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D-brane scattering of $N = 2$ strings

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Abstract

The amplitudes for emission and scattering of $N = 2$ strings off D-branes are calculated. We consider in detail the amplitudes $\langle cc \rangle$ and $\langle occ \rangle$ for the different types of D-branes. For some D-branes we find massive poles in the scattering spectrum that are absent in the ordinary $N = 2$ spectrum.

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1. Introduction

It has become obvious in the last couple of years that D-branes are of utmost importance for our understanding of $N = 1$ string theory. In his pathbreaking paper [1], Polchinski showed that p -dimensional extended objects—the Dp -branes—are the long sought carriers of the Ramond–Ramond charges in $N = 1$ string theory. In the context of scattering amplitude calculations the most important property of D-branes is that their quantum fluctuations are described by open strings moving on the brane and therefore are under good control at weak coupling (for reviews on D-branes see, e.g., [2,3] and literature cited within). This allows to calculate amplitudes for emission and scattering of closed fundamental strings from D-branes by “pre-revolutionary” methods that have been invented more than a decade ago and are well understood [4]. For the $N = 1$ string these computations have been performed, e.g., in Refs. [5–9] and considerably contributed to the understanding of D-brane

physics. It is the purpose of this note to perform a similar analysis for $N = 2$ strings.

That this analysis has not been undertaken so far for the $N = 2$ string finds its reason in the lack of Ramond–Ramond fields in the string spectrum. The $N = 2$ superconformal algebra with $c = 6$ serves as constraint algebra and is powerful enough to remove all string excitations from the spectrum leaving the center of mass motion (which is not tachyonic but massless in this case) as the only physical degree of freedom [10]. This has immediate consequences for possible interactions. All n -point functions vanish [11], the only non-vanishing tree-level amplitude is the 3-point function. The corresponding field theory is self-dual gravity for closed strings and self-dual Yang–Mills theory for the open string sector. The critical dimension of the $N = 2$ string is four but with “unphysical” signature $(2, 2)$, making it possible to identify the four real with two complex dimensions.

Although lacking the necessary Ramond–Ramond fields, it is still possible to formally define D-branes in $N = 2$ string theory by imposing Dirichlet boundary conditions in certain target space directions. The obvious question is then whether the closed $N = 2$ strings feel the presence of the branes. This note gives

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an answer to this question by performing a scattering analysis similar to the one undertaken in [7] for $N = 1$ strings.

2. Conventions

We choose the flat target-space metric as $\eta^{\mu\nu} = \text{diag}(-+-+)$. It is advantageous to subsume the real $(2, 2)$ -vectors into complex $(1, 1)$ -vectors with metric $\eta^{\mu\nu} = \text{diag}(-, +)$.

In detail:

$$X^\pm = (X^{\pm 0}, X^{\pm 1}) = (X^0 \pm iX^2, X^1 \pm iX^3). \quad (1)$$

The $(2, 2)$ -scalar product written in components reads

$$X_1 \cdot X_2 = \frac{1}{2}(X_1^+ \cdot X_2^- + X_1^- \cdot X_2^+).$$

The $(1, 1)$ -scalar product is

$$X_1^+ \cdot X_2^- = \frac{1}{2}(X_1^{++} X_2^{--} + X_1^{+-} X_2^{-+}),$$

where $X^{\pm\pm} = X^{\pm 0} + X^{\pm 1}$ and $X^{\pm-} = -X^{\pm 0} + X^{\pm 1}$.

Moreover, we introduce the matrices J

$$k^+ \cdot p^- = k \cdot p + ik \cdot J \cdot p \quad (2)$$

(J acts as a self-dual complex structure. It is $J_{02} = J_{13} = 1$, $J_{13} = J_{02} = -1$, all other elements = 0), and

$$D^{\mu\nu} = \text{diag}(D^{00}, D^{11}, D^{22}, D^{33}). \quad (3)$$

This matrix D is related to the flat target space metric η by a change of sign in the directions transverse to the D-brane. Example: let x^2 be the only direction transverse to the D-brane. Then $D = \text{diag}(-+++)$.

Emission and scattering off D-branes is conveniently calculated by evaluating correlators between vertex operators on the upper half plane. Open strings are represented by holomorphic vertex operators restricted to live on the real axis whereas closed string vertex operators factorize into holomorphic and antiholomorphic parts,

$$V^{\text{cl}}(z, \bar{z}, p) = :V(z, p/2)::V(\bar{z}, p/2):. \quad (4)$$

Here z lies inside the upper half plane. The sum of each picture number has to add up to -2 inside a non-vanishing scalar product. We will use vertex operators

in the $(-1, -1)$, $(-1, 0)$ and $(0, -1)$ picture [12]:

$$\begin{aligned} V_{(-1,-1)}(k, z) &= e^{-\varphi^- - \varphi^+} e^{ik \cdot X}(z), \\ V_{(-1,0)}(k, z) &= k^+ \cdot \psi^- e^{-\varphi^-} e^{ik \cdot X}(z), \\ V_{(0,-1)}(k, z) &= k^- \cdot \psi^+ e^{-\varphi^+} e^{ik \cdot X}(z). \end{aligned} \quad (5)$$

3. The general calculations

The separate propagators for holomorphic and antiholomorphic fields are standard. However, due to the presence of a worldsheet boundary, there are also non-vanishing correlation functions between holomorphic and antiholomorphic fields [7,13,14]:

$$\begin{aligned} \langle X^\mu(z) X^\nu(\bar{w}) \rangle &= -D^{\mu\nu} \ln(z - \bar{w}), \\ \langle \psi^\mu(z) \psi^\nu(\bar{w}) \rangle &= -\frac{D^{\mu\nu}}{z - \bar{w}}, \\ \langle \varphi^\pm(z) \varphi^\pm(\bar{w}) \rangle &= -\ln(z - \bar{w}). \end{aligned} \quad (6)$$

The example of the fermionic fields shows how this translates in the $\{\pm\}$ -basis:

$$\begin{aligned} k^- \cdot \psi^+(z) p^- \cdot \psi^+(\bar{w}) &\sim -\frac{1}{z - \bar{w}} k^- \cdot (G_+ \cdot p)^+, \\ k^+ \cdot \psi^-(z) p^+ \cdot \psi^-(\bar{w}) &\sim -\frac{1}{z - \bar{w}} k^+ \cdot (G_+ \cdot p)^-, \\ k^+ \cdot \psi^-(z) p^- \cdot \psi^+(\bar{w}) &\sim -\frac{1}{z - \bar{w}} k^+ \cdot (G_- \cdot p)^-, \\ k^- \cdot \psi^+(z) p^+ \cdot \psi^-(\bar{w}) &\sim -\frac{1}{z - \bar{w}} k^- \cdot (G_- \cdot p)^+ \end{aligned} \quad (7)$$

with the definition $G_\pm = D \pm J \cdot D \cdot J$. The D-brane respects the complex structure in target space if $G_+ = 0$, i.e., $D = -J \cdot D \cdot J$.

The new feature here (as compared, e.g., to the mixed amplitudes) is that for Dirichlet boundary conditions in general one gets poles in the operator product expansion between holomorphic and antiholomorphic fields both having a $+$ or $-$ index.

3.1. A_{cc}

It was shown in Ref. [7] that the scattering amplitude of two $N = 1$ closed strings off a D-brane can be obtained from the $N = 1$ open string 4-point function by simply interchanging certain momenta. Thus

the amplitude takes the form of an Euler-Beta-function of the Mandelstam variables that can be expanded as an infinite series of closed string poles in the t -channel or of open string poles in the s -channel and leads to the soft high energy behavior of the amplitude [5]. This result is intuitively clear since the interaction of closed strings with a D-brane is mediated by exchange of closed strings travelling between the passing closed string and the D-brane, or—via worldsheet duality—by open strings moving along the brane. This argument should also be true in $N = 2$ string theory. But it is difficult to imagine what a dual amplitude could look like in a theory with only a single degree of freedom. We therefore expect the scattering amplitude of a closed $N = 2$ string off a D-brane to vanish.¹

The scattering amplitude of two closed strings off a D-brane for the $N = 2$ string is given by the integral of the correlation function of two closed string vertex operators with the correct quantum numbers over the upper half-plane H^+ :

$$A_{cc}(p_1, p_2) \sim \int_{H^+} d^2z d^2w \times \langle V_{(-1,0)}(z, p_1/2) V_{(-1,0)}(\bar{z}, p_1/2) \times V_{(0,-1)}(w, p_1/2) V_{(0,-1)}(\bar{w}, p_1/2) \rangle. \quad (8)$$

Momentum conservation holds only in directions parallel to the brane:

$$\frac{p_1}{2} + \frac{D \cdot p_1}{2} + \frac{p_2}{2} + \frac{D \cdot p_2}{2} = 0, \quad p_1^2 = p_2^2 = 0, \quad (9)$$

where p_1 and p_2 denote the momenta of the incoming and outgoing strings, respectively.

In analogy to 4-particle scattering the amplitude can be parametrized by the following Mandelstam variables:

$$s = \left(\frac{p_1}{2} + \frac{D \cdot p_1}{2} \right)^2, \quad t = \left(\frac{p_1}{2} + \frac{p_2}{2} \right)^2,$$

¹ The D-instanton, of course, is an exception since in this case the s -channel point of view does not make sense and the scattering amplitude is not required to be dual. In fact, as we will see the amplitude falls off as a power of t being typical for a pointlike object.

$$u = \left(\frac{p_1}{2} + \frac{D \cdot p_2}{2} \right)^2. \quad (10)$$

Obviously s is the momentum transfer along the brane and t is the amount of momentum absorbed by the brane. As usual $s + t + u = 0$.

$SL(2, R)$ invariance of the correlation functions on the upper half plane allows us to fix three of the four variables of the vertex operators. For A_{cc} we choose $z = iy$ ($y \in \mathbb{R}^+$) and $w = i$. The correct integration measure is $\int_0^1 dy (1 - y^2)$. The resulting expression can be transformed into well-known integral-representations of Euler-Beta-functions using the “miracle”-substitution

$$y = \frac{1 - x^{1/2}}{1 + x^{1/2}}. \quad (11)$$

The final result is

$$A_{cc} \sim A \frac{\Gamma(s-1)\Gamma(t+1)}{\Gamma(s+t)} + B \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t)} - C \frac{\Gamma(s)\Gamma(t+1)}{\Gamma(s+t+1)} \quad (12)$$

with

$$A = p_1^+ \cdot (G_+ \cdot p_1)^- p_2^- \cdot (G_+ \cdot p_2)^+, \\ B = 4(p_1^+ \cdot p_2^-)^2, \\ C = (p_1^+ \cdot (G_- \cdot p_2)^-)^2.$$

3.2. A_{oc}

The amplitude for two open strings on the brane joining into an outgoing closed string is

$$A_{oc}(k_1, k_2, p) \sim \int_{\mathbf{R}, x < y} dx dy \int_{H^+} d^2z \times \langle V_{(-1,0)}(x, k_1) V_{(-1,0)}(y, k_2) \times V_{(0,-1)}(z, p/2) V_{(0,-1)}(\bar{z}, p/2) \rangle$$

x and y are integrated along the real axis in such a way that x is always left of y . The momenta k_i of the open strings have to be parallel to the brane which implies $k_i = D \cdot k_i$. Momentum conservation in this case reads

$$k_1 + k_2 + \frac{p}{2} + \frac{1}{2} D \cdot p = 0.$$

There is only a single kinematical variable $s = k_1 \cdot k_2 = \frac{1}{4}p \cdot D \cdot p = -\frac{1}{2}p \cdot k_1 = -\frac{1}{2}p \cdot k_2$.

To evaluate the amplitude A_{ooc} we set $z = i$ and $x = -y$ ($x, y \in \mathbb{R}$) and use $2s = -u = -t$. The relevant integrals can all be evaluated with the formulae

$$\int_0^\infty dy \frac{y^a}{(1+y^2)^b} = \sqrt{\pi} \frac{\Gamma(b + \frac{1}{2})\Gamma(a+1)\Gamma(2b-a-1)}{\Gamma(2b)\Gamma(\frac{1}{2}a+1)\Gamma(b-\frac{1}{2}a)}$$

and $\Gamma(a + \frac{1}{2})\Gamma(a) = \sqrt{\pi} 2^{1-2a} \Gamma(2a)$, resulting in

$$A_{ooc} \sim (k_1^+ \cdot p^-) k_2^+ (G_- \cdot p)^- \cdot \frac{\Gamma(1-2t)}{\Gamma^2(1-t)}. \quad (13)$$

This expression is completely analogous to that of the $N = 1$ theory.

4. Evaluating the general results for each D-brane-type

In this section the above amplitudes are explicitly analyzed for each type of D-brane. Due to the special signature $(2, 2)$ of our spacetime we denote the D-branes by $p + q$, where p and q are the number of spatial and time directions, respectively, in which the D-brane lives.

4.1. The $(2+2)$ -brane

4.1.1. A_{cc}

In this case the brane fills all of spacetime and we have ordinary interaction between open and closed strings that has been considered in [15,16]. A_{cc} has the interpretation of the lowest order quantum correction to closed string propagation. Momentum conservation holds in all directions for closed string scattering off the $(2+2)$ -brane. We cannot use our result, though, since by fixing three real parameters before integrating we did not divide out the volume of the whole symmetry group, which is, as we are dealing with a closed string topology $SL(2, \mathbb{C})$ rather than $SL(2, \mathbb{R})$. Naive application of our result (12) would lead to $A_{cc} = 0$, while the real amplitude is known to be constant.

4.1.2. A_{ooc}

For the process of joining of two open strings into a closed string momentum conservation implies that $p \cdot k_1 = p \cdot k_2 = k_1 \cdot k_2 = 0$. Since $s = \frac{1}{4}p^2 = 0$ the amplitude (13) reduces to

$$A_{ooc} \sim (k_1^+ \cdot k_2^-)^2 \quad (14)$$

coinciding with the well-known result [15].

4.2. The $(1+2)$ -brane

4.2.1. A_{cc}

The $(1+2)$ -brane divides spacetime into two halves and is analogous to the 8-brane in $N = 1$ string theory. There is only one transverse dimension which we choose to be the third. Momentum conservation together with the mass-shell condition fixes the momenta in the closed string scattering process almost entirely.

There are two cases; either the uninteresting case of no scattering at all, i.e., $p_1 = -p_2$, or the case

$$\begin{aligned} p_1^0 &= -p_2^0, & p_1^1 &= -p_2^1, \\ p_1^2 &= -p_2^2, & p_1^3 &= p_2^3. \end{aligned} \quad (15)$$

The Mandelstam variables are $s = -t = \frac{1}{2}(p_1^3)^2$ and $u = 0$. What one finds from Eq. (12) is that the first two terms vanish because the denominator diverges at $u = 0$. The third term reduces to

$$\begin{aligned} A_{cc} &\sim -(p_1^+ (G_- \cdot p_2^-))^2 \Gamma(s)\Gamma(1-s) \\ &= -4[(p_1^0)^2 + (p_1^2)^2] \Gamma(s)\Gamma(1-s). \end{aligned} \quad (16)$$

Now using $\Gamma(s)\Gamma(1-s) = \pi/\sin(\pi s)$ we see that this expression has infinitely many simple poles

$$A_{cc} \sim \frac{[(p_1^0)^2 + (p_1^2)^2]^2}{\sin(\pi s)}. \quad (17)$$

4.2.2. A_{ooc}

Again we demand Dirichlet boundary conditions in the 3-direction. $G_- = \text{diag}(0, -2, 0, -2)$. The kinematics read $k_1^3 = k_2^3 = 0$. We have to distinguish between two cases

(a) $k_3^3 = 0$. Here $t = 0$, thus we end up with a finite amplitude:

$$A_{ooc} \sim k_{13}^+ k_2^+ \cdot (G_- \cdot k_3)^-.$$

(b) $k_3^3 \neq 0$. We get

$$\begin{aligned} A_{ooc} &\sim \frac{\Gamma(1-2t)}{\Gamma^2(1-t)} \sim \frac{\Gamma(1/2-t)}{\Gamma(1-t)} \\ &\sim \cos(\pi \cdot (1/2-t)) \Gamma(1/2-t) \Gamma(t). \end{aligned}$$

This amplitude has a tachyonic pole.

4.3. The (1 + 1)-brane

4.3.1. A_{cc}

For this kind of brane the matrix D satisfies the relation $D = J \cdot D \cdot J$ which implies $G_- = C = 0$ and $G_+ = 2D$. The closed string scattering amplitude becomes

$$\begin{aligned} A_{cc} &\sim \left\{ \frac{p_1^+ \cdot (D \cdot p_1)^- p_2^- \cdot (D \cdot p_2)^+}{s-1} t + (p_1^+ p_2^-)^2 \right\} \\ &\times \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t)}. \end{aligned} \quad (18)$$

To further analyze the kinematical prefactor one recalls that in $2+2$ dimensions four momentum vectors with $\sum_1^4 k_i = k_i^2 = 0$ satisfy the relation [11]

$$\begin{aligned} &(k_1^+ \cdot k_3^-)(k_2^+ \cdot k_4^-) k_1 \cdot k_4 \\ &+ (k_1^+ \cdot k_4^-)(k_2^+ \cdot k_3^-) k_1 \cdot k_3 = 0. \end{aligned} \quad (19)$$

It is this equation that is responsible for the vanishing of the 4-point function in open and closed $N = 2$ string theory.

Setting

$$\begin{aligned} k_1 &= p_1, & k_2 &= D \cdot p_2, \\ k_3 &= D \cdot p_1, & k_4 &= p_2 \end{aligned}$$

and using the fact that $(Dp_1)^- \cdot (Dp_2)^+ = p_1^+ \cdot p_2^-$ for this particular form of D , one finds that

$$(p_1^+ \cdot (D \cdot p_1)^- p_2^- \cdot (D \cdot p_2)^+) t + (p_1^+ p_2^-)^2 s = 0.$$

What remains is

$$A_{cc} \sim (p_1^+ \cdot p_2^-)^2 \frac{\Gamma(s-1)\Gamma(t)}{\Gamma(s+t)}. \quad (20)$$

In general this kinematical prefactor does not vanish and again the result has massive poles.

4.3.2. A_{ooc}

Since $G_- = 0$ we have $A_{ooc} = 0$.

4.4. The (0 + 2)-brane

4.4.1. A_{cc}

Two time dimensions with Dirichlet boundary conditions imply that $D = -J \cdot D \cdot J$ and therefore $G_+ = A = 0$ and $G_- = 2D = 2 \text{diag}(++++)$. The scattering amplitude is

$$\begin{aligned} A_{cc} &\sim [(p_1^+ \cdot p_2^-)^2 u + (p_1 \cdot (Dp_2)^-)^2 t] \\ &\times \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t+1)}. \end{aligned} \quad (21)$$

It is remarkable that the prefactor is related to Eq. (19) by making the replacements

$$\begin{aligned} k_1 &\rightarrow p_1, & k_2 &\rightarrow D \cdot p_1, \\ k_3 &\rightarrow p_2, & k_4 &\rightarrow D \cdot p_2 \end{aligned} \quad (22)$$

and using $(Dp_1)^+ \cdot (Dp_2)^- = p_1^+ \cdot p_2^-$. We therefore see that in this case where the brane does not break the complex structure in target space the scattering amplitude vanishes,

$$A_{cc} = 0. \quad (23)$$

4.4.2. A_{ooc}

Open strings on this kind of brane are non-dynamical since the metric on the brane is euclidean such that the masslessness of the open strings implies the vanishing of their momentum. The (0 + 2)-brane should therefore be thought of as a completely rigid object. As one easily sees from inspection of the kinematical prefactor in Eq. (13) the amplitude for closed string emission vanishes: $A_{ooc} = 0$.

4.5. The (0 + 1)-brane

4.5.1. A_{cc}

In this case all three terms in (12) contribute to the scattering amplitude A_{cc} which can be rewritten as

$$\begin{aligned} A_{cc} &\sim \frac{1}{u(s-1)} (A t u + B u(s-1) + C t(s-1)) \\ &\times \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t)}. \end{aligned} \quad (24)$$

We checked that the kinematical prefactor does not vanish for the values of s and t where the Beta-function has its poles.

4.5.2. A_{ooc}

Again this amplitude vanishes trivially.

4.6. The $(0+0)$ -brane/D-instanton

4.6.1. A_{cc}

Dirichlet boundary conditions in all directions imply for the scattering process that there is no relation between the momenta of the incoming and outgoing closed strings. Since $D = -\eta$ the Mandelstam variables and kinematical factors become

$$s = \frac{1}{4} p_1 \cdot D p_2 = 0, \quad t = -u = \frac{1}{4} p_1 \cdot p_2,$$

$$A = 0, \quad B = C = 4(p_1^+ \cdot p_2^-)^2.$$

The scattering amplitude in this case is

$$A_{cc} \sim (p_1^+ \cdot p_2^-)^2 \left(\frac{\Gamma(t)\Gamma(s)}{\Gamma(t+s)} - \frac{\Gamma(t+1)\Gamma(s)}{\Gamma(t+s+1)} \right), \quad (25)$$

leading to a simple $1/t$ pole at $t = 0$. This pole can be obtained by either taking the $s \rightarrow 0$ limit in Eq. (25) or by recalculating the amplitude with $s = 0$ from the very beginning.

The single simple pole at $t = 0$ clearly is due to closed string exchange between the passing closed string and the D-instanton. The kinematical prefactor $(p_1^+ \cdot p_2^-)^2$ is precisely the 3-point function of self dual gravity, as described by the Plebanski equation. From a field theory point of view the process should be considered as the scattering of gravitons off a pointlike (in space and time(s)) source, which can be identified with the D-instanton.

4.6.2. A_{ooc}

Needless to say that the amplitude for emission of a closed string off the D-instanton vanishes.

5. A_{ooc}

For completeness we add our results we obtained calculating the amplitude for emission of a closed string from a D-brane on which three open strings interact.

We find that $A_{ooc} = 0$ for all branes but the $(1+2)$ -brane. In that case we are left with an integral of the

type:

$$\int dx \int dy \frac{1}{y(x+iy)} (x^2 + y^2)^\alpha ((1-x)^2 + y^2)^\beta$$

($\alpha, \beta \in \mathbb{R}$). So far we have not been able yet to solve this integral.

6. Results

We find that if the D-brane breaks the complex structure in target space additional correlation functions appear in the calculation which are absent for the usual Neumann boundary conditions. The result for the amplitude A_{cc} is nevertheless an Euler-Beta-function multiplied by a kinematical prefactor. A closer look at this kinematical factor shows that the amplitude vanishes only for the $(2+2)$ - and the $(0+2)$ -brane and has a single simple pole at $t = 0$ for the D-instanton which is due to closed string exchange. The scattering amplitudes of branes that break the complex structure in target space, i.e., the $(1+2)$ -, the $(1+1)$ - and $(0+1)$ -brane all have poles that do not correspond to states in the spectrum of the $N = 2$ string.

How do we interpret these results? In this Letter we have considered the $N = 2$ string in its gauge fixed NSR formulation. Massive poles in the scattering spectrum seem to be inconsistent with this type of string. The inconsistency can be traced back to the fact the presence of these branes conflicts with $N = 2$ worldsheet supersymmetry. This is due to the fact that a fermion ψ obeying Dirichlet boundary conditions cannot be in the same multiplet as a fermion obeying Neumann boundary conditions. This breaking of $N = 2$ worldsheet supersymmetry hence seems to leave no space for these type of branes in the gauge-fixed NSR formulation of the $N = 2$ string.

This result is also consistent with T-duality. Recall that Neumann and Dirichlet boundary conditions are interchanged upon performing a T-duality transformation in a toroidally compactified spacetime. However, for $N = 2$ string propagation only Ricci-flat Kähler manifolds with $(2, 2)$ signature are allowed. This leaves only the possibility to compactify one or both complex directions. Compactification of one or three real coordinates breaks the complex structure and yields an illegal background. Fortunately the three

relevant branes, namely the $(2+2)$ - and $(0+2)$ -brane and the D-instanton, form a closed set under the action of T-duality in the allowed backgrounds.

Apart from the $(2+2)$ -brane the other relevant branes are non-dynamical since open strings attached to them have vanishing momentum.² This means that *dynamical* D-branes do not exist in the NSR formulation of $N=2$ string theory in accordance with the absence of the corresponding differential forms and solutions of the classical equations of motion. We want to mention, though, that our formulation is not the only one that is able to describe the $N=2$ string. As was shown by Berkovitz and Vafa [17] and Siegel [18] there exists as well a more general formulation of the $N=2$ string in terms of the so-called *topological* $N=4$ string. This formulation admits more degrees of freedom and hence there might be a way how the *forbidden* branes can be consistently incorporated in $N=2$ string theory. But so far no attempt in this direction has been made, leaving room for further work and speculations.

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² Note that dimensional reduction of abelian self-dual Yang–Mills theory to two timelike or zero dimensions yields a vanishing curvature two-form.