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An identification procedure of multi-input Wiener models for the distortion analysis of nonlinear circuits

T. Weng[1], S. Stegemann[1], W. John[2], and W. Mathis[1]

[1]Institute for Theoretical Electrical Engineering, Leibniz University of Hannover, Hannover, Germany
[2]System Integration Laboratory, Paderborn, Germany

Correspondence to: S. Stegemann (stegemann@tet.uni-hannover.de)

Abstract. In this contribution, a system identification procedure of a two-input Wiener model suitable for the analysis of the disturbance behavior of integrated nonlinear circuits is presented. The identified block model is comprised of two linear dynamic and one static nonlinear block, which are determined using an parameterized approach. In order to characterize the linear blocks, an correlation analysis using a white noise input in combination with a model reduction scheme is adopted. After having characterized the linear blocks, from the output spectrum under single tone excitation at each input a linear set of equations will be set up, whose solution gives the coefficients of the nonlinear block. By this data based black box approach, the distortion behavior of a nonlinear circuit under the influence of an interfering signal at an arbitrary input port can be determined. Such an interfering signal can be, for example, an electromagnetic interference signal which conductively couples into the port of consideration.

1 Introduction

In order to ensure the reliability of electronic systems even in harsh electromagnetic environments, it is necessary to analyze the effects of EMI signals on integrated circuits at an early design stage. Generally, a disturbance signal can couple into a nonlinear circuit at an arbitrary port. The output signal will possess interference components, that can significantly distort the circuit's functionality. These interference phenomena arise due to intermodulations between the nominal input signal and the disturbing signal as an effect of the nonlinear device characteristics (Stegemann et al., 2012, 2011). To estimate those interfering products with respect to both inputs, we model the nonlinear circuits under consideration by the two input Wiener model shown in Fig. 1. In this work, the

black box approach is used to identify the considered model, as in this case a rapid solution is more important than a detailed consideration of the circuit structure itself. The model structure is comprised of two linear blocks and a nonlinear static block. Both linear blocks are described by their linear transfer function and independently model the frequency characteristics of the input signal x_{in} and the distortion signal x_{dis}, respectively. The static nonlinear block is given by a multivariate polynomial that models the nonlinear behavior of the circuit. In the proposed system identification procedure for this block oriented model, the parameters of the linear blocks are obtained by adopting a correlation analysis using a white noise input. The artificial and not observable intermediate signals $\tilde{x}_{\text{in}}(t)$ and $\tilde{x}_{\text{dis}}(t)$ are avoided with the assumption of small input signals so that the whole system can be seen as linear. Then, by applying the determined linear transfer functions, a linear system of equations is set up to calculate the parameters of the nonlinear block from frequency domain data. The identified model can be used to calculate characteristic figures of merit for the distortion analysis of nonlinear circuits, such as intermodulation distortion components:

$$\text{IM}_2 = \frac{A_{\text{IM}_2}}{A_{\text{fundamental}}} \tag{1}$$

$$\text{IM}_3 = \frac{A_{\text{IM}_3}}{A_{\text{fundamental}}}. \tag{2}$$

To obtain these values, the output y is analyzed in the frequency domain. Here $A_{\text{fundamental}}$ is the magnitude at the frequency of x_{in} and $A_{\text{IM}_{2,3}}$ are the magnitudes of the intermodulation products of second and third order.

Besides the premise that for small input signals the nonlinearity can be seen as linear, the proposed identification procedure does not need other specific assumptions on the structure of the analyzed circuit. E.g. there is no need to assume an
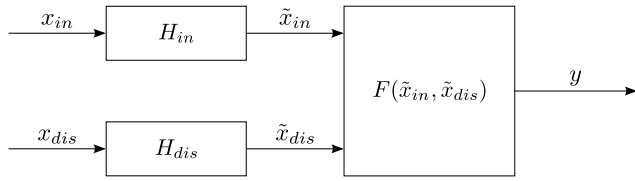
Fig. 1. Block-oriented model structure for identification procedure (cf. Stegemann et al., 2012, 2011).

invertible nonlinearity (cf. Greblicki, 1992) to calculate the intermediate signals for the identification of the linear blocks. Furthermore, each block is characterized with only one test signal, so there is no need for extensive calculations with different or complex input signals (Pintelon and Schoukens, 2001). In comparison with the classical distortion theory described in (Weiner and Spina, 1980) or (Wambacq and Sansen, 1998), the model can be used for EMI induced distortion analysis at nonsignal inputs, e.g. at the supply voltage. The applicability of the procedure for multi input distortion analysis using a simulative approach, is shown in detail for an operational amplifier in the presence of an interference signal at the power supply. In (Stegemann et al., 2012) an analytical approach for the characterization of each block is described. The remainder of the paper is organized as follows. In section 2 the predefined structure of the linear transfer functions and the multivariate polynomial are described, as well as the assumptions needed to identify their parameters. The identification procedure is then described in detail in the Sect. 2.1 for the linear dynamical and Sect. 2.2 for the nonlinear static block. In Sect. 3 the identification procedure is carried out for a two-stage operational amplifier. The results for the characterized blocks are compared with simulations from Cadence Spectre.

2 Identification procedure for distortion analysis

In this section the identification procedure for both, the linear dynamic and the nonlinear static block of the multi-input Wiener model is described. The goal is to characterize each block of the model shown in Fig. 1 by using either simulated or measured input and output data. For this purpose, the transfer characteristics of each block are given as predefined functions, for which we determine a parametrization that approximates the input/output behavior of the circuit. Since we are interested in a frequency domain analysis, we characterize each linear dynamic block by a rational function of the form:

$$H(s) = \frac{\sum_{m=0}^{M} a_m s^m}{\sum_{n=0}^{N} b_n s^n}, \tag{3}$$

i.e. the linear blocks H_{in} and H_{dis} are assumed to be completely described by linear transfer functions. The frequency

domain transforms $\tilde{X}_{\mathrm{in}}(f_{\mathrm{in}})$ and $\tilde{X}_{\mathrm{dis}}(f_{\mathrm{dis}})$ of the artificial intermediate signals $\tilde{x}_{\mathrm{in}}(t)$ and $\tilde{x}_{\mathrm{dis}}(t)$ are then given by:

$$\tilde{X}_{\mathrm{in}}(f_{\mathrm{in}}) = H_{\mathrm{in}}(f_{\mathrm{in}}) \cdot X_{\mathrm{in}}(f_{\mathrm{in}}) \tag{4}$$

$$\tilde{X}_{\mathrm{dis}}(f_{\mathrm{dis}}) = H_{\mathrm{in}}(f_{\mathrm{dis}}) \cdot X_{\mathrm{dis}}(f_{\mathrm{dis}}) \tag{5}$$

This way, the dynamic behavior of a nonlinear circuit is modelled by a linear transfer function for each input, that filters the input signals of the circuit. Note, that from the knowledge of H_{dis}, i.e. the frequency response of the distortion signal, it is possible to estimate the frequency range in which a circuit is most susceptible to EMI as the frequency range in which H_{dis} exhibits its highest gain (Stegemann et al., 2012). The intermodulation products between $x_{\mathrm{in}}(t)$ and $x_{\mathrm{dis}}(t)$ at the output arise due to a nonlinear transformation, which is described by a static nonlinear function $F(\tilde{x}_{\mathrm{in}}, \tilde{x}_{\mathrm{dis}})$. This static nonlinearity is characterized by the K-th degree multivariate polynomial:

$$F(\tilde{x}_{\mathrm{in}}, \tilde{x}_{\mathrm{dis}}) = f(x, y) = \sum_{k=0}^{K} \sum_{l=0}^{K-k} \alpha_{k,l} x^k y^l. \tag{6}$$

The coefficients $\alpha_{k,l}$ are referred to as *nonlinearity coefficients*. For the black box characterization of every block, a relationship between the input of each block to the overall output of the system has to be determined. This can only be done by using suitable test signals (Bai et al., 2007). In Sect. 2.1, the determination of the parameters a_m and b_n of the linear blocks using a Gaussian white noise input and a correlation analysis is described. In order to characterize the linear blocks H_{in} and H_{dis} independently of the nonlinear block $F(\tilde{x}_{\mathrm{in}}, \tilde{x}_{\mathrm{dis}})$, the amplitude of the white noise signal has to be small. When determining the linear blocks, we assume that the output is at most weakly distorted by the nonlinearity. That is, we consider $y(t)$ to be

$$y(t) \approx \mathcal{L}^{-1}\{H_{\mathrm{in}}(s)\} * x_{\mathrm{in}}(t) + \mathcal{L}^{-1}\{H_{\mathrm{dis}}(s)\} * x_{\mathrm{dis}}(t), \tag{7}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform and $*$ denotes the convolution operation. Once the linear blocks are identified, the artificial intermediate signals $\tilde{x}_{\mathrm{in}}(t)$ and $\tilde{x}_{\mathrm{dis}}(t)$ can be calculated by Eqs. (4) and (5). Using sinusoid input functions for the nominal input and the distortion signal, the nonlinearity coefficients of the multivariate polynomial in Eq. (6) can be determined using the spectral approach described in Sect. 2.2.

2.1 Identification of the linear blocks

In the context of linear and time invariant systems, a suitable way to characterize transfer characteristics is by means of white noise (Meyer, 2011). The identification procedure for the linear blocks of the multi input Wiener model is based on the definitions of the autocorrelation $r_{xx}(t_1, t_2)$ and the crosscorrelation $r_{xy}(t_1, t_2)$ of a white noise process, which

can be represented by their mean value and only depends on the time differences (Meyer, 2011):

$$r_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \cdot \int_{-T}^{+T} x(t)x(t+\tau)\,dt, \qquad (8)$$

$$r_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \cdot \int_{-T}^{+T} x(t)y(t+\tau)\,dt. \qquad (9)$$

Here, $x(t)$ is the white noise input, $y(t)$ is the output signal and $\tau = t_2 - t_1$ is the time difference. Using Eq. (8), Eq. (9) can be written as follows (Meyer, 2011):

$$r_{xy}(\tau) = r_{xx}(\tau) \cdot h(\tau). \qquad (10)$$

By transforming Eq. (10) into the frequency domain, we obtain the expression for determining the transfer function:

$$S_{xy}(s) = S_{xx}(s) \cdot H_{corr}(s), \qquad (11)$$

with $s = j \cdot 2\pi f$. From Eq. (11), the characterization of the transfer functions is possible by measuring the response of a linear system to a white noise input and calculating the autocorrelation of the input, as well as the crosscorrelation between the output and input signal. By means of this correlation analysis and by the assumption Eq. (7), we obtain a data set for H_{in} and H_{dis} with respect to frequency, by recording the output wile stimulating the respective input x_{in} or x_{dis} with a white noise signal and zeroing the other input. Note, that we can only expect to obtain approximative results for the transfer functions, since no infinitely long signals can be considered. In order to characterize the parameters of the rational transfer function given in Eq. (3), we adopt a complex curve fitting method to determine the coefficients. A first guess of the degree of the nominator and the denominator polynomial $Z_{est}(s)$ and $N_{est}(s)$, respectively, has to be chosen with the help of the data obtained from the correlation analyis described above. Relying on the approximated characteristics in the frequency domain, we first chose a relatively high degree to overestimate, but to completely describe the system's dynamics (Pintelon and Schoukens, 2001). The coefficients a_m and b_n of (11) will then be determined by a nonlinear fitting problem for which we need a good initial guess. By means of the Gauss-Newton algorithm we obtain the linear transfer functions for both H_{dis} and H_{in}. As a last step, we reduce the order of the transfer function by an iterative approach. Eq. (3) is nonlinearly dependent on the coefficients a_m and b_n of it's nominator and denominator polynomials. That is, the error defined as the difference of the estimated transfer function[1] $H_{est}(s)$ and the characteristics resulting from the correlation analysis $H_{corr}(s)$ at the specific frequencies,

$$e(s) = H_{corr}(s) - H_{est}(s), \qquad (12)$$

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[1]Either being H_{dis} or H_{in} in the general form of Eq. (3).

is nonlinearly dependent on a_m and b_n, too. The goal is to find a set of coefficients that minimizes the error Eq. (12). An approximated solution of this minimization problem can be found using an iterative scheme, such as the Gauss-Newton method, which has been adopted in this work. However, the method requires the knowledge of an initial guess as a starting point for the iteration and, due to the nonlinear dependency on a_m and b_n, it can not be guaranteed that a global minimum will be found (Boyd, 2004) (Dahmen, 2008). In order to determine such an initial solution, we convert the problem into a similar but linear one, which we obtain by multiplying the error function in Eq. (12) by the yet unknown denominator $N_{est}(s)$:

$$\tilde{e}(s) = H_{corr}(s) \cdot N_{est}(s) - Z_{est}(s). \qquad (13)$$

From this definition it is now possible to formulate a linear least squares problem (Tao and Zunde, 1981), which is solved by minimizing the squared error

$$E = \sum_{m=1}^{M} |\tilde{e}(s_m)|^2, \qquad (14)$$

over every frequency $s_m \cdot j\omega_m$. For this linear optimization problem there exists a unique solution, since the problem is convex (Boyd, 2004). Taking this solution as a starting point, the Gauss–Newton iteration is applied to successively improve the solution of Eq. (12). After having found a possible solution for a transfer function, one should try to reduce the order of this function, which is possible for the numerator and the denominator polynomial separately. That is, we wish to find a solution for the linear blocks providing as less poles and zeros as possible with no loss in accuracy. For this purpose, an overall iteration scheme is applied, which successively reduces the degree of the nominator and the denominator, until a predefined error limit is reached. If a rapid increase of the error is observed during order reduction, the solution found up to this point will be used (Pintelon and Schoukens, 2001). A flow-chart of the applied reduction algorithm is depicted in Fig. 2.

2.2 Identification of the nonlinear block

Having successfully identified the linear blocks, both for the input and for the distortion signal, the static nonlinear block can be determined, since the artificial intermediate signals $\tilde{x}_{in}$ and $\tilde{x}_{dis}$ can now be calculated by Eqs. (4) and (5). We calculate the coefficients of the multivariate polynomial in Eq. (6) by simulating or measuring the quasi periodic output for a single frequency excitation at both inputs, as:

$$x_{in}(t) = A_{in} \cos(2\pi f_{in} t + \phi_{in}) \qquad (15)$$
$$x_{dis}(t) = A_{dis} \cos(2\pi f_{dis} t + \phi_{dis}). \qquad (16)$$

The output $y(t)$ will then be composed of various harmonics and intermodulation products of $x_{in}(t)$ and $x_{dis}(t)$. Given
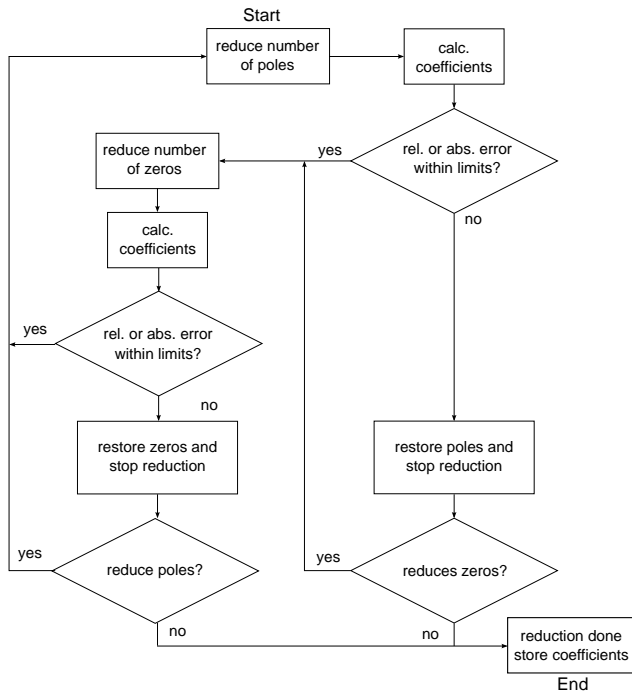
Fig. 2. Flow-chart of the algortihm for order reduction of the identified linear transfer functions.

a predefined order of the polynomial output nonlinearity $F(\tilde{x}_{in}(t), \tilde{x}_{dis}(t))$, we construct a linear system of equations in the complex domain from the output spectrum of $y(t)$. The solution of this system of equations yields the coefficients $\alpha_{k,l}$ for $k, l = 1, \ldots, K$ in (6). We fix the order K by omitting those components in the output spectrum, whose amplitudes do not exceed a predefined threshold. Given the input frequencies f_{in} and f_{dis}, the output frequencies generated by a polynomial nonlinearity are calculated using the concept of module vectors (Weiner and Spina, 1980). That is, the i-th mixing frequency of order $k + l$ is given by:

$$f_{i,k+l} = \boldsymbol{m}_{i,k+l} \cdot \boldsymbol{f}_{exc}^T. \tag{17}$$

Here $\boldsymbol{f}_{exc} = [-f_{in}, -f_{dis}, f_{dis}, f_{in}]$ is the row vector containing the input and the distortion frequency (i.e. the excitation frequencies). The row vector $\boldsymbol{m}_{i,k+l} = [m_{-f_{in}}, m_{-f_{dis}}, m_{f_{in}}, m_{f_{dis}}]$ is the i-th module vector of order $k + l$, whose entries are positive natural numbers and which obey:

$$m_{-f_{in}} + m_{-f_{dis}} + m_{f_{in}} + m_{f_{dis}} = k + l. \tag{18}$$

For a given order $k + l$ there exist[2]

$$i = 1, .., \binom{4 + k + l - 1}{k + l}$$

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[2]Chua and Ng (1979), p. 5, for which $K = 2$ (no. of input frequencies) and $n = k + l$ (order of mixing product).

module vectors and by Eq. (17) the output frequencies are related to the input frequencies. Using this concept, we can construct the following linear system of equations with the coefficients of (6) as unknowns:

$$\mathbf{C}\tilde{\boldsymbol{\alpha}} = \boldsymbol{b}. \tag{19}$$

Here, the vector $\boldsymbol{b}$ on the right hand side contains the complex amplitudes (absolute values and phases) of the simulated or measured output spectrum of $y(t)$ at the frequencies calculated by Eq. (17) which are listed in ascending order. The entries of the matrix $\mathbf{C}$ are calculated as follows: For each frequency from Eq. (17) the resulting signal amplitude and phase is calculated by:

$$q_{k,l,i} = \frac{1}{2^{k+l}} |\tilde{x}_{in}|^k |\tilde{x}_{dis}|^l \exp\left[j\left(k \cdot \arg(\tilde{x}_{in}) + l \cdot \arg(\tilde{x}_{dis})\right)\right]. \tag{20}$$

Since different module vectors can result in the same frequency, there are less unique frequencies than module vectors. Because the vector $\boldsymbol{b}$ contains the complex amplitdues for these unique frequencies only, the values Eq. (20) are connected with the rows i of $\mathbf{C}$ by their frequency, hence the index. The corresponding column j in the matrix $\mathbf{C}$ for each complex value given by Eq. (20) is then calculated by its indices k and l:

$$j = \frac{(k+l) \cdot (k+l+1)}{2} + 1 + l. \tag{21}$$

The fraction is the sum from 1 to $k + l$ which equals the number of all nonlinear coefficients up to the order $k + l - 1$. Thus, it can be seen as the starting index for the coefficients of order $k + l$. As the index l (or k) is unique for each order, since the sum of k and l stays the same, it can be used to obtain the column index. Consequently, the rows of the matrix $\mathbf{C}$ contain the complex amplitudes from Eq. (20) for a specific frequency equal to the frequency in the vector $\boldsymbol{b}$, whereas the complex amplitudes are linked with the columns of $\mathbf{C}$ by their indices k and l according to Eq. (21). Due to the number of frequencies generated by the nonlinearity, Eq. (14) is an overdetermined system of equations, for which a least squares solution can be obtained. After having solved Eq. (19) to get $\tilde{\boldsymbol{\alpha}}$, we obtain the nonlinearity coefficients by division with the corresponding binomial coefficient (see e.g. Weiner and Spina, 1980 or Maas, 2003):

$$\alpha_{k,l} = \frac{k! \cdot l!}{(k+l)!} \tilde{\alpha}_{k,l} \tag{22}$$

3 Modelling a two-stage operational amplifier

The described procedure is now used to model the operational amplifier given in Fig. 3 within a simulation approach. Comparative simulations were made with Cadence Spectre.

The input signal equals the differential signal between the inputs v_{out1} and v_{out2}, where the interfering signal $v_{dis}(t)$
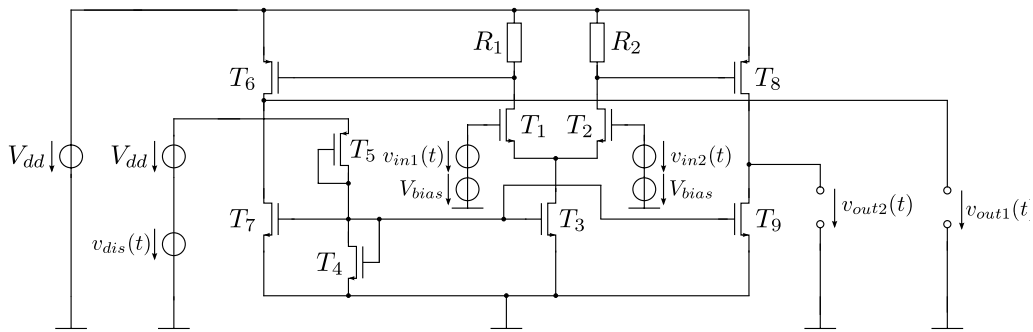
Fig. 3. Operational Amplifier with interferring signal $v_{\mathrm{dis}}(t)$ at the supply voltage
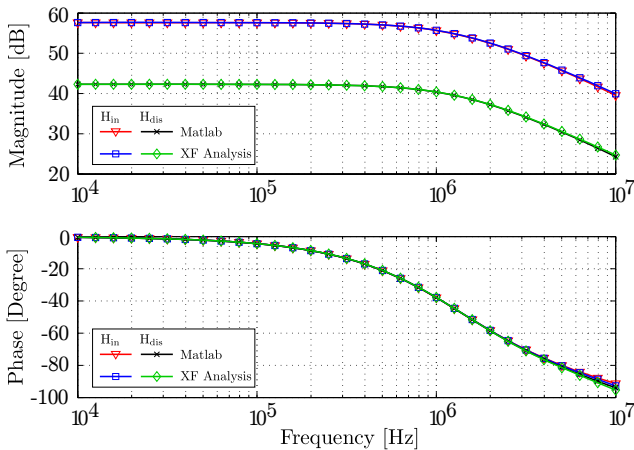

Fig. 4. Transfer functions $H_{\mathrm{in},v_{\mathrm{out1}}}$ and $H_{\mathrm{dis},v_{\mathrm{out1}}}$
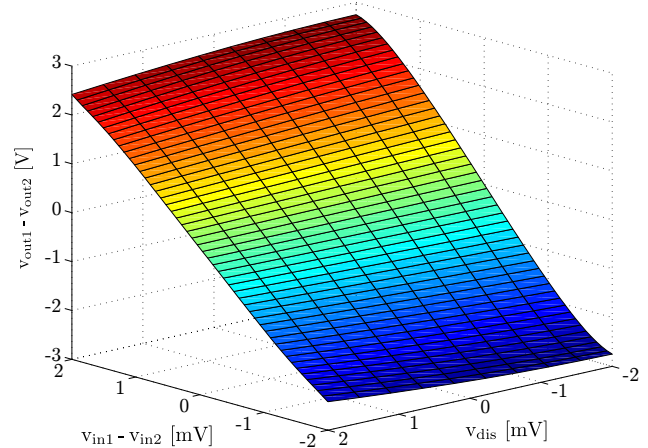

Fig. 5. Nonlinear characteristic in dependency of both input signals.

varies the reference current of the differential stage which leads to a mixing between both inputs. In this case the interfering signal does not directly influence the differential output $v_{\mathrm{out1}} - v_{\mathrm{out2}}$ because of the common mode rejection of the input stage. Thus, two transfer functions are identified for both inputs $H_{\mathrm{in},v_{\mathrm{out1}}}$, $H_{\mathrm{in},v_{\mathrm{out2}}}$, $H_{\mathrm{dis},v_{\mathrm{out1}}}$ and $H_{\mathrm{dis},v_{\mathrm{out2}}}$ according to the outputs v_{out1} and v_{out2}. The resulting bode plots of the transfer functions $H_{\mathrm{in},v_{\mathrm{out1}}}$ and $H_{\mathrm{dis},v_{\mathrm{out1}}}$ are shown in Fig. 4. The procedure yields the same results for the output v_{out2}, with just a 180° phase shift for $H_{\mathrm{in},v_{\mathrm{out2}}}$. These results were achieved by a transient simulation with Gaussian white noise input with a maximum level of 45 μV. Based on the identified frequency response, a frequency range of 10 kHz up to 10 MHz was chosen to conduct the order reduction scheme of 2.1.

For comparison purposes a counter simulation was done, using the XF-analysis in Cadence Spectre. As it can be seen from Fig. 4, there is a very good agreement between the calculated and the simulated transfer functions. The resulting polynomial from Eq. (6) is shown in Fig. 5, which was calculated for a magnitude of 2 mV for both inputs at frequencies of 200 kHz for the nominal input and 18 kHz for the interfering signal, respectively. In this case, the order of the

polynomial was set to $K = 9$. The obtained model is now used for the distortion analysis of the given circuit. As the intermodulation products of second and third order and their figures of merit are of particular interest, these intermodulation components can be calculated from the amplitudes at the frequencies $f_{\mathrm{in}} \pm f_{\mathrm{dis}}$ for the second order and $f_{\mathrm{in}} \pm 2 f_{\mathrm{dis}}$ for the third order components as:

$$\mathrm{IM_2} = \frac{\alpha_{11} \cdot |H_{f\mathrm{dis}}(f_{\mathrm{dis}}) X_{\mathrm{dis}}(f_{\mathrm{dis}})|}{\alpha_{10}} \tag{23}$$

$$\mathrm{IM_3} = \frac{\alpha_{12} \cdot |H_{f\mathrm{dis}}(f_{\mathrm{dis}}) X_{\mathrm{dis}}(f_{\mathrm{dis}})|^2}{\alpha_{10}}. \tag{24}$$

The estimated characteristics of $\mathrm{IM_2}$ and $\mathrm{IM_3}$, in dependency of the distortion amplitude v_{dis}, are depicted in Fig. 6. In this case the magnitude of the nominal input signal was set to 2 mV. The counter simulation was done by a Harmonic Balance simulation of the whole circuit in Cadence Spectre with swept v_{dis}.
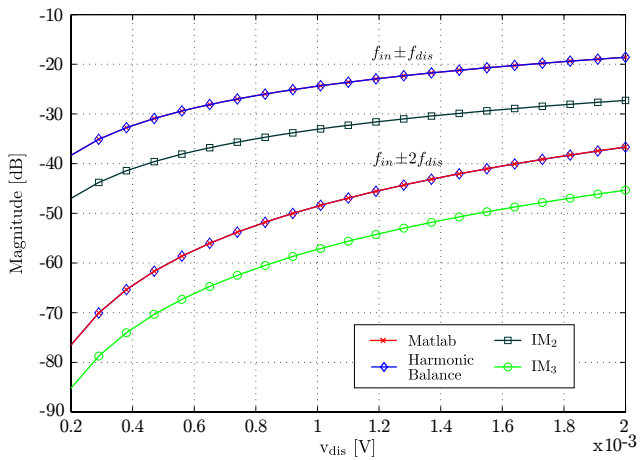
Fig. 6. Second and third order intermodulation distortion in dependency of the distortion amplitude v_{dis}.

4 Conclusion and outlook

In order to derive expressions for analyzing the intermodulation products of an input signal and an interfering signal of a nonlinear circuit, an identification procedure to obtain parametrized transfer functions for a multi input Wiener model from input and output data has been described. The linear transfer functions of the linear blocks for both inputs were derived by a correlation analysis with white noise excitation signals at both inputs and deriving the output by a transient simulation. In this context, an order reduction scheme was proposed to calculate a solution according to specific needs of accuracy or simplicity, regarding the maximum number of poles and zeros of the estimated linear transfer functions. From the simulated output spectrum with single tone stimulation at both inputs, a set of linear equations has been set up to determine the nonlinearity coefficients of the nonlinear block. The proposed identification procedure is suitable to examine the frequency range in which a circuit is most susceptible to a distortion signal, such as an electromagnetic interference signal. Having identified each block of the multi input Wiener model, expressions for the emerging intermodulation products can be obtained. Although the results in this paper were obtained by conducting simulation, the proposed identification procedure can be used in a measurement setup as well. Future work will deal with this measurement set up and consideration of the input signals used for the identification process.

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