries for the years 1952-2016. Survival rates can only take values greater than zero and less than one, so their projections are made through simulation of their logits. A logistic transformation of an ASSSR *s* can be calculated as follows (Johnson, 1949):

$$logit(s) = \ln\left(\frac{s}{1-s}\right) \tag{9}$$

The transformation leads to new unrestricted variables, whereas the underlying ASSSRs cannot take simulation values outside the open interval (0,1) in the forecast. After simulation, the results must be transformed back through the inverse logit to obtain the final ASSSR trajectories.

The solutions of the optimization problem in this case are approximately $\lambda_1 \approx 0.617, \lambda_2 \approx 0.031, \lambda_3 \approx 0.013$, and $\lambda_4 \approx 0.007$, which are also called the eigenvalues (EWs) of the covariance matrix. The sum of the EWs is equal to the sum of the covariance of the original variables; therefore, the EWs are sorted in decreasing order. As mentioned earlier, one of the two reasons to apply PCA is to reduce the original statistical problem into a small number of variables that explain as much of the covariance in the original variables as possible. This means the first eigenvalue (EW) represents the variance that is explained by PC 1. Therefore, we can derive that PC 1 explains approximately 92.4% of the overall covariance of the four time series (TS), whereas PC 1 and PC 2 already explain over 97%.

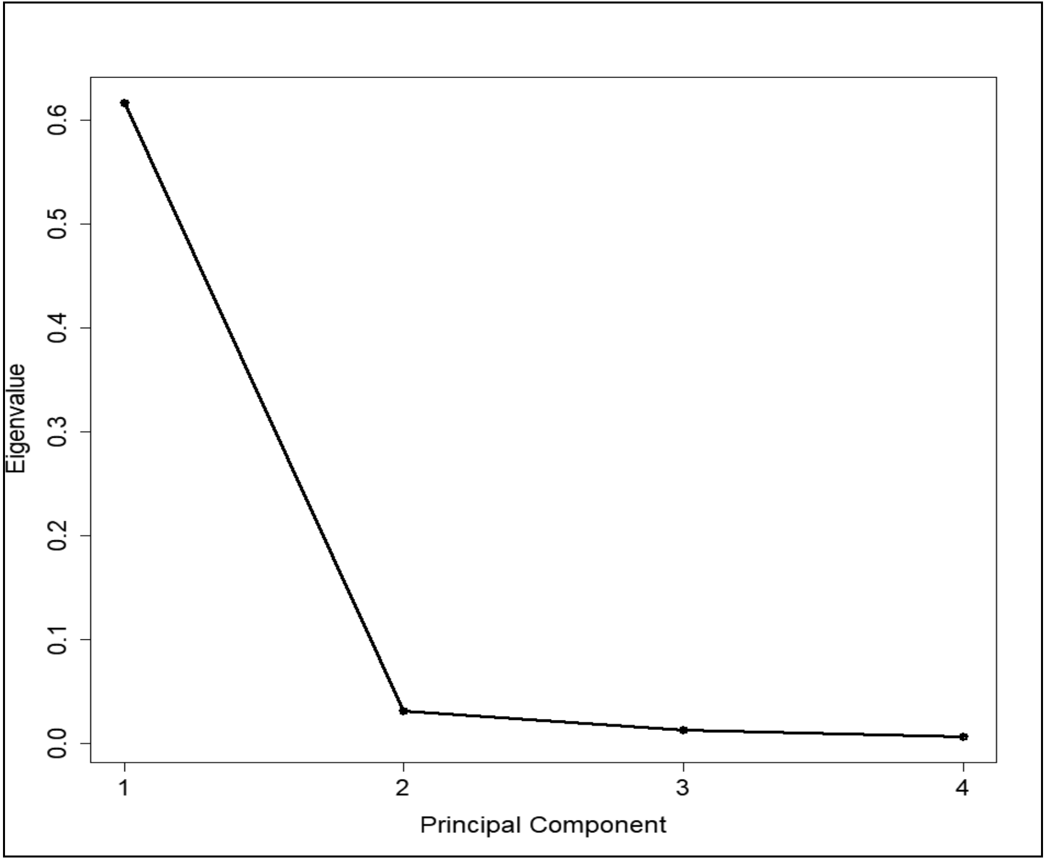
Plugging the EWs into (6) individually leads to the respective EVs, e.g., EV 1:

$$\overrightarrow{e_1} \approx \begin{bmatrix} -0.522 \\ -0.414 \\ -0.539 \\ -0.516 \end{bmatrix}.$$

PC 1 is negatively loaded with all of the four logit-ASSSRs, therefore indicating a type of general mortality index, similar to the Lee-Carter index (Lee and Carter, 1992). The associated PCs can be easily derived by (1).

One important question is the determination of the number of PCs for the analysis. There is no trivial answer; the determination of the number of PCs to use is subjective. Nevertheless, criteria have been proposed to simplify the decision. One possibility is to define a minimum percentage of the variation to be explained. If we would, e.g., target covering at least 95% of the variance in the ASSSRs, we would take the first two PCs into our model. Another common method to select the number of PCs is graphically analyzing the EVs of the covariance matrix with a scree plot (see Handl, 2010), as shown in Figure 1.

Figure 1. *Scree Plot for the Principal Components*



Only the PCs that lie on the left-hand side of the elbow are included in the model; moreover, there is no clear consensus whether the PC at the elbow itself should be included as well. In this case, the scree plot suggests one or two PCs. From a practitioner's point of view, it is generally worthwhile to include the PC at the elbow when making forecasts; otherwise, in many cases, a relatively large share of the variance would be ignored, leading to biased results when constructing prediction intervals (PIs). This result can be observed in many practical applications. Vanella (2017) proposed a simulation method that includes the uncertainty arising from omitting most of the PCs to prevent excessively narrow forecast PIs. This topic is not considered further in this paper.[3] Kaiser's and Jolliffe's criteria are additional alternatives. Kaiser's criterion suggests using only PCs with EWs that are larger than the mean EW (Handl, 2010). Jolliffe (2002) proposed 70% of the mean as the lower limit. Nevertheless, the choice of criterion is subjective.

The focus of this section was the general description of PCA in a semimanual practical application for a better understanding of the method. Nevertheless, PCA can be performed relatively easily using **R**[4].

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[3] For further reading on this issue, see the aforementioned article.

[4] The standard commands **prcomp** and **princomp**, which are pre-installed, can be used for this.

Main Features of Time Series Analysis

In the section, some aspects of TSA, which are highly relevant in the context of PC forecasting, will be explained.

A TS is a variable that generates one observation in each period. The fundamental concept of modern TSA is stationarity, which will be explained briefly. The TS of the ASSSR (in period t) of the 25-year-old males in Germany is defined as a_t. Stationarity (also called weak stationarity in the literature) is sufficiently defined by two conditions: mean stationarity and auto covariance stationarity (Shumway and Stoffer, 2011). The mean stationarity is defined by the equality of the TS mean in each period:

$$E[a_s] = E[a_t] \; \forall \; s, t. \tag{10}$$

Autocovariance stationarity means the theoretical auto covariance between two observations of the TS does not depend on the point in time but on the length of the time interval separating the two observations:

$$Cov[a_{t+h}, a_t] = Cov[a_h, a_0] \forall \; t. \tag{11}$$

Autoregressive integrated moving average (ARIMA) models, developed by Box and Jenkins (Box et al., 2016), are of major importance for practical applications, as subsequently explained (Shumway and Stoffer, 2011). A moving average of order q (MA(q)) is defined as

$$a_t = \omega_t - \sum_{i=1}^{q} \theta_i \omega_{t-i}, \tag{12}$$

where ω_t is a stochastic nuisance parameter in period t, which in practical applications, is normally assumed to follow a Gaussian[5] distribution with a mean of zero and a variance σ^2:

$$\omega_t \sim \mathcal{NID}(0, \sigma^2). \tag{13}$$

The stationarity assumption is beneficial because stationarity allows the assumption that the nuisance parameter is identically distributed in each period. This assumption is especially helpful for running simulations. An MA(q) model thus starts from the premise that the current observation of the variables emerges exclusively as a weighted sum of the last q manifestations of the nuisance parameter and the error in the current period. In this notation, θ_i is the correlation coefficient of the TS with respect to the error in period $t - i$. θ_i is restricted between -1 and 1:

$$|\theta_i| < 1. \tag{14}$$

A feasible alternate representation for an MA(q) process is the *lag notation*, where *L* is the so-called *lag operator*[6]. The lag notation for an MA(q) process is as follows:

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[5] An alternative is to assume a Student's t-distributed nuisance parameter, as proposed by Raftery et al. (2014).

[6] Alternatively, some authors write about the *backshift operator*, which is the same.

$$a_t = \left(1 - \sum_{i=1}^{q} \theta_i L^i\right) \omega_t. \tag{15}$$

The exponent of L indicates which past period is being considered. For example, $L^q \cdot \omega_t$ signifies ω_{t-q}.[7]

Another common type of TS model is an autoregressive model of order p (AR(p)):

$$a_t = \omega_t + \sum_{j=1}^{p} \phi_j a_{t-j}, \tag{16}$$

or in lag notation:

$$\left(1 - \sum_{j=1}^{p} \phi_j L^j\right) a_t = \omega_t. \tag{17}$$

In an AR(p) model, the TS in period t is regressed on its previous p observations (taking the error in period t into account). In this case, $L^p \cdot a_t = a_{t-p}$. Similar to the MA(q) model,

$$|\phi_j| < 1. \tag{18}$$

AR and MA models can also be combined; the combination of an AR(p) model and an MA(q) model produces an ARMA(p,q) model, which is formally defined as follows:

$$a_t = \omega_t - \sum_{i=1}^{q} \theta_i \omega_{t-i} + \sum_{j=1}^{p} \phi_j a_{t-j} \tag{19}$$

or

$$\left(1 - \sum_{j=1}^{p} \phi_j L^j\right) a_t = \left(1 - \sum_{i=1}^{q} \theta_i L^i\right) \omega_t. \tag{20}$$

As mentioned previously, the stationarity assumption is fundamental for ARMA processes. The question is how to identify whether a TS is stationary; graphical analysis is recommended as the first step in the investigation. Figure 2 illustrates a simulated stationary TS[8].

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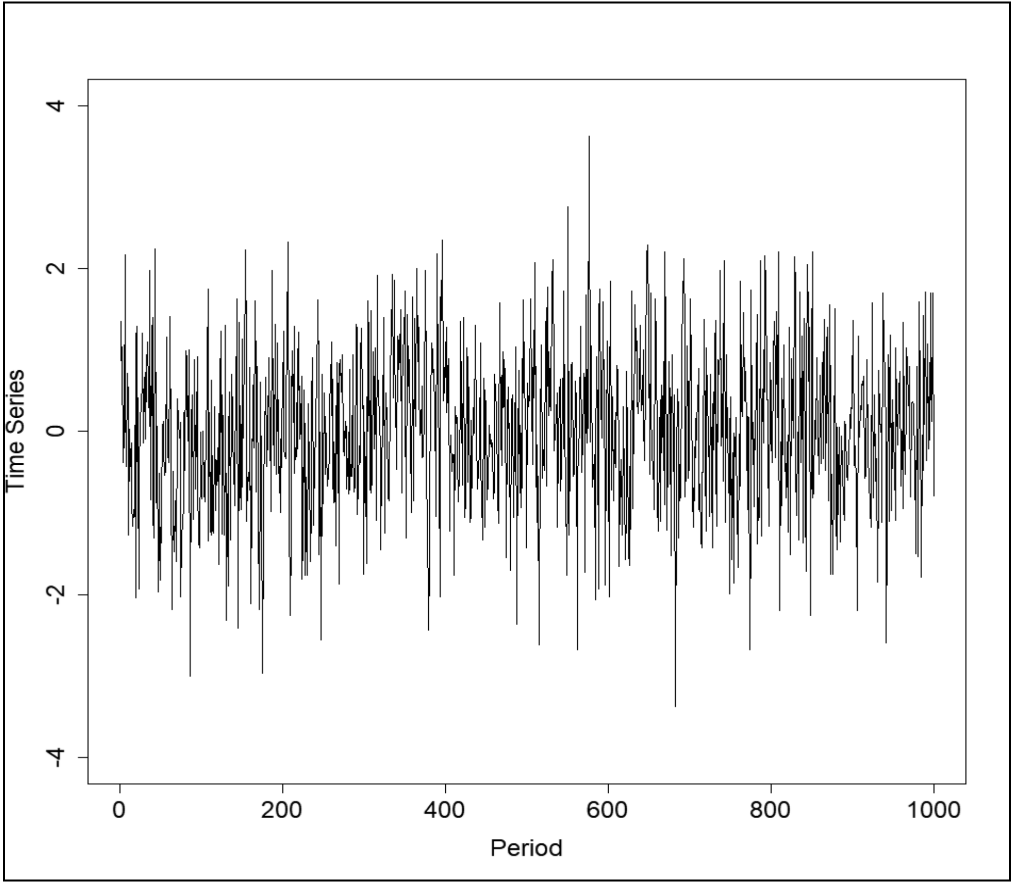
[7] In practical applications, one has to be careful about the explicit definition of the coefficients. Some statistical packages give a slightly different output, e.g., the output in ***R*** changes the sign of the coefficient relative to this contribution.

[8] The TS is generated by 1,000 computer simulations of a Gaussian random variable.

Figure 2. *Stationary Time Series*



It is clear that neither the mean nor the variance show trending behavior. Furthermore, the stationarity hypothesis should be confirmed using statistical tests, such as the augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. The standard ADF test checks the null hypothesis, i.e., whether for the following equation

$$a_t = \rho \cdot a_{t-1} \tag{21}$$

the condition

$$H_0 : \rho = 1$$

holds. This condition corresponds to a *random walk* process (Dickey and Fuller, 1979). Several variants of the test exist. One variant of interest is the one with the alternative hypothesis

$$H_1 : |\rho| < 1,$$

which corresponds to a stationary or asymptotically stationary process. The test statistic in this case is

$$\tau = \frac{\hat{\rho} - 1}{se(\hat{\rho})}, \tag{22}$$

matching the common Student's t-test. However, the test statistic is not compared to the quantiles of a t-distribution but to an empirical distribution produced by

Dickey and Fuller from Monte Carlo simulations (Fuller, 1996).[9] In the example, τ is approximately -10.31, which means that the null hypothesis is rejected at all major significance levels. The statistical evidence indicates stationarity of the TS.

By contrast, the KPSS test is a Lagrange multiplier test with a test statistic

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{SSR/T}, \tag{23}$$

where SSR is the sum of squared residuals of the regression, T is the number of periods and S_t is the sum of the residuals from the regression

$$a_t = \alpha + \beta t$$

until time t. The critical values for the underlying distribution were estimated by Kwiatkowski et al. (1992) through a Wiener process[10]. The KPSS test[11] checks the null hypothesis of stationarity for the TS. Large values lead to rejection of H_0. In the example, the test statistic is approximately 0.0625 for $H_0: \alpha = 0$ and approximately 0.0552 for $H_0: \beta = 0$, well below the critical values at all common confidence levels,[12] so the null hypothesis cannot be rejected in either case. Therefore, the KPSS test does not provide evidence against the assumption of stationarity for the random variable.

Another important test that should be considered is the ARCH-LM test for conditional heteroscedasticity. Given that our TS has the standard deviation σ_t in time *t*, the test for the equation

$$\sigma_t^2 = \alpha_0^2 + \sum_{i=1}^{p} \alpha_i^2 \epsilon_{t-i}^2$$

checks the null hypothesis

$$H_0: \alpha_i = 0 \; \forall \, i \in \mathbb{N}^+$$

where α_0^2 is some constant. If H_0 cannot be rejected, we find no evidence for heteroscedasticity in the TS[13] (Engle, 1982).

If the modeler concludes nonstationarity in the TS based on the statistical tests, a transformation is needed. In this case, it is commonly assumed that the TS was integrated, which is represented by the middle part of the ARIMA notation. A d-times-integrated TS in the simplest case is denoted as an ARIMA(0,d,0) process (Shumway and Stoffer, 2011):

$$(1 - L)^d a_t = \omega_t. \tag{24}$$

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[9] The ADF test is implemented in common statistics software, e.g., in ***R*,** using the command **adf.test** from the package **tseries** (see Trapletti and Hornik, 2018).

[10] The process grows each period by a stochastic value, which is drawn from a Gaussian random variable.

[11] The KPSS test is usually implemented in standard statistics software as well, e.g., in ***R*,** using **kpss.test** from the package **tseries**.

[12] For α=0.1, the critical value is approximately 0.347 for the mean stationarity hypothesis and 0.119 for the variance stationarity hypothesis.

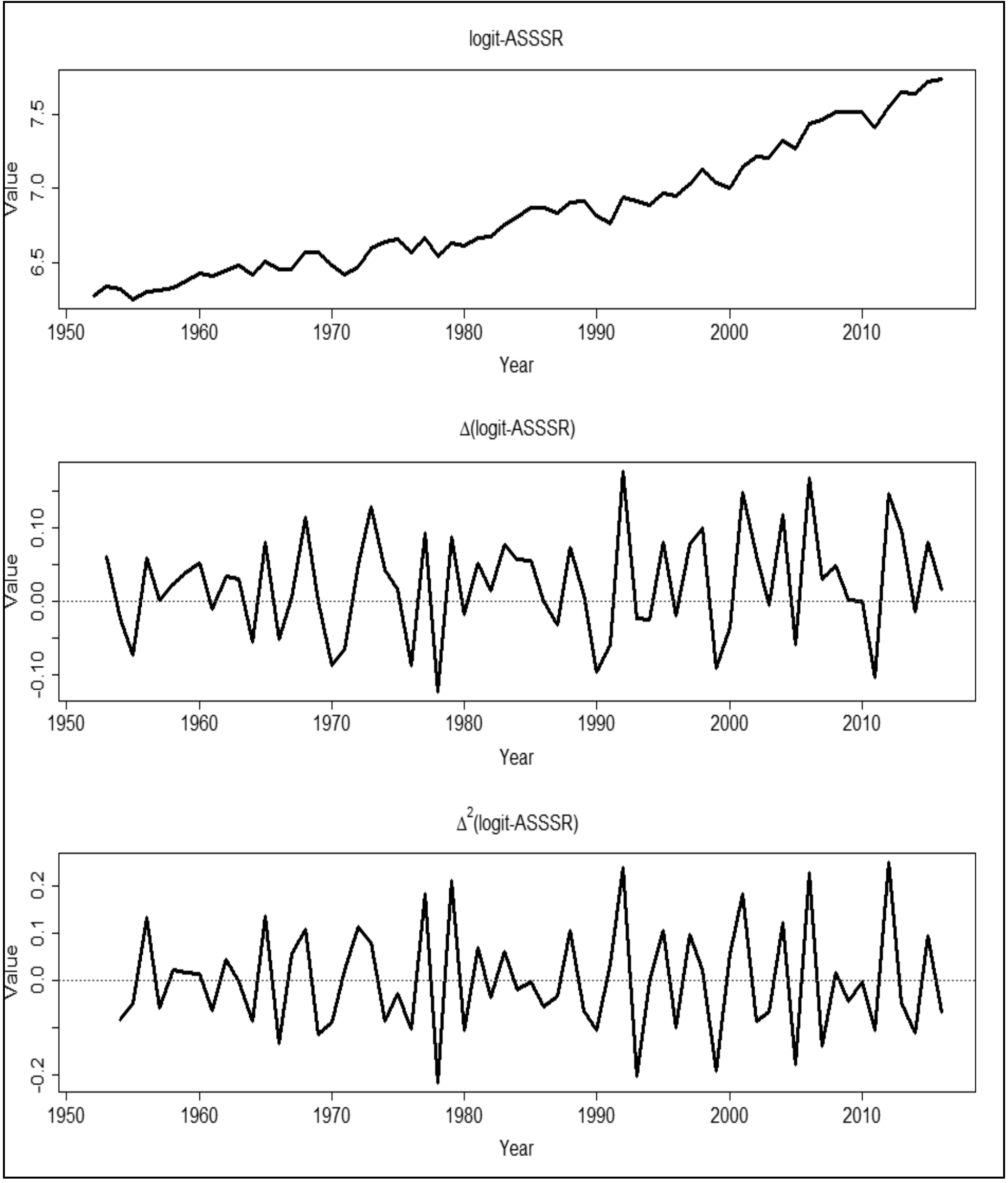
[13] The ARCH-LM Test should be implemented in common statistics software as well, i.e., it can be applied easily by **ArchTest**, included in the package **FinTS** (Graves, 2013).

In principle, a nonstationary TS can be transformed into a stationary TS by differentiating it one or more times (Shumway und Stoffer, 2011). The first difference of a TS is calculated as follows:

$$\Delta a_t = a_t - a_{t-1}. \tag{25}$$

As known from calculus, this operation asymptotically leads to a reduction of the power of the target function (here: the TS) by one. Figure 3 illustrates the result of the differentiation by visualizing the TS of the logit-ASSSR of 25-year-old males in Germany with its first and second difference for the time horizon 1952-2016.

Figure 3. *Logits of ASSSRs of 25-year-old Males with First and Second Differences*



Sources: Destatis, 2005, 2015a, 2015b, 2015c, 2016, 2017a, 2017b, 2018a, 2018b; Own calculations and design.

The trending behavior of the original TS is weakened by differentiation. Graphically, we can conclude that the first difference may already be stationary, and the second difference is even smoother.

The next question to consider for a TS that has been transformed to a supposedly stationary TS is what type of ARMA model best fits the asymptotically stationary TS. Several information criteria can be applied, including Akaike's information criterion (AIC), the Bayesian information criterion (BIC), which is also known as the Schwartz criterion (see, e.g., Greene, 2012), and the Hannan-Quinn criterion (HQC) (Hannan and Quinn, 1979). These criteria follow a similar principle, relying on the log-likelihood as the goodness-of-fit measure. The difference between the criteria is the magnitude of the penalty for model complexity. The best model is the one that minimizes the criterion of choice. The specifics of the criteria are not presented here because they rely heavily on asymptotics. Thus, the reliability of the information criteria strongly depends on the length of the TS (i.e., how much data is available as base data) used as the model input. The availability and quality of data used in typical population studies, especially regarding population forecasts, are relatively poor. Therefore, the information criteria should be considered carefully. Graphical analyses based on the autocorrelation function (ACF) and the partial autocorrelation function (PACF) are recommended for investigating demographic TS[14]. For the logit-ASSSR example, both the ADF and KPSS test suggest one-time differentiation as suitable. From a practical perspective, the KPSS and ADF tests generally show poor performance for short histories, tending to mark stationarity too early. The ARCH-LM test gives a p-value of 0.5128 for the first differences-TS. Thus, the first differences are stationary with a high probability. Figure 4 shows the ACF and the PACF of the first differences for the ASSSR of 25-year-old males in Germany for the period under study.

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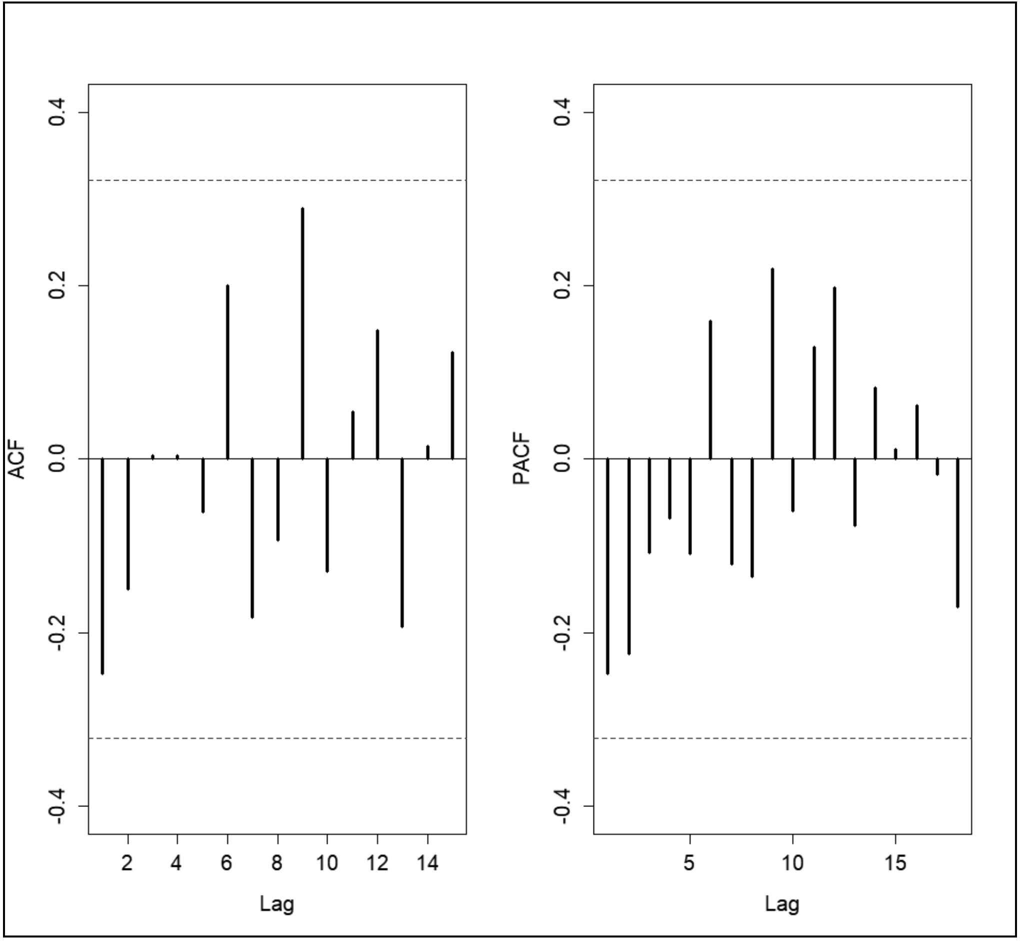
[14] For a more detailed description of the ACF and PACF, see, e.g., Shumway and Stoffer, 2011.

Figure 4. *ACF and PACF of the Lagged Logit-ASSSR*



The graphical representations provide evidence of which lag length to choose and therefore which values of p and q are best. The graphical analysis is not trivial and requires the user to have some experience. However, some basic attributes, which ideally are observable in the figures, are associated with AR and MA processes. First, the dashed line[15] indicates the statistical significance of the lags. An AR(1) process is relatively easy to identify since the related ACF decreases exponentially, whereas the PACF has a large value for the first lag and then abruptly falls to approximately zero. An MA(1) process behaves inversely, with an exponentially decreasing PACF and an ACF with significant values for the first lag only. Figure 4 does not suggest any autocorrelation, as the estimates of the values are not statistically significant at the chosen level, which would suggest that the ASSSR-TS is simply a random walk process. This conclusion can additionally be confirmed by the information criteria.[16]

The use of the lag notation will now be explained. The general ARIMA(p,d,q) process can be described as follows:

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[15] The figures were generated in ***R*** using **acf**() and **pacf**(). The dashed lines are plotted by default, according to the chosen significance level.

[16] Standard optimization algorithms exist. In ***R***, this may be checked using **auto.arima**() in the package **forecast** (see Hyndman et al., 2018).

$$\left(1-\sum_{i=1}^{p}\phi_j L^j\right)(1-L)^d a_t = \left(1-\sum_{i=1}^{q}\theta_i L^i\right)\omega_t. \qquad (26)$$

In the case of an ARIMA(1,1,1) process, the lag notation is

$$(1-\phi L)(1-L)a_t = (1-\theta L)\omega_t,$$

which may be multiplied out to

$$[1-(1+\phi)L+\phi L^2]a_t = (1-\theta L)\omega_t.$$

From the definition of the lag operator, it follows that

$$a_t-(1+\phi)a_{t-1}+\phi a_{t-2} = \omega_t-\theta\omega_{t-1},$$

or equivalently,

$$a_t = (1+\phi)a_{t-1}-\phi a_{t-2}+\omega_t-\theta\omega_{t-1}.$$

Even complicated functional forms can be written in a simple way with the lag operator, which is especially helpful in the context of simulation studies in forecasting.

Forecasting Demographic Rates

This section will explain how the TS methods can be used to forecast the previously identified PCs. A first comparable approach was proposed by Bell and Monsell (1991) for forecasting age group-specific mortality in the United States. That contribution was built upon earlier proposals by Ledermann and Breas (1959) as well as Le Bras and Tapinos (1979) for modeling and projecting age- and sex-specific mortality in France. Lee and Carter proposed a simplified version of the model of Bell and Monsell for mortality (Carter and Lee, 1992; Lee and Carter, 1992) and later fertility (Lee, 1993) forecasting. The Lee-Carter models are currently very popular in mortality and fertility forecasting. Some scientists in Germany have recently used similar models to forecast age- and sex-specific fertility and mortality rates (see, e.g., Fuchs et al., 2018; Härdle und Myšičkova, 2009; Lipps and Betz, 2005, Vanella, 2017). Deschermeier (2015) applied the Hyndman-Ullah (2007) model, which is a PCTS model adjusted for robustness and with functional PCs, allowing for the loadings explained in Section "Introduction to Principal Component Analysis" to vary over time. Vanella and Deschermeier (2018) applied a PC model for migration forecasting in Germany.

Returning to the four TS introduced in Section "Introduction to Principal Component Analysis" (ASSSRs of 25-year-old males in Germany, Italy, Austria, and Switzerland), a simple Lee-Carter model is used to estimate their future course until 2080.[17] The first step in such a forecast should be the identification of the long-term trending behavior. An accurate interpretation of the PCs for this is of high importance, since the forecaster needs to put some qualitative judgment into the initial forecast model as well. Forecasts certainly are best, when they are

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[17] Disclaimer: The simulations presented here are purely of illustrative nature to show the practical application of the methods presented in Sections "Introduction to Principal Component Analysis" and "Main Features of Time Series Analysis" and should by no means be mistaken as actual forecasts.

derived quantitatively, but can be explained qualitatively as well. In our example, the PC in Section "Introduction to Principal Component Analysis" has been identified as a general mortality index. A first graphical analysis of Figure 5 gives an idea of the long-term behavior of the PC.

Figure 5. *Historical Course of Mortality Index*

From the historical course, we might conclude a progressively decreasing course, corresponding to a clear positive trend in the survival rates. The next step is then the smoothing of the index by fitting an appropriate model by ordinary least squares (OLS) estimation[18] to the data. The fit of a quadratic model[19] renders the forecast model for the long-term trend

$$f(t) = -2916.011 + 2.964t - 0.00076t^2, \tag{27}$$

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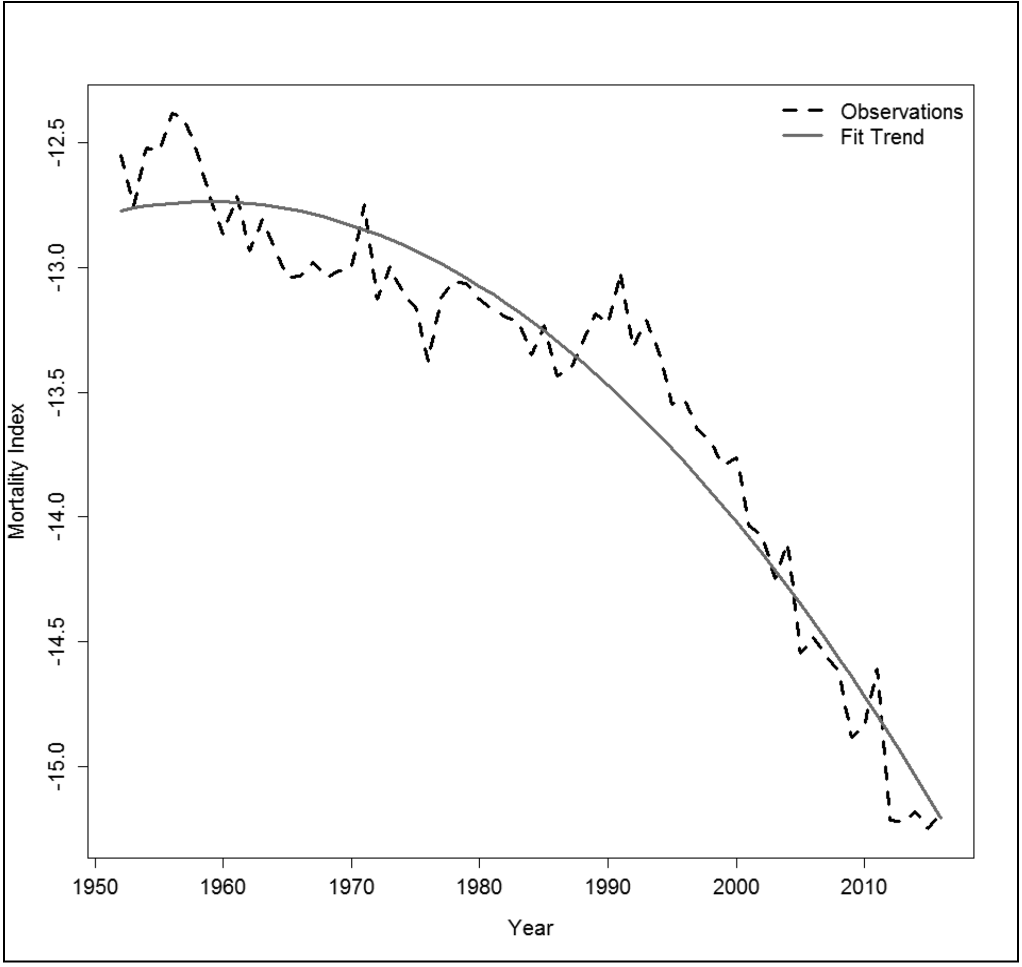
[18] See, e.g. Wooldridge (2013) for an introduction to OLS fitting.

[19] The OLS estimation is easily done in **R** with the **lm**() command.

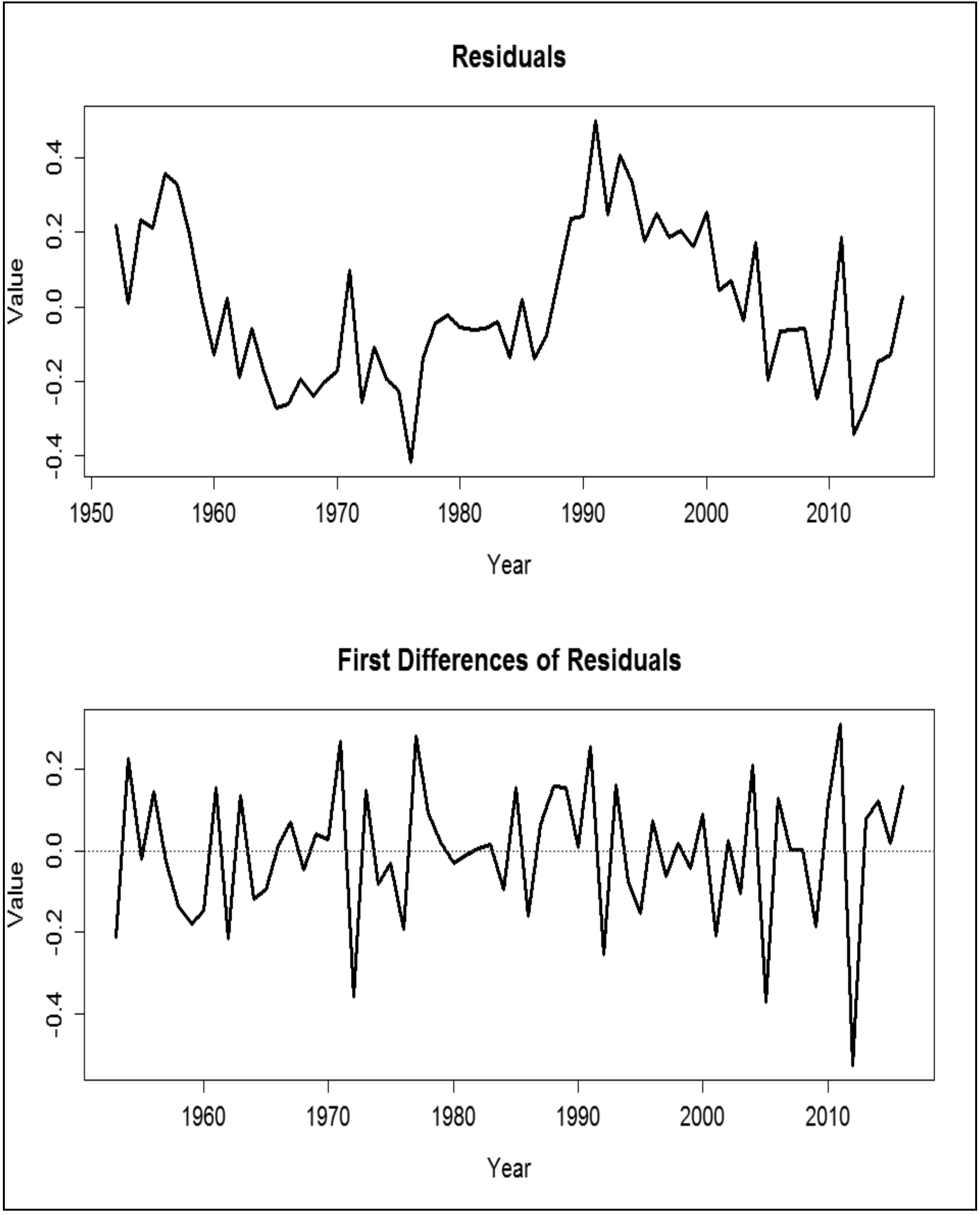
which is statistically highly significant at the individual level (for the coefficients) and due to the overall model significance. The fit is shown in Figure 6.

Figure 6. *Long-Term Trend of Mortality Index*



An ARIMA model is then fit to the resulting residuals. As described in Section "Main Features of Time Series Analysis", we first investigate the residuals for stationarity. The graphical analysis of Figure 7 suggests that the residuals are not stationary, but their first differences might be.

Figure 7. *Residuals and First Differences of Residuals from the Model Fit*



This is confirmed statistically by the ADF test[20] and the ARCH-LM test[21]. The ARMA degrees are determined by the ACF and PACF, which are illustrated in Figure 8.

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[20] $p - value \approx 0.019$.

[21] $p - value \approx 0.417$.

Figure 8. *ACF and PACF of First Differences of Residuals*

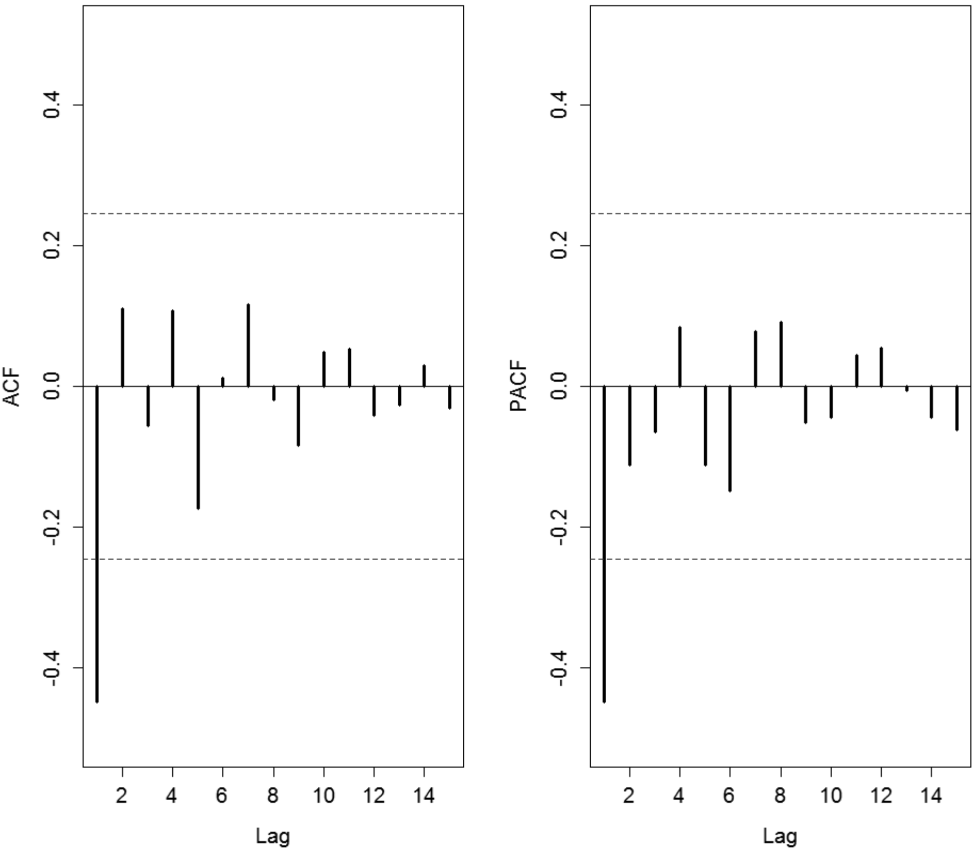

Figure 8 suggests that the first differences of the residuals might follow an AR(1) process with a negative coefficient, since the values alternate and decrease in tendency for the ACF, whereas they are almost zero after the first lag in the PACF. The OLS fit of an ARI(1,1) model to the residuals is highly significant and gives the model

$$(1 + 0.4584L)(1 - L)r_t = \varepsilon_t,$$

which becomes

$$r_t = 0.5416r_{t-1} + 0.4584r_{t-2} + \varepsilon_t, \tag{28}$$

where r_t is the residuum in period *t* and $\varepsilon_t \sim \mathcal{NID}(0, 0.1442^2)$. The combination of (27) and (28) yields

$$p_t = f(t) + r_t. \tag{29}$$

with p_t representing the PCs value in *t*. Equation (29) can then be used for simulation of the future development of the Mortality Index with Wiener Processes[22]. In the example, 10,000 future paths of the PCs are simulated.

The hypothetical history of the PCs is calculated through the matrix notation of equation (1):

$$\boldsymbol{P} = \boldsymbol{VE} \tag{30}$$

Here, ***P*** is a matrix with t rows and s columns. t is the number of observed periods, and s is the number of TS. Consequently, ***P*** has dimensions of 65×4 in the example. ***V*** is the matrix of all logit-ASSSRs and therefore has the same structure as ***P***, i.e., a columnwise collection of all logit-ASSSR-TS. ***E*** is a matrix of columnwise EVs.

Through the reverse transformation of (30), the forecasts for the ASSSRs can be derived from the simulated future values of the PCs:

$$\mathbf{Y}_t = \mathbf{\Pi}_t \boldsymbol{E}^{-1}. \tag{31}$$

In this case, $\mathbf{\Pi}_t\ (10{,}000 \times 4)$ is the simulation matrix[23] of the PCs in year t. $\boldsymbol{E}^{-1}\ (4 \times 4)$ is the inverse of the eigenvector matrix, and the resulting matrix product $\mathbf{Y}_t$ is a $10{,}000 \times 4$ matrix of the simulation values for the logit-ASSSRs, estimated indirectly via PC simulation. Since the PCs are uncorrelated, simultaneous and independent computer simulations for each PC may be done separately to estimate PIs for the ASSSRs. A sufficiently large number of future trajectories must be estimated so that the PCs converge. The author performs 10,000 estimates to simulate the PCs until the year 2080. The simulation of PC 1 is based on (29). Empirical quantiles can be estimated for the PIs based on these trajectories. Finally, we need to inversely logistically transform the logit-ASSSRs to obtain the simulation values for ASSSRs and derive PIs from them. Figure 9 provides the 95% PIs for the ASSSRs for 25-year-old males in Germany, Italy, Austria, and Switzerland.

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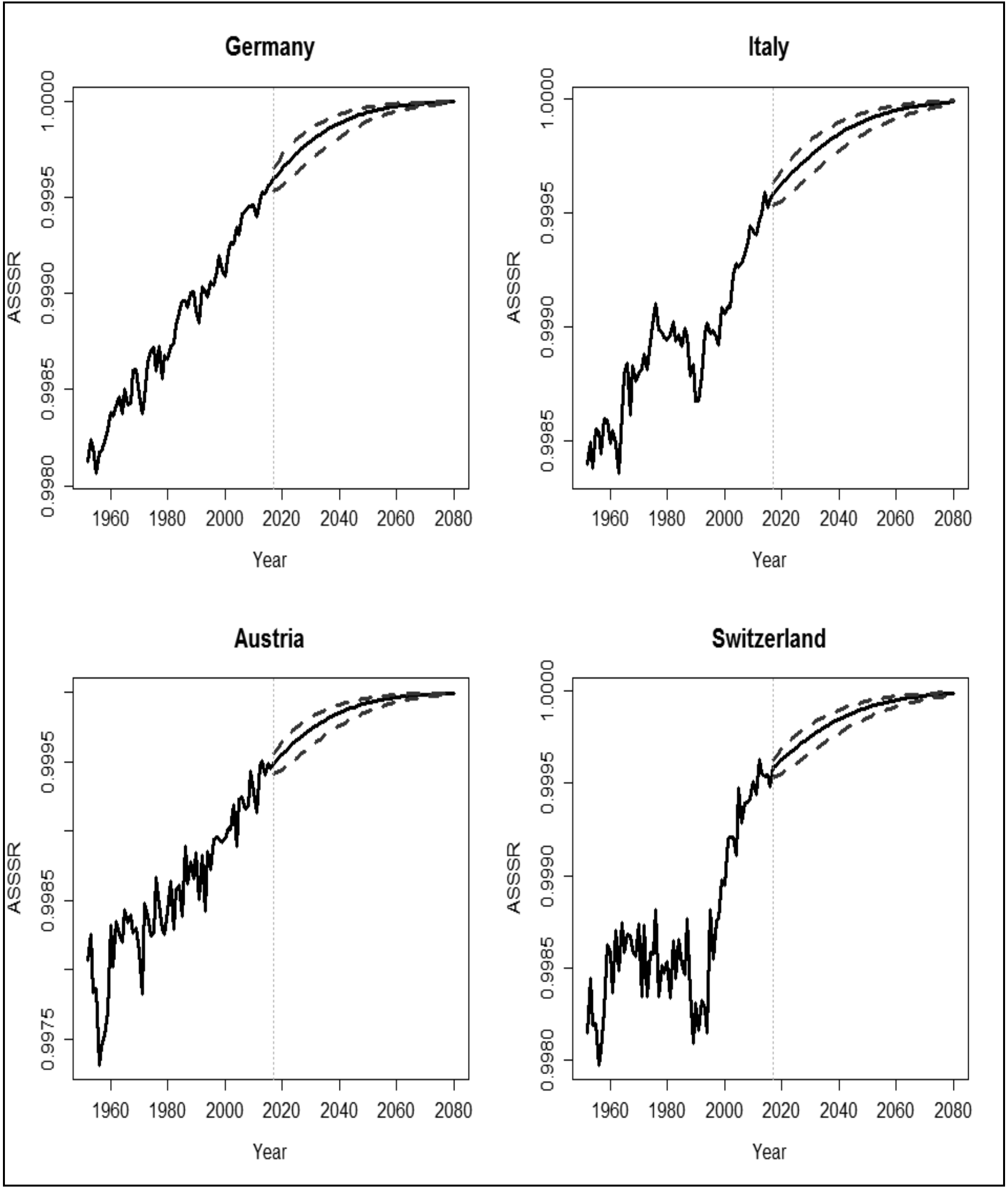
[22] See, e.g., Vanella (2017) on that.

[23] In the example, 10,000 simulations were conducted. Theoretically, this process works for a larger number of iterations, but the computation becomes cumbersome.

Figure 9. *Projected ASSSRs for 25-year-old Males with 95% Projection Intervals*



Sources: Destatis, 2005, 2015a, 2015b, 2015c, 2016, 2017a, 2017b, 2018a, 2018b; Eurostat, 2018; Human Mortality Database, 2018a, 2018b, 2018c, 2018d; Istat, 2018a, 2018b, 2018c, 2018d, 2018e, 2018f; STATcube, 2018; Own calculations and design.

We observe a quite similar future development, as could be expected by the high correlations among the ASSSRs. It should be stressed once more that this is just a simulation study, not a forecast. Counterintuitively, the PIs become narrower over time. In general, PIs will become wider, since uncertainty regarding the far future is greater than that regarding the near future. In this specific case, the result does make sense. Barring that landslide events such as wars or vast pandemics occur, mortality will on average decrease further. Since survival probabilities logically cannot become larger than one, it follows that in the very long term, survival rates will converge towards one in all scenarios, so the intervals become tighter.

Conclusion, Limitations and Outlook

The primary goal of this study has been the presentation of PCA and its practical implementation. On the basis of PCA, arbitrary age- and sex-specific measures, including the quantification of the stochasticity through PIs, may be modeled and forecast without bias, which was illustrated for the ASSSRs of 25-year-old males in Germany, Italy, Austria, and Switzerland. The presented approach is applied internationally on a country level; nevertheless, it may be applied on regional level as well if the required data are available. The example was only mortality for one age group to keep the paper concise, and mortality trends are the easiest expositions due to their clear trends in industrialized countries. Nevertheless, the methods presented can also be applied to other demographic phenomena (fertility, migration) or other fields (e.g., economics; meteorology), depending on the quality of available data.

PCA is a powerful tool for the simplification of complex phenomena and addressing correlation among different variables; however, PCA needs good data to work appropriately. Similar to all quantitative methods, PC forecasts with TSA models cannot address trends that have not been observed in the past. Therefore, forecasting should always assess the possibility of massive structural breaks occurring in the future. Moreover, a qualitative assessment of the PCs is advisable. A PC always represents a composition of the original variables. Therefore, an appropriate interpretation is very important, and PCA results have to be considered judiciously.

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Chapter 2

A principal component model for forecasting age- and sex-specific survival probabilities in Western Europe

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ABHANDLUNG

A principal component model for forecasting age- and sex-specific survival probabilities in Western Europe

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Abstract The assessment of future mortality is of high importance in many areas where the allocation of future resources has to be planned in time, especially in social security and private life insurance. This contribution represents an extension of the classic forecasting approaches of Bell–Monsell and Lee–Carter. Based on a forecast of the first two principal components, age- and sex-specific survival probabilities for 18 Western European countries are predicted simultaneously until the year 2070. In addition to the correlations in the mortality trends between the age groups and the genders, international trends in mortality are captured as well. A major improvement in the classic Lee–Carter models is the adequate quantification of the uncertainty associated with the whole system of variables by stochastic simulation of all remaining principal components with simple time series models. The model's easy applicability to further analyses is illustrated by forecasting the median life span as well as the resulting Gender Gap for Germany, France, and Italy.

Zusammenfassung Die Einschätzung der zukünftigen Mortalität ist für viele Bereiche zur Planung der Ressourcenallokation von hoher Bedeutung, besonders in der Sozialversicherung und der privaten Lebensversicherung. Dieser Beitrag stellt eine Erweiterung der klassischen Prognoseansätze von Bell-Monsell und Lee-Carter dar, wobei auf Basis der Prognose der ersten zwei Hauptkomponenten die alters- und geschlechtsspezifischen Überlebenswahrscheinlichkeiten für 18 westeuropäische Länder simultan bis ins Jahr 2070 prognostiziert werden. Neben den Korrelationen, die bei den Mortalitätstrends zwischen den Altersgruppen und den beiden Geschlechtern bestehen, werden auch internationale Trends aufgefangen. Eine signifikante Verbesserung zu gängigen Lee-Carter-Modellen wird durch stochastische Simulati-

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on der verbleibenden Hauptkomponenten mit simplen Zeitreihenmodellen erreicht, wodurch sich die Unsicherheit im ganzen Variablensystem mit Prognoseintervallen angemessen quantifizieren lässt. Die Flexibilität des Modells für weiterführende Analysen wird anhand der Prognose der medianen Lebensspanne und des resultierenden Geschlechterunterschiedes in dieser Mortalitätskennziffer für Deutschland, Frankreich und Italien illustriert.

1 Introduction

The future development of mortality is of great importance in many areas, particularly for future funding of the statutory pension fund and for pricing purposes in life insurance. Trends in Western European life expectancy are almost monotonously positive since the 1970s. Given that the Total Fertility Rates (*TFR*) for most European countries are significantly below the replacement level fertility of about 2.1 for these regions, c.p. a relatively smaller group of people of working age will therefore need to finance a growing group of retirees. The so-called baby boomers will retire within the next two decades, which will lead to a significant increase in the total number of retirees within a small stretch of time and the swift augmentation of the financial burden for the statutory pension fund. Private life insurance has to deal with a decreasing mortality simultaneously to historically low-interest rate levels. Forecasts of future mortality are therefore an important informational ground for political as well as managerial decision-making. Political decisions on pension fund reforms must consider the future course of mortality in society and the resulting structure of the population. Life insurers must calculate the premiums for their contracts based on good predictions of survival probabilities. The biggest challenge in the forecasting of mortality is the assessment of future longevity. Age- and sex-specific mortality rates (*ASSMR*) for very high ages are commonly not made public in official statistics because of the vulnerability toward errors in the population data for very old people. The first problem arising in the modeling of old-age mortality deals with possible limits of longevity. Specifically, two essential assumptions exist in the literature. Some scientists believe in a specific age, which could not be surpassed biologically. However, there is no empirical evidence for this assumption. The alternative assumes a convergence of the ASSMR toward some asymptote. According to different scientists, this asymptotic probability might lie between 50 and 100%.

In the first step of the analysis, I will approximate the ASSMR, as recommended by Thatcher et al. (1998), for the age group of 90–115 years from historical data with a nonlinear Kannisto model. The base data derived in this manner is used for forecasting of age- and sex-specific survival rates (*ASSSR*) for ages 0–115 for 18 countries in Western Europe up to the year 2070 in one single model. The forecast model represents an extension of the original Bell–Monsell and the simplifying Lee–Carter approach[1], which are time series models based on a principal component analysis (*PCA*). The model results, in addition to the median forecasts (which might

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[1] See Bell and Monsell (1991) as well as Lee and Carter (1992) for a detailed description of the respective models.

be interpreted as c. p. age-specific survival probabilities), in predictive intervals (*PI*) for all ASSSR to any arbitrary significance level. Monte Carlo Simulations of Wiener Processes quantify PI for the ASSSR and potentially for any mortality indicator. Exemplarily, 90%-PI derived this way for the median life span (*MLS*) of females and males in Germany, France, and Italy, which are the three most populous countries in Europe, are illustrated. The proposed modeling approach deals with the problems of cross- and auto-correlation between the age groups, including trends between the two genders as well as European cross-border trends in mortality. Furthermore, the proposed approach deals with the main problem in the classical Lee–Carter models, which underestimate the underlying risk resulting from ignoring the stochasticity of most of the principal components (*PC*). My model will include these risks as well in the simulation.

2 Common mortality forecasts and projections in Europe

In official statistics, future mortality is often estimated by deterministic future projections, with some base scenario and one up to two alternate scenarios assuming a better or worse development in future mortality. Specifically, the national statistical offices in Europe still prefer the scenario technique, although based on historical data (see e.g. Statistisches Bundesamt 2015 on this). Deterministic scenario-based projections only calculate a very small number of possible future scenarios, which have an asymptotic individual probability to occur at zero. Furthermore, there are no individual probabilities assigned to the identified scenarios. Therefore, stochastic forecasts of mortality are preferable for planning purposes of many subjects, e. g. in a wide range of insurance applications, especially financing of the statutory pension fund, care insurance, and health insurance. Booth and Tickle (2008) have pointed out that deterministic models commonly rely on judgmental scenarios, which systematically tend to underestimate the decline in future mortality. Subjective scenarios commonly lead to some bias given that past trends are often believed not to last (Booth and Tickle 2008; Wong-Fupuy and Haberman 2004). Furthermore, these qualitative projections typically assume a maximum in life expectancy, although there is no quantitative evidence for the existence of this upper limit in longevity (Oeppen and Vaupel 2002). Among the countries considered in this study[2], to the author's knowledge, only the Scandinavian countries, Italy, and Portugal use variants of Lee–Carter models. These models are only used for prediction of the baseline scenarios, where the alternatives in these cases are subjective scenarios (Stoeldraijer et al. 2013; Salvini et al. 2006; Instituto Nacional de Estatística 2009; Hansen and Stephensen 2013; Keilman and Pham 2005; Statistiska centralbyrån 2005). Hence, the uncertainty in these forecasts is not considered appropriately. One major goal of my study will be the quantification of uncertainty in mortality forecasting through the application of appropriate statistical methods. The only statistical office offering probabilistic projections for the mortality in Europe is the Population Division of

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[2] Germany, France, Italy, England with Wales, Spain, the Netherlands, Belgium, the Czech Republic, Portugal, Hungary, Sweden, Austria, Denmark, Finland, Slovakia, Scotland, Switzerland and Norway.

the United Nations, which used the Bayesian Hierarchical Model (*BHM*) by Raftery et al. (2014). They proposed a world mortality model for 158 countries, excluding the countries biased by HIV-epidemics or wars. The model projects the future life expectancy for each country as a random walk with drift, which is estimated by a double-logistic function. The six resulting parameters are partly estimated by Markov Chain Monte Carlo simulations, partly set equal to the extreme observations in the countries with the lowest past mortality. The Bayesian modeling requests distributions for the parameters, which are determined for each country by UN analysts. The results are truncated normal distributions for the parameters. Each country-specific mortality is modeled individually and contains global trends given the existence of country-specific as well as world parameters. Monte Carlo simulations finally lead to scenario-based credible intervals. Raftery et al.'s (2013) original model was projecting future mortality for females only. The authors developed the BHM further for males. The approach is modeling the gap between female and male life expectancy by Ordinary Least Squares Regression[3] (*OLS*) using the female life expectancy as a covariate. The gap is then forecasted and simulated with a t-distributed nuisance parameter. The simulated female life expectancy reduced by the simulated gap gives the simulated male expectancy (Raftery et al. 2014). The UN projections are complemented by adjusted projections for the remaining countries over the Rotation model by Li et al. (United Nations 2015; Li et al. 2013). Pampel (2005) found that the decrease in the gender gap, which has been observed in the past in many countries, stems from the increase in female smoking. He forecasted the gender gap in life expectancy with and without the smoking effect until the year 2020. The result indeed was a further increasing gender gap assuming a similar smoking behavior between men and women. This is a very interesting result given that the common belief is a decrease of the gender gap and therefore a convergence of life expectancies between the two sexes as assumed in the Raftery model. This smoking-, or in general behavioral-effect is considered in my model indirectly by using the PCA.

Actuarial applications in life insurance classically tend to rely on projections of age-specific mortality and hazard rates based on mortality reduction factor models, which model the ASSMR by exponential decaying mathematical functions with a mixture of recent trends and subjective assessments. Furthermore, stochasticity in general is not considered in these models. They overestimate future mortality as well (Wong-Fupuy and Haberman 2004). From a methodological point-of-view, private life insurance has become more sophisticated than most statistical offices over the course of years. Lee–Carter models or the two-factor model by Cairns et al. (2006) and its extension by Sweeting (2011) are becoming popular for future estimation of age-specific mortality for the insurers' respective risk portfolios.

The first obstacle, which should be tackled for a complete mortality forecast, is the estimation of old-age mortality in general. Based on the Kannisto–Thatcher Oldest-Old database provided by the University of Southern Denmark at Odense (see Max Planck Institute for Demographic Research 2016 on this), Thatcher et al. (1998) investigated death counts for people over 80 years of age for 13 European countries

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[3] See Wooldridge (2013) for a more detailed description of the optimization technique.

for the period 1960–1990. They tested the fit of six statistical models to the observed mortality and found that for old age mortality, the Kannisto models and logistic models give the best fit because they at least provide unbiased estimates, although mortality rates over 95 years of age still oscillate quite strongly around the estimators. They identified the logistic model as the best fit for modeling old-age mortality, but found the Kannisto model to be a better approximation of mortality in the very high ages. Thatcher et al. (1998) also gave some thought on possible limits of longevity. They found no evidence for a concrete upper limit in life with the maximum observed at 122 years and five months. It is still not clear whether this observation could be identified as an outlier or an indication for potential further future increases for possible maximum life spans. Wolff et al. (2005) came to a similar conclusion on the choice of a statistical model for extrapolation of old-age mortality using life insurance data on insured persons and death counts. They took data on 85–95-year old persons as input data for mortality extrapolation to 120 years of age. They concluded, based on statistical tests and out-of-sample tests for insurance data of 96–99-year-olds, that a logistic or a Kannisto model would be the best fit, but found the logistic approach to be superior over the Kannisto model. My approach for the approximation of old-age mortality builds on the results by Thatcher et al. (1998). The inference of the Kannisto model will be described in detail in the following section, where my forecast modeling approach is explained together with the data used.

3 Data and method

This contribution proposes a joint modeling approach for international forecasting of age- and sex-specific survival rates (*ASSSR*). The countries under study are Germany, France, Italy, England with Wales, Spain, the Netherlands, Belgium, the Czech Republic, Portugal, Hungary, Sweden, Austria, Denmark, Finland, Slovakia, Scotland, Switzerland, and Norway. These countries were chosen by meeting the following criteria:

- Focus on highly developed countries in Europe
- Countries with at least five million inhabitants, thus offering a population big enough to derive valid estimates of age-specific survival probabilities
- Detailed time series available back to the year 1952

It would not be a problem to integrate other developed countries, such as Japan, Australia, or the U.S. into the study, but the analysis was restricted to Europe to keep the simulations on a manageable level computationally. The modeling results might therefore be interpreted as general future developments in Western European mortality. The data were annual unsmoothed ASSMR for the period 1952–2014, downloaded directly from the Human Mortality Database (*HMD*). Missing data were calculated by Eurostat data, with the exception of Germany given that the HMD, due to the reunification for total German mortality, only contains time series since 1990. To minimize aberrations caused by merging two different data sources, the German ASSSR were computed using data on age- and sex-specific death counts and population size provided by the German Statistical Office *Destatis* (GENESIS-Online 2016; Eurostat 2016; HMD 2016; Statistisches Bundesamt 2005; Statistisches Bundesamt 2016). The data provided by Destatis is of high quality and in time. Based on the extracted data, ASSSR for all 18 countries were computed.

One major merit is the unreliability of the data for persons over 90 years of age due to the erroneous and incomplete population data in this age group. My own statistical analysis, as well as the results from the investigation of Thatcher et al. (1998), led to the conclusion that an approximation of the ASSMR for ages 90 years and older with econometric extrapolation using the Kannisto model leads to best estimates for the old-age mortality. The Kannisto model is defined as

$$\mu_x = \frac{a \cdot \exp(bx)}{1 + a \cdot \exp(bx)} \tag{1}$$

with μ_x as the force of mortality, the instantaneous risk of death at age x (Thatcher et al. 1998). In this framework, x may be any unit of age, even seconds or moments. This concept may be generalized to ASSMR. The ASSMR for ages 90 and over are estimated by nonlinear least squares (*NLS*) estimation using the model

$$m_{c,t}(x) = \frac{\alpha_{c,t} \cdot \exp\left(\beta_{c,t} x\right)}{1 + \alpha_{c,t} \cdot \exp\left(\beta_{c,t} x\right)} \tag{2}$$

with $\alpha_{c,t}, \beta_{c,t}$ as time- and country-specific estimators and $m_{c,t}(x)$ as the ASSMR for age x in country c in year t. The ASSMR are approximated until age 115. The estimation leads to higher slopes in the curves for the males compared to the slopes of the females, which denotes the convergence of the ASSMR between the two genders. This result is in line with the often assumed convergence of future life expectancy. To avoid higher estimates for the ASSMR for females than for men in the same age, the male ASSMR used the respective female ASSMR as lower bounds. Mathematically, this may be formulated as

$$\widehat{m}_{m,t,x} = \max(\widetilde{m}_{m,t,x}; \widetilde{m}_{f,t,x}) \tag{3}$$

with $\widehat{m}_{m,t,x}$ as the estimator for the ASSMR of males in year t and age x, $\widetilde{m}_{m,t,x}$ as the ASSMR estimated by the NLS algorithm for males in year t and age x, and $\widetilde{m}_{f,t,x}$ signifying the analogue for females. This was chosen because the female data for older ages is more reliable statistically due to the higher sample size for women than for men due to the bigger female population. After estimating the ASSMR, the approximation of the corresponding ASSSR is straightforward. ASSSR are logically restricted to the interval [0;1] and almost impossibly take values 0 or 1

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[4] An age-specific survival probability of zero would mean that a person has no chance of surviving the coming year, which might be rejected if there is no age limit. An age-specific survival probability of one would mean no risk of death at all, which is clearly impossible.

[5] The logit of a variable x is defined as $ln\left(\frac{x}{1-x}\right)$.

exactly[4]. Therefore, a logistic (or short: logit) transformation[5] (Johnson 1949) of the ASSSR with regard to future simulation was undertaken.

A principal component analysis (*PCA*) is performed for all 18 countries under study simultaneously, which resulted in 4176 time series. The general form of the i^th^ principal component (*PC*) of the covariance matrix of the logit of the ASSSR in this case is:

$$PC_i = \sum_{j=1}^{J} e_{ij} \cdot \text{logit}(\text{ASSSR}_j) \tag{4}$$

with the e_{ij} denoting the coefficient of the j^th^ ASSSR on the PC under study. We call the vector $\overrightarrow{e_i}$ the first eigenvector of the covariance matrix, which corresponds to the i^th^ PC. In matrix notation, the transformation can be denoted as:

$$\boldsymbol{C} = \text{logit}(\boldsymbol{S}) \times \boldsymbol{E} \tag{5}$$

In Eq. (5), **C** denotes the matrix of the principal components, **S** is the matrix of the ASSSR for some time interval, and **E** is defined as the matrix of the eigenvectors of the covariance matrix of the ASSSR. The PCA chooses the PC, which is decreasing by the amount of total variation in the original variables they explain. Therefore, in the first step, the vector $\overrightarrow{e_1}$ is chosen, such that it explains the maximum amount of variation in the ASSSR. PC_2 is determined by $\overrightarrow{e_2}$, which maximizes the explained share of variation in the original variables under the condition that PC_2 is uncorrelated to PC_1 (Chatfield and Collins 1980). Using this procedure, we arrive at 4176 PC for our original variables, descending by importance. The choice on the number of PC to label as sufficient to describe the system is quite subjective. Nevertheless, there exist a couple of criteria, which shall help in choosing the number of PC to explain a sufficient part of the total variation while reducing the dimensionality of the forecasting problem.[6] These criteria are used for identification of the number of PC to model in detail. In the end, the first two PC will be modeled in detail. To avoid the approximation error described in Sect. 1, the remaining PC will be assumed as Random Walk processes with White Noise nuisance parameters. In this way, a level shift is avoided for the first year of the forecast, whereas Random Walks are still relatively easy to compute and automate.

At this point, the PC are forecasted in two steps. First, the time series are smoothed by parametric models. The remaining part is fitted to Autoregressive Integrated Moving Average models (*ARIMA*), which orders are chosen by a combination of graphical analysis, the Kwiatkowski-Philipps–Shin–Schmidt (*KPSS*) Test, the Augmented Dickey–Fuller (*ADF*) Test, the ARCH-LM Test for Conditional Heteroskedasticity, the Autocorrelation Function (*ACF*), the Partial Autocorrelation Function (*PACF*) as well as qualitative analysis of the PC. The models are fitted by OLS estimation.[7]

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[6] Handl (2010) gives a nice and thorough overview of the common criteria; Vanella (2017) illustrates the application for age-specific demographic measures.

[7] Detailed descriptions of the tests and measures are given by Shumway and Stoffer (2011) and Vanella (2017).

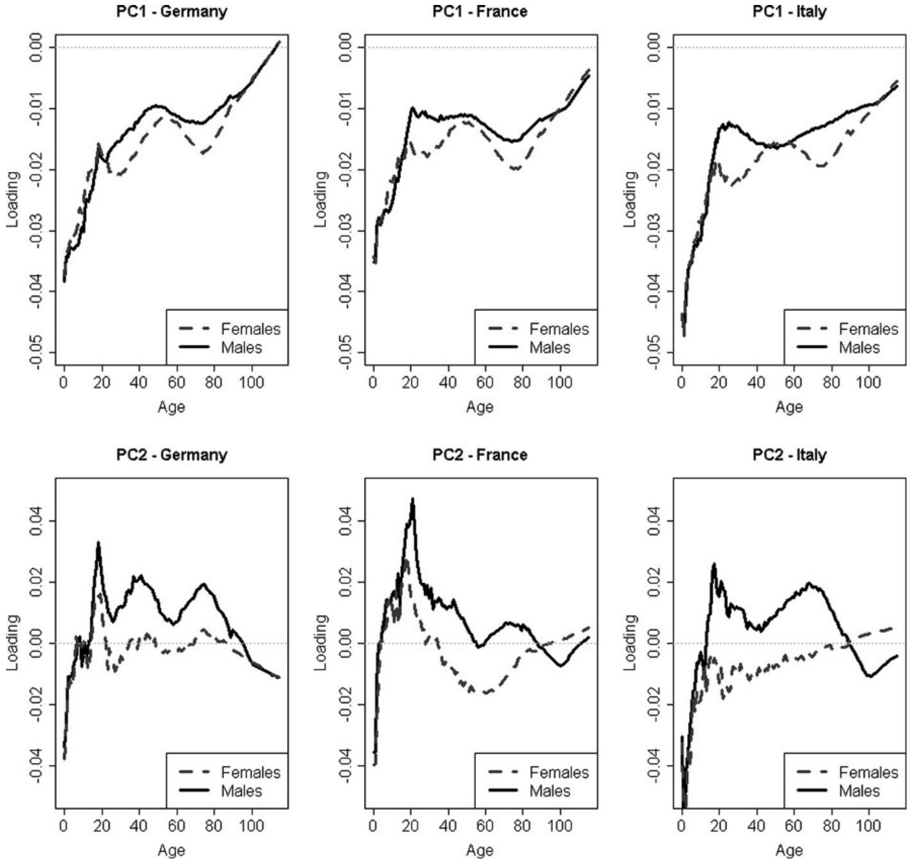

Fig. 1 Correlation between PC and ASSSR. (Source: Own calculation and design)

The resulting time series models are then used for forecasting of the PC until the year 2070. To account for uncertainty, the resulting models are used for 1000 future simulations of the PC, which is achieved by simulating 1000 Wiener processes each with Monte Carlo simulation of the error terms.[8] Transforming the results of these simulations, e. g. for period t, back by the matrix operation[9]

$$\mathrm{logit}^{-1}(\widehat{C}_t \times \widehat{\boldsymbol{E}}^{-1}) = \widehat{S}_t \tag{6}$$

leads to 1000 simulated values for each ASSSR annually. These values might be used for the construction of point-wise predictive intervals (*PI*) covering the future value of each ASSSR until the year 2070 with any probability. The hats indicate the estimated matrices derived from the sample. From the estimated ASSSR, it is easy

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[8] Note that 1000 simulations is a very small number in this context, which is due to keeping the huge number of variables and the long forecasting horizon computable.

[9] logit^{-1} denotes the inverse standard logistic transformation $\frac{\exp(x)}{1+\exp(x)}$ for some x.

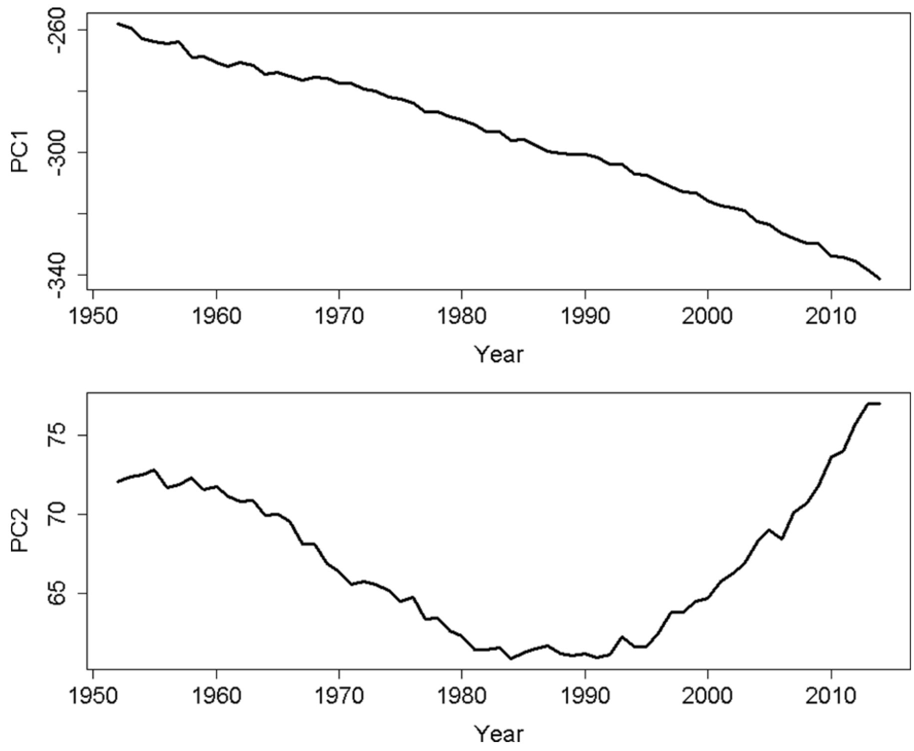

Fig. 2 Historical course of the PC. (Source: Own calculation and design)

to calculate the MLS for females or males at birth in a certain period.[10] Given that the MLS based on the ASSSR can be calculated discretely only on the space of the natural numbers, the MLS will be approximated by linear interpolation between the two adjacent ages in completed years for all simulations.

Fig. 1 illustrates the correlation coefficients of the first two PC for females and males in Germany, France, and Italy.[11] The first PC is a general mortality index, similar to the PC in the original Lee–Carter model. Big negative values for the PC are associated with high improvements in survival probabilities for the respective ages. While the logit-ASSSR for children increases heavily for high negative values of PC1, older people profit less from these improvements. For the very old, this effect diminishes.

The second PC balances the effects of PC1 for small children. Meanwhile, we observe positive correlations for males from the young working ages until around 80 years of age as well as negative correlations with women in the same age groups. Hence, this PC can be interpreted as an indicator for the development of the dif-

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[10] The MLS here is defined by the age, at which half of the population of a certain cohort would be deceased, assuming a constant mortality level similar to the respective period.

[11] This is restricted here to the three mentioned countries to keep it short, but the interpretation may well be translated to the other countries as well, given that the coefficients are quite similar.

ference in the age-specific mortality between the two genders. Increasing values for PC2 are associated with a convergence of the gender gap in life expectancy toward zero.

Fig. 2 illustrates the hypothetical course of the first two PC.

We observe an almost monotonically decreasing mortality trend in PC1. PC2 reflects an increase in the gender gap in mortality until the early 1980s, remaining at a relatively constant level for approximately 15 years, followed by a convergence of mortality between the sexes. This observation is in line with the studies by Pampel, Waldron as well as Trovato and Lalu, who identified an increase in the gender gap in life expectancy due to an unhealthier lifestyle exhibited by men in the late 19th and the 20th century in the Western societies due to smoking and worse nutritional behavior in relation to women. In the late 20th century, the females in part adopted the unhealthier behavior of the men, such that the gender gap decreased again (Pampel 2002; Waldron 1993; Trovato and Lalu 1996). The interesting question at this point is how to predict the future development of these variables in the future. The next section will deal with this problem intensively.

4 Results

At this point, the time series of the PC are fitted, as described above, and forecasted. After the graphical and statistical tests have been performed, the following time series model was identified as the best OLS fit for PC 1:

$$c_1(t) = -261.681 - 0.7615t - 0.0079t^2 + 0.5939\delta_{t-1} + 0.4061\delta_{t-2} + \varepsilon_t \tag{7}$$

with $\delta_t := c_1(t) - \left[-261.681 - 0.7615t - 0.0079t^2\right]$ and $\varepsilon_t \sim N(0, 1.1305^2)\forall t$ being a White Noise nuisance parameter.

The forecasting model for PC 2 needs some assumption on the future course of the variable. Tests show that pure Box–Jenkins models lead to implausible forecasts for the ASSSR. Given that PC 2 is associated with the gender gap in life expectancy, a logistic course, based on the data after the minimum in 1984, appears plausible. This course assumes a further increase, which converges toward some asymptote. This appears in line with the general assumption in the literature that the behavioral part of the mortality converges while the part of the gender gap caused by biological differences in the sexes remains. The OLS fit for this model is shown in Eq. (8):

$$c_2(y) = 60.5228 + 23.6732\text{logit}^{-1}\left(\frac{y}{5.8157}\right) + \omega_{y-1} + \xi_y \tag{8}$$

where $\omega_y := c_2(y)$–$[60.5228 + 23.6732\text{logit}^{-1}(\frac{y}{5.8157})]$ and $\xi_y \sim N(0, 0.5582^2)\forall y$. The scale parameter in the logit-argument has been estimated by Maximum Likelihood minimization of Akaike's Information Criterion (AIC)[12].

Computer simulation of 1000 paths for both variables through Wiener processes leads to the simulated distributions for the future course of the two PCs, which are

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[12] See Shumway and Stoffer (2011) for a more detailed definition of the AIC.

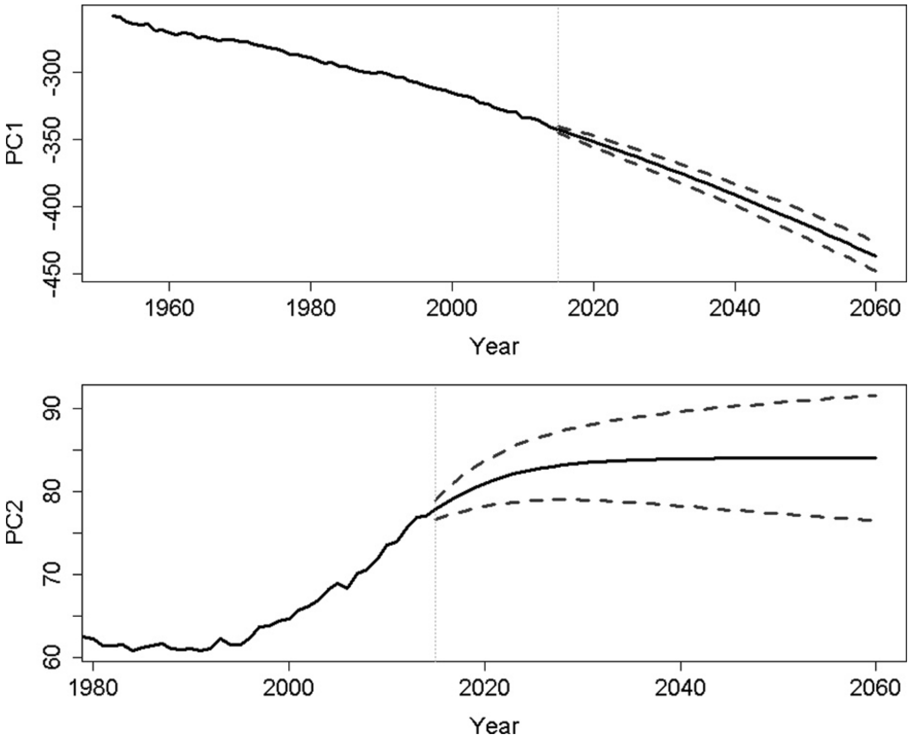

Fig. 3 Forecast of PC1 and PC2 until 2070. (Source: Own calculation and design)

illustrated in Fig. 3 with 90%-PI for the forecast. The dotted gray lines mark the start of the forecasting period, whereas the dashed blue lines are the limits of the PI.

We observe that the general trend in decreasing mortality is going to continue with a clear tendency. A decreasing gender gap is very probable, although at a decreasing rate, as explained previously. The remaining PCs are simulated as Wiener processes of Random Walks 1000 times each. The resulting simulated matrix of the PC was then transformed into simulation matrices for the ASSSR, as displayed in Eq. (6). With these distributions for the ASSSR, it is possible to calculate median life spans as a final step. Multiplication of the ASSSR for some country and gender leads to some density to survive until some age. Mathematically, this may be defined as follows:

$$P\left(a\right)=\prod_{i=0}^{a}\pi_i \tag{9}$$

π_i is the probability for a person of i years of age to survive until age $i + 1$, whereas P(a) denotes the probability of survival at birth until age a. We expect half of the population of one cohort to survive until this age. This means MLS is defined as the value of a, at which $P(a)$ exactly takes the value 0.5. The simulations can be used for estimation of a density function for the MLS for each age, gender, and

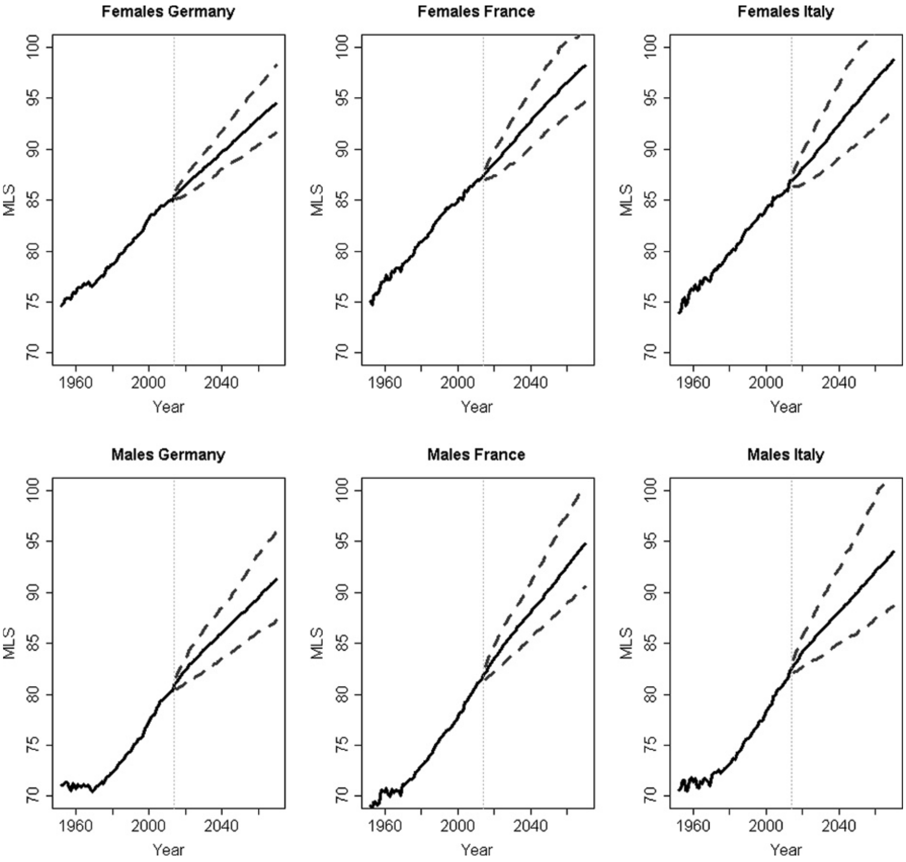

Fig. 4 MLS until the year 2070 with 90%-PI. (Source: Own calculation and design)

country under study. As discussed in Sect. 2, linear interpolation is used for coping with the problem that the measure for age is defined discrete instead of continuous. In this way, out of the original 1000 simulated paths of each variable, 1000 scenarios for the future course of each MLS may be calculated. Fig. 4 illustrates the final results of this estimation.

The analysis shows c. p. a further increase of the median life span until the year 2070 for the countries under study, with a small tendency of a bigger increase for the countries in Southern Europe. It is noteworthy that the uncertainty in the predictions varies between the countries, which is illustrated by 90%-PI in the Fig. 4. It may be concluded from this result that the MLS in Italy are expected to have steeper increases than the MLS in Germany, but the PIs are clearly wider for Italy; therefore, the uncertainty in the prediction is bigger in Italy than it is for Germany. Given that the development of the gender gap is a well-discussed topic in the literature and has an important standing in the UN-forecast as discussed previously, I will illustrate how my model can be used easily to derive distributions for the forecasted gender gap at this point. Given that my model includes the correlations in mortality trends between

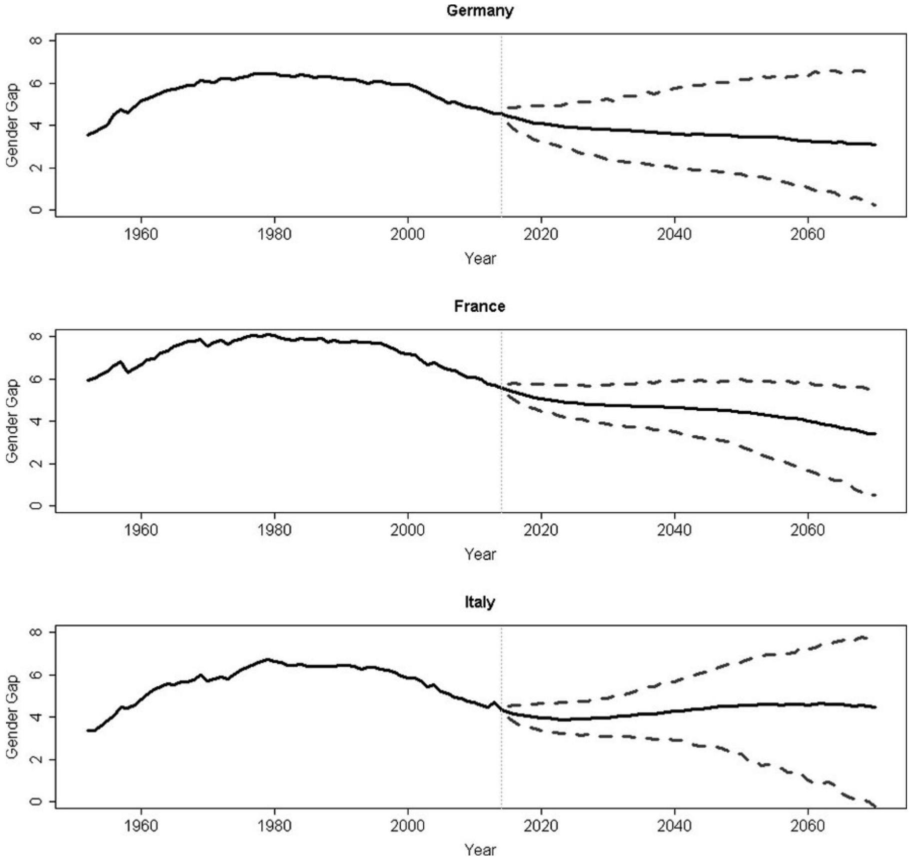

Fig. 5 Forecast for Gender Gap with 90%-PI. (Source: Own calculation and design)

the two sexes, the simulations of the ASSSR can be used for simple differencing as well as to derive the resulting gender gap in each simulation. Fig. 5 describes the forecasts derived in this way for the gender gap in the MLS for Germany, France, and Italy as an illustration.

The trends toward a further decreasing gender gap for Germany and France are quite clear, although the relative improvements are only minor. For Italy, the result is quite inconclusive. The gender gap appears to remain relatively stable in the future due to the very good improvement in the survival probabilities of the females. We need to notice that the PI becomes very large after 2040, which underlines the high stochasticity in the forecast relative to the two other countries. From these results, we can draw some further conclusions. The simulation study leads to a probability of about 2.9% in 2070 for males to have a higher MLS than females. The percentages are 2.8 for France and 5.6 for Italy. These results have to be treated with caution because of the small sample size, but it gives us an idea about the future ratios in mortality between females and males. The probabilities for Germany and France are very close, while the probability for Italy is about twice as big because of the higher uncertainty in the Italian mortality relative to the other two countries. We observe that there is a possibility for men to have a smaller mortality in the future than the women in the same regions and ages, although the probability for this to occur is slim.

5 Conclusions, limitations and outlook

The future course of mortality and survival probabilities is of major importance in demographic research as well as in insurance economics, especially social insurance and life insurance. The size of the older population is the most important driver of expenditures in pensions, health insurance, and care insurance. Therefore, life insurers need to calculate premiums as appropriately as possible based on age- and sex-specific probabilities of survival and death.

This contribution proposed an extension of the Bell–Monsell and the Lee–Carter model. Based on the proposed model, long-term forecasts of age-specific survival probabilities are done. The model showed a relatively easy approach for simultaneous forecasting of mortality in Western Europe. The whole mortality structure in 18 countries for both genders has been quantified by reduction to a two-variable model in essence. This reduction leads to a relatively simple modeling approach, which outputs the future behavior of mortality for many countries, all ages and both genders with one estimation. Furthermore, it includes the correlation in mortality among the countries under study, with consideration of international trends. The results might also be interpreted as a general future development of mortality in Western Europe. Given its easy structure and manageable need for data, the model can be used for regular updating of mortality forecasts. Through the direct forecasting and simulation of age- and sex-specific survival rates, arbitrary mortality measures can be derived easily. To illustrate this process, median life spans and resulting gender gaps were calculated for the three biggest countries in Europe: Germany, France, and Italy. Nevertheless, we need to keep in mind that the model calculates the future development of the mortality in the remaining 15 countries simultaneously as well. To keep it short and comprehensive, I only presented the results for the three mentioned countries.

The most crucial limitation faced in mortality forecasting is the estimation of the old-age mortality, given the lack of reliable data on the population in this age group and a small sample size. Some assumptions on this matter are therefore needed on the general mortality above 90 years of age and the limits in longevity. Owing to further studies on old-age mortality by Thatcher et al. (1998), I assumed logistically increasing age-specific mortality rates for the past data and terminated the investigation until 115 years of age. It is up to the reader to evaluate this choice. Another limitation is that the model cannot predict the unexpected and yet (in the underlying period in the countries under study) to be observed events, such as wars or epidemics, which is common in quantitative time series models. The countries chosen in the model appear quite stable and resistant toward these events. Qualitative models could include these possibilities, but this would be too subjective and uncertain to predict and would take away the strengths of the quantitative model.

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Chapter 3

A stochastic Forecasting Model of international Migration in Germany

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A stochastic Forecasting Model of international Migration in Germany

Abstract

International migration is currently one of the most controversial topics of discussion in our society. Critics of an open migration policy see high immigration figures as a major security risk, and warn of possible displacement effects in the labour market, while its advocates argue that from a demographic viewpoint, for instance, international migration is an opportunity to slow the effects of demographic change by increasing and rejuvenating the population, and above all to increase the available workforce in economic sectors that are already affected by labour shortages. For these reasons, it is especially important to have an unbiased discussion based on empirical results. Against this background, forecasting future migration flows provides a quantitative basis for discussion to support planning in political and business contexts; to date, however, such forecasting has not been entirely satisfactory.

We present a modelling approach for the forecasting of international net migration between Germany and other countries, broken down by gender, age and nationality. Our contribution provides stochastic forecasts of future net migrations using a time-series model based on principal components. In this approach, prediction intervals serve to represent uncertainty with respect to future developments.

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1 Introduction and Motivation

International migration is the most difficult to predict of all the demographic components of population trends. Its development over time is significantly more volatile than that of fertility or mortality. Thus, use of common time series methods based on age-specific demographic rates is problematic for comparable dimensions like age-specific immigration and emigration rates. However, good predictions constitute enormous value here since these forecasts can be used as the basis for many types of economic and business planning.

International migration has been a particularly controversial topic of discussion. Advocates for restrictions on immigration see a major security risk due to an increasing threat of terrorism, or express the concern (although not a rational one, in the context of the minimum wage) that (cheap) foreign labour could threaten native workers' jobs through wage dumping. Advocates of an open migration policy, meanwhile, argue that migration is an opportunity to counteract demographic change by increasing and rejuvenating the population, and thus to combat labour shortages in specific professions. In light of this discussion, it is especially important to have a clear picture of the age and national origin profiles of immigrant and emigrant populations.

In this article, we begin by examining the issues that forecasters face attempting to predict future migration flows. The first of these involves problems with data quality; the second problem is inconsistent data collection and definitions on the international level; and the third lies in the inherent complexity of migration movements. These issues are addressed and discussed in detail in the following chapter. In the third chapter, a number of potential solutions are discussed for the problems mentioned in chapter 2; and in the fourth part of the article, we propose a stochastic forecasting approach for international migration between Germany and other countries. This approach makes it possible not only to estimate the most likely path for future migration figures, but also to quantify the risk in the form of prediction intervals (*PIs*). The results of the forecast can be used in a probabilistic cohort-component model to estimate population trends in Germany. Our model estimates net migration broken down by gender, age in years, and group of nationality. This provides insight not only into how population as a whole will develop, but also into how the distribution of ages and ethnic origins in the population will change through international migration. The results thus offer the potential for further economic and demographic analyses.

2 Issues in the Modelling and Forecasting of international Migration

The first major problem in modelling net migration lies in the inadequate quality of the data. Potential sources of error are present at each stage in the process of compiling statistics. The first errors arise right away in primary data collection. In Germany, there is a general requirement for all people to register at their new place of residence after moving. Similarly, people must unregister from a previous place of residence when moving away. Nevertheless, these migration movements can only be observed to a limited degree, whether within Germany or across borders, e.g. between two EU countries. Although the examples just mentioned involve a legal requirement to register and unregister on a regular basis, this process is often merely voluntary. The result is that in many cases, people may not be recorded in the data at all, or they may be recorded multiple times. So there is often a significant difference – at times quite dramatic – between place of origin and destination in the migration figures derived from these data.

Kupizewska and Nowok (2008: 47) compared statistics on in- and out-migration across EU countries and determined that in cases of migration from one country to another, the figures reported for migrants arriving in the destination country are higher than the corresponding figures reported by the country of origin for migrants moving away to the given destination country. In certain extreme cases, the corresponding figures differed by a factor of over 60. This shows how difficult it is to record migration flows correctly, especially within the Schengen area. We should mention here that Germany tends to have the best migration data in comparison with other countries, if it can be assumed that the odds of counting a migration more than once are lower than the odds of failing to count a given migration at all (cf. Kupiszewska/Nowok 2008: 42-52).

Finally, the amount of detail in the collected data varies to some extent among destination countries because they do not all ask about the same variables, or because incorrect data are recorded due to false statements by migrants, especially with refugee migration (cf. Bewarder/Leubecher 2016). The next stage relates to compiling statistics. There are often additional divergences here between migration figures for the country of origin and destination because different definitions of migrants from one country to the next lead to different ways of counting them. For a few years now, a UN recommendation has been available on the definition of a migrant. It says that migration should only be counted as such when the person in question can demonstrate their presence at the new place of residence for a minimum period of one year (cf. United Nations 1998: 13).

However, there are many countries that do not abide by this recommendation. For example, Belgium requires a minimum residency period of three months, while in Germany, no minimum residency period is set at all. In other words, a person is counted as a migrant from the moment they are first registered in Germany. These different minimum residency criteria can therefore lead to a situation in which an immigration case is registered in a different year than the corresponding emigration (cf. Kupiszewska/Nowok 2008: 58-62).

The third and final stage in which errors occur is in the distribution of migration statistics. The availability and level of detail in the data, as well as the comparability of data from different sources, can vary substantially at times (cf. Kupiszewska/Nowok 2008: 62-68). The statistics are based not only on administrative data, but are sometimes also modified via census data and other surveys. The level of detail in the data varies strongly. For instance, important information like the reasons for migration are often missing from the available statistics. In some databases, data are organised according to country of origin and destination, but do not include age information. Conversely, certain other sources provide information on migrants' ages, but not on country of origin and destination. In addition, migrations are often not recorded directly, but only approximated indirectly via the person's place of residence at two different times. Some databases do not actually include migration figures at all, but only migrant figures (cf. Rogers et al. 2010: 6-7), so the absolute number of migrants in a country at a given time is known, but not when those people migrated. This way of counting also omits any information about how many people have immigrated and emigrated. Instead, only the differences in the migrant stocks at two points in time may be deduced.

Forecasting future migration provides an important basis for political and other decision processes. It is typically mostly deterministic projections of net migration that are proposed based on previously established assumptions (see e.g. Pötzsch/Rößger 2015: 40-41). Besides starting from a baseline scenario that is highly improbable from a statistical point of view, this methodology also provides a few alternative scenarios that are not quantified with probabilities of occurrence. This means uncertainty is not adequately captured in these projections. There is a relatively small number of probabilistic approaches that use statistical methods to generate projections or forecasts for future migration in Germany (cf. Bijak 2011; Bohk 2012; Deschermeier 2016; Dudel 2014; Fuchs et al. 2018; Härdle/Myšičková 2009; Lipps/Betz 2005).

These approaches either do not yet take the substantial structural break of 2015 into account, or do not adequately quantify the uncertainty of their forecasts. And indeed, forecasts of future developments require good data about past patterns — but acquiring such data is difficult, as explained earlier. Furthermore, these data must not only be of good quality in the short term, but must also extend as time series as far back into the past as possible. This is a major problem worldwide, but for Germany and the Schengen area in particular, since German reunification and the various expansions of the EU and the Schengen area have changed borders and barriers to entry many times over the past 30 years.

Conducting migration forecasts for Germany based on pre-1990 data only makes sense to a limited extent, since the situation before that time is not representative of current migration activity. Current data are also problematic for forecasting purposes because they are not available in as detailed and readily combinable a form as would be needed for a comprehensive forecast. For example, the German Federal Statistical Office (*Destatis*) has separate collections of migration data organised by age and gender. There are also data broken down by country of origin and destination, by nationality, or by the purpose of a stay, but no consistent combination of these features. Furthermore, the figures from different data publications are often not consistent with one another.[1] It would be useful to have a consistent compilation and publication of age- and gender-specific migration data, broken down by both nationality and reasons for migration, to be used as underlying data for generating predictions.

Solutions have been proposed to limit the impacts of these data-related problems. The next section examines the most important proposals for the purposes of this article. Alongside the problems with the data, migration forecasting is also made more difficult by the fact that migration movements are influenced by many external political, social, economic and climate factors, resulting in a highly complex set of interactions that is not readily predictable (cf. Bohk 2012: 139). In the context of this article, however, we will put forth a proposal based on a principal components analysis that reduces this difficulty somewhat. We will introduce a modelling approach that statistically quantifies the risk in the forecast; as a positive side effect, the subjective modelling assumptions that necessarily have to be made in this model can in turn be relaxed to some extent.

3 Proposed Solutions for Data Problems and Forecasting Model

In this section, we will first consider possible approaches to a solution for the problems discussed in Section 2. We will then present our own proposal that constitutes the basis of a stochastic forecasting model to predict international migration between Germany and other countries.

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[1] For example, the number of male Germans under the age of 18 (i.e. 14,150) presented in the report on Migrations across Germany's Borders in 2005 by Citizenship and Age Group (Wanderungen über die Grenzen Deutschlands 2005 nach der Staatsangehörigkeit und Altersgruppen) is far from consistent with the cumulative figure (15,473) from the report on Migrations between Germany and Other Countries in 2005 by Individual Years of Age and Gender (Wanderungen zwischen Deutschland und dem Ausland 2005 nach Einzelaltersjahren und Geschlecht). This example was chosen at random, but similar examples abound in this area.

Migration data are often not organised by exact age in years, but are aggregated into age groups. This raises the general question of how to circumvent this restriction if migration data must be broken down by exact ages for the purposes of annual population forecasts. The classic approach to estimating age distributions among migrants is the Rogers-Castro model developed by Andrei Rogers and Luis Castro, a model with up to 13 parameters that estimates the migration cycle across all ages (cf. Rogers/Castro 1981: 5-13). Empirically, the Rogers-Castro model produces a very good fit for international migration figures and internal migrations, and is especially useful for cases in which no information (or only supporting data) is available on the age distribution among migrants. For age-grouped data, Rogers et al. recommend an interpolation of migration figures using supporting data by exact age in years, with the help of the Rogers-Castro model (cf. Rogers et al. 2010: 22-24).

Besides the Rogers-Castro model, approaches that build on the functional data analysis paradigm (Ramsay/Silverman 2001; 2005) are drawing increasing attention. This approach describes a way of thinking about working with data series. The basic idea is that observations in a series are not independent of one another, but follow a functional correlation. On this basis, aggregated demographic rates can be disaggregated into individual years of age via spline regression (zur Nieden et al. 2016; Wood 1994: 27). This proves to be particularly helpful with regional population forecasts, since the required data are generally only available in coarsely aggregated form (Deschermeier 2011: 775). The functional data paradigm serves as the basis for a number of time series models (Hyndman/Ullah 2007; Hyndman/Booth 2008; Hyndman et al. 2013) that are specifically designed for demographic rates. This approach, and the time-series models that build on it, are especially useful for fertility and mortality rates, since both show a high degree of consistency over time.

In theory, it would be possible to produce a forecast based on age- and gender-specific immigration and emigration rates.[2] This approach should not be implemented in practice for immigration (cf. Bijak and Wiśniowski 2010: 777), however, because it would require access to detailed overall population data for all countries. Even if the appropriate data existed, the amount of effort involved in researching the data would be disproportionately high, which would make it inefficient to update the model regularly. Therefore, heavy assumptions are necessary in those applications, which bias the uncertainty of the forecast. Another conceivable option would be to set up a forecasting model with exogenous variables (see e.g. Bijak 2011: 64-72; Bohk 2012: 43-45). However, the disadvantage here is not only that this would complicate the model and increase the amount of data required, but more importantly, that the exogenous variables would also have to be forecast, which in certain cases would be not only very difficult, but practically impossible. For example, an econometric model for immigration might contain explanatory variables like gross domestic product of the countries of origin and destination. One could also imagine using climate variables like earthquake probabilities or temperature variables. Labour market variables like the number of vacant jobs would be possible alternatives as well. To forecast migration movements based on such an explanatory econometric or environmetric model, however, the variables in question would have to be forecast first; this is already very difficult for economic variables, and

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[2] Fuchs et al. (2018) use such an approach.

nearly impossible for certain scientific variables. However, it is possible to approximate this process indirectly through the use of a principal component analysis (*PCA*), as we will explain later in this article.

As mentioned earlier, Destatis has several highly detailed datasets, but they are not always consistent with one another. Rogers et al. (2010: 4-44) recommend creating a synthetic dataset from multiple different sources (e.g. combining census and migration data) through the use of a generalised linear model (*GLM*). In geometric terms, it should be arranged in the form of a cube containing the following three dimensions: *demographic information* (e.g. gender and age), *geographic information* (e.g. country of origin and nationality), and *temporal information* (e.g. the year of migration). However, this assumes that the various data sources are consistent.

In general, the combination of micro data and macro data is an alternative that should be used with caution, since micro data generally raise the question of how representative they are of the population as a whole, and whether they are biased through the over- or underrepresentation of certain groups. Although very good micro data and panel datasets are available (e.g. for Germany, the Socio-Economic Panel or the Micro census), they always rely on random samples and the population is never known, which means that errors of this type cannot be ruled out. In addition, working with micro data is generally more time-intensive, and more expensive as well in some cases (e.g. when working with Micro census data). We will therefore not use this approach here, since one of the features of a good model is that it models reality as well as possible with the least possible effort. We could also imagine using the GLM approach with a combination of different macro data. For example, Destatis has figures on age-specific migration for ages 0-94, but only subdivided into the two nationality groups *"German"* and *"non-German"* or *"foreign"*. Further subdivisions would be of tremendous importance, however; the reasons for migration and barriers to entry for Italians are extremely different to those for Ethiopians, for example, but they are all included in a single group for the purposes of these statistics. These data could not be used as the basis for any meaningful forecast.

On the other hand, Destatis also has migration figures that distinguish by specific nationality. Yet these are subdivided into quite rough age groups at best,[3] and not by exact age. A GLM synthesis of these two datasets would be possible in theory, e.g. by using Micro census data as supporting data, but fails in practice because the two datasets in question do not align, as was shown in chapter 2. One disadvantage that results from excessive differentiation of the population, on the other hand, is that the number of variables to be estimated in an eventual forecast quickly becomes too large to conduct simulations on commonly available hardware configurations. In addition, distinguishing at the level of individual nationalities means that countries which show small net migration flows to Germany will have samples for certain ages that are too small to derive statistically significant estimates from. For these reasons, we initially subdivide our data into seven nationality groups: *Germans (GE), Foreigners from the EU or other Schengen countries (EU), Other Europeans (OE), Asians (AS), Africans (AF), Americans and Oceanians*, referred to in the subsequent discussion as *Overseas (OS)*, and *Other nationalities*[4] *(NA)*.

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[3] Under 18; 18-24; 25-49; 50-64; over 65.

[4] Stateless or unknown nationalities.

The data were broken down according to these distinctions, and differentiated by gender. Since the data were only available in this level of detail for the five age groups mentioned earlier, the missing data must be interpolated. To do this, the migration overviews by age in years were used as supporting datasets, similarly to the approach taken by Rogers et al. To achieve the most precise estimate possible of the age distributions in the subgroups, the age distribution that was found within the five age groups for non-Germans in the report on *Migrations between Germany and Other Countries from 1991-2015 by Individual Years of Age and Gender* (*Wanderungen zwischen Deutschland und dem Ausland 1991-2015 nach Einzelaltersjahren und Geschlecht*) was assumed for all foreign nationality groups. For 1990, no such detailed data are available for Germany as a whole; so the age distribution in 1991 was adapted to fit the age groups for 1990 as well. Distinctions between genders and between immigrants and emigrants were also taken into account to ensure that as much as possible of the information present in the data could be used. However, care was taken to ensure that the estimated distribution for each age group coincides with the values from the migration dataset on *Migrations across Germany's Borders from 1990-2015 by (Selected) Nationalities and Age Groups* (*Wanderungen über die Grenzen Deutschlands 1990-2015 nach (ausgewählten) Staatsangehörigkeit(en) und Altersgruppen*).

The advantage over a classic smoothing approach in accordance with the Rogers-Castro model is that no theoretical age distributions need to be assumed for migrants; rather, these distributions follow the overall distribution of the migrants, resulting in relatively consistent estimation across the two datasets used. Since no detailed data are available for age-specific migration for people older than 94,[5] the grouped data are estimated in part via OLS extrapolation and in part via linear extrapolation to the individual ages up to 100 years old. For ages above 100 years, the model consistently assumes zero net migration. The potential error resulting from this is negligible, since migration levels for this age group are very small in both directions. After preparing our synthetic migration dataset in this way, we use the constructed age- and gender-specific migration flows to generate the directional age-, gender- and nationality-specific net migration figures, resulting in the three-dimensional structure called for by Rogers et al. The result is a 26x1414 matrix with the years (1990-2015) in the rows, and the net migration figures by age (0-100 years), gender (binary) and nationality group (GE, EU, OE, AS, AF, OS, NA) in the columns.[6]

A PCA[7] is then used to generate linear combinations from the net migration figures. We reduce the model to a quasi-two-principal-component model, since these already explain 75.8% of the variation in the age-specific net migration between Germany and other countries, and since the third principal component turns out to be a Random Walk process based on the tests described by Vanella (2017).[8] Figures 1 and 2 illustrate the correlation structures (or component loadings) between the first two principal components and the net migration figures.

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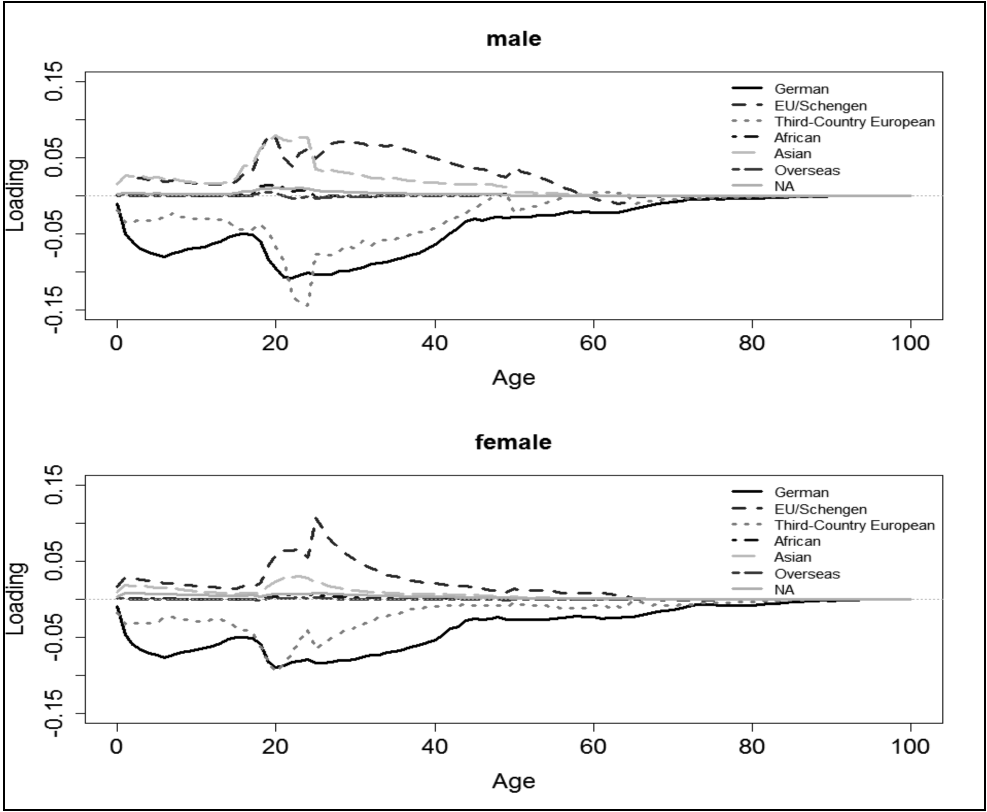
[5] This age boundary varies somewhat between the reports.

[6] Strictly speaking, this actually corresponds to a four-dimensional data structure with dimensions of 26x101x2x7.

[7] See Vanella 2017: 3-9 for a detailed description of the methodology.

[8] See Vanella 2017: 12-17 for a more precise description of the tests used.

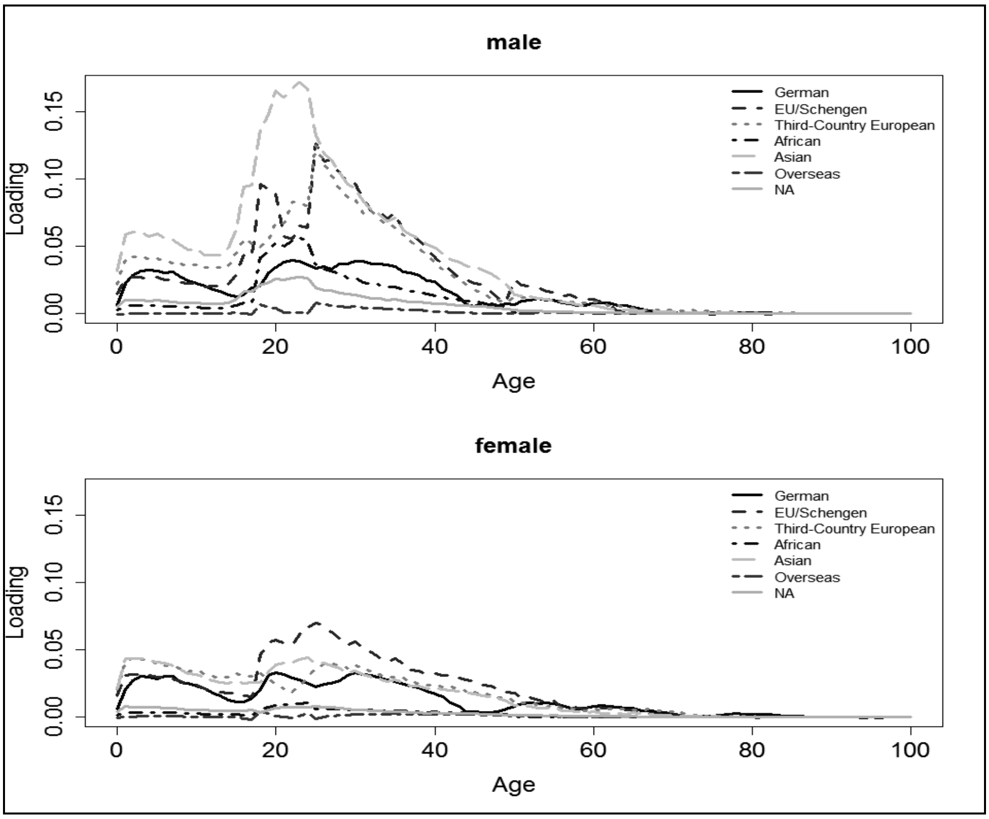
Figure 1: Component loadings for the first principal component



Source: our own calculations and visualisation

It is apparent that the component loadings are especially high for people in an approximate age range of 18 to 50 years old, which suggests that this principal component primarily reflects labour migration. We observe here, for example, that high values for the principal component mean high negative net migration figures for Germans, while (ceteris paribus) high positive net migration numbers are simultaneously generated for EU and Schengen nationalities outside of Germany, as well as for Asians. This suggests that there are replacement effects in the labour market by which emigrating German workers are primarily replaced by foreigners from the EU and Asia, and from India and China in particular, although not one-to-one.

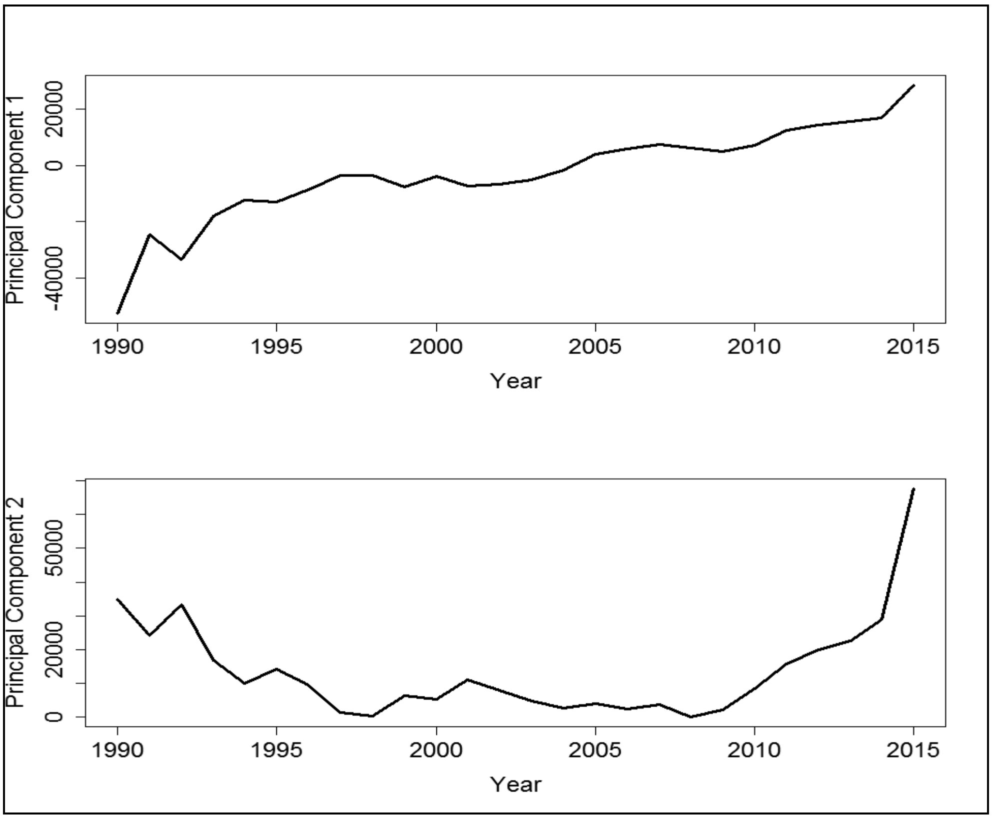
Figure 2: Component loadings for the second principal component



Source: our own calculations and visualisation

The second principal component is essentially positively correlated with net migration; it appears to be a general indicator for the level of immigration. The component loadings are especially high here for male Asians around 20 years of age, and for European foreigners in the mid-20s to early 30s range. This observation can be effectively put in context with the help of the results visualised in Figure 3, which presents the theoretical historical course of the first two principal components since 1990.

Figure 3: Historical course of the first two principal components



Source: our own calculations and visualisation

Principal component 1 increases almost unvaryingly over time, which seems plausible in the context of increasing globalisation. For principal component 2, a nearly U-shaped curve can be observed over the period presented here, with the level gradually declining from very high in the early 1990s due to the fall of the Soviet Union and the Yugoslavian civil wars, then remaining relatively stable until 2008. An increase can then be observed, caused by the euro crisis in connection with the EU expansion to Romania and Bulgaria, since an increasing number of young people have been migrating from crisis-stricken countries in southern and eastern Europe to Germany, and northern Europe in general, since that time. A further boost in migration came in 2014 with the latest expansion of the EU to include Croatia. The effect of the refugee crisis is seen in the 2015 peak, caused by Chancellor Angela Merkel's proposal in the summer of 2015 to make it easier for people from the countries terrorised by the so-called Islamic State to be recognised as refugees in Germany. This led people from other economically underdeveloped countries like Albania and Kosovo to seek refugee status in Germany as well (cf. Bundesamt für Migration und Flüchtlinge 2016: 118-123). Migration from Afghanistan, where the security situation remains as precarious as ever, primarily due to the actions of the Taliban (cf. Heidelberg Institute for International Conflict Research 2017: 71-73), has shot up again as a result of the refugee crisis. The heavy emphasis on male migration is due to the fact that men mostly flee from crisis-stricken regions first in order to bring the rest of their families safely to them later through family reunification procedures (cf. Leubecher 2017). At this point, the question naturally arises as to how to generate forecasts for this highly complex situation. In the following section, we will develop a proposal to address that question.

4 A stochastic Forecasting Approach for Migration between Germany and other Countries

The previous section introduced the underlying data, for which a PCA was performed. These interim results now constitute the basis for a forecast of international net migration in Germany. We will first adjust a parametric time series model to the first principal component, based on the criteria proposed by Vanella (2017), using a Box-Jenkins model[9] to estimate the autocorrelation (see Vanella 2017: 9-17). The resulting model for the time period in question is as follows:

$$c_1(t) = -48213.6744 + 19195.5931 \ln t + 0.3421 \delta_{t-1} + 0.6579 \delta_{t-2} + \varepsilon_t,$$

where $\delta_t = c_1(t) - [-48213.6744 + 19195.5931 \ln t]$ and $\varepsilon_t \sim N(0, 4827.2400^2)\ \forall t$

This assumes logarithmic growth as a long-term trend, which is realistic in light of historical trends.

For principal component 2, the future course is not so easy to derive, since it is strongly linked to current events and crises. We will assume here that due to a gradual recovery of the southern European economy, appropriate measures taken by the German Ministry of the Interior with regard to refugee policy, and military successes in the fight against terror, the crises will gradually stabilise, and that the level of this principal component will therefore converge over the long term to its long-term mean. This initially seems like a very strong assumption for a forecast, but it can be modelled very well stochastically with an AR(1) model, so that the negative growth of the principal component is not chosen arbitrarily. Through an appropriate quantification of PIs, this initial assumption about the future course is quickly relaxed. The forecast model for principal component 2 that results from the OLS estimation is represented as follows:

$$c_2(t) = 13842.6325 + 0.9281 \omega_{t-1} + \xi_t,$$

where $\omega_t = c_2(t) - 13842.6325$ and $\xi_t \sim N(0, 9767.2033^2)\ \forall t.$

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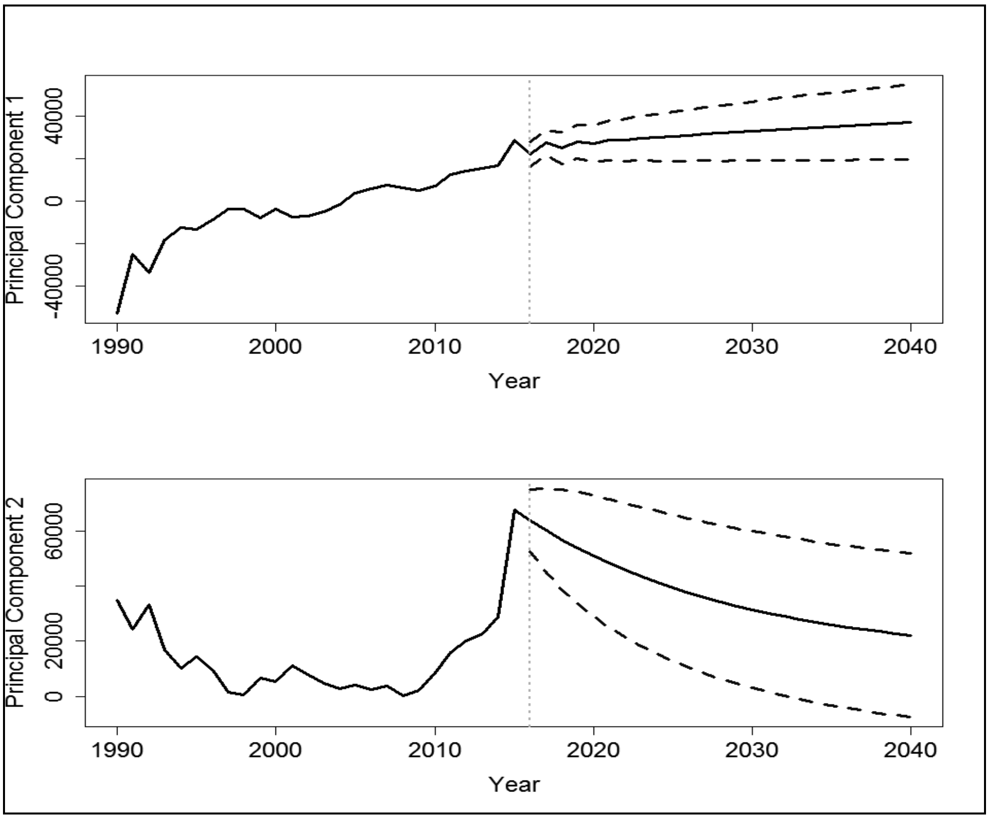
[9] See Box et al. 2016: 88-116 for a detailed description of Box-Jenkins ARIMA models.

The forecast for the first two principal components based on this model is presented in Figure 4, with a 75% prediction interval (*75%-PI*) for purposes of illustration.

Figure 4: Forecast for the first two principal components with 75%-PI



Source: our own calculations and visualisation

We observe that the labour market trends are subject to relatively lower risk, and will continue to trend positively on average. The PI for principal component 2 is relatively wide, so that other courses than the one assumed for the average are appropriately included in the model and associated with probabilities.

This is one of the major advantages of a stochastic analysis over a deterministic one. Even in cases where the trends are less clear, any assumptions which are highly unlikely to occur ex-post are weighted significantly less heavily. The remaining 1,412 principal components are assumed to be Random Walk processes,[10] and are included as such in the estimation of the overall migration risk. Based on the models, a simulation study is now conducted, with all principal components simulated 10,000 times through the year 2040 using Wiener processes[11]. As explained by Vanella, the simulation val-

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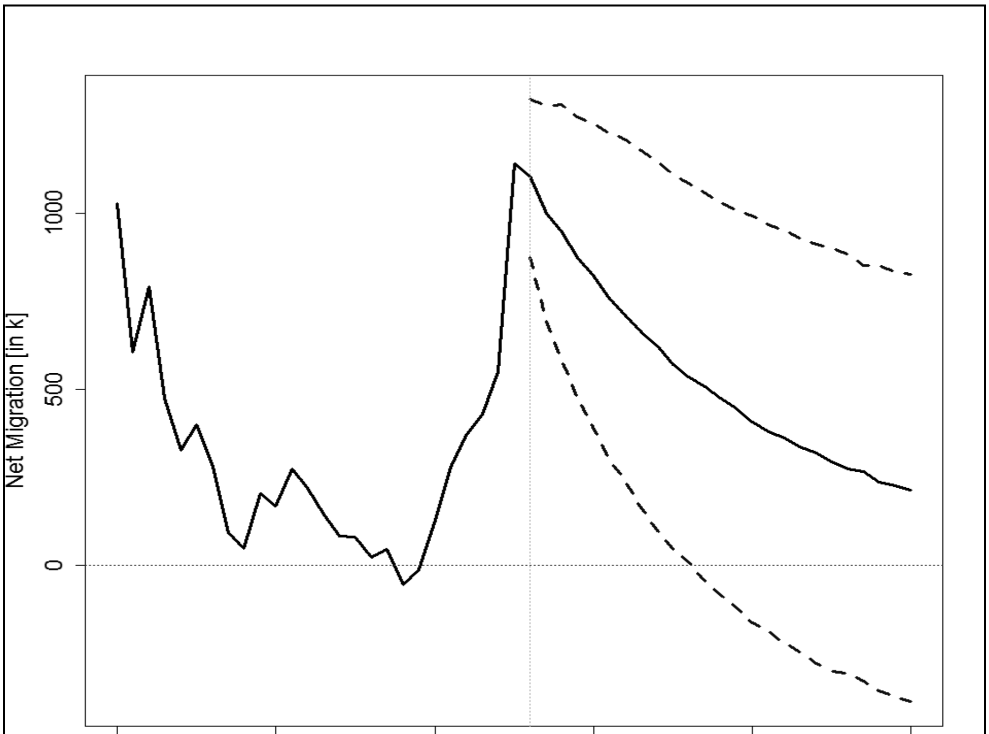
[10] See e.g. Vanella 2017: 12 for the definition of a Random Walk process.

[11] For a detailed explanation of Wiener processes, see e.g. Tavella 2002: 13-17.

ues are now back-transformed (cf. Vanella 2017: 21), so that 10,000 simulated net migration paths are generated for each age from 0-100 years, both genders, and the seven nationality groups. Due to the high dimensionality of the simulation study, a detailed presentation of all forecast results would go well beyond the scope of this article. Below we will therefore look at just a few selected results in detail. Figure 5 illustrates the forecast for total net migration in Germany through the year 2040 with 75%-PI that can be derived from the simulation study.

It should be noted here that at the time this contribution is done, no detailed migration figures are yet available for 2016. The value shown here is therefore a prediction based on the model developed in this article.

Figure 5: Net migration in Germany with 75%-PI

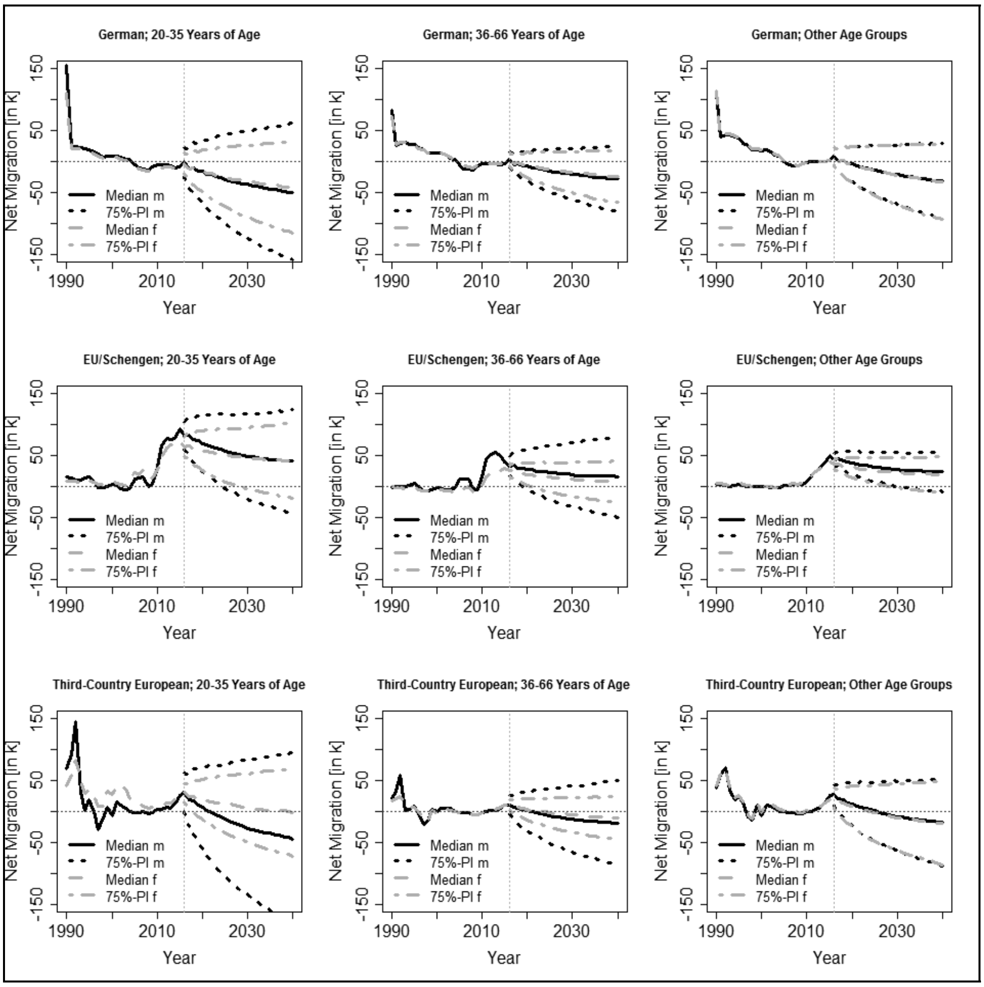


Source: our own calculations and visualisation

We can see that the median level is expected to gradually decline over time, down to 212,419 in 2040. As expected, however, the variance in net migration is very high, since migration is, as mentioned earlier, the demographic component with the highest stochasticity. The 75%-PI is therefore [-387.809; 826.879] in 2040. The median and mean for these results asymptotically converge toward both the forecast of

Deschermeier (2016) and the update to the 13th coordinated population projection based on 2015 data (Statistisches Bundesamt 2017c). Based on our model, a wide range of specific analyses can be conducted. Figures 6 and 7 present derived forecasts for net migration for both genders, three age groups (20-35 years, 36-66 years, and other ages), and the nationality groups. For the sake of clarity in the representation, and because the level of migration between the overseas group and Germany is relatively low, this group is combined with the other nationalities here. Each row in the graphic relates to a particular nationality group, and the forecasts for both genders are represented with 75%-PIs.

Figure 6: Net migration by age groups for Europeans to 2040



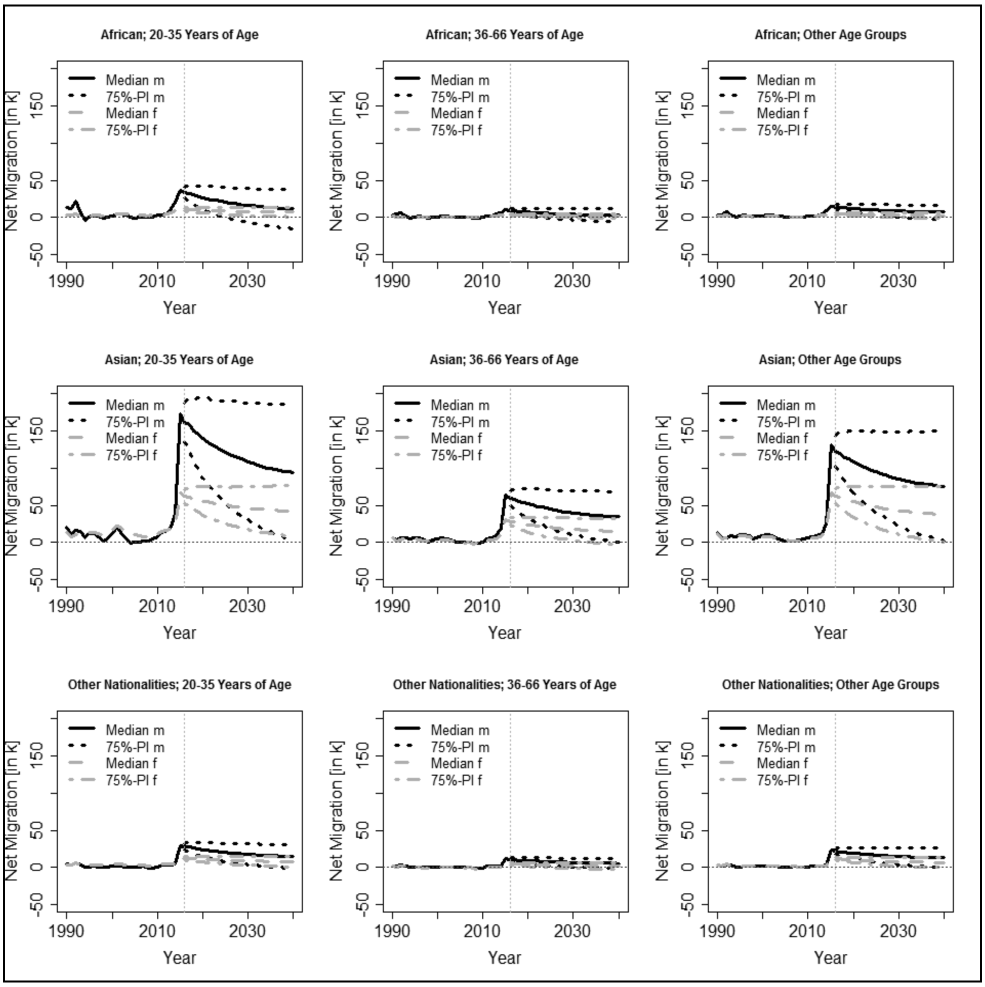
Source: our own calculations and visualisation

For the Germans, a very strong correlation is apparent in the net migration for both genders; a general downward net migration trend is to be expected due to labour migration, studies abroad, and emigration by older Germans to countries with a milder climate or more attractive landscape. This phenomenon was extensively investigated by Rogers et al. in the 1980s for migration movements to southern states within the United States, as well as for international migration (cf. Rogers/Watkins 1987: 490-509; Rogers/Castro 1981: 6). In many migration cycles, peaks were identified at the start of retirement age. The Spanish islands, Austria and Switzerland are especially popular retirement destinations for German emigrants, since these regions have warmer temperatures and/or more attractive landscapes than the migrants' regions of origin in Germany. In addition, German migrants have relatively little difficulty in adapting when they move to these regions, either because there is already a sufficiently large German population living there, or because these countries are themselves German-speaking (cf. Hildebrandt-Woeckl 2015). Nor are there any problems with receiving pension payments even when changing one's place of residence to another EU country (cf. Deutsche Rentenversicherung 2017).

For the EU and Schengen countries other than Germany, the model assumes a moderate decline from the 2015 level. This implies either significant remigration to the southern countries, or a slight reduction in the level of emigration in the crisis-stricken countries. This would assume that the economic situation in those countries will gradually stabilise. This appears realistic in Spain or Portugal, but questionable for Italy and Greece, although optimistic estimates do assume increased stability for those two economies (cf. OECD 2016: 162-238). The high degree of uncertainty in these assumptions is reflected in the very wide PIs, especially for people of working age. We observe here that male migration has risen sharply relative to female migration since the eurozone crisis.

The forecast implies that the two genders will again approach one another for working ages. The effect of crises can be seen very clearly for the other Europeans. In the early 1990s, for example, we observe large migration flows resulting from the collapse of the Soviet Union, and the consequences of the Yugoslavian civil wars among younger age groups. It is striking here that in more peaceful times, the migration level among women consistently lies somewhat above the men's level, but that increased male migration to Germany is observed in periods of crisis. More recently, we also see this after the financial crisis in connection with the German government's policy in the 2015 refugee crisis, which led many men from the poorer Balkan countries to request asylum in Germany (cf. Bundesamt für Migration und Flüchtlinge 2016: 118-123).

Figure 7: Net migration by age groups for non-Europeans to 2040



Source: our own calculations and visualisation

For these nationality groups, we can clearly observe a flight from war, terrorism and poverty in recent years, which has especially increased since the so-called “Arab Spring”, and then as a result of the rise in Islamist terrorism driven by the so-called Islamic State, while the threat posed by the Taliban remains very high. This explains why the differences in migration figures between genders, which were previously almost non-existent, have been so large in recent years. Here again, the forecast assumes that the situation in the crisis-stricken regions will tend to stabilise on average, which would result in a normalisation of the migration level between Germany and these countries. This would require that military successes be achieved against the IS, the Taliban, Boko Haram and other terror groups, which can already be seen in certain places (cf. Heidelberg Institute for International Conflict Research 2017: 189-197), but these changes involve a long-term process in which improvements will be observed only gradually. The fighting may drag on for a long time yet. Some military experts assume that the war against the IS and other Islamist groups will continue for several decades (cf. Page 2014). Even if Islamist terror movements were to be destroyed, a long process of reconstruction would then follow in the affected regions, potentially requiring immense amounts of help from the West, Russia and Iran (cf. World Bank 2016). The wide PIs for net migration reflect the uncertainty of this process. This is especially true for Asian males.

5 Conclusion, Limitations and Outlook

This article addresses the issues that confront scientists with modelling in general, and with forecasts of international migration in particular. We have examined a few potential solutions, discussed our own proposed approach, and implemented that model in a practical application. Based on two datasets provided by the German Federal Statistical Office, a synthetic dataset was developed that contains estimated values for international migration movements in the three dimensions of age, gender and nationality group for the baseline period of 1990-2015. This dataset was used to forecast age-, gender- and nationality-specific net migration figures between Germany and other countries up until 2040. The principal components time series approach suggested by Vanella (2017) was modified and used for detailed simulation of the net migration figures. In addition to the median scenario, PIs were generated for each net migration figure by simulating 10,000 times with Wiener processes. Besides total net migration, the article also illustrated 36 subgroups with PIs. In reality, however, the model actually estimates the future course of all 1,414 variables simultaneously. In the median trajectory, a decline in net migration is expected from the recent historically high level, but migration is not expected to drop as rapidly as assumed in most studies.

The model has a number of limitations. For example, at 26 years, the baseline period is relatively short, since data before 1990 (i.e. before German reunification) would no longer be meaningful for the analysis of current and future migration flows. Furthermore, the data were not available at the level of consistency and completeness needed for the analysis, so interpolations had to be made based on specific assumptions. The high stochasticity of migration and its dependency on exogenous political, social, economic and climate factors makes it difficult in general to predict future migration, and requires assumptions and simplifications to make a forecasting model manageable. Against this background, assumptions had to be made about the future development of the eurozone crisis and the humanitarian crisis in Arab countries. This is fundamentally a subjective process, although it was objectivised to some degree through the adjustment of appropriate statistical models. In general, quantification of risk through the use of PIs relaxes the original assumptions somewhat, thereby improving the validity of the model again.

One general weakness of quantitative models is that they are based solely on the past, and do not take unpredictable shocks into consideration. Subjective assumptions could be used to integrate such possibilities, although rather than providing real probabilities and the associated PIs, this would only provide “credible intervals”, as they are referred to in Bayesian terminology.[12] Because our goal is to generate the best possible forecast, rather than a projection, we decided not to use this type of model.

In general, it is fair to say that the coming years will provide a lot of insight into further developments. Alongside the addition of new data and the resulting larger sample, it will also become clear whether the political steps taken thus far will have long-term results, or whether the refugee crisis will continue. If developments at that time turn out to be fundamentally different than what was encoded in the assumptions or model parameters, it will be scientists' task to update the results or create new models for these developments. After all, existing forecasts and projections do not seek to predict absolute truth. They merely provide decision-makers with quantitative data as a basis for their work.

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[12] See e.g. Lee 2012: 54-55.

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Chapter 4

A Principal Component Simulation of Age-Specific Fertility – Impacts of Family and Social Policy on Reproductive Behavior in Germany

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A Principal Component Simulation of Age-Specific Fertility – Impacts of Family and Social Policy on Reproductive Behavior in Germany

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Abstract

This contribution proposes a simulation approach for the indirect estimation of age-specific fertility rates (ASFRs) and the total fertility rate (TFR) for Germany via time series modeling of the principal components of the ASFRs. The model accounts for cross-correlation and autocorrelation among the ASFR time series. The effects of certain measures are quantified through the introduction of policy variables. Our approach is applicable to probabilistic sensitivity analyses investigating the potential outcome of political intervention. A slight increase in the TFR is probable until 2040. In the median scenario, the TFR will increase from 1.6 in 2016 to 1.63 in 2040 and will be between 1.34 and 1.93 with a probability of 75% under the most realistic policy scenario. Based on this result, it is unlikely that the fertility level will fall back to its extremely low levels of the mid-1990s. Four simple alternate scenarios are used to illustrate the estimated ceteris paribus effect of changes in our policy variables on the TFR as well as the results of simple extrapolations.

Keywords

Fertility, statistical demography, forecasting, family policy, principal component analysis, time series analysis

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1. Introduction

The future development of fertility is of great importance in many areas, particularly regarding planning for and evaluation of future needs for political intervention in family and social policy. The fertility level of a country is commonly represented by the total fertility rate (TFR), which is the sum of the age-specific fertility rates (ASFRs) over all ages during a specific year. Therefore, the TFR can be interpreted as the average number of children a woman bears during her reproductive phase given that the current ASFRs remain constant in the future. As this assumption usually does not hold for any woman, the TFR is considered to be a hypothetical measure (Bongaarts and Feeney 1998). For decades, the TFR in most parts of Europe has been well below replacement-level fertility (Eurostat Database 2018a), which is approximately 2.1 for these regions (Bujard 2015a; United Nations 2015; van de Kaa 1987).

Although births have no immediate effects, for example, on the financial balance of social insurance, they are the most important demographic factor in the long run. First, in a Bismarck-type social security system, the insured working population pays contributions from its labor income in every period. These contributions are then transferred to the recipients of social insurance payments during the same period (Graf von der Schulenburg and Lohse 2014:425–89). An example of this process, which is especially influenced by the aging of the population, is the pension system, where social security contributions are given to the retirees as pension payments. Another example is long-term care insurance, to which the working population regularly contributes, and is provided to the population in need of care. Low fertility in the long run leads to a ceteris paribus (c.p.) smaller workforce and a shift in the age structure in favor of older people. As a result, relatively fewer people of working age shoulder a higher financial burden in terms of social insurance for relatively more elderly people (Bujard 2015a; d'Addio and d'Ercole 2005). This effect is felt approximately 20 years after the birth year of a certain cohort, when it begins to enter the labor market. Small birth cohorts thus lead to shortages in the labor market (Bujard 2015b:136–139). Second, strong birth cohorts, such as the baby boomers in many European countries after World War II, in the long run lead to high demand for social insurance when they reach higher ages. Moreover, morbidity risks increase in old age, leading to higher health costs (World Health Organization 2015:95–98).

Forecasting future ASFRs therefore provides important quantitative information for political decision-making in response to these predicted trends (Zuchandke, Lohse and Graf von der Schulenburg 2014). For example, political decisions about pension system reforms must consider the future course of fertility in society and the resulting population structure. Family policy must attempt to address political measures to increase the TFR for low-fertility countries, which include the majority of Europe and East Asia (Bujard 2015a).

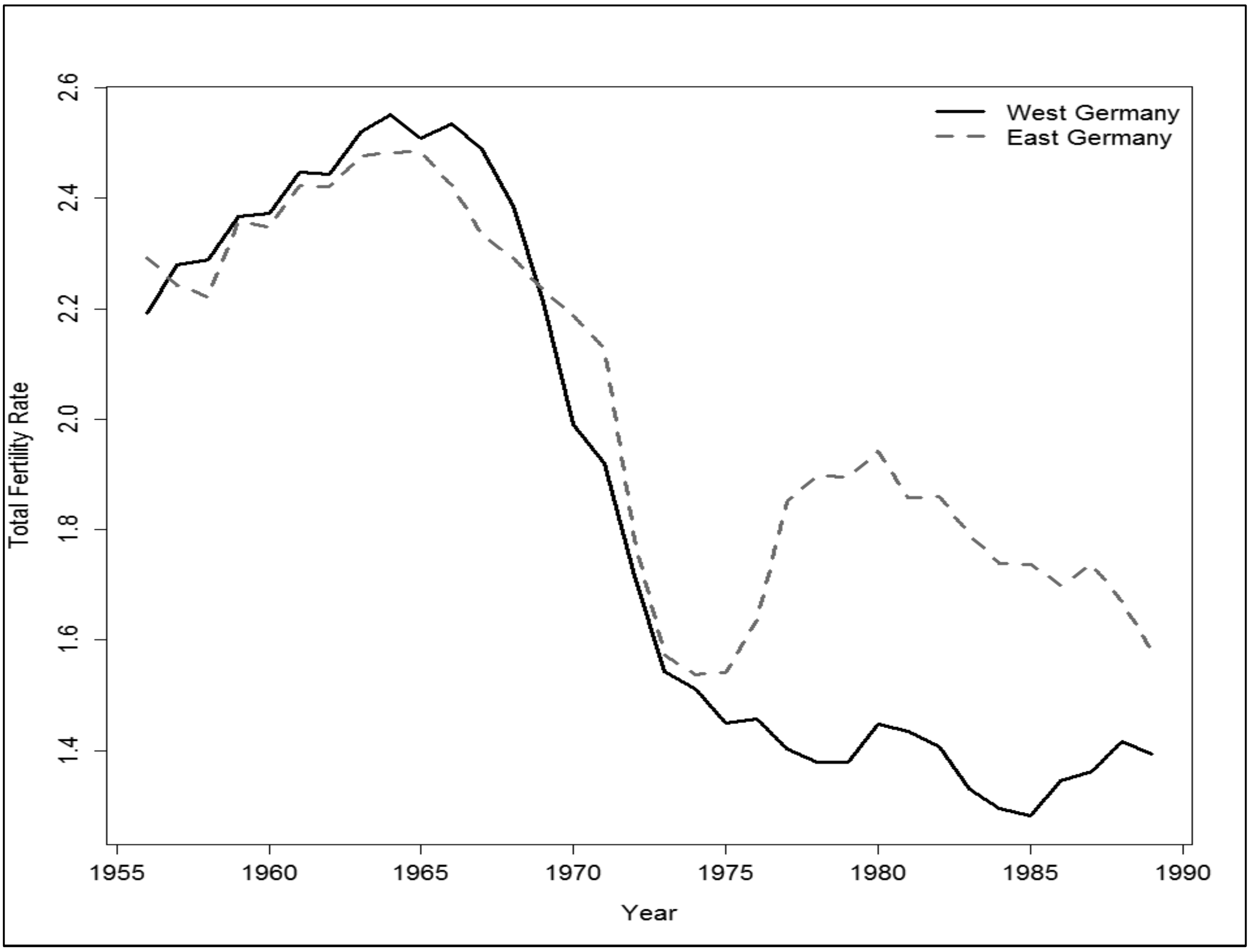
This contribution proposes a conditional forecast approach for the future course of ASFRs. The model framework is based on the Lee–Carter model for fertility (see Lee 1993), which makes use of principal component analysis (PCA) and time series (TS) analysis. We expand the framework by including family policy variables in the PCA, allowing a stochastic estimation of the potential future impact of political measures aimed at increasing a country's reproductive level. The methodology enables the integration of the correlations among the ASFRs and the autocorrelations among each set of ASFRs. Simulations of Wiener processes enable stochastic quantification of the future course of the ASFRs through prediction intervals (PIs). The trajectories of the ASFRs can be cumulated to stochastic forecasts of the TFR, which will be illustrated with 90% PIs.

In the next section, we give an overview of the fertility development in Germany since the mid-1960s, together with some sociological and juridical background. Section 3 provides a short review of past and present approaches for projecting or forecasting fertility in Germany, although many of these models are commonly applied internationally as well. Section 4 discusses the connection between family and social policy and a country's fertility level with a condensed literature review. In Section 5, we propose our forecast model. The model is in essence a two principal component time series model connected to the classical Lee-Carter model for fertility forecasting (see Lee 1992:321–323). The model indicates that fertility trends are strongly affected by the tempo effect in fertility (Bongaarts and Feeney 1998). Moreover, we conclude that the quantum of fertility can be influenced by adequate political measures. The model is applied for probabilistic sensitivity analyses of reforms in family policy based on a selection of variables available for Germany. However, the framework can be applied to other countries and should work even better for countries with longer TS.

2. Past fertility trends and reforms in family and social policy in Germany

In societies that follow a Bismarck-type principle in social security, low fertility rates over longer horizons are quite problematic. Small birth cohorts mean small cohorts entering the labor market when they reach the working age, which results in shortages in the labor market. Since that generation is obliged to pay the biggest part of the retirement income of the elderly population via contributions from their labor income, this leads to a high financial burden on the working population. This is especially difficult when small cohorts follow a phase of very high fertility, as has been the case for many European countries in the late 1960s and early 1970s. Figure 1 illustrates this situation by the sharp declines in the TFRs of East *(Deutsche Demokratische Republik or DDR)* and West Germany *(Bundesrepublik Deutschland or BRD)* after the mid-1960s.

Figure 1. Total fertility rates in West and East Germany



Sources: Human Fertility Database 2016a, 2016b; own design.

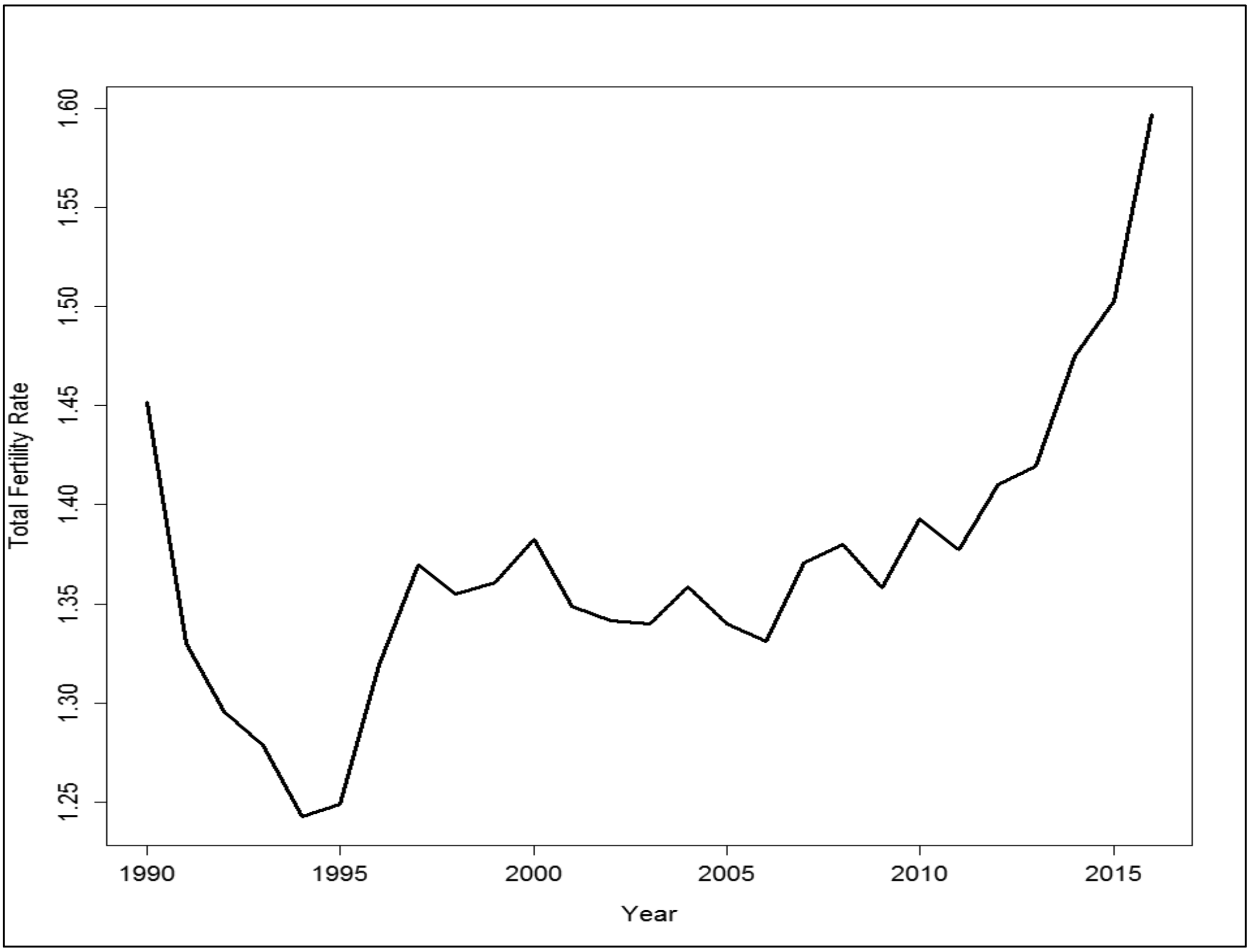
We observe increasing TFRs for both parts of Germany after World War II, reaching a climax in the mid-1960s, after which the TFRs decrease quite heavily until the early 1970s. The main reason for this was the second wave of the women's rights movement beginning in 1968, which was especially distinct in West Germany (Hertrampf 2008). The feminist movement has led to a postponement of births to an older age (Gustafsson 2001; Lesthaeghe 2010). Whereas this trend is persistent, it is important to determine the extent to which these postponed births are being recuperated by births in older age groups. Although the introduction of the birth control pill is not causally linked with the decline in births, the active decision to conceive children was facilitated by its market release at the beginning of the 1960s (Bundeszentrale für politische Bildung 2015). This trend persisted in the BRD until the late 1970s, and the TFR decreased to 1.38 in 1978 after reaching 2.54 in 1966. By contrast, the TFR in the DDR declined to 1.54 children by 1974, after which it increased to 1.94 in 1980.

Such strong fluctuations are problematic for countries following a Bismarck-style social security system, as explained above. Therefore, it is important to discuss options in family policy to influence the level of fertility. Whether these efforts indeed have an influence on fertility is subject to debate. This controversy exists in part because the effects of certain measures are difficult to specify since their impact can be observed directly only in the long run or not at all (Bujard 2011; d'Addio and d'Ercole 2005). For instance, statistical testing of the impact of policy in Germany is extremely difficult for several reasons. First, German reunification in 1990 caused a structural break. Second, time series (TS) of policy variables are not always available for long past horizons (e.g., due to reforms and the resulting structural breaks in the data).

There is a vast discussion in the literature about the effects of family policy on fertility. The conclusions differ strongly depending on the data source, the geographical units or countries under study, the methodology and the variables, as Bujard (2015a) has shown. For instance, the divergence of the trends in the two parts of Germany after the mid-1970s has been discussed in the literature. Temporal and infrastructural support in family policy in the DDR since 1972 were hypothesized to have an effect on the TFR. These effects nevertheless appeared to vanish after 1980 (Büttner and Lutz 1990; Höhn and Schubnell 1986). In 1979, the BRD, whose family policy had previously been restricted to financial support, began to alter its policy with the passing of the Maternity Leave Act (*Gesetz zur Einführung eines Mutterschaftsurlaubs)*. This law ensured maternity leave of up to six months after childbirth for employed women. Furthermore, the law forbade employers from releasing female employees during their maternity leave (BGBl I 1979/32). The next milestone in the undertaking of raising the TFR was the Federal Child-Raising Allowance Act *(Bundeserziehungsgeldgesetz or BErzGG*) in 1985. The BErzGG gave one parent, independent of sex, the opportunity to take paid parental leave of up to 10 months after the birth of a child (BGBl I 1985/58). This period was lengthened in the following years up to the first three years after birth (BGBl I 1989/32; BGBl I 1991/64). Figure 1 (previous page) shows an increase in TFR in 1980 and after 1985 – the years after the mentioned reforms – implying a positive effect of the family policy measures on reproductive behavior, although the effect might weaken after a certain period. Figure 2 (next page) shows the TFR for reunited Germany since 1990. After reaching its minimum in 1994 at approximately 1.24, the TFR has since recovered slowly.

In 2007, the BErzGG was annulled with the introduction of the *Bundeselterngeld- und Elternzeitgesetz (BEEG).* Whereas the BErzGG offered a constant amount as financial compensation for the time spent raising a child, the parental allowance *(Elterngeld)* varies depending on the wage the person taking parental leave earned during the twelve months before the parental leave and can be up to two-thirds of the average wage of the person during that time span. The *Elterngeld* can be paid for up to 14 months

Figure 2. Total fertility rate in Germany



Sources: Destatis 2015a, 2015b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

for each couple, whereas the payment for one of the partners is limited to 12 months. An additional innovation was that the parental leave can be split among the first years of life of the child. Currently, the total allowable parental leave is up to three years taken over the first eight years after childbirth (BGBl I 2006/56). Thus, whereas classic measures in family policy have focused on financial compensation, measures since the late 1970s identified time as another important factor. The goal has shifted to giving employed potential parents the option to take time off work to raise a child without the risk of losing their jobs.

Following the *Trias of Family Policy* concept (Bujard 2011), family policy started to offer infrastructural support in childcare in addition to financial compensation and time for child raising. Since 1996, all parents in Germany have a claim for a childcare placement for their children over three years old (Spieß 2014). Whereas child care opportunities were rare in the past, especially for very small children under three years of age, in 2008, the *Kinderförderungsgesetz (KiföG)* was passed. Since August 2013, the *KiföG* has guaranteed childcare placement for children under three years of age if both parents are employed or in school (BGBl I 2008/57). Moreover, the government ran the *Zukunft Bildung und Betreuung* project (Future Education and Care), which, between 2003 and 2009, subsidized the construction of full-time schools and the evolution of normal schools to full-time schools (Bundesministerium für Bildung und Forschung 2018). These initiatives demonstrate the increasing awareness of the importance of giving both parents the flexibility to return to work relatively quickly after childbirth, and therefore, the high priority of coping with low fertility in family politics. The TFR has recently reached 1.60, its highest level since the early 1970s (for both parts of Germany combined). The question arises as to whether this trend stems from effective political measures.

3. Fertility forecasts and projections for Germany

Future projections for fertility in Germany are often based on deterministic scenario analyses. Germany's federal statistical office, *Destatis,* assumes a constant TFR in its 13th coordinated population projection for Germany. The underlying assumptions are that ASFRs will be decreasing for younger women under 30 years of age and that those losses will be balanced by increasing ASFRs for older women. This effect is known as the tempo effect (Bongaarts and Feeney 1998). These assumptions are based on survey data of the 2008 and 2012 micro-censuses on childlessness and the number of children mothers conceive in combination with historical trends of the final number of children for older cohorts. The trends between these two points in time represent the trends mentioned above. In an alternate scenario, which the authors classify as realistic, based on expert opinion rather than on empirical facts, they assume a slight increase in the TFR. This increase would result from a larger increase in the ASFRs of women over 30 years of age and constant fertility rates for younger women. In this case, we would not only see the tempo effect but also a quantum effect, resulting in an increasing TFR. The TFR in this scenario will increase to 1.6 in 2028 and remain constant thereafter[1] (Pötzsch and Rößger 2015). While the model assumptions might be based on reasonable arguments, they appear to be too restrictive for deriving long-term trends through 2060. Forecasting future trends into the far future is not advisable based on the very recent past. The longer the forecast horizon, the wider is the range of possible future scenarios since uncertainty arising in each period accumulates over time (see, e.g., Box, Jenkins, Reinsel and Ljung 2016:136–147). In simple terms: We know far more about the demographic behavior in the next year than about that in 30 years. Therefore, deterministic assumptions about future fertility become less and less likely the more distant into the future we are looking (Lee 1998:166–167). Furthermore, the large range of possible scenarios is neither identified nor quantified with individual probabilities.

The United Nations (UN) proposes a Bayesian hierarchical model (BHM) for stochastic projection of TFRs (United Nations 2015, 2017; Raftery, Alkema and Gerland 2014; Alkema et al. 2011). The BHM is a global model composed of 158 countries, which are classified into one of three possible cases: high-fertility countries (Phase I), countries transitioning from high to low fertility (Phase II), and low-fertility countries (Phase III). Germany is classified as a Phase III country because its TFR has been below the replacement level of 2.1 children since the early 1970s, as illustrated in Figure 1 and Figure 2. The TFR for Germany is assumed to slowly recover and converge toward 2.1 in the long run and is modeled by an autoregressive model of order one [AR(1)]. The quinquennial TFR is stochastically simulated 60,000 times with Markov chain Monte Carlo algorithms to identify the median scenarios with PIs through 2100. In the median scenario, the TFR in Germany is expected to exceed 1.6 through the mid-21st century and to exceed 1.7 by the end of the century. The trajectories for the TFR are thereafter distributed over the reproductive ages leading to trajectories of the ASFRs. These schedules are weighted averages of the past experience of low-fertility countries and Germany's latest historical development with respect to age-specific fertility. The fertility schedule is assumed to converge in the long term toward the global age-specific fertility schedule (Ševčíková et al. 2015). The UN model has some interesting features. It quantifies uncertainty via stochastic simulations while including both national and international trends. One might wonder whether the mathematical assumptions in the model are too restrictive in assuming a global convergence of international fertility trends. Similar points can be made about the ASFRs. The model proposed in our paper takes correlations among age groups into consideration without imposing excessively strict assumptions about their future behavior.

Alders, Keilman and Cruijsen (2007) attempt to combine the strengths of quantitative and qualitative models with TS models and perform TFR forecasting for 18 European countries, including Germany.

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[1] Note that the TFR in Germany in 2016 has already reached this level, as explained in Section 2.

For Germany, they use data from 1950–2000 to estimate a generalized autoregressive conditional heteroscedasticity model resulting in point forecasts and 80% PIs from 3,000 trajectories until 2050. Given the estimated TFR, age schedules are used to estimate the ASFRs. The plausibility of the results of the quantitative forecasts are qualitatively assessed by two fertility experts. Although the technique appears to consider all necessities, the estimated 80% PI for the TFR in Germany is between 0.88 and 2.21, which is too wide for valuable policy implications. While we think it is better to overestimate future uncertainty instead of underestimating it, the 80% PI for the TFR appears to be overly conservative for Germany, where it has never been below 1.2 historically (Rahlf 2015) and has not surpassed a value of 2.2 since the early 1970s (see Figure 1). Another caveat is the assumption of an age schedule, which ignores the tempo effect.

Alho (1990) proposes indirect estimation of the TFR through forecasting the average ASFR. He constrains the average ASFR to upper and lower bounds through a modified logistic transformation. We will borrow that idea to some degree, as will be explained in Section 5. Bozik and Bell (1987) propose applying a PCA dimensionality reduction in estimating future ASFRs based on ARIMA forecasts of the TFR. Lee (1993) calculates a fertility index for indirect estimation of the ASFRs, which he derives from a PCA for the ASFRs. He integrates Alho's transformation into his forecast model to constrain the TFR to within certain bounds. To include uncertainty in the forecast, Lee applies a simple autoregressive moving average model [ARMA(1,1)], with which he simulates 1,000 trajectories for the fertility index. Approximate 95% PIs for the TFR can be derived with this process. Härdle and Myšičková (2009) apply Lee's model to forecast the TFR in Germany until 2060 with PIs. That method has some flaws. First, it assumes that the mean TFR remains constant at its last observed level, thereby ignoring the current fertility trends. Second, the PIs are rather narrow and have an unintuitive structure because they are wider for the first few periods and stagnate thereafter. Realistically, the risk in future predictions should increase for more distant points in the future, as explained above. Moreover, the restriction to one simple fertility index completely ignores the uncertainty associated with the remaining PCs, which leads to a systematic underestimation of future risks. Fuchs, Söhnlein, Weber and Weber (2018) use a similar approach to forecast the labor force in Germany until 2060. They distinguish between nationals and foreigners, thereby including possible effects of international migration on fertility. Hyndman and Ullah (2007) propose a robust adjustment to Lee's model that is insensitive to past outliers due to extreme events, e.g., wars or epidemics. Deschermeier (2015) applies this approach to forecast the population of Germany until the year 2030. A caveat of this approach is that outliers are assumed to be one-time events that cannot be repeated in the future. Because events that occurred in the past should not be ruled out for the future and are assumed to have zero probability, this approach leads to underestimation of future uncertainty in the forecast, which is discussed further in Section 5.

Since the 1980s, forecast models for cohort, rather than period, fertility have gained popularity. One such approach is the cohort autoregressive integrated moving average model of de Beer (1985). A modern approach based on Bayesian statistics is presented by Schmertmann, Zagheni, Goldstein and Myrskylä (2014). Although the cohort perspective has its advantages and is well justified, we prefer a period perspective because period effects can be observed in a more timely manner. Summary measures, such as the cohort fertility rate, provide clear information about the average number of children a cohort of females has given birth to. We are able to analyze these results only *after* the cohort's reproductive phase; therefore, there is a large time lag to consider.

4. Connections between fertility and family policy

Section 2 illustrated the complexity of family policy and the resulting difficulty of evaluating it for the case of Germany. Past studies have addressed these questions from theoretical or applied perspectives.

Economic factors have been found to effect the reproductive level of couples – at least since an investigation by Jaffe (1940) of United States (U.S.) Census Bureau data, which showed differences in fertility rates with U.S. census and tax data for the years 1800–1840. On a descriptive level, evidence has surfaced showing lower fertility in the wealthier strata of the population, estimated by the worth of personal properties the families owned (Jaffe 1940). Kiser and Whelpton (1953), in 1941, produced a more thorough investigation of the factors influencing fertility, entitled the *Indianapolis Study of Social and Psychological Factors Affecting Fertility*. The authors conducted surveys with 1,444 couples from which they tentatively tested 23 previously stated hypotheses on the connection of certain socio-economic, demographic, psychological and contraception factors on the planned family size. Becker (1960) built a microeconomic framework stemming from classical utility theory. Within this framework children are defined as a 'good' that provides utility to a household but also incurs costs (either direct monetary costs or indirect costs, such as time needed for child-raising). Assuming perfect control over the number of children a couple conceives, the partners will maximize their expected net utility by creating a portfolio composed of children and other goods. Walker (1995) discusses the effects of investment in social security systems on the TFR in a low-fertility setting, using the case of Sweden in the 1980s. For this, he proposes a life-cycle model in which, quite similar to Becker's approach, the expected utility of females is maximized through optimal choice of timing of births and labor periods, which maximizes the present value of their lifetime utility, taking direct and indirect costs of children as well as statutory subsidies into account.

Given the presented theories alongside the extensive knowledge and accessibility of contraceptives, we would assume that couples on average would conceive the exact number of children they wish. Deviations from that number may be caused by biological infertility (Greil and McQuillan 2018:42–3) or infrastructural restrictions, such as excessive (opportunity) costs (Andersen, Drange, and Lappegård 2018; Testa and Bolano 2018) or the lack of childcare support (Andersen, Drange, and Lappegård 2018:900–2). Family policy could aim at facilitating these restrictions to help families reach their preferred number of children. Bujard (2015a) provides an overview of a range of international studies concerning the effects of family policy on fertility. Mostly, these studies attempt to estimate the impact via econometric models with the TFR as the endogenous variable.

Because a vast literature on this topic exists, we keep our focus on studies of fertility in Germany, especially in light of the lack of comparability for the impact of family policy across countries (see Kalwij 2010 on this). Cigno and Rosati (1996) investigated the influence of macroeconomic factors (income, male and female wage, interest rates) and family transfers in the form of child benefits and social security coverage on the TFR for Italy, the BRD, the USA and the UK using cointegration analysis. However, the analysis was largely incomplete because the model for the BRD did not include child benefits as an explanatory variable. One interesting result of the analysis was that social security coverage was negatively correlated with the TFR, indicating that a greater supply of social insurance has a negative effect on reproductive behavior. In a follow-up paper, Cigno, Casolaro and Rosati (2003) elaborated on the earlier model with a vector autoregressive model, using child benefit, social security coverage, pension gap, interest rate and mean real wage for both genders as explanatory variables. They identify positive correlations between the TFR and the lagged social security deficit. Moreover, the study supports the results of the earlier study; therefore, the generally good social security coverage in Germany leads to a reduction in the TFR, whereas a larger gap between working-age income and the received retirement pension supports the decision to procreate. In an investigation of micro-level data for 16 Western European countries, Kalwij (2010) applies a proportional hazard model to estimate the age-specific risk of child conception based on demographic, economic, and country-specific explanatory variables. He concludes that, among other effects, parental leave has a positive impact on the probability of bearing a first child, while subsidized daycare opportunities for employed mothers increase the likelihood to conceive an additional child if the mother had previously given birth. The results on financial benefits are ambiguous, which Kalwij traces to spurious regression when not controlling for the overall fertility, represented by the TFR and the crude birth rate, in the population under study.

Gauthier and Hatzius (1997) conduct regression of the TFR on economic and family policy variables for a pool of 22 industrialized countries and conclude that cash benefits have a positive effect on the TFR, whereas maternity leave opportunities have no effect on fertility. They attribute these results to the small variation in the maternity leave data. Adserà (2004) estimates a series of panel data models for data on 23 OECD countries for the years 1960-1997. She identifies, among other results, a highly significant positive effect of the length of maternity leave after birth on the TFR. Bujard (2011) performs a series of regression analyses for all OECD countries and finds that the financial benefits, the length of paternal leave and the rate of children under three years of age in daycare (lagged by one year) has a positive effect on the TFR. Furthermore, the analysis finds evidence that the costs of daycare are negatively correlated with the TFR, indicating that the marginal costs of children have a negative effect on birth rates (d'Addio and d'Ercole 2005). Bauernschuster, Hener and Rainer (2013) find, based on German panel data for the years 1998-2010, that a high degree of childcare coverage is associated with increased fertility rates for all age groups.

Using a micro-simulation approach based on panel data, Abiry et al. (2014:44–195) estimate the effect of different family policy measures on the birth numbers and cohort fertility alongside the female occupational behavior in Germany. For the *Kindergeld,* they quantify a positive, statistically significant effect on the short-term number of births as well as a long-term effect on the cohort fertility using a life-cycle model. Bujard and Passet (2013) estimate the effects of the *Elterngeld* reform in 2006 on fertility using SOEP data. They do not find a general statistically significant impact of the *Elterngeld* but conclude that it has a positive effect on the fertility of females over 35 years of age as well as for females with an academic degree. The reform therefore appears to have had an encouraging effect for recuperation of births in an older age group, especially for those with a high level of education, whose schooling takes a long time. Abiry et al. (2014:155–76) even derive a positive effect of that reform on fertility over all age groups with their micro-simulation. The authors, moreover, show that the insufficient supply of daycare spots and the costs these create for the parents lead to a significant decrease of the birth level in comparison to that envisioned by the potential parents.

The presented studies give an indication of the positive effects of different family policy measures on certain ASFRs and the TFR in general. These studies contribute to an understanding of reproductive behavior, but they cannot be operationalized in forecast studies that predict future reproductive behavior. The present study aims to fill this gap.

5. Method and data

The data used for this study are cumulated from multiple data sources to obtain a broad basis for our modeling approach. First, the live birth numbers by single years of age of the mother were obtained. Data for births since 1992 can be downloaded from Destatis' database, GENESIS-Online (GENESIS-Online 2018b). For the years 1968-1990, the age-specific birth numbers for the two parts of Germany were provided by Destatis on request (Destatis 2015b, 2018a, 2018b). The data for 1991 were also provided by Destatis for East Germany (Destatis 2018c), and the data for West Germany were taken from the Statistical Yearbook of the Federal Republic of Germany (Destatis 1993). The birth numbers for the years 1968 to 1991 were merged to avoid a structural break in the TS due to different geographical bases in the data. The population data by age and sex for Germany as a whole were provided by Destatis on request for the years 1967-2011 (Destatis 2016). The data for 2012-2016 were downloaded from GENESIS-Online (2018a). All the demographic statistics used were provided by Destatis, ensuring that no error in the data due to different sources exists. For the policy variables used in the next section, we extracted data on public expenditures for daycare from the *Kinder- und Jugendhilfestatistiken*, provided online by Destatis (Destatis 2004a, 2004b, 2005a, 2005b, 2005c, 2005d, 2005e, 2005f, 2005g, 2006a, 2006b, 2006c, 2006d, 2006e, 2009a, 2009b, 2009c, 2010; GENESIS-Online 2018e). Rates of inflation were calculated from the consumer price indices in Germany, which were downloaded from GENESIS-Online (2018c). The remaining policy variables were derived from the laws presented in Section 2.

Birth numbers in the data are broken down by single years of age and are cumulated for the upper age group of “50 years and older”. To take very late births into account without overestimating the ASFRs for 50-year-old mothers, the ASFRs for females aged 50-54 years were extrapolated by cubic splines estimation under the conditions that the splines cut the ASFRs of the 49-year-olds, that the ASFRs of the 52-year-olds (as median age of the group) are equal to the overall ASFR for the age group 50-54, and that the ASFRs equal zero at age 55. Cubic splines interpolation (technically, we perform an interpolation between age 49 and age 55) over a certain interval fits a process assuming a cubic parametric function

$$f(a) = \beta + \gamma * a + \delta * a^2 + \varphi * a^3, \tag{1}$$

whose parameters (in Greek letters) need to be defined. A unique solution requires four conditions, of which two are given by the two knots (left-hand and right-hand sides of the curve). The third and fourth parameters are obtained by running an optimization algorithm. In our case, we apply the Hyman filter of the Forsythe-Malcolm-Moler algorithm.[2] The Hyman filter (Hyman 1983) ensures monotonicity of the spline, which is crucial for our approximation to avoid negative estimates for the ASFRs. Cubic splines are often found optimal for smoothing over age groups in demographic modeling since they give not only curves passing certain points but also smooth fits where adjacent splines have identical slopes and curvatures in the knots (McNeil, Trussell and Turner 1977:246–252; Deschermeier 2011:775–777). Live births to 13- and 14-year-olds were approximated in the cases where they were not available following an assumed geometric decrease:

$$\tilde{B}_{14-\alpha,y} = B_{15,y} * \gamma_y^{\alpha}, \alpha = 1,2 \tag{2}$$

Past live births $\tilde{B}$ for 13- and 14-year-old mothers in year y were estimated in this way. In this case, γ is the growth rate resulting from the following condition:

$$\textstyle\sum_{\alpha=1}^{2} B_{15,y} * \gamma_y^{\alpha} = B_{14-,y}, \tag{3}$$

where $B_{14-,y}$ denotes the cumulated number of births to women aged 14 years or younger. The γ_y values were calculated mathematically under the condition that the estimators of the live births, which were rounded to zero decimal places, were equal to the known number of births in the age group.

The ASFR for females aged a in year y was estimated as the ratio of live births to mothers aged a $(B_{a,y})$ over the mean female population aged a during year y. We have no daily, but only annual age-specific population data. To address this limitation, we assumed constant probabilities for death and migration over the course of the year, which allowed us to estimate the mean female population aged a in y $(\bar{F}_{a,y})$ as the mean between female population aged a-1 at the end of year y-1 $\left(F_{a-1,y-1}\right)$ and the female population aged a at the end of year y $\left(F_{a,y}\right)$:

$$ASFR_{a,y} = \frac{B_{a,y}}{\bar{F}_{a,y}} = \frac{B_{a,y}}{\frac{1}{2}\left(F_{a,y} + F_{a-1,y-1}\right)} \tag{4}$$

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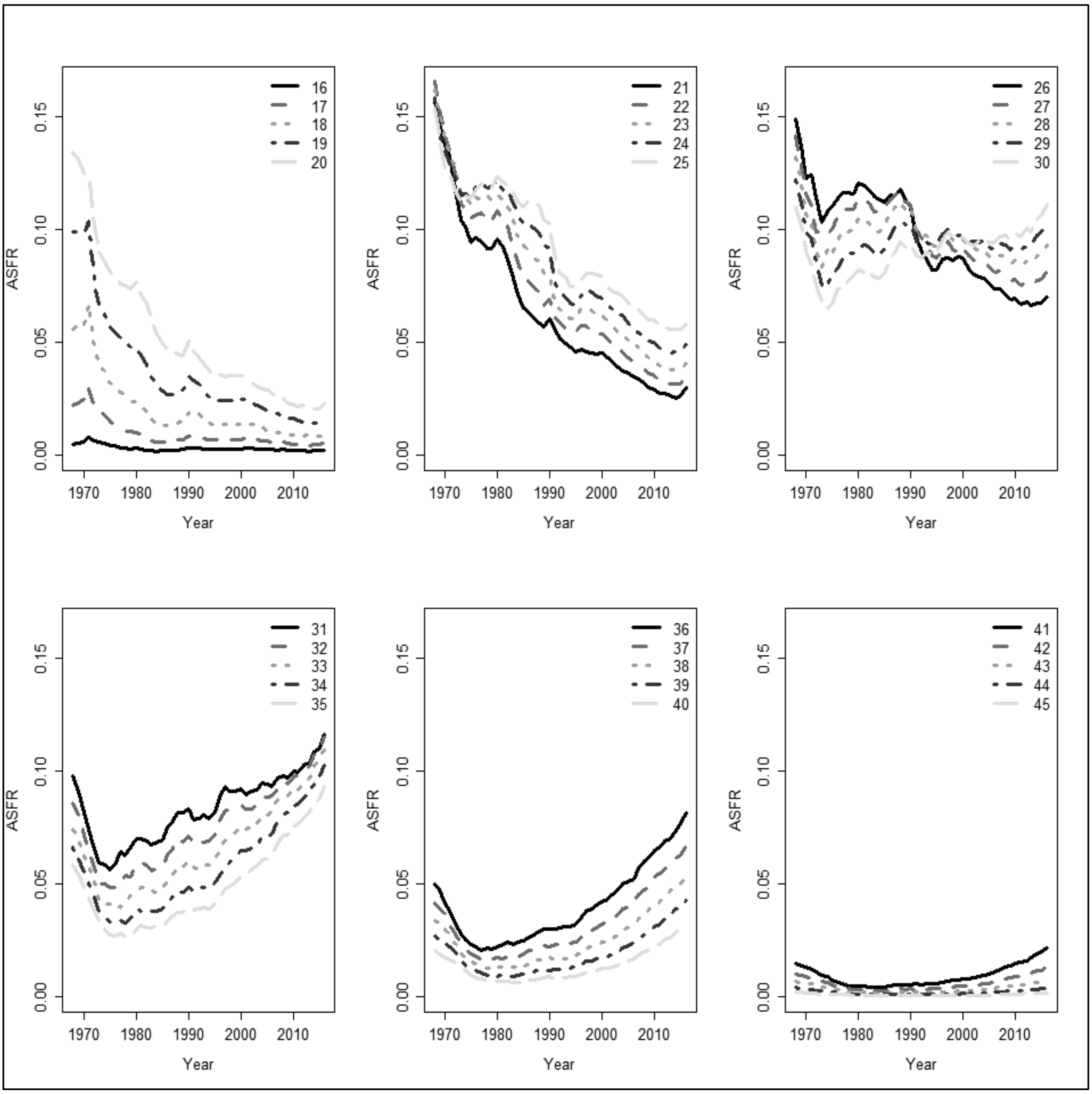
[2] This is easily done using the **splinefun** command in **R**.

Figure 3 illustrates the ASFRs for 16- to 45-year-old females derived in this way.[3]

Figure 3. Age-specific fertility rates for females in Germany aged 16-45 years since 1968



Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

We observe a general decline in fertility until the middle of the 1970s. Thereafter, the ASFRs for females in their mid-20s and older increased slightly, which might be associated with the reforms of 1972 and 1976 in the DDR (Büttner and Lutz 1990; Höhn and Schubnell 1986). The effect of these reforms appears to vanish after 1980. Since 1979, family policy in the BRD started to follow the concept of the *Trias* instead of providing pure financial compensation. Figure 3 shows that the positive trend in reproduction for females aged 30 years and older continued, but it remains unclear whether this is a result of the policy reforms since 1979.

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[3] The ASFRs were derived for 13-54 year olds. For the sake of comparability, not all time series are shown here. ASFRs for very young and old females are too small for graphical illustration at the chosen scales.

PCA was performed as the next step. The use of the log or logit transformation of ASFRs and TFRs is popular in the literature (see, e.g., Alho 1990; Lee 1992) because these transformations ensure that future forecasts or projections remain within certain limits, as explained in Section 3. A standard logit transformation produces values in the interval (0,1) for the original variable.[4] We follow the logit approach for the ASFRs with upper and lower bounds according to Alho (1990:524). Since the historical ASFR maximum observed in our data (during the baby-boom period) was approximately 0.165, it appears reasonable to set the upper bound for the forecasts at $\frac{1}{6}$, meaning that annually, not more than every sixth female born in a certain period will have a live birth. Mathematically, this transformation is then

$$logit(r_{a,y}) = ln\left(\frac{r_{a,y}}{{}^{1}/_{6}-r_{a,y}}\right), \tag{5}$$

with $r_{a,y}$ being the ASFR of females in age a in year y. Algebraically, the PCA transformation is then

$$\boldsymbol{C} = logit(\boldsymbol{F}) \times \boldsymbol{E} \tag{6}$$

where $\boldsymbol{F}$ is a 49x42 matrix of the ASFRs (49 periods in the rows, 42 years of age in the columns), $\boldsymbol{E}$ is a 42x42 matrix of the loadings (each column is one eigenvector) and $\boldsymbol{C}$ is the theoretical TS of the principal components (a 49x42 matrix). Figure 4 (next page) shows the loadings of the first two PCs.

The first PC is loaded negatively for females under 29 years old, and the correlations become positive for females aged 30 years and older. Increases in PC 1 are therefore associated with decreasing ASFRs for younger females and increasing ASFRs for women over 30 years. Therefore, this PC is associated with the tempo effect. Increasing values for PC 1 show a strong trend toward shifting births from younger ages to older ages. Keeping in mind that a PC is simply a linear combination of the underlying variables (here: the ASFRs), we might also call it an index. Therefore, throughout the rest of this paper, PC 1 will be called the *Tempo Index*. PC 2 is generally non-negative. It takes negative values for the age group 25-27, but these are very small in absolute value (approximately -0.03). This PC therefore represents changes in overall fertility, which is why we refer to it as the *Quantum Index.*

Indices of the ASFRs enable the calculation of hypothetical past values of the PCs through plugging the past observations of the ASFRs annually into (6). To put the PCs into historical context, their hypothetical courses are illustrated in Figure 5 (p. 91).

Figure 5 shows an increase in the Tempo Index, which represents the persistent trend of birth postponement since the late 1960s. As described above, the postponement was a result of the second wave of the women's rights movement since 1968, manifesting as a female desire for emancipation and self-participation in the labor market. Gustafsson (2001) conducted a literature review and concluded from the analyzed studies and the TS of mean age at birth that the timing of births may have a large impact on the lifetime earnings and lifetime human capital of the mother. She concluded that the mother's childbearing tax was lower in cases of either a high or very low age of first conception in comparison to cases of intermediate ages of first conception. Therefore, giving birth at an age over 30 years is more attractive than giving birth at approximately 25 years if the mother chooses to pursue her own career.

The bottom picture in Figure 5 gives the course of the Quantum Index; the years 1979, 1985, 1992 and 2007 are marked. Major family policy reforms were introduced during these years (see Section 2). We observe increases in the index since 1985. After reunification in 1990, the Quantum Index decreased again until 1994, after which it mostly increased to the current level. Therefore, it appears to be associated with family policy reforms (see the overview of family policy reforms in Section 2).

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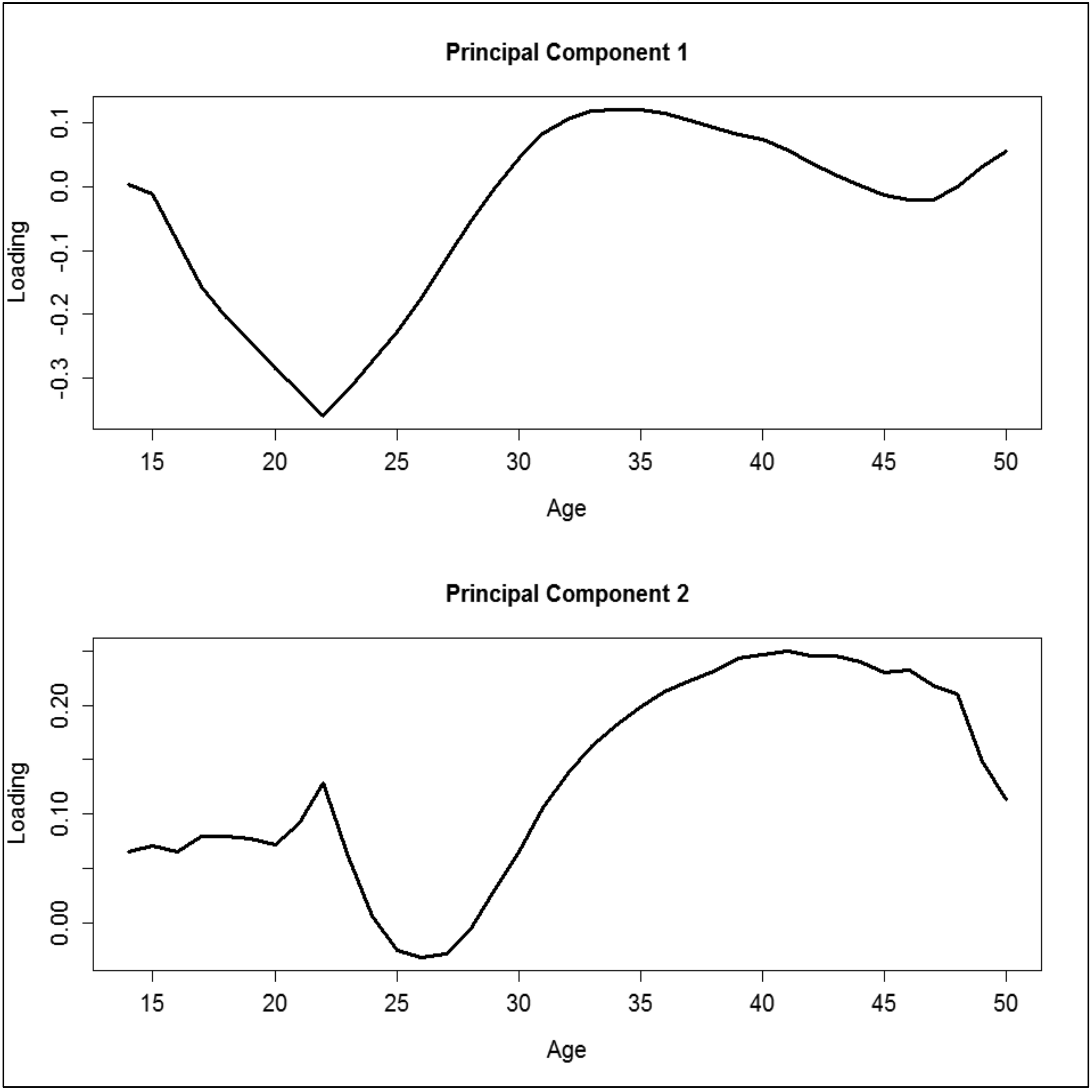
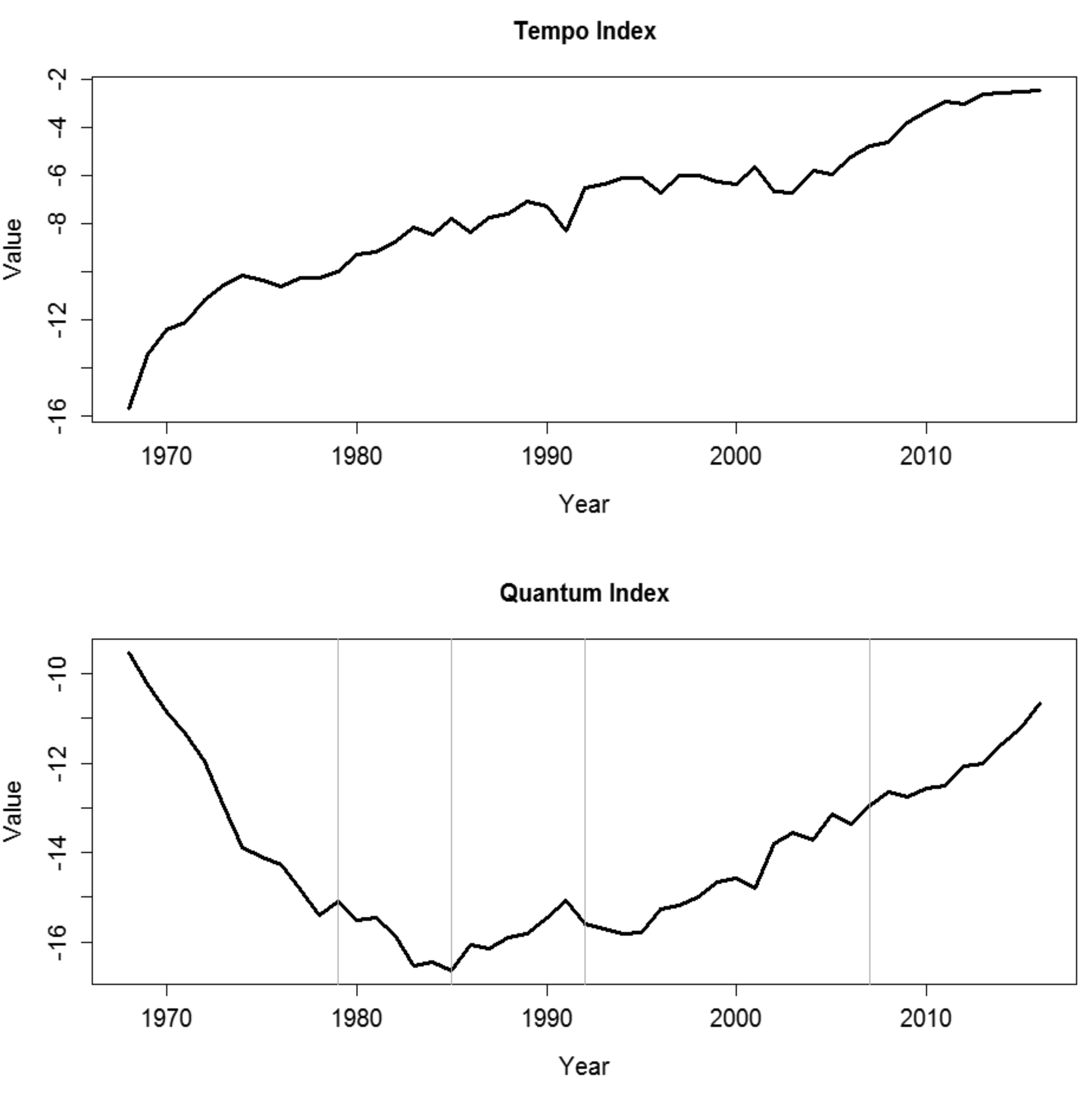
[4] The standard logit of a variable x is $ln\left(\frac{x}{1-x}\right)$; see, e.g., Johnson (1949).

Figure 4. Loadings of the first two principal components for the logits of the age-specific fertility rates



Source: Own calculation and design.

Figure 5. Past courses of the first two principal components



Source: Own calculation and design.

Three more scenarios will be considered; the reader can judge which scenario might fit reality best. For these scenarios, we perform a regression analysis with six possible explanatory variables to estimate the association of the Quantum Index to policy measures. The first category of explanatory variables includes financial benefits: the average annual child benefit *(Kindergeld – **KG**)* and financial benefits associated with parental leave with the old *Erziehungsgeld **(ErzG)***, which was discontinued in 2007. The amounts were calculated in Euros for the first year after birth, inflation-adjusted to 2010 prices. Its successor, the *Elterngeld,* was included as well. The *Elterngeld* is difficult to operationalize in a model because it depends on the previous wages of the persons receiving it, and it is also subject to upper and lower bounds. We attempted to measure its effect as a binary variable (***EG***) that takes the value of one for every year since the *Elterngeld*'s introduction and a value of zero otherwise. Finally, the pension entitlements for child raising were considered as indirect financial benefits (*Rentenpunkte – **RP***).We also tested an infrastructural variable. On the basis of the studies mentioned in Section 2, we assumed daycare to have an important impact on fertility. Whereas complete data on daycare are rare, TS on statutory expenditures for daycare coverage exist back to at least 1991. We used the real expenditures in billion Euros, adjusted to 2010 prices (*Betreuungsausgaben – **BA***). Finally, the impact of the length of parental leave *(Elternzeit – **EZ**)* in months as a time variable was included as well. The Quantum Index was regressed on the first lags of the explanatory variables because we can assume a time horizon of approximately one year between a couple's decision to conceive a child and its birth (see, e.g., Bujard 2011).

The quantified explanatory model could then be used for conditional forecasting of the Quantum Index under different policy assumptions. Three different c. p. scenarios were forecast. Scenario 3 (*Status Quo*) assumed constancy in the RPs and the EZ, which is very realistic.[5] All monetary variables were assumed to be held at their 2016 inflation-adjusted levels. This assumption is not totally unrealistic; moreover, this scenario gives us the opportunity to estimate the sensitivity of the final target variables, namely, the ASFRs and the TFR, toward investments in the respective variables. We addressed this question in the two remaining alternate scenarios, where we adjusted the future spending to estimate their effect on the Quantum Index and the TFR. Scenario 4 (*Child Benefit Adjustment)* assumed that the average *Kindergeld* would be adjusted annually with regard to inflation at the average rate since 1991. Scenario 5 *(Daycare Push)* presumed that the real investment in subsidization of daycare would be intensified at the same growth rate as has been the case since 1991. Both were c.p. scenarios assuming the effect on fertility to remain at the level quantified by the explanatory model, which might be doubted, as has been pointed out for the case of the Simple Extrapolation Scenario. The results will be presented in Section 6.

6. Results

Table 1 shows the results of the tested models. The estimated coefficients are presented for each variable along with their associated standard errors in brackets.

Table 1. Predictive model specifications for Quantum Index

Variable	Model 1	Model 2
(Intercept)	-21.474*** (0.5575)	-21.5131*** (0.5891)
Average Monthly Child Benefit [in €] (*Kindergeld – KG*)	0.0011*** (0.0002)	0.0009*** (0.0002)
Fixed Monthly Child-Raising Benefit [in €] *(Erziehungsgeld – ErzG)*	0.0005*** (0.0001)	0.0005*** (0.0001)
Income-based Monthly Child-Raising Compensation [in €] *(Elterngeld – EG)*	3.2602*** (0.6475)	3.1642*** (0.6824)
Total Pension Entitlement per Child [in Pension Points] *(Rentenpunkte – RP)*	- 0.34249* (0.19)	-
Annual Statutory Expenditures for Daycare Coverage [in billion €] (*Betreuungsausgaben – BA)*	0.16387*** (0.0254)	0.1551*** (0.0264)
Total Parental Leave Entitlement per Child [in Months] *(Elternzeit – EZ)*	0.051*** (0.0165)	0.042** (0.0166)

Source: Own calculation and design.

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[5] Reforms of the parental leave are subject to political discussion, not regarding its total amount but, rather, how to split it.

A single asterisk next to an estimated coefficient means statistical significance at the 10% level, two asterisks are associated with statistical significance at the 5% level, and three asterisks denote significance at the 1% level.

In general, the regression results show strong connections between the policy variables and the Quantum Index. Model 1 shows a high impact of all regressors on the Quantum Index other than the pension entitlements, whose coefficient was significant only at 10% level, suggesting a lack of correlation. Moreover, a negative impact of this variable on fertility is theoretically implausible, since more RP for childbearing mean smaller marginal costs of the child, implying either a positive effect on the probability to conceive a child or no effect at all. A negative impact would economically make no sense. Omitting this variable in Model 2, all regressors show highly significant correlation to the Quantum Index with p-values of 0.02 (for **EZ)** and below. We therefore choose to take this model as a forecast model of that index. A large portion of the increase in the TFR observed during the last 20 years therefore appears to be attributed to effective family policy. Our estimates confirm the study results of Abiry et al. (2014) and Bujard and Passet (2013) on the effects of the *Kindergeld*, the *Elterngeld* and supply with statutory subsidized daycare. The positive effect of parental leave estimated here is in line with the international results reported by Kalwij (2010). As an OLS model of this type cannot completely explain the connection between the explanatory variables and the dependent variable, especially given the small data base and the lack of a control group usually given in this kind of research method, the errors in the fit are carried over into the nuisance of the forecast model, which is modeled using an ARIMA model, as will be explained below.

We simulate the ASFRs indirectly through conditional forecasting of the PCs. The Tempo Index has a stable positive long-term trend, which is statistically quantified by an OLS model with a logarithmic as well as a linear trend. The remaining disturbance after fitting that trend to the data is modeled with an ARIMA model[6]. The degrees of the ARIMA model are determined by graphical analysis of the hypothetical TS and its differences, the autocorrelation function, the partial autocorrelation function, the augmented Dickey–Fuller test and the ARCH-LM test for conditional heteroscedasticity.[7] The fitted model for the Tempo Index t is

$$t(y) = -14.7244 + 0.12135y + 1.4748 \ln y + e_{y-1} + \varepsilon_y, \varepsilon_y \sim \mathcal{NID}(0, 0.51635^2), \qquad (7)$$

where e is a random walk process[8] and y is the year. The uncertainty of the forecast is accounted for by simulating 10,000 paths of a Wiener process until the year 2040, following Vanella (2018). We can then derive quantiles from the simulation results. Figure 6 shows the median simulation as well as the estimated annual 90% PIs of the Tempo Index.

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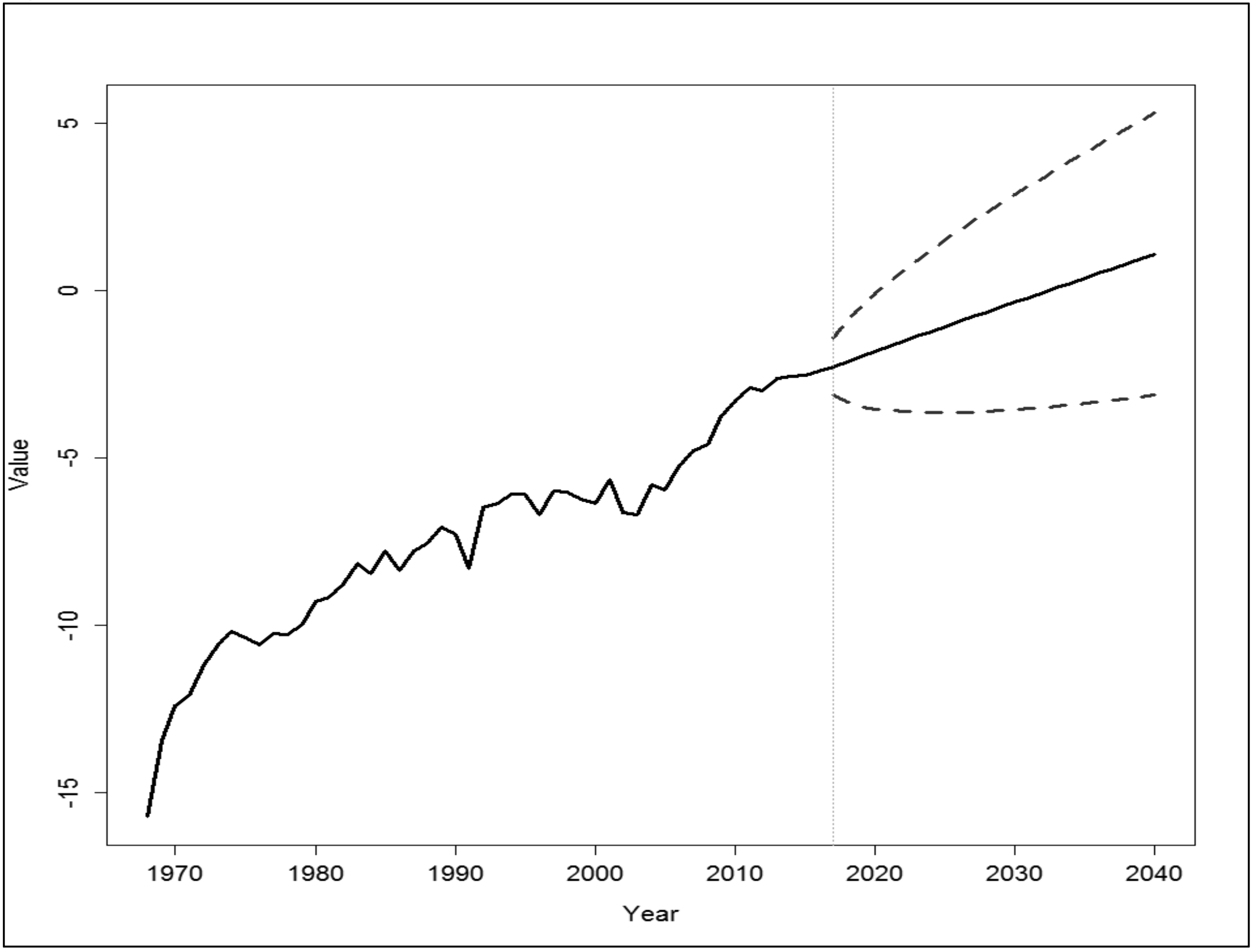
[6] See, e.g., Box, Jenkins, Reinsel and Ljung 2016:47–126; Shumway and Stoffer 2011:83–162; Vanella 2018:228–9 on ARIMA models.

[7] See, e.g., Vanella 2018:230–5 on these tests.

[8] A random walk is an ARIMA(0,1,0) process.

Figure 6. Forecast of the Tempo Index



Source: Own calculation and design.

We now turn to the conditional forecasts of the Quantum Index. For the Simple Extrapolation Scenario, the long-term trend and the stochasticity are fit using a similar procedure as for the Tempo Index. This assumes a progressive long-term trend, which is fit by a quadratic model; the resulting residuals by the tests mentioned above describe a random walk process:

$$q_1(y) = -0.4595y + 0.00315y^2 + f_{y-1} + \varphi_y, \varphi_y \sim \mathcal{NID}(0, 0.29194^2), \tag{8}$$

Regarding the Convergence Scenario, there are two questions to be answered before conducting the OLS regression. First, the inflection point of the logistic trend needs to be defined. Using graphical analysis of the Quantum Index and its second difference, which is approximately its curvature, no inflection point is found. The general trend of the curve is convex; therefore, we assume the inflection point to be the last observation in 2016. Second, the scale parameter α in Equation (9) needs to be estimated:

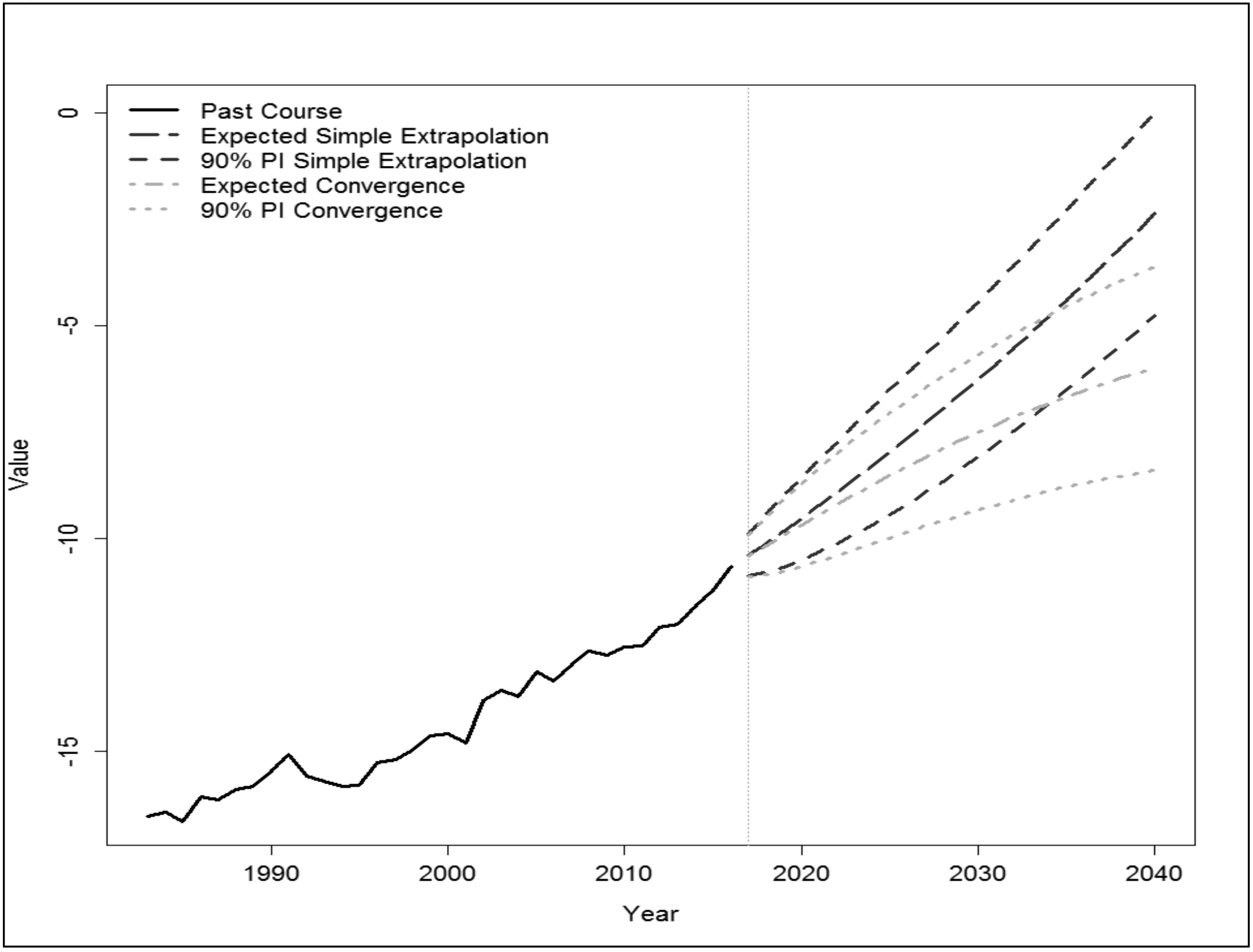
$$logit(x) = ln\left(\frac{x/\alpha}{1-x/\alpha}\right). \tag{9}$$

This can be achieved with maximum likelihood estimation to obtain the optimal scale with regard to the data. In this case, an α of 12.81756 leads to the best fit of the curve to the data. Using these two results, the OLS estimate for the forecast function is

$$q_2(y) = -17.34054 + logit^{-1}\left(\frac{y}{12.81756}\right) + g_{y-1} + \gamma_y, \gamma_y \sim \mathcal{NID}(0, 0.2927^2). \tag{10}$$

The slightly larger standard error in comparison to Scenario 1 represents the slightly worse fit of the model to the data. The conditional forecasts of the Quantum Index under these two scenarios with 90% PIs are given in Figure 7.

Figure 7. Conditional forecast of Quantum Index under scenarios 1 and 2



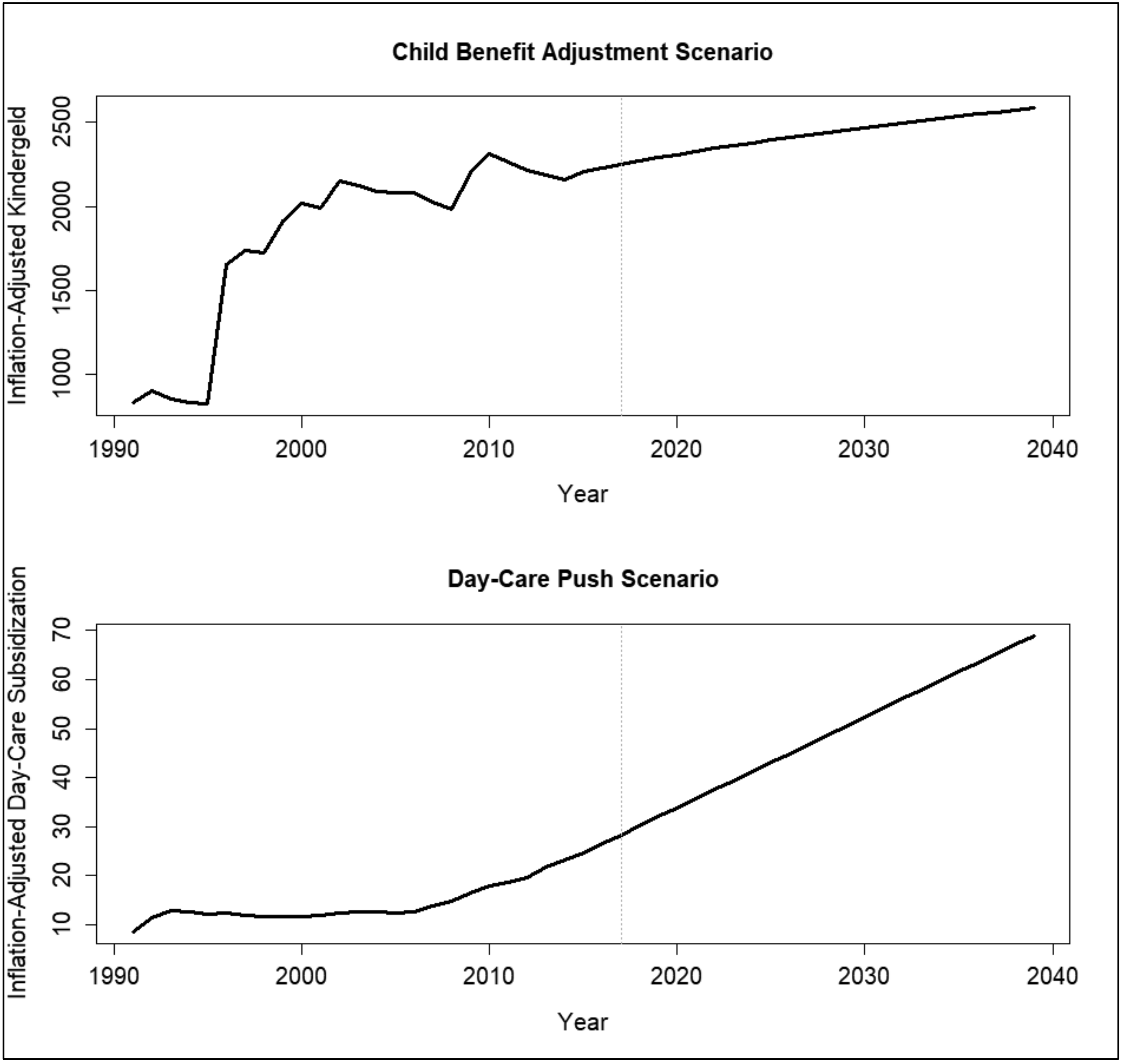
Source: Own calculation and design.

We now use the previously fit explanatory model to simulate the Quantum Index used for the remaining three scenarios. In this case, the nuisance parameter is again best fit by a random walk. Therefore, the forecast model of the Quantum Index is

$$q_y = -21.51312 + 0.00087kg + 0.00051erzg + 3.16415eg + 0.15511b + 0.04197ez + f_{y-1} + \xi_y, \quad \xi_y \sim \mathcal{NID}(0, 0.29747^2), \tag{11}$$

where f is a random walk process, kg is the annual average child benefit, $erzg$ is the annual *Erziehungsgeld* payment, and b is the money spent on subsidizing daycare in billion Euros. These monetary variables are all inflation adjusted to 2010 prices. eg is the *Elterngeld* dummy variable and ez the total entitlement to parental leave in months. All explanatory variables are lagged by one year. In the Status Quo Scenario, we estimate a conditional forecast assuming that the child benefit and daycare subsidization are held inflation-adjusted constant at their last observed levels in 2016. This assumption appears to be plausible and allows further sensitivity analyses. Scenarios 4 and 5 extrapolate the historical trends of the *Kindergeld* and the costs of child-care subsidization, respectively, inflation adjusted to prices in 2010. These inputs are predicted similarly as has been described above. These predictions are illustrated in Figure 8.

Figure 8. Assumptions about investment in child benefit and daycare for scenarios 4 and 5

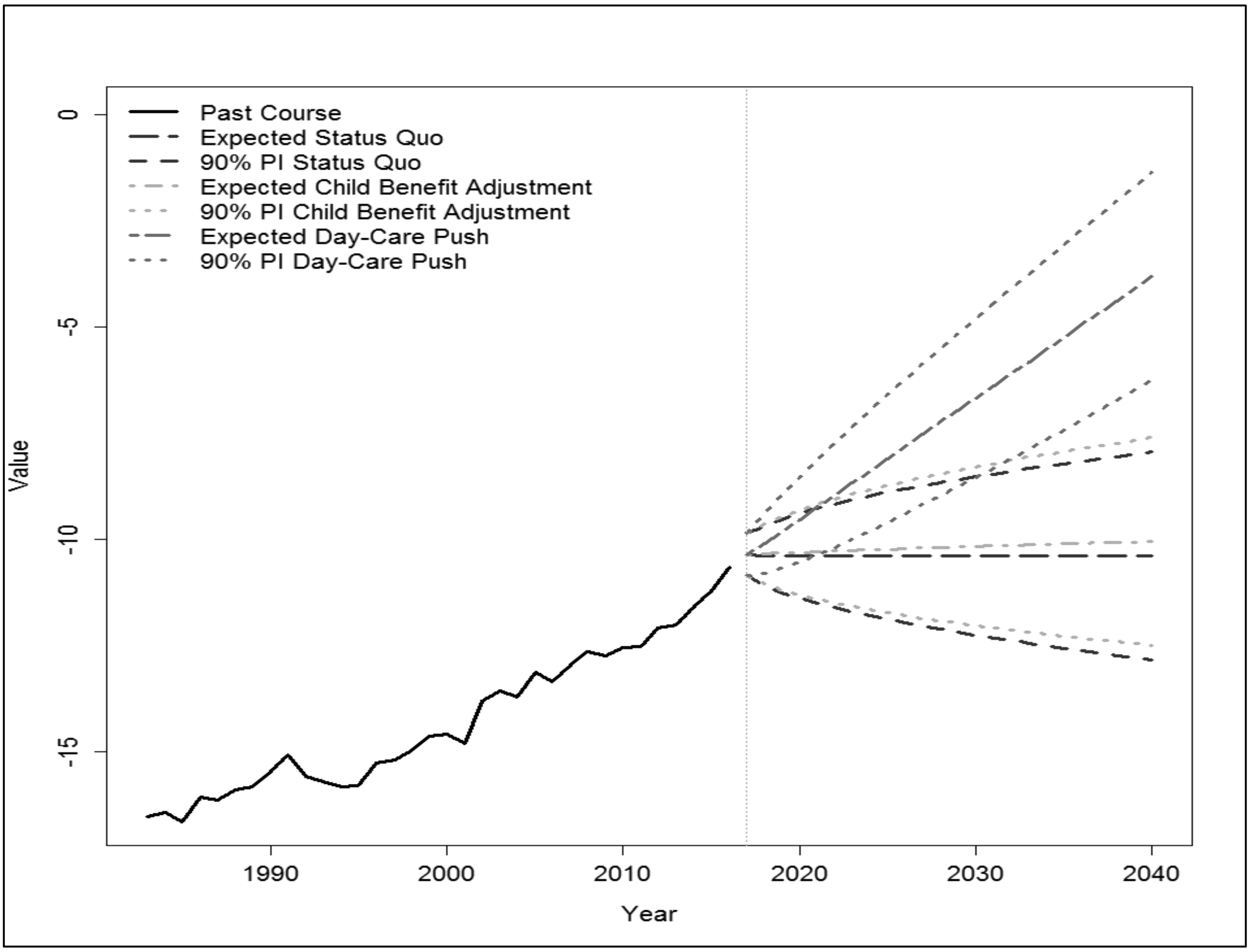


Sources: Destatis 2004a, 2004b, 2005a, 2005b, 2005c, 2005d, 2005e, 2005f, 2005g, 2006a, 2006b, 2006c, 2006d, 2006e, 2009a, 2009b, 2009c, 2010; GENESIS-Online 2018c, 2018d, 2018e; own calculation and design.

We should keep in mind that these predictions are simply examples to test the sensitivity of the Quantum Index toward adjustments in family policy. The policy measures are exogenous by nature, and an investment nearly three times as great as the current level into daycare is certainly questionable.

The conditional forecasts resulting from the simulation study for Scenarios 3-5, including the median outcome and the 90% PIs, are shown in Figure 9.

Figure 9. Conditional forecast of Quantum Index under scenarios 3-5



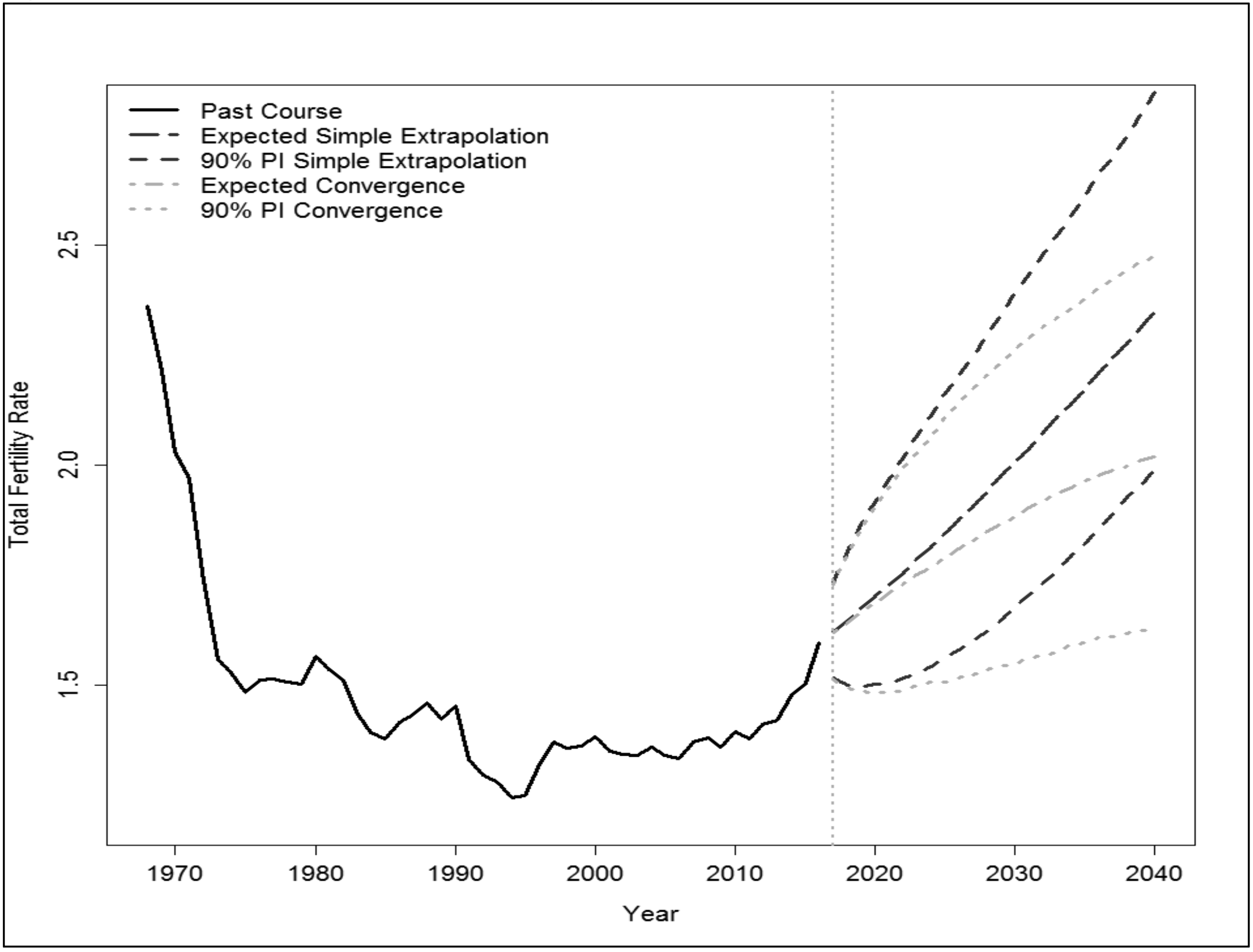
Source: Own calculation and design.

The first two PCs explain approximately 82% of the variance in the ASFRs for the base time horizon of 1968–2016. The Tempo Index explains the largest share (59.6%), whereas the Quantum Index explains 22.4% of the variance in the ASFRs. The remaining 40 PCs are simulated as random walk processes to consider their risk as well to some extent. This is an improvement on the classical Lee-Carter model, which would choose, e.g., the first two PCs and ignore the rest as irrelevant for the analysis. This is not problematic if the goal of the modeler is to predict the mean course, but if one wants to estimate the future uncertainty of the forecast as well, that model would in this case ignore about 18% of the total variance, leading to a systematic underestimation of future risk. Our model at least includes that uncertainty to some extent in the simulation study. A smaller number of PCs would lead to tighter PIs because a portion of the variance is simply assumed to be zero. Because the Tempo Index and the Quantum Index explain a large proportion of the variance in the data, the possible error arising from our random walk assumption is negligible. The resulting simulations of the PCs can be plugged back into (6) and solved for $\boldsymbol{F}$ to simulate the ASFRs. The simulation matrix of the ASFRs in year y is

$$\widehat{\boldsymbol{F}}_y = logit^{-1}\left(\widehat{\boldsymbol{C}}_y \times \widehat{\boldsymbol{E}}^{-1}\right) \quad (12)$$

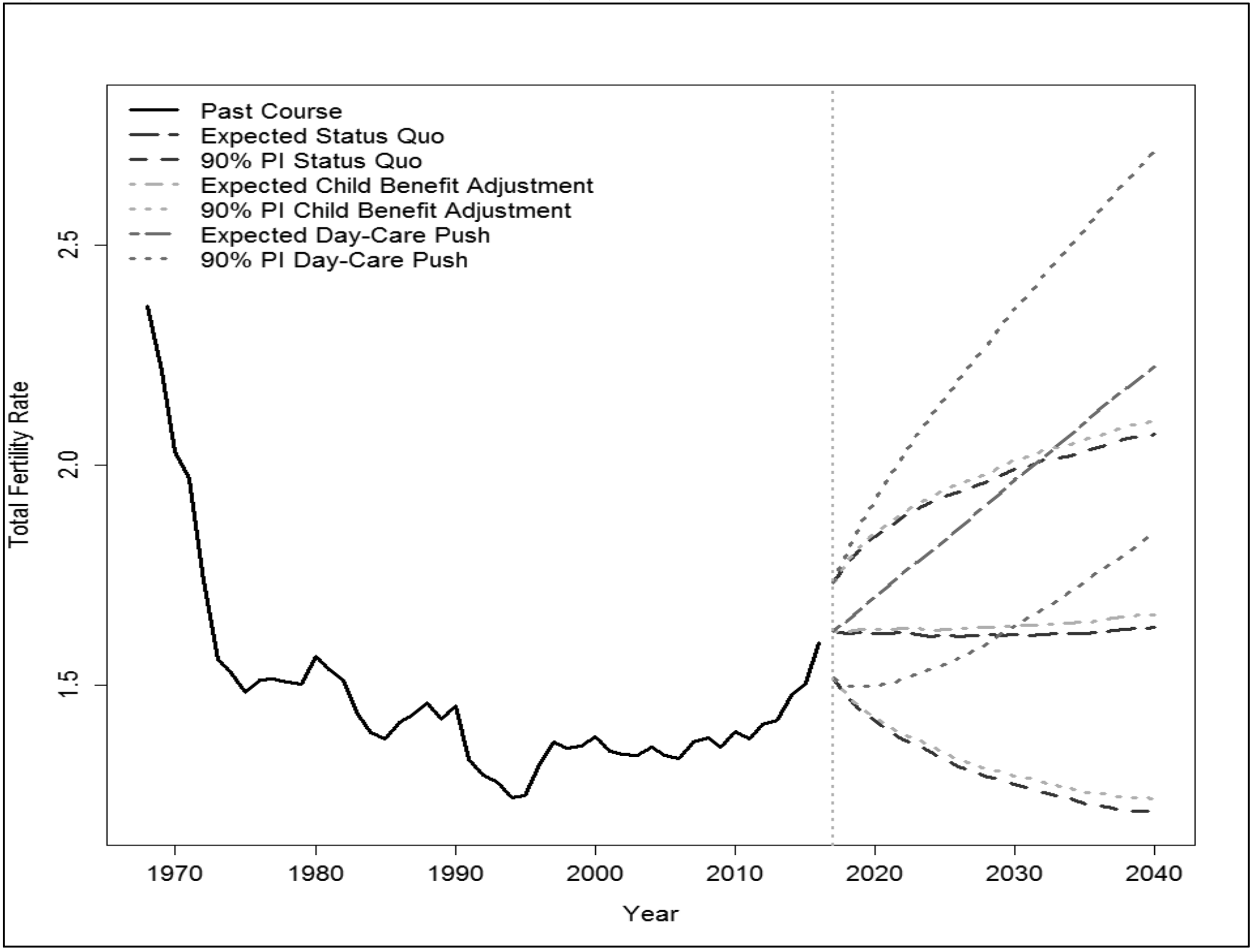
The hats over the matrix notations denote empirical matrices derived from simulations rather than theoretical matrices. The TFR is a synthetic measure that provides an idea about the general fertility level during a certain period. Therefore, we use our ASFR simulations to forecast the TFR, which is simply the sum of all the ASFRs for a specific year. Thus, summing over the rows of $\widehat{\boldsymbol{F}}_y$ yields 10,000 trajectories for the TFR in year y, from which we can obtain arbitrary quantiles. The conditional forecasts of the TFR with 90% PIs until 2040 under the five specified policy scenarios are illustrated in Figure 10 and Figure 11.

Figure 10. Conditional forecasts of the Total Fertility Rate with 90% PIs for scenarios 1 and 2



Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

Figure 11. Conditional forecasts of Total Fertility Rate under scenarios 3-5

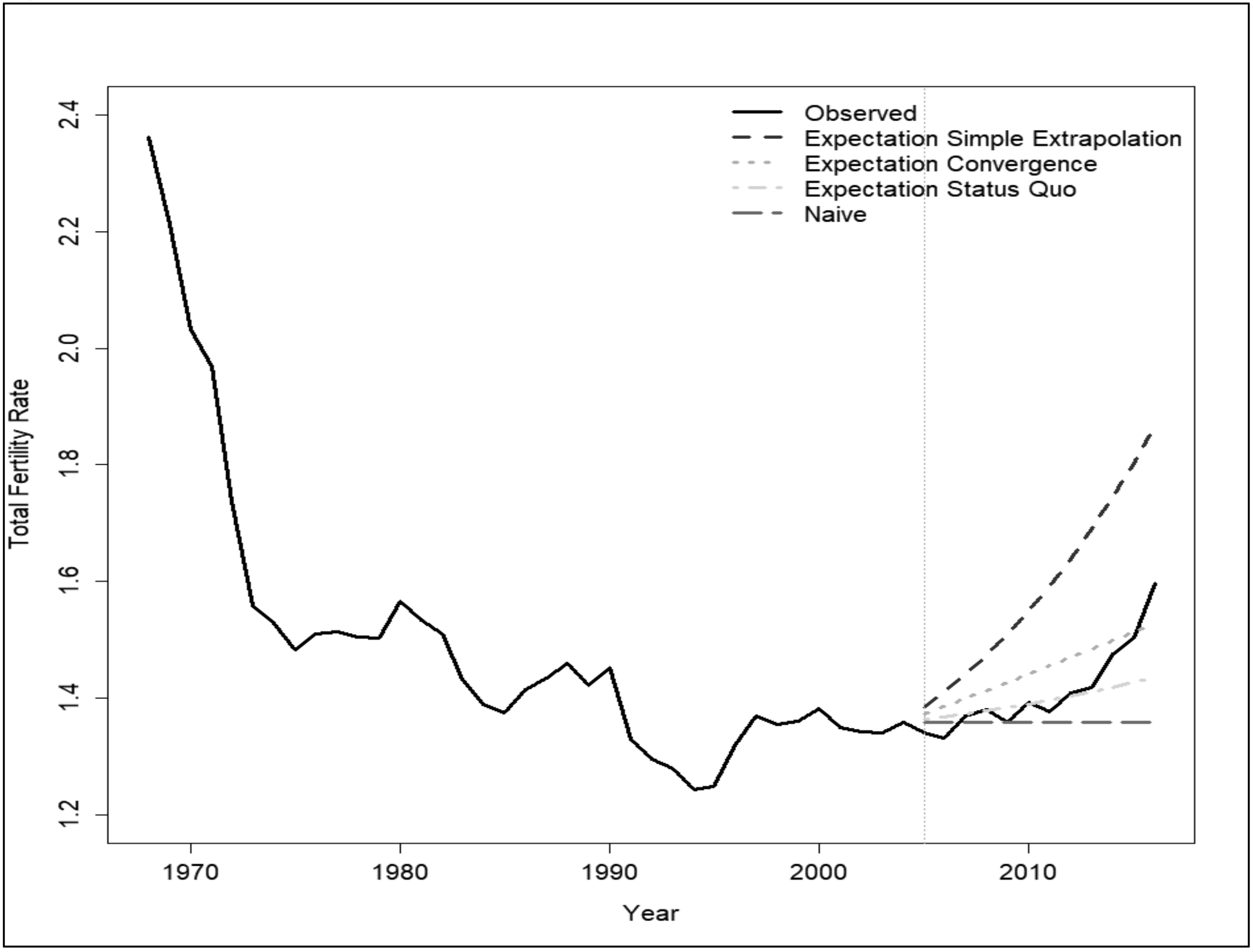


Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

Whereas Scenarios 3 and 4 lead to a slight increase in the TFR until 2040 (1.63 or 1.66) from its level in 2016 (1.6), the remaining Scenarios 1, 2 and 5 show a huge increase to over 2 children per female (2.35, 2.02 and 2.22, respectively). Intuitively, Scenarios 3 and 4 appear more reasonable, given that the TFR in Germany has not exceeded a value of 2 for more than four decades. Moreover, the difference in the TFR between the level in 2016 and the all-time low of 1.24 in 1994 is approximately 0.36. Thus, a median increase of over 0.4 for the forecast to 2040 seems too high in the median scenario.

To tentatively assess the forecasting accuracy, we compare the predicted TFRs by backtesting following our Scenarios 1–3 and a naïve forecast to the observed TFRs for the years 2005–2016. In official fertility forecasting, naïve forecasts assuming a constant TFR are often preferred (see, e.g., Pötzsch and Rößger 2015:31–3). Therefore, we use a naïve forecast as the basis for comparison to our models. Scenarios 4 and 5 cannot be tested here because during the time interval used for the backtest (1991–2004), the EZ was payed and the EG was not yet introduced. Thus, the coefficients of our explanatory model for the questionable variables cannot be estimated. Figure 12 shows the historical course of the TFR with the predictions of the four mentioned approaches.

Figure 12. Predicted TFR for the years 2005–2016 due to different scenarios



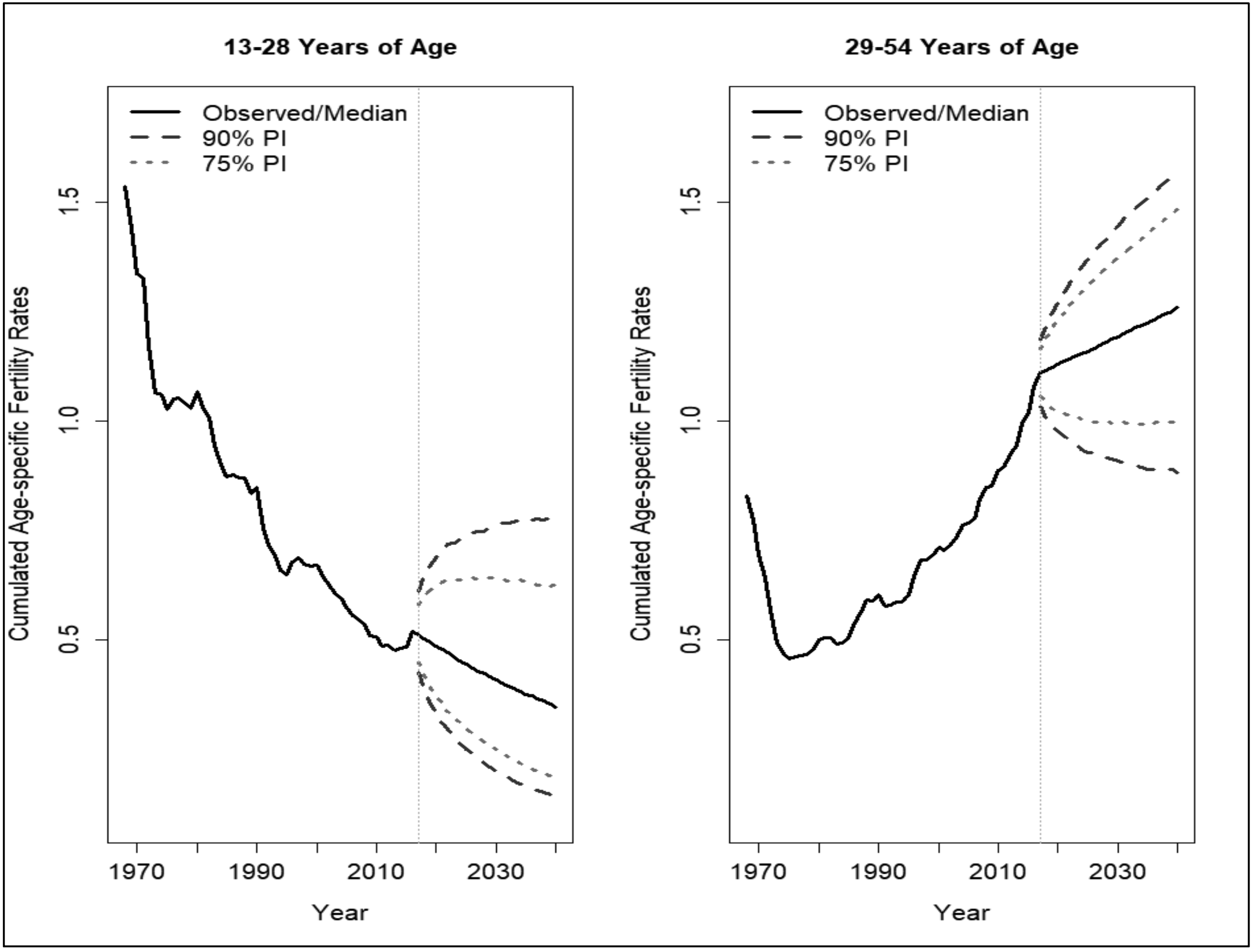
Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

Graphically, the Convergence and the Status Quo Scenarios give the best fit to the data. Whereas the Status Quo Scenario overall predicts the course closest to the TFR observed in reality, the Convergence Scenario models the last years the best. A naïve forecast does not give a good fit, the Simple Extrapolation of the time series correctly predicts the direction, but completely overestimates the TFR, as assumed earlier. The standard errors of the four different models to the observed TFR are approximately 0.0914 for the naïve forecast, 0.1995 for the Simple Extrapolation Scenario, 0.0519 for the Convergence Scenario and 0.0556 for the Status Quo Scenario.

The reader can judge which scenario appears the most plausible for the fertility outlook. If our small test of accuracy gives some indication, the Convergence and the Status Quo Scenarios are the most accurate to forecast the future ASFRs in Germany. As noted above, a TFR of 2.02, as predicted under the Convergence Scenario, seems too high considering the historical course since the early 1970s and other recent projections of the TFR in Germany (e.g., Fuchs, Söhnlein, Weber and Weber 2018:45–6; Eurostat Database 2018c; United Nations 2017; Pötzsch and Rößger 2015:31–3; Vanella 2016:17). Therefore, we consider the Status Quo Scenario in greater detail. Under the assumed family policy regime, an additional slight increase in the TFR is probable over the forecast horizon. In the median scenario, the TFR increases from 1.6 in 2016 to 1.63 in 2040. Furthermore, the TFR will be between 1.21 and 2.06 with a probability of 90% and between 1.34 and 1.93 with a probability of 75%. Based on this result, it is unlikely that fertility will fall back to its extremely low level of the mid-1990s. An increase to the replacement level of 2.1 also appears to be unlikely, but an increase toward the Northern European level of approximately 1.8 (Eurostat Database 2018a) is realistic, though improbable.

Figure 3 (p. 88) showed that the long-term trends of birth rates are negative for women under 29 years of age and positive for women over 29 since the mid-1970s. These trends are expected to continue in the future, as shown by our forecast. Figure 13 presents the forecasts of the cumulated ASFRs for two age groups: 13-28 years of age and 29 years or older, underlying the Status Quo Scenario.

Figure 13. Forecast of fertility by age



Sources: Destatis 1993, 2015b, 2016, 2018a, 2018b, 2018c; GENESIS-Online 2018a, 2018b; own calculation and design.

We see that the postponement of births is likely recuperated at older ages, and the median result indicates an increase in the TFR. The slight increase in fertility for the younger group during the last two years is mainly associated with the high international immigration into Germany since 2014. In 2014 (Eurostat Database 2018b), 60,873 children in Germany were born to foreign mothers under 30 years of age out of the total of 236,413 mothers in that age group (a ratio of approximately 0.257). That same ratio increased to 71,146 out of 231,918 (0.307) in 2015 and even further to 92,581 out of 232,476 in 2016 (0.398). Because immigration is expected to decrease slowly in the future (see, e.g., Fuchs, Söhnlein, Weber and Weber 2018; Vanella and Deschermeier 2018), this increase in the ASFRs for younger females is expected to quickly revert to its long-term trend and the ASFRs will continue to decrease.

7. Conclusions, limitations and outlook

The future evolution of fertility is the strongest demographic driver for the long-term stability of social security systems and labor markets. Small birth cohorts ceteris paribus are associated with small cohorts entering the labor market, when they come of age, leading to shortages in labor supply. Moreover, in a Bismarck-type social security system a stable population structure is desirable since the birth cohorts after entering the labor market provide social security payments. These shortages in the labor market mean financial shortages in the social systems, which have to shoulder the burden of financing the older cohorts, which are then relatively strong since originating from stronger birth cohorts. Therefore, the importance of good fertility forecasts as a quantitative basis for political planning should not be underestimated. Possible political measures in family policy must be planned carefully, weighing the possible effects on fertility and the direct costs. Stochastic time series modeling combined with a simulation approach can help to visualize uncertainty via PIs. Official fertility projections usually implement a scenario technique, which does not provide information on the probability of occurrence. Political planning based on scenarios is limited to choosing the alternative that is most in line with the political agenda. The quantification of uncertainty makes the forecasting results of stochastic approaches less vulnerable to subjective treatment.

We proposed a principal component time series model for conditional forecasting of future ASFRs. The method is rooted in Lee–Carter modeling and accounts for autocorrelation and cross-correlation among the variables, thereby taking trends among the ASFRs and over time into account. This approach is a blend of the mentioned statistical methodology with commonly used qualitative scenario approaches, which take possible societal and political trends into consideration that are not covered by the past data completely. We have shown that a major portion of the fertility trends in recent decades can be attributed to the postponement of birth due to the second wave of the women's rights movement in combination with the development of the birth control pill. The postponement has also resulted in a recuperation of births in older age groups. Moreover, although there have been some effective reforms in Germany during the last 40 years, we conclude that good family policy measures have the power to compensate for social trends and can increase the probability of birth recuperation for women over 30 years of age. Our model also included family policy variables in the analysis and indirectly estimated their effect on fertility by age of the females via PCA. Our approach provides the opportunity for sensitivity analyses of possible family policy measures.

We want to stress that the analysis is not conclusive. The time series for the policy variables are relatively short, and the model fit will naturally improve as more data become available. Some variables, such as daycare supply, that would provide a more direct estimate of the effects of political measures on fertility are simply not available as regularly as needed. Other reforms, such as the BEEG in 2006, are difficult to operationalize because their structures are too complex to be covered completely in an econometric model. The model does not claim to completely cover all influences on the TFR; rather, it proposes an approach for further improvements in fertility forecasting in low-fertility countries by taking political intervention into account. Therefore, Scenarios 3 to 5 must be interpreted as conditional forecasts in the case that our identified explanatory model holds. This limitation is included to some degree in the model though since we consider the uncertainty of the forecast, represented by PIs. Other factors such as international migration can have an influence on fertility as well. These effects, which are not modeled explicitly by our model, are included implicitly in the simulations of the remaining 40 PCs, which were not explained and modeled in detail but are simulated as random walk processes. Thus, uncertainty arising from those factors is considered in our model as well to some degree. The implications of migration for fertility are difficult to assess in a time series framework due to the short time series available for policy variables and valid for migration in Germany and Europe. Thus, long-term trends of migration cannot be covered, at least for a data source such as Germany. Analyses for other countries or other regional units might consider that point as well if long enough time series without structural breaks are available.

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Chapter 5

A Probabilistic Cohort-Component Model for Population Forecasting – The Case of Germany

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A Probabilistic Cohort-Component Model for Population Forecasting – The Case of Germany

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Abstract

The future development of population size and structure is of importance since planning in many areas of politics and business is conducted based on expectations about the future makeup of the population. Countries with both decreasing mortality and low fertility rates, which is the case for most countries in Europe, urgently need adequate population forecasts to identify future problems regarding social security systems as one determinant of overall macroeconomic development. This contribution proposes a stochastic cohort-component model that uses simulation techniques based on stochastic models for fertility, migration and mortality to forecast the population by age and sex. We specifically focused on quantifying the uncertainty of future development as previous studies have tended to underestimate future risk.

The model is applied to forecast the population of Germany until 2045. The results provide detailed insight into the future population structure, disaggregated into both sexes and age groups. Moreover, the uncertainty in the forecast is quantified as prediction intervals for each subgroup.

Keywords Demography · Forecasting · Stochastic simulation · Cohort-component method · Principal component analysis · Time series analysis · Monte Carlo simulation

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Introduction

The future development of the population structure is of immense importance since planning in many areas of politics and business is done based on expectations about the future composition of the population. Countries with low fertility and decreasing mortality rates, as is the case for most countries in Europe, particularly need accurate population forecasts since these demographic changes transform the long-term age distribution of the population in favor of older persons. These changes result in widely discussed future problems, e.g., for the social security systems as well as the labor market as a whole. The public discussion about the demographic change in Germany and its challenges is mostly tinged with negative undertones (Deschermeier 2011: 669). Nevertheless, the transformation of a society also represents a very positive aspect: people are getting older while experiencing more healthy and active years of life compared to those in previous generations (Schnabel et al. 2005: 3).

As a result of the European debt crisis, which hit countries in Southern and Eastern Europe especially hard, alongside expansions of the European Union with the inclusion of economically weak countries in southeastern Europe (Brücker et al. 2017a: 3), as well as increasing aggressiveness by the Taliban in Afghanistan (Bundesministerium des Innern 2011: 107; International Organization for Migration 2014: 104), net migration into Germany has increased monotonically since 2009. The net migration was amplified by the so-called *Arab Spring*, which started in 2011 in Tunisia. Since then, Islamists have gained massive amounts of power due to the power vacuum appearing after the end of dictatorships in the affected countries (Council on Foreign Relations 2012). The so-called *Islamic State (IS)* in 2014 had rapid and surprising military success, especially in Syria and Iraq, where they proclaimed a caliphate (Heidelberg Institute for International Conflict Research 2017: 189). Many people subsequently fled from these regions, leading to record refugee migration into Germany in 2015. People from Syria were essentially guaranteed legal refugee status. These changes subsequently motivated many people in places such as Serbia, Albania, Kosovo and Iran to try their chances as refugees as well. Some of these refugees even immigrated illegally using false identification documents to pose as Syrians (Aust et al. 2015; Bewarder and Leubecher 2016; Bundesamt für Migration und Flüchtlinge 2016: 14–50; Zeit Online 2015).

Against this background, this paper provides a stochastic population forecast of the year-end population in Germany through the year 2045. The population in each year of the forecast is broken down by sex and age for the range 0 to 102 and 103 years for males and females, respectively. We use stochastic modeling approaches developed in past contributions (Vanella 2017; Vanella and Deschermeier 2018, 2019) to forecast the demographic components of the population development. These forecasts are used to estimate the growth in the age- and sex-specific population, starting from the estimated population on December 31, 2017. In this way, we generate 10,000 sample paths for the future population by simulating a probabilistic cohort-component model by Monte Carlo simulation of Wiener processes of the demographic components.

Stochastic approaches are gaining popularity as an alternative to the common deterministic population projections that use scenarios to address future uncertainty (Keilman et al. 2002: 410). Planners and decision makers need to know which future path is most likely to occur. Moreover, stochastic forecasts based on simulations are less prone to subjective decision-making, since the results show a wide range of possible scenarios and quantify them probabilistically. As a result, the risk of personal misjudgment by the modelers is reduced. Our model returns not only the median age- and sex-specific population up to the year 2045 but also quantifies the uncertainty in the forecast, illustrated with 75% and 90% prediction intervals (*PIs*) for each year, age and sex.

The next section presents a condensed historical overview of the evolution of the cohort-component method for population updating and past advances in population projection, starting with the first deterministic models and continuing with improvements to these models through probabilistic forecasting. Our study primarily focuses on Germany; therefore, our overview gives special emphasis to population projections for Germany. In Section 3, we describe the population forecast process in detail by explaining how the demographic components fertility, migration and mortality are forecast and how these individual forecasts are combined into an overall population forecast for Germany via a probabilistic cohort-component model. Section 4 presents and discusses the results, and Section 5 provides an outlook and discusses the limitations of the presented approach.

Selected Population Forecasts and Projections with Special Emphasis on Germany

Future population projections are often conducted by deterministic cohort-component models. To the best of the authors' knowledge, this method dates to 1863, when the Census Bureau of England and Wales (1863) ran a projection of the population in England and Wales for the year 1881 by 20-year age groups. Births, deaths and migrations were identified as the components of demographic development. The population was projected by making assumptions about changes in birth rates, mortality rates and net migration for each age group or cohort. Cannan (1895: 508–515) further developed the method by taking ten-year age groups and assuming trends in age-specific fertility derived from recent census data. He projected the population in England and Wales until the year 1951. Whereas Cannan's approach implicitly modeled international migration in combination with deaths, Whelpton (1928: 255–270) incorporated expectations of migration in a forecast of the U.S. population by age group, sex and ethnicity until the year 1975, setting the stage for modern cohort-component modeling.

Deterministic methods quantify a limited number of scenarios whose likelihoods of occurrence are not quantified by probability. Therefore, stochastic methods are recommended for population forecasting (Alho and Spencer 2005: 2–3; Bomsdorf et al. 2008: 125; Keilman et al. 2002: 410–412; Lee 1998: 157–170; Lutz and Scherbov 1998: 83).

Ledermann and Breas (1959: 637–681) proposed the transformation of age-specific mortality rates *(ASMRs)* into indices through singular value decomposition, which was developed geometrically by Pearson at the beginning of the twentieth century (1901: 559–563). They were thus the first to use principal component analysis (*PCA*) to reduce the high dimensionality in demographic processes. Le Bras and Tapinos (1979: 1405–1449) elaborated on the preliminary work of Ledermann and Breas 20 years later in the first principal component (*PC*)-based population projection for France until the year 2075.

Bozik and Bell (1987) proposed a groundwork for stochastic modeling by applying autoregressive integrated moving average (*ARIMA*) models to forecast age-specific fertility rates *(ASFRs)* in the United States. Bell and Monsell (1991: 156–157) applied this method to forecasting age-specific mortality rates *(ASMRs)*. Lee and Carter simplified the Bozik-Bell and Bell-Monsell approaches to forecast age-specific mortality (Lee and Carter 1992: 660–668) and fertility rates (Lee 1993: 190–199) in the U.S. Since then, various modifications of the *Lee-Carter model* have been proposed (see, e.g., Booth 2006: 554–562; Booth et al. 2006: 290–304 for an extensive overview), maybe most notably the functional PC approach of Hyndman and Ullah (2007: 4945–4952).

Many population projections and forecasts[1] have been made for Germany during the past half-century; the best known is the *coordinated population projection* from the German Federal Statistical Office (*Destatis*). The first version was published in 1966. Since then, 13 updates have been made with improved techniques. The basic principle involves making a set of assumptions about the long-term development of life expectancy, total fertility rate (*TFR*) and net migration (currently two to three alternatives for each) to derive age-specific statistics. These different assumptions are combined to create different realistic scenarios for the course of the future population until the year 2060 (Pötzsch 2016: 37; Pötzsch and Rößger 2015: 7–41). The European Union (2015: 14–29) also uses deterministic methods.

Probabilistic population forecasts for Germany are rare. To the best of the authors' knowledge, the first approach was undertaken by Lutz and Scherbov (1998: 83–91). Their idea was to pool a large number of earlier deterministic projections and to approximate the distributions of the parameters by assuming Gaussian distributions. Lutz and Scherbov investigated nine population projections for Germany and derived distributions for the TFR, life expectancy and net migration. On the basis of these summary statistics and assumptions about the distributions of the age-specific rates, they calculated empirical quantiles for the population size via scenario-based simulation to obtain projection intervals through 2050. This method is very attractive when a sufficient statistical basis for inference is lacking but appears rather subjective since it is built upon the scientists' assessment of the future course of the demographic components. Subjective judgment generally has a high potential for error since it is not necessarily connected to statistical data. Furthermore, individuals experience difficulties in translating their qualitative judgment about realistic future scenarios into quantitative probabilities (Lee 1998: 168–170).

Lipps and Betz (2005: 11–38) produce separate forecasts for the population in West and East Germany for the period 2002–2050, assuming convergence of the mortality and fertility rates in the East towards the levels in the West. They simulate 500 trajectories for a mortality index, the TFR and net migration. The age-specific mortality rates are derived through the classic Lee-Carter index, and the TFR is assumed to follow a *random walk process*.[2] Age-specific fertility rates (*ASFRs*) are deduced from the TFR with a variable Gaussian ASFR distribution. The net migration is modeled as an autoregressive process of order one (*AR(1)*). Age-specific migration is then calculated via a distributional assumption. The simulation of the time series processes

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[1] For further reading on the distinction between forecasts and projections, see, e.g., Bohk (2012: 21–25).

[2] See, e.g., Dickey and Fuller 1979: 427; Vanella 2018: 230 for a definition of a random walk.

produces 500 trajectories with PIs of the age- and sex-specific populations of West and East Germany.

This contribution was a major improvement on previous approaches. A general limitation of models using a fixed age schedule for the ASFR, as assumed by Lipps and Betz, is that they ignore the tempo effect in fertility, which describes the postponement of child-bearing into later points in life (e.g. Vanella and Deschermeier 2019). They assume that the mother's mean age at birth will converge to 31.45 years in the long run. This approach is quite restrictive and, at least from today's perspective, not realistic at 31.45 years.[3] Quantification of the PIs for this statistic seems problematic since the variance in the forecast is apparently constant and has the same value for 2002 and 2050. Uncertainty about the far future is probably greater than that for the near future (Box et al. 2016: 129–147).

Bomsdorf et al. (2008: 125–128) use ARIMA models to forecast the TFR and the net migration in Germany. They use these summary measures to derive ASFRs and age-specific migration via age schedules, namely, a Beta distribution for the ASFRs. Age- and sex-specific measures for mortality and net migration are obtained from the Lee-Carter model, and 5000 simulations of the time series models produce empirical PIs. Härdle and Myšičková (2009: 4–26) apply the Lee-Carter models for mortality and fertility to estimate these two components for Germany. Furthermore, they forecast immigration to and emigration from Germany with separate AR(1) models to estimate the population in Germany until the year 2057.

Dudel (2014: 95–216) non-parametrically forecasts the populations of West and East Germany until 2060 using historical simulation techniques based on 1000 trajectories. His method, although statistically interesting, has a few caveats. First, the mortality model assumes a perfect correlation between the two genders, which statistically is unlikely (see, e.g., Vanella 2017: 543–552). The main trends in mortality reduction result from advances in medicine and better education among the population with regard to health and hygiene, improvements from which females and males both benefit (Pötzsch and Rößger 2015: 34). However, different developments in mortality are evident for both sexes, mostly arising from different smoking (Pampel 2005: 461–463; Trovato and Lalu 1996: 31–35; Waldron 1993: 458–460) and nutritional (Luy and Di Giulio 2006: 1–8; World Health Organization 2015) behaviors. Second, Dudel rejects trajectories for the TFR under 1 and over 3, censoring the total density. A pre-specified transformation would have mitigated this problem from the very beginning. Third, the overall migration model can be criticized because it assumes a fixed age schedule (which is unlikely) and the PIs' width remains almost constant over time instead of increasing, which has been identified as a limitation for earlier studies as well.

Deschermeier (2015, 2016) forecasts the population of Germany until 2035. He uses the model by Hyndman and Ullah (2007) to forecast the ASFRs and applies an advanced version of Hyndman et al. (2013) to forecast ASMRs and net migration. Although the model appears promising, it also underestimates the uncertainty in the forecast. Hyndman's approach smooths the data against outliers, which may be reasonable in some cases to obtain better estimates for the mean prediction. The problem

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[3] The mean age at childbirth in 2015 was 31, and long-term increases were nearly linear per annum for almost two decades (see GENESIS-Online Datenbank 2018).

with this method is that this smoothing ignores the probability of future outliers and therefore effectively underestimates the future uncertainty by simply stating that already observed outliers cannot appear again in the future.

Fuchs et al. (2018: 44–54) forecast the population until the year 2060 using time series methods for the PCs of the demographic rates. This method is the most complex population forecast in Germany to date, since it is an almost full stochastic model, taking correlations in demographic rates into account and considering the effects of migration on fertility and mortality as well. Nevertheless, the authors appear to underestimate the uncertainty, as the PIs of the TFR and net migration remain essentially constant after 2020. Our approach follows a similar principle, and the main difference is the underlying migration model. We address the age-specific net migration directly and explicitly cover the uncertainty in the migration forecast, which was not done satisfactorily in previous studies, including that by Fuchs et al. As migration is very difficult to forecast, predicting the future uncertainty and covering and quantifying the range of possible scenarios become even more important. These problems are accentuated by our study.

Approaches that try to combine the strengths of quantitative and qualitative forecast approaches based on Bayesian statistics have become more popular in recent years. One such popular approach is the Bayesian hierarchical model, developed by Raftery et al. and applied by the United Nations (*UN*) for forecasting quinquennial life expectancy and TFR projections for all countries (Alkema et al. 2011: 818–829; Raftery et al. 2014a, b: 60–65; Raftery et al. 2013: 780–786; Raftery et al. 2014a, b: 801–806; United Nations 2015: 15–33, 2017). Migration is not addressed stochastically, which results in underestimation of the uncertainty in the population projections in this case, as migration is the greatest source of uncertainty. Azose and Raftery (2015: 1631–1634) propose a probabilistic global international migration approach based on the Bayesian hierarchical model for the age- and sex-specific net migration rates of the countries. The forecasts are then transformed into net migration numbers of all countries. Azose et al. (2016) integrate that model into the UN population projection model, applying it to probabilistic projections for all countries until the year 2100, giving Germany special mention alongside a small number of other countries. Wisniowski et al. (2015) propose a blend of the Lee-Carter approach with Bayesian statistics, applied in a population forecast for the United Kingdom until 2024. Whereas their approach is not fully developed and lacks powerful prior information, it is certainly intriguing regarding future studies.

A general problem of many studies is the probable underestimation of the future risk in the population forecasts. Some models quantify risk by qualitative judgment, which is very difficult to translate into mathematical numbers as shown earlier. On the other hand, the presented quantitative studies mostly use the Lee-Carter model for forecasting, which is mostly sufficient for the mean but naturally leads to underestimation of future risk, as this model only considers a small amount of the PCs. The risk explained by the other PCs is thus ignored in the analysis, leading to a systematic underestimation of the future uncertainty. Many models do not quantify the uncertainty in migration at all, which is especially problematic, as international migration is the most uncertain of all demographic components. The overview of the relevant literature shows that approaches for population forecasting for the case of Germany that model all three demographic components by age and sex stochastically do not yet exist, with the exception of Azose et al. (2016). Our contribution is to propose an approach that is not only fully probabilistic but also considers the autocorrelations and cross-correlations of the demographic rates.

Method and Data

In this section, we propose a population forecast based on a probabilistic cohort-component model. The partial models for the demographic components shall be explained shortly.[4] First, the age-, sex-, and nationality-specific net migration *(ASNSNM)* figures are forecast as in Vanella and Deschermeier (2018). The data used are synthetic net migration figures per years of age (0–104 for females, 0–103 for males), sex (binary) and nationality group, which are estimated by the authors using two data sets provided by Destatis for that study. The nationalities are split into seven groups: Germans, EU- or Schengen-citizens excluding Germany, Third-Country Europeans, Africans, Asians, Citizens from the Americas or Oceania *("Overseas")*, and finally persons with no clear information on their citizenship, either because it is unknown or they have none *("NA")*. The synthetic[5] data used for that study are estimated through two datasets provided by Destatis; the first includes age-specific migration data by sex, divided by Germans and non-Germans (Destatis 2015, 2016a, 2017a, 2018a, f), and the second dataset is disaggregated by nationality and five age groups (Destatis 2017b, 2018b, g).[6] Vanella and Deschermeier (2018: 266–267) derived the synthetic dataset used for the analysis from these two provided datasets. The base time period is 1990–2017. We run a principal component analysis (PCA) on the derived 1463 ASNSNM figures. We address the net migration instead of modeling the immigration and emigration directly for three reasons. First, the number of variables is obviously halved when addressing net migration. This makes our model more efficient. Second, the need for a transformation of the original data is avoided. Gross migration numbers cannot have negative values. Conversely, Monte Carlo simulation results in negative as well as positive results. This would result in the need for transforming the original data, which can bias the original distribution of the data. Third, the number of emigrants out of Germany for certain age groups is very small, resulting in less reliable statistical estimates of the migration numbers for these ages. This can lead to biased loadings in the PCA as well. Because we are not interested in the share of the migrants in the population, our net migration model is efficient for our purposes.[7] The loadings of the first two PCs are for both sexes and the different nationality groups given in Figs. 1 and 2.

Vanella and Deschermeier (2018: 268) identified the first PC as an index of labor migration due to the high positive loadings on European and Asian net migration alongside high negative loadings on Germans in the working age group.

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[4] The original sources serve as a more detailed description of the models and their results.

[5] Our dataset does not exist as such but is rather estimated from different sources used by Vanella and Deschermeier (2018) in their study. Therefore, we call it a *synthetic dataset.*

[6] The exact method for deriving the synthetic data is outlined in Vanella and Deschermeier (2018: 264–271).

[7] Dividing the population into migrants and natives may be of interest for many research questions, such as for the labor market. However, the population data for Germany by nationality are of too low quality and the corresponding time series are too short to derive representative base data on this issue.

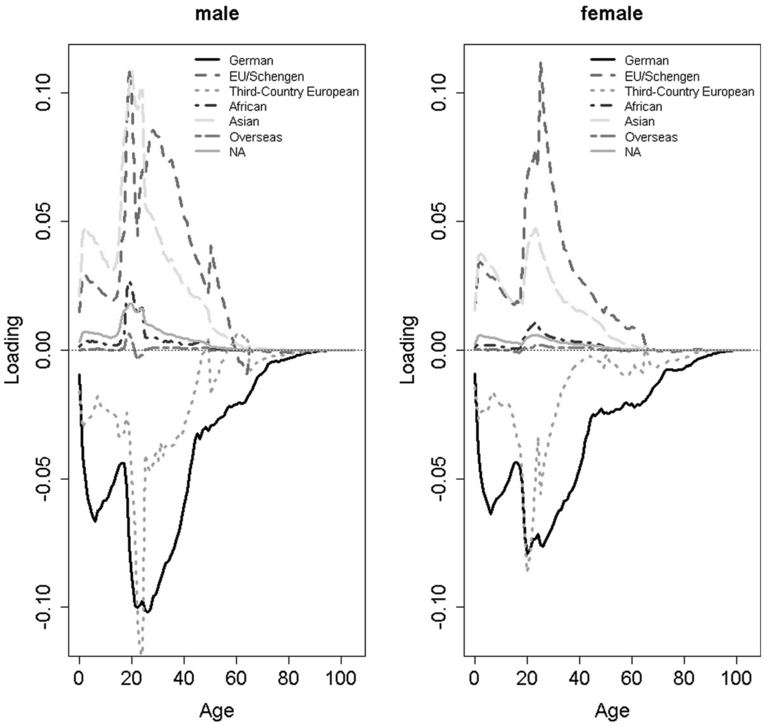

Fig. 1 Loadings of the Labor Market Index of the Migration Model. (Source: Own calculation and design)

The loadings of the second PC are non-positive, thus addressing the overall net migration level. In combination with the historical course, Vanella and Deschermeier (2018: 269–270) argue that the absolute value of the PC is especially large in times of significant crises, therefore addressing it as a *Crises Index.*

The historical course together with the forecast of these two variables through 2045 is plotted in Fig. 3.

The Labor Market Index has an increasing long-term trend on average and includes cyclical effects, which are typical for labor markets. The Crises Index is assumed to converge towards its median during the base period due to a lack of better knowledge. The models are fit via ordinary least squares regression in the first step. This serves the derivation of the long-term trend in the index, which is then extrapolated in the forecast. The resulting noise is estimated with ARIMA models, which are then used for future simulation to consider the uncertainty in the forecast. Using this procedure, we can easily assess which trends are stable in the long run and which changes in the PCs result from stochastic elements. ARIMA models are very suitable to reflect the uncertainty in a forecast since future risk accumulates over time and typically becomes greater for points that are more distant in time. ARIMA models emulate this phenomenon very well.

The remaining 1461 PCs are assumed to be random walk processes and are simulated accordingly. This assumption accounts for the overall uncertainty in the migration risk. Previous PC-based forecast approaches tend to omit the majority of the PCs from the analysis for simplification. On the downside, this results in a systematic underestimation of future risk since the risk of the omitted variables is ignored. Since our objective is to represent the future uncertainty as realistically as possible, we follow the random walk

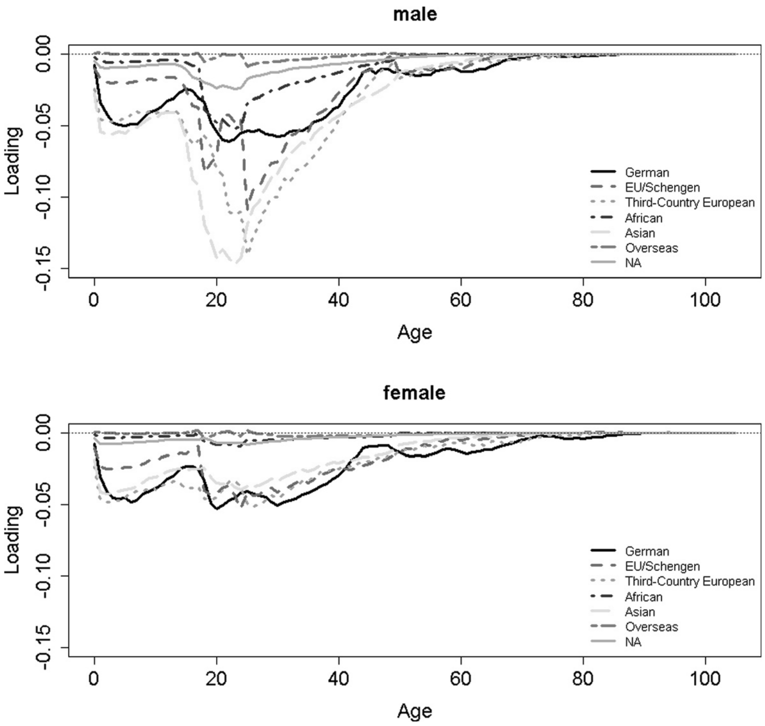

Fig. 2 Loadings of the Crises Index of the Migration Model. (Source: Own calculation and design)

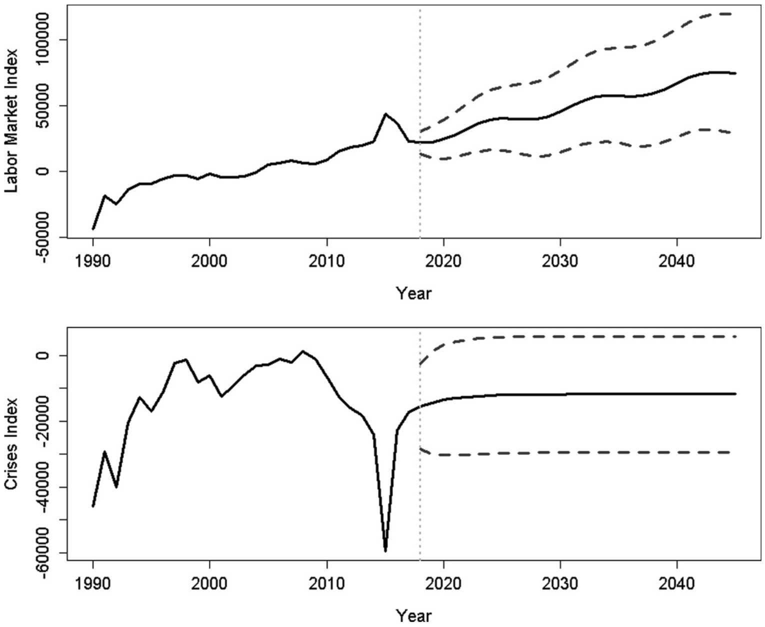

Fig. 3 Forecasts of the First Two Principal Components in the Migration Model. (Source: Own calculation and design)

approach. Random walks are very easy to simulate and represent the true behavior of the 1461 variables reasonably well. In our case, the Labor Market Index and the Crises Index alone explain 76% of the overall variance in the ASNSNM for 1990–2017. Thus, completely omitting the remaining 1461 PCs would systematically underestimate the future risk by 24%. Therefore, we have a reasonable trade-off between simplifications of the dimensionality of our forecast problem while still considering the true risk in the forecast. The resulting 10,000 trajectories of the future course of the PCs are transformed back into forecasts of the ASNSNMs through 2045, which are then finally aggregated by sex and age for the net migration forecast, as the final population forecast does not discriminate by nationality. The results of the forecasts are presented in Section 4 among other simulation outcomes.

The model for mortality is based on Vanella (2017). Death numbers by year of age and sex have been provided by Destatis for the years 2000–2017 (Destatis 2017c, 2018c, 2019a). Since Destatis does not provide detailed death count data for the age group 100+ for years before 2000, we took the estimated age- and sex-specific death numbers for the period 1956–1999 from the Human Mortality Database (2019a, b, c) to produce time series for the period 1956–2017 of all age- and sex-specific death variables. For 1956–1989, the data are split by West and East Germany and were therefore aggregated. We do not use the data by actual age reached at time of death but, rather, use the difference between year of death and birth year as age. Since the Human Mortality Database *(HMD)* data on deaths in essence originate from Destatis as well (Scholz et al. 2018: 2), we have no conflicting inputs that might result from meshing different data sources.

In addition, year-end age- and sex-specific population estimates were downloaded from the HMD for the period 1956–2017 (Human Mortality Database 2019d, e, f). The population data from the HMD are based on Destatis data as well but contain estimated values up to 109 years of age. Moreover, the data back to 1989 are adjusted to the 2011 census following Klüsener et al. (2018). The original Destatis data overestimated the old-age population heavily since the previous censi for the two parts of Germany had been conducted in the 1980s, which led to massive errors in the data due to the German reunification and large errors in the data on international migration (Scholz et al. 2018: 2–5). Based on these data, we estimated adjusted age- and sex-specific mortality rates *(ASSMRs)* as follows:

$$m_{x,y,g} = \frac{D_{x,y,g}}{P_{x,y,g} + D_{x,y,g}},$$

with $m_{x,y,g}$ being the ASSMR of persons aged x years of sex g at the end of year y. $D_{x,y,g}$ addresses the corresponding number of people who have died over the course of year y, and $P_{x,y,g}$ is the total population of same age and sex living at the end of year y. In this way, the mortality rates calculated are computed recursively for the period under study instead of based on the population at risk at the end of the period before that. Our way of quantifying the mortality risk has the advantage that migration occurring in the year under study is considered in the overall mortality. Moreover, mortality rates above one, which may appear in the common definition,[8] are not possible since we base our

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[8] A mortality rate for a particular year is generally calculated by dividing the number of deaths that occurred during that year by the number of persons at risk of dying in that same cohort who are still alive at the end of the previous year.

mortality risk on the hypothetical population that would have been estimated if no deaths had occurred. The timing of migration and births over the year is represented in the data as well; therefore, we use the term *adjusted* mortality rates. Deaths for age group 103+ for males and 104+ for females are aggregated since the population estimates for these age groups are too small and therefore lead to unrepresentative estimates. As a result, the ASSMRs represent mortality for these age groups accordingly. Age- and sex-specific survival rates *(ASSSRs)* result from subtracting the corresponding ASSMRs from 1. A PCA is performed for the ASSSRs. The loadings of the first two PCs are illustrated in Fig. 4.

Vanella (2017: 543–548) identified the first two PCs resulting from the PCA as a classical Lee-Carter Mortality Index and a Behavioral Index regarding nutritional and smoking behavior, respectively, which explain the gender gap in mortality to some extent.

Figure 5 gives the forecasts of these two indices:

The development of the Lee-Carter Index shows a general trend of decreasing mortality over all age groups, which has slowed in recent years. The expected increase in the Behavioral Index reflects convergence in nutritional and smoking behaviors between males and females. The width of the PIs of the Behavioral Index forecast shows the high uncertainty associated with this convergence trend. For both variables, the long-term trends are estimated by logistic models, which are specified by maximum likelihood estimation.

Regarding fertility, we use data on age-specific births among individuals aged 15 to 49 for the years 1968 to 2017 provided by Destatis directly or downloaded from GENESIS-Online (GENESIS-Online Datenbank 2019a; Destatis 2007, 2014a, b, 2018d, e) together with the age-specific data on the female population of reproductive age. Specific birth data on younger or older mothers are not available; therefore, births to mothers under 15 and mothers over 49 years of age are estimated as one age group each. Following Vanella and Deschermeier (2019), we assume 13 as the minimum age at birth and 54 as the maximum. We derive age-specific fertility rates (*ASFRs)* by dividing age-specific births by the corresponding mean age-specific female population for the respective year. As proposed by Vanella and Deschermeier (2019: 86–100), we run a PCA on the logistically transformed ASFRs for mothers aged 15–49 years and for the age groups 13–14 and 50–54 for the base period 1968–2017.[9] This time horizon was proposed in that paper because it shows fertility developments after the second wave of the feminist movement (Hertrampf 2008). Vanella and Deschermeier (2019: 89–95) show that the first PC represents the tempo effect in fertility, whereas the second PC is associated with the general quantum of fertility and is to some extent influenced by family policy. Fig. 6 illustrates the loadings of these two PCs.

Figure 7 shows the historical courses of these two variables with the forecast until the year 2045.

The forecast of the Quantum Index, which addresses the quantum of fertility, is influenced by family policy,[10] as Vanella and Deschermeier (2019: 91–100) have

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[9] The authors propose, based on the historical data and further considerations, 1/6 as the upper bound for the ASFRs (Vanella and Deschermeier 2019: 89).

[10] Several studies show positive effects of family policy on the fertility level; see, e.g., Kalwij (2010) on the effects of parental leave entitlements and daycare opportunities, and Gauthier and Hatzius (1997) on the impact of cash benefits on fertility.

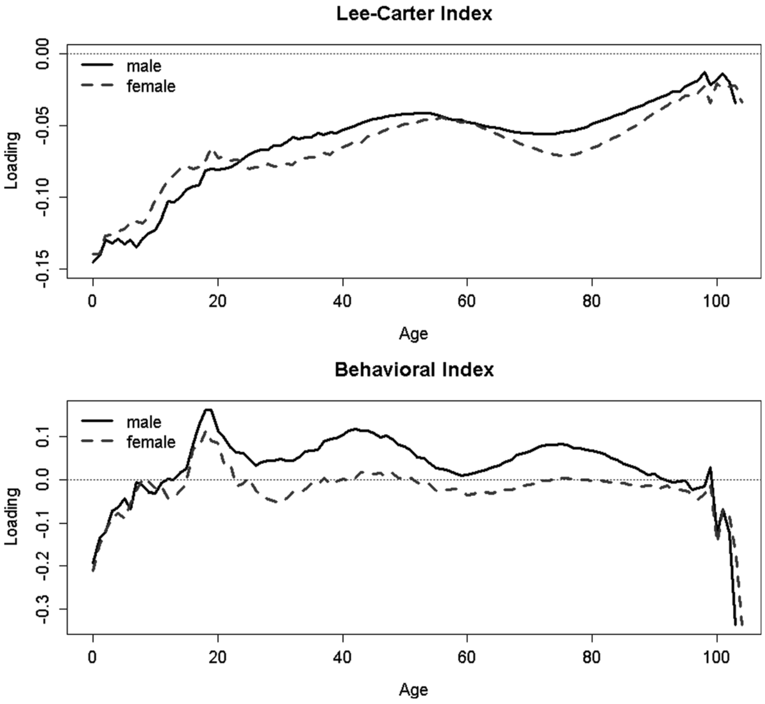

Fig. 4 Loadings of the First Two Principal Components in the Mortality Model. (Source: Own calculation and design)

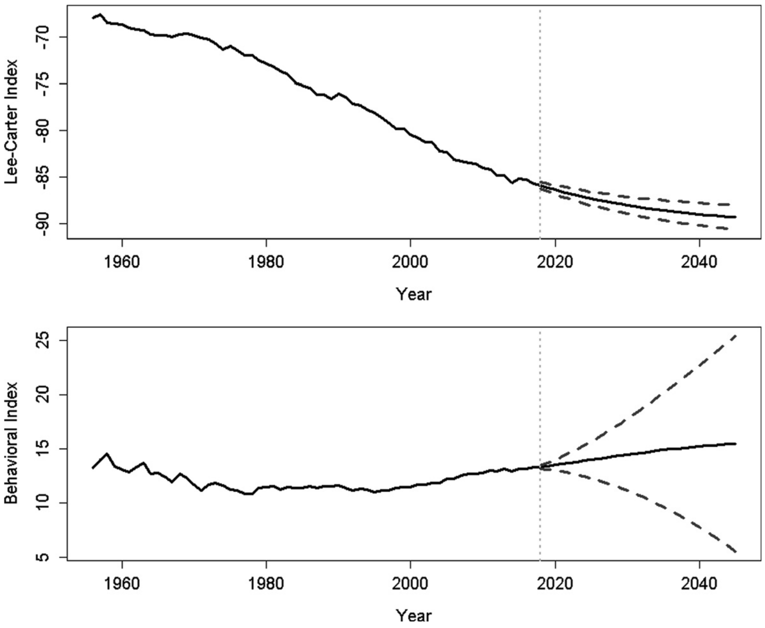

Fig. 5 Forecasts of the First Two Principal Components in the Mortality Model. (Source: Own calculation and design)

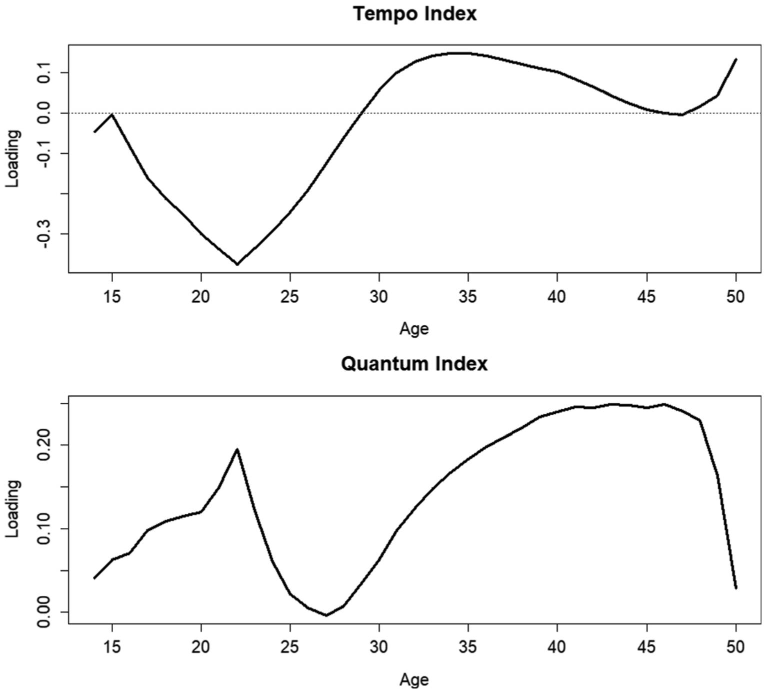

Fig. 6 Loadings of the First Two Principal Components in the Fertility Model. (Source: Own calculation and design)

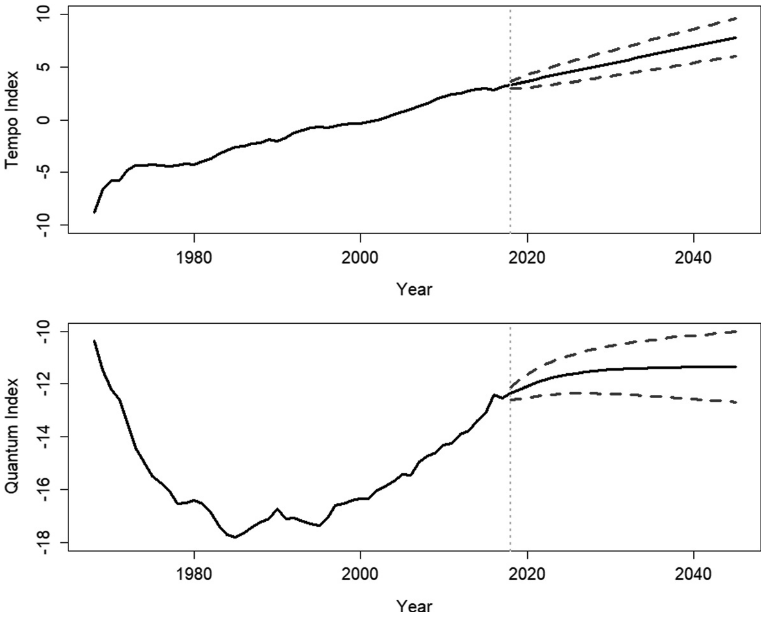

Fig. 7 Forecasts of the First Two Principal Components in the Fertility Model. (Source: Own calculation and design)

shown. The authors discuss a variety of models to fit to the Quantum Index. Their backtest for forecasting the TFR results in good fits of the so-called *Status Quo Scenario* and *Convergence Scenario*. Whereas the Status Quo Scenario poses specific assumptions on the future statutory financial investment into family policy, the Convergence Scenario extrapolates the Quantum Index by a logistic process. The implicit underlying assumption here is that further investment into family support through financial benefits, parental leave opportunities and daycare coverage is undertaken and that these have a positive impact on fertility but that this impact is not equal to the past increases and decreases over time. The assumptions appear reasonable, and the resulting mean forecast of the TFR resulting from our calculation (1.67 in 2045) leads to similar results as in Vanella and Deschermeier (2019: 99) and are in the interval classified as realistic for the TFR by Destatis (2019b: 15). Therefore, we take the Convergence Scenario as the basis of our forecast of the Quantum Index.

The gender of the children is simulated after computing the birth numbers. Therefore, we calculate the ratio of males among all live births annually based on the sex-specific birth numbers in Germany from 1950 to 2017 extracted from GENESIS-Online (GENESIS-Online Datenbank 2019b). We then fit a logarithmic ARIMA model to the data for simulation of the birth ratio until 2045. The ratio's historical course alongside the median forecast and 75% PIs is given in Fig. 8.

An apparent trend of a decreasing ratio of male births is evident over the analyzed horizon. This trend can also be observed in other industrialized countries since at least the 1970s (Davis et al. 2007: 941–943; James 2000: 1179–1182). Although various studies individually report some evidence that environmental factors such as weather

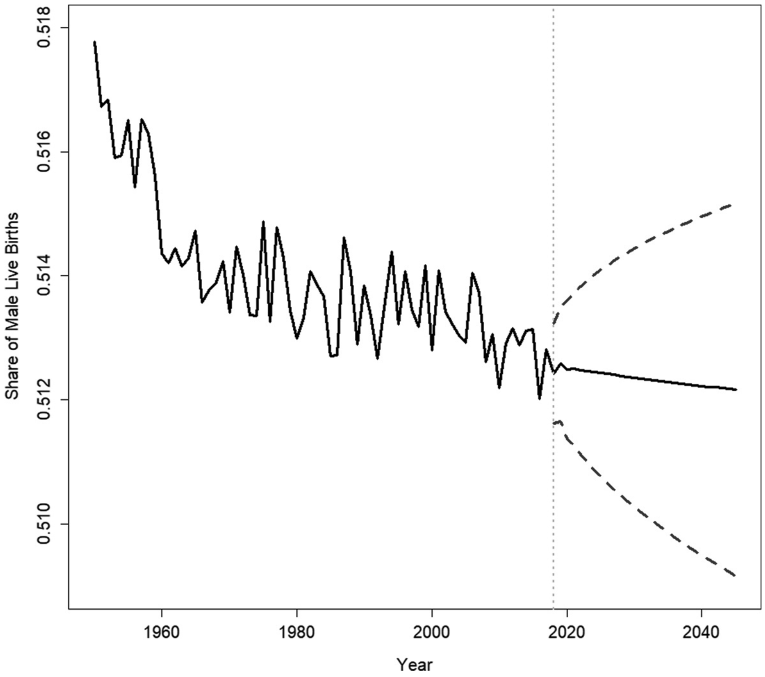

Fig. 8 Forecast of the Share of Males Among Live Births. (Sources: GENESIS-Online Datenbank 2019b; Own calculation and design)

(Helle et al. 2008), exposure to toxins (see, e.g., Davis et al. 2007: 941–942), and nutritional behavior (Mathews et al. 2008: 1662–1666) have some influence on a baby's sex, none of the findings explain the observed trends of decreasing ratios of male births. However, considering the apparent basic trend since 1950, assuming that the trend will continue over the forecast horizon is plausible.

All described models are based on principal component time series models and thus include autocorrelations in the time series alongside cross-correlations among the age- and sex-specific demographic rates and numbers.

We now describe the procedure for population forecasting with our model. Let $P_{x,y,g,t}$ denote the population aged x years at the end of year y for sex g in trajectory t. The population update is performed through the following step-wise process.

Step I:

The forecast begins with an adjustment of the base population concerning international migration flows in the first forecast year $y+1$**.** The addition of international net migration aged $x+1$ years of sex g during year $y+1$ and in trajectory t ($M_{x+1,y+1,g,t}$) to $P_{x,y,g,t}$ leads to the hypothetical subpopulation $\hat{P}_{x+1,y+1,g,t}$ at the end of year $y+1$ without any deaths:

$$\hat{P}_{x+1,y+1,g,t} = P_{x,y,g,t} + M_{x+1,y+1,g,t}.$$

Step II:

The actual number of survivors from $\hat{P}_{x+1,y+1,g,t}$ at the end of $y+1$ is calculated through multiplication with the adjusted ASSSR $s_{x+1,y+1,g,t}$[11] for persons aged $x+1$ years of sex g in year $y+1$ and in trajectory t:

$$P_{x+1,y+1,g,t} = \hat{P}_{x+1,y+1,g,t} * s_{x+1,y+1,g,t}.$$

Step III:

The mean female population by age in $y+1$ in the reproductive age group is approximated:

$$F_{x,y+1,w,t} = \frac{P_{x-1,y,w,t} + P_{x,y+1,w,t}}{2}.$$

Step IV:

The live births $B_{y+1,t}$ are estimated:

$$B_{y+1,t} = \sum_{x=u14}^{50+} F_{x,y+1,w,t} * f_{x,y+1,t},$$

where $f_{x,y+1,t}$ denotes the ASFR for females aged x years in year $y+1$ in trajectory t.

Step V:

The numbers of the male $B_{y+1,m,t}$ and female $B_{y+1,w,t}$ live birth numbers are calculated:

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[11] Note that, since our definition of mortality and survival rates strictly restricts them to the interval [0; 1], $m_{x,y,g,t} + s_{x,y,g,t} = 1 \; \forall \, x, y, g, t$.

$$B_{y+1,m,t} = B_{y+1,t} {}^{*} r_{y+1,m,t},$$

with $r_{y+1,\ m,\ t}$ representing the share of male live births in year $y+1$ in trajectory t. Subsequently, the female birth numbers are

$$B_{y+1,w,t} = B_{y+1,t} {}^{*} \left(1 - r_{y+1,m,t}\right).$$

Step VI:

The number of survivors among the children born in $y+1$ is calculated:

$$P_{0,y+1,g,t} = B_{y+1,g,t} {}^{*} s_{0,y+1,g,t}.$$

We reiterate that our model includes the timing of births, migration and deaths over the year implicitly as well since the input data are adjusted as such. Assuming that the future timing of these demographic variables is similar to the timing observed in the past data, the timing of the demographic movements is included in the forecast as well. Migrants and newborns enter the population under study distributed over the year. Therefore, they are not at risk of death for the full year in the population under study. We account for this through our adjusted data, as our past data used for the forecast include this information as well through our cohort perspective. If, e.g., children on average are born in the middle of the year, the ASSSRs in our model indeed do not represent the actual probability to survive the whole year, as would the case for most models, but instead would give the probability of surviving the semi-year in which they lived in Germany until the end of the period, on average.

In this way, the population by sex and age in year $y+1$ in trajectory t is obtained. This process is then used to stochastically forecast the population by sex and age until the year 2045. The algorithm is illustrated in Fig. 9.

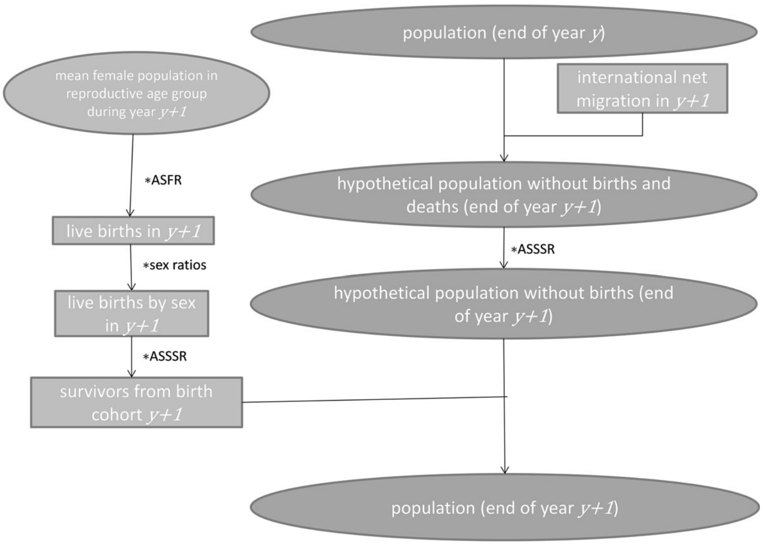

Fig. 9 Process of Annual Population Update. (Source: Own design)

Population Development in Germany Until 2045

The combination of the resulting trajectories for the demographic components as explained in Section 3 results in a probabilistic cohort-component forecast of the age- and sex-specific population for the ages 0–102 years for males and 0–103 years for females. The respective age groups beyond these limits are treated as one age group each. The initial population for the forecast is the age- and sex-specific population estimate from the HMD for December 31, 2017 (Human Mortality Database 2019d).

In Section 3, we described the models for forecasting of the demographic components. Now, we provide a selection of the results from the forecast. The overview is kept short since more detailed results for the age-specific measures can be found in previous papers. The fertility model results in 10,000 trajectories for all ASFRs. By multiplication of the ASFRs with the corresponding female population, the birth forecast is completed. The results are given in Fig. 10.

The increasing trend in births, as witnessed since 2012, is expected to continue until 2024. Birth numbers will probably subsequently decrease moderately because most children are born by mothers over 29 years of age, as shown by Vanella and Deschermeier (2019: 101). This decrease can therefore be explained by the decreasing number of births at the beginning of the 1990s, as shown at the left-hand side of the graph. The median increase during the second half of the 2030s stems from a slightly increasing TFR[12] together with almost stagnating birth numbers during the cohorts 2005 to 2011, which by then will be in their reproductive phase.

Similarly, the death numbers are derived from the ASSSRs and the population update. As shown in Fig. 9, deaths can be derived by simulating the hypothetical age- and sex-specific population at the end of some period in some trajectory without deaths and then multiplying this number with the respective adjusted ASSMR to derive the actual number of deaths among this group. The resulting death numbers are illustrated in Fig. 11.

As a result of the aging of the population, a further increase in deaths, as witnessed since the mid-2000s, is probable until the end of the forecast horizon.

By subtracting the death numbers from the birth numbers, we calculate the natural population growth, whose forecast can be derived indirectly from the birth and death forecasts as well. The results of the natural population growth forecast are illustrated in Fig. 12. A clear negative tendency is probable. The deaths will almost certainly exceed the births over the whole forecast horizon.

Counterbalancing the shrinking population due to natural population decrease is the international net migration. The forecast method for the ASNSNM numbers has been explained in Section 3, the results of the simulation are cumulated into the total net migration for illustration purposes in Fig. 13.[13]

The median scenario gives a slightly decreasing net migration, whereas some cyclic course due to economic cycle is probable. In general, the high uncertainty in migration forecasting is obvious, but in general a positive net migration is very likely. The median of net migration in 2045 is 271,689 persons. This is a higher balance than most previous projections provide, which had been calculated before the record influx of 2015. As many

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[12] Our model predicts a slight increase in the median TFR from its initial value of 1.56 in 2016 to 1.67 in 2045.

[13] More detailed results, although based on the jump-off year 2015, can be found in Vanella and Deschermeier (2018: 274–276).

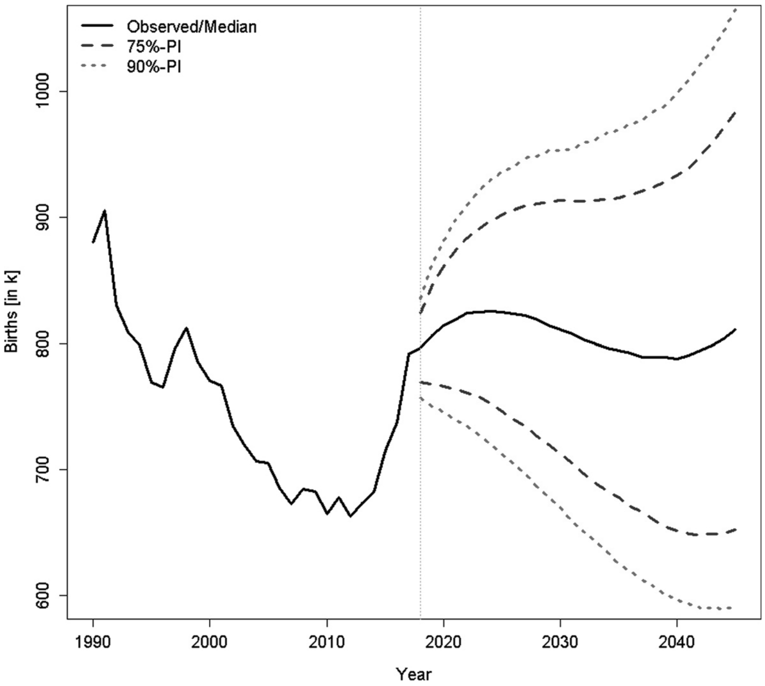

Fig. 10 Forecast of Birth Numbers. (Sources: GENESIS-Online Datenbank 2019b; Own calculation and design)

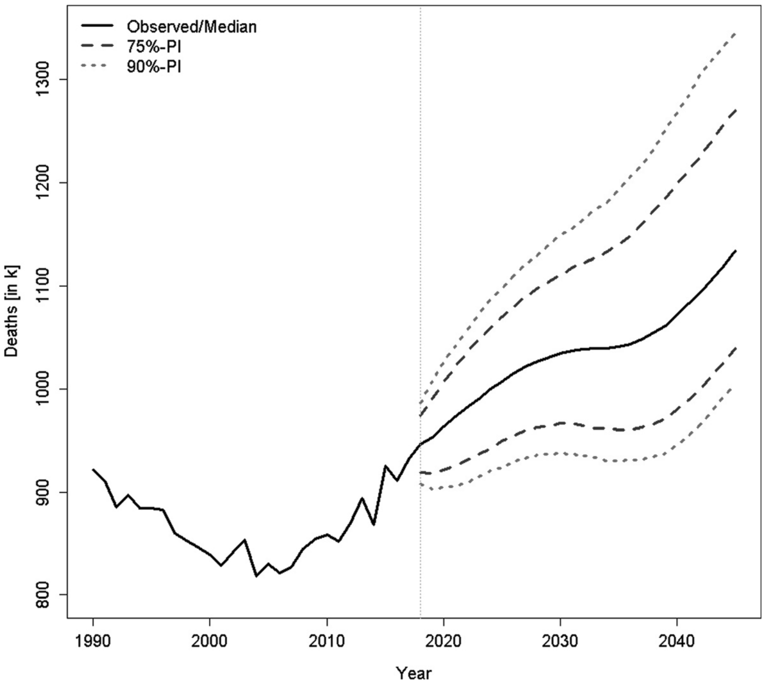

Fig. 11 Forecast of Deaths (Sources: GENESIS-Online Datenbank 2019c; Own calculation and design)

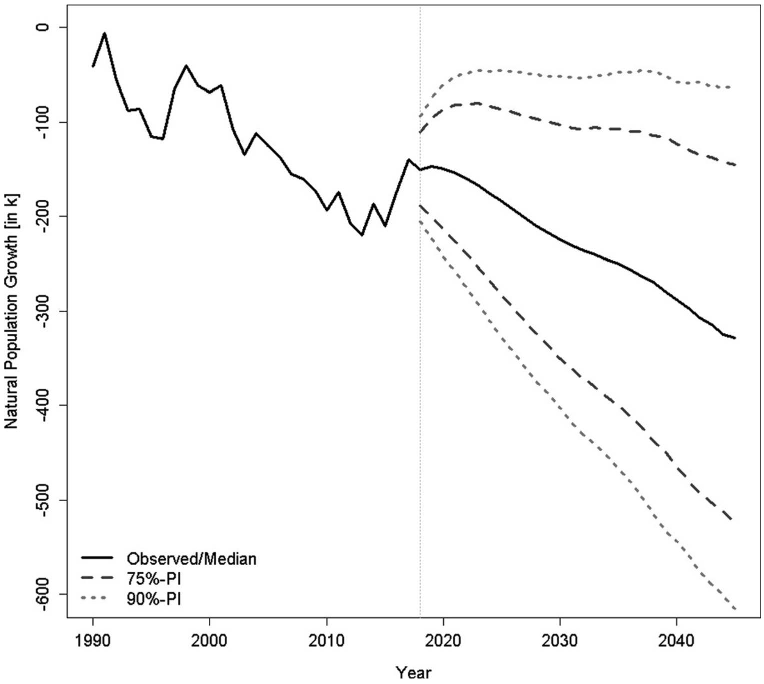

Fig. 12 Forecast of Natural Population Growth. (Sources: GENESIS-Online Datenbank 2019b, c; Own calculation and design)

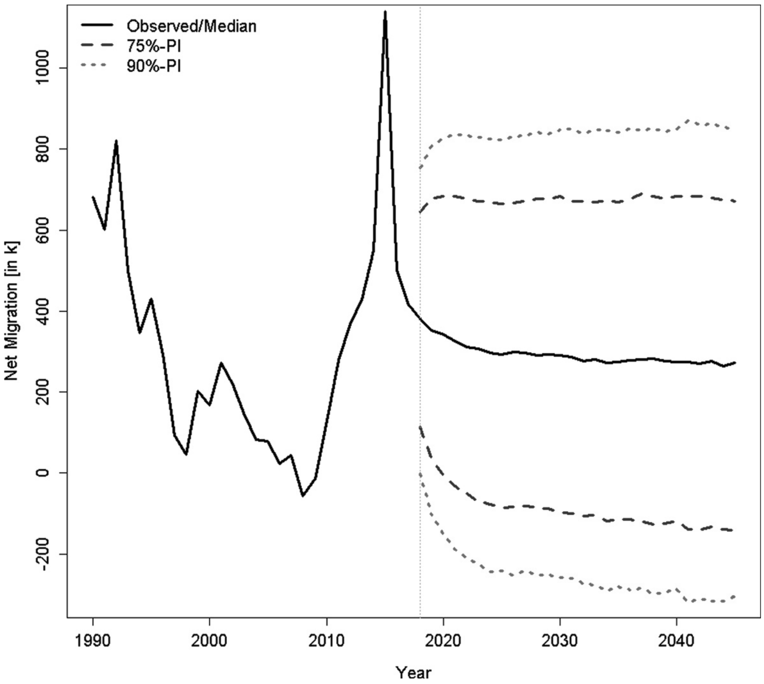

Fig. 13 Forecast of Net Migration. (Sources: GENESIS-Online Datenbank 2019d; Own calculation and design)

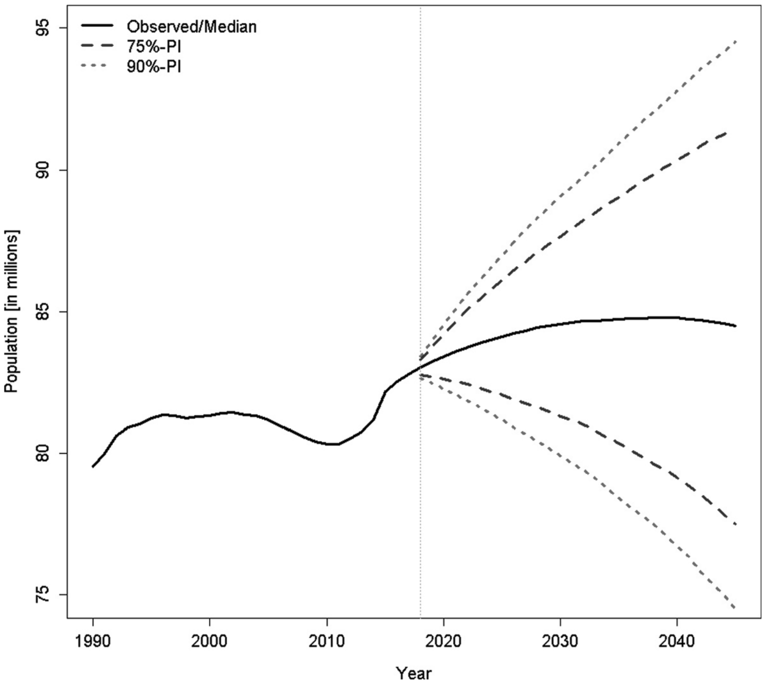

Fig. 14 Forecast of Population Size. (Sources: Human Mortality Database 2019d; Own calculation and design)

bigger cities of origin of the refugees, especially in Syria, are mostly devastated by war (McKenzie 2018; Pleitgen 2017), it seems unlikely that there will be a mass emigration out of Germany in the years to come, as one might expect due to experience from past refugee crises. Furthermore, the results reflect the strong past development of the economy in Germany. This trend is probable to remain stable in the future (OECD 2017: 130–133). The attractive labor market is likely to attract more people in the future (Fuchs et al. 2018: 49–54), especially within the EU due to the unrestricted free movement of workers (Vanella and Deschermeier 2018: 274–277). Moreover, some sending economies in Southern and Eastern Europe still struggle heavily economically since the financial crisis and do not appear likely to recover in the near future (World Bank 2019: 100–101). Total net migration in 2045 is estimated to be above zero at 77.62% probability.

The high importance of positive net migration, especially in the younger ages, shall be mentioned to fill the shortages occurring in the labor market due to overaging. We stress that the effect of migration on the labor market and the social security system very much depends on the skill level and education of the immigrants. Especially in cases of refugee migration, where education is often either relatively low or not accepted by German standards, it usually takes a long time for the immigrants to fully integrate into the labor market (Brücker et al. 2017b).

Figure 14 shows the forecast of total population until the year 2045 with 75% and 90% PIs.

In contrast to many earlier studies on Germany (see Section 2), the population is expected to increase moderately over the forecast horizon due to high, yet decreasing, net migration, an increasing TFR and decreasing mortality.

Our median forecast of the total population is substantially above those of the other studies. Most of the presented studies were conducted before the refugee crisis since 2014 and the above average net migration since 2010 caused by the European debt crisis. These developments mark significant changes in migration. As such remarkable events often are ignored in migration forecasts, our model considers the probability of future crises, which might lead to high migration influx, through the Crises Index explained in Section 3. The underlying assumption is that crises have a similar probability to occur in the future as they have been observed in the past data. Since our historic data cover two longer periods of international crises, the future risk should be covered appropriately by our model. One advantage of our approach becomes clear by investigating the PIs of our backtest in Appendix C. According to our forecast, the ceteris paribus population in 2045 will be between 74.5 and 94.5 million people at a 90% probability level, with a median outcome of 84.5 million.

In many cases (like in social security), the structure of the population is of higher importance than its size per se. Therefore, Fig. 15 gives an overview of the age structure of the population in 2017 compared to the forecast in 2045 with PIs for both sexes.

We observe the almost 30-year shift in the population. In general, there is greater uncertainty for males. Whereas the retirement-age population can be predicted relatively well, the uncertainty in the future working-age population is rather large for males due to the higher uncertainty in the migration forecast for males relative to females (Vanella and Deschermeier 2018: 274–277). The uncertainty in the population of persons under 30 years of age mostly arises from the fact that this portion of the population has not yet been born but also from the relatively high uncertainty in international migration.

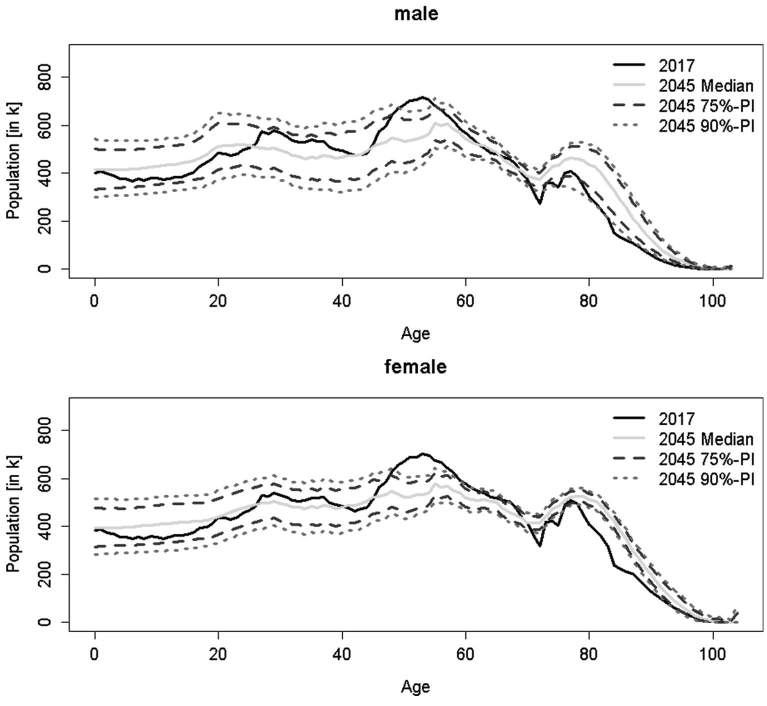

Fig. 15 Population by Sex and Age in 2017 and 2045. (Sources: Human Mortality Database 2019d; Own calculation and design)

The aging of the population is of high social and economic importance. Therefore, in addition to the overall age structure, the median age of the male and female populations is considered as a summary indicator for the future age distribution of the population. The median age of the population can be obtained from the simulation results because it is the exact age that cuts the population in half. This computation for all 10,000 trajectories can be used to extract PIs for the median age, similar to the computation of median life span conducted by Vanella (2017: 548–552). The results of our analysis are shown for both sexes in Fig. 16.

We observe a rejuvenation effect for the upcoming years due to the high net migration witnessed during the most recent years, as illustrated in Fig. 13, and to increasing birth numbers, as shown in Fig. 10. The high net migration around the year 2015 combined with the high forecast values for the upcoming years leave a mark in the age structure of Germany. This can be seen in the age structure for the male and female populations in the year 2045. By that time, the majority of the population that immigrated during the high influx phase will be approximately 55 years old, while the baby boomer generation will be in their eighth decade of life. Over the forecast horizon, the median age traces this development by a rejuvenation effect for men and women. The probable decrease in the number of births after the early 2020s and decreasing net migration and mortality (Vanella 2017: 550) lead to an aging of the population structure, as represented by the increasing median ages after that point. After the mid- to late 2030s, another phase of rejuvenation can be expected due to the increasing number of births resulting from the

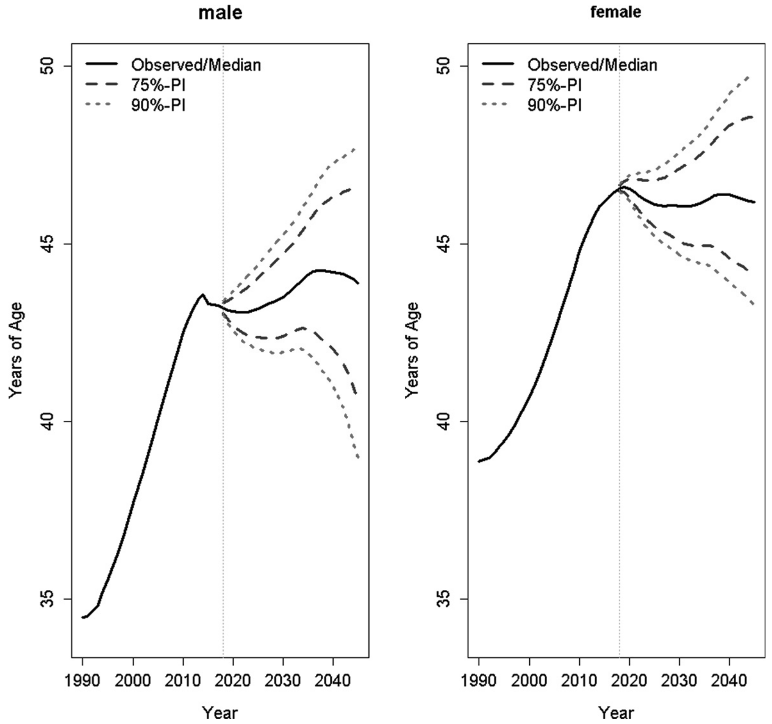

Fig. 16 Median Age by Sex until 2045. (Sources: Human Mortality Database 2019d; Own calculation and design)

stronger birth cohorts after 2010 (see Fig. 10), which will then enter their most fertile period, and the large increases in death numbers, as the baby boomers will witness high mortality after that period (see Fig. 11). More results of the forecast can be found in Appendix B, where we have listed selected predictions for the following three age groups: under 20 years, which is referred to as the young population, persons aged 20–66, which is the main working age population and persons aged 67 and older, the population of the standard legal pension age (Fuchs et al. 2018: 50).

Appendix C shows the results of a backtest we performed for indicating the performance of our model procedure. The interested reader may feel free to look for more details on this topic there.

Our model provides a wide range of detailed analyses targeting specific topics of interest, as we have shown based on certain important measures. The forecast results offer the possibility for a wide range of future studies, e.g., analyzing the effects of population changes on social security, the labor market or housing demand. The selected results in Appendix B give a small impression of the future demographic structure from a labor market perspective.

Conclusions, Limitations and Outlook

This paper proposed a probabilistic cohort-component approach for population forecasting by sex and age. It was applied to predict the population of Germany until the year 2045. Germany witnessed a record migration influx in 2015 due to the refugee movement, especially from Syria, Iraq and Afghanistan, in combination with the challenging economic situation in many countries in Southern and Eastern Europe. The record net migration marks a considerable event for Germany's demographic development. The strong long-term decrease in the population expected by many does not appear to hold based on our findings. The results provide essential data on the consequences of the current trends for decision makers, planners and scientists.

The model predicts the population of Germany by age and sex until the year 2045. The forecast is conducted as a composite of three time series models based on PCA for the three demographic components fertility, international migration and mortality by sex and age. The method is specified for Germany, but it can also be applied to other countries or regional units, for which sufficiently long time series data for the demographic components are available. Stochastic modeling of the population produced point estimates of the future population in addition to a measure of the future uncertainty via prediction intervals. The results may be disaggregated or aggregated almost arbitrarily regarding sex, age and level of uncertainty.

The model is well suited for regular updating and does not require large amounts of data input since it is restricted to demographic variables and uses official statistics provided by Destatis and the HMD. One interesting result is the detailed reporting and probabilistic quantification of the disaggregated population for all ages and both sexes; therefore, the results offer many possibilities for future forecast studies that require disaggregated population data as inputs, e.g., research on social security, life insurance, the labor market or housing demand.

Our method is restricted to quantitative methods; therefore, past unobserved trends are not considered in the future. Nevertheless, for all demographic variables, the input data span at least as long of a time horizon as is forecast; thus, we believe that all realistic trends that might be observed during the time horizon are included in the model. The addition of expert knowledge would be possible, if the forecaster thinks the past trends insufficiently cover the possible future outcome. The model suffers from a relatively small input time horizon because the migration data are restricted back to the year 1990. Older data is not representative because of the overall very different geopolitical situation in Eurasia back then. Furthermore, fertility is difficult to forecast, since it is significantly influenced by policy as well. We tried to include this effect to some extent into the model as well, following a convergence assumption in family policy to avoid bias to the extent possible. Our forecast horizon is 2045 and not 2060 or 2100, as in other studies, since we do not intend to create misinterpretations for the far future, for which reliable forecasts are not possible with the available data.

A larger forecast period would be interesting but cannot be achieved via responsible statistical modeling. Thus, the future availability of input data suited for model estimation will improve the quality of our models and allow for longer forecast horizons. Even with a forecast horizon that reaches only until 2045, the uncertainty is rather large. Most of the risk stems from the uncertainty about future net migration. Although the net migration model performs reasonably well, a possible extension of the model would be to separately estimate in- and out-migration. Joint estimation of birth rates, survival rates and migration numbers (or rates in the case of out-migration) would represent another possible extension. The forecast model could theoretically discriminate by nationality for the mortality and fertility forecast, resulting in a nationality-specific population forecast. However, this would require detailed time series data on mortality and fertility by nationality alongside data on naturalizations. Moreover, the jump-off population would be needed by nationality as well. None of this information is available.

Empirical updating might be required if the development in the upcoming years differs from our forecast due to political or economic developments. Those structural breaks are not implemented in our simulation approach.

Acknowledgments We would like to thank the anonymous reviewers for their helpful remarks, which contributed to substantive improvements in the paper. Moreover, we appreciate the support by the Federal Statistical Office, who provided us with much of the input data for our study.

Appendices

Appendix A: Forecast Functions of Principal Components

Migration Model

Labor Market Index (Principal Component 1):

$$\begin{aligned} l(y_l) = -23,943.514 + 1,942.419 * y_l \\ + 3,646.207 * \sin(0.698 * y_l - 3.316) + u_l(y_{l-1}) + e_l(y_l) \end{aligned}$$

- y_l being the year under study, with $y_l = 0$ corresponding to the year 1990
- $u_l(y_l)$ being the autoregressive part in the ARIMA model specifying the difference between the observation in y_l and the deterministic long-term trend specified by the model:

$$u_l(y_l) := l(y_l) - [-23,943.514 + 1,942.419 * y_l + 3,646.207 * \sin(0.698 * y_l - 3.316)]$$

- $e_l(y_l)$ being the nuisance parameter of the ARIMA model with $e_l \sim \mathcal{NID}(0; 7,307.992^2) \forall y_l$

Crises Index (Principal Component 2):

$$c(y_c) = -11,777.96 + 0.687 * u_c(y_{c-1}) + e_c(y_c)$$

with

$$e_c \sim \mathcal{NID}(0; 11,180.7^2) \forall y_c$$

Principal Components 3–1463:

$$pc_i^M(y) = pc_i^M(y-1) + e_i^M(y)$$

with

$$e_i^M \sim \mathcal{NID}\left(0; \sigma_i^{M2}\right), i = 3, 4, \ldots, 1463$$

Mortality Model

Lee-Carter Index (Principal Component 1):

$$m(y_m) = -66.518 - 23.881 * \frac{\exp\left(\frac{y_m}{14.901}\right)}{1 + \exp\left(\frac{y_m}{14.901}\right)} + 0.618 * u_m(y_{m-1}) + e_m(y_m)$$

with

- $y_m = 0$ corresponding to the year 1995

$$e_m \sim \mathcal{NID}(0; 0.299^2) \forall y_m$$

Behavioral Index (Principal Component 2):

$$b(y_b) = 10.3 + 8.341 * \frac{\exp\left(\frac{y_b}{14.025}\right)}{1 + \exp\left(\frac{y_b}{14.025}\right)} + 1.298 * u_b(y_{b-1}) + 0.405 * u_b(y_{b-2})$$

$$-0.702 * u_b(y_{b-3}) + e_b(y_b)$$

with

- $y_b = 0$ corresponding to the year 1990

$$e_b \sim \mathcal{NID}(0; 0.158^2)\,\forall y_b$$

Principal Components 3–209:

$$pc_j^D(y) = pc_j^D(y-1) + e_j^D(y)$$

with

$$e_j^D \sim \mathcal{NID}\left(0; \sigma_j^{D^2}\right), j = 3, 4, \ldots, 209$$

Fertility Model

Tempo Index (Principal Component 1):

$$t(y_t) = -7.592 + 0.155 * y_t + 0.742 * \ln(y_t) + u_t(y_{t-1}) + e_t(y_t)$$

with

- ln() denoting the natural logarithm
- $y_t = 0$ corresponding to the year 1967

$$e_t \sim \mathcal{NID}(0; 0.289^2)\,\forall y_t$$

Quantum Index (Principal Component 2):

$$q\left(y_q\right) = -17.186 + 5.758 * \frac{\exp\left(\frac{y_q}{5.167}\right)}{1 + \exp\left(\frac{y_q}{5.167}\right)} + u_q\left(y_{q-1}\right) + e_q\left(y_q\right)$$

with

- $y_q = 0$ corresponding to the year 2010

$$e_q \sim \mathcal{NID}(0; 0.216^2)\,\forall y_q$$

Principal Components 3–37:

$$pc_k^F(y) = pc_k^F(y-1) + e_k^F(y)$$

with

$$e_k^F \sim \mathcal{NID}\left(0; \sigma_k^{F^2}\right), k = 3, 4, \ldots, 37$$

Appendix B: Selected Forecast Results for Three Age Groups

Table 1 Forecast Population (in millions) for Selected Years and Three Age Groups with 75% PIs

Year	Young Median	Young 75% PI Lower Bound	Young 75% PI Upper Bound	Working Age Median	Working Age 75% PI Lower Bound	Working Age 75% PI Upper Bound	Old Median	Old 75% PI Lower Bound	Old 75% PI Upper Bound
2017	15.252			51.804			15.736		
2021	15.573	15.354	15.799	51.588	51.178	52.012	16.428	16.357	16.500
2025	16.122	15.675	16.563	50.755	49.958	51.568	17.233	17.064	17.398
2029	16.642	15.948	17.337	49.396	48.224	50.559	18.446	18.181	18.715
2033	17.031	16.065	17.985	47.845	46.346	49.325	19.812	19.439	20.188
2037	17.086	15.853	18.311	47.065	45.233	48.871	20.630	20.127	21.118
2041	16.974	15.510	18.481	47.352	45.211	49.513	20.389	19.744	21.013
2045	16.835	15.124	18.609	47.829	45.340	50.382	19.844	19.020	20.603

Sources: Human Mortality Database 2019d; Own calculation and design

Explanations:

- "Young" population means the population younger than age 20
- "Working Age" addresses the population aged 20–66
- "Old" population means persons aged 67 and older

Appendix C: Backtest for Forecast Accuracy

We have used the population data described in the data section until the year 2008 only, the year before the big migration influx described in the introduction occurred. Avoiding the structural break, which occurred in the data due to the 2011 Census (see also Section 3), we base the test on the last census before that, which occurred in 1987. The base population is the age- and sex-specific population on December 31, 2008. Since the dataset provided does not include detailed information for the population aged 95 and older, we estimated the distribution of the population in that age group by using the population estimates for December 31, 1999 and updating the respective cohorts using the age- and sex-specific numbers of deaths by cohort for the years 2000–2008. Migration in this case is ignored for the sake of simplicity. All of these data have been provided by Destatis on demand (Destatis 2016b, 2017c). Our model is now used for forecasting the age- and sex-specific population until December 31, 2018, following the same approach as that described in Section 3. To put the results in perspective, some selected results of Destatis' (2009) 12th coordinated population projection, namely, the stated most probable middle assumptions alongside the two extreme scenarios, are compared with the hypothetical population numbers. We stress that this is not the actual population number, but rather the estimate of what the population number would have been, based on updating the 1987 Census estimate. To update this estimate, we annually increase the population by birth numbers and net migration, while subtracting the death numbers, starting at our estimate for the 2008 population numbers.

Figure 17 provides all these estimates for the period 2009 to 2018 alongside our model estimates of the median population and the 90% PIs.

Our median forecast would have performed similarly as poor as Destatis' projection (not even their extreme scenarios have been able to capture any of the real population developments). However, Fig. 17 underlines the advantage of our probabilistic approach, as eight of the ten population estimates from the update are captured by our 90% PI, and even the two values outside the interval in 2015 and 2016 are just slightly outside of our interval bounds. Our model even managed to identify the extreme migration of the year 2015 as a possible, yet unlikely, scenario. This stresses the message advocated by us: It is extremely difficult to predict the future demographic development, especially when with regard to migration, but an appropriate stochastic approach covers all possible outcomes and quantifies them.

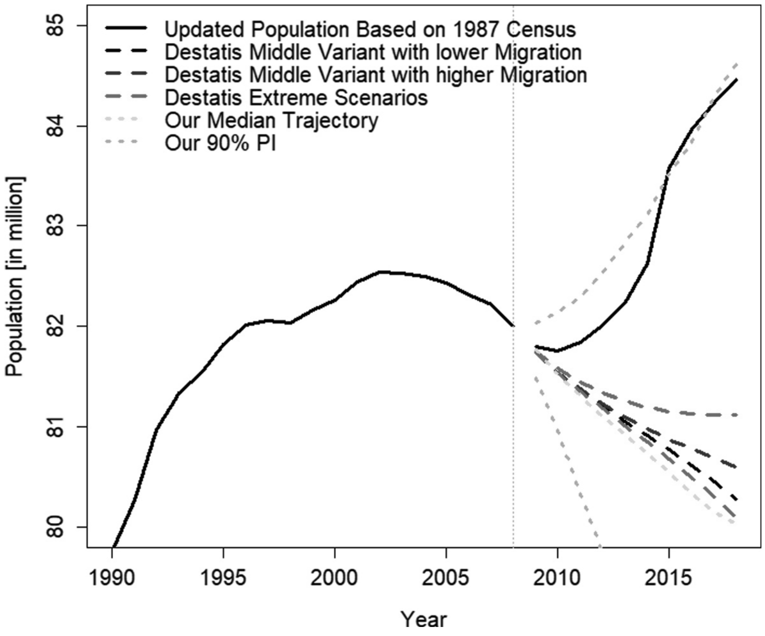

Fig. 17 Backtest Results for 2009–2018 with Destatis Projection and Population Update. (Sources: Destatis 2009, 2016b, 2017c; Own calculation and design)

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Chapter 6

The Impact of Population Aging on the German Statutory Pension Insurance

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The Impact of Population Aging on the German Statutory Pension Insurance – A Probabilistic Approach

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Abstract

The demographic transition is a phenomenon affecting many industrialized societies. These economies are experiencing a decline in mortality alongside low fertility rates – a situation that puts social security systems under severe pressure. To implement appropriate reform measures, adequate forecasts are needed of the future population structure and, correspondingly, of the pension insurance. In this article, we use a probabilistic approach to forecast the numbers of pensioners aged 60 and above in Germany up to 2040, considering trends in population development, labor force participation and early retirement as well as the effects of further pension reforms. Principal component analysis is used for dimensionality reduction and consideration of cross-correlational effects between age- and sex-specific pension rates for old-age as well as disability pensions. Time series methods enable the inclusion of autocorrelation effects in the model and the simulation of future uncertainty. The model predicts that in the median, the numbers of old-age pensioners will increase by almost 9 million persons (from 18.1 million in 2016 to 26.8 million for old-age pensions) alongside increases in disability pensions by 2040 given the raising of the legal retirement ages following the introduced regulations. This result shows a clear need for further reforms if the German statutory pension insurance is to be sustainable in the long run.

Keywords: Population Aging; Stochastic Forecasting; Principal Component Analysis; Time Series Analysis; Applied Econometrics; Public Pension Systems; Social Policy

JEL: C53, H55, J11

1 Introduction

Countries with low fertility and decreasing mortality rates struggle with an aging population structure and negative natural population growth (OECD 2018). Decreasing mortality means longer periods of pension claims as long as retirement ages are not adjusted parallel to the increases in life expectancy. Ceteris paribus (c.p.), low fertility results in a smaller workforce in the long run (Zuchandke et al. 2014). Mortality in Western Europe has been decreasing almost monotonically since the 1970s (Vanella 2017), whereas replacement-level fertility has not been reached (Vanella and Deschermeier 2019). For countries applying a Bismarck-type (or *pay-as-you-go*) pension system[1], this particular demographic development results in double financial distress: The elderly are at increasing risk of suffering from old-age poverty, while a growing share of labor income generated by the working population has to be transferred to the elderly (Goffart 2018). Demographic aging combined with pay-as-you-go schemes thus affects the financial sustainability of pension systems if that trend is not averted by policy reforms.

In the case of Germany, payments out of the statutory pension insurance *(Deutsche Rentenversicherung – DRV)* are based on this original Bismarckian principle (Graf von der Schulenburg and Lohse 2014). Public pension payments constitute the largest share of retirement income in Germany at approximately 63% (Federal Ministry of Labour and Social Affairs 2016). Therefore, future old-age income will depend heavily on changes in the size and structure of the population, which are essential for the financial stability of the DRV. The DRV should ensure a certain living standard for its pensioners while not overloading the working population with excessively heavy contributions to the pension insurance (Vogt 2017).

Since the late 1980s, the German government has passed a series of pension reforms as countermeasures to the demographic aging process. In 1989, the *Rentenreformgesetz 1992 (RRG 1992)* was the first reform with a clear demographic agenda: it raised the legal retirement age for female and unemployed persons from 60 years to 65 (at that time the standard legal retirement age for men) until 2008 and was one important measure for containing the number of future retirees. Moreover, the exceptional early retirement age of 63 years for persons who had been employed for at least 35 years was abolished[2] (RRG 1992). Furthermore, whereas early retirement without sanction had previously been possible, the reform introduced financial repercussions for retiring early. Since that reform, for every month of pension claim before the individual legal retirement age, the monthly pension payment is reduced by 3‰. In addition, incentives for exceeding the legal retirement age were increased by introducing a bonus system. Each month of delayed pension claims beyond the individual legal retirement age is rewarded with an increase of 5‰ in monthly pension payments (Wilke 2009). The *Wachstums- und Beschäftigungsförderungsgesetz (WFG)* in 1996 accelerated the increases in the legal retirement ages for unemployed and females even further, underlining the urgency of the policy measures. Due to the RRG 1992, the increase in the retirement age of these two groups would have ended

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[1] Pension payments are redistributed from the labor force to the pensioners within the same period (see, e.g., Graf von der Schulenburg and Lohse 2014 on this).

[2] In 2014, the German government returned to a similar measure, with a legal retirement age of 63 years for persons who have 45 years of social security payments (Bundesregierung 2013). This change is considered in our model as well.

at the target age of 65 in 2018 (RRG 1992, §41 I); however, the WFG required that mark to be reached in 2007 for unemployed men and in 2010 for women (WFG 1996). The legal retirement age for severely disabled persons, previously 60 years, was increased to 63 years between the years 2000 and 2006 following the *Rentenreformgesetz 1999* (RRG 1999). According to the so-called *Riester reform* of 2001, a new pension formula was introduced that linked individual pension payments to the overall development of labor income and savings in society over the period. In 2004, the pension formula was adjusted by adding the so-called *sustainability factor,* which is directly connected to the old-age dependency ratio[3]; this change therefore directly considers overall demographic development when determining pension payments (Wilke 2009). The *RV-Altersgrenzenanpassungsgesetz* from 2007 was the latest reform aimed at responding to the demographic aging process. The standard legal retirement age will now increase gradually until 2031, when it will reach 67 years. The legal retirement age for severely disabled individuals is adjusted accordingly from 63 to 65 years and for mineworkers from 60 to 62 years (RV-AltAnpG 2007). The different retirement ages find direct consideration in our forecast model; therefore, the average annual retirement ages for different groups of people over the past and forecast time horizons are illustrated in Appendix A.

Further possible reforms should be based on new and adequate forecasts of the future development of the DRV (Zuchandke et al. 2014). The present contribution provides a stochastic forecast of the year-end old-age and disability pensioner numbers aged 60 and above receiving payments from the DRV through the year 2040. We use principal component analysis (PCA) for dimensionality reduction and consideration of cross-correlation between the age- and sex-specific pension rates (ASSPRs), and the connections among retirement, disability and legal retirement age are covered as well. Time series models include autocorrelation of the ASSPRs, providing the methodological framework for quantification of future uncertainty in these predictions. Combined with a fully probabilistic population forecast model developed by Vanella and Deschermeier in earlier studies (Vanella 2017; Vanella and Deschermeier 2018, 2019, 2020), a forecast of the future numbers of pensioners for this age group is elaborated. The model takes trends in labor force participation and early retirement, along with demographic trends such as decreasing mortality and morbidity, into consideration implicitly by time series analysis (TSA). The effects of the pension reform of 2007 are captured to some extent by an econometric model in the forecast. The simulation not only returns the median age- and sex-specific retired

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[3] The ratio of number of persons exceeding a certain age (mostly 65 years) over the number of persons in the assumed working-age, e.g. 15-64 years (Wilke 2009).

and officially disabled population over 60 years of age up to the year 2040 but also quantifies the uncertainty in the forecast, illustrated with 75% and 90% prediction intervals (PIs) for each year, age and sex.

The next section presents a literature review on forecasts and projections for statutory pension systems with special emphasis on stochastic approaches on the one hand and studies of Germany on the other hand. We will then describe the method and data used for our analysis and present a selection of the results generated by our forecast. The model is applied to Germany but is in principle applicable to other countries as well, especially those that apply a Bismarck-type social security system. The paper will then conclude with a discussion of the results and limitations, giving an outlook of opportunities for further research.

2 Forecasts of Pension Demand with Special Emphasis on Germany

Population forecasts are rarely of a probabilistic nature; instead, deterministic projections are mostly conducted (see, e.g., European Commission 2018; OECD 2018; Pötzsch and Rößger 2015). These approaches result in equally deterministic pension projections because they rely on the underlying population predictions (see, e.g., Vogt 2017; Werding 2011; Wilke 2009).

The timing of pension claims is basically an individual decision. However, social policy can try to influence retirement decisions by bonus-malus systems to affect retirement behavior as well as labor force supply (Gruber and Wise 2000). Germany's pension reform of 1989 followed that approach. There is strong evidence for the effects of policy reforms on social security and retirement decisions or expectations (Börsch-Supan 1992, 2000; Coppola and Wilke 2014; Buchholz et al. 2013). Nies (2010) uses the *Versichertenkontenstichprobe (VKST)*[4] to estimate the risk of retirement from the German labor market through econometric modeling with socioeconomic factors as predictor variables.

Wilke and Börsch-Supan (2009) simulate the labor force in Germany until 2050 using scenario analyses of the development of the population and trends in labor force participation. The combination of two demographic and four different labor market scenarios gives eight trajectories of the labor force in Germany by 2050. Börsch-Supan and Berkel (2004) estimate individual probabilities of retirement under different socio-economic criteria in an econometric framework using data from the German Socio-Economic Panel Study (SOEP). Bucher-Koenen and Wilke

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[4] The VKST is a micro data set on retired persons in Germany provided by the DRV.

(2009) apply the results of these two studies to estimate the long-term effect of the RV-AltAnpG 2007 by simulating different scenarios for the labor force participation rates and the population's adjustment of its average retirement age to the increasing legal retirement ages of the 2007 reform.

Estimation of future financial development and thus the potential need for intervention in the DRV is generally based on deterministic projections of the population, combined with the legal retirement age and assumptions on the development of the labor market. Mostly, simple statistics such as the old-age dependency ratio are consulted for this (e.g., Pötzsch and Rößger 2015; European Commission 2018). Wilke (2009) and Holthausen et al. (2012) propose a detailed model for long-term projections of the future financial outlook for a wide range of German social security reforms until the year 2100. These analyses include many factors, such as population development, labor force participation, and policy reforms. They introduce a complex model that is able to show the future demand for social security based on subjective assumptions about future demographic and economic development. Werding (2011, 2013) projects the financial outlook of the DRV using the population projections by Destatis, deriving possible trends in labor force participation from micro census data and modeling future macroeconomic growth with a Cobb-Douglas production function. From these partial models, the scenarios for the future old-age dependency ratio and the resulting financial expenses for old-age pensions until 2060 are derived. Vogt (2017) uses a similar model for a more current projection. The EU and the OECD offer similar projections for their respective member countries on an annual basis (see, e.g., European Commission 2018; OECD 2018).

All models are very detailed and provide suggestions for further model advances. They are quite restrictive in their assumptions, however, which is inevitable for deterministic models. Furthermore, the assumptions on fertility and migration development are in many cases questionable because the total fertility rate (TFR) is generally assumed to be constant. Vanella and Deschermeier (2019) show that a naïve forecast[5] of the TFR for Germany performs rather poorly. A drawback of Vogt's (2017) simulation is that it does not include the increase in the legal retirement ages as a result of the pension reform of 2007, thereby overestimating the number of old-age retirees. Deterministic methods generally have some limitations because they are restricted to a limited number of scenarios whose respective probabilities of occurrence are mostly not quantified. Thus, stochastic forecasts in demographic research are gaining popularity as an alternative to common deterministic projections that use scenarios to address future

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[5] Assuming a constant TFR long-term.

uncertainty (Istat 2018; Keilman et al. 2002; Lee 1998). Stochastic forecasts based on simulations are less prone to subjective decision-making and provide a huge number of possible future outcomes while being able to quantify their likelihood. Keilman et al. (2002) propose a probabilistic population forecast model, which is applied to Norway until 2050. Fertility is forecast using a multivariate autoregressive moving average (ARIMA) model, including the TFR, the mean age at childbearing (MAC), the variance in the MAC and the minimum reproductive age as four parameters. Alho and Spencer (2005) propose a probabilistic forecast approach to the old-age dependency ratio, based on a stochastic model of population forecasting for Finland. Their method could be helpful for formulating social policy reforms that include flexible adjustments of the legal retirement age. Li et al. (2009) estimate the aging effect in the Chinese population as a proxy of the pension demand by deriving the future old-age dependency ratio. They do so by constructing a probabilistic population forecast through stochastic modeling of the demographic components fertility, mortality and international migration. These partial forecasts are performed through a combination of quantitative and qualitative model assumptions as baseline scenarios for future development. Uncertainty is quantified by assuming a similar future risk for the demographic components in China compared to a pool of European countries over the distant past, as proposed in Alho and Spencer (2005). The net migration in the mean is assumed according to the UN projection (see United Nations 2007). The uncertainty of future migration is assumed to be similar to past trends for Europe, as given in Alho and Nikander (2004). The resulting population forecast is used to estimate simulations of the future old-age dependency ratio. Ahn et al. (2005) apply a similar method for a stochastic projection of the financial outlook of the Spanish pension insurance through 2050. Giang and Pfau (2008) generate a partially probabilistic projection of the financial pension outlook for Vietnam until 2100. Fertility and mortality are estimated by the popular Lee-Carter models for these two components (see Carter and Lee 1992; Lee and Carter 1992; Lee 1993), whereas the modeling procedure for international migration is not clearly described in the paper and appears to be deterministic. Assuming stationarity for the labor force participation rates, the authors derive estimates for the age- and sex-specific numbers of contributors to the social security system. Giang and Pfau (2008) extract the projections of future pensioner numbers. These entities are used for a stochastic estimation of the future old-age dependency ratio. Forecasting a range of economic factors, the future outlook for contributions into the pension insurance as well as the demand for pension entitlements is approximated.

Lipps and Betz (2005) forecast the population in Germany until 2050 stochastically by running 500 trajectories. Mortality and fertility are estimated for West and East Germany separately.

Age-specific mortality rates are forecast using the Lee-Carter model for mortality, while the TFR is assumed to be a random walk process. The age schedule for the fertility rates is assumed Gaussian, with a converging MAC over the long term. Under these assumptions, the distribution of age-specific fertility rates (ASFRs) is simulated. The total net migration is assumed to be an autoregressive process of order one (AR(1)) (see, e.g., Shumway and Stoffer 2011 on AR processes). Simulating the net migration and the age distribution, the future population is estimated as well. The trajectories for the population are used for computation of the old-age dependency ratio, in the respective paper defined as the ratio of people over 60 years of age to the population between 20 and 59. Härdle and Myšičková (2009) propose a probabilistic cohort-component forecast for the population in Germany through 2058. Age-specific mortality and fertility rates are forecast by applying the respective Lee-Carter models. International migration is modeled separately for immigration and emigration, where the total numbers for both statistics are estimated by AR(1) models. The age structures of the migrants are approximated by Kernel density estimation.[6] As a result of the population forecast, the authors forecast the old-age dependency ratio for retirement ages 65 and 67. Using a status quo assumption, the authors derive a stochastic projection of the future social insurance premium rate and the average replacement rate.

More details on the data and methods used in the more significant studies on pension forecasting mentioned in this section are given in Appendix C. One common merit of the presented studies is the indirect derivation of the retired population over the labor force participation and the resident population. That does not include the population living abroad while receiving pension payments in the country under study. In the case of a country such as Germany, where the net migration of the native population above retirement age is mostly negative (Federal Ministry of Labour and Social Affairs 2018; Vanella and Deschermeier 2018), this leads to systematic underestimation of the retired population.

3 Method and Data

In this section, we propose a joint probabilistic forecast model for the number of old-age pensions and disability pensions by sex and age of the pensioners. In the first step, past age- and sex-specific pension rates (ASSPRs) for old-age and disability pensions are estimated. The data have been accumulated from three sources: the German Federal Statistical Office *Destatis*, the

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[6] See, e.g., Härdle et al. 2004 on that.

DRV and the Federal Health Reporting Service *(gbe-bund),* provided by Destatis and the Robert Koch Institute (RKI). Thus, we used the year-end sex-specific stocks of old-age pensioners by age (in years)[7] for the years 1992-2015 from the gbe-bund database (Destatis 2018a). It is not advisable to use data prior to 1992 because the integration of pensions for citizens from the former German Democratic Republic *(DDR)* into the DRV after the German reunification did not happen before 1992 (RÜG 1991). Therefore, data until 1991 are available for West Germany only. Furthermore, the DRV was reformed in 1992, transforming disability pensions for persons who had already passed their personal legal retirement age into old-age pensions (RRG 1992). The data for 2016 were provided by the DRV on demand (Deutsche Rentenversicherung Bund 2017a). Because the gbe-bund data originate from the DRV as well, we ensure that our data are consistent. We estimate the pension rates for ages 60 to 69. The pensions for persons aged 70 and older are cumulated, which does not bias the results because retirement risk should not change significantly among this age group (see, e.g., the predicted retirement age curves in Börsch-Supan and Berkel 2004). There is still some difference due to the undercounting of international migration (Vanella and Deschermeier 2018), but the grouping decreases the dimensionality of the data and mitigates the error naturally arising from the population updating in the old-age population (Vanella 2017). One advantage of our approach is that we take into account the numbers of persons residing abroad who receive pension payments from the DRV; previous approaches have not done this. The disability pension numbers have been collected from three DRV sources. We extracted the data by age for ages 60 to 64+ by sex, discriminating between full and partial disability pensions. The data for the years 2010-2016 were downloaded from the statistical database of the DRV (Statistikportal der Rentenversicherung 2018), and the data for 2000-2009 are available at the DRV research homepage (Forschungsportal der Deutschen Rentenversicherung 2018). The data for 1992-1999 were provided by the DRV on demand (Deutsche Rentenversicherung Bund 2018).

Year-end population estimates by sex and age based on the 2011 census have either been provided by Destatis on demand or downloaded directly from Destatis' homepage for the years 2010-2016 (Destatis 2014, 2015a, 2015b, 2015c, 2016a, 2016b, 2018b). Avoiding a possible structural break due to different census bases, we estimate the population from 1992-2009 by reverse updating the population aged 60 years and older starting from 2010 backwards with Destatis data on deaths and net migration by age and sex (Destatis 2005, 2015d, 2017).

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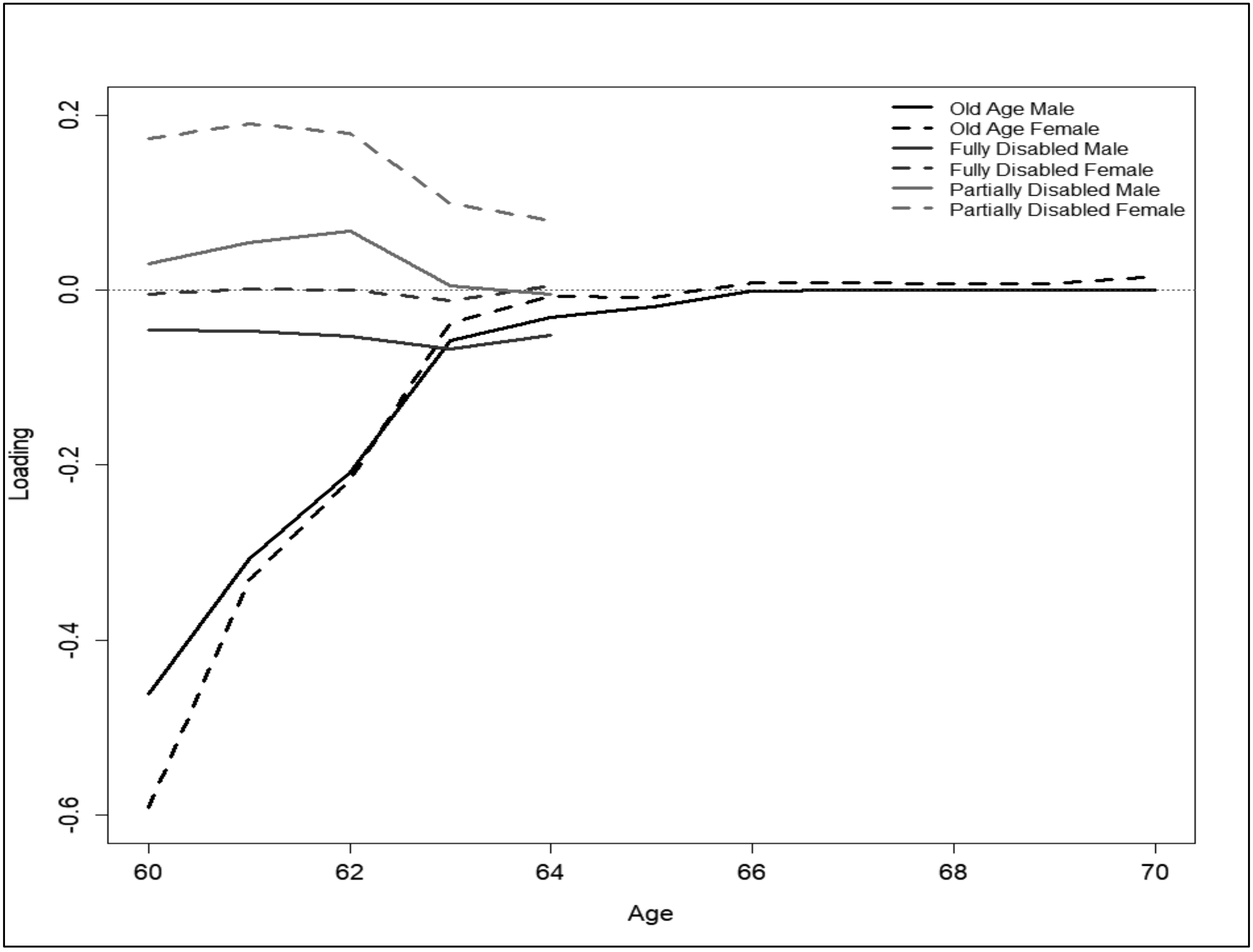
[7] Ages 60-99 annually, over 99 grouped.

The pension counts are divided by the population estimates, allowing us to calculate annual ASSPRs for the period 1992-2016. The resulting data matrix has 42 columns as conglomerates of 42 time series of ASSPRs. Basing the model on the ASSPRs has the advantage of including the possibility of a return into the labor force indirectly in our data. Earlier approaches tend to estimate labor force participation rates first and derive pension rates from those (see Section 2). That approach has a major limitation; it ignores the population receiving pension payments while living abroad. For Germany, this leads to a systematic underestimation of the pension numbers because the number of persons living abroad after retiring is certainly larger than vice versa (Deutsche Rentenversicherung Bund 2017b; Vanella and Deschermeier 2018).

We apply principal component analysis (PCA) to the matrix of the logarithmized ASSPRs (see, e.g., Chatfield and Collins 1980; Handl 2010; Vanella 2018 for a comprehensive description and application of the method). This approach allows us to minimize the effective dimension of the data while also covering the correlations between the time series in our model (Vanella 2018). The principal components (*PCs)* are linear combinations of all ASSPRs, which are correlated to these while being uncorrelated to each other (Chatfield and Collins 1980; Vanella 2018). The correlations (or *loadings*) between the first PC and the ASSPRs are illustrated in Figure 1.

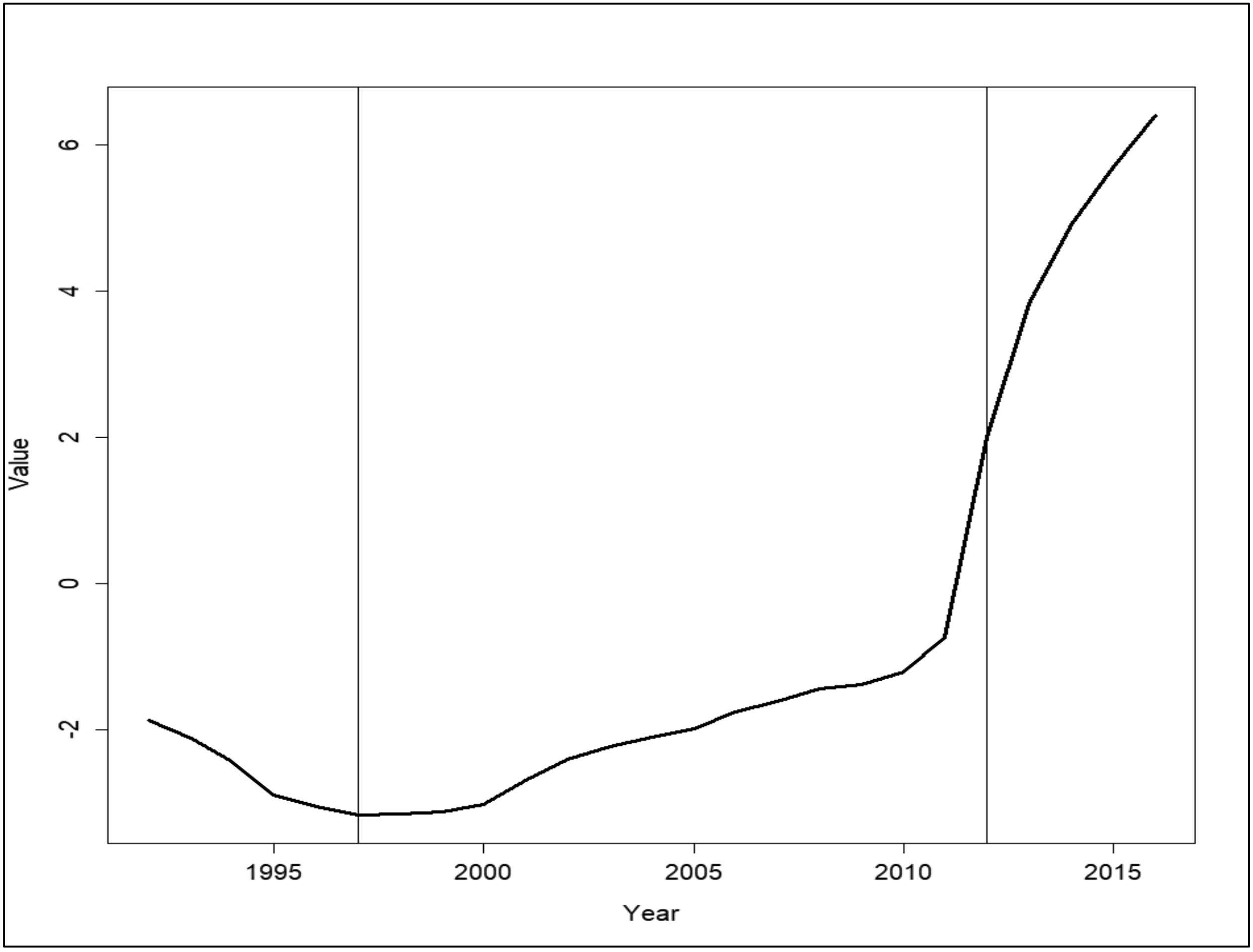
Figure 1. Loadings of the first Principal Component



Source: Own calculation and design

Principal Component 1 *(PC1)* is negatively correlated with the rates of old-age pensioners in the pre-legal retirement ages and with the rates of fully disabled persons. Moreover, its loadings are positive for partially disabled persons. Positive trends in PC1 are therefore, c.p., associated with decreases in early retirement rates. Moreover, the PC appears to represent trends in the rejection of claims for full disability pensions, while instead claiming partial disability pensions (Bäcker 2012). PC1 explains approximately 85.61% of the total variance in the log-ASSPRs. Figure 2 shows the historical course of PC1. The years 1997 and 2012 are marked by vertical lines, since in these years the effects of the RRG 1992 and the RV-AltAnpG 2007 started to kick in (see Appendix A).

Figure 2. Past Course of Principal Component 1



Source: Own calculation and design

PC1 has a decreasing trend until the late 1990s. It increases almost monotonically shortly after 1997 and has an even steeper slope since 2012, strongly implying a connection of PC1 to the past pension reforms that introduced raises in the legal retirement ages, as explained in Section 1.

To test our a priori stated hypothesis about PC1 to some extent and to integrate the effects of the legal retirement ages on it for our forecast model of the future pensioner numbers, we iteratively fit an explanatory model for PC1 with the mean annual retirement ages as exogenous variables. Those variables are derived from the sources presented in Section 1. The results of the different iterations are given in Table 1, with standard errors for the coefficients in brackets. For informative purposes only, we give the R^2 and adjusted R^2 for each model.

Table 1. Model Estimates for Principal Component 1[8]

Retirement Age	Model 1.1	Model 1.2	Model 1.3	Model 1.4	Model 1.5	Model 1.6
Intercept	- 572.91** (213.7)	- 618.5** (244.97)	- 591.56** (218.25)	- 642.71*** (130.34)	- 637.72*** (129)	- 603.22*** (19.39)
Standard	- 0.34 (4.73)	0.96 (5.41)	0.31 (4.72)	1.27 (3.34)	0.88 (3.27)	-
35 Years Long Insured	0.5 (0.52)	0.04 (0.56)	- 0.08 (0.34)	- 0.16 (0.2)	-	-
45 Years Long Insured	- 0.1 (0.25)	- 0.07 (0.29)	- 0.08 (0.27)	-	-	-
Severely Disabled	0.11 (0.5)	- 0.15 (0.56)	-	-	-	-
Unemployed	- 0.47** (0.18)	-	-	-	-	-
Women	0.55*** (0.17)	0.45** (0.19)	0.41*** (0.1)	0.4*** (0.1)	0.33*** (0.04)	0.33*** (0.04)
Mineworkers	9.25*** (1.77)	8.95*** (2.03)	9.25*** (1.68)	9.05*** (1.51)	9.29*** (1.47)	9.68*** (0.35)
R^2	0.9919	0.9887	0.9886	0.9886	0.9882	0.9882
Adj. R^2	0.9886	0.9849	0.9856	0.9863	0.9865	0.9871

Source: Own calculation and design

The models all show high joint significance. Regarding the individual significance of the coefficients, the base model 1.1 shows insignificant estimates for some and even one significant negative estimate (unemployed persons), which is implausible. Considering Figure 1, a negative coefficient would mean that an increase in the legal retirement age for unemployed persons would lead to a decrease in PC1 and, c.p., an increase in the prevalence of early retirement, which can be clearly rejected from the theoretical framework. We are probably dealing with a spurious regression caused by the multicollinearity in the past observations of the explanatory variables (see Appendix B on that). Omitting the variables iteratively, we finish with Model 1.6, which gives plausible and highly significant estimates. Moreover, it gives the highest adjusted R^2 among models 1.2 – 1.6, representing the best tradeoff between model fit and complexity. The final model accentuates the effects of the legal retirement ages of females and mineworkers on PC1, with special emphasis on the mineworkers. The PC stresses developments in very early retirement between age 60 and 62, an age group where most retirement stems from mineworkers. For example, in 2016, over 70% of the pensioner numbers among

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[8] One asterisk means statistical significance on a 10% level against $H_0\colon \beta_x \leq 0$, with β_x being the x-th coefficient. Two asterisks indicate a 5% significance level and three asterisks mean 1%.

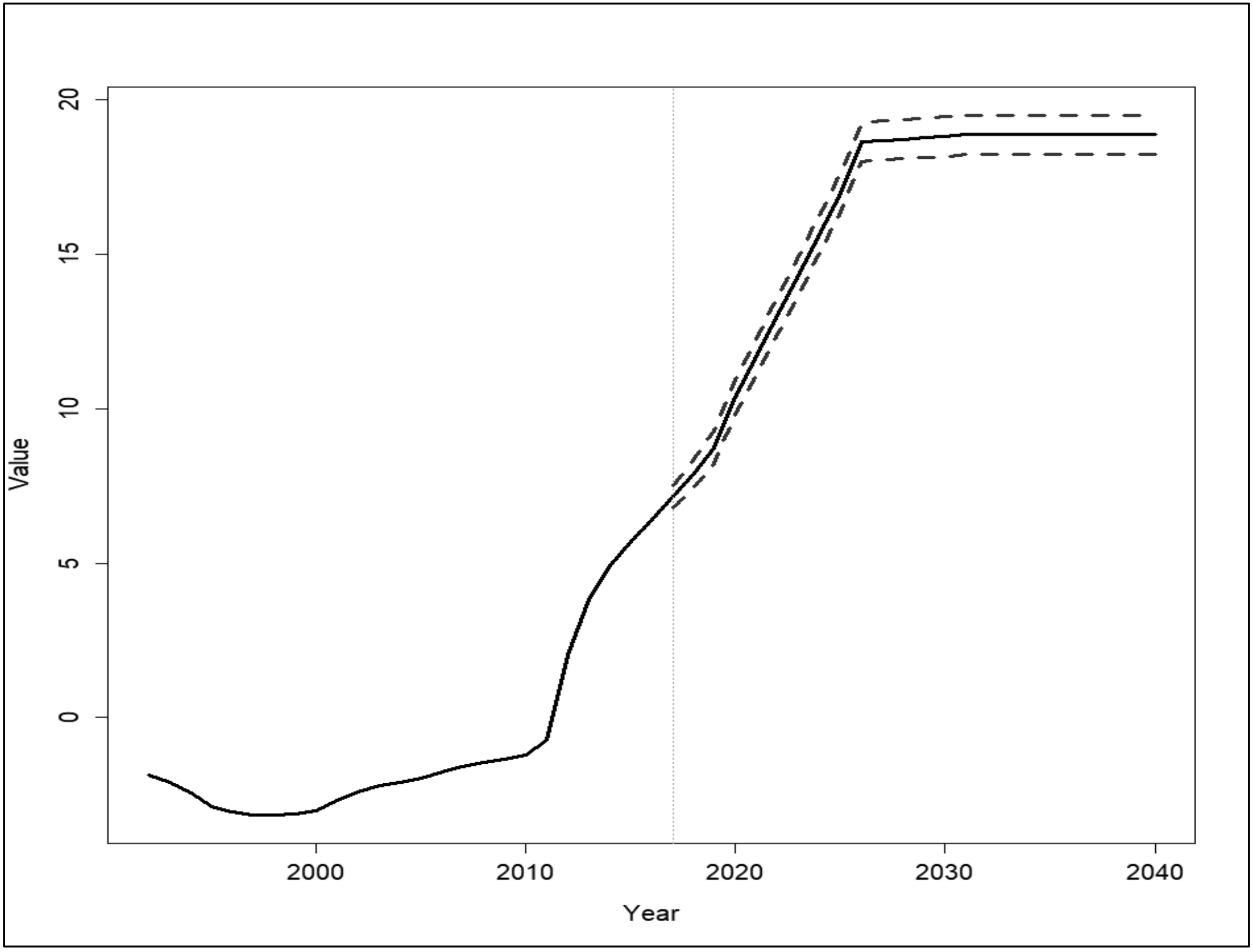
male 60-year-olds were mineworkers (Deutsche Rentenversicherung Bund 2017a). We should keep in mind that the legal retirement age of the females represents the males as well because their retirement ages have been identical since 2010 (see Appendix A). After smoothing the data to the quantified model, we fit a Box-Jenkins time series model to the data (see Box et al. 2016). Based on the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF), we identify an autoregressive model of order one (AR(1)) as the most appropriate model for the error term (see, e.g., Shumway and Stoffer 2011 on ARMA processes, ACFs and PACFs). The forecast model for PC1 is

$$p_1(y) = -603.22 + 0.33f_y + 9.68b_y + 0.83u_{y-1} + \varepsilon_y, \tag{1}$$

with $\varepsilon_y \sim \mathcal{NID}(0, 0.22^2)$, f_y being the mean legal female retirement age and b_y being the mean legal retirement age of mineworkers in year y, as calculated in Appendix A. u_y is the difference between the actual value of PC1 in period y and its mean estimate according to Model 1.6.

The forecast of PC1 with 75% predictive intervals (PIs) is illustrated in Figure 3:

Figure 3. Forecast of Principal Component 1 with 90% PI

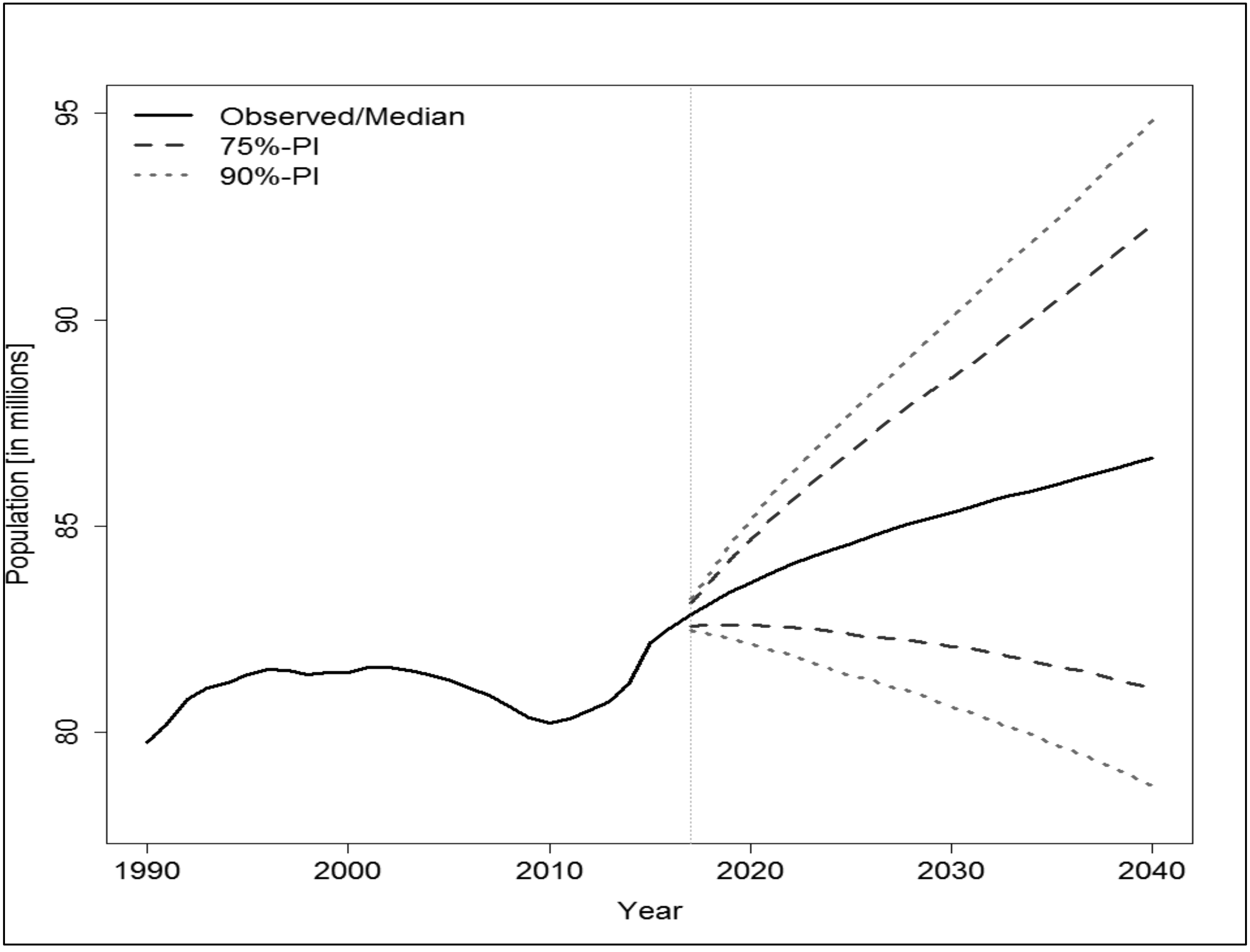


Source: Own calculation and design

The remaining 41 PCs are assumed to be random walk processes. The fitted PC models are used for future simulation of the 10,000 trajectories until 2040 via Wiener processes (see, e.g., Vanella 2018 on these). In this way, the stochasticity of all variables is considered in the forecast model (Vanella 2017). The trajectories of the PCs can easily be re-transformed into trajectories of the ASSPRs (Vanella 2018). These are multiplied by the trajectories resulting from the probabilistic population forecast for Germany conducted by Vanella and Deschermeier (2020). In this way, trajectories of the pensioner numbers are derived through 2040.

4 Results

Figure 4. Population until 2040 with 75% and 90% PIs

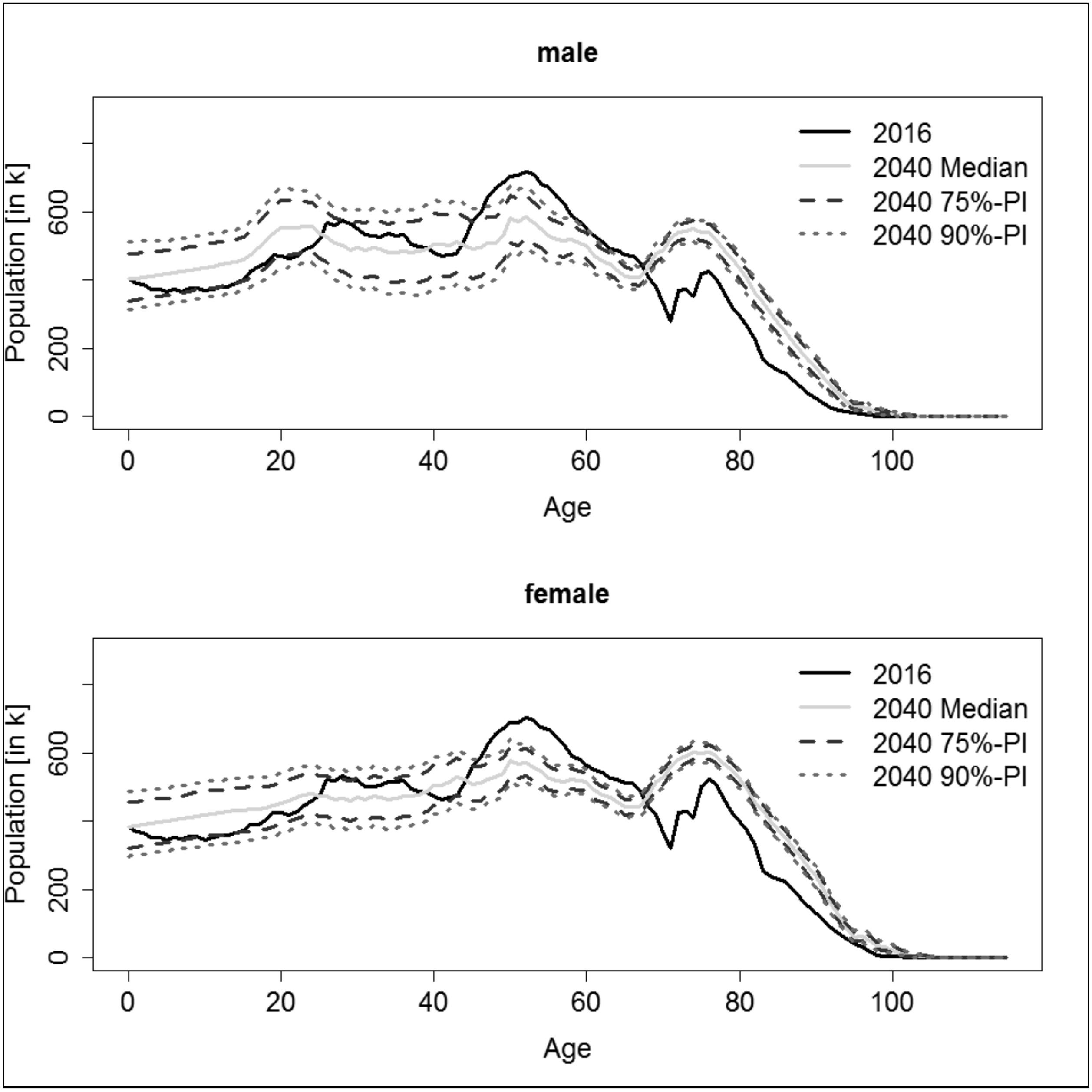


Sources: Vanella and Deschermeier (2020); Own calculation and design

We will now present selected results of the population forecast by Vanella and Deschermeier (2020), which constitutes the basis of the pension forecast conducted in the present contribution. Figure 4 shows the forecast of the future total population through 2040 with 75% and 90% PIs.

There is a high probability that the total population will increase over the forecast horizon. In 2040, the probability is over 70% that it will be larger than the population in 2016, as can be seen in the PIs. The population in the median forecast for December 31st, 2040 will be slightly below 87 million. In light of the pension fund, the population structure is of high relevance. Figure 5 compares the estimated age- and sex-specific population on December 31st, 2016 with the median forecast and the 75% and 90% PIs for 2040 for each age and both sexes:

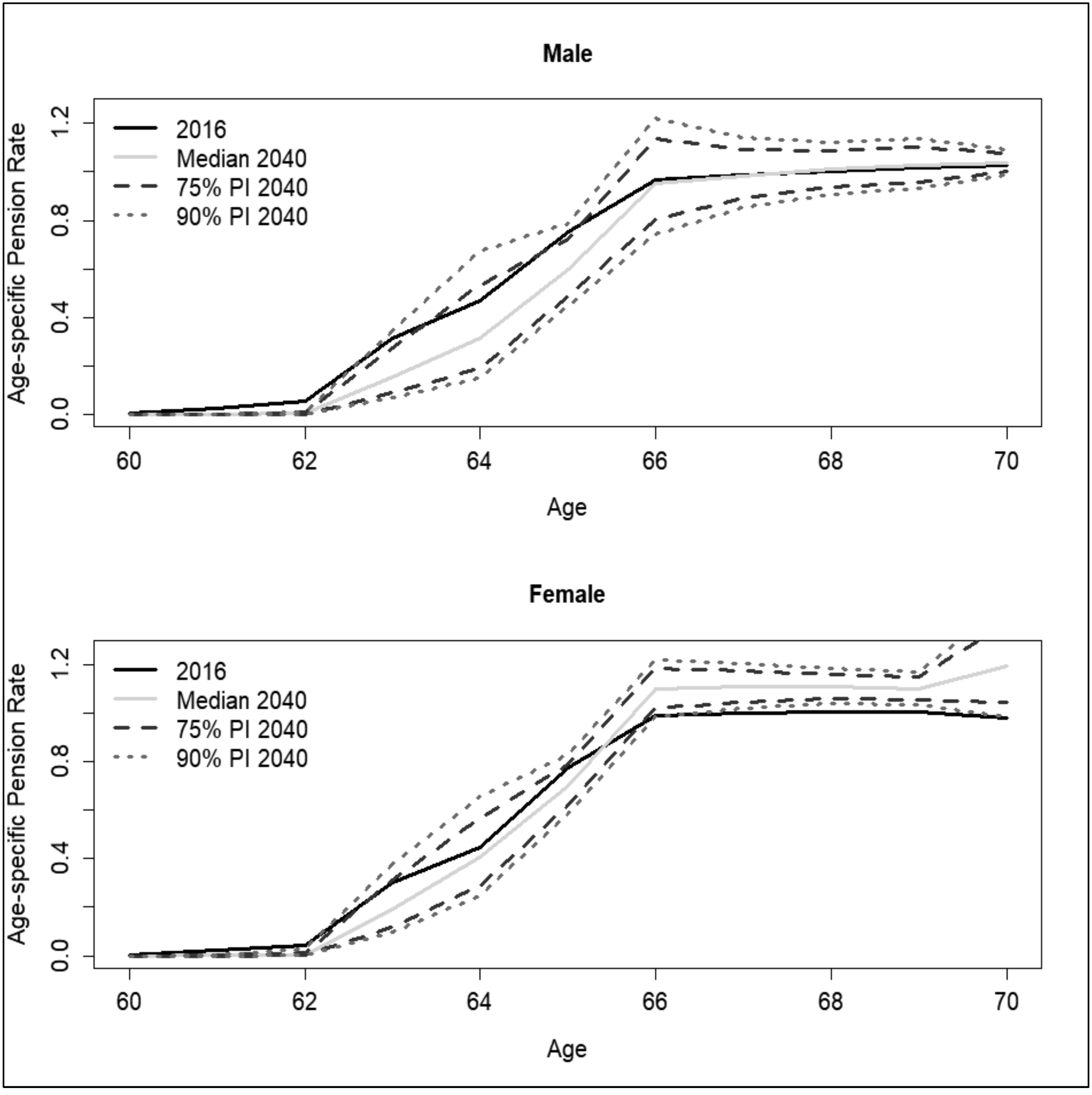
Figure 5. Population by Sex and Age on Dec 31, 2016 and 2040



Sources: Vanella and Deschermeier (2020); Own calculation and design

Obviously, the increase in the population results from clear growth in the population in the pension age group, whereas the population in the typical labor age group is expected to decrease by then. The old-age dependency ratio resulting from the population structure is often used as a representative statistic for future pressure on the DRV, as illustrated in Section 2. Our modeling approach provides more insight into the actual pension numbers because the predicted population at this stage is multiplied by the age- and sex-specific risks of pension claim estimated by our PC time series method. The trajectories of the PCs are transformed back into trajectories of the ASSPRs, as mentioned above. The trajectories can be used to estimate quantiles of the forecast to construct PIs. Figure 6 illustrates the ASSPRs for old-age pensions at year-end 2016 in comparison to the predicted ASSPRs in the median trajectory with 75% and 90% PIs at the end of the forecast horizon.

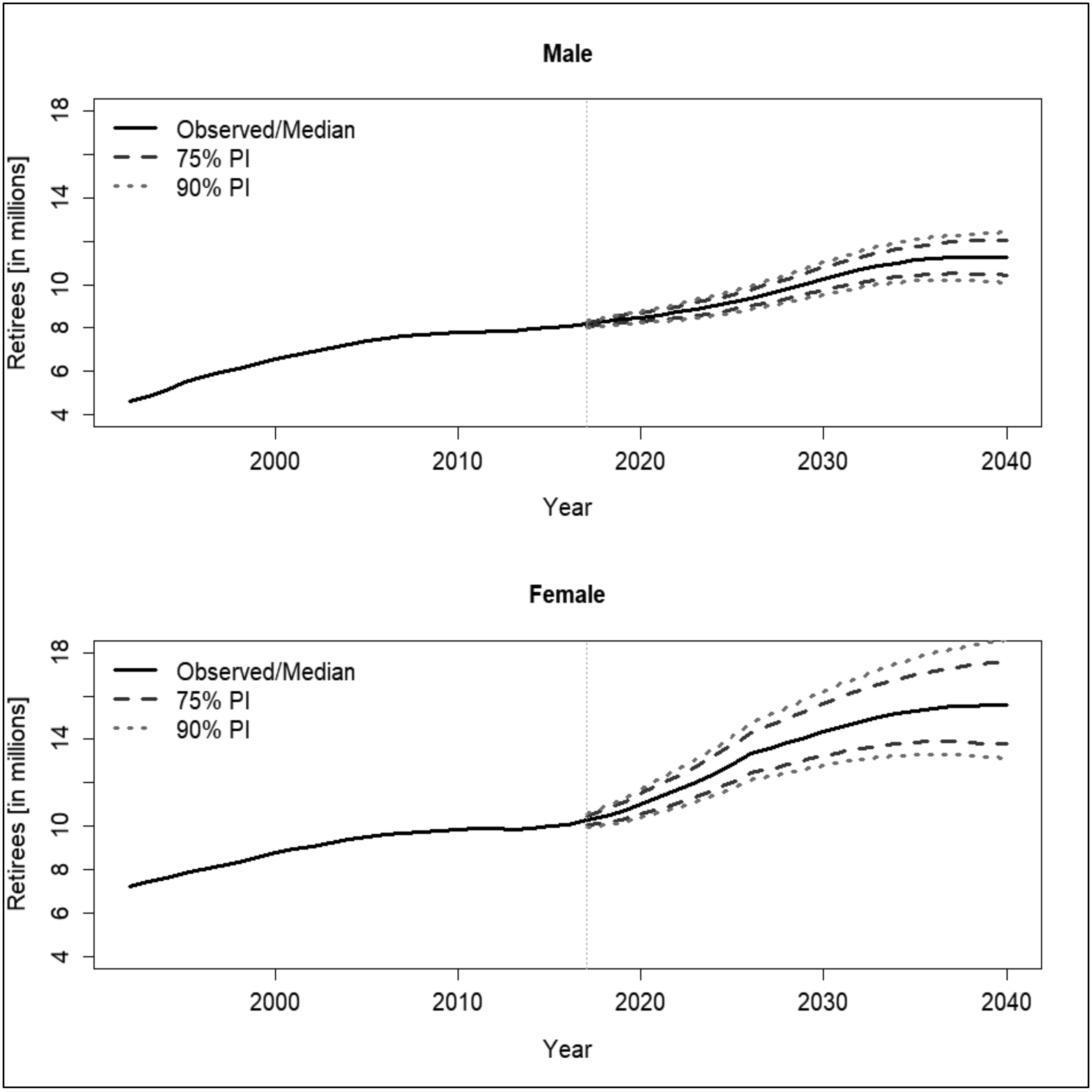
Figure 6. Age- and Sex-specific Pension Rates in 2016 and 2040



Source: Own calculation and design

For the age group of people under 66 years, a decrease in the prevalence of old-age pension claims is probable as a result of the pension reform of 2007 described in Section 1. For the age group older than that, the changes for the males will be subtle, whereas the ASSPRs of the females will almost certainly increase. This stems from the high labor force participation rates of the female population born since the baby-boom years (Fuchs et al. 2018). The preceding generations participated less in the labor market because their primary profession was mostly motherhood and housekeeping (Hertrampf 2008).

Figure 7. Old-Age Pensioners by Sex until 2040

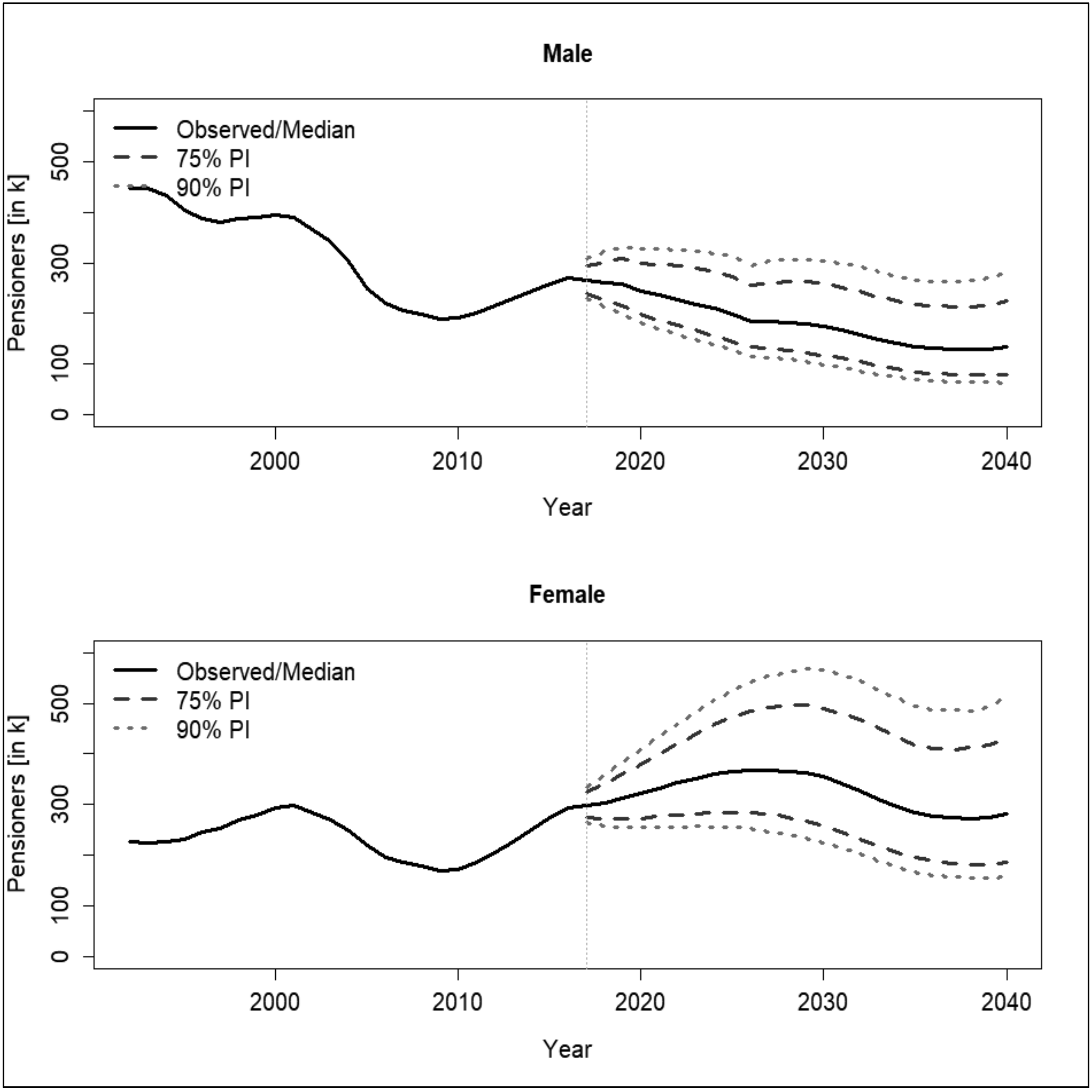


Sources: Destatis 2018a; Deutsche Rentenversicherung Bund 2017a; Own calculation and design

Multiplication of the ASSPRs derived in this study with the age- and sex-specific population estimates results in forecasts of the future pension numbers. Figure 7 illustrates the resulting forecast of the total numbers of old-age retirees by sex. In the mean, we observe a monotonically increasing number of old-age pensioners for both sexes over the whole time horizon. The increase is especially large until the late 2020s, the period in which the strongest birth cohorts reach their respective retirement ages. After this point, there is a high probability that the total numbers of retirees will increase further, but at decreasing rates. This trend is caused by slightly decreasing birth cohorts entering their retirement ages, combined with the effects of the pension reforms since 1992. Overall, we see that the number of old-age pensioners will increase from 8.1 million to 11.2 million in the median, with an associated 90% PI between 10.1 and 12.4 million for the males and from 10.1 to 15.6 million (90%-PI: 13.1 to 18.6 million) for the females between 2016 and 2040. These results include demographic trends and the labor market participation effect. These results show the massive increase in retirees occurring over the forecast horizon. The increase in legal retirement ages by two years obviously does not suffice to address demographic development from the perspective of the DRV.

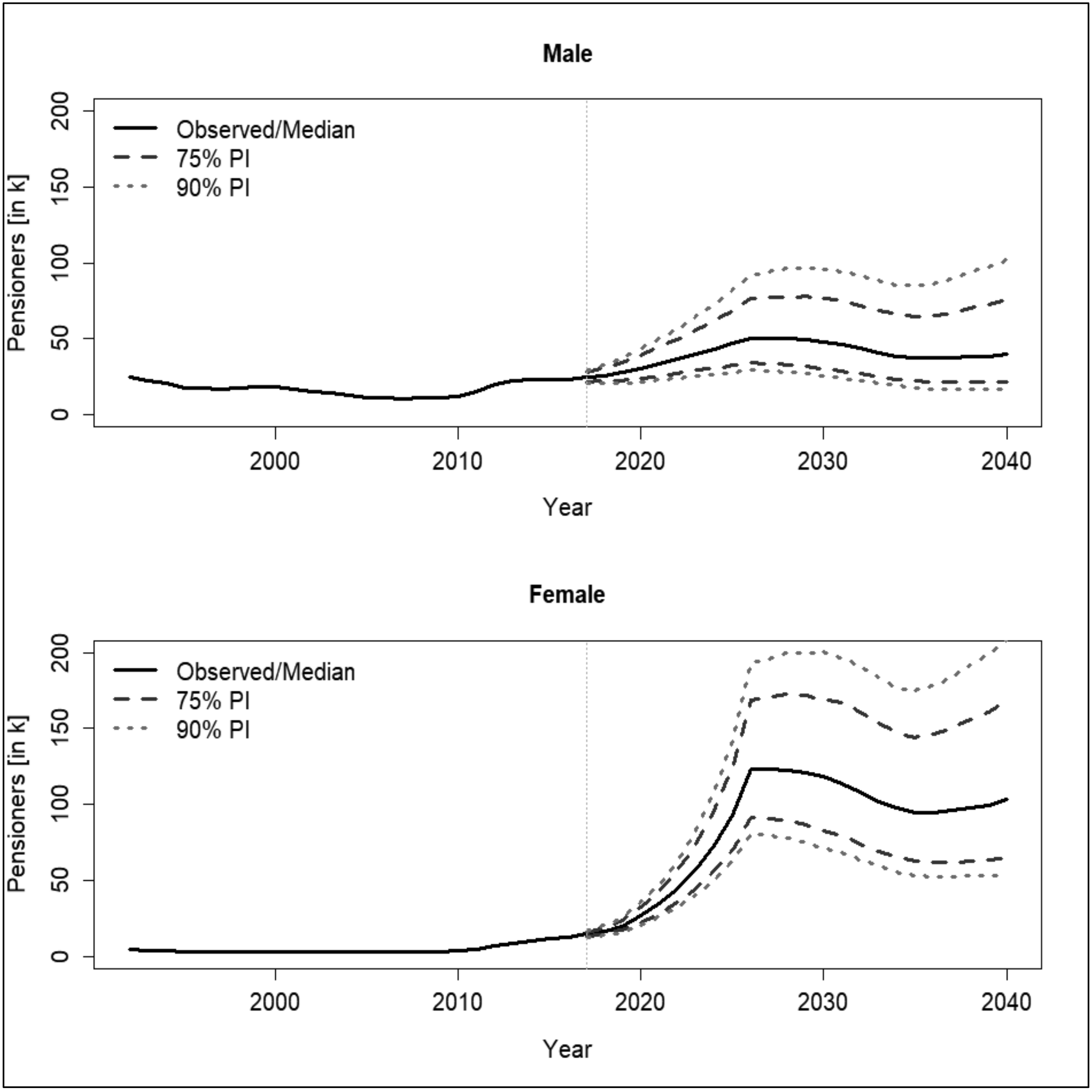
Figures 8 and 9 show the cumulated forecasts of the numbers of disability pensions for fully and partially disabled persons by sex, respectively. The long-term trend for males is negative because the age-specific prevalence of disability evidently decreases. On the other hand, the increase in the legal retirement age means, c.p., an increase in the risk of disability pension claims. These trends are superimposed on the demographic trends for the females; therefore, increasing numbers of retirees can be expected until the mid-2020s. After that point, the strong birth cohorts enter the legal retirement age, so the disability pension numbers will probably decrease again slightly because of the decrease in the population numbers in the respective age group.

Figure 8. Forecast of Full Disability Pensions



Sources: Deutsche Rentenversicherung Bund 2018; Forschungsportal der Deutschen Rentenversicherung 2018; Statistikportal der Rentenversicherung 2018; Own calculation and design

Figure 9. Forecast of Partial Disability Pensions



Sources: Deutsche Rentenversicherung Bund 2018; Forschungsportal der Deutschen Rentenversicherung 2018; Statistikportal der Rentenversicherung 2018; Own calculation and design

It can be concluded that the pension reforms that increase the legal retirement rates not only contain the increase in old-age retiree numbers but also increase the numbers of disability pensions until the late 2020s. Especially for the females, the increase in the legal retirement age will lead to a sharp increase in the number of cases in which a disability pension will be claimed. This is an effect of increasing female labor force participation rates in combination with the increasing legal retirement age, as there will be more women active in the labor market and therefore “eligible” for disability pensions; in the past, these women might have retired earlier.

To conclude, we see that a trivial analysis based on simple statistics such as the old-age dependency ratio does not suffice for a thorough forecast of the demand for statutory pension payments. An age-specific and joint forecast of old-age and disability pensions is needed for a full understanding of the real sensitivity of the pension system to reforms and demographic developments.

5 Discussion and Outlook

The present study showed the effect of future demographic development in Germany on the numbers of old-age and disability pensioners aged 60 years and older. Due to the aging of the baby-boom generation, we expect the numbers of old-age pensioners to increase by almost 9 million persons, from 18.1 million in 2016 to 26.8 million in 2040. An increase holds even under increasing legal retirement ages, as adopted in the 2007 pension reform. Stochastic modeling not only in the median scenario but also for trajectories with high mortality rates shows increasing pensioner numbers. The pension reforms targeting obvious demographic trends help mitigate the effects of the aging process to some extent but are far from sufficient.

Further reforms by adjustment of the three basic parameters of the DRV in Germany are inevitable: the pension contribution rate, the pension level and the legal retirement age. Specific economic incentives, such as higher financial losses for early retirement or incentives for immigration of high-skilled labor, are possible reforms as well. Furthermore, proposals regarding the financing option, such as a shift to a tax-funded system or the implementation of state-owned funds, are likewise being discussed. Furthermore, demography and labor market policy could offer another option for long-term stabilization of the pension system. A larger number of retirees means that there is a need for a proportional increase in the labor force, assuming that the labor market offers enough jobs to support this increase. Because fertility influences the labor market only after approximately 20 years, a short- or mid-term effect can only be achieved by either decreasing emigration of the labor force or increasing the immigration of qualified workers, who can be integrated into the labor market quickly.

Still, our approach has some drawbacks. The population under study is aged 60 and older. The PCA indicated some relationship between the legal retirement age and the probability of claiming a disability pension for younger persons, and this relationship could be rejected. While this restriction has no effect on the old-age pension estimates, we systematically underestimate the numbers of disability pensioners, cutting those under 60 years. In 2016, the number of people under 60 years of age claiming disability pensions was 1.2 million. Therefore, this is a significant number not considered here. Moreover, the model does not include widow and orphan pensions. There are three reasons for this: First, regarding disabled persons under 60, inclusion in the model could give false indication of sensitivity to retirement ages. Those effects of course do not exist because persons do not “decide to die” based on the pension policy regime. Second, the data for this type of pension are not available in the form needed to fit our model. Third, we would need data or strong assumptions on nuptial behavior, eradicating the advantages of the chosen probabilistic approach to some degree.

Further studies might include these types of pensions in their analyses. To provide a full picture of not only the numbers of pensions but also their volumes, a full pension model should include all kinds of pensions covered by the DRV as well as the development of the labor market. Moreover, the present contribution was restricted to persons instead of economic entities such as monetary units. Such deeper analyses require forecasts of economic development as well. Because our pension model is fully probabilistic, the associated economic model should also be probabilistic. Drawing the stochasticity from one source only, as done in previous studies, would create a biased picture of reality by creating some kind of pseudo-stochasticity. Further research might add forecasts addressing the financial effects using a probabilistic economic model and might elaborate on the approaches presented in Section 2 within a probabilistic framework.

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Appendix A: Mean Retirement Ages

Table 2. Past, Current, and Future Mean Annual Legal Retirement Ages

Year	**Standard**	**35 Years Insured**	**45 Years Insured**	**Severely Disabled**	**Unemployed**	**Women**	**Mineworkers**
1992	65.000	63.000	63.000	60.000	60.000	60.000	60.000
1993	65.000	63.000	63.000	60.000	60.000	60.000	60.000
1994	65.000	63.000	63.000	60.000	60.000	60.000	60.000
1995	65.000	63.000	63.000	60.000	60.000	60.000	60.000
1996	65.000	63.000	63.000	60.000	60.000	60.000	60.000
1997	65.000	63.000	63.000	60.000	60.292	60.000	60.000
1998	65.000	63.000	63.000	60.000	60.792	60.000	60.000
1999	65.000	63.000	63.000	60.000	61.292	60.000	60.000
2000	65.000	63.292	63.292	60.292	61.792	60.292	60.000
2001	65.000	63.792	63.792	60.792	62.292	60.792	60.000
2002	65.000	64.292	64.292	61.292	62.792	61.292	60.000
2003	65.000	64.792	64.792	61.792	63.292	61.792	60.000
2004	65.000	65.000	65.000	62.292	63.792	62.292	60.000
2005	65.000	65.000	65.000	62.792	64.292	62.792	60.000
2006	65.000	65.000	65.000	63.000	64.792	63.292	60.000
2007	65.000	65.000	65.000	63.000	65.000	63.792	60.000
2008	65.000	65.000	65.000	63.000	65.000	64.292	60.000
2009	65.000	65.000	65.000	63.000	65.000	64.792	60.000
2010	65.000	65.000	65.000	63.000	65.000	65.000	60.000
2011	65.000	65.000	65.000	63.000	65.000	65.000	60.000
2012	65.083	65.083	65.083	63.000	65.083	65.083	60.292
2013	65.159	65.159	65.159	63.000	65.159	65.159	60.538
2014	65.235	65.235	63.000	63.000	65.235	65.235	60.614

2015	65.311	65.311	63.000	63.292	65.311	65.311	60.689
2016	65.386	65.386	63.167	63.538	65.386	65.386	60.765
2017	65.462	65.462	63.333	63.614	65.462	65.462	60.841
2018	65.538	65.538	63.500	63.689	65.538	65.538	60.917
2019	65.614	65.614	63.667	63.765	65.614	65.614	61.000
2020	65.689	65.689	63.833	63.841	65.689	65.689	61.167
2021	65.765	65.765	64.000	63.917	65.765	65.765	61.300
2022	65.841	65.841	64.167	64.000	65.841	65.841	61.433
2023	65.917	65.917	64.333	64.167	65.917	65.917	61.567
2024	66.000	66.000	64.500	64.300	66.000	66.000	61.700
2025	66.167	66.167	64.667	64.433	66.167	66.167	61.833
2026	66.300	66.300	64.833	64.567	66.300	66.300	62.000
2027	66.433	66.433	65.000	64.700	66.433	66.433	62.000
2028	66.567	66.567	65.000	64.833	66.567	66.567	62.000
2029	66.700	66.700	65.000	65.000	66.700	66.700	62.000
2030	66.833	66.833	65.000	65.000	66.833	66.833	62.000
2031	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2032	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2033	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2034	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2035	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2036	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2037	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2038	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2039	67.000	67.000	65.000	65.000	67.000	67.000	62.000
2040	67.000	67.000	65.000	65.000	67.000	67.000	62.000

Sources: RRG 1992; WFG 1996; RRG 1999; RV-AltAnpG 2007; Bundesregierung 2013: 72; RVLeistVerbG 2014; Own calculation and design

Appendix B: Correlation Matrix of Mean Retirement Ages

Table 3. Correlation Matrix of Different Legal Retirement Ages over the Period 1992 to 2016

	Standard	**35 Years Insured**	**45 Years Insured**	**Severely Disabled**	**Unemployed**	**Women**	**Mineworkers**
Standard	1	0.87	0.52	0.82	0.75	0.78	0.96
35 Years Insured	0.87	1	0.72	0.99	0.97	0.96	0.89
45 Years Insured	0.52	0.72	1	0.7	0.72	0.64	0.47
Severely Disabled	0.82	0.99	0.7	1	0.99	0.98	0.86
Unemployed	0.75	0.97	0.72	0.99	1	0.98	0.79
Women	0.78	0.96	0.64	0.98	0.98	1	0.84
Mineworkers	0.96	0.89	0.47	0.86	0.79	0.84	1

Source: Own calculation and design

Appendix C: Comparison of Selected Pension Projections

Table 4. Overview of Selected Studies on Pension Forecasting

Study	Baseline Data	Methods and Assumptions	Results	Countries	Forecast Horizon
Alho and Nikander (2004)	Smoothed Age- and sex-specific mortality rates (ASSMRs) over preceding 30-year period Smoothed/interpolated/extrapolated Age-specific fertility rates (ASFRs) for females aged 15-49 in 2002 Estimated overall net migration and age pattern for 1990-2000 Estimated Jump-off population on January 1, 2003	Age-, sex-, and country-specific rates of decline in the ASSMRs assumed by linear extrapolation until 2030, after that constant ASSMRs in point forecast Assumed future total fertility rates (TFRs) in 2050, linear interpolation for the intermediate years; Mean age at childbearing (MAC) assumed to increase to 31 years by 2017, constant age schedule thereafter Net migration constant for ten years, then linear increase to presumed ultimate level Simulate 3,000 trajectories for each demographic component by AR(1) models including auto- and cross-correlations	Stochastic forecast of age- and sex-specific population (ASSP) Stochastic forecast of age-dependency ratio	19 EU and Schengen countries	2004-2050
Ahn, Alonso-Meseguer and García (2005)	Smoothed ASSMRs 1998-2002 ASFRs in 2002 Estimated overall net migration and age pattern for 1990-2000 Estimated Jump-off population on January 1, 2003	Population forecast model similar to Alho and Nikander (2004) with 1,500 trajectories LFPR assumed to increase exponentially until a stated maximum (average of EU countries), thereafter kept constant Derive working-age population from population and LFPR forecasts	Stochastic forecast of ASSP Deterministic projection of macroeconomic development	Spain	2004-2050

	Labor force participation rate (LFPR), employment and unemployment rate in 2000 Data on unemployment benefits in 2000 from Spanish Labor Force Survey	Unemployment rate assumed to decrease linearly to 4.5% in 2015, constant thereafter Derivation of employed and unemployed population Labor productivity growth assumed to increase to 2 in 2019, constant thereafter Projection of GDP as sum of growth rate in employed population number and labor productivity Wage increases assumed equal at same rate as labor productivity Forecast of persons entering old-age pension or other types of pensions annually Calculation of pension contributions as share of labor income	Pseudo-probabilistic projection of financial balance of pension system		
Lipps and Betz (2005)	Numbers of deaths by age and sex 1954-2000 Numbers of births by mother's age (15-49) 1973-2000 Population by sex and age 1954-2000	Forecast of ASSMRs following Lee and Carter (1992) TFR assumed random walk process; MAC forecast by logistic growth model; age schedule assumed Gaussian Net migration number assumed AR(1) process 500 trajectories sampling from estimated demographic models	Stochastic forecast of ASSP Stochastic forecast of old-age dependency ratio (OADR)	East and West Germany separately	2001-2050

Giang and Pfau (2008)	Mortality rates by five-year age groups 1999-2005 Fertility rates by five-year age groups (15-49) 1990-2005 Net migration structure of Japan 2005 Estimated Jump-off population 2005 by five-year age groups and sex Population active contributor age structure from Vietnam Household Living Standards Survey 2004 LFPR from annual surveys 1996-2005 Quinquennial Urbanization projections by UN 2005-2050 Inflation rates 1994-2005 Real investment return for pension fund assets 1996-2005 Real wage growth 1992-2005	Forecast of ASSMRs following Lee and Carter (1992) Forecast of ASFRs following Lee (1993); sex ratio of births assumed 1.06:1 for boys to girls as derived from past data; long-term TFR assumed 1.85, following UN assumptions Net migration constant LFPRs from 2005 assumed constant over forecast horizon Projected active labor force derived from population forecast and LFPRs Statutory pension coverage rate assumed to increase to 66% by 2105 Mean retirement ages assumed 57 and 51 years for males and females, respectively Average length of employment from past data Active contributors at average retirement ages assumed to change status into pensioner Economic variables stochastically simulated with log-normal distributions	Pseudo-stochastic pension fund forecast (1,000 iterations); migration and labor market variables deterministic	Vietnam	2005–2105
Härdle and Myšičková (2009)	ASSMRs 1956-2006 ASFRs (15-49) 1950-2006	Forecast of ASSMRs following Lee and Carter (1992) Forecast of ASFRs following Lee (1993)	Stochastic population forecast (5,000 iterations)	Germany	2007-2058

	Age- and sex-specific immigration and emigration ASSP on January 1, 2007	Total immigration and emigration by sex modeled by fit AR(1) models Age structure of migrants estimated by estimated kernel density Status Quo scenario for labor market participation, income pension system	Stochastic OADR, taking pension reform of 2007 into account Projection of pension premium rate and average pension level deterministic by nature, taking stochastic population into account		
Wilke (2009)	ASSMRs after World War II ASSP on December 31, 2005 Population projections by Destatis LFPRs from micro census Danish LFPRs	Different scenarios regarding future development of mortality, fertility and migration Different scenarios about labor force participation Different scenarios about unemployment rates Different assumptions about economic growth and development of wages Different assumptions about retirement age Different scenarios for disability risks	Wide range of possible deterministic scenarios for future development of pension fund, contribution rates and pension levels taking German pension system fully into account	Germany	2006-2100

Werding (2013)	ASSMRs 2000-2008 ASFRs (15-49) 2008 Age- and sex-specific immigration and emigration 2008 ASSP on December 31, 2008 LFPRs by sex and age (15-64) 1991-2010 LFPRs by sex and age (65-74) 2000, 2005 and 2010 Labor force, social insured employed and unemployed persons by qualification from micro census 1991-2010 Data on wages by sex and qualification Estimated unemployment rates by qualification Economic data	Forecast of ASSMRs following Lee and Carter (1992) TFR assumed constant on 2008 level; fixed age schedule following Gaussian function for ASFRs Total immigration and emigration assumed to increase to 800,000 and 650,000 to 2020 respectively, constant thereafter; age schedules similar to 2008 Female LFPRs converge to males' Deterministic projection economic, labor market and educational variables	Deterministic projections of numbers of active insured and number and structure of pensioners, contribution rates and pension levels	Germany	2009-2060
European Commission (2018)	Demographic, labor market and economic data	Scenario analyses for the variety of variables	Deterministic pension expenditure projections	28 EU member states	2016-2070
OECD (2018)	Demographic, labor market and economic data	Scenario analyses for the variety of variables	Deterministic pension expenditure projections	OECD-/G20-countries	2017-2060

Our study	Deaths by sex and age 1952-2016 Births by age of mother (13-52) 1968-2016 Age- and sex-specific immigration and emigration 1990-2016 Immigration and emigration by age group, sex and nationality ASSP 2010-2016 Pension numbers by age, sex and type (old-age, disability) 1992-2016 Forecast of age- and sex-specific survival rates following Vanella (2017) Forecast of ASFRs following Vanella and Deschermeier (2019) Forecast of age- and sex-specific net migration following Vanella and Deschermeier (2018) Forecast of ASSP from Vanella and Deschermeier (2020)	Forecast of pension rates by age, sex and pension type (old-age, full disability, partial disability) by time series forecast of principal component model Derivation of pension numbers from forecast pension rates and population Monte Carlo simulation (10,000 trajectories)	Stochastic forecast of pension rates by age and sex for old-age pensions, full disability pensions and partial disability pensions Stochastic forecast of numbers of old-age pensioners, fully disabled and partially disabled aged 60 and older	Germany	2017-2040

Chapter 7

A probabilistic projection of beneficiaries of long-term care insurance in Germany by severity of disability

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A probabilistic projection of beneficiaries of long-term care insurance in Germany by severity of disability

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Abstract

Demographic aging puts social insurance systems under immense pressure as frailty risks increase with age. The statutory long-term care insurance in Germany (GPV), whose society has been aging for decades due to low fertility and decreasing mortality, faces massive future pressure. The present study presents a stochastic outlook on long-term care insurance in Germany until 2045 by forecasting the future number of frail persons who could claim insurance services by severity level with theory-based Monte Carlo simulations. The simulations result in credible intervals for age-, sex- and severity-specific care rates as well as the numbers of persons for all combinations of age, sex and severity by definition of the GPV on an annual basis. The model accounts for demographic trends through time series analysis and considers all realistic epidemiological developments by simulation. The study shows that increases in the general prevalence of disabilities, especially for severe disabilities, caused by the demographic development in Germany are unavoidable, whereas the influence of changes in age-specific care risks does not affect the outcome significantly. The results may serve as a basis for estimating the future demand for care nurses and the financial expenses of the GPV.

Keywords Long-term care · Social insurance · Monte Carlo simulation · Stochastic forecasting · Disability risks · Social policy

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1 Introduction

Demographic aging is a phenomenon affecting many industrialized societies. Decreasing mortality risk ceteris paribus (c.p.) leads to a larger share of the population in the old-age group (European Union 2017: 25; Vanella 2017: 550) and consequently to an overall higher number of people in need of care, since frailty risks increase with age (see, e.g., Fuino and Wagner 2018a: 56–57; Kochskämper 2018: 9). As a result, the financial long-term sustainability of care systems is in danger (Majeed and Khan 2019: 183–184; Nguyen et al. 2019: 132; Fuino and Wagner 2020: 151–152).

However, empirical studies investigating the trends in frailty and dementia are very scarce, and thus far, almost no studies have analyzed long-term frailty trends. Nevertheless, projections of the future number of frail persons are of immense value for estimating the future demand for nursery home or ambulant care (Bowles 2015: 94–99). Due to the lack of reliable and consistent data, little research has been conducted on the outlook of persons in need of care based on sound statistical grounds. Moreover, past deterministic projections rely on relatively strong assumptions on morbidity risks and demographic development and do not sufficiently quantify future uncertainty. Different trajectories are commonly based on varying assumptions of the connection between increasing life expectancy and the development of care risks (see Kochskämper 2018: 12–14, among others). Taking Germany as an example, we propose a new and innovative method to project the number of persons in need of care. Germany presents an interesting case study because its population is aging quite rapidly, as is the case for most countries in Europe (United Nations 2019). As the largest country in Europe, Germany deserves investigation.

Statutory long-term care insurance in Germany (GPV) is a special case and is considered a role model in this regard by a group of other European countries (Wild 2010: 13–14). It was introduced in 1995 as a response to the expected future increase in demand for long-term care (Bowles 2015: 189–193). The GPV is organized as a pay-as-you-go social insurance with only partial coverage and is financed by social contributions that amount to 3.05% of the income, of which half is paid by the employer and half by the employee[1] (Klie 2005: 39). Services are provided to insured persons who are in need of care as ascertained by official medical personal. In addition, it evaluates how high the demand for care is, which then determines how high the benefits are (Klie 2005: 124). Initially, the amount of care–and, thus, the benefits–were categorized into four care levels (*Pflegestufen*): I, II, III and level III with hardship. In 2013, care level 0 was added for persons with no physical limitations who suffered dementia or another major mental illness. The decision of whether a person was "officially" in need of care and the allocation to the care level was mainly based on physical limitations in activities of daily living (Richter 2017: 98). These were assessed in four areas: personal hygiene (e.g., showering, brushing teeth, food preparation and consumption (cooking, eating food)), mobility (e.g., using stairs, getting up from bed) and domestic tasks (e.g., groceries, cleaning). The more limited a person is in these four areas, the higher the care level.

In 2016, the *Zweites Pflegestärkungsgesetz* (*PSG II*) was introduced. This law significantly changed the definition of the need for care. The major innovation was the acceptance of dementia as a health state qualifying for insurance benefits. Dementia had not been captured

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[1] There are some exceptions: self-employed people and pensioners pay the full 3.05% themselves, and those without children pay an additional contribution of 0.25% (Richter 2017, 17).

in care data bases before 2013. The definition of the severity of frailty has been changed from care levels 0, I, II, III and level III with hardship to "care degrees" (*Pflegegrade*) 1–5. Allocation to the care degrees is based on the time the caring activity takes. The time is assessed in six areas: mobility (e.g., using stairs, getting up from bed), ability to communicate (e.g., understanding, orientation), psychological problems (e.g., anxiety, aggressive behavior), self-supply (e.g., preparing food, washing), ability to cope with tasks stemming from sickness and disability (e.g., intake of medication, communication with medical staff) and ability to structure daily routine (e.g., initiating contacts with other persons). This reform has obviously led to a structural break in the data, creating difficulties in identifying the long-term trends in the severity of frailty based on its legal definition.

The present study simulates long-term trends in age- and sex-specific care risks by severity type. We use the *Pflegestatistik* from the micro census for 2017 in combination with population data to quantify the care rates. The age-, sex-, and severity-specific care rates until 2045 are projected by Monte Carlo simulation of Wiener processes, and the uncertainty in the projections is addressed and illustrated via credible intervals (CIs). Thus far, there exists only one stochastic study on the outlook of the GPV (see Sect. 2.3). Otherwise, all studies have been conducted deterministically and therefore have some limitations. First, deterministic projections impose rather hard assumptions about future development. From a statistical point of view, respective trajectories are highly improbable (Keilman et al. 2002: 410). Second, the analysis is limited to a rather small number of scenarios (see, e.g., Rothgang and Schmähl 1995) that do not sufficiently consider the future uncertainty. Third, the specified scenarios are seldom quantified with the likelihood of occurring. Fourth, the projections are not necessarily rooted in statistical data but rather in the judgments of a limited number of experts. These judgments tend to be rather subjective and therefore have some bias because of the personal opinions of the persons interviewed. Even good experts tend to perform worse on judgment-based forecasting than on forecasts that are conducted on solid statistical ground. Moreover, even when experts have good ideas about realistic future developments, they experience difficulties in translating their subjective assessments into probabilities (Lee 1998: 156–170). Furthermore, there is no previous study that projects the future numbers of disabled persons by the new definition of care degrees. Our study addresses both limitations in the literature simultaneously, filling a significant gap.

Based on our results, the future financial outlook of the GPV and the demand for care may be derived, among further applications. The remainder of the paper has the following structure: The next section provides an overview of previous studies on the link between mortality risks and morbidity risks, with an emphasis on disabilities and dementia. Moreover, some important past projection studies for the GPV are presented. Section 3 presents the underlying data and proposes a stochastic forecast model that is based on age-, sex- and severity-specific care rates (ASSSCRs), which results in a probabilistic projection of the number of care recipients until 2045. A selection of the model results is then presented in Sect. 4 before we conclude with a discussion of our paper and an outlook for future potential research.

2 Literature review

2.1 Approaches to estimating the connection of disability to mortality risk

There is an ongoing discussion on the impact of mortality changes on morbidity[2] and how to merge the two concepts to forecast morbidity based on forecasts of mortality. An early approach was proposed by Sullivan (1971: 351), who split life expectancy into healthy and disabled life expectancy.

Since the late 1970s, two theories have been dominant in the discussion. The first is the so-called *expansion thesis,* originally formulated by Gruenberg in 1977. He hypothesized that medical advances led to an extension of life with disabilities or chronic diseases (Gruenberg 2005: 781). Under this premise, an increase in life expectancy is associated with an increased portion of the lifetime spent with sickness or disability,[3] while the time lived in full health is unaffected.

The second extreme hypothesis is the *compression thesis* by Fries (1980: 132–134). Fries stated that factors such as improved nutrition, better hygiene and advanced medical treatment led to a decrease of acute illness and concluded that the life span in disability could be compressed into a time span before death (Fries 1980: 132–134). The consequence is that increases in life expectancy are associated with a pure gain in healthy life expectancy, whereas the portion of life spent with sickness or disability remains constant.

Many studies have tried to find statistical evidence for one theory or the other and have produced ambiguous results. Schoeni et al. point out that these studies rely on very limited data comprising extremely short time series and therefore might reach incorrect conclusions. The authors provide evidence for a positive effect of decreasing mortality on frailty with a combination of graphical and statistical analysis for a cohort study of people over age 75 in the United States using a synthetic dataset derived from various surveys (Schoeni et al. 2001: S206–S216). Manton et al. (2006) use multiple American survey datasets for a projection study of life expectancy and active life expectancy following the Sullivan method. The data provide some evidence that an increasing life expectancy expands the healthy life expectancy but not on a one-to-one basis; this suggests that the real connection between mortality and disability lies between the two fundamental hypotheses.[4] Perenboom et al. (2004) show positive trends for years lived with disability overall and increasing life expectancy for the Netherlands. Their closer analysis of disability trends separated by disability severity shows strong increases for lower severity, whereas more severe disability rates tend to decrease with higher life expectancy. Lin et al. (2012) conduct an age-period-cohort analysis of American survey data on limitations in performing *activities of daily living* (*ADL*) and *instrumental activities of daily living* (*IADL*) over the 1982–2009 period. They show a clear age trend in the probabilities of heavier ADL disabilities as well as lighter IADL disabilities, which is unsurprising. Moreover, the period trends are mostly negative until the end of the last millennium. Since then, IADL have stagnated, whereas there is no clear trend for ADL. This result shows shrinking probabilities for lighter

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[2] Morbidity generally refers to the incidence or prevalence of a certain disease (Schröder and Würtz 2003: 58; CDC 2012: 3–10).

[3] A disability is a limiting health condition of either physical or mental nature (CDC 2019). Therefore, being disabled always means being morbid, whereas morbidity is not necessarily connected to disability.

[4] This case is also known as *dynamic equilibrium* (Crimmins and Beltrán-Sánchez 2010: 75–76).

disabilities over the time horizon, whereas heavier disabilities do not seem to be affected. The cohort trends are negative for both disability types.

The World Health Organization (WHO) applies Sullivan's method for calculating life expectancy and health-adjusted life expectancy (HALE) for its member countries. The WHO reports a clear positive correlation between life expectancy and HALE, but the correlation coefficient is smaller than one (Mathers and Ho 2014: 10–14). This gives evidence that in reality, neither the expansion nor the compression theses hold completely. Increases in life expectancy appear to cause increases in healthy as well as unhealthy life expectancy.

The presented studies show that there is still no clear understanding of the real connection between decreasing mortality rates at high ages and frailty risks. This lack of understanding stems from different definitions of disability and limitations in the data and methodology used to date. The slight tendency seems to give the most evidence in support of the dynamic equilibrium hypothesis. Based on the evidence from the literature, we assume that the connection between mortality and morbidity trends to be stochastic, which will be covered via Monte Carlo simulation. This is explained further in Sect. 3 of this study.

2.2 Studies on the link between trends in mortality and dementia

Since the restructuring of the GPV in 2016, dementia has been defined as a disabling state; this designation allows individuals affected by dementia to receive financial compensation for care (Wingenfeld 2017: 39–46). Thus, a forecast of the future financial burden on the GPV needs to take the future development of dementia into account. Similar to the link between classical disabilities and morbidities, the connection between changes in mortality and the risk for dementia must be assessed. Since most studies either do not investigate time trends in dementia or are rather underwhelming methodologically,[5] a gap in the literature exists. Manton et al. (2005) pooled American survey data on persons aged 65 and older for 1982–1999 and found some evidence for decreasing c.p. trends in severe cognitive impairment. A major limitation of that study is that age is not given in years but rather is split into two age groups: persons aged 65–79 and persons over 79. Therefore, there might be some bias due to the age group structure of the sample. Nevertheless, the results appear plausible.

Satizabal et al. (2016) conducted a more advanced study using panel data to compare four five-year cohorts by their age-specific incidence of dementia and calculating hazard ratios while also considering educational level. They confirmed the results of Manton et al. (2005) but found that the differences in dementia risk were statistically significant only for persons with higher education.[6] The study implies that an overall decline in age-specific dementia risk does exist but is caused by increases in education rather than by pure increases in life expectancy.[7]

Wu et al. (2014) conducted a review of 70 studies about the age- and sex-specific prevalence of dementia in China, Hong Kong and Taiwan over the period 1980–2012, and they pooled these results to estimate the possible effects of age-, period-, and cohort-effects on this prevalence. In addition to observing the obvious age effect, they concluded that there

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[5] See Prince et al. (2015: 13–15) on this.

[6] Defined by having received a high school diploma.

[7] There is strong evidence for higher education leading to higher c.p. life expectancy, as well (Meara et al. 2008: 354).

was a possible cohort effect on dementia, stemming from common life circumstances for the specified cohorts, whereas they did not identify a period effect on dementia. Thus, they could not conclude that there was a general effect of higher life expectancy on the risk of dementia. In a follow-up paper, Wu et al. (2015) conducted another review of international studies on the prevalence of dementia. They concluded that the effect of increasing life expectancy on the age-specific prevalence of dementia strongly depended on the development level of the country and its population. The authors' observation suggested that countries with a heavily expanding life expectancy, mediocre education, and a weak understanding of healthy lifestyles likely have unhealthy conditions with higher risk of mental disorders. In contrast, countries that already have a very high life expectancy that increases at a lower rate are associated with a very high degree of development. This leads to higher education and a healthier lifestyle, resulting in increases in healthy life expectancy, as expected by the compression thesis. The authors include the Western and Northern countries as well as North America in the latter category. Fastame et al. (2015: 2159–2161) confirmed the mitigating effect of high education on the risk of cognitive disorders in an experimental setting.

The presented studies show no clear evidence for a direct connection between life expectancy and dementia, and long-term studies are basically non-existent, especially for Germany (Ziegler and Doblhammer 2010: 96). We implicitly model trends in dementia within our probabilistic projection of care rates.

2.3 Projections of demand for long-term care in Germany

Projections of the future demand for long-term care and long-term care insurance in Germany have thus far been conducted almost exclusively by deterministic modeling.

The first projection on the future outlook of the GPV was conducted by Rothgang and Schmähl (1995), who estimated the number of persons in need of care in Germany until the year 2030 directly after the installment of the GPV using the 7th coordinated population projection of the German federal statistical office (*Destatis*) as the assumed future population. The authors applied age- and sex-specific care risks extracted from a series of survey studies by Infratest (1992, 1993) as well as Krug and Reh (1992). Assuming that 90% of frail persons were covered by the GPV and using three scenarios of claim behavior regarding cash and service transfers, the possible financial expenditure of the GPV was estimated given the regulations on financial benefits. Rothgang and Vogler (1997) elaborated on that approach by adding sensitivity analysis of three scenarios of realistic future population development with different assumptions on migration and mortality. The future development of mortality was estimated using the model by Bomsdorf and Trimborn (1992). Rothgang (2001) developed that approach further using data on GPV beneficiaries from the *Pflegestatistik,* which was introduced in 1999. In that study, he added further scenarios addressing the future trends in morbidity and the type of care services claimed by persons in need of care. For morbidity, he added a scenario in which the dynamic equilibrium thesis held to the baseline scenario of constant care risks. For each scenario, he multiplied the number of beneficiaries by type of service and severity of disability with the respective costs to estimate the future outlook of the GPV.

Blinkert and Klie (2001) projected the number of persons in need of care based on persons claiming nursing home or domiciliary care services along with future informal care potential[8] until 2050 under four different scenarios. The analysis considered demographic development and trends in family networks and in the labor market, especially female labor market participation. Based on their results, the authors derived the potential demand on the labor market for caregivers, as well.

Bowles (2015) proposed a detailed model for simulating the financial demand placed on the GPV until 2080. He stressed that this was not a forecast but rather a demonstration of possible scenarios for the future outlook of the GPV. He projected the population by sex and age under certain assumptions regarding fertility, migration, and mortality. Moreover, he used data from federal health reporting provided by the Robert Koch Institute (*RKI*) and Destatis on recipients of services in long-term care for 2011 detailed by sex, age group, level of disability (I; II; III), and type of care (domiciliary or stationary) on December 31, 2011, as well as age- and sex-specific population estimates by Destatis for that very same day for calculating care rates differentiated by the mentioned criteria. These rates were then multiplied by the projected population, similar to the method proposed by Rothgang and Schmähl, to simulate the future population in need of care by sex, age, degree of disability, and type of received care through 2080. Future uncertainty was considered to some degree by scenario analyses measuring the sensitivity to changes in the input variables. Mortality changes were included by projecting the future total life expectancy, which was split into healthy life expectancy and life expectancy in need of care, as proposed by Sullivan (1971: 351). Bowles' approach separated disabled life expectancy into life expectancy in care levels I, II, or III. Regarding morbidity, three scenarios were considered. In baseline scenario 1, Bowles (2015: 151) assumed that the care risks remained constant at their 2011 level. In combination with the Sullivan model, this means that the increase in life expectancy affects only life expectancy under disability and not healthy life expectancy. Therefore, the baseline scenario represented the outcome in the event that the expansion thesis held. Scenario 2 assumed the compression of disabled life expectancy into the last years before death. Thus, increases in life expectancy translated into healthy life expectancy exclusively. Scenario 3 followed the premise that half of the growth in life expectancy would translate into healthy life expectancy, while the other half would be under disability. Finally, Bowles gave a financial outlook of the GPV under the simulated trajectories. To do so, the estimated numbers of age- and sex-specific persons in need of care were multiplied by historic rates of service claims from the GPV and then multiplied by the corresponding financial costs of these services. The simulations were then varied by alternate scenarios, as well.

The presented studies were all deterministic and thus were relatively inflexible and vulnerable to errors. Deterministic projections in general have the limitations of being based on rather strong assumptions and thus have a low individual probability of occurring in the future. Moreover, risk is considered only by scenario analyses, which offer a small number of alternate trajectories of the future and are generally not quantified probabilistically (Lee 1998: 165–167).

Bomsdorf et al. (2008) gave the only stochastic projection of the future number of frail persons in Germany by severity level. They estimated the future population based on the mortality model by Babel et al. (2008), the fertility model by Babel et al. (2006) and a stochastic model for net migration based on the Lee-Carter model for mortality (see Lee and Carter 1992). The authors calculated age- and sex-specific care risks by severity from the Pflegestatistik 1999–2005 and extrapolated the estimated trends into the future. In this way,

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[8] Persons who provide home care services, identified from the personal networks of care recipients.

Bomsdorf et al. (2008) simulated the number of people in need of care through 2050 by severity with 90% prediction intervals. This approach was quite sophisticated but is unfortunately not applicable due to the recent reform of the German care system.

Thus far, no studies have included the effects of PSG II in their future care projections. Our study not only gives a future outlook based on the current long-term care system but does so stochastically. We propose a stochastic model that to some degree builds on the approaches by Rothgang and Vogler (1997) and Bowles (2015).

3 Data and methods

3.1 Data

Our first objective is to simulate the long-term trends in care risks. In the first step, we estimate ASSSCRs. Although we have the care statistics data at hand, it is not feasible to use time series for estimating trends, since changes in legal regulations lead to structural breaks in the data (see introduction). In particular, the change from care levels (*Pflegestufen*–PS) to care degrees (*Pflegegrade*–PG) and the inclusion of the definition of dementia as a disability in the legal sense significantly change the number of people in need of care, thus clearly limiting the use of time series methods. To avoid such a structural break, we base our analysis on the care statistics of 2017 (RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017). The dataset covers all persons receiving benefits from the GPV[9] (who are thus officially assigned to a care degree) on December 15, 2017, by birth cohort, gender, care degree and type of service they receive, among other variables. The total number of persons in the dataset who can be identified by these criteria is approximately 3.41 million, with the moderate care degrees being more dominant. Among this total number, approximately 1.57 million persons have PG2, 1.02 million have PG3 and 0.55 million are in PG4. The extreme cases of PG5, with less than a quarter million persons, and PG1, with less than 50 thousand, are much less frequent. The data span all ages, with a median age of 81 years. Due to their higher life expectancy (Vanella 2017: 551) and their c.p. higher care risk (see Fig. 3), the share of females in the dataset is quite high at almost 63%. The microdata are cumulated to serve as counts of age-, sex-, and severity-specific care cases. We define age by year (cohort-based) for the age group 60–93. Cases aged 59 and younger are cumulated in one age group, since the care risks in this age group are relatively low and smoothing of the observed data can be prevented in this way. Theoretically, we can assume that care risk increases with age, or mathematically speaking, the curve should be monotonically increasing. This is not strictly the case in observed rates under 60 years, implying that there are stochastic residuals in the data for the year under study. The error arising from our pooling of this age group is negligible, since the overall care risk is small and the slope of the frailty curve over that age group is not steep. The 94+ age group is also cumulated, as the number of cases, especially for PG1, is too small to derive representative estimates for the associated care risks in these age groups from the data.

We estimate the ASSSCRs by dividing the specific cases by the corresponding population estimates, which have been downloaded from the Human Mortality Database (2019).

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[9] Including persons living abroad.

The raw population data in the Human Mortality Database (HMD) originate from Destatis, but the published data at the HMD are of higher quality and are more detailed for the old-age population. The official population statistics are truncated above age 100 and are very susceptible to errors from the annual population update, as Scholz et al. (2018: 2–5) have shown. This stems from estimation errors in the census in addition to unregistered migration, which leads to relatively large errors for the old-age population, which is naturally smaller than the younger population. Therefore, estimation errors have a relatively larger impact on the estimates of mortality or disability risks. The HMD data are smoothed for the age group above 80 to account for these problems (Scholz et al. 2018: 4–5). Figure 1 illustrates the ASSSCRs in 2017 for Care Degrees 3, 4 and 5.

Obviously, the age-specific care risk increases with increasing age, as one would expect. Figure 2 visualizes the prevalence of PG1 and PG2.

In contrast, these “less severe” care risks show decreasing prevalence for the oldest people. This appears plausible, as in this age group, the worsening of already present frail conditions is probable and associated with the transition into a higher PG (Fuino and Wagner 2018a: 56–69, b: 329).

The ASSSCRs diverge after the eighth decade of life, with females’ risks becoming much larger than males’. This trend becomes even clearer considering the estimates of the overall care rates, which are the sums of the rates from Figs. 1 and 2 by sex and age, as illustrated in Fig. 3.

This phenomenon is also observed in other investigations (e.g., Fuino and Wagner 2018a: 57; Kochskämper 2018: 9) and may be associated with the fact that the female partner in a married couple is commonly younger than the male partner (GENESIS-Online Datenbank 2019). Combined with the lower life expectancy of males in comparison to females (Vanella 2017: 550–552), we conclude that females in most cases survive their spouses. Thus, a male who becomes frail is often cared for by his spouse (Hank and Stuck 2008: 1288; Cheema 2013: 2406). This may lead to a tendency of underestimating care risks in the statistics, since it is realistic that not all persons giving

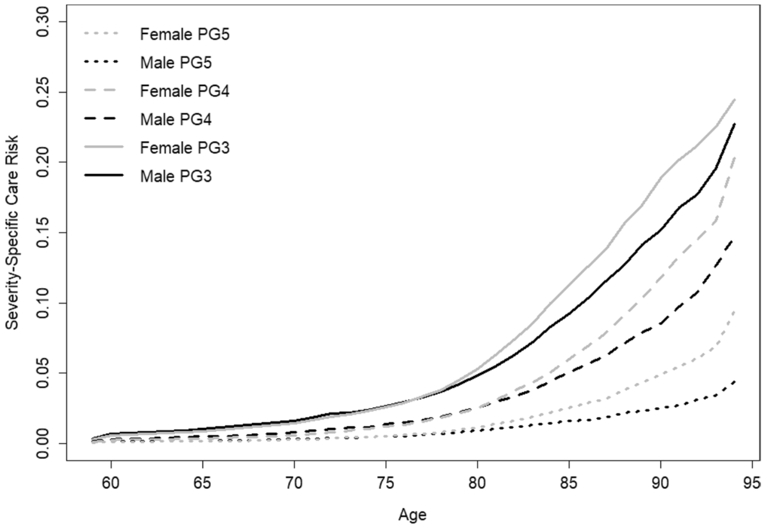

Fig. 1 Age- and sex-specific care rates for care degrees 5–3 in 2017. *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; own calculation and design

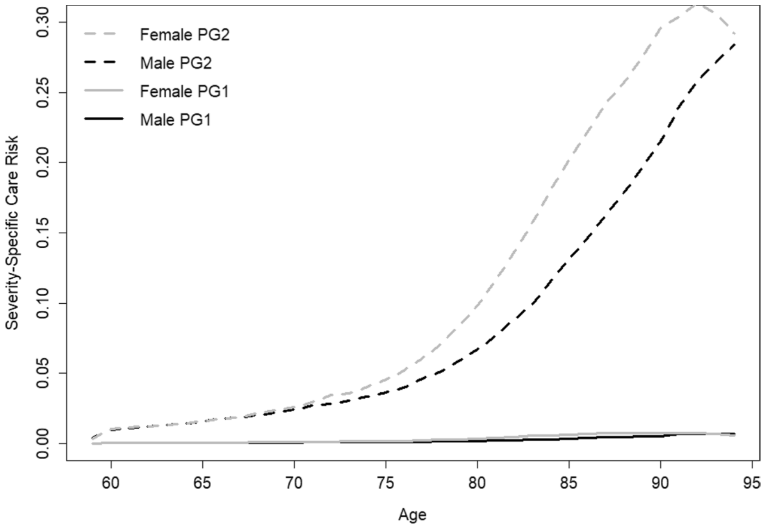

Fig. 2 Age- and sex-specific care rates for care degrees 2 and 1 in 2017. *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; own calculation and design

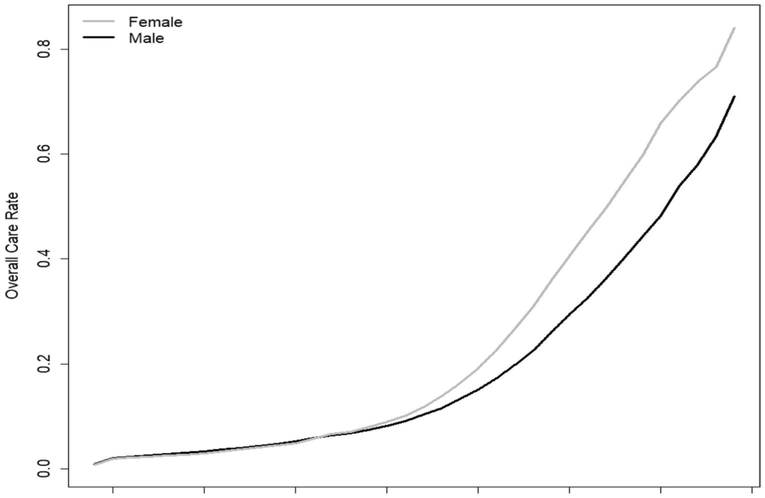

Fig. 3 Care rates in 2017 by age and sex. *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; own calculation and design

informal care claim financial support for doing so. Those persons will not appear in the statistics. Since frailty for females often appears when they are already widows or their husbands are too old to care for them, women have a higher tendency to claim professional help (Destatis 2018a: 9). Moreover, on average, females have a higher c.p. life expectancy than males but tend to live under worse health than males at the same age, as other studies have shown (Nguyen et al. 2019: 137–140).

3.2 Methods

After having estimated the baseline data for our analysis, the forecast of the ASSSCRs is conducted. Due to the absence of valid time series on the ASSSCRs (see Sect. 1), we estimate the prevalence trends following the idea of the Sullivan method presented in Sect. 2. The literature overviewed in that section postulates a non-negative correlation between increases in life expectancy and increases in life expectancy with disability; however, the results of the studies do not give a conclusive picture of the real connection between the two variables. Nevertheless, some conclusions can be drawn from the literature. The two extreme hypotheses of expansion and compression give a plausible frame for a realistic connection. Moreover, much of the literature proposes that the two classical hypotheses do not fully apply; rather, some kind of dynamic equilibrium in between gives the real correlation between mortality and morbidity.

From these observations, our approach is inspired by a Bayesian approach to apply so-called *non-informative prior distributions*; i.e., we know little or nothing about the actual probabilities of certain outcomes or the true parameter, and we therefore do not impose heavy assumptions on the true distribution of the parameter (Lynch 2007: 55). Knowing that a decrease in mortality leads to some c.p. decrease in morbidity, we can assume that a decrease in the mortality risk by d % for males aged x in year y is associated with a decrease in the care risk for males in that very same age and year by 0% to d %. Generally speaking, the correlation coefficient between mortality risk m and care risk c for persons aged x years of gender g in year y is

$$\rho_{m,c,x,g,y} \sim U(0,1), \tag{1}$$

with $U(0,1)$ indicating a uniform distribution with minimum 0 and maximum 1 (Lynch 2007: 55). A uniform distribution with these limits states that the probability of the random variable taking a certain value in that interval is equal for all values. This is why the term *non-informative* is used frequently because there is no pre-defined modus for the distribution of the variable. Figure 4 presents the probability density function of $\rho_{m,c,x,g,y}$ to illustrate this principle.

$\rho_{m,c,x,g,y} = 0$ would signify no connection between the two variables; thus, increases in life expectancy (i.e., decreases in age-specific mortality rates) would not result in decreasing care rates, thus increasing the frail life expectancy. This scenario would follow the expansion thesis. In the other extreme scenario, a correlation coefficient of 1 would be associated with decreases in care rates according to mortality rates. Therefore, increases in life expectancy would be lived completely disability-free, following the compression thesis. Values between the two boundary values would represent some mixture of life expectancy in good and poor health, according to the dynamic equilibrium thesis. For the projection, 10,000 random numbers are drawn annually from a uniform distribution according to (1). These are taken as trajectories of the Sullivan parameter.

Age- and sex-specific mortality rates (ASSMRs) are simulated until 2045 according to Vanella's (2017) version of the Lee-Carter model for both genders. The model proposes applying principal component analysis to the variance matrix of the time series of the

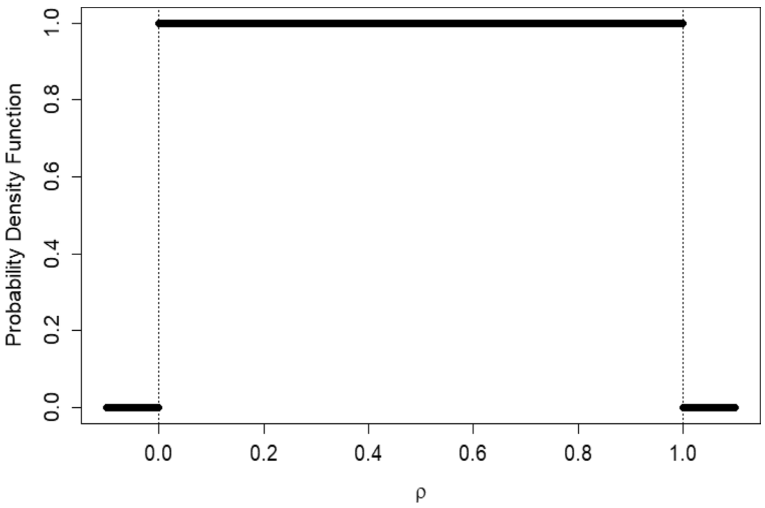

Fig. 4 Probability density function of correlation coefficient between mortality and disability change. *Source*: Own calculation and design

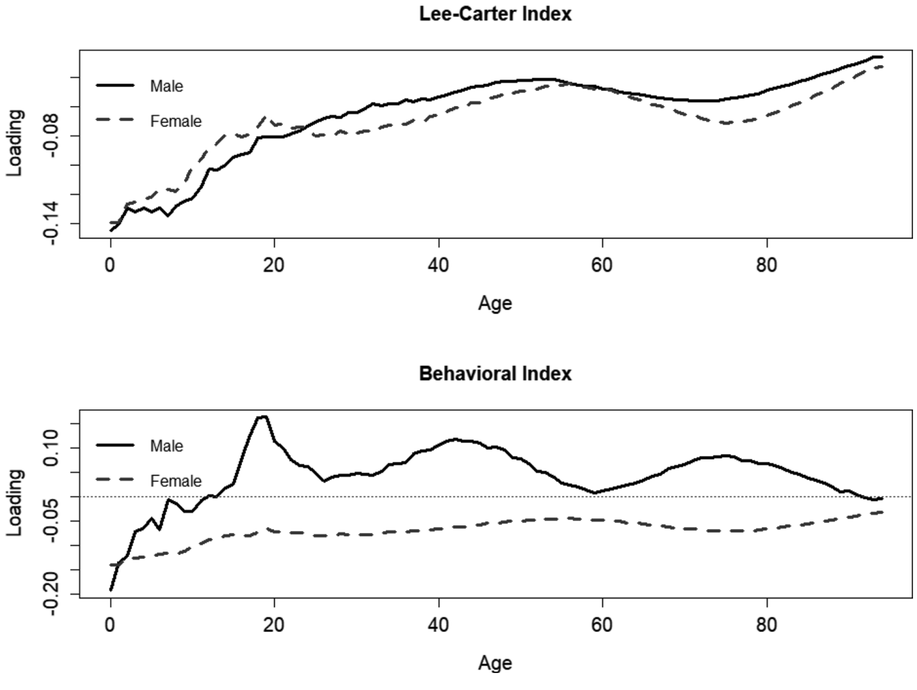

Fig. 5 Loadings of the first two principal components of the mortality model. *Source*: Own calculation and design

logistically transformed age- and sex-specific survival rates of a collection of countries. Lee and Carter (1992: 662–663) used a similar approach to American mortality rates to

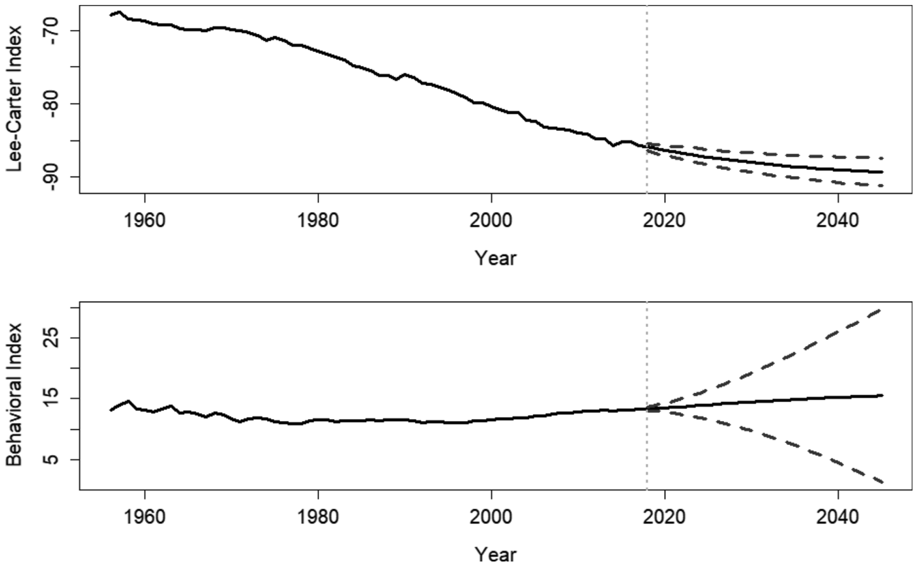

Fig. 6 Forecast of first two principal components in the mortality model with 90% PIs. *Source*: Own calculation and design

show that the first principal component explains a large share of the mortality development because mortality improvements involve common factors such as advances in the understanding of hygiene (Pötzsch and Rößger 2015: 34) and medical technologies (World Health Organization 2015: 51). The entire population benefits from these improvements. Therefore, many have identified the approach as the Lee-Carter model. Vanella extended the Lee-Carter model by identifying the second principal component as some type of behavioral index that explains different mortality improvements between both genders to some extent (Vanella 2017: 547–548). Moreover, that study addressed the major limitation of the Lee-Carter model, which is the systematic underestimation of future uncertainty, resulting in overly narrow prediction intervals. Figure 5 presents the loadings of the first two principal components, which can be interpreted as the correlation coefficients between the ASSSRs and the respective principal component.

Parametric functions are fit by OLS to the time series of these two principal components to estimate their long-term trends. Autoregressive Integrated Moving Average models present the remaining nuisance.[10] This procedure results in forecast models of the first two principal components, which are then simulated 10,000 times annually as Wiener processes to estimate 10,000 trajectories for their future development. The theoretical mean forecasts with 90% prediction intervals for the two principal components until 2045 are illustrated in Fig. 6.

The forecast in the mean shows a decreasing overall mortality trend (the Lee-Carter Index is negatively correlated to the ASSSRs, and thus decreases in it mean increases in the ASSSRs) and a convergence of male ASSSRs to female ones, although this connection

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[10] We recommend Shumway and Stoffer (2016: 77–137) for further reading on this model family.

is highly stochastic, as the width and direction of the prediction interval indicate.[11] The remaining principal components are assumed random walk processes and simulated as such. The resulting 10,000 trajectories of all principal components are then transformed back into 10,000 trajectories of the logit-ASSSRs and finally ASSSRs.

From these trajectories, the relative change in some ASSMR in trajectory t can be computed:

$$\partial m_{x,g,y,t} = \frac{m_{x,g,y,t}}{m_{x,g,y-1,t}} - 1. \tag{2}$$

From (1) and (2) follows the relative change in the ASSSCRs in year y in trajectory t and with PG p:

$$\partial c_{x,g,p,y,t} = \partial m_{x,g,y,t} * \rho_{m,c,x,g,y,t}, \tag{3}$$

from which the ASSSCRs in trajectory t in year y

$$c_{x,g,p,y,t} = c_{x,g,p,y-1,t} * \left(1 + \partial c_{x,g,p,y,t}\right) \tag{4}$$

result.

The ASSSCRs are then multiplied with the trajectories of a probabilistic age and sex-specific population forecast for Germany generated based on a stochastic cohort-component algorithm proposed by Vanella and Deschermeier (2020). In this way, we derive stochastic age-, sex- and severity-specific care numbers by 2045:

$$N_{x,g,p,y,t} = c_{x,g,p,y,t} * B_{x,g,y,t}, \tag{5}$$

with $N_{x,g,p,y,t}$ being the number of persons in care aged x years of sex g with care degree p at the end of year y in trajectory t and $B_{x,g,y,t}$ being the population in Germany of persons of the same age and sex in the same year and trajectory.

The results of our analysis are reported in Sect. 4.

4 Results

Following (2), the simulated ASSMRs are used to compute 10,000 trajectories of the future relative change in the ASSMRs. The median trajectories for a selection of years are given in "Appendix A". These trajectories are then multiplied with 10,000 random draws from a uniform distributed density for ρ, as explained in (3). The relative change in the ASSSCRs simulated in this way is then plugged into (4) to derive the corresponding ASSCRs in that period. This process is then reiterated annually, starting with the baseline year 2017. In this way, we derive 10,000 trajectories of each ASSSCR in each year until 2045, the end of the forecast horizon. These trajectories can be used to estimate CIs[12] for every variable

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[11] For more details on the interpretation of the index and its forecast, see Vanella (2017: 547–552).

[12] We refer to CIs as in Bayesian statistics rather than using the common term prediction intervals from frequentist statistics. In doing so, we stress that the uncertainty in care risk is not derived from time series data but rather estimated from theory. Therefore, we find the presented intervals "credible" based on our analysis while not knowing the real intervals.

non-parametrically. Because of the magnitude of the results, we present only a small portion of the results here.

"Appendix B" compares the ASSSCRs in 2017 to their respective median trajectories and the estimated 90% CIs in 2045.[13] Basically, small overall improvements in age-specific care risks can be expected due to improvements in mortality. Due to realistic increases in ASSMRs, the ASSSCRs might also increase in the future, although decreasing risks are more probable. Figure 7 illustrates the estimated overall care rates (i.e., the sums of all age–specific care rates over all care degrees) by sex as observed in the data for 2017 compared to the median trajectory with 90% CIs in 2045.

While overall frailty risk remains almost constant for the younger ages (i.e., under age 70), slight decreases are probable until approximately age 90, as further improvements in mortality are likely to be reached by this age group. As mortality trends for the oldest persons in the past do not show clear trends, the ASSMRs in the mean and the median will not

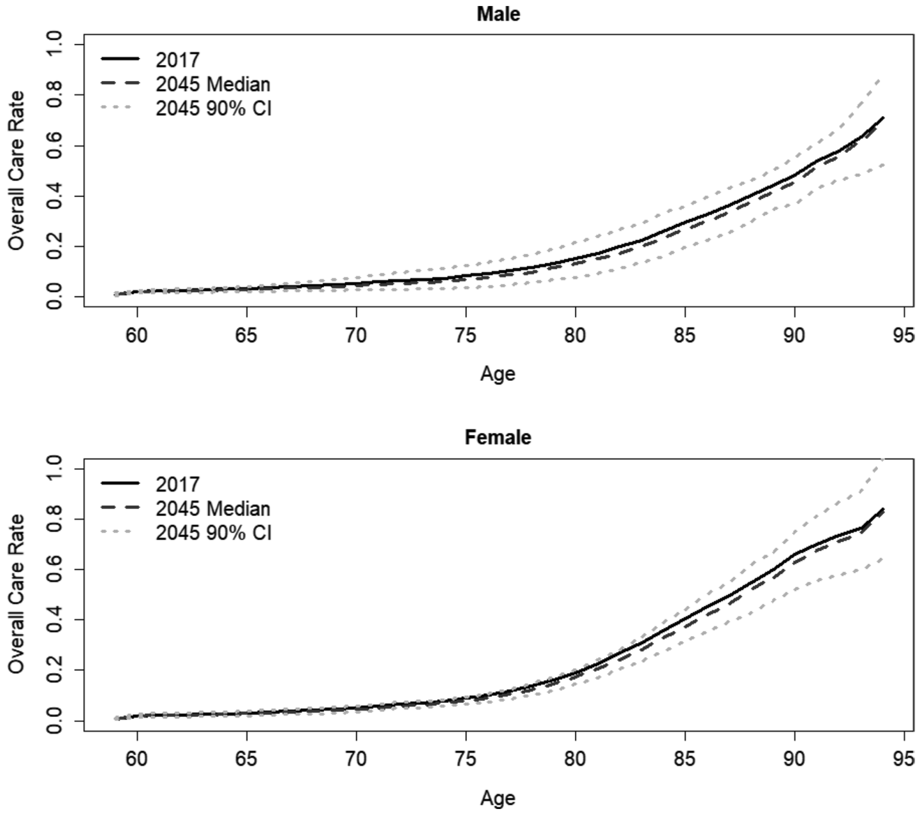

Fig. 7 Overall care rates in 2017 and 2045 with 90% CIs. *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; Vanella (2017); Vanella and Deschermeier (2020); own calculation and design

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[13] This is not done graphically, since the differences are quite small and would be difficult to identify in a graphical representation.

change much in the future. In general, we see broadening CIs with increasing age, representing higher uncertainty with regard to future development.

In addition to the development of age-specific care prevalence, the overall development of the size and age structure of the population is another important determinant of overall frailty prevalence. Vanella and Deschermeier (2020) conducted a probabilistic forecast of the age- and sex-specific population in Germany until 2045. We borrow their results for our study. A slight increase in the population size to almost 85 million persons by 2040 is predicted in the median. Even more important than the population size is the age structure of the population. For both genders, we observe an almost certain increase in the population size from age 70 onwards, which is the age group in which the risk of being in need of care starts to increase significantly, as illustrated in Fig. 7. "Appendix C" gives the forecast of the sizes of two old-age groups to illustrate the increase in the population size in these age groups, which are very susceptible to care risks.

Multiplying the 10,000 annual trajectories for the age- and sex-specific population provided by Vanella and Deschermeier (2020) with the 10,000 trajectories for the ASSSCRs estimated by us, we follow (5) and derive 10,000 trajectories of the number of persons claiming care services by age (−59,60,…,93,94+), sex and care degree. Figures 8 and 9 illustrate the future development of the number of persons claiming care insurance services by care degree, without discriminating by sex and age, with 90% CIs.

The number of persons in PG1 is rather negligible, whereas the number of persons in PG2, PG3 and PG4 will increase significantly, as given in Table 1.

The increase in PG5 is not as strong. This can be explained first by the overall smaller risk of being classified into PG5 rather than into a less severe state. Second, we see from "Appendix B" that the risk of suffering from severe disability after age 80 increases strongly. Since the size of the population decreases heavily at that point, the part of the

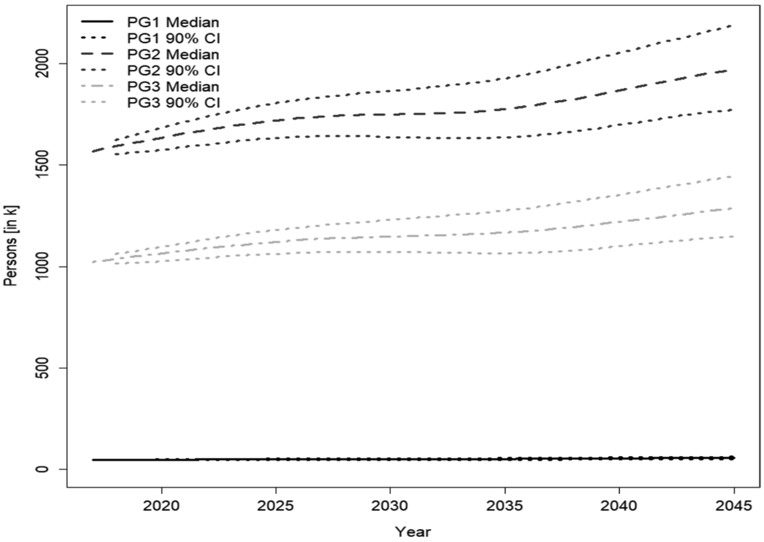

Fig. 8 Persons receiving care insurance services PG1-3 until 2045. *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; Vanella (2017); Vanella and Deschermeier (2020); own calculation and design

population at severe risk of these kinds of disabilities is rather small. Hence, the resulting number of persons in PG5 remains relatively small.

5 Discussion and outlook

The present study showed the effect of future demographic development in Germany on the number of frail persons claiming services from the GPV. Due to the aging process, we expect these numbers to increase heavily until 2045, especially for the moderately severe care degrees 2–4, with no mitigation of this trend in sight. This development implies large increases in the financial pressure on the GPV and in the demand for nurses to care for those in need. Furthermore, this development will probably strain the possibilities of reconciling work and care for many employed caregivers, as approximately 70 percent of caring in Germany is done in families (Ehrlich et al. 2019: 1)

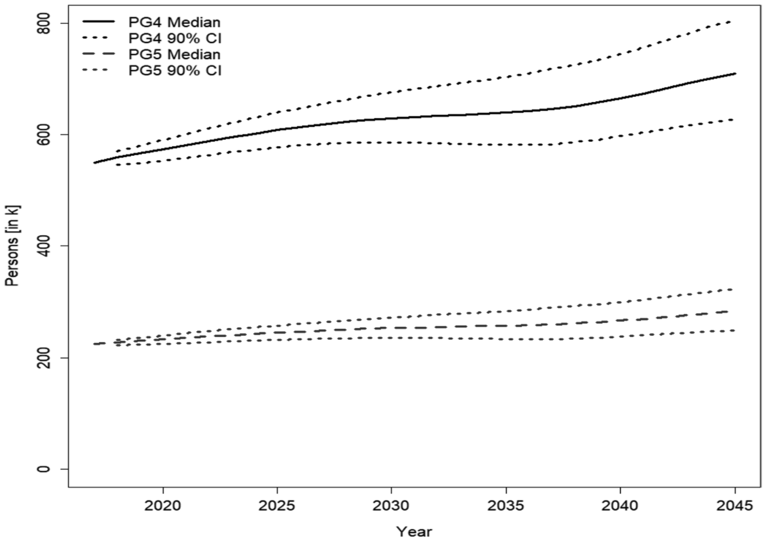

Fig. 9 Persons receiving care insurance services PG4 and PG5 until 2045 *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; Vanella (2017); Vanella and Deschermeier (2020); own calculation and design

Table 1 Million persons claiming services from statutory long-term care insurance, 2017 and 2045. *Sources*: RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; own calculation and design

Care degree/ PG	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
1	0.04	0.05	0.06	0.06
2	1.57	1.77	1.97	2.19
3	1.02	1.15	1.29	1.45
4	0.55	0.63	0.71	0.80
5	0.22	0.25	0.28	0.32

Using theory-based Monte Carlo simulation, we estimated median trajectories for the future number of persons claiming benefits of the GPV by severity of disability, and we derived 90% CIs. We thus provide a realistic frame for future development as a sound basis for future planning in politics and business.

When interpreting the results of our study, limitations must be acknowledged. The definition of severity in the framework of insurance changed significantly in 2016 with the change from care levels to care degrees and the inclusion of dementia in the specification of disability severity. As statistics before 2013 did not include measurements of dementia and the new care degree 1 has no predecessor, it is not possible to construct valid time series for the care prevalence in Germany. Therefore, it is not possible to estimate autocorrelations for the time series or cross-correlations among the time series of the various prevalence rates, making more sophisticated analyses not yet feasible. Hence, we had to make simplifying assumptions based on the theoretical connection between mortality and morbidity. Future studies should be able to address this issue better when time series for the problem under study are available. We stress that the intervals presented in our study cannot account for the correlations mentioned above to the full extent; therefore, we labeled them “credible intervals” instead of the more common term “prediction intervals”. Moreover, the reader should keep in mind that our study exclusively covers persons receiving care insurance benefits. Therefore, we might underestimate the total prevalence in the population to some degree, as persons who have claimed services but have not been assigned a specific care degree are not included in the study. Moreover, some persons might be frail but do not claim services, as their family members care for them without receiving any compensation for doing so. Finally, long-term care policies in Germany are undergoing several reforms currently, and thus, the institutional framework and regulations of who is official defined of being in the need of care might change yet again.

However, our study provides an informative and detailed outlook from which scientific as well as societal implications can be drawn. Based on our results, further studies regarding the future demand for care nurses and the financial expenses of long-term care insurance in Germany should be conducted. In addition, our methodological approach could be applied to other countries with comparable long-term care systems. From a societal perspective, policy makers, employers, trade-unions, providers of long-term care and other stakeholders must acknowledge the challenges stemming from the extremely likely increasing need for professional care nurses and financial expenses for the GPV. They have to develop strategies and measures on the national, regional and local level to tackle these challenges.

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Appendix A

See Tables 2 and 3.

Table 2 Predicted relative annual change in age-specific mortality rates of males. *Sources*: Destatis (2018b, 2019); Human Mortality Database (2019); Vanella (2017); own calculation and design

Age	2017	Median 2020	Median 2025	Median 2030	Median 2035	Median 2040	Median 2045
– 59	– 0.0032	– 0.0158	– 0.0153	– 0.0134	– 0.0100	– 0.0081	– 0.0076
60	– 0.0420	– 0.0103	– 0.0093	– 0.0068	– 0.0047	– 0.0035	– 0.0030
61	0.0033	– 0.0114	– 0.0097	– 0.0079	– 0.0054	– 0.0043	– 0.0029
62	0.0114	– 0.0115	– 0.0100	– 0.0084	– 0.0048	– 0.0042	– 0.0033
63	– 0.0354	– 0.0113	– 0.0115	– 0.0089	– 0.0065	– 0.0051	– 0.0032
64	– 0.0028	– 0.0127	– 0.0106	– 0.0099	– 0.0060	– 0.0054	– 0.0044
65	– 0.0383	– 0.0122	– 0.0114	– 0.0100	– 0.0069	– 0.0059	– 0.0046
66	0.0029	– 0.0131	– 0.0131	– 0.0109	– 0.0070	– 0.0064	– 0.0045
67	– 0.0283	– 0.0138	– 0.0137	– 0.0117	– 0.0082	– 0.0061	– 0.0048
68	0.0047	– 0.0148	– 0.0154	– 0.0122	– 0.0092	– 0.0074	– 0.0059
69	0.0220	– 0.0154	– 0.0151	– 0.0127	– 0.0095	– 0.0076	– 0.0055
70	– 0.0340	– 0.0158	– 0.0155	– 0.0125	– 0.0102	– 0.0077	– 0.0063
71	– 0.0012	– 0.0163	– 0.0171	– 0.0142	– 0.0118	– 0.0079	– 0.0057
72	0.0173	– 0.0163	– 0.0179	– 0.0150	– 0.0117	– 0.0089	– 0.0054
73	0.0236	– 0.0167	– 0.0177	– 0.0149	– 0.0116	– 0.0092	– 0.0059
74	0.0045	– 0.0172	– 0.0175	– 0.0149	– 0.0112	– 0.0081	– 0.0060
75	0.0008	– 0.0179	– 0.0177	– 0.0146	– 0.0112	– 0.0090	– 0.0058
76	– 0.0162	– 0.0184	– 0.0184	– 0.0144	– 0.0104	– 0.0088	– 0.0056
77	0.0180	– 0.0166	– 0.0172	– 0.0139	– 0.0096	– 0.0087	– 0.0051
78	– 0.0239	– 0.0165	– 0.0167	– 0.0131	– 0.0092	– 0.0079	– 0.0057
79	– 0.0125	– 0.0155	– 0.0163	– 0.0127	– 0.0086	– 0.0068	– 0.0051
80	– 0.0152	– 0.0146	– 0.0157	– 0.0126	– 0.0081	– 0.0066	– 0.0057
81	– 0.0203	– 0.0134	– 0.0145	– 0.0122	– 0.0083	– 0.0070	– 0.0047
82	– 0.0065	– 0.0126	– 0.0139	– 0.0108	– 0.0076	– 0.0059	– 0.0043
83	– 0.0101	– 0.0115	– 0.0135	– 0.0104	– 0.0067	– 0.0059	– 0.0036
84	– 0.0213	– 0.0108	– 0.0121	– 0.0092	– 0.0065	– 0.0052	– 0.0036
85	0.0057	– 0.0101	– 0.0105	– 0.0083	– 0.0060	– 0.0039	– 0.0036
86	– 0.0013	– 0.0091	– 0.0098	– 0.0076	– 0.0050	– 0.0037	– 0.0029
87	– 0.0066	– 0.0076	– 0.0086	– 0.0066	– 0.0042	– 0.0031	– 0.0035
88	0.0065	– 0.0079	– 0.0077	– 0.0067	– 0.0041	– 0.0031	– 0.0024
89	0.0235	– 0.0060	– 0.0068	– 0.0049	– 0.0030	– 0.0019	– 0.0025
90	0.0196	– 0.0059	– 0.0066	– 0.0048	– 0.0032	– 0.0029	– 0.0018
91	0.0298	– 0.0053	– 0.0045	– 0.0040	– 0.0028	– 0.0014	– 0.0014
92	0.0351	– 0.0049	– 0.0041	– 0.0030	– 0.0019	– 0.0015	– 0.0015
93	0.0405	– 0.0041	– 0.0035	– 0.0025	– 0.0015	– 0.0009	– 0.0006
94+	0.0117	– 0.0035	– 0.0030	– 0.0020	– 0.0017	– 0.0017	– 0.0001

Table 3 Predicted relative annual change in age-specific mortality rates of females. *Sources*: Destatis (2018b, 2019); Human Mortality Database (2019); Vanella (2017); own calculation and design

Age	2017	Median 2020	Median 2025	Median 2030	Median 2035	Median 2040	Median 2045
–59	0.2220	− 0.0120	− 0.0102	− 0.0064	− 0.0068	− 0.0013	− 0.0047
60	− 0.0274	− 0.0068	− 0.0045	− 0.0032	− 0.0019	− 0.0004	− 0.0017
61	0.0222	− 0.0080	− 0.0053	− 0.0032	− 0.0005	− 0.0008	− 0.0013
62	0.0412	− 0.0079	− 0.0051	− 0.0035	− 0.0014	− 0.0017	− 0.0017
63	− 0.0193	− 0.0086	− 0.0062	− 0.0041	− 0.0022	− 0.0016	− 0.0018
64	− 0.0124	− 0.0086	− 0.0055	− 0.0040	− 0.0024	− 0.0014	− 0.0016
65	− 0.0087	− 0.0087	− 0.0058	− 0.0057	− 0.0024	− 0.0015	− 0.0021
66	− 0.0224	− 0.0092	− 0.0059	− 0.0058	− 0.0024	− 0.0028	− 0.0021
67	− 0.0145	− 0.0093	− 0.0079	− 0.0063	− 0.0031	− 0.0022	− 0.0027
68	− 0.0191	− 0.0100	− 0.0078	− 0.0063	− 0.0041	− 0.0034	− 0.0022
69	0.0065	− 0.0108	− 0.0096	− 0.0075	− 0.0041	− 0.0037	− 0.0030
70	− 0.0250	− 0.0112	− 0.0096	− 0.0079	− 0.0040	− 0.0042	− 0.0031
71	0.0152	− 0.0115	− 0.0099	− 0.0077	− 0.0054	− 0.0040	− 0.0028
72	0.0365	− 0.0126	− 0.0117	− 0.0088	− 0.0060	− 0.0049	− 0.0026
73	0.0705	− 0.0134	− 0.0121	− 0.0086	− 0.0062	− 0.0051	− 0.0029
74	0.0185	− 0.0142	− 0.0122	− 0.0096	− 0.0066	− 0.0048	− 0.0034
75	0.0077	− 0.0151	− 0.0126	− 0.0099	− 0.0072	− 0.0054	− 0.0037
76	0.0126	− 0.0150	− 0.0125	− 0.0092	− 0.0063	− 0.0053	− 0.0033
77	0.0162	− 0.0143	− 0.0120	− 0.0093	− 0.0064	− 0.0052	− 0.0033
78	− 0.0230	− 0.0144	− 0.0120	− 0.0087	− 0.0058	− 0.0044	− 0.0028
79	− 0.0089	− 0.0137	− 0.0118	− 0.0085	− 0.0062	− 0.0046	− 0.0030
80	− 0.0194	− 0.0130	− 0.0110	− 0.0080	− 0.0055	− 0.0045	− 0.0032
81	− 0.0146	− 0.0128	− 0.0107	− 0.0076	− 0.0060	− 0.0042	− 0.0027
82	− 0.0168	− 0.0116	− 0.0099	− 0.0074	− 0.0052	− 0.0040	− 0.0032
83	0.0050	− 0.0113	− 0.0093	− 0.0067	− 0.0045	− 0.0036	− 0.0031
84	− 0.0041	− 0.0111	− 0.0090	− 0.0061	− 0.0048	− 0.0032	− 0.0035
85	0.0087	− 0.0098	− 0.0084	− 0.0060	− 0.0040	− 0.0027	− 0.0026
86	0.0158	− 0.0094	− 0.0076	− 0.0047	− 0.0036	− 0.0024	− 0.0025
87	− 0.0091	− 0.0088	− 0.0073	− 0.0048	− 0.0035	− 0.0027	− 0.0022
88	0.0046	− 0.0078	− 0.0063	− 0.0042	− 0.0034	− 0.0018	− 0.0021
89	− 0.0010	− 0.0068	− 0.0054	− 0.0038	− 0.0026	− 0.0018	− 0.0017
90	0.0193	− 0.0066	− 0.0048	− 0.0035	− 0.0025	− 0.0020	− 0.0019
91	0.0362	− 0.0054	− 0.0035	− 0.0026	− 0.0021	− 0.0012	− 0.0010
92	0.0238	− 0.0050	− 0.0034	− 0.0028	− 0.0024	− 0.0015	− 0.0009
93	0.0312	− 0.0044	− 0.0025	− 0.0014	− 0.0018	− 0.0011	− 0.0008
94+	0.0316	− 0.0022	− 0.0018	0.0000	− 0.0016	− 0.0001	0.0000

Appendix B

See Table 4.

Table 4 Estimated age-, sex- and severity-specific care rates in 2017 and 2045 with 90% credible intervals. *Sources*: Human Mortality Database (2019); RDC of the Federal Statistical Office and Statistical Offices of the Länder, Pflegestatistik, survey year 2017; own calculation and design

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
− 59	Male	1	0.0001	0.0000	0.0001	0.0001
60	Male	1	0.0004	0.0003	0.0003	0.0004
61	Male	1	0.0004	0.0003	0.0004	0.0005
62	Male	1	0.0004	0.0003	0.0004	0.0005
63	Male	1	0.0005	0.0003	0.0004	0.0006
64	Male	1	0.0005	0.0003	0.0005	0.0006
65	Male	1	0.0006	0.0004	0.0005	0.0007
66	Male	1	0.0006	0.0004	0.0005	0.0008
67	Male	1	0.0006	0.0003	0.0005	0.0008
68	Male	1	0.0007	0.0003	0.0006	0.0009
69	Male	1	0.0007	0.0003	0.0006	0.0009
70	Male	1	0.0007	0.0003	0.0006	0.001
71	Male	1	0.0008	0.0004	0.0007	0.0011
72	Male	1	0.0009	0.0004	0.0007	0.0013
73	Male	1	0.0009	0.0004	0.0008	0.0014
74	Male	1	0.0009	0.0004	0.0008	0.0014
75	Male	1	0.001	0.0004	0.0008	0.0015
76	Male	1	0.0011	0.0005	0.0009	0.0016
77	Male	1	0.0013	0.0006	0.0011	0.0019
78	Male	1	0.0015	0.0007	0.0013	0.0022
79	Male	1	0.0017	0.0008	0.0014	0.0024
80	Male	1	0.0017	0.0009	0.0015	0.0024
81	Male	1	0.0022	0.0012	0.0019	0.003
82	Male	1	0.0024	0.0013	0.0021	0.0032
83	Male	1	0.0027	0.0016	0.0024	0.0035
84	Male	1	0.0029	0.0018	0.0026	0.0037
85	Male	1	0.0035	0.0023	0.0032	0.0043
86	Male	1	0.004	0.0027	0.0036	0.0048
87	Male	1	0.0044	0.0031	0.004	0.0052
88	Male	1	0.0048	0.0035	0.0044	0.0055
89	Male	1	0.0051	0.004	0.0048	0.0057
90	Male	1	0.0053	0.004	0.005	0.0061
91	Male	1	0.0065	0.0052	0.0063	0.0074
92	Male	1	0.0068	0.0055	0.0066	0.0078
93	Male	1	0.0069	0.0053	0.0067	0.0083
94+	Male	1	0.0067	0.0049	0.0065	0.0083
− 59	Male	2	0.0037	0.0017	0.0032	0.0059
60	Male	2	0.0098	0.0071	0.009	0.0112
61	Male	2	0.0111	0.0077	0.0101	0.0129
62	Male	2	0.0119	0.0081	0.0107	0.0141
63	Male	2	0.0131	0.0085	0.0118	0.016
64	Male	2	0.0145	0.0093	0.013	0.0177

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
65	Male	2	0.0156	0.0097	0.0139	0.0196
66	Male	2	0.0175	0.0103	0.0154	0.0227
67	Male	2	0.0183	0.0104	0.016	0.0243
68	Male	2	0.0201	0.0105	0.0174	0.0282
69	Male	2	0.0217	0.0111	0.0186	0.0306
70	Male	2	0.0239	0.0121	0.0204	0.0343
71	Male	2	0.0269	0.0128	0.0228	0.0398
72	Male	2	0.0283	0.0131	0.0238	0.0426
73	Male	2	0.0307	0.0141	0.0257	0.046
74	Male	2	0.0332	0.0152	0.0279	0.0502
75	Male	2	0.0363	0.0164	0.0304	0.0553
76	Male	2	0.04	0.0183	0.0337	0.0605
77	Male	2	0.046	0.0221	0.0391	0.0676
78	Male	2	0.0511	0.0249	0.0435	0.0749
79	Male	2	0.0587	0.0298	0.0504	0.0834
80	Male	2	0.0667	0.0342	0.0577	0.0952
81	Male	2	0.0766	0.0406	0.0666	0.1068
82	Male	2	0.088	0.0489	0.077	0.1193
83	Male	2	0.0993	0.059	0.0881	0.1291
84	Male	2	0.1152	0.0712	0.1029	0.1464
85	Male	2	0.1317	0.0869	0.1192	0.1603
86	Male	2	0.1467	0.0998	0.1338	0.1777
87	Male	2	0.1628	0.1151	0.1503	0.1922
88	Male	2	0.1789	0.133	0.1664	0.2055
89	Male	2	0.1961	0.1533	0.1852	0.2216
90	Male	2	0.215	0.1645	0.203	0.2474
91	Male	2	0.2401	0.1915	0.2295	0.2722
92	Male	2	0.2579	0.2065	0.2489	0.2964
93	Male	2	0.2711	0.2077	0.2639	0.3275
94+	Male	2	0.2839	0.2093	0.278	0.3547
− 59	Male	3	0.0031	0.0014	0.0027	0.005
60	Male	3	0.0065	0.0047	0.006	0.0075
61	Male	3	0.0072	0.0051	0.0066	0.0085
62	Male	3	0.008	0.0055	0.0072	0.0095
63	Male	3	0.0083	0.0054	0.0075	0.0102
64	Male	3	0.0092	0.006	0.0083	0.0113
65	Male	3	0.0105	0.0065	0.0093	0.0132
66	Male	3	0.0112	0.0066	0.0099	0.0146
67	Male	3	0.0124	0.007	0.0108	0.0164
68	Male	3	0.0134	0.007	0.0116	0.0188
69	Male	3	0.0149	0.0076	0.0127	0.021
70	Male	3	0.0163	0.0082	0.0139	0.0234
71	Male	3	0.0183	0.0087	0.0155	0.0271
72	Male	3	0.021	0.0097	0.0176	0.0315

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
73	Male	3	0.0217	0.0099	0.0182	0.0325
74	Male	3	0.0238	0.0109	0.02	0.0359
75	Male	3	0.0264	0.0119	0.0221	0.0402
76	Male	3	0.0292	0.0134	0.0246	0.0442
77	Male	3	0.0328	0.0158	0.0279	0.0482
78	Male	3	0.0367	0.0179	0.0312	0.0537
79	Male	3	0.0422	0.0214	0.0362	0.0599
80	Male	3	0.0485	0.0248	0.0419	0.0692
81	Male	3	0.0547	0.029	0.0476	0.0762
82	Male	3	0.0629	0.0349	0.055	0.0852
83	Male	3	0.0722	0.0429	0.064	0.0938
84	Male	3	0.0831	0.0514	0.0743	0.1057
85	Male	3	0.0925	0.061	0.0837	0.1125
86	Male	3	0.103	0.0701	0.0939	0.1247
87	Male	3	0.1152	0.0815	0.1064	0.136
88	Male	3	0.1271	0.0945	0.1183	0.146
89	Male	3	0.1412	0.1104	0.1334	0.1596
90	Male	3	0.1517	0.1161	0.1432	0.1746
91	Male	3	0.1677	0.1338	0.1603	0.1901
92	Male	3	0.1773	0.142	0.1712	0.2038
93	Male	3	0.1957	0.15	0.1905	0.2365
94+	Male	3	0.2276	0.1677	0.2228	0.2843
− 59	Male	4	0.0016	0.0007	0.0014	0.0026
60	Male	4	0.0029	0.0021	0.0027	0.0033
61	Male	4	0.0033	0.0023	0.003	0.0039
62	Male	4	0.0036	0.0024	0.0032	0.0042
63	Male	4	0.004	0.0026	0.0036	0.0049
64	Male	4	0.0044	0.0028	0.0039	0.0054
65	Male	4	0.0049	0.003	0.0043	0.0061
66	Male	4	0.0052	0.0031	0.0046	0.0068
67	Male	4	0.0058	0.0033	0.005	0.0076
68	Male	4	0.0065	0.0034	0.0056	0.0091
69	Male	4	0.0071	0.0036	0.006	0.01
70	Male	4	0.0079	0.004	0.0067	0.0113
71	Male	4	0.009	0.0043	0.0076	0.0133
72	Male	4	0.0101	0.0047	0.0085	0.0152
73	Male	4	0.0112	0.0051	0.0094	0.0168
74	Male	4	0.0114	0.0052	0.0096	0.0172
75	Male	4	0.0135	0.0061	0.0113	0.0205
76	Male	4	0.015	0.0069	0.0127	0.0227
77	Male	4	0.0169	0.0081	0.0143	0.0248
78	Male	4	0.0188	0.0091	0.016	0.0275
79	Male	4	0.0225	0.0114	0.0193	0.032
80	Male	4	0.0252	0.0129	0.0217	0.0359

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
81	Male	4	0.0292	0.0154	0.0254	0.0406
82	Male	4	0.0334	0.0185	0.0292	0.0452
83	Male	4	0.038	0.0226	0.0337	0.0494
84	Male	4	0.0444	0.0275	0.0397	0.0564
85	Male	4	0.0509	0.0336	0.0461	0.0619
86	Male	4	0.0563	0.0383	0.0513	0.0681
87	Male	4	0.0627	0.0443	0.0579	0.074
88	Male	4	0.0707	0.0526	0.0658	0.0812
89	Male	4	0.079	0.0617	0.0746	0.0893
90	Male	4	0.0852	0.0652	0.0804	0.0981
91	Male	4	0.0973	0.0776	0.093	0.1103
92	Male	4	0.1071	0.0857	0.1033	0.123
93	Male	4	0.126	0.0966	0.1227	0.1522
94+	Male	4	0.1479	0.109	0.1448	0.1847
− 59	Male	5	0.0007	0.0003	0.0006	0.0011
60	Male	5	0.0012	0.0008	0.0011	0.0013
61	Male	5	0.0014	0.0009	0.0012	0.0016
62	Male	5	0.0015	0.0011	0.0014	0.0018
63	Male	5	0.0017	0.0011	0.0015	0.002
64	Male	5	0.0018	0.0011	0.0016	0.0022
65	Male	5	0.0019	0.0012	0.0017	0.0023
66	Male	5	0.0021	0.0012	0.0019	0.0027
67	Male	5	0.0023	0.0013	0.002	0.003
68	Male	5	0.0024	0.0012	0.002	0.0033
69	Male	5	0.0026	0.0013	0.0022	0.0036
70	Male	5	0.0031	0.0016	0.0026	0.0044
71	Male	5	0.0034	0.0016	0.0029	0.005
72	Male	5	0.0037	0.0017	0.0031	0.0056
73	Male	5	0.0039	0.0018	0.0033	0.0059
74	Male	5	0.0043	0.002	0.0036	0.0065
75	Male	5	0.0049	0.0022	0.0041	0.0075
76	Male	5	0.0053	0.0024	0.0044	0.008
77	Male	5	0.0062	0.003	0.0053	0.0091
78	Male	5	0.0068	0.0033	0.0058	0.0099
79	Male	5	0.008	0.0041	0.0069	0.0114
80	Male	5	0.0091	0.0047	0.0079	0.013
81	Male	5	0.0103	0.0055	0.009	0.0144
82	Male	5	0.0116	0.0064	0.0101	0.0157
83	Male	5	0.013	0.0077	0.0116	0.0169
84	Male	5	0.0146	0.009	0.013	0.0185
85	Male	5	0.0161	0.0106	0.0145	0.0195
86	Male	5	0.0168	0.0114	0.0153	0.0203
87	Male	5	0.0184	0.013	0.017	0.0218
88	Male	5	0.0216	0.016	0.0201	0.0248

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
89	Male	5	0.0231	0.018	0.0218	0.0261
90	Male	5	0.0251	0.0192	0.0237	0.0289
91	Male	5	0.0269	0.0215	0.0257	0.0305
92	Male	5	0.0309	0.0248	0.0299	0.0355
93	Male	5	0.0342	0.0262	0.0333	0.0413
94+	Male	5	0.0437	0.0322	0.0427	0.0545
− 59	Female	1	0.0001	0.0000	0.0001	0.0001
60	Female	1	0.0003	0.0002	0.0003	0.0004
61	Female	1	0.0004	0.0002	0.0003	0.0005
62	Female	1	0.0004	0.0003	0.0004	0.0005
63	Female	1	0.0004	0.0003	0.0004	0.0005
64	Female	1	0.0005	0.0003	0.0005	0.0006
65	Female	1	0.0005	0.0003	0.0005	0.0007
66	Female	1	0.0006	0.0004	0.0006	0.0008
67	Female	1	0.0007	0.0005	0.0007	0.0009
68	Female	1	0.0008	0.0005	0.0007	0.001
69	Female	1	0.0008	0.0006	0.0008	0.001
70	Female	1	0.0009	0.0006	0.0008	0.001
71	Female	1	0.001	0.0007	0.001	0.0012
72	Female	1	0.0013	0.0009	0.0012	0.0014
73	Female	1	0.0013	0.001	0.0012	0.0015
74	Female	1	0.0014	0.001	0.0012	0.0015
75	Female	1	0.0016	0.0012	0.0014	0.0016
76	Female	1	0.0019	0.0014	0.0017	0.0019
77	Female	1	0.0021	0.0016	0.0019	0.0022
78	Female	1	0.0025	0.0019	0.0022	0.0027
79	Female	1	0.003	0.0022	0.0026	0.0032
80	Female	1	0.0034	0.0026	0.0031	0.0036
81	Female	1	0.0041	0.0031	0.0037	0.0044
82	Female	1	0.0045	0.0035	0.0041	0.0047
83	Female	1	0.0054	0.0042	0.0049	0.0058
84	Female	1	0.0057	0.0044	0.0052	0.0062
85	Female	1	0.006	0.0047	0.0056	0.0065
86	Female	1	0.0067	0.0052	0.0062	0.0074
87	Female	1	0.0071	0.0056	0.0067	0.0079
88	Female	1	0.0072	0.0057	0.0068	0.0082
89	Female	1	0.0072	0.0057	0.0068	0.008
90	Female	1	0.0072	0.0057	0.0069	0.0082
91	Female	1	0.0072	0.0057	0.007	0.0084
92	Female	1	0.0071	0.0056	0.0069	0.0084
93	Female	1	0.0071	0.0055	0.0069	0.0084
94+	Female	1	0.0058	0.0045	0.0058	0.0072
− 59	Female	2	0.0033	0.0019	0.003	0.0048
60	Female	2	0.0101	0.007	0.0096	0.0133

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
61	Female	2	0.0113	0.0078	0.0108	0.0152
62	Female	2	0.012	0.0083	0.0114	0.0155
63	Female	2	0.0131	0.0091	0.0124	0.0169
64	Female	2	0.0139	0.0094	0.0131	0.0183
65	Female	2	0.0158	0.0106	0.0149	0.0209
66	Female	2	0.017	0.0114	0.0159	0.0221
67	Female	2	0.0191	0.0129	0.0178	0.0244
68	Female	2	0.021	0.0145	0.0194	0.0259
69	Female	2	0.0234	0.0164	0.0214	0.0279
70	Female	2	0.0257	0.018	0.0233	0.03
71	Female	2	0.0295	0.0207	0.0268	0.0344
72	Female	2	0.0345	0.0251	0.0308	0.0379
73	Female	2	0.0355	0.0258	0.0316	0.0384
74	Female	2	0.04	0.0298	0.0354	0.0417
75	Female	2	0.0454	0.0339	0.0401	0.047
76	Female	2	0.0519	0.0388	0.0459	0.054
77	Female	2	0.0604	0.0447	0.0535	0.0635
78	Female	2	0.0709	0.0525	0.0631	0.0754
79	Female	2	0.0835	0.0612	0.0747	0.0903
80	Female	2	0.0981	0.0745	0.088	0.1038
81	Female	2	0.1161	0.0875	0.1046	0.1243
82	Female	2	0.1364	0.1049	0.1236	0.1443
83	Female	2	0.1576	0.1214	0.1438	0.1697
84	Female	2	0.1801	0.1383	0.1651	0.1961
85	Female	2	0.2016	0.1574	0.1862	0.2185
86	Female	2	0.2221	0.1722	0.2073	0.2473
87	Female	2	0.2425	0.1916	0.2269	0.2675
88	Female	2	0.2571	0.2005	0.2427	0.2907
89	Female	2	0.2754	0.2187	0.2608	0.3084
90	Female	2	0.296	0.2343	0.2822	0.3364
91	Female	2	0.3042	0.2414	0.2926	0.3524
92	Female	2	0.313	0.2439	0.3018	0.368
93	Female	2	0.3063	0.2402	0.2987	0.3649
94+	Female	2	0.2922	0.2241	0.2882	0.3623
− 59	Female	3	0.0025	0.0014	0.0023	0.0036
60	Female	3	0.0058	0.004	0.0055	0.0076
61	Female	3	0.0065	0.0044	0.0062	0.0086
62	Female	3	0.0067	0.0046	0.0063	0.0086
63	Female	3	0.0073	0.0051	0.0069	0.0094
64	Female	3	0.0081	0.0055	0.0077	0.0107
65	Female	3	0.0088	0.0058	0.0082	0.0115
66	Female	3	0.0098	0.0065	0.0091	0.0127
67	Female	3	0.0107	0.0073	0.01	0.0138
68	Female	3	0.0118	0.0081	0.0109	0.0145

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
69	Female	3	0.0129	0.009	0.0118	0.0154
70	Female	3	0.0141	0.0099	0.0128	0.0165
71	Female	3	0.0167	0.0117	0.0152	0.0195
72	Female	3	0.0189	0.0138	0.0169	0.0208
73	Female	3	0.0204	0.0148	0.0182	0.0221
74	Female	3	0.023	0.0172	0.0203	0.024
75	Female	3	0.0258	0.0193	0.0228	0.0267
76	Female	3	0.0287	0.0214	0.0254	0.0298
77	Female	3	0.0333	0.0247	0.0296	0.0351
78	Female	3	0.0381	0.0282	0.0339	0.0405
79	Female	3	0.0448	0.0329	0.0401	0.0485
80	Female	3	0.0531	0.0403	0.0476	0.0562
81	Female	3	0.0628	0.0473	0.0566	0.0672
82	Female	3	0.0736	0.0566	0.0667	0.0779
83	Female	3	0.0851	0.0656	0.0777	0.0916
84	Female	3	0.0995	0.0764	0.0912	0.1083
85	Female	3	0.1127	0.088	0.1041	0.1221
86	Female	3	0.1259	0.0976	0.1175	0.1401
87	Female	3	0.1378	0.1089	0.1289	0.152
88	Female	3	0.156	0.1217	0.1473	0.1765
89	Female	3	0.1695	0.1346	0.1605	0.1898
90	Female	3	0.1886	0.1493	0.1798	0.2144
91	Female	3	0.202	0.1603	0.1942	0.234
92	Female	3	0.2118	0.165	0.2042	0.2491
93	Female	3	0.2251	0.1765	0.2195	0.2682
94+	Female	3	0.2444	0.1875	0.2411	0.303
− 59	Female	4	0.0013	0.0007	0.0012	0.0019
60	Female	4	0.0024	0.0017	0.0023	0.0032
61	Female	4	0.0026	0.0018	0.0025	0.0035
62	Female	4	0.0028	0.002	0.0027	0.0037
63	Female	4	0.003	0.0021	0.0028	0.0038
64	Female	4	0.0031	0.0021	0.0029	0.0041
65	Female	4	0.0034	0.0022	0.0032	0.0044
66	Female	4	0.0038	0.0025	0.0036	0.0049
67	Female	4	0.0042	0.0029	0.0039	0.0054
68	Female	4	0.0047	0.0032	0.0043	0.0057
69	Female	4	0.0051	0.0036	0.0047	0.0061
70	Female	4	0.0059	0.0041	0.0054	0.0069
71	Female	4	0.0068	0.0048	0.0062	0.008
72	Female	4	0.0079	0.0058	0.0071	0.0087
73	Female	4	0.009	0.0065	0.008	0.0097
74	Female	4	0.01	0.0075	0.0088	0.0104
75	Female	4	0.0118	0.0088	0.0104	0.0122
76	Female	4	0.0132	0.0098	0.0116	0.0137

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
77	Female	4	0.0155	0.0114	0.0137	0.0162
78	Female	4	0.0183	0.0136	0.0163	0.0195
79	Female	4	0.0215	0.0158	0.0192	0.0233
80	Female	4	0.0255	0.0194	0.0229	0.027
81	Female	4	0.0304	0.0229	0.0274	0.0326
82	Female	4	0.0372	0.0286	0.0337	0.0393
83	Female	4	0.0431	0.0332	0.0393	0.0464
84	Female	4	0.0507	0.0389	0.0465	0.0552
85	Female	4	0.06	0.0469	0.0554	0.065
86	Female	4	0.0694	0.0538	0.0648	0.0773
87	Female	4	0.0783	0.0618	0.0732	0.0863
88	Female	4	0.0913	0.0712	0.0862	0.1033
89	Female	4	0.1032	0.0819	0.0977	0.1156
90	Female	4	0.1178	0.0932	0.1123	0.1339
91	Female	4	0.1336	0.106	0.1285	0.1547
92	Female	4	0.145	0.113	0.1398	0.1705
93	Female	4	0.1579	0.1238	0.154	0.1881
94+	Female	4	0.2039	0.1564	0.2011	0.2528
− 59	Female	5	0.0006	0.0003	0.0005	0.0009
60	Female	5	0.0011	0.0008	0.0011	0.0015
61	Female	5	0.0012	0.0008	0.0011	0.0016
62	Female	5	0.0013	0.0009	0.0012	0.0016
63	Female	5	0.0014	0.001	0.0013	0.0018
64	Female	5	0.0015	0.001	0.0014	0.002
65	Female	5	0.0015	0.001	0.0014	0.002
66	Female	5	0.0017	0.0011	0.0015	0.0021
67	Female	5	0.0019	0.0013	0.0017	0.0024
68	Female	5	0.0021	0.0015	0.002	0.0026
69	Female	5	0.0023	0.0016	0.0021	0.0028
70	Female	5	0.0027	0.0019	0.0024	0.0031
71	Female	5	0.003	0.0021	0.0027	0.0035
72	Female	5	0.0036	0.0026	0.0032	0.0039
73	Female	5	0.0043	0.0031	0.0038	0.0047
74	Female	5	0.0047	0.0035	0.0041	0.0049
75	Female	5	0.0053	0.0039	0.0047	0.0055
76	Female	5	0.006	0.0045	0.0053	0.0062
77	Female	5	0.0069	0.0051	0.0061	0.0072
78	Female	5	0.0079	0.0059	0.0071	0.0084
79	Female	5	0.0095	0.007	0.0085	0.0103
80	Female	5	0.0114	0.0087	0.0103	0.0121
81	Female	5	0.0136	0.0102	0.0122	0.0145
82	Female	5	0.0161	0.0124	0.0146	0.017
83	Female	5	0.0189	0.0145	0.0172	0.0203
84	Female	5	0.0219	0.0168	0.0201	0.0238

Table 4 (continued)

Age	Sex	Care degree	2017	90% CI 2045 lower bound	Median 2045	90% CI 2045 upper bound
85	Female	5	0.0253	0.0198	0.0234	0.0274
86	Female	5	0.0289	0.0224	0.027	0.0322
87	Female	5	0.0317	0.025	0.0296	0.035
88	Female	5	0.0379	0.0295	0.0358	0.0428
89	Female	5	0.043	0.0341	0.0407	0.0481
90	Female	5	0.0492	0.0389	0.0469	0.0559
91	Female	5	0.0544	0.0432	0.0523	0.063
92	Female	5	0.0607	0.0473	0.0585	0.0713
93	Female	5	0.0686	0.0538	0.0669	0.0817
94+	Female	5	0.0939	0.0721	0.0927	0.1165

Appendix C

See Fig. 10.

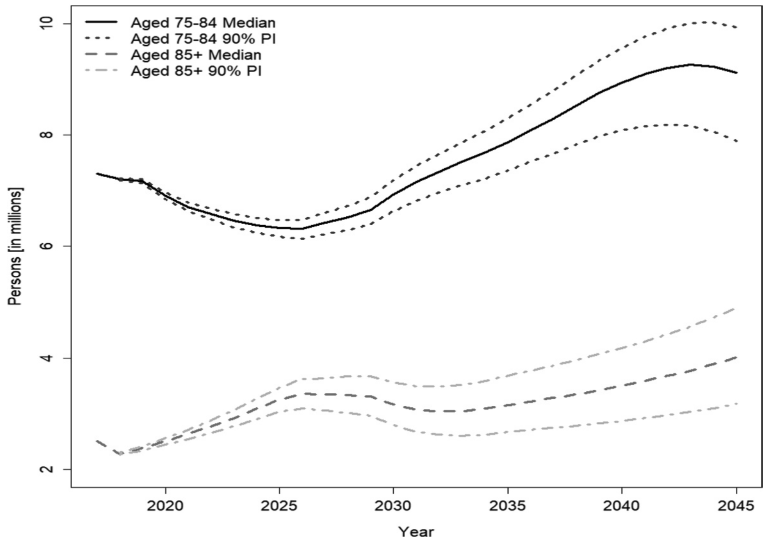

Fig. 10 Old-age population by 2045 by age group. *Sources*: Human Mortality Database (2019); own calculation and design

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