

## New actions for modified gravity and supergravity

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**ABSTRACT:** We extend the  $f(R)$  gravity action by including a generic dependence upon the Weyl tensor, and further generalize it to supergravity by using the super-curvature ( $\mathcal{R}$ ) and super-Weyl ( $\mathcal{W}$ ) chiral superfields in  $N = 1$  chiral curved superspace. We argue that our (super)gravitational actions are the meaningful extensions of the phenomenological  $f(R)$  gravity and its locally supersymmetric generalization towards their UV completion and their embedding into superstring theories. The proposed actions can be used for study of cosmological perturbations and gravitational instabilities due to a nonvanishing Weyl tensor in gravity and supergravity.

**KEYWORDS:** Classical Theories of Gravity, Supergravity Models, Superspaces, Cosmology of Theories beyond the SM

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**1 Introduction**

The  $f(R)$  gravity theories, whose Lagrangian is given by the function  $f$  of the spacetime scalar curvature  $R$ ,

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} f(R), \tag{1.1}$$

are the particular class of modified gravity theories which can provide the *geometrical* description of inflation in the early universe and acceleration of the present universe due to gravity alone — see e.g., refs. [1, 2] for a review — in agreement with all known observations. The  $f(R)$  gravity is known to be classically equivalent to the scalar-tensor gravity [3–6], so that in the context of inflation or dark energy it amounts to quintessence. The “fifth force” at present due to exchange of the extra scalar (dubbed *scalaron* in the context of  $f(R)$  gravity) can be effectively *screened* on local scales (like the Solar system) but can allow the enhancement of gravity on cosmological scales due to the so-called *chameleon* effect [7, 8]. Gravitational instabilities in  $f(R)$  gravity can also be avoided by demanding the proper signs of the first and second derivatives of the function  $f$ , thus making it free of ghosts and tachyons [2]. The coupling of  $f(R)$  gravity to matter fields after a transformation to the Einstein frame gives rise to the couplings of inflaton (scalaron) to all matter fields and thus leads to the *universal* reheating after inflation in the early universe [9–11]. All the successes of the  $f(R)$  gravity theory are related to the FLRW backgrounds.

The  $f(R)$  gravity models currently have the *phenomenological* status, i.e. they are *not* (yet) derivable from any fundamental theory of gravity (like superstrings). Because of that any  $f(R)$  gravity model needs *fine-tuning* of its parameters, in order to meet observations. Moreover, the  $f(R)$  gravity is *neither* UV-complete *nor* renormalizable. The renormalizability can be restored by adding the higher-curvature terms containing the Weyl tensor,

like e.g., the conformal gravity term proportional to the Weyl tensor squared [12, 13]. Yet another way to improve the status of  $f(R)$  gravity is to find its embedding into the fundamental framework of superstring theory. It should be mentioned that the Weyl-tensor-dependent terms are known to appear in the (perturbative) superstring gravitational effective action indeed [14]. Hence, at the best, the  $f(R)$  gravity may be considered as merely *part* of the gravitational effective action which is presumably derivable from a fundamental theory of quantum gravity (like superstrings). The  $f(R)$  gravity part is responsible for the evolution of the scale factor in the FLRW metric of the universe, however, it is *not* enough for treating gravitational (tensor) perturbations. For example, the  $f(R)$  gravity-based models of dark energy can only be distinguished from the standard ( $\Lambda$ CDM) Cosmological Model by studying cosmological perturbations [15]. Our interpretation makes it clear that the full gravitational action should have other terms beyond the  $f(R)$  action.

From this perspective it is natural to extend  $f(R)$  gravity action (1.1) to a more general one, namely,

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} f(R, C), \tag{1.2}$$

having a generic dependence upon the spacetime Weyl tensor  $C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{2}(g_{\mu\rho}R_{\nu\sigma} - g_{\nu\rho}R_{\mu\sigma} - g_{\mu\sigma}R_{\nu\rho} + g_{\nu\sigma}R_{\mu\rho}) + \frac{1}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})R$  also. Since the indices of the Weyl tensor have to be contracted in the Lagrangian, the  $C$ -dependence is actually given by

$$\begin{aligned} f(R, C) &= f(R, C^2, C^3, C^4, \dots) \\ &= f_0(R) + f_2(R)C^2 + f_3(R)C^3 + f_4(R)C^4 + \mathcal{O}(C^5) \end{aligned} \tag{1.3}$$

where the  $C^n$  denote the scalar products of the Weyl tensor, and the dots may also include the contracted covariant derivatives of  $R$  and  $C$  as the additional arguments of the  $f$ -function. In the case of  $f(R)$  gravity, adding the covariant derivatives of  $R$  leads to a classically equivalent scalar-tensor gravity with *more* scalars [16, 17]. In what follows we ignore the terms with the covariant derivatives of  $R$  and  $C$  for simplicity. The FLRW background has  $C_{\mu\nu\rho\sigma}^{\text{FLRW}} = 0$  so that an arbitrary  $C$ -dependence in the action (1.2) does *not* affect the Friedman equation for the FLRW metric, and hence, keeps the cosmological achievements of  $f(R)$  gravity. But, for example, the Schwarzschild solution and the black hole physics will be modified [18].

When compared to a generic gravitational action, our action (1.2) is distinguished by the absence of manifest dependence upon the Ricci curvature tensor. At the *quadratic* level with respect to the curvatures its only possible contribution, which is proportional to the Ricci tensor squared, can always be eliminated via the Gauss-Bonnet (topological) combination in favor of the  $C^2$  term. A generic dependence of the gravitational action upon the Ricci tensor can lead to the extra propagating massless spin-2 mode [19, 20].

Our action (1.2) can also be considered as the *alternative* to the popular  $f(R, G)$  gravity where  $G$  is the Gauss-Bonnet combination,  $G = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{2}{3}R^2$ . The  $G$ -combination is a total derivative in four dimensions, so that the linear term in  $G$  does not affect the equations of motion, thus leading to a ghost-free  $f(R, G)$  theory. The spectrum of the linearized  $(R + C^2)$  action has a massive spin-2 ghost particle in addition to a massless

graviton [12, 13]. Presumably, this ghost violates unitarity in a quantized  $(R + C^2)$  field theory.<sup>1</sup> The unitarity issue is crucial for a fundamental theory of gravity, but does not arise when treating the  $C^2$  term as a perturbation in the action. It may also be possible that the conformal gravity ghost is an artifact of the truncation of some highly non-linear (with respect to the curvature) action to a four-derivative action. See ref. [23] for the possible ghost-free completion of the conformal gravity by the partially massless bimetric gravity.

However, our main reasoning for the absence of the manifest Ricci tensor dependence in the gravitational effective action is supersymmetry. We are going to demonstrate that our action (1.2) allows a locally  $N = 1$  supersymmetric extension as a *chiral* supersymmetric invariant in curved superspace. Indeed, if such an action is to arise from superstrings, it must be in a supersymmetric context, while the chirality of the gravitational effective action would guarantee stability of its cosmological solutions against the higher-order quantum corrections due to the well known non-renormalization theorems in supersymmetry and supergravity [24–27]. As is well known in superspace supergravity [24–26, 28], the relevant superfield containing the Ricci tensor as one of its field component in the superfield is *not* chiral, whereas the supergravity superfields containing the  $R$  and  $C$  tensors *are* chiral (see section 3 below for more details).

A supersymmetrization of eq. (1.2) can also be considered as a supersymmetric generalization of the  $F(\mathcal{R})$  supergravity action [29] that is the manifestly  $N = 1$  supersymmetric extension of the  $f(R)$  gravity action in  $N = 1$  chiral curved superspace,

$$S_F = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{H.c.}, \tag{1.4}$$

in terms of the analytic function  $F(\mathcal{R})$ .<sup>2</sup> Besides having the manifest local  $N = 1$  supersymmetry, the action (1.4) has the so-called *auxiliary freedom* [31, 32] because the auxiliary fields do *not* propagate in this theory. It distinguishes the action (1.4) from other possible supersymmetric extensions of eq. (1.1). A calculation of the real function  $f(R)$  in eq. (1.1) from a given holomorphic function  $F(\mathcal{R})$  in eq. (1.4) requires solving an algebraic equation of motion for the auxiliary field  $M$ . It is the non-trivial task in general, unlike the usual supergravity whose dependence upon the auxiliary fields is always Gaussian. The component structure of the bosonic sector of  $F(\mathcal{R})$  supergravity was systematically investigated in refs. [33–37] on the simplest examples.

Some physical applications of the  $F(\mathcal{R})$  supergravity theory to the early universe cosmology, inflation and reheating were systematically studied in refs. [38–41]. In particular, a successful embedding of the chaotic slow roll (Starobinsky) inflation into the  $F(\mathcal{R})$  supergravity is based on the following *Ansatz* [38]:

$$F(\mathcal{R}) = -\frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 - \frac{1}{6}f_3\mathcal{R}^3 \tag{1.5}$$

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<sup>1</sup>See, however, refs. [21, 22] challenging the standard lore about non-unitarity of conformal gravity.

<sup>2</sup>The field construction of the  $F(\mathcal{R})$  supergravity theory by using the  $N = 1$  superconformal tensor calculus was given in ref. [30].

whose coefficients are given by

$$f_1 = \frac{3}{2}M_{\text{Pl}}^2, \quad f_2 = \sqrt{\frac{63}{8}} \frac{M_{\text{Pl}}^2}{m}, \quad \text{and} \quad f_3 = \frac{15M_{\text{Pl}}^2}{M^2} \quad (1.6)$$

in terms of the scalaron masses:  $M$  in the high curvature regime and  $m$  in the low curvature regime, respectively [38, 40]. We have temporarily restored the Planck mass dependence here, in order to show the (mass) dimensions of the  $f$ -coefficients. A possible connection between the  $F(\mathcal{R})$  supergravity and the *Loop Quantum Gravity* was investigated in ref. [42].

In this paper we generalize the  $F(\mathcal{R})$  supergravity to a more general theory whose bosonic sector includes an  $f(R, C)$  gravity action (1.2).

Our paper is organized as follows. In section 2 we rewrite the bosonic action (1.2) to the Einstein frame where the  $R$ -dependence is reduced to the standard Einstein-Hilbert term, in the presence of the propagating scalaron and the Weyl tensor. In section 3 we construct a new manifestly supersymmetric extension of the bosonic action (1.2) by using curved superspace of the (old) minimal superspace supergravity. Section 4 is devoted to rewriting our new supergravity action to the more conventional form, in terms of the Kähler potential and the “superpotential”. In section 5 we derive the bosonic part of the simplest non-trivial model in our new family of modified supergravity theories. We discuss the possible origin of our new supergravity actions in section 6. We use the natural units  $c = \hbar = M_{\text{Pl}} = 1$  where  $M_{\text{Pl}}$  is the reduced Planck mass, and the  $(1 + 3)$ -dimensional space-time signature  $(+, -, -, -)$ .

## 2 $f(R, C)$ gravity in Einstein frame

The action (1.2) is the extension of (1.1) with an extra dependence upon the Weyl tensor. Hence, as long as the Weyl tensor vanishes, all the results of  $f(R)$  gravity can be reproduced. For instance, the vacuum solutions in both theories with  $R = R_0$  satisfy the equation

$$R_0 f'(R_0) = 2f(R_0). \quad (2.1)$$

The generalized action (1.2) can be transformed to the Einstein frame, like the  $f(R)$  gravity action (1.1). Let us rewrite the action (1.2) to the form

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} [f'(\phi, C)(R - \phi) + f(\phi, C)] \quad (2.2)$$

where the new scalar field  $\phi$  has been introduced. The primes denote the derivatives with respect to the *first* argument. On the one side, the equation of motion for the new scalar is algebraic,

$$f''(\phi, C)(R - \phi) = 0. \quad (2.3)$$

Assuming that  $f'' \neq 0$ , we get  $\phi = R$  and, hence, recover the original action (1.1) back.

On the other side, let us define a new metric

$$\tilde{g}_{\mu\nu} = f'(\phi, C)g_{\mu\nu} \quad (2.4)$$

in the action (2.2), where the scalar function  $f'$  is given by

$$f'(\phi, C) = f'(\phi, 0) + \left. \frac{df'}{d(C^2)} \right|_{C=0} C^2 + \mathcal{O}(C^4). \quad (2.5)$$

Though eq. (2.4) is not a standard Weyl transformation because the Weyl tensor  $C = C(g)$  is metric-dependent, it can still be considered as the (non-canonical) local field redefinition of the metric, under which the Weyl tensor transforms *covariantly*,

$$\tilde{C}_{\mu\nu\rho\sigma} \equiv C_{\mu\nu\rho\sigma}(\tilde{g}) = f'(\phi, C) C_{\mu\nu\rho\sigma}(g). \quad (2.6)$$

As a result, the action (2.2) takes the form

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} \tilde{R} + \frac{3}{4(f')^2} \tilde{g}^{\mu\nu} \partial_\mu f' \partial_\nu f' - V(\phi, C) \right] \quad (2.7)$$

where we have introduced the scalar function

$$V(\phi, C) = \frac{f(\phi, C) - \phi f'(\phi, C)}{2f'(\phi, C)^2}. \quad (2.8)$$

The new metric  $\tilde{g}$  can be considered as the metric in the Einstein frame. After the scalar field redefinition

$$\sigma = \sqrt{\frac{3}{2}} \ln f'(\phi, C) \quad \text{or} \quad f'(\phi, C) = \exp \left[ \sqrt{\frac{2}{3}} \sigma \right], \quad (2.9)$$

the scalar kinetic term in the action (2.7) takes the *canonical* form, and the action itself in terms of the new fields  $\sigma$  and  $\tilde{g}_{\mu\nu}$  reads

$$S[\sigma, \tilde{g}] = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} \tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma, \tilde{C}) \right] \quad (2.10)$$

with the scalar function

$$V(\sigma, \tilde{C}) = \frac{1}{2} e^{-2\sqrt{2/3}\sigma} f(\phi(\sigma, \tilde{C}), e^{-\sqrt{2/3}\sigma} \tilde{C}) - \frac{1}{2} e^{-\sqrt{2/3}\sigma} \phi(\sigma, \tilde{C}) \quad (2.11)$$

where  $\phi(\sigma, \tilde{C})$  is the solution to the algebraic equation (2.9).

As a non-trivial simple example, let us consider the following action:

$$f(R, C) = R - \frac{R^2}{6M^2} - bRC_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \quad (2.12)$$

with the real parameter  $b$ . We find

$$f'(\phi, C) = 1 - \frac{\phi}{3M^2} - bC^2 = e^{\sqrt{2/3}\sigma}, \quad (2.13)$$

which can be easily solved for

$$\phi = 3M^2 \left[ 1 - e^{\sqrt{2/3}\sigma} - be^2 \sqrt{2/3} \tilde{C}^2 \right]. \quad (2.14)$$

Hence, the transformed action in the Einstein frame takes the form

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma, \tilde{C}^2) \right] \quad (2.15)$$

with the scalar function

$$V(\sigma, \tilde{C}^2) = \frac{3}{4}M^2 \left( 1 - e^{-\sqrt{2/3}\sigma} + be^{\sqrt{2/3}\sigma}\tilde{C}^2 \right)^2. \quad (2.16)$$

The inflaton scalar potential  $V(\sigma)$  at  $C = \tilde{C} = 0$  is known to be quite suitable for slow roll inflation at large (positive)  $\sigma$  [43, 44]. However, a nonvanishing Weyl tensor in eq. (2.16) may destabilize the slow-roll even at a small value of the parameter  $b$  due to the large exponential factor in front of the  $\tilde{C}^2$  term.

As is clear from our derivation, the classically equivalent actions (1.2) and (2.10) are *not* equivalent in quantum theory because they are related via the non-trivial field redefinition which results in the non-trivial field-dependent Jacobian in the path integral.

Due to the presence of the  $\tilde{C}^2$  term in the action (2.15), this quantum gravity theory is formally *renormalizable* but has *ghosts* in Minkowski background [12, 13]. In the context of a fundamental theory of quantum gravity any presence of ghosts is unacceptable [45] so that the  $C^2$  term may not be allowed. However, in the perturbative framework, when the gravity spectrum is determined by the leading (Einstein-Hilbert) action while all the other higher-derivative terms are considered as the interaction, the presence of the  $\tilde{C}^2$  term is not a problem. For *linear* gravitational perturbations around the FLRW background, the whole  $C$ -dependence in the action (1.2) is irrelevant.

### 3 $F(\mathcal{R}, \mathcal{W})$ supergravity in superspace

In this section we demonstrate that our action (1.2) has a simple chiral locally  $N = 1$  supersymmetric extension in four spacetime dimensions. For that purpose we use the chiral version of the curved superspace in the (old) minimal formulation of  $N = 1$  supergravity [24–26]. The curved superspace is the most powerful, concise and straightforward method of constructing general couplings in supergravity, in the manifestly supersymmetric way. We use the notation of ref. [2] and briefly comment on its relation to the more standard notation of ref. [25] in section 5.

To reduce the off-shell field contents of superfield supergravity to the minimal set, one imposes certain off-shell constraints on the supertorsion tensor in curved superspace [24–26]. An off-shell supergravity multiplet has the auxiliary fields of noncanonical (mass) dimension, in addition to the physical spin-2 field (graviton)  $e_\mu^a$  and spin-3/2 field (gravitino)  $\psi_\mu$ . In the old minimal setting the auxiliary fields (in a WZ-type gauge) are given by a complex scalar  $M$  and a real vector  $b_\mu$ . It is worth mentioning that imposing the off-shell constraints is independent upon writing a supergravity action.

The chiral superspace density reads

$$\mathcal{E}(x, \theta) = e \left[ 1 + i\theta\sigma^a\bar{\psi}_a - \theta^2 \left( M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b \right) \right] \quad (3.1)$$

where  $e = \sqrt{-\det g}$ ,  $\psi_a^\alpha = e_a^\mu \psi_\mu^\alpha$  is chiral gravitino,  $M = S + iP$  is the complex scalar auxiliary field. We use the lower case middle Greek letters  $\mu, \nu, \dots = 0, 1, 2, 3$  for curved spacetime vector indices, the lower case early Latin letters  $a, b, \dots = 0, 1, 2, 3$  for flat (target) space vector indices, and the lower case early Greek letters  $\alpha, \beta, \dots = 1, 2$  for chiral spinor indices.

A solution to the superspace Bianchi identities together with the constraints defining the  $N = 1$  Poincaré-type minimal supergravity theory reduce all the super-curvature and super-torsion tensor superfields to only *three* covariant tensor superfields,  $\mathcal{R}$ ,  $\mathcal{G}_a$  and  $\mathcal{W}_{\alpha\beta\gamma}$ , subject to the off-shell relations [24–26]:

$$\mathcal{G}_a = \bar{\mathcal{G}}_a, \quad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)}, \quad \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{R} = \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{W}_{\alpha\beta\gamma} = 0, \quad (3.2)$$

and

$$\bar{\mathcal{D}}^{\dot{\alpha}} \mathcal{G}_{\alpha\dot{\alpha}} = \mathcal{D}_\alpha \mathcal{R}, \quad \mathcal{D}^\gamma \mathcal{W}_{\alpha\beta\gamma} = \frac{i}{2} \mathcal{D}_\alpha{}^{\dot{\alpha}} \mathcal{G}_{\beta\dot{\alpha}} + \frac{i}{2} \mathcal{D}_\beta{}^{\dot{\alpha}} \mathcal{G}_{\alpha\dot{\alpha}}, \quad (3.3)$$

where  $(\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_{\alpha\dot{\alpha}})$  stand for the curved superspace  $N = 1$  supercovariant derivatives, and the bars denote Hermitian conjugation.

The covariantly chiral complex scalar superfield  $\mathcal{R}$  has the scalar curvature  $R$  as the coefficient at its  $\theta^2$  term, the real vector superfield  $\mathcal{G}_{\alpha\dot{\alpha}}$  has the traceless Ricci tensor,  $R_{\mu\nu} + R_{\nu\mu} - \frac{1}{2}g_{\mu\nu}R$ , as the coefficient at its  $\theta\sigma^a\bar{\theta}$  term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield  $\mathcal{W}_{\alpha\beta\gamma}$  has the self-dual part of the Weyl tensor  $C_{\mu\nu\rho\sigma}$  as the coefficient at its linear  $\theta^\delta$ -dependent term.

As is clear from eqs. (3.2) and (3.3), building a *chiral* superspace action (without using the covariant derivatives) is only possible with the superfields  $\mathcal{R}$  and  $\mathcal{W}_{\alpha\beta\gamma}$ .

Hence, the  $F(\mathcal{R})$  supergravity action (1.4) admits a natural extension in the chiral curved superspace because of the last equation (3.2), namely,

$$S = \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}, \mathcal{W}) + \text{H.c.} \quad (3.4)$$

with an extra dependence upon the totally symmetric spinor  $N = 1$  covariantly-chiral Weyl superfield  $\mathcal{W}_{\alpha\beta\gamma}$  of the old minimal  $N = 1$  superspace supergravity. Since the  $\mathcal{W}_{\alpha\beta\gamma}$  is anti-commuting and has only four independent components, an expansion of the superfield  $F(\mathcal{R}, \mathcal{W})$  in the  $\mathcal{W}_{\alpha\beta\gamma}$  terminates as

$$F(\mathcal{R}, \mathcal{W}) = F_0(\mathcal{R}) + F_2(\mathcal{R})\mathcal{W}^2 + F_4(\mathcal{R})\mathcal{W}^4 \quad (3.5)$$

in terms of the (complex) scalar products of the Weyl superfield  $\mathcal{W}_{\alpha\beta\gamma}$ . For definiteness, we confine ourselves to the concrete supersymmetric model defined by

$$F(\mathcal{R}, \mathcal{W}) = -\frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 - \frac{1}{6}f_3\mathcal{R}^3 + g\mathcal{R}\mathcal{W}^2 \quad (3.6)$$

with the real parameters  $(f_1, f_2, f_3, g)$ , which is the simplest  $\mathcal{W}$ -dependent extension of eqs. (1.5) and (1.6). The (mass) dimension of the new coupling constant  $g$  in eq. (3.6) is negative  $(-1)$ .



#### 4 Kähler potential and “superpotential” out of $F(\mathcal{R}, \mathcal{W})$

In this section we show that the most general  $F(\mathcal{R}, \mathcal{W})$  supergravity action (3.4) can be transformed in curved superspace (i.e. in the manifestly supersymmetric way) to the more conventional form, in terms of the Kähler potential and the “superpotential”. After that going to the Einstein frame merely requires the standard procedure of Weyl transformations for the component fields [25] or the super-Weyl transformations of the superfields [46].

First, the action (3.4) is classically equivalent to

$$S = \int d^4x d^2\theta \mathcal{E} [-\mathcal{Y}\mathcal{R} + Z(\mathcal{Y}, \mathcal{W})] + \text{H.c.} \quad (4.1)$$

where we have introduced the new (independent) covariantly chiral scalar superfield  $\mathcal{Y}$  and the new analytic function  $Z(\mathcal{Y}, \mathcal{W}) = \mathcal{Y}\mathcal{R}(\mathcal{Y}) + F(\mathcal{R}(\mathcal{Y}), \mathcal{W})$  as the Legendre transform of the function  $F(\mathcal{R}, \mathcal{W})$  with respect to its first argument: the functional form of  $\mathcal{R}(\mathcal{Y})$  is the inverse of eq. (4.2). In fact, the equation of motion for  $\mathcal{Y}$  is

$$\mathcal{Y} = Z'^{-1}(\mathcal{R}, \mathcal{W}) = -F'(\mathcal{R}, \mathcal{W}) \quad (4.2)$$

where derivatives (denoted by primes) and the inverse are with respect to the first arguments, considering the second argument  $\mathcal{W}$  as a parameter, and we assume  $Z''(\mathcal{Y}, \mathcal{W}) \neq 0$  or equivalently  $F''(\mathcal{R}, \mathcal{W}) \neq 0$ . Substituting the solution  $\mathcal{Y}(\mathcal{R}, \mathcal{W})$  back into the action (4.1) reproduces the original action (3.4).

Now we treat  $\mathcal{Y}$  as a dynamical superfield. The kinetic terms of  $\mathcal{Y}$  in the action (4.1) are obtained by using the (Siegel) identity

$$\begin{aligned} \int d^4x d^2\theta \mathcal{E} \mathcal{Y}\mathcal{R} + \text{H.c.} &= \int d^4x d^4\theta E^{-1}(\mathcal{Y} + \bar{\mathcal{Y}}) \\ &= -\frac{3}{8} \int d^4x d^2\theta \mathcal{E} (\bar{\mathcal{D}}^2 - 8\mathcal{R}) e^{-K/3} + \text{H.c.} \end{aligned} \quad (4.3)$$

where  $E^{-1}$  is the full curved superspace density. and  $K$  the Kähler potential of the superfields  $(\mathcal{Y}, \bar{\mathcal{Y}})$ ,

$$K = -3 \ln(\mathcal{Y} + \bar{\mathcal{Y}}) + 3 \ln 3. \quad (4.4)$$

It gives rise to the “no-scale” kinetic terms

$$\mathcal{L}_{\text{kin}} = \left. \frac{\partial^2 K}{\partial \mathcal{Y} \partial \bar{\mathcal{Y}}} \right|_{\mathcal{Y}=\bar{\mathcal{Y}}} \partial_\mu \mathcal{Y} \partial^\mu \bar{\mathcal{Y}} = 3 \frac{\partial_\mu \mathcal{Y} \partial^\mu \bar{\mathcal{Y}}}{(\mathcal{Y} + \bar{\mathcal{Y}})^2}. \quad (4.5)$$

These kinetic terms (4.5) represent the *non-linear sigma model* [14] with the hyperbolic target space of (real) dimension two, whose metric is known as the standard (Poincaré) metric having the  $\text{SL}(2, \mathbf{R})$  isometry.

Next, consider the remaining term  $Z(\mathcal{Y}, \mathcal{W})$ . For example, as regards our superfield Ansatz (3.6) with the notation (1.6), we find

$$Z(\mathcal{Y}, \mathcal{W}) = \frac{7M^2}{40m^2} \mathcal{R}(\mathcal{Y}, \mathcal{W}) + \left( \mathcal{Y} - \frac{3}{4} + g\mathcal{W}^2 \right) \left[ \frac{\sqrt{14}M^2}{60m} + \frac{2}{3} \mathcal{R}(\mathcal{Y}, \mathcal{W}) \right] \quad (4.6)$$

where

$$\mathcal{R}(\mathcal{Y}, \mathcal{W}) = \frac{\sqrt{14} M^2}{20 m} \left[ 1 - \sqrt{1 + \frac{80m^2}{21M^2} \left( \mathcal{Y} - \frac{3}{4} + g\mathcal{W}^2 \right)} \right]. \quad (4.7)$$

As is clear from eq. (4.1), the holomorphic function  $Z(\mathcal{Y}, \mathcal{W})$  plays the rôle of the *superpotential*. The truly scalar superpotential is given by  $Z(\mathcal{Y}, 0)$ .

In conclusion, the  $F(\mathcal{R}, \mathcal{W})$  supergravity action can be rewritten to the form of the standard matter-coupled supergravity action — see eq. (4.1) — as a sum of eqs. (4.3) and (4.6), in terms of the chiral scalar superfield  $\mathcal{Y}$  and the chiral spinor superfield  $\mathcal{W}$ .

## 5 Bosonic sector of $F(\mathcal{R}, \mathcal{W})$ supergravity

The superfield action (3.4) of  $F(\mathcal{R}, \mathcal{W})$  supergravity leads to the following field theory action in terms of the superfield components:<sup>3</sup>

$$\begin{aligned} \mathcal{L} &= \int d^2\theta \mathcal{E} F(\mathcal{R}, \mathcal{W}) + \text{H.c.} \\ &= -\mathcal{E} \mathcal{D} \mathcal{D} F | - 2 \mathcal{D}^\alpha \mathcal{E} \mathcal{D}_\alpha F | - \mathcal{D} \mathcal{D} \mathcal{E} F | + \text{H.c.} \end{aligned} \quad (5.1)$$

where the vertical bars stand for the lowest field components of each superfield in its expansion with respect to the anti-commuting superspace coordinates. By using the results of refs. [25, 28] we find the bosonic part of the action above in the form

$$\begin{aligned} \mathcal{L}_b &= -e \left( -\frac{1}{3} R + \frac{2}{3} i e_a{}^\mu \mathcal{D}_\mu b^a + \frac{4}{9} M^* M - \frac{2}{9} b^\mu b_\mu \right) \frac{\partial F}{\partial \mathcal{R}} \Big| - 4e M^* F | \\ &\quad - \frac{e \epsilon^{\eta\lambda}}{576} \left( \sigma_{\alpha\beta}^{ab} \sigma_{\gamma\lambda}^{cd} C_{abcd} - i \epsilon_{\lambda\alpha} \sigma_{\beta\dot{\eta}}^\mu \mathcal{D}_\mu b_{\dot{\eta}} \right) \left( \sigma_{\delta\epsilon}^{ef} \sigma_{\zeta\eta}^{gh} C_{efgh} - i \epsilon_{\eta\delta} \sigma_{\epsilon\dot{\kappa}}^\nu \mathcal{D}_\nu b_{\dot{\kappa}} \right) \frac{\partial^2 F}{\partial \mathcal{W}_{\delta\epsilon\zeta} \partial \mathcal{W}_{\alpha\beta\gamma}} \Big| \\ &\quad + \text{H.c.} \end{aligned} \quad (5.2)$$

where all the fermionic contributions are ignored. The necessary formulae needed to derive eq. (5.2) are collected in appendix A. The equation of motion for the auxiliary complex scalar field  $M$  reads

$$\begin{aligned} 0 &= \frac{\partial \mathcal{L}_b}{\partial M^*} \\ &= -e F | + \frac{e}{6} M \frac{\partial F^\dagger}{\partial \mathcal{R}^\dagger} \Big| - \frac{e}{9} M \left( \frac{\partial F}{\partial \mathcal{R}} \Big| + \frac{\partial F^\dagger}{\partial \mathcal{R}^\dagger} \Big| \right) \\ &\quad + \frac{e}{36} \left( -\frac{1}{3} R + \frac{2}{3} i e_a{}^\mu \mathcal{D}_\mu b^a + \frac{4}{9} M^* M - \frac{2}{9} b^\mu b_\mu \right) \frac{\partial^2 F^\dagger}{\partial \mathcal{R}^{\dagger 2}} \Big| \\ &\quad - \frac{e \epsilon^{\dot{\eta}\lambda}}{13824} \left( \bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{ab} \bar{\sigma}_{\dot{\gamma}\dot{\lambda}}^{cd} C_{abcd} + i \epsilon_{\dot{\lambda}\dot{\alpha}} \sigma_{\dot{\eta}\dot{\beta}}^\mu \mathcal{D}_\mu b_{\dot{\eta}} \right) \left( \bar{\sigma}_{\dot{\delta}\dot{\epsilon}}^{ef} \bar{\sigma}_{\dot{\zeta}\dot{\eta}}^{gh} C_{efgh} + i \epsilon_{\dot{\eta}\dot{\delta}} \sigma_{\dot{\kappa}\dot{\epsilon}}^\nu \mathcal{D}_\nu b_{\dot{\kappa}} \right) \frac{\partial^3 F^\dagger}{\partial \mathcal{W}_{\dot{\delta}\dot{\epsilon}\dot{\zeta}} \partial \mathcal{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}} \partial \mathcal{R}^\dagger} \Big|. \end{aligned} \quad (5.3)$$

<sup>3</sup>When comparing our notation to that of ref. [25], one should take into account that in the latter the space-time signature is  $(-, +, +, +)$ , the normalization of chiral integration over the anti-commuting superspace coordinates  $\Theta$ 's differs from ours by the factor of 4, and the definition of the Riemann curvature differs by the sign.

As regards our model (3.6), eq. (5.3) takes the form

$$\begin{aligned}
 & f_3 M^3 + 3f_3 M M^{*2} + 6f_2 M^2 + 12f_2 M M^* - 72f_1 M \\
 & - f_3 (2b_\mu b^\mu + 6ie_a^\mu \mathcal{D}_\mu b^a + 3R) M^* - 6f_2 (2b_\mu b^\mu + 6ie_a^\mu \mathcal{D}_\mu b^a + 3R) \\
 & - \frac{27}{4} g \left[ 2 \left( C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - i C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} \right) - \frac{4}{3} \left( F_{\mu\nu} F^{\mu\nu} + i F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \right] = 0 \quad (5.4)
 \end{aligned}$$

where  $F_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$  is the field strength of the auxiliary vector field  $b_\mu$ ,  $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$  is its Poincaré dual, and  $\tilde{C}^{\mu\nu\rho\sigma} = \frac{1}{2} \epsilon^{\mu\nu\xi\pi} C_{\xi\pi}{}^{\rho\sigma}$ . For many physical applications (as well as initial study) the imaginary parts of the scalar fields may be ignored, so that  $M$  is real. Then the above equation is simplified to

$$\begin{aligned}
 & f_3 M^3 + \frac{9}{2} f_2 M^2 - \left( \frac{3}{4} f_3 R + \frac{1}{2} f_3 b_\mu b^\mu + 18f_1 \right) M \\
 & - \frac{9}{2} f_2 R - 3f_2 b_\mu b^\mu - \frac{27}{4} g \left( C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{2}{3} F_{\mu\nu} F^{\mu\nu} \right) = 0. \quad (5.5)
 \end{aligned}$$

It is always possible to get the real roots of this cubic equation by demanding positivity of its discriminant (e.g., via the standard Cardano-Vieta method) in the case of a sufficiently high scalar curvature and a sufficiently small contribution of the last term in eq. (5.5). However, the corresponding formulae appear to be long and not very illuminating. Here we confine ourselves to the much simpler case when  $f_2 = f_3 = 0$ . Then eq. (5.3) can be easily solved as

$$M = -\frac{g}{16f_1} \left[ 3 \left( C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - i C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} \right) - 2 \left( F_{\mu\nu} F^{\mu\nu} + i F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \right]. \quad (5.6)$$

Substituting the result with  $f_1 = 3/2$  to the bosonic part of the Lagrangian (5.2), we find

$$\begin{aligned}
 e^{-1} \mathcal{L}_b = & -\frac{1}{2} R - \frac{1}{3} b_\mu b^\mu + \frac{g^2}{432} \left[ \frac{9}{4} (C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma})^2 - \frac{9}{2} C_{\mu\nu\rho\sigma} C^{\rho\sigma\zeta\pi} C_{\zeta\pi\tau\phi} C^{\tau\phi\mu\nu} \right. \\
 & + 9 C_{\mu\nu\rho\sigma} C^{\rho\sigma}{}_{\zeta\pi} C^{\mu\zeta}{}_{\tau\phi} C^{\tau\phi\nu\pi} - 3 C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} F^{\zeta\pi} F_{\zeta\pi} - 6 F^{\mu\nu} C_{\mu\nu\rho\sigma} C^{\rho\sigma\zeta\pi} F_{\zeta\pi} \\
 & \left. + 12 F^{\mu\zeta} F^{\nu\pi} C_{\mu\nu\rho\sigma} C_{\zeta\pi}{}^{\rho\sigma} - (F^{\mu\nu} F_{\mu\nu})^2 + 4 F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} F_{\sigma\mu} \right]. \quad (5.7)
 \end{aligned}$$

## 6 Discussion

In this section we briefly comment on the possible *origin* of the Weyl-tensor-dependent terms in the gravitational effective action.

The obvious source of those terms is provided by the standard *Weyl anomaly* in the quantum field theory of massless matter in a gravitational background, which is given by

$$\langle T \rangle_g = \frac{c}{16\pi^2} C^2 - \frac{a}{16\pi^2} G \quad (6.1)$$

where we have introduced the trace  $T$  of the matter energy-momentum tensor in the gravitational background ( $g$ ), the central charge  $c$ , the Gauss-Bonnet combination  $G$ , and the so-called  $a$ -coefficient. Equation (6.1) suggests us to consider even more general actions of the type

$$S = -\frac{1}{2} \int d^4 x \sqrt{-g} f(R, C, G). \quad (6.2)$$

However, as was already mentioned above, the truly chiral supersymmetric extension is not compatible with the  $G$ -dependence. It implies that any dynamics (or gravity models) relying on the  $G$ -dependence of the gravitational action may be unstable, being not protected against quantum corrections by supersymmetry.

The supergravity corrections in the  $\alpha'$ -expansion of the gravitational superstring effective action arise as the loop corrections in the quantized supergravity, though with *finite* coefficients. Since the supergravity counterterms are usually given by the *full* superspace integrals, it is unlikely that our action (3.4) can be generated in the *perturbative* superstring theory. However, it may well be generated *non-perturbatively*. The kinetic terms of the dilaton-axion (complex) scalar in superstring theory are precisely given by the non-linear sigma-model (4.5) indeed, whereas the dilaton-axion superpotential can only be generated non-perturbatively in superstring theory.

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## A The lowest components of the superfields

The leading (in the zeroth order with respect to the Grassmann superspace coordinates) field components of various superfields can be obtained by using refs. [25, 28]. Here we list the leading terms of the relevant bosonic superfields used in the main text:

$$\mathcal{E}| = e, \tag{A.1}$$

$$\mathcal{D}\mathcal{D}\mathcal{E}| = 4eM^* + 4e\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu, \tag{A.2}$$

$$\mathcal{R}| = -\frac{1}{6}M, \tag{A.3}$$

$$\begin{aligned} \mathcal{D}_\alpha\mathcal{D}_\beta\mathcal{R}| &= \frac{1}{2}\epsilon_{\alpha\beta}\left(-\frac{1}{3}R - \frac{2}{3}i\bar{\psi}^\mu\bar{\sigma}^\nu\psi_{\mu\nu} - \frac{1}{12}\epsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\mu\bar{\sigma}_\nu\psi_{\rho\sigma} + \psi_\mu\sigma_\nu\bar{\psi}_{\rho\sigma})\right. \\ &\quad \left. + \frac{2}{3}ie_a{}^\mu\mathcal{D}_\mu b^a + \frac{4}{9}M^*M - \frac{2}{9}b_\mu b^\mu - \frac{1}{3}\bar{\psi}^\mu\bar{\psi}_\mu M + \frac{1}{3}\psi_\mu\sigma^\mu\bar{\psi}_\nu b^\nu\right), \end{aligned} \tag{A.4}$$

$$\begin{aligned} \mathcal{D}_\delta\mathcal{W}_{\alpha\beta\gamma}| &= \frac{1}{8}\left[-2\sigma_{\alpha\beta}^{ab}\sigma_{\gamma\delta}^{cd}C_{abcd} - i\psi_{\gamma\epsilon\delta}\bar{\psi}_{\beta\eta\alpha}\dot{\eta}^\epsilon - i\bar{\psi}_{\gamma\epsilon}\dot{\epsilon}\psi_{\delta\eta\alpha}\dot{\eta}^\epsilon - \psi_{\alpha\epsilon\beta}\bar{\psi}_{\gamma\dot{\epsilon}}\dot{\eta}^\epsilon b_{\delta\dot{\eta}} - \psi_{\gamma\epsilon\delta}\bar{\psi}_{\alpha\dot{\eta}}\dot{\eta}^\epsilon b_{\beta\dot{\epsilon}}\right]_{\text{t.s.}} \\ &\quad - \frac{i}{4}\epsilon_{\delta(\alpha}\hat{D}_{\beta\dot{\epsilon}}\dot{\epsilon}b_{\gamma)\dot{\epsilon}}, \end{aligned} \tag{A.5}$$

where we have introduced the notation

$$\begin{aligned} \hat{D}_{\beta\dot{\epsilon}}b_{\gamma\dot{\eta}} &= \mathcal{D}_{\beta\dot{\epsilon}}b_{\gamma\dot{\eta}} \\ &\quad + \frac{3}{2}\psi_{\beta\dot{\epsilon}}\alpha\left[\frac{1}{4}\bar{\psi}_{\gamma\dot{\beta}}\dot{\beta}_{\alpha\dot{\beta}\dot{\eta}} + \frac{1}{12}\epsilon_{\alpha\gamma}\bar{\psi}^{\beta\dot{\beta}}\dot{\beta}_{\beta\dot{\eta}\dot{\beta}} - \frac{i}{6}\psi_{\gamma\dot{\eta}\alpha}M^* + \frac{i}{6}\bar{\psi}_{(\gamma\dot{\beta}}\dot{\beta}b_{\alpha)\dot{\eta}} - \frac{i}{12}\bar{\psi}_{\alpha\dot{\beta}}\dot{\beta}_{\dot{\eta}}b_{\gamma\dot{\beta}}\right] \end{aligned}$$

$$-\frac{3}{2}\bar{\psi}_{\beta\dot{\epsilon}}^{\dot{\alpha}}\left[\frac{1}{4}\psi^{\beta}_{\dot{\alpha}\beta\dot{\eta}\dot{\gamma}}+\frac{1}{12}\epsilon^{\dot{\alpha}\dot{\eta}}\psi_{\dot{\gamma}}^{\dot{\beta}\beta}_{\dot{\beta}\beta}+\frac{i}{6}\bar{\psi}_{\dot{\gamma}\dot{\eta}\dot{\alpha}}M-\frac{i}{6}\psi_{\beta(\dot{\alpha}}^{\beta}b_{\dot{\gamma}\dot{\eta}})+\frac{i}{12}\psi^{\beta}_{\dot{\alpha}\dot{\gamma}}b_{\beta\dot{\eta}}\right]. \quad (\text{A.6})$$

The subscript t.s. in eq. (A.5) denotes the total symmetrization of the undotted indices inside the square brackets, and  $\psi_{\mu\nu} = \mathcal{D}_{\mu}\psi_{\nu} - \mathcal{D}_{\nu}\psi_{\mu}$ . The Hermitian vector field  $b_a$  is defined as the lowest component of the superfield  $\mathcal{G}_a$ ,  $|\mathcal{G}_a| = -\frac{1}{3}b_a$ .

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