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Investigation of the skive hobbing process by applying a dexel-based cutting simulation

V. Böß, B. Denkena, S. Henning*

Institute of Production Engineering and Machine Tools, Leibniz Universität Hannover, Germany

* Corresponding author. Tel.:+49 511 762 5751; fax: +49 511 762 5115. *E-mail address:* henning_s@ifw.uni-hannover.de

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Abstract

In this paper, a novel approach for cutting simulation of skive hobbing is presented. Skive hobbing is a process applied for finishing of already hardened gears. The process is characterized by varying tool engagements and very small chip thicknesses. The paper describes the process-modeling and the analysis of the workpiece-tool contact. For efficient modeling, a novel dexel-based method is presented for the description of a gear segment with discretized stock allowance. Characteristic of the method is an analytical description of the target workpiece contour which allows to describe points of the surface by Cartesian coordinates on the winding off. Dexel are oriented orthogonal to the surface of the final workpiece contour. The initial lengths of the dexel describe the stock allowance. By cutting these dexel with a tool, a time- and position-dependent prediction of material removal values, e.g. width of undeformed chip or undeformed chip thickness, is possible. Selected results are presented and interpreted.

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Keywords: skive hobbing; cutting simulation; dexel-based approach

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1. Introduction

The gear wheels in modern gear systems of plant and automotive engineering have to comply wide requirements with regard to noise and friction properties as well as transmittable power and lifetime. To reach sufficient high strengths of the tooth flanks, gear wheels often get hardened subsequent to the main material removal process. Afterwards, the removal of occurring distortions as well as the setting of the required surface qualities take place during the final processing, which is also called hard finishing. An economic process for hard finishing of gears with middle surface quality, like it is used in agricultural machinery, is the skive hobbing. In comparison to other hard finishing processes, skive hobbing is remarkable for two technological characteristics. Firstly, a negative tip rake angle has to be mentioned, whereby the removal of material occurs due to the so-called “husking cut”. On the other hand, only the tooth flanks are machined during skive hobbing and the area of the tooth base will be avoided [1,2]. Furthermore, skive hobbing is remarkable for very small removal of material, that must be big enough to remove the distortion due to heat treatment on the one hand, but is not allowed to exceed the hardening depth on the other hand, like every process of hard finishing [3].

Because of the material removal during skive hobbing, the residual stresses on the surface and in the surface layer of the tooth flanks are affected. Because this residual stress behavior has major influence on bearing capacity and fatigue strength of the gear [4,5], a prediction of the residual stresses, based on a material removal simulation provides high potential in process optimization.

During skive hobbing, an involute gear is produced by rolling the reference profile of the tool with the workpiece. Due to a corresponding relative movement the involute is generated [3,6]. Tool and workpiece resemble the involved bodies of a worm drive. The kinematics of the process is composed of a rolling of the skive hobbing tool and the workpiece as well as an axial feed movement during which material is removed.

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Therefore, the necessary relative motion between the tool and the workpiece is realized by two rotational and one translational axis (see Fig. 1).

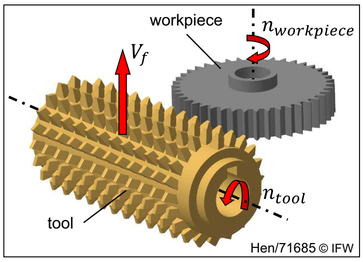


Fig. 1. Kinematics of skive hobbing process

Due to the complex primary motion during cutting no statements for the undeformed chip parameters and the cutting direction in a concrete position of the generated flank could be made so far [3]. Previous research work in mathematical modeling of tool-workpiece-systems and contact zone analysis in hard finishing of gears focus on gear grinding and gear honing. In the following section, a brief overview on research activities on the mentioned topic is given.

In [7] a simulation approach for analyzing the tool engagement in gear grinding and calculating the material removal rate is presented. The approach is based on a workpiece discretization by Cartesian Dexel models. The workpiece contour of the prefinished gear (tooth with stock allowance) is created from a simple cuboid by simulating material removal operations due to the relative movement between this cuboid (workpiece) and an appropriate tool model for prefinishing.

In [8] a simulation based approach for calculating the normal- and tangential forces in gear grinding is presented. Here, initially the calculation of the contact situation between tool and workpiece in form of cutting depth, cutting width and the cross-section of undeformed chip is done. Based on this information, an empirical process force model is derived.

In [9] a simulation of the contact conditions in gear grinding of beveliod gears is developed. It is based on a geometrical description of the tool and workpiece and the geometrical intersection of these objects. Subsequent simulations of the finished gear rolling with another gear are performed to determine contact conditions in operation.

[10] presents simulation-based studies in gear honing. Investigated are the effects of different gear diameters to the contact conditions and the actual cutting speeds on the tooth surface.

In [11] a comprehensive process model is developed to represent the gear honing. It consists of a workpiece model, a tool model and a machine model. The discretization of the workpiece contour is achieved by triangulation. The approach allows the calculation of the tool-workpiece contact, local cutting speeds and process forces.

The presented short review of previous research work shows that in the sector of hard finishing of gears, mainly the manufacturing processes gear grinding and gear honing are investigated in case of material removal behavior. Because of the varying process kinematics and tool engagement conditions, the mentioned results and scientific findings are not applicable for the process of skive hobbing.

Based on existing results of a current research project funded by the German Research Foundation, in this paper a simulation based approach is presented about the time-resolved and position-sensitive forecast of the removal of material during skive hobbing.

In the following chapters of this paper, an innovative approach in material removal simulation of skive hobbing is presented. On the one hand, the mathematical formulation of the mentioned manufacturing process is described. On the other hand, selected results from the contact zone analysis between workpiece and tool are presented. For efficient modeling, a novel dexel-based method for the mathematical representation of a gear segment with discretized stock allowance is developed. The idea of the method is an analytical description of the final workpiece contour (chapter 2) in combination with a discrete description of the stock allowance, realized by dexel that are oriented orthogonal to the final surface of the tooth (chapter 3). The method allows a time-resolved and position-sensitive prediction of material removal values like cutting width, cutting depth or material removal volume. Afterwards, selected results are presented and interpreted (chapter 4). Finally, a conclusion and an outlook on possible applications is given (chapter 5).

2. Process modeling

For calculating the process-specific material removal values, the skive hobbing machining process has to be modeled in an appropriate simulation environment. Therefore, the system “CutS”, specially developed at the Institute of Production Engineering and Machine Tools in Hannover, is used. CutS is a simulation environment, which can represent cutting operations between tool and workpiece by workpiece discretization [7,12,13]. The workpiece, the tool and the process kinematics are the essential process-specific elements to model. Furthermore, is it necessary to develop suitable analysis algorithms.

To create a flexible and universally valid model for hard finishing of gears, the target final contour of the workpiece is described analytically. Concerning this, the curve shape of a single tooth gap, consisting of entry side and outgoing side flank as well as tooth base and tooth crest is modeled firstly. In the simplest case, an involute tooth flank can be described according to DIN 3960 [14] by the following mathematic equation in Cartesian coordinates:

$$P(\varepsilon) = M * r_b * \begin{pmatrix} \cos(\eta+\varepsilon) + \varepsilon * sin(\eta+\varepsilon) \\ \sin(\eta+\varepsilon) - \varepsilon * cos(\eta+\varepsilon) \end{pmatrix} \qquad (1)$$

$$for\ \varepsilon_{min} \leq \varepsilon \leq \varepsilon_{max}$$

The function variable ε describes the off-winding of the involute from its generator circuit, here the base circle of the gearing with radius r_b (cp. Fig. 2 left hand side). The angle η is an offset angle, which shifts the starting point of the involute on its generator circuit. The involute of the gearing which has to be modeled is limited by its intersection with the root form circle r_{Ff} of the gearing (function angle ε_{min}) on the one hand and the intersection with the crown circle ε_{max} (function angle ε_{max}) on the other hand (cp. Fig. 2 right hand side).

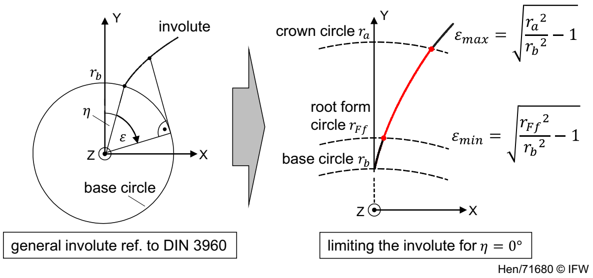


Fig. 2. Mathematical description of an involute tooth flank

For a complete mathematical description of a workpiece segment in case of a tooth gap, an idealized description of the tooth base and tooth crest is required furthermore (Fig. 3). In this context, the mentioned idealized descriptions are sufficient, because in skive hobbing only the tooth flanks are machined.

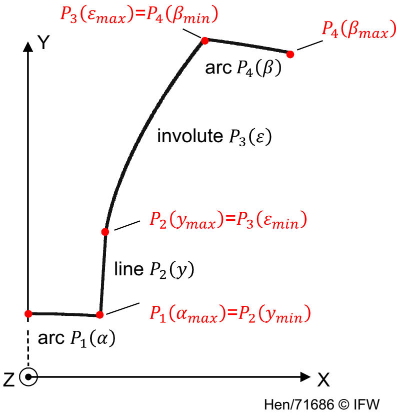


Fig. 3. Mathematical functions describing the final workpiece contour

The mathematical description of the tooth base consists of a circular arc $P_1(\alpha)$ as a function of a rotational coordinate α and of a line $P_2(y)$ as a function of a translational coordinate y. The subsequent tooth flank is described by the already explained (cp. equation 1) involute function (now called $P_3(\varepsilon)$). For transforming this involute in proper position and orientation (cp. Fig. 3) in the Cartesian coordinate system of the workpiece (origin on the rotation axis), an appropriate transformation matrix is necessary. Therefore, the involute segment shown in Fig. 2 is shifted with its start-point in the origin of the workpiece coordinate system (translational matrix $T_1(\varepsilon_{min})$), before it is rotated by the tooth gap-half-angle (α_{max}) of the modeled gear (rotational matrix $R(\alpha_{max})$). To complete the transformation, the rotated involute is shifted to the end-point of the idealized tooth base by the translational matrix $T_2(y_{max})$. The calculation of the resulting transformation is shown in equation 2.

$$M(y_{max}, \propto_{max}, \varepsilon_{min}) = T_2(y_{max}) \cdot R(\alpha_{max}) \cdot T_1(\varepsilon_{min}) \qquad (2)$$

The matrices (defined in homogenous coordinates) for computing the mentioned transformation are shown in equations 3 - 5.

$$T_1(\varepsilon_{min}) = \begin{pmatrix} 1 & 0 & -P_{3,x}(\varepsilon_{min}) \\ 0 & 1 & -P_{3,y}(\varepsilon_{min}) \\ 0 & 0 & 1 \end{pmatrix} \qquad (3)$$

$$R(\alpha_{max}) = \begin{pmatrix} cos(-\alpha_{max}) & -sin(-\alpha_{max}) & 0 \\ sin(-\alpha_{max}) & cos(-\alpha_{max}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (4)$$

$$T_2(y_{max}) = \begin{pmatrix} 1 & 0 & P_{2,x}(y_{max}) \\ 0 & 1 & P_{2,y}(y_{max}) \\ 0 & 0 & 1 \end{pmatrix} \qquad (5)$$

The idealized description of the tooth crest is realized by another arc segment $P_4(\beta)$ as function of a rotational coordinate β and with the radius of the crown circle of the modeled gear. The definition of the above mentioned mathematical functions $P_1(\alpha)$, $P_2(y)$ and $P_4(\beta)$ as well as the calculation of the characteristic coordinate values α_{max}, y_{min}, y_{max}, β_{min} and β_{max} (cp. Fig. 3) are not further explained in this paper. For completion of the tooth gap, the mentioned set of functions (representing a tooth half-gap) is mirrored with reference to the y-axis.

Based on this analytical description and by extruding the front surface along the rotation axis of the workpiece (z-axis), a CAD model is generated in the simulation system CutS. The surface of this geometric model illustrates the final target workpiece contour of the gear's tooth gap after the final skive hobbing operation.

The skive hobbing tool is modeled based on the reference profile in DIN 3972 [15] and other relevant technological tool parameter like diameter, axial slots, tool orthogonal rake angle and tool orthogonal clearance. This publication is not dealing with the model of the tool more detailed.

3. Discretization of the stock allowance

3.1. Modeling approach

The above described workpiece model has to be discretized with a suitable procedure to get a time-resolved and position dependent prediction of the material removal. This discretization is the basis for different geometric analysis based on the workpiece surface, such as the calculation of material removal volumes, depths of cut, undeformed chip thicknesses or widths of cut. The well-known Cartesian dexel models are unsuitable for this simulation because only a small stock allowance within the range of micrometers has to be removed during skive hobbing. Simultaneously, this small removal of material has to be described with high accuracy.

To achieve a suitable modeling accurateness, a new procedure is developed to arrange the dexel in defined distances orthogonal to the final surface of the work piece. This procedure maximizes the possible discretization accuracy in reference to the depth of cut.

Therefore, first the analytical description of the final workpiece contour is discretized by points. To reach an equidistant arrangement of this points on the final surface, the set of functions describing the final shape of the tooth gap are converted in unit speed curves.

For converting the mathematical functions to unit speed curves, the set of functions describing the whole tooth gap contour are winded off and the original function variables are replaced by a common variable t, representing the total length of the whole contour. Thus, altogether four section wise defined functions are developed that are all dependent on the same variable t. Because only the involute tooth flanks are machined during skive hobbing, the modeling of the stock allowance is also necessary only on this flanks. Due to this, in the following section only the parametrization of the function describing the involute ($P_3(\varepsilon)$) is explained.

For converting the involute function into a unit speed curve, its original variable ε (cp. equation 1) has to be replaced with a general variable that describes the position on the involute. Therefore, a suitable relation between the function angle ε and the corresponding winding off length must be formulated, see equation 6.

$$t = \frac{1}{2} * r_b * \varepsilon^2 \tag{6}$$

The offset resulting from the arc length of the tooth root (t_2) must be added to the involute length to make a continuous description of the tooth gap possible, according to the total arc length of the contour (beginning at the y-axis of the workpiece coordinate system, which is the symmetry axis of the tooth gap). Furthermore, the part of the involute which is not element of the tooth flank has to be considered when calculating the winding off length. The mentioned section is the part of the involute which is described by the function angle below the intersection point with the root form circle of the gear ($0 \leq \varepsilon \leq \varepsilon_{min}$). The arc length of this section is subtracted from the involute length that is forming the tooth flank contour. The corrected arc length is specified in equation 7.

$$t = t_2 + \frac{1}{2} * r_b * (\varepsilon^2 - {\varepsilon_{min}}^2) \tag{7}$$

Then the desired correlation follows from elementary transformations to substitute the function angle ε with the position t on the involute (equation 8).

$$\varepsilon = \sqrt{\frac{2 * (t + \frac{1}{2} * r_b * \varepsilon_{min}^2 - t_2)}{r_b}} \quad for \;\; t_2 \leq t \leq t_3 \tag{8}$$

The interval lower limit t_2 for the area of the involute flank arises from the sum out of the arc length from P_1 and P_2 (describing the tooth root, cp Fig. 3). The interval upper limit t_3 is the summation out of the interval lower limit t_2 and the arc length of the involute section formed from the tooth flank. The involute section is bounded by function angles ε_{min} and ε_{max} (cp. Fig. 2).

Attending this developed correlation as well as the general involute function (equation 1) and the transformation rule (equation 2), points along the contour line of the tooth flank can be described position-resolved in the Cartesian workpiece coordinate system in dependency on the arc length (variable t) (Fig. 4 left hand side). The position along the flank line can be described directly by the position on the z-axis in the workpiece coordinate system (variable z) in case of a spur bevel gear.

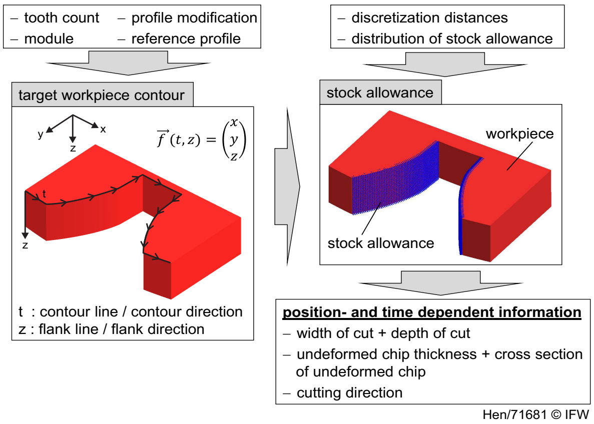


Fig. 4. Parametrical workpiece model and discretized stock allowance

It is necessary to calculate the surface orthogonal direction vector in every discretized point ($P_3(t,z)$) to arrange the dexel on the tooth flank. Therefore, the calculation of the direction vectors along the contour line after equation 9 has to be carried out first.

$$\vec{p}(t,z) = \frac{dP(t,z)}{dt} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ 0 \end{pmatrix} \tag{9}$$

The cross products of this vectors and the unit vector of the z-axis equates to the surface orthogonal direction vectors (equation 10). These vectors determine the orientation of the dexel (cp. Fig. 5).

$$\vec{n}(t,z) = \frac{\vec{p}(t,z) \times \vec{e_z}}{|\vec{p}(t,z) \times \vec{e_z}|} \tag{10}$$

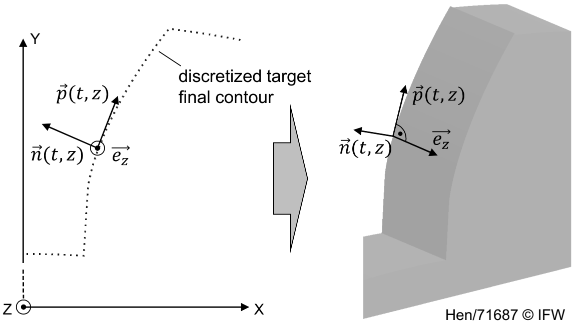
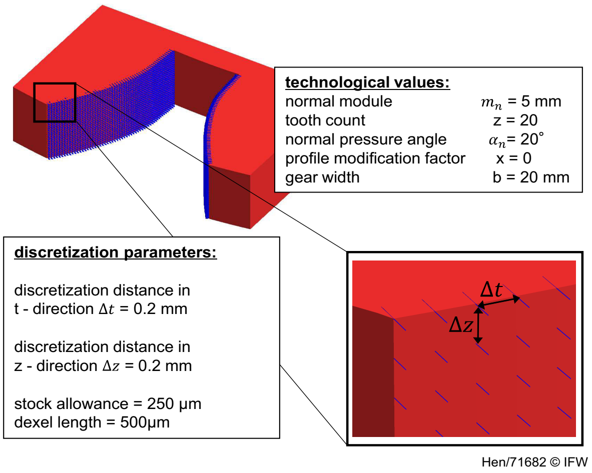


Fig. 5. Direction vectors of the discretized stock allowance

Using this information consisting of the surface point in the Cartesian coordinate system as dexel starting point and the associated direction vector for the stock allowance, dexel with specified lengths could be generated to describe the allowance (cp. Fig. 4 right hand side). In addition, a definition of the length in negative direction along the direction vector is also possible. This definition in negative direction is reasonable to represent a possible undersize in the machined final workpiece surface, which can arise by reason of process errors.

3.2. Application for illustrating the method

Fig. 6 shows a tooth gap which is discretized with the described vector field model. The technological values of the tooth gap are specified in the figure. The length of the winding off along the contour line is 36.4 mm in this case. Together with the gear width of 20 mm a tooth gap surface of 728 mm² results (36.4 mm x 20 mm). The gear width corresponds to the length of the flank line in the shown spur bevel gear.



technological values:
normal module m_n = 5 mm
tooth count z = 20
normal pressure angle α_n= 20°
profile modification factor x = 0
gear width b = 20 mm

discretization parameters:

discretization distance in t - direction Δt = 0.2 mm

discretization distance in z - direction Δz = 0.2 mm

stock allowance = 250 µm
dexel length = 500µm

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Fig. 6. Modeling the stock allowance by surface-orthogonal dexel

The surface of the tooth is discretized along the contour line (surface coordinate t) and along the flank line (surface coordinate z) with a step size of 0.2 mm in this example. The calculation with equation 11 results in a total number of 18200 dexel.

$$Dexelcount = \frac{t_{max}}{\Delta t} * \frac{z_{max}}{\Delta z} \tag{11}$$

A constant stock allowance of 250 µm was chosen across the entire tooth gap. This was achieved with an offset of 250 µm in negative direction of the surface normal with a dexel length of 500 µm.

The above described dexel based vector field model is able to quantify material removal at the tooth flanks, position- and time-resolved. Due to the orthogonal dexel orientation in relation to the final workpiece surface, cutting operations during process simulation can be represented with highest accuracy.

4. Results

The capability of the developed approach will be presented with a case study below. The machining of a spur bevel gear without profile correction will be simulated. The technological values of the gear and the discretization parameters of the corresponding simulation model are the ones that were described in the example in the previous section (cp. Fig. 6). The modeled constant stock allowance of 250 µm is standard practice for such a gearing in finish cutting [1].

A skive hobbing tool with the corresponding module with an effective reference profile No.1 after DIN 3972 is used. The tool has an external diameter of 80 mm, 10 axial slots and a tip rake angle of -20°. The minor number of only 10 axial slots (resulting from 10 teeth per tool revolution) for this size of a milling cutter is chosen because in this case only one cutting edge is engaged in the tooth gap at the same time. This makes it easier to interpret the results presented here.

A cutting speed of 80 m/min and an axial feed of 1 mm per workpiece revolution at parallel feed are chosen as process parameters. The skive hobbing tool is positioned along the feed axis centric to the gear.

Fig. 7 shows the material removal in case of a full section cut after one cycle of the tooth gap through every position in the joint rolling motion. This material removal results from the tool's feed motion during the sum of every position in the joint rolling motion. The figure presents the winded-off surface of the tooth gap with the position resolved distribution of the total material removal. Therefore, the amount of length reduction of every dexel during the whole simulation (which means the cycle of every position with cutting operations of one workpiece revolution) is displayed.

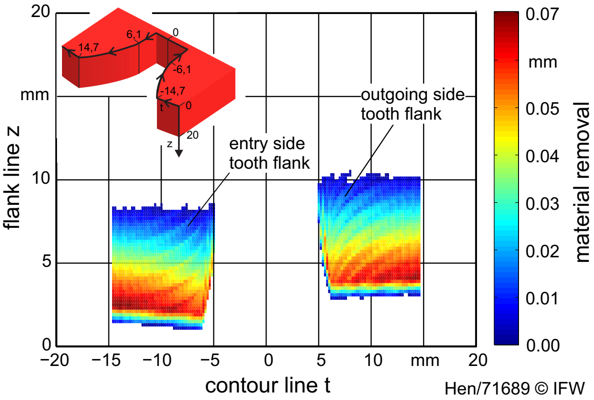

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Fig. 7. Material removal by tool's feed motion

The offset in flank direction between entry side tooth flank (left) and outgoing side tooth flank (right) of the machined tooth gap results from the helix angle of the skive hobbing tool to compensate the pitch of the envelope screw of the tool and the resulting overturning of the single cutting edges. The form of evaluation in Fig. 7 is practicable for every single simulation step. Consequently, it is possible, for instance, to analyze the current material removal at a specified position. It is also possible to display the whole material removal of a specific single cutting edge of the tool.

To calculate the cross-sections of undeformed chips, a special algorithm has been developed for the presented dexel model. This algorithm relies on the assumption that the vector describing the width of cut of every single cutting edge of the tool (contemplated in the dexel field of the winded off tooth gap surface) is approximately parallel to the t-coordinate of the winded off workpiece surface at any time. This requirement is given if the diameter of the milling tool is sufficiently large.

For each simulation step, every cut dexel is identified. The amount of reduction of all these identified dexel are written in a matrix. The column indices correspond to the t-coordinates of the tool surface. The row index corresponds to the z-coordinate of the surface. Afterwards, the maxima of the columns are ascertained out of this matrix. An increment of the cross-section of undeformed chip is calculated for each of these maximal values. This is done by multiplication of the amount of reduction with the distance to the neighbor dexel in contour direction (Δt, cp. Fig. 6). The summation of all this increments gives the cross-section of undeformed chip for the contemplated simulation step.

The cross-sections of undeformed chips of the entry side tooth flank and the outgoing side flank are displayed over the process time in Fig. 8. It can be seen that both – the entry side flank and the outgoing side flank – in the contemplated process are generated of altogether 19 single cutting edge engagements. The machining of the entry side flank takes place from the tooth crest to the tooth root. The cross-section of undeformed chip is affected by a high increase during the course of the first three engagements of cutting edges and a following decreasing characteristic interrupted by a second peak close to the end of machining the entry side flank. In contrast to this, the outgoing side flank is machined in the direction from tooth root to tooth crest. The time dependent characteristics of undeformed chip is therefore mirrored compared to the entry side flank.

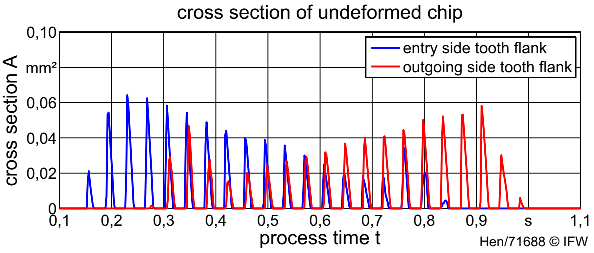

Fig. 8. Cross-sections of undeformed chips at the tooth flanks

5. Conclusion and Practical Applicability

The novel approach presented in this paper allows a particularized contemplation and analysis of the geometric correlations during skive hobbing. Therefore, the final workpiece contour was described analytically and discretized in defined distances along the contour line and the flank line. The description of the stock allowances with orthogonal dexel occurs in this discretized points. By modeling the skive hobbing process kinematics, it is possible to analyze the material removal during machining. Selected results of sample processes were shown.

Based on the perceptions of the presented approach for example process force calculations or simulation-based forecasts of residual stresses at the tooth flank surface can be executed. It is also possible to determine the effects of process anomalies – e.g. the deficient centering of the tool-workpiece-combination – on different geometric material removal values.

A concrete practical applicability of the presented approach is the combination of simulation of process anomalies with an appropriate residual stress modeling. Hereby, the effects of different process anomalies on the residual stress state, caused by the machining process, can be predicted. So, it is possible to define tolerable centering- and radial runout errors for a particular skive hobbing process within the residual stresses on the tooth flank surface are not significantly affected, and therefore the durability and life expectancy of the gears produced is not affected. Thus, the presented approach provides input for the usage in process design and optimization as well as for the usage in tool development.

Acknowledgements

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