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Inverse Determination of Constitutive Equations and Cutting Force Modelling for Complex Tools Using Oxley's Predictive Machining Theory

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Abstract

In analysis of machining processes, finite element analysis is widely used to predict forces, stress distributions, temperatures and chip formation. However, constitutive models are not always available and simulation of cutting processes with complex tool geometries can lead to extensive computation time. This article presents an approach to determine constitutive parameters of the Johnson-Cook's flow stress model by inverse modelling as well as a methodology to predict process forces and temperatures for complex three-dimensional tools using Oxley's machining theory. In the first part of this study, an analytically based computer code combined with a particle swarm optimization (PSO) algorithm is used to identify constitutive models for 70MnVS4 and an aluminium-alloyed ultra-high-carbon steel (UHC-steel) from orthogonal milling experiments. In the second part, Oxley's predictive machining theory is coupled with a multi-dexel based material removal model. Contact zone information (width of cut, undeformed chip thickness, rake angle and cutting speed) are calculated for incremental segments on the cutting edge and used as input parameters for force and temperature calculations. Subsequently, process forces are predicted for machining using the inverse determined constitutive models and compared to actual force measurements. The suggested methodology has advantages regarding the computation time compared to finite element analyses.

1. Introduction

Modelling of mechanical and thermal loads during machining has become increasingly important in order to decrease the cost of experimental investigations for tool and process optimisation in metal cutting. Especially, finite element analyses are widely used for this task nowadays. Despite significant progress in the field of numerical modelling, 3-D simulation of more complex machining processes like external turning or milling leads to extensive computation time. In contrast to this, analytical models offer fast computation, but are often restricted to 2-D analysis. Another option are empirical models that are practical and easy to develop, but involve time-consuming and costly experiments. Additionally, the developed models are only valid for the analysed process.

According to Arrazola et al., hybrid modelling may offer a solution to overcome the drawbacks of individual approaches [1]. The combination of analytical and numerical or empirical and numerical modelling is expected to reduce the computation time considerably. However, numerical and analytical modelling require flow stress data of the work material as a function of strain, strain rate and temperature. Since chip formation is a result of a large strain ($\varphi > 1$), high strain rates ($\dot{\varphi} = 10^3 - 10^5$ s^{-1}) and high temperatures ($T = 200 - 1000$ °C), it is necessary to determine parameters for constitutive models under these severe conditions [2, 3]. Even high-speed impact compression tests are limited to strains $\varphi < 1$ and strain rates $\dot{\varphi} < 10^3$ s^{-1}. Moreover, these tests are relatively complicated and expensive [3, 4]. Aiming to obtain flow stress data at higher strain, strain rates and temperature gradients Oxley suggested

an inverse modelling by use of the machining process itself [5]. Further research has led to several implementations of the approach [3, 4, 6]. Still, the developed approaches are mostly compared to orthogonal cutting experiments of well-known work materials. Consequently, there is a need for a practical methodology that combines the advantages of inverse modelling of constitutive equations as well as hybrid modelling of machining operations. This article presents a methodology to determine constitutive parameters of the Johnson-Cook's (J-C) flow stress model by inverse modelling in combination with an approach to predict process forces and temperatures for complex three-dimensional tools using Oxley's machining theory. Subsequently, the methodology is validated for 70MnVS4 and a novel aluminium-alloyed UHC-steel [7].

Nomenclature

γ	rake angle
δ	thickness ratio at the interface
$\bar{\varepsilon}$	effective strain
$\dot{\bar{\varepsilon}}$	effective strain rate
θ_r	tool rotation angle
κ	tool cutting edge angle
λ	conductivity
ρ	density
σ	flow stress
ϕ	shear angle
A	material constant
A_5	failure strain
a_p	depth of cut
B	strain sensitivity parameter
C	strain rate sensitivity parameter
C_{AB}	shear strain constant
c_p	heat capacity
F_i	process forces
f	feed
f_z	feed per tooth
l	length of the shear plane AB
n	strain hardening parameter
m	thermal softening parameter
R_e	yield strength
R_m	tensile strength
Δs_1	thickness of the primary shear zone
Δs_2	thickness of the secondary shear zone
T	temperature
t_1	uncut chip thickness
t_2	chip thickness
v_c	cutting speed
w	width of cut
w_{F_i}	weighting factor

2. Analytical model of orthogonal cutting

Since the beginning of the last century, many researchers have developed and improved analytical models, including slip-line models, aiming to predict directly cutting forces and temperatures. The parallel-sided shear zone model for orthogonal machining by Oxley and his co-workers [5] shown in Fig. 1 can be considered as one of the most important contributions in the field of analytical cutting models.
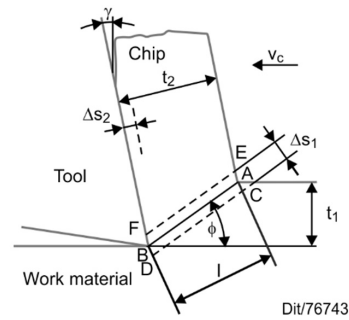

Fig. 1. Parallel-sided shear zone model [5]

The model allows the prediction of cutting forces, average temperatures and stresses based on input values for thermal properties, flow stress data as a function of strain, strain rate and temperature, tool geometry and cutting conditions. In contrast to the widely used model from Ernst and Merchant [9], Oxley considered a shear zone extending on both sides of the shear plane AB. Thus, strain hardening of the material during chip formation can be taken into account. Oxley assumed plane strain, uniform normal and shear stresses at the tool-chip interface and a perfectly sharp tool. In contrast to FEM, average temperatures and stresses are calculated instead of distributions. Moreover, the model considers only continuous chip formation.

In order to predict the shear angle and subsequently the process forces, two constants (C_{AB} and δ) are required. The constant C_{AB} relates the shear strain rate to the length of AB (l). The ratio of the plastic zone at the tool-chip interface (Δs_2) and the chip thickness (t_2) is represented by the constant δ. Flow charts depicting the algorithm can be found in the original work of Oxley [5] as well as in an article by Shatla et al. [3].

In the early stages of the theory the material flow stress was modelled by the power law equation. More recently, Oxley's predictive machining theory has been coupled with the J-C constitutive equation as shown in equation (1).

$$\sigma = \left(A + B\bar{\varepsilon}^n \right)\left(1 + C \cdot ln\frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}_0} \right)\left(1 - \left(\frac{T - T_0}{T_m - T_0} \right)^m \right) \qquad (1)$$

With A, B, C and m as material constants, $\dot{\bar{\varepsilon}}_0$ as reference strain rate, T_m as melting temperature and T_0 as initial workpiece temperature The strain-hardening is described by the constant n [6, 8]. Lalwani et al. [8] presented an extension of Oxley's predictive theory for the J-C material model by introducing a new constant n_{eq} (equivalent strain hardening exponent) which can derived from equation (2).

$$n_{eq} \approx \frac{n \cdot B \cdot \bar{\varepsilon}^n}{A + B \cdot \bar{\varepsilon}^n} \qquad (2)$$

Where $\bar{\varepsilon}$ is the effective strain at the shear plane AB. The extension permits calculating forces and temperatures based on J-C flow stress models. Moreover, it can be used to determine parameters of the J-C constitutive equations via an inverse modelling approach.

3. Inverse modelling of constitutive equations

The applied methodology for the inverse analysis of the constitutive equations consists of an experimental and a computational part (see Fig. 2).
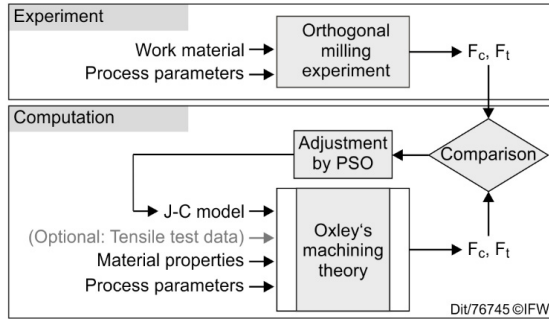

Fig. 2. Flow chart of the presented methodology for inverse modelling

The actual cutting and thrust forces are measured in an orthogonal slot milling experiment. In order to ensure a high quality of the inverse analysis it is crucial to satisfy several experimental conditions. Shatla et al. [3] described the conditions in detail and only a brief summary is given in the following. Since plane strain is assumed, the workpiece thickness has to be at least 10 times the maximum uncut chip thickness. Additionally, the cutting speed needs to be set high enough to avoid formation of a build-up edge and a continuous chip formation must be ensured.

Oxley's machining theory including the extensions given by Lalwani et al. [8] is programmed with Visual Basic .NET. Additionally, the program contains a subroutine that compares calculated and measured process forces and identifies suitable parameters of the J-C flow stress model. The optimisation of the parameters is based on a weighted least mean square fit described by equation (3).

$$Min \left\{ \sum \sqrt{\left[w_{F_c} \cdot \left(\frac{F_{c,sim} - F_{c,exp}}{F_{c,exp}} \right) \right]^2 + \left[w_{F_t} \cdot \left(\frac{F_{t,sim} - F_{c,exp}}{F_{c,exp}} \right) \right]^2} \right\} \qquad (3)$$

The minimization of the so-called cost function by adjusting the parameters A, B, C, n and m of the J-C flow stress model can be considered as a multi-dimensional optimization problem. In the present work, a particle swarm optimisation (PSO) similar to that described by Özel et al. [10] is used. The swarm's initial population evolves over several generations striving toward an optimum. Each particle of the swarm has a certain position and velocity within the problem space that is updated in every generation based on the best position of the particle and the best position of the entire swarm. The optimisation is stopped when the iteration number reaches a predetermined limit. Within the present study, a number of 50 iterations at a swarm size of 10 has been identified as sufficient with respect to the quality of the optimisation. Instead of optimising all five material constants of the J-C constitutive model, it is also possible to obtain the constants A, B and n from tensile tests and implement them into the algorithm.

4. Inverse determination of J-C constitutive equations

4.1. Experimental setup

The orthogonal milling experiments are carried out on a milling center (Heller MCi 16). As tools uncoated cemented carbide (WC-Co) cutting inserts are used. The inserts are mounted on a tool holder with a diameter of $d = 40$ mm. Aiming to avoid overlapping of force signals only one insert is used for cutting. Different rake angles ($\gamma = 3°$ and $13°$) are realized by grinding the rake face of the insert. Due to the assumption of sharp cutting tools in Oxley's machining theory, sharp cutting edges are used in this study only. The cutting speed and feed per tooth are varied in a range of $v_c = 30\ldots120$ m/min and $f_z = 0.05\ldots0.1$ mm, respectively. The experimental specifications of the experiment are summarized in table 1. Each experiment is repeated once. A preliminary test of the process parameters was conducted to ensure stable cutting conditions according to the assumptions made by Oxley (no build up edge, continuous chip formation).

Table 1: Experimental specification for orthogonal milling

Machining process	Orthogonal milling		
Cutting speed v_c [m/min]	30	75	120
Feed per tooth f_z [mm]	0.05	0.075	0.1
Rake angle γ [°]	3		13
Coolant	None		

The milling experiments are carried out with 70MnVS4 and an aluminium-alloyed UHC-steel as described by Denkena et al. [7]. In order to satisfy the plane strain assumption the width of cut (thickness of the sample) is set to $w = 6$ mm. Table 2 compares the mechanical and thermal properties of the machined materials.

Table 2: Comparison of mechanical and thermal properties [7, 11]

	70MnVS4	UHC-steel
Yield strength R_e [MPa]	700 – 750	730 – 760
Tensile strength R_m [MPa]	1,058	930 – 950
Failure strain A_5 [%]	9.5	5 – 7
Conductivity λ [W/(mK)]	52	14
Heat capacity c_p [J/(kgK)]	490	500
Density ρ [g/cm³]	7.8	6.9 – 7.2

The process forces are measured with a Kistler 9257B piezoelectric 3-component dynamometer. All force measurements are taken at a sampling rate of 25 kHz. Subsequently, the forces, which are measured within the coordinate system of the workpiece (F_x, F_y), are transferred into the coordinate system of the tool (F_c, F_t) and resolved for each tool rotation angle. Aiming to minimize experimental scatter 20 milling cycles are taken from each measurement and averaged automatically with a MATLAB tool. From this data, process forces at rotation angles of 30°, 60° and 90° are selected for the inverse modelling of the J-C constitutive model. The weighting factors of equation (3) are set to $w_{Fc} = w_{Ft} = 1$. In order to assess the influence of optional tensile test data on the quality of the calculated constitutive equation two modelling approaches are investigated. Firstly, all five material constants of the J-C constitutive equation are obtained from inverse modelling. Secondly, the parameters *A*, *B* and *n* are identified from curve fitting tensile test data.

4.2. Results

Table 3 summarizes the obtained parameters of the J-C flows stress model. Parameters derived from curve fitting of tensile test data are shaded. Due to the different constraints, the obtained parameters differ significantly.

Table 3: Inverse determined J-C constitutive equations

Material		*A*	*B*	*C*	*n*	*m*
70MnVS4	w/o tensile data	819.8	140.3	0.02	0.22	6.79
	w tensile data	741.6	709.1	0.03	0.22	0.84
UHC-steel	w/o tensile data	137.9	402.8	0.05	0.09	5.75
	w tensile data	752.3	282.3	0.04	0.17	2.14

In order to evaluate the quality of the calculated material model the deviations to the actual cutting forces ΔF_c while orthogonal milling are summarized in the box-plot depicted in Fig. 3. Every model is tested for the cutting forces of all 36 experiments at three different tool rotation angles (30°, 60° and 90°).
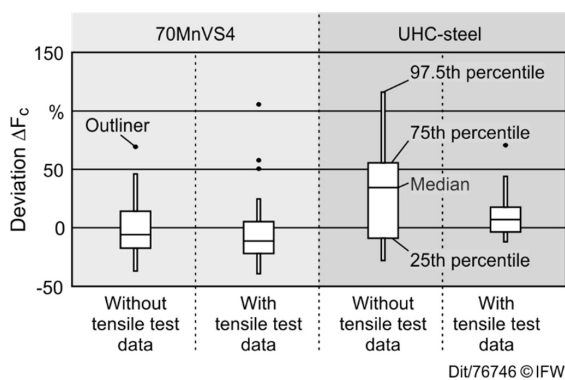
Fig. 3. Box-plot of the deviation between measured and simulated cutting forces in orthogonal milling using different modelling approaches

It can be seen that the modelling approach with additional tensile test data results in a lower variation and in case of the aluminium-alloyed UHC-steel in a significantly reduced median of the cutting force deviation ΔF_c. This can be explained by the reduced solution space in case of the modelling approach with additional tensile test information. With an increasing number of variables the optimisation becomes more difficult and the implemented algorithm can be trapped inside a local optimum more easily. Another advantage of the approach including tensile test data is to link the obtained constitutive model closer to actual material properties.

A comparison of the actual cutting forces and the cutting forces simulated with the modelling approach including optional tensile data is shown in Fig. 4. The simulation results are in good agreement with measured cutting forces. However, in case of the aluminium-alloyed UHC-steel the cutting force exhibits a slightly unsymmetrical behaviour which is not accounted for in the simulation.
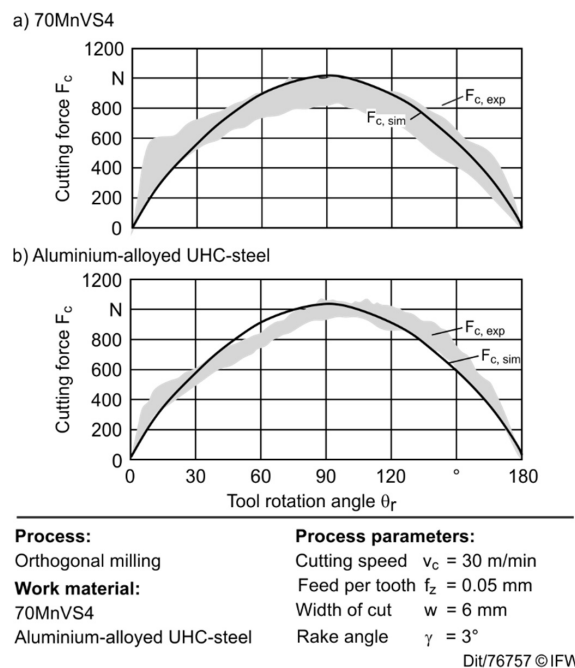
Fig. 4. Comparison of measured and simulated forces in orthogonal milling of a) 70MnVS4 and b) an aluminium-alloyed UHC-steel

5. Multi-Dexel approach for complex tool geometries

With respect to practical application, Oxley expanded his model to oblique cutting [5]. In order to expand Oxley's predictive machining theory to even more complex tool geometries like intricate turning or milling tools the analytical model has been coupled with a numerical approach. In the following, the developed approach is presented exemplary for external turning. However, the methodology can also be transferred to milling operations.

The material removal process and the contact zone analysis are based on a multi dexel numerical NC-Simulation-System called CutS [12]. At first, the tool, which described by a STEP-

file, is discretized along the cutting edge in equidistant areas (see Fig. 5). During the material removal simulation the contact zone is identified by the intersection of the modelled rake face and dexel elements. The detected dexel cut points (DCPs) are analysed afterwards. The DCPs are transformed in a cutting edge segment individual coordinate system (b; h) which is orthogonal to the cutting edge. The undeformed chip thickness for a single cutting edge element is defined as the maximum distance of a DCP orthogonal to the cutting edge. Thus, it is possible to calculate the undeformed chip thickness even for complex tool geometries. The processed information also allows a detailed analysis of the tool shape. More details on the cutting edge contact-zone analysis can be found in [13].
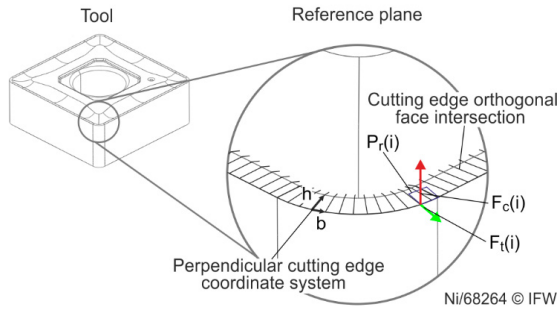

Fig. 5. Contact-zone analysis for complex tools

Besides the undeformed chip thickness, the rake angle, the cutting speed and the width of cut are derived for every cutting edge element and used as input parameters for Oxley's predictive machining theory. For the prediction of cutting forces, local cutting force coordinate systems of every discretization object along the cutting edge are used. The cutting force F_c is aligned with the direction of the cutting speed of a rake face segment. The cutting normal/tangential force F_t is orientated perpendicular to F_c and the cutting edge. Finally, the cutting and thrust forces of all cutting segments are integrated into the global process forces F_c, F_f and F_p.

Using the presented hybrid approach Oxley's predictive machining theory has been expended from orthogonal cutting to more complex tool geometries and processes. However, the presented approach contains some simplifications that are addressed in the following. Firstly, the approach does not consider chip flow. Thus, forces caused by friction between the workpiece and chips are not taken into account. Secondly, the presented approach assumes that each segment of the cutting edge can be regarded individually. Thirdly, since the approach is based on Oxley's machining theory, it implicitly adopts all assumptions made in the original theory including a sharp tool and steady state cutting conditions.

6. Inverse determination of J-C constitutive equations

6.1. Experimental setup

The external turning experiments are carried out on a Gildemeister MD10S lathe. Force measurements are conducted with a Kistler 9129A piezoelectric 3-component dynamometer. Multilayer coated (TiCN+Al$_2$O$_3$+(TiN)) cemented carbide inserts (SNMG 120408) are applied as cutting tools. The rake

angle was set to $\gamma = -6°$ and the cutting edge angle to $\kappa = 45°$. The cutting speed and feed rate are varied in the range of $v_c = 30...100$ m/min and $f = 0.2...0.4$ mm, respectively. The specifications of the experiment are shown in table 4. As work material 70MnVS4 and the aforementioned aluminium-alloyed UHC-steel are used.
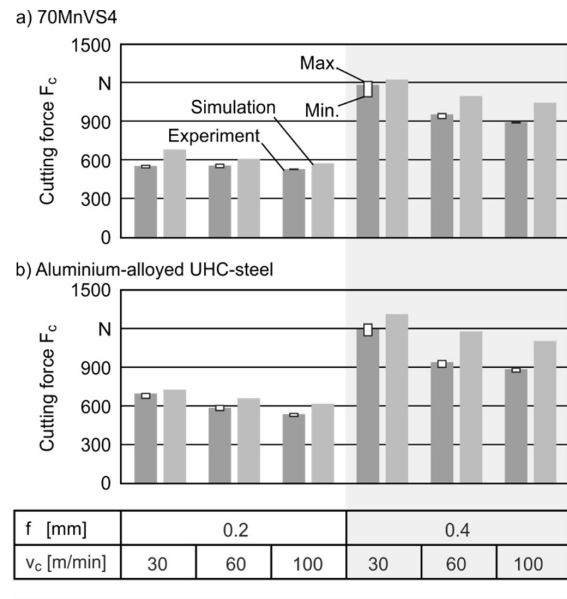
Table 4: Experimental specification for external turning

Machining process	External turning		
Cutting speed v_c [m/min]	30	60	100
Feed f [mm]	0.2		0.4
Coolant	None		

All external turning experiments are modelled with the multi-dexel approach presented in section 5. Based on the results shown in section 4.2, the J-C constitutive equations including tensile test data for 70MnVS4 and the alloyed UHC-steel are chosen for modelling.

6.2. Results

Figure 6 a) compares the simulated and measured cutting forces for turning 70MnVS4. The simulation results are in reasonable agreement with the actual force measurements. Effects from the cutting speed and feed on the cutting force are simulated correctly. It can be seen that rising cutting speeds result in lower cutting forces.

Fig. 6. Predicted and actual process forces for turning a) 70MnVS4 and b) an aluminium-alloyed UHC-steel

A higher feed increases the cutting force. The deviation between simulated and actual cutting force ranges between 4% and 24%. Generally, there is tendency to overestimate the cutting force. Despite an improvement compared to most FE-analyses the thrust force is still underestimated in most cases. In terms of the resultant force the deviation is calculated from -5% to 11%.

The results for the aluminium-alloyed UHC-steel are summarized in figure 6 b). Comparable to the prior shown results, the effects of cutting speed and feed on the process forces are predicted appropriately. The deviations of the cutting force are in a range between 4% and 26%. Similarly to turning 70MnVS4, the predicted cutting forces are overestimated especially in case of f = 0.4 mm. The resultant force deviation for a feed of f = 0.2 mm ranges from 1% to 7%. At a higher feed of f = 0.4 mm the failure increases up to 21%.

A comparison of the simulated temperatures at the tool-chip interface reveals that turning of UHC-steel leads to substantial higher temperatures ($\Delta T_{int} \approx$ 300 K) than turning of 70MnVS4. Thus, a higher thermal load is expected to act on the tool resulting in accelerated tool wear. These results are supported by observations presented in [7].

It can be concluded that the presented hybrid approach predicts process forces for machining operations with complex tool geometries properly, despite several assumptions made. The combination of a multi-dexel contact zone analysis and Oxley's predictive machining theory connects the prediction of process forces to actual material properties. The calculation of the shown external turning operation takes approximately 5 – 10 min, which represents a significant reduction compared to FE-analyses. The systematic deviations of the cutting and thrust force can be explained by the choice of the weighting factors in the preceding inverse determination of the J-C parameters. Since Oxley's predictive machining theory seems to underestimate the thrust force slightly, the algorithm compensates the systematic error by adjusting the constitutive equation. Thus, the deviation of the resultant force fluctuates around the actual resultant force, but the cutting forces are slightly over- and the thrust forces underestimated. The higher failure of prediction for a higher feed of f = 0.4 mm can be explained with the area of validity of the inverse determined constitutive equation, which can be enlarged by additional orthogonal milling experiments at higher feed rates.

7. Conclusion

An approach to determine constitutive parameters of the Johnson-Cook's flow stress model by inverse modelling as well as a methodology to predict process forces and temperatures for complex three-dimensional tools using Oxley's machining theory has been introduced. Flow stress models have been derived from orthogonal milling experiments and material data for 70MnVS4 and an aluminium-alloyed UHC-steel. It could be shown that a combination of the inverse determination with tensile test data improves the quality of the

identified constitutive equation. Subsequently, the derived J-C flow stress models are used for predicting process forces and temperatures in external turning via a hybrid modelling approach. It has been demonstrated that the predicted forces are in good agreement for turning 70MnVS4 and an alloyed UHC-steel. Due to the combination of a multi-dexel contact zone analysis and a fast analytical force model, the computation time has been reduced considerably compared to widely used FE-analyses. The presented methodology allows gaining fast and inexpensively information on the mechanical and thermal loads of the tool during machining even for novel materials. Further research will focus on minimization of the predictive failure and a validation of the presented methodology for milling operations.

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